# THE MONETARY MECHANISM AND ITS INTERACTION WITH REAL PHENOMENA

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THIS paper traces the major developments of lasting value in our understanding of the monetary mechanism and its role in the economy which have occurred since the early forties, when the process of digesting the General Theory [11] and integrating it with the earlier streams of thinking had been more or less completed. I consider here primarily the state of monetary thought as of about the mid-fifties, leaving to another time consideration of the evolution that has occurred since.

The middle of the 1950's provides, in my view, a useful landmark in reviewing the evolution of monetary and macroeconomic theory, since the developments that occurred up to that time are, on the whole, rather different in purpose and approach from those that have occurred later. The development in the first-mentioned period seems to me to consist largely of refinements, clarifications, and developments of the basic framework, which had already been laid out by the early forties. The most significant contributions in the more recent period, on the other hand, have tended to approach monetary issues in terms of the theory of asset management and portfolio decisions, exploiting concurrent advances in the theory of saving, in managerial economics and, most importantly, in the theory of choice under uncertainty. These developments, on the whole, have not led to any major revision or rejection of the positions reached by the mid-fifties, but have rather been concerned with providing a better understanding of the determinants of the demand for money and its relation with the demand and supply for various other assets, physical as well as financial, by final transactors and by "financial intermediaries."

This paper is, in essence, a summary of a longer and more rigorous statement which I expect to complete later, and which, together with an exploration of the developments since the 1950's, will be published elsewhere when completed. Because it is in the nature of a summary, I have frequently found it necessary to sacrifice rigor and to omit nearly all of the proofs and many of the references.

The material is divided into six major sections. The first is a brief comparison of a basic model of money and the economy which I believe would have been widely accepted about 1944, with a corresponding model as of the middle of the 1950's. The following sections, then, examine in some detail the implications of the mid-fifties model for the relationship to crucial variables in the economy, and for major lines of monetary and fiscal policy.

## I

### The Mid-50's Model and Its Relation to the 1944 Model

I hope I may be permitted the liberty of using the macroeconomic model presented in my 1944 article, "Liquidity Preference and the Theory of Interest in Money" [13], as a representation of where monetary economists thought money stood in relation to the economy at about that period. This model is reproduced in Table 1 (with very minor modifications of notation), together with a revised version labeled the "Mid-50's model." This is essentially the model that I would have used had I been writing a comparable article at that time (and did actually use in my class lectures, cf. [15]).

While no systematic attempt is made here at justifying in detail the equations of the model, a task which was substantially performed in the 1944 paper, in this section we will review the main differences as well as similarities between the original and the revised model.

The two models are basically identical in spirit: they both treat the economy at a highly aggregative level in terms of four goods and corresponding markets. Two of the goods are physical commodities, labor and real output,

and two are money-fixed claims, money and bonds. For the sake of concreteness, real output is visualized as consisting of a single homogeneous commodity which, in the mid-fifty model, I choose to label the *MM* (read mum). This output can be either used for current consumption ($C$) and thereby disappears, or can be instead devoted to investment ($I$), thereby becoming part of the capital stock ($K$). Thus, $X, C, I, S$ are measured in *MM*/time, and $K$ in *MM*'s. Similarly, labor ($N$) is measured in man-hours/time. The two money-fixed claims on the other hand are measured in units of money assumed to be the dollar, and bonds are to be regarded as one-period loans or claims to future (next-period) money. The amount of bonds held by a transactor may be positive or negative; in the latter case it represents the transactor's debt.

Corresponding to the four commodities there are three independent prices or terms of trade with money: the price of output — or general price level — $P$ ($/*MM*); the price of labor, $W$ ($/man-hour); and the price of a dollar, next period, $(1/1+r)$, where $r$ is the rate of interest. In addition, both models, at least at the outset, assume:

A.1: certainty

A.2: absence of money illusion (which is an implication of rational behavior)

A.3: unit elasticity of price expectations (and independence of interest rate expectations from current prices)

The main differences between the two models can be summarized under five headings:

1. *Explicit reliance on a general equilibrium formulation.* The mid-fifty model is explicitly structured in terms of markets, one for each

TABLE 1.

COMPARISON OF 1944 WITH MID-50's MODEL

*DEFINITIONS AND CLASSIFICATION OF VARIABLES*

I. *Endogenous variables*

(a) *Flow variables:* (1) $X$, Real income; (2) $C$, Real consumption; (3) $I$, Real investment; (4) $S$, Real saving; (5) $X^d$, Real aggregate demand; (6) $N$, Employment (man-hours per unit of time); (7) $N^s$, Labor supply; (8) $Y$, Money income

(b) *Price variables:* (9) $P$, Price of output; (10) $W$, Price of labor (wage rate); (11) $r$, Rate of interest

(c) *Money market variables:* (12) $M^d$, Demand for money by private sector; (13) $B^p$, Demand for bonds by non-bank private sector; (14) $B^b$, Demand for bonds by banks; (15) $M^b$, Supply of bank money

II. *"Initial conditions":* $K_o$, Real stock of capital; $v_o^j$, Net worth of $j^{th}$ household; $V_o = \sum_j v_o^j$, Aggregate net worth of private sector; $[V_o] = (v_o^1, v_o^2, \ldots v_o^j, \ldots)$.

III. *Other parameters:* $W_o$, Rigid money wage rate; $N^f$, "Full employment" labor supply; $M$, Money supply; $M^*$, Supply of government money; $M^{*p}$, Government money outside banks; $M^{*b} = M^* - M^{*p}$, Government money held by banks; $G$, Supply of government bonds.

*MODEL I* (1944)

(3) $PS = S(r, Y)$
(2) $PI = I(r, Y)$
(6) $X = X(N)$
(7) $X_N(N) = W/P$
$W = \alpha W_o + (1-\alpha)F^{-1}(N) \times P$
(9) $\alpha = \begin{cases} 1, \text{ if } N < N^f \\ 0, \text{ otherwise} \end{cases}$
(4) $PS = PI$
(8) $PC = Y - PS$
(5) $Y = PX$
(1) $M = L(r, Y)$

*MODEL II* (mid-50's)

(1) $C = C(X, NW/P, r, [V_o/P])$
(2) $I = I(r, X, K_o)$
(3) $X^d \equiv C + I$
(4a) $X = X(N, K_o)$ or (4b) $X = s\ (P/W, K_o)$
(5a) $X_N(N, K_o = W/P$ or (5b) $N = n^d(W/P, K_o)$
(6) $\begin{cases} N^s = n^s(W/P), & \text{if } n^d(W_o/P, K_o) > n^s(W_o/P) \\ W = W_o & \text{if } n^d(W_o/P, K_o) \leqslant n^s(W_o/P) \end{cases}$
(7) $X^d = X$
(8) $N^s = N$
(9) $S \equiv X - C$
(M.1) $Y \equiv PX$
(M.2) $M^d = L(r, Y, P, [V_o]) = PL(r, Y/P, [V_o/P])$
(M.3) $M^d = M$
(M.4) $M = M^b + M^{*p}$
(M.5) $M^b = B^b + M^{*b}$
(M.6) $B^p = B(r, X, N, P, W, [K_o], [G_o + M_o^*])$
(M.7) $B^p + B^b = G$

*MEMOS*

(i) $B_o^p + B_o^b = G_o$
(ii) $M_o = M_o^b + M_o^{*p} = B_o^b + M_o^*$
(iii) $M_o + B_o^p = G_o + M_o^*$
(iv) $V_o = PK_o + M_o + B_o^p = PK_o + G_o + M_o^*$
(v) $K_1 = K_o + I$

commodity, with each market in turn described by (a) supply conditions, (b) demand conditions, and (c) clearing of market or equilibrium conditions, of which one is redundant (Walras' law).

The main advantage of the general equilibrium framework is that it insures a systematic and, at least initially, symmetrical treatment of all markets.[1] For the commodity market, the demand conditions are described by equations (1) to (3); the supply conditions by (4b) and the clearing conditions by (7). In the labor market the supply is given by (6), to be reviewed more closely below; the demand by (5); and market clearing by (8). The remaining two markets are described under the next heading, 2.

2. *Explicit treatment of the bond market and more precise formulation of the relation between the demand and supply of money and bonds and the banking system.* In the 1944 model, the money market was described very summarily by a single equation (I.1) and the bond market was omitted altogether. In the mid-1950's model, the money market is again described by a demand (M.2), a supply (M.4), and a clearing of market equation (M.3). Furthermore, two sources of money supply to the private sector are recognized: bank money ($M^b$) and government money ($M^{*b}$). The bond market is described by the last three equations. (M.5) is a streamlined consolidated balance sheet of the banking system (including the central bank, if any); the left-hand side is the liability side or bank money supply, which must be equal to the asset side consisting of government money ($M^{*b}$) and bank credit ($B^b$) which can also be regarded as the banking system demand for bonds. (M.6) gives the *net* demand for bonds by the public, the difference between lending (gross demand) and borrowing (gross supply). (M.7) is the clearing of market condition; the left-hand side is the aggregate net demand by the private sector and the right-hand side is the *net* supply of bonds to the private sector, which would be zero in the absence of government debt, and is otherwise equal to this debt, $G$.

While the bond market is thus given explicit treatment, it is still permissible to treat this market as the redundant one and we shall find it convenient to do so. Furthermore, for the purpose of this paper, we shall regard the two components of the money supply and hence their sum $M$ as exogenously determined by the government or through the monetary authority's control over the banking system, or both, without concerning ourselves with the many possible institutional devices through which this control can be exercised. Under these conditions it is found that the last four equations, (M.4) to (M.7) describing the banking system and the bond market can be disregarded, so long as one is interested in the functioning of the rest of the system. This is because the first twelve equations, consisting of the nine "real equations" (1) to (9) and the three monetary equations (M.1) to (M.3), form a determined subsystem in the first twelve variables listed at the top of Table 1, namely the eight flow variables, the three price variables, and the demand for money. The analysis that follows, therefore, is primarily concerned with this subsystem.

3. *Improvements in the consumption and investment function and in particular more adequate recognition of the role of stocks.* Since the mid-forties the consumption function has received a great deal of attention at both the theoretical and the empirical levels. Many of the contributions, and notably the "permanent income" hypothesis of Friedman [7] and the "life cycle hypothesis" of Modigliani, Brumberg and Ando [14] [16] [1], have de-emphasized the role of current income and emphasized the role of other variables, particularly long-term income expectations and wealth. The consumption function (1) is sufficiently general to be consistent both with the original simple-minded Keynesian version and with the more recent but still controversial formulations. In particular, it is consistent with the life-cycle hypothesis according to which aggregate consumption can be approximated by a linear homogeneous function of aggregate labor in-

[1] Without the benefit of this crutch, in my 1944 paper I was led in the formal model of section 2, to drop any explicit reference to the bond market — implicitly treated as the redundant one — which, in turn, caused some unfortunate and unnecessary misunderstandings [9]. More seriously, I forgot altogether the existence of this commodity in the analysis of section 13, which resulted in some outright wrong statements, as was shown later by Patinkin [20], [21].

come $\frac{NW}{P}$ and net worth $(V_o/P)$, with coefficients depending in principle on the rate of return on capital $(r)$ [1].

Similarly, the investment function, or investment component of aggregate demand (2) can be shown to be consistent with a wide variety of approaches and hypotheses, such as the crude acceleration principle, the so-called "flexible accelerator" or gradual adjustment hypothesis, or even the more traditional formulation relying upon the production function and the marginal productivity of capital. Every one of these approaches implies that investment demand must be an increasing function of the level of output and a decreasing (or at least nonincreasing) function of the initial stock of capital and of the cost of capital.

