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# 5 A Competitive Entrepreneurial Model of a Stock Market

Richard E. Kihlstrom and Jean-Jacques Laffont

## 5.1 Introduction

In the study of the stock market's role in the economy, two closely related questions arise. First, is the allocation of risk and capital that results from competitive trading in firm shares and debt efficient in some appropriate sense? Second, what is or should be the objective of the firm?

The second question arises because the traditional profit maximization hypothesis cannot be implemented when profits are uncertain as they will be when contingent claims markets are incomplete. Incompleteness of these markets is, in general, a feature of economies in which the only institution for the exchange of risks is a stock market for firm shares. The question of what a firm maximizes is not merely of intrinsic interest. In fact, the study of stock market efficiency requires a model of firm behavior. If, in particular, we are interested in the efficiency of competitive stock markets, the firm's behavior must be competitive in an appropriate sense.

The present paper is intended as a contribution to the recent literature which focuses on the above-mentioned questions. Section 5.9 contains a brief survey of this literature. A more complete survey is provided by Baron (1979).

The initial work on the efficiency of a stock market is that of Arrow (1953), which was subsequently elaborated by Debreu (1959). Arrow

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assumed that contingent claims markets were complete and showed that the stock market allocation of risk was efficient in a first-best sense. Debreu's extension of Arrow's work incorporated production. Because he retained Arrow's assumption of complete contingent claims markets, Debreu could assume that firms maximized profits. Like Arrow, Debreu was able to show that the competitive equilibrium was efficient.

The first paper to study stock market economies with production in which the implied contingent claims markets were incomplete was by Diamond (1967). Diamond suggested a concept of second-best efficiency which he showed to be appropriate for use in judging the optimality of a stock market. Using this concept, he demonstrated the second-best efficiency of the stock market allocation of risk and productive resources. By restricting the technology to satisfy a condition that he called *stochastic constant returns to scale*, Diamond was able, in an incomplete market setting, to formalize the hypothesis of stock market value maximization as an extension of profit maximization. He therefore solved the problem of specifying a criterion for a firm maximization by limiting the technologies under consideration. A subsequent paper by Leland (1974) showed that under Diamond's assumption of stochastic constant returns to scale, stockholders unanimously agree that stock market value maximization should be the firm's objective.

For our paper, as for virtually all of the "post-Diamond" literature, Diamond's paper serves as the point of departure. We adopt his concept of restricted efficiency and attempt to obtain analogues of his results in situations characterized by technologies which do not exhibit stochastic constant returns to scale. We assume that firms are created and run by expected utility maximizing entrepreneurs who simultaneously make portfolio decisions on their own account and operating and financial decisions on their firm's account. This hypothesis of firm behavior is the basis for our extension of Diamond's concept of equilibrium. In assuming that entrepreneurs maximize expected utility we are following Kihlstrom and Laffont (1979). In fact, the economy studied in this paper differs from that of Kihlstrom and Laffont (1979) in only one respect: the presence of a stock market which permits entrepreneurs to share the risks associated with their firms.

In our theory the firm is competitive in the sense that entrepreneurs take all prices as given. They are able to obtain capital, the only productive resource in our model, in a competitive bond market or by issuing firm shares in a competitive stock market. Bond market competition implies that the price of debt is treated as given. Competition in the stock market is reflected in the fact that the relationship between a firm's operating and financial decisions and its share value is treated parametrically by all entrepreneurs.

In order to ensure that the model is competitive, we assume the existence of a large number of individuals each of whom can create a firm and become an entrepreneur. Following Aumann (1964), we introduce the large numbers assumption formally by assuming a continuum of individuals. The technology required by entrepreneurs is assumed to be available to all, at a cost. Thus, entry is unlimited but not costless.

Other conditions appear to be necessary to ensure competition. Specifically, each entrepreneur who chooses a specific operating and financial decision for his firm should face stock market competition for investor capital. This will be true if there is a large number of entrepreneurs who make the same choices and whose outputs are, in some sense, statistically dependent. If outputs of two identical firms are independent, shares in these firms are not perfect substitutes, and this tends to reduce competition. To ensure competition in the stock market, we assume that the returns across firms are statistically dependent. We also assume that individuals are divided into types and that there is a large number of individuals of each type.

The formal structure of the model is presented in section 5.2.

The third section describes the roles played in our theory by two classical results and by the arguments traditionally employed to establish these results. One of these results is the Modigliani and Miller (1958) theorem, which asserts that, in equilibrium, financial decisions are irrelevant. The other well-known result is that the equilibrium value of an existing firm is the market value of the productive resources invested in its creation and operation. These results arise as necessary conditions for the equilibrium defined in section 5.2. They imply that, in equilibrium, all individuals are indifferent between becoming entrepreneurs or remaining nonentrepreneurs (capitalists, in our terminology). The proof given in section 5.3 that these classical results are necessary for equilibrium is an adaptation to our formal structure of the traditional arguments normally used to obtain them. The fundamental idea is that, in equilibrium, any possibility of arbitrage profits must be eliminated. The Modigliani-Miller theorem is necessary to prevent profitable arbitrage between debt and equity. When the stock market value of a firm equals the value of the resources it employs, arbitrage strategies involving firm entry or exit will, of necessity, fail to earn profits.

The third section also discusses the role of price expectations in the model. Since the shares of firms which are actually observed in equilibrium are traded, their expected price can be assumed to equal their actual price. There will, however, be many operating and financial decisions which could have been chosen but are not. These are identified with potential firms about which individuals are assumed to have share price expectations. For these potential (as opposed to existing) firms, the possibility of free entry determines only upper bounds on the expectations. The expectations at which these upper bounds are attained even for potential firms will be referred to as *classical expectations*. They will play a crucial role in the analysis which follows section 5.3.

