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Comment Jan de Haan

Hedonic regression has now become one of the standard tools for statistical agencies to adjust their CPIs for quality changes in markets with a high turnover of differentiated models such as PCs. The authors address an important question, namely the difference between "hedonic imputation indexes" and time dummy hedonic indexes, which are the two main approaches to estimating hedonic price indexes (in the academic literature). They provide a novel exposition of the factors underlying the difference between these approaches, both for the unweighted and the preferred expenditure-share weighted case. In particular, the authors derive three conditions under which the two approaches lead to identical results: constancy (over time) of the average characteristics, constancy of the estimated characteristics parameters (used in the imputation approach), and constancy of the characteristics variance-covariance matrix. As the authors rightly claim, the third condition is somewhat unanticipated. Apart from being a valuable contribution to the literature, the chapter seems highly relevant for the work of statistical agencies. However, the use of matrix algebra makes the exposition very

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technical, and the implications may not be readily understood by a typical price statistician (though the empirical illustration is certainly helpful). Following I present some of the authors' findings in a simplified way by avoiding matrix notation, comment on them, and take the opportunity to make a few additional observations. I focus on the unweighted case, just for the sake of simplicity, but spend a few words on weighting also.

A couple of choices have implicitly been made in the chapter right from the start. For example, it is assumed that the hedonic regressions are run on the price data that are collected for the CPI. There may be statistical offices that perform hedonic regressions on a different data set and then use the estimated coefficients to adjust the raw CPI data for quality changes (which, I agree, is a problematic approach). More importantly, the chapter discusses a specific type of hedonic imputation. Let S^0 and S^1 be the samples of items in periods 0 and 1; $S_M = S^0 \cap S^1$ is the matched sample with size n_M, S_D^0 the subsample of disappearing items, and S_N^1 the subsample of new items. For simplicity I assume a fixed sample size n; thus $n - n_M$ is the number of disappearing and new items. I distinguish three types of unweighted symmetric imputation indexes—single imputation (SI), double imputation (DI), and full imputation (FI) indexes, as follows:

(1) $$\hat{P}_{SI} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{1/n} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{1/2n} \prod_{i \in S_N^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{1/2n};$$

(2) $$\hat{P}_{DI} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{1/n} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{1/2n} \prod_{i \in S_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{1/2n};$$

(3) $$\hat{P}_{FI} = \prod_{i \in S_M} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{1/n} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{1/2n} \prod_{i \in S_N^1} \left(\frac{\hat{p}_i^1}{\hat{p}_i^0} \right)^{1/2n},$$

where p_i^t denotes the price of item i in period t ($t = 0,1$) and $\hat{p}_i^t$ an imputed (predicted) price. If I am correct, the authors seem to consider (at least implicitly) a fourth type of imputation index, namely

(4) $$\hat{P}_{HI} = \left[\prod_{i \in S^0} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{1/n} \prod_{i \in S^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{1/n} \right]^{1/2},$$

(in case of a fixed sample size). They use a log-linear hedonic model that explains the logarithm of price p_i^t from a set of K characteristics z_{ik} and an intercept term α^t:

(5) $$\ln(p_i^t) = \alpha^t + \sum_{k=1}^{K} \beta_k^t z_{ik} + \varepsilon_i^t,$$

where β_k^t is the parameter for z_{ik}. By assumption the random errors ε_i^t have expected values of zero and constant (identical) variances.[1] Equation (9) is

1. Characteristics have no superscript for time t as an individual item is supposed to be of constant quality so that its characteristics are fixed over time.

