APPENDIX I: PART G

NOTES ON MEASURING CAPACITY BY CENSUS ENUMERATION

By Almarin Phillips

University of Pennsylvania
NOTES ON MEASURING CAPACITY BY CENSUS ENUMERATION

INTRODUCTION

The use of the census of manufactures for measuring industrial capacity and the degree of capacity utilization has been handicapped by two factors. First, there has been no generally agreed upon definition of capacity for this purpose and, second—though not unrelated to the first—it has proven difficult to frame a set of questions which would at once be unambiguous and provide indicia of the desired capacity magnitudes.

These notes attempt to develop an operational concept of capacity through the use of a linear programming approach to the firm. In addition to providing a clear definition of capacity, this mode of presentation has other advantages. The meaning of balanced and unbalanced facilities is clarified, output restraints imposed by capital stocks are distinguished from those imposed by other factors, and possible differences between capacity measured in terms of output and capacity measured in terms of capital stocks are illustrated.

The paper represents no more than an exploratory venture. The purpose is to search for pitfalls rather than to propose definitive procedures. While it seems feasible to obtain capacity estimates from census surveys, no attempt is made to formulate the questions which census forms might pose. A list of relevant areas to which questions might be directed is included in the final section.

CAPACITY OF THE INDIVIDUAL ESTABLISHMENT

A census establishment can be viewed as a collection of \( m \) processes (or types of capital goods) capable of producing various quantities of \( n \) goods. Output is restrained by the existing stocks of capital goods, by the more or less technically determined production characteristics of the capital goods, and by economic, technical, and social considerations affecting the time-intensity of capital usage (i.e., the length of the workday and workweek, number of shifts, amount of downtime, etc.). From the combination of these, the output restraints imposed by the capital stocks can be expressed:

\[
A \cdot X \leq Y,
\]

in which \( A \) is a matrix of \( a_{ij} \) coefficients describing the technically determined production characteristics, \( X \) is a row vector \((x_1, x_2, \ldots x_n)\) of output rates for the products, and \( Y \) is a column vector \((y_1, y_2, \ldots y_m)\) reflecting the combined effects of the sizes of the stocks of capital and the time-intensity of their use. The inequalities in (1) form an \( n \)-dimensional polyhedron, the outer surface of which is here defined as the "capacity" of the establishment. Given \( A \), capacity is a function of only the capital stocks and their time-intensity usage.
While capacity is defined by the capital stocks, other types of restraints are typical. Of particular relevance to capacity measures are restraints associated with demand. Given prices, demand restrictions would appear as:

$$X \leq X,$$

where $X$ is a vector indicating the maximum amounts of each of the $n$ goods that can be sold.

Another typical restraint arises from the limited availability of input materials and services. These exist because of fixed factor supplies and because of the capacity limitations of supplying firms and would appear as:

$$B \cdot X \leq Z,$$

where $B$ is a matrix of $b_{kj}$ coefficients showing the amount of each of $r$ inputs used per unit of output of the several products and $Z$ is a vector $(z_1, z_2, \ldots z_r)$ giving total input limitations.

A necessary (but not sufficient) condition for the simultaneous full utilization of all process capacities is that the $m$ individual surfaces comprising the entire outer surface of (1) intersect at a unique point in $n$-space. Stated alternatively, it is necessary that:

$$A \cdot X = Y$$

have a solution. If this condition does not hold, there is no output mix which will fully utilize the capital stock. “Slack” will exist in at least one process and may exist in as many as $(m-2)$ processes by reason of the capital stock restraints alone.

This provides further definition. An establishment has balanced capacity when an output mix exists which would simultaneously utilize all processes fully [i.e., when (4) has a solution]. An establishment has unbalanced capacity when this condition cannot be met. Unbalanced capacity can take two forms. If the surface formed by one of the individual equations implied by (4) lies outside the limits of the other equations of the system in all $n$ dimensions, the process to which this equation refers is redundant. With redundant unbalance, in addition to the inability to use fully all processes simultaneously, there is at least one process which individually cannot be fully utilized no matter what the output mix. In nonredundant unbalance, where all that is lacking is a unique point of intersection, some output mix will fully utilize each of the processes even while no single mix will fully utilize them all simultaneously. The importance of the distinction between redundant and nonredundant unbalance is that in the former there is one type of capital that does not enter the meaningful capacity definition.