For the case of classical expectations, section 5.4 establishes the existence of an equilibrium and describes its structure. The existence proof is simplified by the fact that the model exhibits several special properties. These are described in the propositions and lemmas of section 5.4, which precede the existence theorem. In order to prove the existence theorem, the special properties of the model are used to show that an equilibrium can be identified with the equilibrium of a simple two-good pure trade competitive economy.

The first result in section 5.4 is proposition 3, which asserts that optimal portfolios of all individuals have an extremely simple form. Specifically, it asserts that any entrepreneur's portfolio contains only shares in his own firm or in firms operated in the same way as his firm. Nonentrepreneurs, i.e., capitalists, are shown to hold shares in only those firms which are operated in the same way that they themselves would operate a firm if they were to become entrepreneurs. The optimality of nondiversification established in proposition 3 depends crucially on two assumptions: the concavity of the production function in the variable or operating inputs, and the statistical dependence of the outputs of all firms.

The next important result in section 5.4 is lemma 2, which implies that the entrepreneur's maximization problem has a unique solution. This is important for the proof of existence. The proof of lemma 2 is slightly complicated because of a nonconvexity introduced by the fact that the entrepreneur makes a portfolio decision for his own account as well as a production decision on his firm's account. A related nonconvexity was first observed by Drèze (1974).

The simplified structure of an equilibrium implied by proposition 3 is described in proposition 4. In the equilibrium, there are as many types of firms as there are types of individuals. There is, in essence, a type of firm created for each type of investor. Each type of individual holds shares in only the firms created by entrepreneurs of his type. This clientele effect results in unanimity among the shareholders of every firm about the goals the firm should pursue. This effect was discussed earlier by Smith (1970).

The remainder of section 5.4 transforms the equilibria with the simple structure described in proposition 4 into equivalent equilibria of a two-good pure trade competitive economy. Theorem 1 of this section uses this transformation to obtain an equilibrium. This transformation is also the basis for the efficiency theorem of section 5.5. In theorem 2 of that section it is shown that the equilibrium is efficient in the second-best sense defined by Diamond (1967). The marginal conditions for Arrow-Debreu first-best efficiency are stated in proposition 7 of section 5.5. These conditions are used to explain why, in general, the equilibrium is inefficient in the Arrow-Debreu sense. Proposition 8 uses the marginal conditions to prove that an equilibrium is first-best efficient if there are sufficiently many risk neutral individuals or if all individuals are alike.

Section 5.6 argues that the equilibrium studied in this paper is an appropriate generalization of Diamond's model to the case in which there may not be stochastic constant returns to scale. To support this assertion, it is shown that the equilibrium studied here coincides with Diamond's when the technology exhibits stochastic constant returns to scale. Section 5.6 also interprets our equilibrium as a generalization to the case of uncertainty of the classical model of perfect competition in which firms produce at the point of minimum average cost. Our model is shown to reduce to the perfectly competitive equilibrium when there is no uncertainty.

Section 5.7 is the only section to consider equilibria with nonclassical expectations. It conjectures that such equilibria exist and are, in general, inefficient.

Section 5.8 observes that our results can be interpreted as a demonstration that the need for a stock market only arises when there are fixed costs to entry, uncertainty, and risk aversion on the part of a large number of investors. It is shown that if one of these features is absent, the market in firm shares is unnecessary since the economy can achieve the same allocation without a stock market. In a closely related sharecropping context in which there are no fixed costs, the irrelevance of a market for output shares has been demonstrated by Stiglitz (1974) and Newbery (1977).

Section 5.9 contains a brief survey of recent literature and describes where our work fits within this literature. This survey includes a discussion of an externality which arises in stock market economies because, as Smith (1970) and Drèze (1974) pointed out, the firm's production decision is a public good for the shareholders.

## 5.2 Introduction to the Entrepreneurial Stock Market Model

There is a continuum of individuals each of whom is associated with a point in the interval $[0,1]$. There are two goods: "capital," an input; and a consumption good, "income." Each person begins with an endowment of $A > 0$ units of income and one unit of capital. These endowments of capital and income are received without uncertainty. The fact that capital is purely an input and not a consumption good is reflected in the fact that income is the only argument in the individual utility functions. Each individual $\alpha$ is assumed to have a twice differentiable von Neumann–Morgenstern utility function $u(I,\alpha)$ defined on $[0,\infty)$ with positive and nonincreasing marginal utility of income at all nonnegative income levels;

i.e., $u' > 0$ and $u'' \leq 0$. At times, we will find it convenient to assume that $u$ is strictly concave or even that $u'' < 0$.

We will also make frequent use of the following assumption.

*A*1. *There are n types of individuals, where n is a finite integer. For all individuals of type i, there is a common utility function* $u(\bullet,i)$. *Thus, if* $\alpha$ *is of type i, we will let* $i(\alpha) = i$ *denote his type and his utility function will be*

$$u(\bullet,\alpha) = u(\bullet,i(\alpha)).$$

*The measure of the set of individuals of type i is denoted by* $\mu_i$. *For each* $i = 1,\ldots,n$, $\mu_i > 0$.