estimated separately in periods 0 and 1, that is on the data of the samples S^0 and S^1. Ordinary least squares (OLS) regression of equation (5) yields parameter estimates $\hat{\alpha}^t$ and $\hat{\beta}_k^t$ and predicted prices $\hat{p}_i^t = \exp[\hat{\alpha}^t + \Sigma_{k=1}^K \hat{\beta}_k^t z_{ik}]$. However, because the OLS regression residuals $e_i^t = \ln(p_i^t) - \ln(\hat{p}_i^t) = \ln(p_i^t/\hat{p}_i^t)$ sum to zero in each period (i.e., $\Sigma_{i \in S^t} e_i^t = 0$), the index given by equation (4) coincides with the full imputation index (3). In appendix A the authors show that this approach is equivalent to what is often called the characteristics prices approach. The full imputation index (3), and thus equation (4), can be related to the single imputation index (1) in the following way:

$$\hat{P}_{FI} = \hat{P}_{SI}\left[\frac{\exp(\bar{e}_N^1)}{\exp(\bar{e}_D^0)}\right]^{(1-f_M)/2}, \tag{6}$$

where $f_M = n_M/n$ denotes the fraction of matched items and $1 - f_M = (n - n_M)/n$ the fraction of unmatched items; $\bar{e}_D^0 = \Sigma_{i \in S_D^0} e_i^0/(n - n_M)$ and $\bar{e}_N^1 = \Sigma_{i \in S_N^1} e_i^1/(n - n_M)$ are the average residuals for the disappearing and new items. Dividing equation (2) by equation (1) yields

$$\hat{P}_{DI} = \hat{P}_{SI}\left[\frac{\exp(\bar{e}_D^0)}{\exp(\bar{e}_N^1)}\right]^{(1-f_M)/2}, \tag{7}$$

which relates the double imputation index (2) to the single imputation index. Equations (6) and (7) show that the choice of imputation method matters if the average residuals of the disappearing and new items differ, especially if they have different signs (and f_M is relatively small). For example, $\hat{P}_{DI} < \hat{P}_{SI} < \hat{P}_{FI}$ if $\bar{e}_D^0 < 0$ and $\bar{e}_N^1 > 0$. This happens if disappearing items are sold at prices that are unusually low given their characteristics, perhaps due to "dumping," and new items are introduced at unusually high prices.

At first sight it is not obvious why we would prefer the full imputation index (3), and thus equation (4), to the other imputation methods. A drawback seems to be that the observed prices are replaced by model-based estimates: in general this increases the variance of the hedonic index as it adds model variance to the matched-item part and may give rise to unnecessary bias if the hedonic model would be misspecified (which almost certainly happens to some extent in practice). But this view might be too simplistic. It can easily be shown that the following expression applies to the full imputation index:

$$\hat{P}_{FI} = \frac{\prod_{i \in S^1}(p_i^1)^{1/n}}{\prod_{i \in S^0}(p_i^0)^{1/n}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k^{01}(\bar{z}_k^0 - \bar{z}_k^1)\right], \tag{8}$$

with $\hat{\beta}_k^{01} = (\hat{\beta}_k^0 + \hat{\beta}_k^1)/2$ and where $\bar{z}_k^t = \Sigma_{i \in S^t} z_{ik}/n$ is the average sample value of the k-th characteristic in period t ($t = 0,1$). By taking logs of equation (8) the authors' equation (34) is obtained (for a fixed sample size). Since the average characteristics of matched items are the same across periods, by denoting the average characteristics of the disappearing and new items by

$\bar{z}^0_{Dk} = \Sigma_{i \in S^0_D} z_{ik} / (n - n_M)$ and $\bar{z}^1_{Nk} = \Sigma_{i \in S^1_N} z_{ik} / (n - n_M)$, respectively, expression (8) can be rewritten as

$$(9)\ \hat{P}_{FI} = \left[\prod_{i \in S_M} \left(\frac{p^1_i}{p^0_i} \right)^{1/n_M} \right]^{f_M} \left\{ \frac{\prod_{i \in S^1_N} (p^1_i)^{1/(n-n_M)}}{\prod_{i \in S^0_D} (p^0_i)^{1/(n-n_M)}} \exp\left[\sum_{k=1}^{K} \hat{\beta}^{01}_k (\bar{z}^0_{Dk} - \bar{z}^1_{Nk}) \right] \right\}^{1-f_M}.$$