A lack of balance in capital stocks causes a lack of correspondence between excess capacity measured by output and excess capacity measured by the degree to which capital stocks are utilized. With balanced capacity, and the assumption of linear production relations, the ratio of actual output to capacity output would be equal to the ratio of capital stock being utilized to total capital stock so long as the relative mix of output is the same as the mix at full balanced capacity utilization. But, without attempting precisely to define the term, as the degree of unbalance increases, the correspondence between the ratio
of actual to capacity output and the ratio of stocks utilized to total stocks tends to disappear. The latter cannot be unity even when the former is. One way of measuring unbalance, then, is to compare these two ratios. Note, however, that the ratios could be different because the output mix is other than that compatible with simultaneous full utilization or because of nonlinear production functions.

If excess capacity is measured by the ratio of stock utilized to total stock, unbalance will make an excess appear even when all the demand restrictions in (2) are redundant. Similarly, estimating capacity output by dividing this ratio into actual output may give an incorrect result since, when the restrictions in (2) are redundant (and no other noncapital restraints are in operation), total output cannot actually be increased despite the existence of unused facilities. Even when the demand restrictions in (2) intersect the capacity surface, they do not cause excess capacity until such intersection precludes the establishment from selecting a point on the capacity surface which yields higher profit.

When demand restrictions are severe enough to cause actual output to be below the capacity function in at least one dimension, there is no obvious way in most circumstances to separate the amount of the excess associated with the levels of demand from that associated with unbalance. In the simple case of balanced capacity and demand restrictions which yield the same relative mix as that of the capacity optimum, the ratios of stocks utilized to total stocks in each process will be equal and, again assuming linearity, equal to the ratio of actual to capacity output for each and all products. But this is the only simple case. Even with balanced capacity, demand restrictions which cause the relative mix of output to be different from the balanced capacity mix will cause varying ratios of capital stock utilization in the several processes. The ratio of actual to capacity output will vary depending on the complex of demand restrictions and the assumptions made with respect to product mix [i.e., the direction used in moving from actual output to the capacity function]. When excess capacity—measured in terms of stocks—is caused jointly by demand restrictions and by an unbalanced capacity function, a precise separation of the two effects appears to be impossible.

At the level of the individual establishment, excess capacity which results from supply constraints parallels completely that caused by demand limitations. Nonetheless, supply restrictions do add to the complex of reasons for the existence of excess capacity and to the complications in segregating its causes.

**CAPACITY OF AGGREGATES OF ESTABLISHMENTS**

For purposes of capacity measurement, the best of possible worlds would be that in which all establishments were vertically integrated from the hire of factors of production through to the supplying of final demand. In such circumstances the restraints on output deriving from the stocks of capital would be conceptually simple to formulate. The “capacity” of the economy would be analogous to that of the establishment in (1), with the \( \bar{Y} \) summed over all establishments with capital stocks which, actually or potentially, could be used to produce any good, with the \( m \) set increased to include all types of capital and
the \( n \) set increased to include all goods. The result is the same as the usual production possibility curve of economics, considering capital as the sole scarce productive factor and limited by linearity assumptions. Similarly, any number of establishments could be grouped on the basis of the types of goods they produce or the nature of their capital stocks to derive "industry" capacity functions.

Estimates of capacity derived from individual establishments would not yield the above type of aggregate, however. The principal reason is that neither capital stocks nor goods in the process of production are completely mobile among establishments. The excess capacity caused by unbalanced facilities within establishments would tend to disappear if stocks could be reallocated or if goods in the production process could be costlessly moved among establishments. While the market mechanism does operate to affect such adjustments over time, both capacity and excess capacity estimates based on ratios of stocks utilized to total stocks tend to underestimate the theoretical potential of the economy.

In this hypothetical world of fully integrated plants, it is only through such things as the possible "dovetailing" of unbalances that the stocks of one establishment interrelate with those in others. But as soon as the integration assumption is relaxed and intermediate production by separate establishments is permitted, interrelations among establishments must be considered for other reasons. The problem is that even if capacity is always balanced within establishments it may be unbalanced among them. The capacity of buying establishments may be redundant in terms of the supply restrictions imposed by the stocks of supplying establishments and vice versa. Aggregation of capacity measures based on establishment reports are not apt to reflect these interrelations and, hence, to overstate the possible total industrial capacity output.

Conceptually this type of interrelation can be accounted for by a combination of input-output analysis and linear programming. This analytic framework is extremely complicated, however, if detailed input-output coefficients for each product of multiproduct establishments are included and if heterogeneity among establishments precludes a rather massive grouping into "industries."