Note that our assumptions imply that

$$\sum_{i=1}^{n} \mu_i = 1.$$

Each individual can choose to be an entrepreneur or a capitalist. If he becomes an entrepreneur, his capital endowment is expended on entrepreneurial activities. By investing his capital in this way, the entrepreneur obtains the income producing technology described by the production function $g(K,\tilde{x})$. The first argument $K$ in this function represents the operating or variable capital employed by the entrepreneur in his firm. It does not include the one unit of entrepreneurial capital which is the fixed or set up cost of the firm. Thus, the total capital employed by the firm is $K + 1$, the operating capital plus the entrepreneurial capital. The second argument $\tilde{x}$ is a random variable which influences the output of all firms.

A number of assumptions will now be made about $g$ and $\tilde{x}$. First, to avoid technicalities associated with differentiation under the integral, we hypothesize that $\tilde{x}$ takes value in a finite subset $X = \{x_1,\ldots,x_S\}$ of some finite dimensional subspace. Next, we assume that regardless of the level of operating capital $K$, the worst that can happen is that there is no output. Thus, for all $x$ in $X$ and all $K \geq 0$, $g(K,x) \geq 0$. If $K = 0$, output must be zero for all possible $x$ values; i.e., $g(0,x) = 0$ for all $x \in X$. In addition, there is assumed to be some $x \in X$, say, $x = \underline{x}$, for which $g(K,\underline{x}) = 0$ for all $K \geq 0$. The assumption of existence of such an $\underline{x}$ is not essential to the analysis. It is used primarily to simplify the discussion. We also assume that for all $K \geq 0$ and all $x \neq \underline{x}$ the marginal product is positive and decreasing; i.e., $g_K(K,x) > 0$ and $g_{KK}(K,x) < 0$, if $K \geq 0$ and $x \neq \underline{x}$. It will be necessary to assume that for each $x \neq \underline{x}$, there exists a capital level $K(x)$ at which

$$g(K(x),x) = g_K(K(x),x)[K(x) + 1]. \tag{1}$$

For a specific $x$, $K(x)$ is as shown in figure 5.1. Note that $K(x)$ maximizes

$$\frac{g(K,x)}{(1 + K)},$$

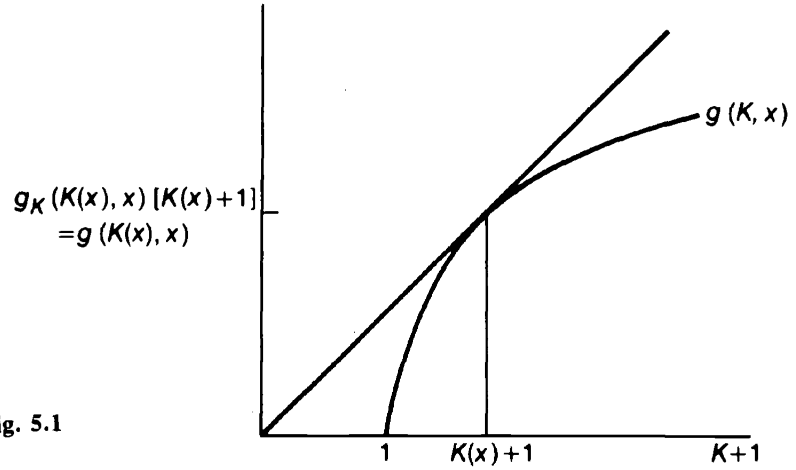


Fig. 5.1

which is output per unit of capital. Similarly for any capital price $r$, $K(x)$ minimizes

$$\frac{r(1+K)}{g(K,x)},$$

which is the firm's average cost. Thus, by assuming that $K(x)$ exists we are in effect assuming that for each $x$, there exists an output level, specifically $g(K(x),x)$, at which average cost is minimized.

The random variable $\tilde{x}$ is assumed to be the same for all firms. It is furthermore hypothesized that all individuals agree about the distribution of $\tilde{x}$. This distribution is denoted by $\rho$. (In fact, this assumption can be weakened somewhat when assumption A1 holds. Specifically, it is possible to assume only that all individuals of each type $i$ agree about the distribution of $\tilde{x}$.)

The expenditure of entrepreneurial capital is, in essence, the payment of a set up cost which transfers to the entrepreneur the technology described by the production function. The entrepreneur thus becomes the sole owner of the firm and is free to make all productive and financial decisions associated with his firm's operation. Specifically, he can choose the firm's investment level $K$ as well as the debt-equity level. In addition to making decisions on the firm's account, the entrepreneur will also be faced with financial decisions to be made on his own account. He can, for instance, sell some or all of the shares in his firm. The proceeds from this sale can then either be held as what is essentially a safe asset or be used to purchase shares in firms operated by other entrepreneurs. Because the entrepreneur is initially the sole owner of the firm, he can make all of these decisions, whether for his own account or the firm's, so as to further his own personal interests, i.e., so as to maximize his own expected utility.