4. *Correction of the faulty formulation of the homogeneity properties of the consumption, investment, and demand for money functions.* The new formulation of equations (1) and (2) and of (M.2) implies that the real demand for consumption, investment, and money is homogeneous of zero degree in money income, wealth, and prices, or the corresponding money demand is homogeneous of first degree in the same three variables. This property is an implication of the assumption of rational behavior. In the 1944 model I had intended to make this same assumption but in fact did not formulate it properly; I assumed instead that the three money demands were homogeneous of first degree in *money income alone.* This formulation leads to the incorrect implication that a change in money income has the same effect on money consumption whether due to a change in real income with prices constant, or to a change in prices with real income constant; and similarly for the other two variables. This error in turn led to a peculiar and objectionable property of the model, to wit that the first four equations of the model form a closed subsystem in the four variables $(PX)$ (or $Y$), $PS$, $PI$ and $r$, involving in particular $M$ as a parameter. This dichotomization implies that the equilibrium values of the rate of interest and the money flow variables are independent of the real variables of the system, and in particular of the form of the production function and of the level at which the rigid money wage $W_o$ is set. This ceases to be true once the consumption and investment function are properly [2] formulated as in Model II. In particular, as shown below, for a given production function (including in it the initial stock of capital), and given $M$, an increase in $W_o$ will normally tend to result in an increase of $r$, as well as of $P$, money income, investment and consumption, and in a fall in employment, real income and the other real flows.[3]

5. *Use of a more convenient and effective device for expressing the hypothesis of wage rigidity.* This device embodied in equation (6) relies on the notion of a "potential" supply function $n^s(W/P)$, expressing the maximum supply of labor available at each real wage. By combining this function with the demand function we can determine the excess supply (or demand) at any real wage. The hypothesis of wage rigidity then states that the money wage will not be bid below the rigid level $W_o$, even if there is an excess supply at this level. This hypothesis is formalized in equation (6). In fact, this equation together with (5b) and (8) simultaneously implies that $W$ and $N$ are determined by the intersection of the demand function (5b) and the potential supply function, if this intersection determines a value of $W$ larger than $W_o$; otherwise $W = W_o$ and the level of employment is determined by the demand function alone. The difference between this level of employment and the potential supply at $W_o$ is then "involuntary unemployment" in the Keynesian sense. Note finally that equation (6) can also be used to formalize the classical assumption of wage flexibility by merely setting $W_o$ equal to zero or, at any rate,

[2] By "properly," I do not of course mean empirically correct but merely consistent with the hypothesis I intended to formalize.

[3] I might add, in partial defense of my 1944 construction and for solace to those who may have accepted it, that, if the production function is given and if $W_o$ is treated as an unchangeable parameter, then the system *does* imply the existence of a unique relation between money consumption or investment, and money income; but then it implies also a unique relation between these variables and $r$, so that even under the stated conditions equation (I.1) and (I.2) could not be regarded as behavior equations but, at best, as reduced forms. Furthermore, the functional form of these reduced forms would shift around with changes in $W_o$ or the form of the production function. Thus, even though in many cases my original system leads to "correct" inferences, it is an unreliable tool of analysis.

sufficiently low to insure that the first line will apply in the relevant range.

From this brief review it should be apparent that most of the differences between the 1944 and the mid-fifties model are matters of elegance, clarity, and minor improvements — with the main exception of the correction of the error under (4) above. Even the explicit recognition of the role of stocks examined under (3) is not crucial, if we regard the model as focusing on the determinants of short-run equilibrium, since, in general, stocks are initial conditions which can be treated as given parameters. An exception to this statement must be made with respect to the inclusion of wealth in the consumption function, for, in the presence of a government debt fixed in money terms, real wealth, $V_0/P$, cannot be regarded as a given initial condition. As will presently be shown, the resulting "wealth effect" has interesting logical implications, although it is of practical relevance *only* under the assumption of wage and price flexibility, an assumption which has little, if any, empirical substance.

The remaining sections of the paper are devoted to examining various implications of the model, focusing in part on issues which have been prominently debated in the period under review. In the two sections immediately following, the analysis rests entirely on the mid-fifties model discussed so far. Then, in sections IV and V, this model will be broadened and amended to allow explicitly for the role of government and for certain imperfections in the capital market. Finally, in the concluding section VI, I endeavor to summarize the results with special reference to the importance of money and monetary factors as a determinant of the real variables.

## II

### Implications of the Model Under Price Flexibility — Homogeneity, Dichotomy, Neutrality, and the Role of the Supply and Demand for Money

We proceed first to a summary of some implications of the model within the classical framework of price and wage flexibility. Its main justification is the hope of disposing for good of a controversy, connected with the names of Pigou and Patinkin, which has plagued the profession, draining the resources into what strikes me as a largely barren endeavor.

In essence, the Patinkin controversy revolves around two main issues concerning the properties of an economic system relying on a token money as medium of payment, namely (1) the neutrality of money and (2) the validity of the dichotomy of monetary and real economics. It will be useful to define more precisely the nature of these issues and their implications.

*Neutrality.* Money is said to be neutral if the equilibrium value of the real variables of the system — in our model the flow variables, $X$, $C$, $I$, $N$, $S$, the price ratio $P/W$, and $r$ — is independent of the money supply $M$. If money is neutral, then at most $M$ can affect the actual level of prices, or terms of trade between current commodities and money — $P$ and $W$ in our model. A special case of neutrality is where, in addition, prices are proportional to $M$, the so-called "quantity theory of money." [4]

*Dichotomy.* This issue can be defined in many alternative ways, as is apparent from Patinkin [21] where many possible meanings of the word are considered and carefully divided into good ones (valid) and bad ones (invalid). The problem I am concerned with here might be defined in terms of certain mathematical properties of systems of simultaneous relations or, instead, in terms of the substantive economic issues involved.

If we take the first approach, we may say that the dichotomy holds if the system of simultaneous relations purporting to describe the functioning of the economy has the following property: the entire set of relations can be "dichotomized" into two subsets, one of which, containing the relations describing the functioning of all markets except the money and bond market, forms a determinate subsystem of the entire system and is therefore sufficient to determine all the real variables of the system. Obviously, if the dichotomy in the above sense holds, then the equilibrium value of the real variables is independent of the money supply — i.e., dichotomy implies neutrality. But in ad-

[4] This definition of the quantity theory of money is obviously quite different from the one adopted by Friedman in [6] and is, I believe, more consistent with generally accepted terminology.

dition these equilibrium values are also independent *of the form of the demand-for-money equation.* This last implication is, of course, the significant one from the economic point of view, for it justifies separating the study of the functioning of a free market economy into a "monetary" and a "real" branch. In particular, when concerned with the real variables we need pay no attention to monetary habits and institutions (although the converse need not be true).

This then suggests an alternative and probably more fruitful definition of dichotomy, namely, as a warranted separation of monetary and real economics (or value theory). Specifically, we can say that the dichotomy holds if the equilibrium value of the real variables of the system is *independent of both the supply and the demand for money.* Note that in this second sense it is quite possible for the dichotomy to hold in the long but not in the short run (or, conceivably, vice versa).

Of course, to the extent that the equilibrium of the system (whether in the short or in the long run) can be identified with the solution of a system of simultaneous relations, dichotomy in the first sense implies dichotomy in the second. But our second definition is broader and is a proposition about the real world, even though in passing judgment about its validity we may have to rely on the mathematical properties of certain formal descriptions thereof. Note finally that, in terms of our second definition, we can meaningfully say that the dichotomy is "approximately" valid, in the sense that it provides a good first approximation to observed phenomena.

In terms of these definitions, Patinkin's basic contention can be summarized as follows: in an economy relying on a token money as a medium of exchange, the dichotomy does not hold, but under certain conditions money will be neutral. In particular the following two assumptions, in addition to assumptions A.1 to A.3 above, are generally sufficient for the neutrality to hold (cf. [21], especially chapters IV, VIII, and Mathematical Appendix 4).

A.4, Concerning the functioning of markets: absence of price rigidity in any market

A.5, Concerning tastes: the market demand and supply for each commodity is invariant under a redistribution of wealth among transactors [5]

It is found that Patinkin's contention is basically unwarranted, although no attempt at rigorous proof is possible here. Furthermore, while the argument runs in terms of the aggregative model of Section I, it holds as well if we recognize the existence of many different commodities and kinds of labor. Specifically, the following propositions can be established (for a closed economy).

(1) *If the money supply consists entirely of bank money and there is no government debt,*[6] *the dichotomy is valid.*[7] Basically the reason is that under these conditions, and assumptions A.1 to A.5, all the demand and supply equations for commodities are homogeneous of zero degree in the prices, $P$ and $W$, and hence can be expressed as functions of their ratio, $W/P$. This holds in particular for the consumption function (1). In fact, A.5 implies that wealth affects consumption only through its aggregate value $V_o/P$. But $V_o/P$ in turn can be replaced by the initial stock of physical capital $K_o$, since the money component of aggregate net worth is exactly offset by the indebtedness to the banking system (cf. Memo iv in Table 1). Thus $P$ appears in the consumption function only in the form $W/P$.[8] It can be readily verified that

[5] Patinkin has been especially concerned with what may be called "neutrality in principle," i.e., neutrality with respect to a "neutral" change in the money supply, which is defined as one that leaves undisturbed the relative distribution of cash balances. For this type of neutrality, Patinkin does not need assumption A.5. However, as will be apparent from (2) and (3) below, when neutrality does not hold with assumption A.5, "neutrality in principle" does not hold either.

[6] By government debt is meant here any claim (positive or negative) of the private sector (including banks) on the government, whether interest bearing or not. Hence, the assumption of no government debt implies that $M^{*p}$, $M^{*b}$ and $G$ are all zero, which, using (M.4), (M.5) and (M.7), in turn implies $M = M^b = B^b = -B^p$. In other words, the money supply consists entirely of what Gurley and Shaw call "inside money" (cf. [8] p. 73).

[7] This conclusion, which was stated in my 1960 Postscript to [13], has recently been acknowledged by Patinkin [19] in his very useful review of Gurley and Shaw [8], which also contains some of the results summarized under (3) below.

[8] Note that the consumption demand — as well as labor supply — of *individual* households will generally not be homogeneous of zero degree in prices. Indeed changes in the price level affect *individual* wealth by determining the real value of money fixed claims and debts. However, since *aggregate* wealth is unaffected, such changes only result in

under these conditions the first nine equations of the model form a determined subsystem in all the real variables including the price ratio $W/P$. This subsystem is therefore sufficient to determine these variables without any reference to the supply of money or to the demand for money equation (M.2). The only role of the monetary part is to determine the price level $P$, and hence finally also $W$. Furthermore, if assumption A.5 is extended to the demand for money, then it also turns out that $P$ is strictly proportional to $M$ so that the quantity theory also holds.

It may be useful to try to state in plain English the major forces determining equilibrium in the system described by Model II under the stated assumptions. This mechanism can be summarized roughly as follows:

i. Given the production function, including the initial stock of capital, and hence consumers net worth, and given the preferences for current and future consumption as compared with leisure, there is a unique real-wage rate $W/P$ that clears the labor market, and leads in turn to a unique level of output $X$: this corresponds to the simultaneous solutions of the four equations (4) to (6) and (8) in the four variables, $P/W$, $N$, $N^s$ and $X$.

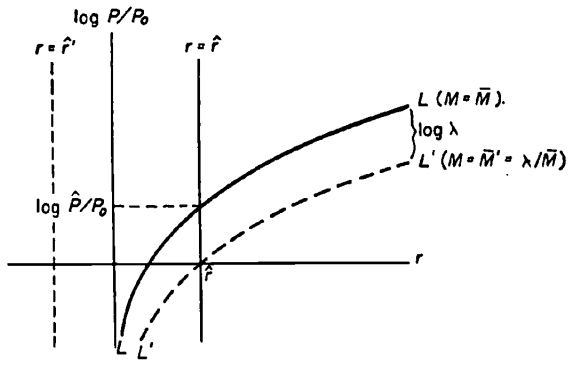
ii. For given $X$, preferences for current versus later consumption (including in later consumption planned bequests) and the terms of trade between them, represented by $r$, determine the size of current consumption $C$ and the desired carry-over of resources, $S = X - C$. It is the function of the rate of interest to insure that investors will be induced to add to the stock of capital an amount $I$ equal to the desired carry-over $S$. These forces are described by the remaining five real equations of the system, (1) to (3), (7) and (9), and complete the determinants of the real variables.

iii. Given the real solution, the price level together with certain institutional factors (such as length of the income period, synchronization of receipt and expenditure, transaction costs, etc.) determine the demand for money arising from transaction requirements and, possibly, also on asset account. There is then a unique price level that equates the demand to the exogenous supply $M$. This same price level also induces the private sector to issue, on balance, enough claims against itself to satisfy the banking system demand for bonds, which banks pay with $M$.[9]

The determination of the price level can be exhibited graphically as in Figure 1, in terms

FIGURE 1

THE ROLE OF THE SUPPLY OF MONEY IN THE DETERMINATION OF THE PRICE LEVEL AND THE RATE OF INTEREST: THE CASE OF PURE BANK MONEY




of the coordinates of the point of intersection of two curves in the $(r,P)$ plane. (Actually, on the ordinate it is convenient to plot the logarithm of $P/P_o$, where $P_o$ may be thought of as the previously ruling price.) One of these curves is the graph of the line $r = \hat{r}$, perpendicular to the $r$ axis, stating that the rate of interest must be at the level $\hat{r}$ that clears the commodity market and which is determined by the real part of the system. The other, denoted by $LL$, is the graph of the equation that must be satisfied in order for the money market to be cleared, namely: $P \times L(r, \hat{X}, K_o) = \overline{M}$. Treating $\overline{M}$ as a given parameter and $\hat{X}$ as already determined by the real part of the system, this equation can be looked upon as a relation between the two variables $r$ and $P$. Its graph shows, in effect, the be-

a redistribution of wealth between households, which by A.5 does not affect *aggregate* demands and supplies.