Equation (9) shows that this hedonic index is a weighted average of the matched-item index $\Pi_{i \in S_M} (p^1_i / p^0_i)^{1/n_M}$ and a (quality-adjusted) index for the unmatched items. The latter index adjusts the ratio of geometric mean prices of new and disappearing items for differences in the average characteristics of those items. Equation (9) further shows that the matched items' price relatives are implicitly left unchanged. Thus, matching where possible remains the basic principle even if a hedonic index would be estimated that, at first glance, does not seem to rely on matching.[2]

Another advantage of the full imputation approach is its comparability with the time dummy approach. In its standard form the time dummy hedonic model reads

$$(10)\qquad \ln(p^t_i) = \alpha + \delta D^t_i + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon^t_i,$$

where D^t_i is a dummy variable that takes on the value of 1 if i is sold period t (i.e., for $i \in S^1$) and 0 otherwise (for $i \in S^0$). A pooled OLS regression yields predicted prices $\hat{p}^0_i = \exp[\hat{\alpha} + \Sigma^K_{k=1} \hat{\beta}_k z_{ik}]$ and $\hat{p}^1_i = \exp[\hat{\alpha} + \hat{\delta} + \Sigma^K_{k=1} \hat{\beta}_k z_{ik}]$. It follows that

$$(11)\qquad \hat{P}_{TD} = \exp(\hat{\delta}) = \frac{\hat{p}^1_i}{\hat{p}^0_i} = \frac{\prod_{i \in S^1} (p^1_i)^{1/n}}{\prod_{i \in S^0} (p^0_i)^{1/n}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k (\bar{z}^0_k - \bar{z}^1_k) \right],$$

using the fact that, since an intercept term is included in equation (10), the residuals again sum to zero in both periods. Expression (11) is well known (see, e.g., Triplett 2004) and is quite similar to equation (8). This means that the time dummy index can also be written in the form of equation (9) when we replace $\hat{\beta}^{01}_k$ by $\hat{\beta}_k$. Using this result we obtain

$$(12)\qquad \hat{P}_{TD} = \exp[(1 - f_M) \sum_{k=1}^{K} (\hat{\beta}_k - \hat{\beta}^{01}_k)(\bar{z}^0_{Dk} - \bar{z}^1_{Nk})] \hat{P}_{FI},$$

which makes clear that the difference between the time dummy index and the hedonic imputation will particularly be small if the set of matched items is large, the (average) regression coefficients from both approaches are close

2. This is of course not to say that statistical agencies should try to match as much as possible. Samples preferably reflect the population of items at any point in time, possibly by using (or trying to mimic) PPS sampling. The fact that the current period sample may differ substantially from the base period sample is what makes the use of hedonics so important.

to each other, and the differences in the average characteristics of the new and disappearing items are small.

Finally I turn to weighted hedonic price indexes. The authors choose expenditure shares pertaining to the single period as regression weights. The advantage is obvious: it is a straightforward generalization of the unweighted approach, yielding an estimator of the (full) imputation Törnqvist price index. The same set of regression weights is used for the weighted time dummy approach, so that both weighted approaches can easily be compared. The disadvantage, on the other hand, is that WLS regression might increase the variance of the estimated parameters compared to OLS (especially if, as the authors assume, the errors have identical variances). The use of the single imputation Törnqvist index

$$\hat{P}_{T,SI} = \prod_{i \in S_M} \left(\frac{p_i^1}{p_i^0} \right)^{(s_i^0 + s_i^1)/2} \prod_{i \in S_D^0} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{s_i^0/2} \prod_{i \in S_N^1} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{s_i^1/2}, \tag{13}$$

or its double imputation counterpart is, however, more flexible: explicit weighting makes it possible to apply all kinds of regression weights when estimating model (5), including equal weights. Moreover, the authors' weighted time dummy index violates the (weak) identity test in a matched-item context (without new or disappearing items), a property that was mentioned already by Diewert (2003).[3] My conclusion would be that the issue of weighting in hedonic regressions is still unresolved.

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3. De Haan (2004), following up on Diewert (2003), proposed using regression weights for the WLS time dummy approach that are identical to the weights of the price relatives in equation (14). In that case the time dummy index can be interpreted as a single imputation Törnqvist index, and of course, in a matched-item situation the matched-item Törnqvist will be obtained.