Since our aim is to construct a competitive model of the stock and capital markets, we assume that the prices in these markets are fixed by supply and demand and taken as given by all traders. Thus, a typical entrepreneur $\alpha$ can either purchase all of the capital $K(\alpha)$ he decides to employ at a price $r$ (denominated in income terms), or obtain all or part of this capital by the sale of his firm's shares in the competitive market for these shares. If all of the firm's capital is purchased, its debt is $rK(\alpha)$ and its profits will be

$$g(K(\alpha),\tilde{x}) - rK(\alpha).$$

If, in addition, all of the shares to the firm are retained by the initial owner, i.e., the entrepreneur, then he will receive all of these profits. If, however, the entrepreneur decides to retain only $\gamma(\alpha)$ of the shares in his firm, he can sell the remaining $1 - \gamma(\alpha)$ shares for $M(\alpha)[1 - \gamma(\alpha)]$ units of capital, where $M(\alpha)$ is the competitively determined market price for a share in the firm. (Notice that we have assumed that the price of shares is denominated in capital terms.) The capital proceeds from this sale either can be retained for the firm's account and used in production or can be sold for the entrepreneur's private account. If it is sold for private account, the entrepreneur may receive payment in different forms. On the one hand, he can sell the capital directly in the capital market and receive the price $r$. On the other hand, he can sell it in the markets for shares to firms operated by other entrepreneurs. For example, he can buy a share in the firm run by $\beta$ at a cost of $M(\beta)$ capital units, where $M(\beta)$ is the competitive price of a share in $\beta$'s firm.

If $S(\alpha)$ units are retained for production on the firm's account, the firm's debt is reduced from $rK(\alpha)$ to $r[K(\alpha) - S(\alpha)]$. The amount $K(\alpha) - S(\alpha)$, which will usually be denoted by $B(\alpha)$, is the amount of capital the firm raises by incurring debt. The firm's profits will then be

$$\tilde{\pi}\big(K(\alpha), B(\alpha)\big) = g\big(K(\alpha),\tilde{x}\big) - rB(\alpha) \tag{2}$$

of which entrepreneur $\alpha$ will receive

$$\gamma(\alpha)\tilde{\pi}\big(K(\alpha), B(\alpha)\big). \tag{3}$$

Equation (2) makes it clear that two firms run by different entrepreneurs, say, $\alpha$ and $\beta$, will generate identical profit distributions if $K(\alpha) = K(\beta)$ and $B(\alpha) = B(\beta)$. For this reason, we will assume that in an equilibrium there is a function $N$,

$$N : [0, +\infty) \times (-\infty, +\infty) \to (-\infty, +\infty),$$

which relates a firm's capital input and debt capital levels to the price of the firm. Thus, if a firm employs $K$ units of capital and raises $B$ capital units by issuing debt, its price will be $N(K,B)$. We are explicitly assuming that $N$ is the same for all entrepreneurs. Thus, $M(\alpha)$ depends on $\alpha$ only through $K(\alpha)$ and $B(\alpha)$; i.e.,

$$M(\alpha) = N(K(\alpha), B(\alpha)). \tag{4}$$

In the remainder of the paper $M(\alpha)$ will always be given by (4).

Suppose then that $\alpha$ employs $K(\alpha)$ units of capital and sells $[1 - \gamma(\alpha)]$ shares to the firm to obtain $M(\alpha)[1 - \gamma(\alpha)]$ units of capital. Also, suppose that he finances $B(\alpha)$ units of capital purchase by issuing debt at a cost of $rB(\alpha)$ to the firm. He must then retain $K(\alpha) - B(\alpha)$ of the capital obtained by the sale of shares for use in production by the firm. If the remaining

$$M(\alpha)[1 - \gamma(\alpha)] - [K(\alpha) - B(\alpha)]$$

units of capital are then sold on his personal account for income at the price $r$, the entrepreneur's income is

$$A + r\{M(\alpha)[1 - \gamma(\alpha)] - [K(\alpha) - B(\alpha)]\} + \gamma(\alpha)\tilde{\pi}[K(\alpha), B(\alpha)]. \tag{5}$$

Note that this expression (5) for entrepreneur $\alpha$'s final wealth will also hold if $B(\alpha)$ exceeds $K(\alpha)$, i.e., if the entrepreneur issues debt on the firm's account which exceeds that required to finance the firm's operating capital. If the excess capital $[B(\alpha) - K(\alpha)]$ thereby obtained is then sold on private account, the entrepreneur's final wealth is increased by $r[B(\alpha) - K(\alpha)]$ so that, as stated, (5) does represent final wealth. It should be clear that the entrepreneur could use this procedure to increase his wealth indefinitely unless either he is explicitly prohibited from using it or the market responds to it by revaluing the firm when $B(\alpha)$ is increased. Modigliani and Miller have argued that the latter will happen, and we will return to discuss how their arguments imply a relationship between $M(\alpha)$ and $B(\alpha)$ which permits an equilibrium to exist.

We now want to assume that the entrepreneur buys shares in other firms. In the discussion of this case, $\alpha$'s holding of his own firm is treated symmetrically with his holding of other firms. For the purpose of representing $\alpha$'s portfolio problem, let $E$ be the (Lebesgue measurable) set of entrepreneurs. If $E'$ is a Lebesgue measurable subset of $E$, $\Gamma(\alpha, E')$ will denote the (nonnegative) number of shares of firms in $E'$ held by $\alpha$. The portfolio held by $\alpha$ is therefore represented by the nonnegative measure $\Gamma(\alpha)$ defined on $E$ and its Lebesgue measurable subsets.

In the discussion which follows, we will find it convenient to omit the adjective "Lebesgue measurable" when referring to subsets of $E$ and functions with domain $E$. The reader should keep in mind, however, that *all* such subsets and functions are assumed to be Lebesgue measurable. In particular, for every Lebesgue measurable $E' \subseteq E$, the function $\Gamma(\cdot, E') : E \to [0,\infty)$ is assumed to be Lebesgue measurable.

In the terminology just introduced, the entrepreneur's portfolio problem is to choose a nonnegative measure $\Gamma(\alpha)$. The restriction to nonnegative measures is an embodiment of the assumption that short sales are impossible. This restriction is imposed to avoid the possibility that infinite profits can be obtained by using strategies which involve short sales. This problem is known to arise in models similar to that being described here if short sales are not ruled out or at least limited.