[9] It is apparent from the above that even in a system relying entirely on "inside money" the determinancy of the price level can be established without reference, explicit or implicit, to Gurley and Shaw "portfolio balance" or "diversification" mechanism ([8], especially Chap. V). Indeed that mechanism is intimately related to their very objectionable treatment of firms as separate and self-contained entities. In our model the net worth of firms is treated as a component of households' wealth.

havior of the so-called income velocity of circulation (more precisely its logarithm) as a function of the interest rate, except for a proportionality factor, $\overline{M}/\hat{X}$. It can be expected to rise from left to right, since a priori considerations abundantly supported by empirical evidence (see, e.g., [12], [29], [2], [30]) suggest that, as $r$ rises, velocity rises, the real demand for money falls (since $\hat{X}$ is given) and hence $P$ must rise to reduce the real supply. The position of $LL$ depends, of course, on the money supply $\overline{M}$. A change in this supply from $\overline{M}$ to say $\overline{M}'$ would shift it parallel to itself by a distance log $(\overline{M}'/\overline{M})$.

While the general shape of the $LL$ graph is clear within the empirically observed range of variation of $r$, there may be some doubts as to its behavior for extreme values. For instance, as $r$ grows indefinitely the graph may rise indefinitely or may instead approach some asymptote. Similarly, there is some question as to whether $r$ can ever be bid down to zero. But whatever views one might hold about the possibility of a zero interest rate, one thing is certain, namely that in perfect markets, and as long as money has negligible storage costs, the money market can never be cleared with a negative $r$. For, even if banks were prepared to lend at a negative rate, everybody would wish to borrow "infinitely large" amounts and hold the money so borrowed to earn the premium now paid to borrowers. Hence the demand for money (as well as the supply of bonds) must become infinite at rates more than negligibly negative; and for any finite supply of money $\overline{M}$ there would be an excess demand for money and the money market would not be cleared. This means that the locus of $LL$ lies entirely in the half plane to the right of a line perpendicular to the abscissa and coinciding with the ordinate axis, or possibly negligibly to the left of it, on account of the storage cost of money, if any.

As can be seen from the figure, if the money authority is concerned with maintaining a constant price level, then it should set the money supply at a level such that the corresponding $LL$ curve intersects the other curve on the $r$ axis, where log $(P/P_o) = 0$ or $P = P_o$. If we visualize the mechanism of price determination as one in which prices stay put at some given level as long as at that level the market is cleared, and rise (or fall) if at that level there is an excess demand (or an excess supply), then the task of the money authority in maintaining a stable price level can be stated in a somewhat different and not uninteresting form. That is, instead of enforcing directly the money supply $\overline{M}^{*\prime}$ of Figure 1, it may endeavor, through appropriate devices, to enforce the rate of interest $r' = \hat{r}$ and then let the money supply seek its own level in response to the demand generated by the initial price level $P$ and the given value of $r$. If it succeeds in picking and enforcing a rate $r'$ equal to the natural rate $\hat{r}$, then the money supply will be precisely $\overline{M}'$ as we can see from the figure. Of course, if $r'$ is set too low there will be excess demand which will raise $P$ and cause an expansion in $M$, and this process will tend to continue as long as $r'$ is enforced — this is simply the familiar Wicksellian cumulative process of inflation. Conversely, if $r'$ is set too high there will be cumulative deflation and contraction in $M$. The only way of stopping the cumulative inflation is either to raise $r'$ to the right level or to clamp down on the money supply letting $r$ seek its equilibrium level, at $\hat{r}$.

The above analysis is subject to one important qualification, namely that the two equations graphed in the figure may fail to possess a solution. This failure is bound to occur if $\hat{r}$, the value of $r$ that is consistent with clearing the commodity market, is negative; and there is nothing impossible in principle about $\hat{r}$ being negative, whatever might be the factual relevance of this possibility. Under these conditions the value of $r$ that clears the commodity market cannot clear the money market, and conversely. In graphical terms, if $r$ is negative, then the line $r = \hat{r}$ lies to the left of the ordinate, and therefore cannot have a point of intersection with $LL$ which must lie entirely to its right.

This possibility, which is, of course, the "Keynesian case," or liquidity trap, might occur even with $\hat{r}$ positive, if sufficiently small,

but is bound to occur if $\hat{r}$ is negative. The economic system, then, does not possess a resting point except through wage rigidity, or appropriate interferences with the market process, including devices for making money sufficiently expensive to store.

In summary, with pure bank money the classical conclusions about neutrality and dichotomy are valid, and so is the Keynesian contention that a market system may not possess a position of equilibrium.

(2) *If the money supply consists entirely of government money and there is no other form of national debt,*[10] *then money is neutral but the dichotomy does not hold,* i.e., the equilibrium value of the real variables is independent of the *supply* but not of the *demand* for money. This is the assumption underlying Patinkin's analysis and his conclusion stands in this case.

The dichotomy breaks down because the variable $V_0/P$ now can be expressed at $K_0 + M/P$, and hence $P$ appears in the consumption function (1) separately and not merely in the form of a price ratio $W/P$. Thus, the first nine equations of the system now involve 10 unknowns and cannot be solved separately. However, this subsystem can be solved for the nine real variables in terms of $M/P$, the "real money supply." In particular, let $r(M/P)$ and $X(M/P)$ be the solution for $r$ and $X$. By inserting this solution into the clearing condition for the money market one obtains

$$\frac{M}{P} = L(r(M/P),\ X(M/P),\ K_0 + \frac{M}{P})$$

This equation in the single variable $M/P$ yields a solution, say $(\hat{M/P})$, which in turn can be used to obtain the solution for all the remaining real variables. This solution implies that the *real* money supply and all the real variables are independent of the *nominal* money supply. In other words, money is neutral in that a change in $M$ merely changes $P$ in proportion $(P = \frac{M}{(\hat{M/P})})$ but leaves the equilibrium value of the real variables unchanged. Yet these equilibrium values are not invariant under a shift in the *demand function* for money expressed by $L$, for in general a change in this function would give rise to a different solution for $M/P$, and, hence, for the real variables.

(3) *In the presence of national debt or with a mixed money supply, neither the dichotomy nor the neutrality holds.* This conclusion can be shown to follow from the previous two. Just how a change in the money supply or in the demand for money will affect the real variables of the system depends on which component of the supply is changed and on the relative size of the components and of the national debt. As an illustration, consider the empirically most relevant case where the money supply consists entirely of bank money and the national debt is positive. Then an increase in the money supply, by increasing $P$ and reducing real wealth, will tend to increase saving and reduce the rate of interest to the extent necessary to produce a matching increment in investment. But different conclusions would hold under other assumptions, which need not be analyzed in this summary.

The results summarized under (1) to (3) above are helpful in assessing the theoretical and empirical relevance of explicitly recognizing the dependence of consumption on wealth, to which attention was first called by the well-known contributions of Scitovsky and Pigou [27], [22], [23]. From (1) above it is apparent that this recognition has no significant implication unless there exists in the economy some net money fixed claims on (or to) the government,[11] whether in the form of money or in the form of interest-bearing debt. However, when such claims do exist, as is usually the case, then the wealth effect has significant implications. In particular, a system satisfying assumptions A.1 to A.5 will generally possess a position of full employment equilibrium, contrary to Keynes' conclusion. This is because $V_0/P$ now includes a component which is inversely related to the price level $P$. Thus, by making $W$, and hence $P$, sufficiently small, it is generally possible to make real wealth and consumption so large and the rate of saving and investment so small that the commodity market can be cleared with a positive rate of interest, consistent with the clearing of the money market. In terms of Figure 1,

[10] In the terminology of Gurley and Shaw, the money supply consists entirely of "outside" money (cf. [8] pp. 72–73).

[11] Or foreigners, if we consider an open economy.

the value of $r$ clearing the commodity market is no longer a constant $\hat{r}$ but, instead, a decreasing function of $P$, i.e., a curve falling from left to right. At least for sufficiently low $P$ this curve may be expected to lie to the right of the ordinate axis and to intersect there the $LL$ curve.

This conclusion is not without interest from the standpoint of the history of economic controversies. But it does not imply by any means that one can rely on the Pigou-Scitovsky effect mechanism as a practical stabilization device, and that, if only wages were sufficiently flexible, a market economy could never be plagued by lack of effective demand. For one thing, the conclusion is valid only so long as we assume that redistributional effects can be neglected, and that the elasticity of price expectation is no larger than unity. Furthermore, in depressed situations in which a system without a government-money fixed liability would have no solution under wage flexibility, the size of deflation required to re-establish full employment might well be such as to produce even more damage than widespread unemployment. Thus, even if the cure could be counted on not to kill the patient — which is doubtful — there would be a great deal to be said for a less gruesome remedy such as fiscal policy.

Another implication of the wealth effect is that under the conditions of case (3), which are empirically by far the most relevant, the convenient classical proposition that "money is but a veil" is not warranted. But to keep the role of the wealth effect in proper perspective, it should be noted that the same conclusion would hold even in the absence of this effect as soon as we drop any of the heroic assumptions A.1 to A.5. In short, while there is really no ground for holding that in the "real world" money is ever strictly neutral, the lack of neutrality is due only in negligible portion to the Pigou-Scitovsky effect. Indeed, within the range of variation of prices characterizing a normal healthy economy, that wealth effect is likely to be so negligible that, were it not for other forces, especially wage-price rigidities, money would be very nearly neutral. In particular, interest rates would be nearly unaffected by monetary policy, and prices would respond roughly in proportion to the money supply. Since in the long (and even not so long) run, rigidities can be neglected, I would conclude that neutrality and the quantity theory — in the sense of a stable relation between the money supply and the value of output at any given interest rate — is a good long-run approximation, subject, however, to the stricture that in the long run monetary institutions may gradually change. In the case of really rapid inflation, the Pigou effect may be a little less negligible but still of secondary importance in comparison with the redistributional effect. Furthermore, given a large change in $M$, the proposition that the price level will change roughly in proportion is likely to provide a good approximation, once the change in $M$ has come to an end. Thus, the significance of the wealth effect appears to be primarily technical, but its empirical relevance would seem to be small, at least under normal conditions.

## III

### Implications of the Model Under Wage Rigidity

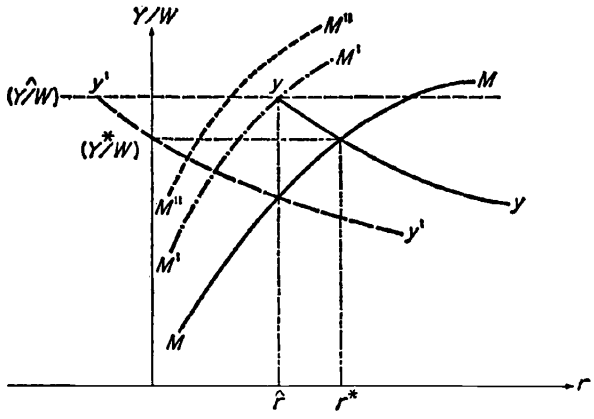
A. *The Strict Keynesian Version.* As already noted, wage rigidity can be said to exist if the wage rate $W$ will not be bid below some level $W_0$ even though at this wage rate there is an excess supply of labor; and the rigidity is "effective" if the market clearing value of $W$ is less than $W_0$. The implications of wage rigidity will be examined first in terms of what may be called the "strict Keynesian version" of the model, which assumes pure bank money and no national debt (and thus no Pigou effect) and competitive behavior in the commodity and the capital markets and on the demand side of the labor market.

Under these assumptions in the absence of wage rigidity the dichotomy holds and the real system (1) to (9) would possess a solution for all the real variables. Let $\hat{X}$, $\hat{N}$, $\hat{r}$, and $(\hat{P/W})$ denote this solution for the corresponding variables and let us label it the "full employment" solution. With wage rigidity, however, the dichotomy breaks down, for though $P$ does not appear separately in (1), $W$ appears

separately in (6), and thus the nine real equations contain ten unknowns. The equilibrium value of the real variables depends then on the money supply $M$, as well as on the rigid wage $W_0$, and more specifically on their ratio $M/W_0$.