Finally, it should be noted that interrelations may exist with respect to other factors of production which several establishments demand in common. A factor which appears to exercise no restraint from the point of view of each establishment may be restrictive from the point of view of all of them. Again, a form of input-output analysis is necessary to handle this problem.

**Areas for Census Questions**

While measurement difficulties are indicated, the concept of capacity is itself operational, both at the establishment and "industry" or economy levels. The following appear to be relevant areas for census questions:

1. The time-intensity of capital usage.—In (1), the \( Y \) parameters depend on "normal" work schedules as well as on the size of capital stocks. At a given time, these schedules may be functions of demand restrictions or reflective of practices designed to overcome unbalances.
in the facilities of the various processes. Capacity estimates should be based on the schedules which would be used if demand were redundant for all products, but with whatever degree of unbalance as actually exists. Capacity questions, then, should stipulate not the existing work schedules, but those that would be used with no demand restrictions and the present facilities.

2. Capacity in output terms.—With work schedules defined as above, and with the assumption that the existing relative output mix is retained and that there are neither demand nor supply restrictions, the ratio of current to maximum possible output could be ascertained. Maximum possible output is that at which the first facilities "bottleneck" occurs.

3. Capacity in terms of capital stock.—The ratio of plant and equipment in use to total plant and equipment carried on the books could also be sought, but this poses severe measurement problems. If value measures are used, it must be decided whether depreciated or undepreciated values are the more appropriate, and in addition, methods will have to be developed to convert the reported values to constant dollar terms. If physical quantity measures are employed, methods will have to be developed to aggregate heterogeneous capital items.

It would be presumptive as well as impossible to attempt the resolution of capital stock measurement difficulties here. Work done in the last decade on the deflation to constant values of capital stock—illustrated by Daniel Creamer's pioneering work at the National Bureau and at the Conference Board—suggests that pragmatic methods of deflation are available, at least for broad industry groups.

Whether depreciated or undepreciated values are preferable for capacity measures will depend in part on the purpose of the measurement. If the capital stock is to be valued in terms of the least cost alternative method for producing goods, the depreciated values seem the better. But while capital stocks depreciate in terms of alternative cost valuations with age, this is not necessarily because the older stock items produce physical output at a slower rate. Length of life does not typically reflect capital being used up—in the sense that it disappears—but rather that continually higher maintenance expenses are necessary to maintain its ability to produce.

If depreciated values are used, and if it may be assumed that newer capital which is less costly to operate and maintain tends to be kept in use and older, more costly capital to be shut down first, the ratio of depreciated value capital in use to total depreciated value capital in the establishment will tend to underestimate the relative amount of unemployed physical capital and to underestimate the amount by which physical output could be expanded with existing stocks. It may be argued, then, that the depreciated values are better if one wants the economic value of capital in use relative to the economic value of total capital, but that undepreciated values are better if one wishes to estimate via capital stock measures the amount by which gross output could be increased.

There is another possibility for measuring capital utilization rates through census reports. If the plant and equipment of an establishment can be subdivided into reasonably homogeneous items of physical capital corresponding to the major processes of the establishment, ratios of physical stock in use to total stock might be found for each.
These would be useful in themselves—particularly for estimating the extent to which transfers of capital among establishments might reduce the amount of unbalance in facilities—and could conceptually be aggregated into a ratio for the entire establishments with the use of value (depreciated or undepreciated?) weights.

4. The degree of balance.—With the caveats obvious from the above, a comparison of the actual to capacity output ratio with the ratio of stock in use to total stock should provide some indication of the extent to which unused facilities are due to unbalance. In view of the problems inherent in the capital ratios and since the two ratios may differ due to nonlinear production functions, it might also be asked what ratio would obtain between stocks in use and total stock at the maximum possible output as defined above. To estimate whether the existing mix is the cause of unbalance, questions could also be asked to determine whether some other mix would more fully utilize facilities. To check for redundant unbalance it could be asked whether some of the facilities would not be fully used regardless of mix.

5. Supply restrictions.—Questions could be asked to determine whether the ratio of actual to capacity output is the result of limited material or factor supplies. If supply restrictions appear, it might be asked whether these influence the mix as well as the level of output and what the ratio of actual to capacity output would be in the absence of supply restrictions.

6. Demand restrictions.—Questions similar to those for supply restrictions might be asked, but the answers are implied by previous answers to the supply and balance questions.