For the purpose of further interpreting $\Gamma(\alpha)$, suppose that $\alpha$ chose to invest in only two firms: his own and that run by $\beta$. Then $\Gamma(\alpha,\{\alpha\}) = \gamma(\alpha) > 0$ would be the number of shares $\alpha$ holds in his own firm and $\Gamma(\alpha,\{\beta\}) > 0$ would be the number of shares he holds in firm $\beta$. Since he holds only these firms, $\Gamma(\alpha,E\sim\{\alpha,\beta\}) = 0$. A second example arises if $\alpha$ holds shares in all firms and there exists a "density function" $\gamma(\alpha,\beta) > 0$ such that $\Gamma(\alpha,E') = \int_{E'}\gamma(\alpha,\beta)d\mu(\beta)$, where $\mu$ is the Lebesgue measure. Then $\gamma(\alpha,\beta)$ can be interpreted as $\alpha$'s holding in firm $\beta$.

Using the notation just introduced, the entrepreneur's random income is

$$\tilde{W}_E(K(\alpha),B(\alpha),\Gamma(\alpha)) = A + r\{[M(\alpha) - \int_E M(\beta)\Gamma(\alpha,d\beta)] - [K(\alpha) - B(\alpha)]\} + \int_E \tilde{\pi}(K(\beta),B(\beta))\Gamma(\alpha,d\beta), \tag{6}$$

where, for each $\beta$, including $\alpha$, $\tilde{\pi}(K(\beta,B(\beta))$ is defined by (2) and $M(\beta) = N(K(\beta),B(\beta))$. If one recalls that our notation treats $\alpha$'s holding of shares in his own firm symmetrically with his holding of shares in other firms, it will be clear that (5) is the special case of (6) in which the entrepreneur retains only shares in his own firm.

Entrepreneur $\alpha$'s problem then is to choose $K(\alpha)$, $B(\alpha)$, and $\Gamma(\alpha)$ so as to maximize

$$Eu(\tilde{W}_E(K(\alpha),B(\alpha),\Gamma(\alpha)),\alpha). \tag{7}$$

In order to avoid the problems associated with bankruptcy, this choice is assumed to be made subject to the constraint that (6) be nonnegative with probability one. Since $g(K,\underline{x}) \equiv 0$, this will be true if and only if $(K(\alpha),B(\alpha),\Gamma(\alpha))$ satisfies

$$A + r\{M(\alpha) - \int_E [M(\beta) + B(\beta)]\Gamma(\alpha,d\beta) - [K(\alpha) - B(\alpha)]\} \geq 0. \tag{8}$$

Note that in solving this problem, $\alpha$ is assumed to recognize that the price $M(\alpha)$ of his firm will be related to its capital input and debt level by (4). He also takes the share price $M(\beta)$ and the decisions $\big(K(\beta),B(\beta)\big)$ of all other entrepreneurs $\beta \in E$ as given.

Capitalists face a decision problem which is analogous to the entrepreneurs'. Specifically, either capitalists can sell their capital for income, receiving $r$ income units for each capital unit sold, or capital can be exchanged for shares in firms at the competitive prices $M(\beta) =$

$N(K(\beta), B(\beta))$. When the portfolio $\Gamma(\alpha)$ is chosen, capitalist $\alpha$'s random income is

(9) $\tilde{W}_C(\Gamma(\alpha)) = A + r[1 - \int_E M(\beta)\Gamma(\alpha, d\beta)] + \int_E \tilde{\pi}(K(\beta), B(\beta))\Gamma(\alpha, d\beta)$

and he chooses $\Gamma(\alpha)$ to maximize

$$Eu(\tilde{W}_C(\Gamma(\alpha)), \alpha). \tag{10}$$

For capitalists, the problem of bankruptcy is avoided by assuming that $\Gamma(\alpha)$ is chosen subject to the constraint that (9) be nonnegative with probability one. This is true if and only if

$$A + r[1 - \int_E [M(\beta) + B(\beta)]\Gamma(\alpha, d\beta)] \geq 0. \tag{11}$$

Note that in (9) and (10) $\alpha$ is *not* included in $E$; thus, he must take as given the profit variables $\tilde{\pi}[K(\beta), B(\beta)]$ of *every* firm in which he may invest. This situation is to be contrasted with that represented by (6) and (7) in which $\alpha \in E$ and can make a simultaneous choice of $\tilde{\pi}(K(\alpha), B(\alpha))$ and $\Gamma(\alpha)$.

Individual $\alpha$ will be an entrepreneur if and only if

(12) $\max_{\substack{K(\alpha)\geq 0 \\ B(\alpha) \\ \Gamma(\alpha)\geq 0}} \mathrm{E}u(\tilde{W}_E(K(\alpha), B(\alpha), \Gamma(\alpha)), \alpha) \geq \max_{\Gamma(\alpha)\geq 0} Eu(\tilde{W}_C(\Gamma(\alpha)), \alpha)$;

he will be a capitalist if and only if the inequality in (12) is reversed. Having defined $E$ as the set of entrepreneurs, we will let $C$ be the set of capitalists.