This conclusion, and its implications, can be illustrated by the simple graphical apparatus of Figure 2, if we are prepared to make a few

FIGURE 2
WAGE RIGIDITY AND MONETARY POLICY




convenient simplifying assumptions. In particular, suppose that, to a first approximation, the demand for money can be treated as homogeneous of first degree in money income and not significantly affected by wealth, i.e., $M^d = L(r,Y) = YL^*(r)$. Then in order for the money market to be cleared we must have $YL^*(r) = M$, which implies

$$(k.1) \qquad Y/W = \frac{M/W}{L^*(r)}.$$

Taking $M$ and $W$ as given parameters, this is a relation between $Y/W$ (income in wage units) and $r$, shown in Figure 2 as the $MM$ curve. (Note that the Keynesian device of using labor as the numeraire is a very natural one since the rigid wage provides a stable unit of measurement.) This curve can again be regarded as the graph of income velocity as a function of interest rates, except for a proportionality factor, $M/W$. Thus, a change in the ratio $M/W$ will cause the curve to shift up or down in proportion to the change.

Next, from the real part of the system one can derive a second relation between these same variables which must be satisfied for the commodity and labor markets to be cleared. For this purpose we first derive from (4b) a relation between $P/W$ and $X$, which represents in essence the Marshallian short-run supply function for commodities (short run, because $K_0$ is fixed). By means of this relation and (5b), consumption demand given by (1) can be expressed in terms of $X$ and $r$ only, say, $C = C^*(X,r)$. Substituting this result and (2) into (3) and (7) one obtains an expression of the form

$$C^*(X,r) + I\ (X,r) = X$$

(the initial condition $K_0$ being subsumed in the functional form), which can be solved for $X$ in terms of $r$, say

$$(k.2) \qquad X = x(r).$$

For any given value of $r$ this relation yields that level of output for which the sum of the corresponding consumption and investment demands just equal output, and the commodity market is cleared. Finally, by multiplying $X$ by the corresponding supply price $P/W$, we arrive at the desired relation between $Y/W = (P/W)\ X$ and $r$, say

$$(k.3) \qquad Y/W = y(r).$$

It is represented in Figure 2 by the $yy$ curve and is shown only for values of $Y/W \leq (\hat{Y/W})$, since for larger values there would be an excess demand for labor, i.e., the labor market would not be cleared (nor could the rigidity be effective).

Since the assumption of competition in the commodity markets precludes the possibility of a falling short-run supply function — i.e., $P/W$ must be a nondecreasing function of $X$ — the slope of the function $y(r)$ depends on that of the function $x(r)$. This slope referred to the $r$ axis in turn can be shown to be given by the expression $(C_r^* + I_r)\ \dfrac{1}{I - C_x^* - I_x}$. Here $C_x^*$ and $I_x$ denote respectively the marginal propensity to consume and invest with respect to changes in real income, and $C_r^*$ and $I_r$ are the corresponding propensities with respect to changes in $r$. The first factor $(C_r^* + I_r)$ can be taken to be negative since $C_r^*$ is negative and $I_r$, whatever its sign, is most unlikely to outweigh it. The second factor is a generalization of the conventional multiplier, sometimes called the "supermultiplier," and is generally assumed to

be positive. If so, $X$, and hence $Y/W$ will be a declining function of $r$ as shown in the figure. It should be observed, however, that, contrary to a common view, there is really nothing impossible or unstable about the supermultiplier being negative (or zero) in some regions and the graph of $yy$ rising from left to right (or being parallel to the ordinate).

The intersection of $MM$ and $yy$ gives the equilibrium values, say $(\overset{*}{Y/W})$ and $r^*$, and these values clearly depend on $M/W_0$ which controls the position of $MM$. The equilibrium value of real income $X^*$, can then be obtained by substituting $r^*$ into (k.2), $(P/W)^*$ from $X^*$ and the supply function and so on; and they must all be functions of $M/W_0$.

In our graph the equilibrium position is one of less than "full employment." Full employment could be reached only with a larger money supply in wage units which would shift up the $MM$ curve to a position where it intersects the $yy$ curve at the full employment point $[\hat{r},(\hat{Y/W})]$. The money supply required to reach this position can be seen from (k.1) to be $(\hat{M/W}) = (\hat{Y/W})\ L^*\ (\hat{r})$. Under wage flexibility the adjustment would of course occur through the market process as unemployment would cause $W$ to fall, and hence $M/W$ to rise, until the labor market is cleared. But under wage rigidity it can only come about by an expansion of the *nominal* money supply to the level $M' = (\hat{M/W}) \times W_0$. This expansion can be achieved by the monetary authority either by directly enforcing the correct money supply $M'$ or by picking and enforcing the correct rate of interest $\hat{r}$, and letting the money supply seek its appropriate level $M'$. (Both processes may, of course, go on simultaneously.) Note that in the present model both monetary expansion and money wage reductions act only through shifts in the $MM$ curve and lead to the same value of $Y/W$ and $r$, and to differences only in money prices and income. This "Keynesian" result depends, however, on our having assumed away a "Pigou-Scitovsky" effect, by supposing $G$ to be zero.

We need still to consider what would happen if the initial money supply were such that the $MM$ curve passes above the full employment point, as is the case for the curve $M''M''$ in Figure 2. Here, of course, if wages are only rigid downward, the classical mechanism takes over; the rate of interest being initially too low, there is excess demand in the commodity market which bids up prices and wages; the rise in wages eventually reduces the effective money supply $M/W$, shifting down the $MM$ curve, until it intersects the $yy$ curve at the full employment point. The process just described corresponds to what is usually called "demand-pull" inflation.

Oversimplified as our model is, it brings into sharp focus the predicament of the money authority in a system with rigid wages, namely, to continuously pick and enforce the correct monetary policy (the money supply $M'$, or the interest rate $\hat{r}$, or both) under inadequate knowledge of the relevant portion of the $MM$ and $yy$ curves and their shift through time. Also the consequences of errors are asymmetric. If it follows a more "loose" policy, it generates demand-pull inflation or price rises which, under wage rigidity, are largely irreversible. If it follows a "tighter" policy it engenders unemployment and not merely deflation — presumably a somewhat lesser evil as long as contained within limits. In reality the problem is further complicated by the fact that the monetary authority may be expected to pursue the double goal of full employment *and* price stability. Unfortunately, these two objectives will be inconsistent with each other if the exogenously determined wage rate is such that, when combined with the full employment price-wage ratio $(\hat{P/W})$, it implies a full employment price level, $\hat{P} = (\hat{P/W}) \times W_0$, higher than the historically received one. This is of course the "cost push" case. If the monetary authority pursues full employment it cannot prevent a rise in prices; while if it refuses to go along and expand the money supply enough to insure full employment at the given wage, it will certainly cause unemployment, while it may not even succeed in preventing some rise in prices.

The dilemma is even more dramatic if the wage rate is controlled by a mechanism whereby money wages tend to rise at a rate depending on the level of unemployment, and this

rate of change is larger than the "rate of increase of productivity" (the rate at which $P/\hat{W}$ falls) at rates of unemployment larger than what might be regarded as unavoidable frictional unemployment (see, e.g., [26]). According to some views this is *the* predicament of our times, but I don't propose here to assess this claim or, even less, to propose remedies.

In concluding this section it is well to call attention to two types of situation where to reach and maintain full employment is beyond the power of the monetary authority. One is the well-known case where the full employment rate of interest $\hat{r}$ is negative (or possibly very close to zero), represented in Figure 2 by the curve $y'y'$. It corresponds to a situation where, in an economy with flexible wages, there would be no set of prices and interest rate capable of simultaneously clearing all markets. The system can have a solution only if wages are rigid and unemployment is allowed to develop, permitting the elimination of the excess supply that would otherwise arise at any positive rate of interest. In fact, the fall in employment is accompanied by a reduction in *supply* $X$. To be sure, this fall tends to reduce also the *demand*, but only to a smaller extent, as consumption, at least, is kept up by initial wealth and the expectation that the fall in income is but transitory in nature. (In Keynesian terms, the marginal propensity to consume is less than 1.) Thus, at some sufficiently low level of employment and output, the excess supply tends to disappear, and the system finds a resting point. A very aggressive monetary policy might at best bring output close to the ordinate of the point at which the $y'y'$ curve cuts the vertical axis.[12] Only fiscal policy can get the system back to full employment (cf. section IV).

The other case where monetary (as well as fiscal) policy is powerless is the case of "real wage rigidity." Here, because of union pressure, legislation, minimum subsistence levels, or otherwise, the real wage cannot be reduced below some level, say $(W/P)^m$, and this level exceeds the marginal product of labor corresponding to the given production function (including the stock of capital) and full employment of labor. (Full employment in this connection might be defined in terms of some standard labor force participation and work week, or in terms of the potential supply available at the fixed real wage.) There will then be an effective ceiling on output, $X$ and on $Y/W$, below the full employment ceiling exhibited in Figure 2. Any attempt to expand employment beyond that ceiling, through expansion of the money supply (or fiscal policy), will only succeed in increasing prices and wages without increasing output and employment. The situation just described frequently takes the form of a "balance of payments problem." The attempt to expand employment and income increases imports, forcing a devaluation, or equivalent measures, which increase the price of imports, and lead finally to a rise in money wages and prices. This type of situation, which some hold to be common in underdeveloped economies, can be remedied only if the wage rigidity can be broken or productivity increased through technological progress and capital accumulation, provided this does not immediately result in a commensurate rise in the rigid real wage.

B. *Modifications of the Keynesian Model: Imperfections in the Commodity Market and Wealth Effects.* The assumption of a competitive commodity market, which justifies the rising supply function of the previous section, may be replaced by an alternative one, possibly more realistic and certainly more convenient. It is the assumption that prices tend to represent a roughly constant markup on unit labor cost, possibly reflecting the prevalence of market imperfections of the oligopolistic type. An alternative formulation of this same hypothesis is that labor income $WN$ is a fairly stable share of total income $PX$, at least in the neighborhood of full employment, $\hat{X}$ [31]. Indeed, a constant markup on unit labor cost means:

$$P = (1 + m)\ (WN/X),$$

where $m$ is the markup.

This equation in turn implies:

[12] Even the most aggressive monetary policy can at best force $r$ to zero, and cannot force the money supply above the demand arising at this interest rate and the price level corresponding to the given wage rate and the level of output $X = x(0)$. To make monetary policy more effective would require very special devices such as making money sufficiently expensive to store.

$$WN = \frac{1}{1+m}(PX),$$

where $1/(1+m)$ is the constant share of labor income in total income. The empirical evidence seems, on the whole, to support such a stability, at any rate in the medium run. To be sure, as output fluctuates below, and up to, full employment the share of profits in income tends to fluctuate with output. But total property income is the sum of profits, interest, and rent income, and the share of the latter two components moves in the opposite direction, imparting stability to the share of total property income and hence also of labor income.

Suppose further that in the short run $N$ is approximately proportional to $X$, i.e., that the elasticity of output with respect to labor input is close to 1. This hypothesis does not seem to be grossly inconsistent with the empirical evidence, at least in the neighborhood of full employment. (It is, however, inconsistent with the assumption that the real wage is equal to the marginal product of labor, for then the elasticity of $X$ with respect to $N$ would be equal to the labor share of income, which is well below unity.) Under this further assumption $N/X$ can be approximated by a constant, say, $n$, and consequently $P/W$ becomes itself a constant or

$$(4.i) \qquad P/W = (1+m)n \equiv \pi,$$

an equation that replaces the original (4b). The parameter $\pi$ is of course a constant only at a given point of time. It may be expected to fall over time as productivity, $X/N$, rises through technical progress and the accumulation of capital, and hence labor input per unit of output, $n$, falls. If (4.i) holds, then the demand for labor is no longer given by (5) but directly by solving the production function (4a) for $N$ in terms of $X$ and $K_o$. Finally we may wish to recognize that, in the short run, the labor supply may be rather inelastic with respect to the real wage rate, and hence may be adequately approximated by a constant, say $N^f$. Then equation (6) is replaced by

$$(6.i) \qquad \begin{cases} N^s = N^f \text{ if } n^d(X, K_o) \geq N^f \\ W = W_o \text{ if } n^d(X, K_o) < N^f \end{cases}$$

These various modifications do not change the count of equations or unknowns of Model II. However, the resulting system — call it II.i — is somewhat easier to see through. For, so long as $W$ is rigid at $W_o$, $P$ itself can be regarded as given exogenously (since it is proportional to $W_o$). Hence, instead of solving the system for $Y/W$ in terms of $r$, we can solve it for $Y/P$. But $Y/P$ is $X$ and hence its solution in terms of $r$ is simply (k.2). Similarly, equation (k.1) can be rewritten as $Y/P = (M/P)/L(r)$ and Figure 2 can be redrawn with the more conventional and convenient variable $Y/P$ measured on the ordinate, and the money supply stated in "real" terms, $M/P$.

The implications of this model are roughly the same as those discussed in the previous section, but easier to comprehend and expose. In particular, the price level $P$ may be regarded as determined by the rigid wage (together with labor productivity and the constant mark-up) provided that the money supply is no larger than what is required to transact a full employment income at the price level corresponding to $W_o$. But if the money supply is larger, so that the first line of equation (6.i) holds, then $P$ is determined by the money supply, $P^* = M/L(r, \hat{Y/P})$, and the wage rate is determined by $P$ and hence, indirectly, again by $M$.

The implications of the model are also not appreciably affected if one recognizes the existence of a money-fixed government debt, $G$. In terms of the graphical analysis of Figure 2, the main effect of $G$ is found to be that equation (k.2) must be changed to

$$(k.2') \qquad X = Y/P = x(r, G/P).$$

Accordingly, the graph of this equation — the $yy$ curve — is not independent of the value of $P$ and hence of $W_o$. In particular, a fall in $W_o$, by reducing $P$ and increasing real wealth and consumption, would require a smaller rate of investment and hence a larger value of $r$ to clear the commodity market for any given $X$. It tends therefore to shift the $yy$ curve to the right. The full employment level of $Y/P$ might also be affected, via the labor supply, but presumably not significantly, at least for reasonable changes in $P$. Similarly the money market clearing condition becomes

(k.1′) $L(r,V/P,G/P) = M/P.$

Hence, the position of $MM$ depends not only upon the real money supply $M/P$ but also directly on $P$ and hence $W_0$. Given $M/P$, a fall in $W_0$ and $P$ should tend to increase the demand for money, i.e., reduce velocity for any given $r$, shifting $MM$ downward.

The main implication is that monetary expansion and money wage cuts need not have a symmetrical effect on output and the other real variables, since the equilibrium value of these variables, while still a function of $M$ and $W$, no longer depends merely on their ratio. Pure monetary expansion still affects the equilibrium by shifting up the $MM$ curve. But a wage cut of the same proportion would shift up $MM$ somewhat less (because of the wealth effect on the demand for money), and would also shift $yy$ to the right. If, as seems likely, the wealth effect on the commodity market is larger than that on the demand for money, a wage cut will tend to result in a somewhat larger increase in $X$ than would monetary expansion. But the difference may be expected to be altogether negligible: a priori considerations as well as empirical estimates [1] suggest that the shift in either curve from the wealth effect is likely to be minor, for reasonable changes in $W_0$ and reasonable assumptions about the ratio of $G$ to aggregate net worth — say, 0 to 20 per cent. In part, this is due to the fact that the change in wealth resulting from the "deflation" is likely initially to affect consumption only moderately, the rest of the effect being spread in time.

Note, finally, that because wage deflation tends to shift the $yy$ curve to the right — provided it does not generate expectations of further fall in prices — it could conceivably lead to an expansion of employment even in situations where monetary policy as such is powerless. Hence, across-the-board wage cuts might appear to provide a possible alternative to fiscal policy. But, as already noted at the end of section II, this view has little practical merit. For even if we wave aside the problem of enforcing an over-all wage cut, this approach provides at best a weak and unreliable tool, inconsistent with the maintenance of a stable price level.

## IV

### The Role of Government Monetary and Fiscal Operations

The essential implications of government fiscal operations can be formalized by the addition of two equations to Model II and certain modifications in some of the remaining ones.

The main modifications are the introduction of total tax receipts and other fiscal parameters in the consumption function and possibly in some other equations such as (2), (4), and (5); the addition of government purchases of goods $F$ to the definition of aggregate demand (3), and the purchases of labor services, $F_n$, to the demand for labor on the right-hand side of (8); and the addition of the government stock of capital, $K_0^g$ in (2), (4) and (5). The two additional equations are: (i) a tax collection equation, which may be considered as part of the real system, of the general form

(10) $T = t(X, V_0/P, P, [\tau]),$

where $T$ denotes tax receipts net of transfer payments measured in real terms (i.e., measured in $MM$'s) and $[\tau]$ denotes the relevant set of tax parameters; (ii) the government budget identity, to be added to the monetary set,

(M.8) $PF + WF_n - PT = \Delta G + \Delta M^*$

The left-hand side is the budget deficit in money terms, denoted hereafter by $D$. On the right-hand side, $M^*$ may be usefully defined as "net government money" or the difference between the amount of money issued by the government (if any) and the amount of bank-created money held by the government. It is also convenient for present purposes to replace (M.4) and (M.5) by the single equation

(M.4g) $M = B^b + M^*,$

while dropping the variable $M^b$.

After these additions and modifications, the set of equations (1) to (10) plus (M.1) to (M.3) turns out again to form a closed subsystem in the original variables plus $T$, but involving now a new set of "fiscal policy parameters," to wit, $F$, $F_n$, and $[\tau]$. Hence in examining the implications of the "government model," it is permissible to concentrate on this subset and to disregard again the bond market and the other monetary relations. However,

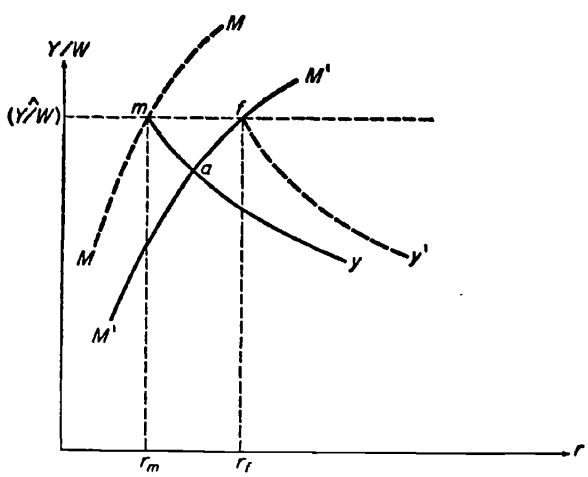
some of these relations and in particular (M.4g) and (M.8) are useful to clarify the relations between monetary and fiscal policy, which are essentially linked through debt management. In particular, from these two equations we can derive the relation

$$\Delta M = (\Delta B^b - \Delta G) + D.$$

This relation serves to make clear that, in principle, through appropriate monetary and debt management, monetary and fiscal policies can be made entirely independent of each other. That is, a given deficit (or surplus) $D$ can be made consistent with any change in $M$ through appropriate changes in $B^b$, the amount of credit extended by the banking system to the rest of the economy, or in $G$, the amount of government debt held by the private sector. It is for this reason that, in our model, we can continue to treat the money supply $M$ as exogenously given even in the presence of government fiscal operations. And we define monetary policy as the control of the money supply and not merely of the amount of bank credit.

The role of fiscal policy and its relation to monetary policy can again be analyzed through a graphical apparatus analogous to that of Figure 2 and set out in Figure 3.

FIGURE 3
MONETARY VERSUS FISCAL POLICY




The money market clearing conditions are basically unchanged and so is the nature and interpretation of the $MM$ curve. Its position is again controlled by the real money supply $M/W$ and possibly also to a minor extent directly by $W$, through the wealth effect. As for the commodity market, through a series of substitutions of the type described earlier and a few suitable simplifications and approximations — such as neglecting direct government purchases of labor — we can obtain a condition stating the equality of demand and supply in this market of the form:

$$\text{(g)} \qquad C^*(X, r, G/W, T, [\tau]) + I^*(X, r, [\tau]) + F = X.$$

If we further use (10) to eliminate $T$, the above equation can be solved for $X$ in terms of $r$ and fiscal parameters. From this solution in turn we can derive the relation

$$\text{(k.3g)} \qquad Y/W \equiv (P/W)X = (P/W)x^*(r, F, [\tau], G_o/W),\ X \leq \hat{X}.$$

For given values of the fiscal policy "parameters," $F$ and $[\tau]$, this equation can be looked at as a relation between $Y/W$ and $r$, represented in our graph by the $yy$ curve. Its position in the plane depends of course on fiscal policy, and this dependence can be conveniently approximated by a series of fiscal "multipliers" describing the upward (or downward) shift of the curve in terms of the change in $Y$ (given $r$) per unit change in the indicated parameter. The formulae below, obtained by total differentiation of equation (g), above, or (10) or both, provide a sample of such multipliers' effect on real income $X$. The effect on $Y/W$ can be obtained by multiplying these formulae by $P/W$, (if $P/W$ can be taken as a given parameter). The symbol $C^*_T$ denotes here the marginal effect on consumption of an increase in tax payments (usually assumed to equal the marginal propensity to consume with respect to income, $C^*_x$ with sign reversed) and $t_x$ is the marginal change in tax receipts per unit change in income before taxes.

(i) Effect of unit change in expenditure, $F$:

$$\frac{dX}{dF} = 1/(1 - C^*_x - C^*_T t_x - I^*_x)$$

(ii) Effect of an increase in tax payments $T$ or, equivalently, of a decrease in deficit $D$, expenditure constant (under the approximation that consumption depends only on the tax liability and not on the specific form of taxes):

$$\frac{dX}{dT}\Big|_{F \text{ constant}} = C^*_T/(1 - C^*_x - I^*_x)$$

(iii) Effect of an increase in deficit, through increased expenditure, tax schedules unchanged:

$$\frac{dX}{dD} = 1/[1 - C^*_x - I^*_x - (1 + C^*_x)t_x]$$ [13]

(iv) Effect of an increase in expenditure matched by an equal increase in taxes (balanced budget multiplier):

$$\frac{dX}{dF} \mid dT = dF = (1 + C^*_T)/(1 - C^*_I - C^*_x)$$ [14]

If the relevant marginal effects are roughly independent of the magnitude of $X$ (and $T$), then a given fiscal change would shift the $yy$ curve parallel to itself; however, this need not be the case in general. If one drops the assumption that $P/W$ is independent of $X$, then each of the above expressions must be multiplied by $(1 + E)(P/W)$ where $E$ is the elasticity of $P/W$ with respect to $X$, which may also vary with $X$.

This analysis of how various possible fiscal operations shift the curve up or down must, however, be qualified in one important way: it is strictly valid only so long as the shifted curve remains below the full employment ceiling $\hat{Y/W}$. In principle, this ceiling might itself respond to fiscal policy, though the responsiveness is likely to be slight in the short run and may be ignored for present purposes. Once the shifted curve bumps against the ceiling, further changes in the fiscal variable will obviously have zero multiplier effects (in real terms); and to see how they affect the economy one needs a very different kind of analysis, much along the traditional lines of public finance.

But before taking up this point, one may review the more or less conventional analysis as to the relation between fiscal and monetary policy as stabilization devices. Suppose that, for an initially given set of policies, the $MM$ curve and the $yy$ curve can be represented by the two solid curves of Figure 3, intersecting at point $a$, below full employment. This situation may be visualized as one actually prevailing, or as the one that would tend to come about under a *status quo* policy. There are then two "pure" policies and very many mixed policies that might be used to shift equilibrium to (or at least toward) the full employment ceiling. One is pure monetary policy, which would consist in expanding the money supply or enforcing a lower interest rate or both, shifting the $MM$ curve to the position $M'M'$ and establishing full employment at $m$. Pure fiscal policy on the other hand would consist in manipulating taxes or expenditure or both, shifting the $yy$ curve to the position $y'y'$ and reaching full employment at $f$.

Wherein lies the difference between position $m$ and $f$, aside from the difference in $r$, and in the income velocity of circulation, which can be picked up from the graph? Since, by assumption, both are full employment positions, output $X$ and employment $N$ will be the same. The difference consists therefore in the utilization of output as between private consumption, public consumption, and capital formation.

1. Suppose, first, that fiscal policy took the form of increased expenditure, tax structure constant. Then, at least to a first approximation, consumption $C$ will be the same at $m$ and $f$, and the difference will be found in the utilization of $X - C$. Clearly, $f$ will involve a smaller amount of *private* capital formation, $I$, and a larger use of $X$ by the government. *Total* capital formation will depend further on the way $F$ is divided between $F^c$ (government capital formation) and $F - F^c$ (expenditure on current account).

2. If, on the other hand, the shift in $yy$ is brought about entirely by personal tax reductions, $F$ constant, then higher consumption replaces a portion of private capital formation which is repressed through the higher interest rate (higher and lower here always mean relative to $m$ and not to $a$).