*Definition 1. A* competitive stock market entrepreneurial equilibrium (CSMEE) *is a partition* $\{E, C\}$ *of* $[0,1]$, *a capital price* $r \in [0, +\infty)$, *and functions* $N$: $[0, +\infty) \times (-\infty, +\infty) \to (-\infty, +\infty)$, $K$: $E \to [0, +\infty)$, $B$: $E \to (-\infty, +\infty)$, $\Gamma$: $[0,1] \to$ *the set of nonnegative measures on the Lebesgue measurable subsets of E, such that* (i) (12) *holds for each* $\alpha \in E$ *and* (12) *is reversed for each* $\alpha \in C$, (ii) $(K(\alpha), B(\alpha), \Gamma(\alpha))$ *maximizes* (7) *subject to* (8) *for each* $\alpha \in E$, (iii) $\Gamma(\alpha)$ *maximizes* (10) *subject to* (11) *for each* $\alpha \in C$, (iv) $\int_0^1 \Gamma(\alpha, E')\mu(d\alpha) = \mu(E')$ *for each subset* $E'$ *of* $E$, *and* (v) $\int_E K(\beta)\mu(d\beta) = \mu(C)$.

In this definition, condition (iv) expresses the equality of supply and demand in the market for shares for firms. If, for example, there is a density function $\gamma(\alpha,\beta)$ such that, for each $\alpha$ and $E'$,

$$\Gamma(\alpha, E') = \int_{E'} \gamma(\alpha,\beta)\mu(d\beta),$$

then, the supply equals demand condition

$$\int_0^1 \gamma(\alpha, \beta)\mu(d\alpha) = 1, \qquad \text{for each } \beta,$$

implies that

$$\int_0^1 \int_{E'} \gamma(\alpha, \beta)\mu(d\beta) = \mu(E')$$

as required by (iv).

Condition (v) in definition 1 asserts that capital supply equals capital demand.

## 5.3 The Function $N$ and the Modigliani-Miller Theorem

In this section we will use adaptations of familiar arguments to obtain restrictions on the function $N$ which are necessary for the existence of an equilibrium. These conditions will on the one hand facilitate the interpretation of the function $N$ and on the other hand simplify the following discussion of existence of equilibrium. In this discussion, there is one particular $N$ function which will play a crucial and classical role. This function will be denoted by $N_C$ and is defined by

$$N_C(K,B) = 1 + K - B.$$

When this function describes the market valuation of firms employing any operating capital level $K$ of which any amount $B$ is raised as debt, the equilibrium exhibits two classical properties. First, it is impossible to earn arbitrage profits by setting up a firm and then selling all of the shares at the market price. In fact, this type of arbitrage will be unprofitable when and only when $N(K,B) \leq N_C(K,B)$. For if a firm is set up which employs $K$ units of operating capital and issues debt to raise $B$ of these capital units, the entrepreneur will have invested $1 + K - B$ units of equity capital. If $N(K,B) \leq N_C(K,B)$, the market value (in capital terms) of the firm created with this investment is no larger than $1 + K - B$. Thus, an investment of $1 + K - B$ units of equity capital has resulted in the creation of an asset with a capital value smaller than or equal to the investment. The result is no profits. Profits are possible, however, if $N(K,B)$ exceeds $N_C(K,B)$. For in this case an investment of $1 + K - B$ units of equity capital creates a firm with a market value in excess of $1 + K - B$.

The second well-known result implied by the fact that $N(K,B) = N_C(K,B)$ for all $(K,B)$ is the Modigliani-Miller theorem, which has two important consequences. The first is that $N(K,B) + B$, the value of the firm (in capital terms), is constant for all firms employing the same level of operating capital $K$. In fact,

$$N_C(K,B) + B = 1 + K.$$

This equality asserts that the capital value of the firm equals the total capital invested in it, a simple restatement of the no arbitrage profits condition just discussed. The second consequence of Modigliani and Miller's theorem is that all individuals—the entrepreneur as well as all potential and actual investors—are indifferent about $B$, the debt financed capital level. Since $B$ is the sole financial decision made by the entrepreneur on the firm's account, the firm's financial policy is irrelevant to all individuals. This universal indifference to $B$ is a simple consequence of the expressions for $W_C$ and $W_E$ which emerge when $1 + K - B$ is substituted in (6) and (9). In particular, when $N(K,B) = N_C(K,B)$ for all $(K,B)$, then

$$(13)\quad \tilde{W}_E(K,B,\Gamma) = \tilde{W}_C(\Gamma) = A + r[1 - \int_E \Gamma(d\beta)] + \int_E [g(K(\beta),\tilde{x}) - rK(\beta)]\Gamma(d\beta) = A + r[1 - \int_E [1 + K(\beta)]\Gamma(d\beta)] + \int_E g(K(\beta),\tilde{x})\Gamma(d\beta)$$

for all $(K,B,\Gamma)$. Notice that for no firm $\beta$ does $B(\beta)$ appear in the expressions given for $\tilde{W}_E(K,B,\Gamma)$ and $\tilde{W}_C(\Gamma)$.