3. Finally, tax reductions might partly take the form of "investment incentives." Then consumption is likely to expand somewhat less and investment to contract somewhat less than in case 2. However, some reduction in investment relative to $m$ is still almost sure to occur. In fact, a reduction in taxes, no matter what its form, always increases income net

[13] This expression could be negative, even though $1 - C^*_x - I^*_x$ is positive, if $dX/dF)t_x > 1$ in which case an increase in $F$ will increase $T$ by a larger amount, reducing the deficit. Then an increase in $D$ can be obtained only by reducing $F$, and hence $X$.

[14] This multiplier is usually stated to be unity, but this conclusion is seen to hold only under the special assumptions $C^*_T = -C^*_x, I^*_x = 0$.

of taxes — disposable income at $f$ is higher than at $m$. For $I$ to be as high as at $m$, the tax inducement to invest, which increases yields permitting the higher interest rate at $f$, would have to induce larger saving out of a given disposable income, and sufficiently so to offset the increased disposable income. This is a most unlikely outcome, especially if one believes that a higher rate of interest is more likely to reduce than to increase saving. Thus, paradoxically, reliance on tax inducements to invest instead of on monetary policy to stimulate demand is likely to generate a *lower* rate of investment.[15] In addition, it will tend to produce higher yields on investment and higher market interest rates. In general, one may conclude that the main differential effect of using tax incentives instead of monetary policy to stimulate investment, when either method could be effective, is to produce higher yields which are in turn consistent with higher market interest rates. This difference might be desirable in the context of certain balance of payments problems which are, however, beyond the scope of the present "closed economy" model.

The conclusion to be drawn from this brief analysis is that, in so far as full employment could be maintained by purely monetary devices — i.e., where the initial $yy$ curve lies entirely sufficiently within the positive quadrant — the choice between monetary policy and various types of fiscal policy to achieve the appropriate level of aggregate demand must be based on traditional considerations. These are: the relative merits of private versus public consumption in choosing between $C$ and $F - F^c$; the relative "social" yield of private versus government capital formation in choosing between $I$ and $F^c$; and finally, on "intergeneration" comparisons in choosing between total current "consumption" $C + F - F^c$, on the one hand, and total capital formation $I + F^c$ on the other. As I have argued in some detail in [18], by pushing capital formation at the expense of consumption we increase the stock of capital and real income available to the community in the future, at the expense of the current generation. I have also argued that a neutral policy might be regarded as one that makes the current generation pay for the government services it is currently receiving, and that such a policy requires, by and large, collecting currently in taxes an amount equal to $F - F^c$. Lower taxes make future generations pay for the services enjoyed by the current generation, while higher taxes, in essence, make the current generations pay for services enjoyed by future generations.

There remain to consider briefly two cases. The first, when we start out from a position of full employment, requires very little additional comment. Here, an increase in government expenditure must clearly be justified on grounds other than maintenance of full employment. If the increased expenditure is not accompanied by higher taxes, then consumption will be unchanged and hence the whole increase in $F$ must come from a reduction in $I$, as the government taps private saving that would otherwise have gone into capital formation. This is the standard case on which rests the "classical" argument that deficit financing shifts the burden to "future generations" (cf. [3] and the references cited there, and [18]). Since the reduction in $I$ and expansion of government borrowing will tend to be accompanied by higher interest rates, an appropriate restrictive monetary policy will be called for. In terms of Figure 3, this situation could be represented as a shift of the commodity market curve from $yy$ to $y'y'$. It requires a shift of the money curve from $MM$ to $M'M'$, in order to offset the higher velocity of circulation accompanying the higher interest rate.

If the increased expenditure is accompanied by a matching increase in taxes there will still be some shift to the right of the $yy$ curve, as consumption will tend to fall by less than taxes (the multiplier effect of a balanced budget). Hence, again, private capital formation $I$ will have to be restricted somewhat through a higher interest rate and a tighter monetary policy. The maintenance of private investments would, in the short run, require instead an ap-

[15] Some might question the empirical relevance of this conclusion on the ground that I am ignoring here the corporate form, and that tax inducement to corporations might increase corporate saving and thus total saving and private investment. Basically, my position on this point is that corporate saving, except possibly in the very short run, is a substitute for, and not an addition to, saving out of conventional disposable income. This conclusion follows readily from the *M-B-A* consumption function in combination with the argument set forth in [17].

propriate budget surplus put at the disposal of investors, to offset the reduced private saving.

There is, second, the "Keynesian" case where the *yy* curve crosses over to the second quadrant. Here (aside from the rather impractical monetary cures mentioned earlier) fiscal policy is the only remedy, at least to the extent necessary to shift the *yy* curve to a position where it can make contact with monetary policy. Even in this situation a case can be made in principle for favoring government capital formation over other ways of stimulating demand, since, as I have argued in [18], there will otherwise be a "burden" on future generations, although, in any event, the burden will be small in relation to the benefits accruing to the current generation.

This analysis of the *modus operandi* of monetary and fiscal policies and their implications suggests, at least to me, that the case for a currently balanced budget, and hence for relying on monetary rather than on fiscal policy as a first line of defense in counteracting shifts in the forces controlling aggregate demand, is somewhat stronger than might have appeared some time ago. However, the choice of a proper mix involves many more aspects than those we can develop here, including considerations of reliability of the tools and of feasibility in a given concrete institutional setting. In particular, exclusive reliance on monetary policy, whenever this policy could, in principle, do the job, might require swings in the money supply and interest rates of a size that might prove unsettling to the working of the economy. (See, however, the argument of the next section.) These considerations support a policy of built-in stabilizers with reasonably high marginal tax take $t_x$, (which, remember, includes transfers). Such stabilizers tend to moderate the swings in time in the position of the *yy* curve resulting from shifts in the investment or consumption function or both, thus reducing the burden imposed on monetary policy. On the same grounds, a good case can be made for some countercyclical variation in expenditure and tax parameters, although at least from present evidence one might have reservations about the suitability of tax cuts announced to be but temporary (cf. [18] section IX). But there still remains a prima-facie case for balancing the budget over a suitable span of time (cyclically balanced budget) in so far as this is consistent with full employment, unless a convincing case can be made for discriminating between generations.

## V

### Imperfections in the Capital Markets — the Availability Doctrine

The models on which we have relied so far assume, at least implicitly, a well-functioning competitive capital market in which investments are limited and brought into line with saving through the mechanism of the rate of interest or cost of capital. In such a model there exists a single short-run equilibrium rate of interest which measures both the return to lenders and the cost to borrowers, and also equals (or at least is not less than) the internal marginal rate of return to all units.[16] There is also no need to give separate treatment to financial intermediaries: all loans may be regarded as extended directly from the lending or surplus units to the final borrowers needing funds to finance their expenditure.

That this assumption is unrealistic probably no one would have disputed seriously. It was, however, the merit of the availability doctrine, advanced in the postwar period, that it made a convincing case for the proposition that disregard of certain institutional imperfections of the capital market leads to an unsatisfactory and seriously distorted view of the *modus operandi* of monetary policy and its consequences (see, e.g., [24] [25] [4] [28]). Actually, the promoters of this doctrine seem to have been largely motivated by a specific issue of monetary policy: the advisability of abandoning the policy of pegging the yield of government securities, which in turn made it impossible to maintain close control over the money supply. They were primarily interested in establishing that, even if one accepted the then-prevailing view that aggregate demand was very inelastic with respect to interest rates (i.e., the elasticity

[16] Note that even if we wished to recognize the existence of a plurality of maturities, under our present assumption of certainty, the current return — interest plus capital gain — would be the same on all maturities and equal to *the* short rate.

of our $y(r)$ function was close to zero), abandonment of that policy and re-establishment of effective limitations on the quantity of money would not result in a sharp rise in interest rates, consequent collapse of the price of government securities, and soaring cost of servicing the national debt. To support this contention, they advanced a number of arguments, varying considerably in generality and persuasiveness. Here we shall be concerned primarily with one argument which seems to have the greatest validity and general applicability: the proposition that interest rates charged to borrowers by financial intermediaries are largely controlled by institutional forces and slow to adjust at best; and that the demand for funds is accordingly limited not by the borrowers' willingness to borrow at the given rate but by lenders' willingness to lend — or, more precisely, by the funds available to them to be rationed out among the would-be borrowers.

The implications of this proposition can be grasped most easily by considering one limiting case. Suppose the task of making credit available to units in need of financing requires specialized knowledge and organization and is therefore carried out exclusively by specialized institutions which we may label financial intermediaries. That is, surplus units, whose wealth exceeds their holding of physical assets and who carry the balance of their wealth in the form of claims on other units, do not lend directly to the deficit units, but instead lend to, or acquire claims on, the intermediaries. The intermediaries in turn lend to the final debtors of the economy at some rate, say $r'$, which, at least in the short run, may be taken as institutionally given, and adjusts at best only slowly to market conditions, as indicated below. Let us also assume initially that the rate $r'$ is such that the flow of net demand for credit from intermediaries (gross borrowing less repayments) exceeds the net flow of funds acquired by them and that the two are brought into equality by rationing the available supply among the potential borrowers. The rate $r'$ in turn also controls the rate intermediaries pay to their creditors or depositors, say $r_{in}$.

Under these conditions the flow of borrowing and borrowers' demand for commodities is limited not by the cost of borrowing $r'$ but by the flow of funds made available to intermediaries by primary lenders. Also the single rate $r$ of the perfect market model, measuring simultaneously (i) the return to primary lenders, (ii) the cost to final borrowers, (iii) the internal marginal rate of return from investments, and (iv) the opportunity cost of holding money, is replaced by a plurality of rates. Accordingly, the demand for money can no longer be regarded as a function of the rate of interest $r$, but will depend instead on the opportunity cost which will vary between $r_{in}$ for lending units and the internal rate for rationed units. Unfortunately, the internal rate is no longer obtainable from market quotations, nor is it otherwise directly observable.

Some measure, however, or index of prevailing internal rates might be derived from the investment function, equation (2) of our model. Suppose we solve this equation to express $r$ as a function of $I$ and $X$:

$$(3a) \qquad r_s = R(I,X), \frac{dr_s}{\partial I} < 0, \frac{dr_s}{\partial X} > 0$$

Under perfect markets this function would give the internal rate corresponding to the given value of $I$ and $X$. The same would be true under rationing, if rationing were "efficient," i.e., if the flow of investment were allocated among units in the very same way in which it would be distributed through the price mechanism under perfect markets. Since rationing cannot be perfectly efficient, the opportunity cost will presumably vary from unit to unit, but $r_s$ might still provide a reasonable indicator of prevailing internal rates. Accordingly, the demand for money might be approximated by replacing in equation (M.2) the variable $r$ with the variables $r_{in}$ and $r_s$, or also $r'$ and $r_s$ on the ground that $r_{in}$ is itself a function of $r'$. The same substitution must of course be made in equation (1).

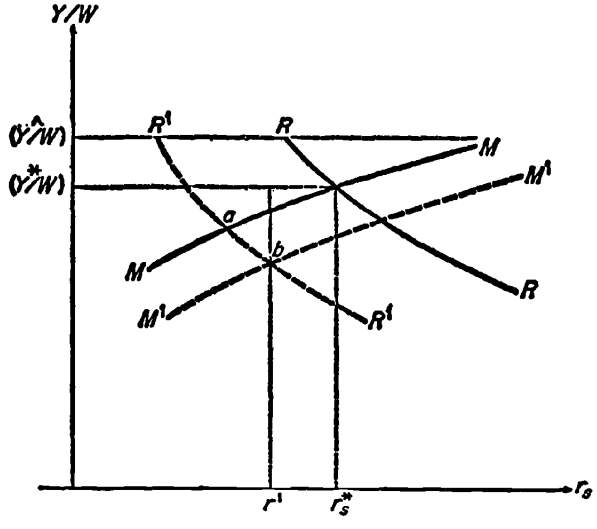
Except for these modifications of equations (1) and (M.2) and the re-interpretation of (3), or its equivalent (3.a), as defining the index $r_s$, our original system of equations (1) to (M.3) of Model II can still be used to formalize the functioning of an economy with capital rationing of the type described. Furthermore, if we treat $r'$ and $r_{in}$ as exogenously given, these equations still form a determined system

in the twelve original endogenous variables (except that $r$ is replaced by $r_s$).

The working of this system can again be clarified by a graphical analysis of the type of Figure 2, and exhibited in Figure 4. Specifically, from the first nine equations in ten endogenous variables we can derive again a relation between $X$ and $r_s$; which it is now convenient to write as $r_s = R\ (X)$. This equation expresses the relation between the index of internal rates and the level of output when the commodity and labor markets are cleared at the corresponding level of output, which means in particular that the rate of investment equals the rate of saving. Stated differently and somewhat less precisely, it shows the internal marginal rate of return prevailing when the level of aggregate demand is $X$ and the flow of resources available for investment is equal to the rate of saving prevailing at this level of output. Since to each value of $X$ there corresponds a value of $P/W$ we can also obtain a corresponding equilibrium relation between $r_s$ and $Y/W$, which is shown in Figure 4 as the $RR$ curve. This curve is shown as falling from left to right on the same grounds on which the $yy$ curve was drawn with a negative slope in Figures 2 and 3; a larger income makes possible a larger rate of investment which in turn implies a lower marginal rate of return.

FIGURE 4

SOME IMPLICATIONS OF THE "AVAILABILITY DOCTRINE"




As for the money market, by assuming as a convenient approximation either that $P/W$ can be treated as a constant and $G = 0$, or that $M^d$ is homogeneous of first degree in money income and not significantly affected by wealth, we can write the market clearing condition as: $L(Y/W, r', r_s) = M/W$. For given values of $M/W$ and $r'$, this condition yields a second relation between $Y/W$ and $r_s$, shown again as the $MM$ curve. The general shape of this curve must be similar to that of the corresponding $MM$ curve of Figure 2; however, its elasticity with respect to the variable on the abscissa must be smaller, since $r_s$ is only one of the rates affecting the velocity of circulation while the other, $r'$, is constant by assumption. The intersection of the two curves yields the equilibrium value of $Y/W$ and $r_s$, say $(Y/W)^*$ and $r_s^*$, from which the equilibrium value of $X$ and other variables can be inferred. The assumed value of the parameter $r'$ is also shown in the figure. Under certain assumptions, the gap between $r'$ and $r_s$ can be taken as an indicator of the size of the rationing gap (or fringe of unsatisfied borrowers), the size of the demand for credit unsatisfied at the lending rate $r'$.

It is apparent from our figure that the workings of a model with capital rationing of the type considered are not radically different from those of the original Model II. In particular, if we start from a position of full employment equilibrium, an upward or downward shift in the position of the $RR$ curve, reflecting, e.g., an improvement or deterioration of investment opportunities, would lead, respectively, to "inflation" or unemployment, unless offset by appropriate changes in the money supply. Also, in a situation of less than full employment equilibrium, such as the one assumed in our figure, unemployment could be cured by an appropriate monetary expansion. However, under capital rationing this outcome could come about without any change in the lending rate $r'$.

The mechanics of this operation are not difficult to trace out. The expansion of the money supply is initially accomplished by a relaxation of rationing by banks and a consequent expansion of lending. However, as income expands in response to the direct increase in investment and the induced expansion of consumption, the

higher rate of investment can be sustained without further monetary expansion through the increased flow of saving, which in part results also in an increased flow of credit available from intermediaries.

Similarly, an increase in $W$,$M$ constant would tend to result in higher prices and a fall in the rate of real investment and income, partly moderated by a rise in the velocity of circulation under the influence of the increase in $r_s$. In terms of our figure, the $MM$ curve shifts down and $(Y/W)^*$ falls as the rise in $Y$, induced by the higher $r_s$, is proportionally smaller than the increase in $W$.

While $r'$ may be taken as given in the short run, it may be expected to adjust gradually over time, tending toward some normal relation to $r_s^*$. But because this adjustment is a slow one, we may infer that even if $r_s^*$ swings sharply and rapidly over time in response to cyclical and other forces, $r'$ will tend to fluctuate over a much smaller range. Thus, the capital rationing mechanism provides a plausible way of reconciling moderate fluctuations in market rates with a widely shifting and interest-inelastic investment schedule.

We must, however, stop to consider what might be expected to happen if, as the result of a rapid decline in investment opportunities, the $RR$ curve were to shift downward to a position such as $R'R'$ in our figure. This new curve intersects $MM$ at $a$, but this intersection could not possibly describe a position of equilibrium. It implies in fact an equilibrium value of $r_s$ smaller than $r'$ which is impossible since, clearly, for every borrower the internal rate must be no less than $r'$. What is involved here is that at $a$, with the lending rate $r'$ borrowers are unwilling to borrow all that the intermediaries would have available to lend and the rate of investment is less than the rate of saving, so that income must fall.

In order to exhibit the new equilibrium position, let us assume at first that the rationing gap can dwindle to zero, intermediaries being willing to lend to anybody prepared to pay $r'$, and that under these conditions $r_s$ can be as low as $r'$. Then, as far as the commodity market is concerned, equilibrium could be reached at $b$, where the perpendicular through $r'$ intersects $R'R'$. At this point, in fact, $r_s$ equals $r'$ and the commodity market is cleared, since $b$ is on the $R'R'$ curve. (In other words income at $b$ is such that the rate of investment demand at $r'$ equals the rate of saving.)

What happens to the money market condition? As intermediaries are unable to lend all the funds they receive, they accumulate cash balances, reducing the supply of money outside intermediaries, which is the relevant measure of $M$, and causing the $MM$ curve to shift down to the point where it also intersects the $R'R'$ curve at $b$. This is, however, only one possible outcome; the other possibility is that the banking system will be unable to maintain the money supply at the initial level as the flow of debt repayment exceeds the flow of credit demanded at $r'$. Thus the fall in $M$ in the hands of the public may occur in part through intermediaries' accumulation and in part through a reduction of total money supply. Of course, the untapped lending power of both banks and intermediaries may gradually put downward pressure on $r'$ leading to a rise in $X$, a reactivation of idle intermediaries' balances or an expansion in bank money, or both.

Insofar as lending institutions may not be willing to lend to all borrowers prepared to pay the rate $r'$, a "minimal" rationing gap must exist in the new equilibrium position; hence the equilibrium point may tend to fall somewhat below and to the right of $(b)$ on the $R'R'$ curve, and correspondingly the $MM$ curve must shift down further till it goes through this point.

It appears from the above analysis that the recognition of the role of intermediaries and market imperfections in the guise of sluggish lending rates and of direct rationing rather than price rationing has certain significant implications. First, it helps to account for fluctuations in market lending rates which appear rather modest in relation to likely cyclical swings in the return from investment. Second, it implies that monetary policy may affect aggregate demand without appreciably affecting lending rates, at least in the short run. Third, it suggests that monetary policy — understood now as the control over the power of banks to create money rather than over the actual money supply — may break down under less stringent conditions than those of the original Keynesian

case. Because of sticky lending rates monetary policy may become powerless even when the value of $r_s$ corresponding to a full employment output is well above zero.

The model we have used can be considerably enriched by relaxing various oversimplifications. For instance, one can allow for a class of "prime borrowers," who are able to borrow directly from the public as well as from intermediaries and banks in a roughly competitive market, at a rate $r_p$, which, in contrast to $r'$, will tend to be sensitive to variations in the internal rate $r_s$. While these refinements must be passed by here, it is worthwhile to review briefly the relevance of the imperfections described to the problem which provided the original motivation for the availability doctrine, namely, the consequences of pegging or dropping the peg on government securities. This task can be accomplished by merely adding borrowing by the government (a "prime" borrower) to the model we have discussed, and hence a stock of government securities. In addition, we must take into account some institutional features of the American monetary system, and notably that the money supply is controlled by the "Central Bank" through the size of its demand liabilities.

Clearly, under perfect capital markets the yield on government securities, $r_g$, must coincide with the rate of interest $r$. Any attempt on the part of the Central Bank to impose a lower yield could only result in its having to acquire the outstanding stock with a corresponding increase in its liabilities and in the potential money supply. Consider now the situation under the imperfect market model. Here government bonds can be held either by primary lenders or by banks or by intermediaries. Since primary lenders have the choice of holding either governments or claims on intermediaries, their demand for governments, at a given point of time (when their total portfolio can be assumed as given), must be an increasing function of $r_g$ and a decreasing function of $r_{in}$. Similarly, the demand of intermediaries and banks must be an increasing function of $r_g$ and a decreasing function of $r'$. Hence, treating $r_{in}$ as a function of $r'$, the total demand for governments might be written as

$$G^d = G(r', r_g).$$

(This function might well include additional variables, such as the size of the rationing gap. However, the simplest formulation above is adequate for present purposes. Note also that the demand depends on initial conditions subsumed in the functional form.) Hence, clearing of market requires $G(r', r_g) = G$. With $r'$ exogenously given, this condition yields the equilibrium value of $r_g$. In general one might expect $r_g < r' < r_s$, implying that market imperfections of the type under consideration tend to reduce the cost of government borrowing.

To examine the consequence of pegging and unpegging in a context similar to that in which the issue was debated, let us suppose we start from an initial position of full employment equilibrium with the yield on governments at $r_g^o$. Suppose next that there occurs an upward shift in $RR$, leading in turn to some rise in $r'$. Then if the money supply were kept unchanged (or somewhat decreased to offset the rise in velocity) the equilibrium value of $r_g$ would also rise. Suppose, however, the Central Bank tried to peg $r_g$ at the initial level $r_g^o$. As under perfect market conditions, this pegging could only be accomplished at the cost of permitting an expansion in the money supply, since at $r_g^o$ there would arise an excess supply of government bonds which the Central Bank would be forced to acquire. The expansion in $M$ would, of course, result in an expansion of lending and of aggregate demand, which under full employment would imply inflationary price increases.

Consider next the consequence of dropping the peg and putting an end to monetary expansion. Under perfect market conditions this would result in all rates and yields including $r_g$ moving to that rate $\hat{r}$ which limits real investment demand to the full employment flow of saving; $\hat{r}$ is of course the same as the full employment rate $\hat{r}_s$, given by the intersection of the shifted $RR$ curve with the full employment line $Y/W = (\hat{Y/W})$. Especially if investment demand, and hence the $RR$ curve, is very inelastic, the upward shift in $RR$ would result in a large increase in $\hat{r}_s$, implying a correspondingly sharp increase in $r_g$. But, with market im-

perfections of the type considered, $r_g$ is controlled by $r'$ and $r_{in}$ and not directly (or at least not significantly) by $r_s$. Hence, so long as $r'$ does not respond much, at least initially, to the shift in $RR$ and the main effect of the shift is to increase the rationing gap, $\hat{r}_g$ will not be appreciably higher than $r_g^o$. Hence dropping the pegging policy will not result in sharp change in $r_g$, at all commensurate to the shift in $\hat{r}_s$, as claimed by the supporters of the availability doctrine. Actually, this conclusion may be considered as a special case of a more general result: because $r_g$ is tied to $r'$, and not directly to $r_s^*$, rationing not only tends to reduce on the average the cost of government borrowing but also tends to reduce the amplitude of fluctuations in $r_g$, as compared with what they would be under perfect market conditions.

## VI

### Some Concluding Remarks: The Role of the Money Supply and the False Dichotomy Between Quantity Theory and Income-Expenditure Theory

By way of conclusion and partial summary of this survey I propose to examine in this section what are the implications of the analysis for the critical question: how important is the role of monetary factors, and particularly of the money supply, as a determinant of the level of money income, output, and prices? Interest in this long-standing issue has been rekindled by a number of recent writings, and particularly by a challenging contribution of Friedman and Meiselman [5] in which the authors suggest that prevailing views in this matter readily fall into two opposing camps. One camp, which may be identified with the quantity theorists, holds that the quantity of money "is a key factor in understanding and even more controlling economic change"; and presumably from this view it is but a short and unavoidable step to accept Friedman's recommendation that discretionary monetary management be replaced by the simple rule of expanding the money supply at a constant rate. The other camp holds that "the stock of money matters little" and is supposed to consist of those embracing the "income-expenditure theory." From the test carried out by Friedman and Meiselman to assess the relative merits of the two points of view, it turns out that the "income-expenditure theory" is operationally defined as the hypothesis that current measured consumption is a linear function of measured disposable income, plus corporate saving, plus corporate profit inventory valuation adjustment, plus a couple of further adjustments.[17]

It should be readily apparent that the view of the monetary mechanism which emerges from this survey — and which I like to think is widely shared at the present time, at least in its broad outline — cannot possibly be forced into either of these camps. Nor is this surprising. In the first place the "income-expenditure theory" as operationally defined above is in no way inconsistent with the quantity theory, at least as defined by Friedman himself, as the hypothesis of stable demand for money (cf. [6], especially page 16). And in the second place both accepting and rejecting either of these theories is perfectly consistent with a wide range of views about the importance of the money supply as a determinant of income, including at one end the view that it is the key factor and, at the other end, the view that it matters not at all.

We suggest that the Friedman and Meiselman analysis, as well as many of the arguments over the importance of money, suffer from a failure to distinguish clearly between endogenous and exogenous forces and between structural relations and "reduced forms."