Another important consequence of $N(K,B) = N_C(K,B)$ is the equality of $\tilde{W}_E(K,B,\Gamma)$ and $\tilde{W}_C(\Gamma)$ expressed in (13). In writing this equality, it was implicitly assumed that $K \in \{K(\beta)\colon \beta \in E\}$, which means, in effect, that the set $\{K(\beta)\colon \beta \in E\}$, and therefore the availability of investment opportunities, is the same for both entrepreneurs and capitalists. This equality implies that once $\{K(\beta)\colon \beta \in E\}$ is determined, all individuals are indifferent between being entrepreneurs and capitalists. Of course entrepreneurs are able to choose $K$ and $B$ for their own firms and thereby influence $\{K(\beta)\colon \beta \in E\}$, the set of investment opportunities. Furthermore, the $(K,B)$ choice is made simultaneously with the choice of a portfolio. This is a choice not available to capitalists as long as they remain capitalists. Because entrepreneurs have an option not open to capitalists, it appears that, in spite of (13), some individuals may strictly prefer the former role to the latter. But capitalists who are not satisfied with the investment opportunities available from existing firms can always choose to become entrepreneurs and thereby create new firms in which to invest. Thus, not only are $K$, $B$, and $\Gamma$ chosen simultaneously by entrepreneurs, but all individuals choose $\Gamma$ at the same time that they decide whether to be entrepreneurs or capitalists. Any individual who is a capitalist in equilibrium has chosen not to exercise the option to become an entrepreneur. This choice indicates satisfaction with the investment opportunities made available by other entrepreneurs. Thus, in an equilibrium characterized by $N(K,B) = N_C(K,B)$, the entrepreneur's ability to make a $(K,B)$ choice not available from other entrepreneurs is superfluous, and all individuals are indifferent between being entrepreneurs and capitalists.

The equilibria on which this paper focuses are those in which $N(K,B) = N_C(K,B)$ for all possible $(K,B)$ choices. There are, however, other equilibria in which this equality may fail to hold at some $(K,B)$ choices. But even in these other equilibria $N$ and $N_C$ are closely related. This relationship will be described by two propositions which are stated and proved in the remainder of the present section. The first proposition asserts that $N_C$ serves as an upper bound above which the equilibrium $N$ can *never* rise. This proposition is proved by demonstrating that if $N$ rises above $N_C$ for any $(K,B)$ choice, then every individual will, in preference to remaining a capitalist, create a firm employing $K$ units of operating capital and raising $B$ of these capital units with debt. As noted above, this type of arbitrage operation is profitable if and only if $N(K,B)$ exceeds $N_C(K,B)$. When all individuals attempt to exploit these profitable arbitrage possibilities, all capital is used as entrepreneurial capital; none remains to satisfy the demand for operating capital. Thus, equilibrium in the capital market is inconsistent with $N(K,B) > N_C(K,B)$ at any possible $(K,B)$ choice.

The second proposition is somewhat weaker. It asserts that at the equilibrium $(K,B)$ levels, $N(K,B)$, in fact, equals $N_C(K,B)$. To be perfectly correct, it can only be said that for almost all entrepreneurs $\alpha$, $N(K(\alpha),B(\alpha)) = N_C(K(\alpha),B(\alpha))$. We will now state and prove these results and then return to a discussion of the difficulties created by the fact that, in some equilibria, $N$ may differ from $N_C$.

*Proposition 1 (Modigliani-Miller). In a* CSMEE,

$$N(K,B) \leqq 1 + K - B \tag{14}$$

*for all possible* $(K,B)$ *choices (not just the equilibrium choice).*

*Proof.* If (14) fails because at some $(K,B)$

$$N(K,B) > 1 + K - B\,, \tag{15}$$

then (15), (6), and (9) imply that, for any $\Gamma(\alpha)$,

$$\begin{aligned}\tilde{W}_C(\Gamma(\alpha)) &= A + r[1 - \textstyle\int_E M(\beta)\Gamma(\alpha,d\beta)] + \int_E \tilde{\pi}(K(\beta),B(\beta))\Gamma(\alpha,d\beta)\\ &< A + r\{[N(K,B) - \textstyle\int_E M(\beta)\Gamma(\alpha,d\beta)] - [K - B]\}\\ &\quad + \textstyle\int_E \tilde{\pi}(K(\beta),B(\beta))\Gamma(\alpha,d\beta) = \tilde{W}_E(K,B,\Gamma(\alpha))\,.\end{aligned} \tag{16}$$

Thus, the random income achievable by becoming an entrepreneur is sure to exceed the random income obtainable as a capitalist for every choice of $\Gamma(\alpha)$. In making this statement, it was assumed that $\alpha$ chose the same portfolio after he became an entrepreneur that he chose before becoming an entrepreneur. This is clearly possible for an entrepreneur even though the existence of his own firm creates a new investment opportunity, i.e., one which was not available when he was a capitalist. It is possible because both as an entrepreneur and as a capitalist, $\alpha$ takes the capital input choices of other firms as given and because the entrepreneur can always sell the one share in his own firm for $N(K,B)$ to obtain the portfolio. Inequality (16) asserts that $\alpha$ will be sure to have more income after becoming an entrepreneur, employing $K$, issuing $rB$ as debt on the firm's account, and then selling the firm to buy $\Gamma(\alpha)$. Because of this, *every* individual will prefer to be an entrepreneur rather than to remain a capitalist, but then $\mu(C) = 0$ and condition (v) in the definition of an equilibrium must fail to hold. Thus, $N$ could not be an equilibrium price if (15) holds for some $(K,B)$. QED.

As remarked earlier, proposition 1 implies that, for all $(K,B)$,

$$N(K,B) \leq N_C(K,B).$$

We can now show that for those $(K,B)$ which appear in the equilibrium the upper bound becomes effective. The proof proceeds by demonstrating that no entrepreneur will ever choose a $(K,B)$ at which $N(K,B)$ is actually less than $N_C(K,B)$. He will always prefer to be a capitalist instead. A complication arises here which did not appear in the proof of proposition 1. The problem occurs because an individual $\alpha$ who would choose $(K(\alpha), B(\alpha))$ if he were an entrepreneur may be unique in preferring that choice. If he chooses to be a capitalist, he eliminates an important investment opportunity. This problem is avoided if assumption A1 holds. Under this assumption, no individual $\alpha$ is ever unique in his preference for a specific $(K(\alpha), B(\alpha))$. Thus, no individual who chooses to be a capitalist eliminates an investment opportunity which other entrepreneurs will not provide for him.