To make this point clear let us take as a starting point the model underlying the analysis of section IV. It consists of a system of thirteen equations in as many endogenous variables which will, in general, admit of a solution for all the endogenous variables in terms of exogenous variables and the param-

[17] The test actually carried out by Friedman and Meiselman consists in correlating consumption, $C$, with "offset to saving," $A$, which is the sum of investment, government deficit, and net exports. But these two variables will be linearly related if and only if $C$ is a linear function $C + A$, which, using well-known accounting identities and definitions, can be readily shown to be equal to the sum of disposable income, corporate savings, corporate inventory valuation adjustment, excess of wage accruals over wage disbursements, and statistical discrepancy (cf. [5], appendix A). We are not aware of any author's having advanced such a formulation of the consumption function and Friedman and Meiselman unfortunately have not provided the reader with any specific reference.

eters of the structural equations. Consider in particular the solution for income $Y$. So long as wage rigidity holds, we can write this solution as

(i) $$Y = f(M, F, [\tau]; W, [p])$$

where $F$ and $[\tau]$ are fiscal policy parameters and $[p]$ is the set of relevant parameters of the structural equations, reflecting technology, tastes and initial conditions. Alternative points of view about the importance of money and the real cleavage of opinions can be profitably stated and clarified in terms of the properties of the "reduced form" function $f$ of equation (i) implied by the underlying set of structural relations.[18]

To say that output and prices are totally unaffected by monetary factors means that $f$ does not include $M$ among its arguments. We shall refer to this point of view as the "effective demand only" theory abbreviated as EDO. Clearly, for EDO to be valid the system of equations obtained after deleting (M.2) and (M.3) should contain a determinate subsystem involving $P$, $X$, and $Y$. Furthermore, this subsystem must not involve $r$; for if it did its solution would also determine uniquely the demand for money (cf. equation M.2) and the value so determined would in general not be equal to the supply; in other words the entire system would then be inconsistent. Among the implications of this result the following are relevant for present purposes.

First, EDO is not equivalent to what is usually called the "theory of effective demand" — the assertion that the level of output is determined by the effective demand for it and not by the productive capacity of the economy. That proposition is hardly more than a truism, even if a fruitful one — just like the proposition that a change in effective demand can affect income only through a change in $M$ or in the velocity of circulation, or both. The essence of EDO is the proposition that effective demand is *totally unaffected by the supply of money either directly or indirectly*.

Second, EDO is perfectly consistent with the quantity theory, as it requires no special assumptions about the demand for money except that it should not be a function of $X$ and $P$ only.

Third, EDO bears no relation whatever to the "income-expenditure theory" tested by Friedman and Meiselman. For it requires no special assumption about the form of the consumption function or its stability, except that consumption should not depend on the rate of return from assets. It is true that the "elementary model" frequently used for introducing students to Keynes relies on a linear consumption function, say $C = c_0 + cY$, and exogenously given investment, $I$. This model falls under EDO, but because of the assumption about $I$ and not about the consumption function.

The implications of EDO can be conveniently visualized in terms of Figures 2 and 3. Since the commodity market equations now determine a unique value of $Y$, say $Y^*$, totally unrelated to the value of $r$, the $yy$ curve degenerates to straight line parallel to the abscissa and at a distance $Y^*/W_0$ above it. Of course, the value of $Y^*$ and hence the position of $yy$ depends on parameters of the commodity market equations and on fiscal parameters and this dependence is in fact described by the reduced form $f$.[19] For instance, in the "elementary classroom model" we have

$$Y^* = (c_0 + I)/(1 - c) = f(c_0, c, I)$$

if the government is excluded, and a somewhat more complex expression involving fiscal parameters if the government is included. The intersection of $yy$ and $MM$ determines the rate of interest, which therefore depends on the money supply and is in fact the only variable that monetary policy can affect (cf. [6], p. 17).

Our analysis leads us to reject the EDO theory and hence to the conclusion that $M$ appears as an argument of the reduced form $f$. For it accepts the view that, in general, an increase in $M$ will result in an increase in effective demand, basically by way of increasing investment demand. This increase in turn may come about partly because the expansion of $M$ will initially tend to reduce the cost of capital (though this reduction may be only

[18] The idea of relying on the reduced form $f$ for contrasting alternative points of view was first suggested to the author by Albert K. Ando.

[19] It might be noted in passing that the EDO model runs into some logical difficulties if the $yy$ curve were to be above the full employment line — i.e., if effective demand were to exceed full employment output.

transitional) and partly because it permits a relaxation of rationing and a larger flow of investment expenditure, financed initially through newly created money and subsequently through the larger flow of money saving.

Consider next the view that income is completely determined by monetary forces, i.e., by the demand for and supply of money, independently of conditions in the "commodity" markets. We shall label this point of view the "money only" theory and abbreviate it as MO. Clearly, for MO to be valid, equations (M.2) and (M.3) must form a *determinate* subsystem involving $Y$, and $M^d$. But a two-equation system can be determined only if it contains no more than two unknowns. It follows that a necessary and sufficient condition for the MO theory to be valid is that the demand for money should be a function of $Y$ and $Y$ only, or say $M^d = L(Y)$. Thus, MO is not equivalent to the quantity theory but only to a very special form of it; and it is perfectly consistent with the view that consumption is a linear function of current income and current income only. In fact, it requires no special assumption about any of the other equations of the system, except that $r$ must appear somewhere in these equations.[20]

By equating demand and supply one finds: $M = L(Y)$, implying

(ii) $$Y = L^{-1}(M),$$

where $L^{-1}$ is the inverse of the function $L$. Comparing this result with (i) we see that under MO the function $f$ is simply $L^{-1}$, the inverse of the demand function for money. Accordingly, stability of the demand for money implies stability of the function $f$.

In terms of our figures 2 or 3, MO implies that the $MM$ curve degenerates to a straight line parallel to the abscissa and at a height $(Y/W) = \dfrac{L^{-1}(M)}{W}$. With income already determined by the monetary part, the only function left for the commodity market curve $yy$ is to determine the rate of interest at the point where it crosses the above line. Thus changes in the commodity market, and in particular in the consumption and investment function, or in fiscal parameters do not affect income at all, but only the rate of interest.[21]

One special case of the MO theory is the "elementary model" frequently used for introducing students to the quantity theory, which takes the form $L(Y) = (1/V)Y$, where the constant $V$ is the velocity of circulation. Then equation (i) becomes $Y = VM$. Under the further assumption that (1) $V$ is in fact constant in time, at least up to a stochastic component which is both unpredictable and uncorrelated with any of the remaining variables of the system; (2) full employment output grows at an approximately constant rate $g$; and (3) it is desired to make money income grow at a rate which is consistent with the maintenance of full employment, provided prices are stable; one is finally led to the Friedman rule that monetary management should consist exclusively in expanding the money supply at $g$ per cent per year.[22]

Our analysis rejects MO because it acknowledges that the demand for money depends *also* on the rate of interest or, more generally, on the rate of return obtainable by exchanging money for other assets. This dependence, which even Friedman accepts in principle, is amply supported by empirical evidence. To admit that $r$ enters in (*ii*) — or somewhat loosely that the velocity of circulation depends on $r$ — may appear to require no more than a minor amendment to the MO theory. For, it may be argued, equations (M.2) and (M.3) still imply a relation between income and the

[20] If $r$ were to appear nowhere in the remaining 11 equations these would also form a determinate subsystem in 11 unknowns (the original thirteen minus $M^d$ and $r$), and its solution for $Y$ would generally be inconsistent with the solution generated by (M.2) and (M.3).

[21] It will be noted that the MO theory as stated is only a theory of the determinants of money income; in order to derive from it propositions about $P$ and $X$ one needs some theory of the relation between these three variables which can only be derived from other equations of the system. In so far as this relation can be established without reference to the "commodity demand equations" (1) and (2), as is true for instance in our own model II (cf. equation 4.b), it can still be said that the level of income is independent of the state of effective demand. And this conclusion remains valid under the somewhat more general demand for money equation $M^d = L(P,X)$.

[22] This policy prescription can be shown to be valid under somewhat more general assumptions than those stated in the text. In particular, if the demand for money can be approximated by the form $L(P,X) = KPX^a$, as Friedman has suggested, then the required expansion of the money supply is $\dot{M}/M = a\ \dot{X}/X = ag$.

money supply, namely $M = L(r,Y)$. In particular suppose for the sake of argument that $L(r,Y)$ is homogeneous in $Y$ and can therefore be written as $L^*(r)Y$. Then solving the above equation for $Y$ we can write

(iii) $$Y = V(r)M,$$

so that $Y$ is still proportional to $M$, except that the proportionality factor depends on $r$. Or, in other words, $Y$ is still controllable through $M$ except that $M$ must be adjusted to offset changes in velocity.

This line of argument however is worthless. For equation (iii) in contrast to (i) is not a reduced form equation; it contains the endogenous variable $r$ which like all other variables is a function of all parameters, including $M$. Thus whether and to what extent a change in $M$ affects $Y$ depends on its effect on $r$ and $V(r)$, and this effect cannot possibly be inferred from (iii) alone. It depends on what relation exists between $Y$ and $r$, a relation that can only be derived through the commodity markets and is embodied in the $yy$ curve of Figures 2 and 3. It is only through this relation — which is well defined if EDO does not hold — that we can eliminate $r$ from (iii) and obtain a solution for $Y$ in terms $M$ and other parameters. This solution is, of course, simply the reduced form $f$.

Because our analysis implies that $M$ appears as an argument of the function $f$, it agrees with Friedman and Meiselman that the money supply is an important factor in understanding and even in controlling the level of income. But because it also implies that the function $f$ is not merely the inverse of a stable demand function for money but rather the result of a complex interaction of monetary and real forces it leads equally to rejection of the view that the money supply is the only device for controlling $Y$, that it is always an adequate device, and most of all that it is in any meaningful sense the "cause" of economic instability.

The usefulness of $M$ as a stabilization device depends critically on the nature and form of $f$. Suppose we fix the value of the fiscal parameters at some stated level, and consider the set of values of $Y$ achievable by varying $M$ (for given $W_o$). This set may not include the full employment value ($\hat{Y}/W$). In terms of Figures 2 and 3 this will happen whenever the $yy$ curve intersects the full employment line sufficiently far to the left — the so-called liquidity trap. There will then be a ceiling to $Y/W$ short of $\hat{Y}/W$, either because $r$ and $Y$ approach some asymptote, or because beyond a point the money authority loses the power of expanding the money supply. We suggest that the real cleavage of expert opinion is not at all between those who hold the MO doctrine and those holding the EDO doctrine but rather revolves around whether $\hat{Y}$ is achievable by monetary policy nearly all of the time, only some of the time, or hardly ever. We are inclined toward the first-mentioned view, at least for reasonable values of the fiscal parameters — say values implying an approximately balanced budget in the neighborhood of full employment. But we are ready to admit that this view is debatable and that, in any event, the past is not necessarily a good guide to the future.

In so far as $\hat{Y}$ is achievable by monetary policy, if a larger or smaller value of $Y$ is allowed to develop one might be justified in saying that the accompanying unemployment or price rise results from an inadequate or excessive money supply. But this is quite different from saying that therefore the behavior of the money supply is the *cause* of instability. In the first place, because money is not the only possible tool for stabilization and not necessarily the best (cf. section IV), failure to avoid fluctuations could be attributed to fiscal policy as well as to inadequacies in the money supply. Second, and much more important, recognition that the relation between $Y$ and $M$ embodied in the function $f$ depends not only on the demand for money but also on the remaining equations of the system has widespread implications. If $f$ were simply the inverse of the demand for money equation, as asserted by MO, and if one also accepted the various other assumptions that justify the Friedman rule, then it would indeed follow that departures of $Y$ from its stable path could be attributed only to autonomous departure of the money supply from the growth path implied by the rule. But this conclusion becomes

invalid, even if the demand for money is quite stable, once we recognize that the function $f$ will shift around under the impact of shifts in the demand functions for commodities. Such shifts will cause deviations of $Y$ from the desired path even though $M$ is on the path. Even if these deviations could be offset by appropriate changes in $M$, we cannot say that $M$ is the cause of instability any more than we can say that the fact that headache can be avoided by taking aspirin makes aspirin the cause of headaches. The cause of the instability lies in these shifts and not in autonomous changes in $M$. On the contrary such changes are necessary if the shifts in the commodity markets are to be effectively offset. Thus, just because this analysis agrees with Friedman's on the importance of money and on the stability of the demand for money it leads to a categorical rejection of the notion of entrusting the control of the money supply to his simple mechanical rule.

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