Since $N(K(\alpha), B(\alpha)) = N_C(K(\alpha), B(\alpha))$ for almost all entrepreneurs $\alpha$, (13) holds and almost all individuals are indifferent between being workers and capitalists.

*Proposition 2 (Modigliani-Miller). Suppose that Assumption* A1 *holds. In a* CSMEE,

$$N(K(\alpha), B(\alpha)) = 1 + K(\alpha) - B(\alpha) \tag{17}$$

*for almost all (Lebesgue measure)* $\alpha \in E$. *Furthermore, in equilibrium almost all individuals of the same type have the same expected utility; i.e., if* $u(\cdot,\alpha) = u(\cdot,\alpha')$ *and* $\alpha \in E$ *while* $\alpha' \in C$, *then*

$$\max_{\Gamma(\alpha')} Eu(\tilde{W}_C(\Gamma(\alpha')),\alpha') = \max_{\substack{\Gamma(\alpha)\\K(\alpha)\\B(\alpha)}} Eu(\tilde{W}_E(K(\alpha),B(\alpha),\Gamma(\alpha)),\alpha) \tag{18}$$

*or if* $u(\cdot,\alpha) = u(\cdot,\alpha')$ *and* $\alpha$ *and* $\alpha'$ *are in* $E$, *then*

$$\max_{\substack{\Gamma(\alpha)\\K(\alpha)\\B(\alpha)}} Eu(\tilde{W}_E(K(\alpha),B(\alpha),\Gamma(\alpha)),\alpha) = \max_{\substack{\Gamma(\alpha')\\K(\alpha')\\B(\alpha')}} Eu(\tilde{W}_E(K(\alpha'),B(\alpha'),\Gamma(\alpha')),\alpha'). \tag{19}$$

*Thus, condition* (12) *must hold with an equality for all individuals who will therefore be indifferent between being entrepreneurs and capitalists.*

The proof of this proposition is contained in appendix 1.

Because $N = N_C$ only at equilibrium choices for $(K,B)$, it is not always true that investors are indifferent about a firm's $B$ choice. If a significant number of firms employ $K(\alpha)$ units of operating capital and choose to raise $B(\alpha)$ capital units through debt, then

$$N(K(\alpha), B(\alpha)) = 1 + K(\alpha) - B(\alpha).$$

But there may be some other $B$ level at which

$$N(K(\alpha), B) < 1 + K(\alpha) - B.$$

In that case, $B(\alpha)$ is definitely preferred to $B$. If, however, there is also a significant number of firms which choose $(K', B')$ with $K' = K(\alpha)$, then

$$N(K(\alpha), B(\alpha)) + B(\alpha) = N(K(\alpha), B') + B' = 1 + K(\alpha)$$

and all individuals are indifferent between the financial decisions $B(\alpha)$ and $B'$.

As mentioned above, the analysis to follow will concentrate on the case in which

$$N(K,B) = N_C(K,B) \tag{20}$$

for all $(K,B)$. We will demonstrate the existence of such equilibria. We will also show in section 5.5 that the equilibria in which (20) always holds are efficient in the restricted sense of the term introduced by Diamond (1967). It will furthermore be seen in section 5.7 that there may exist many other equilibria in which (20) does not always hold and that many of these equilibria may be inefficient. These facts underline the central role of the function $N_C$ as well as the importance of providing a satisfactory interpretation for the function $N_C$ to justify the assumption that $N = N_C$.

For $(K,B)$ levels which are observed in a particular equilibrium, $N(K,B)$ is easily interpreted as the observed price of shares of firms employing $K$ operating capital units and raising $B$ of these units as debt. If, in equilibrium, no firm chooses $(K,B)$, then $N(K,B)$ must be interpreted as the price individuals expect to prevail if such firms are introduced. It might be thought that if the expectations $N$ are to be "rational" or self-fulfilling, then $N$ must equal $N_C$ everywhere. Whether this is true depends on the sense in which the expectations are rational. Suppose, first, that $N$ is rational in the sense that if $N(K,B)$ is expected to be the price of shares in a firm which chooses $(K,B)$ and if a significant number of entrepreneurs (a set of positive Lebesgue measure) do, in fact, choose $(K,B)$, then the actual equilibrium price of their firms is $N(K,B)$. This is a rather weak form of the rational expectations hypothesis, since it only requires actual fulfillment of expectations at $(K,B)$ levels observed in equilibrium. For $(K,B)$ pairs not chosen by a significant number of firms, the expectation $N(K,B)$ is never confirmed by observation because shares of firms choosing $(K,B)$ are never exchanged at $N(K,B)$. But the expectation $N(K,B)$ is never refuted either, since shares of these firms are never exchanged at prices other than $N(K,B)$.

Using this rational expectation interpretation of $N$, proposition 1 asserts that an expectation of $N(K,B) > N_C(K,B)$ can never be fulfilled. Proposition 2 asserts that for $(K,B)$ choices observed in equilibrium, the only $N$ which is rational is $N(K,B) = N_C(K,B)$. In view of these propositions, an expectation $N(K,B) < N_C(K,B)$ can be rational in the sense just described if no significant number of firms choose to employ $K$ units of operating capital and raise $B$ of these units with debt. And, in fact, an equilibrium in which (20) fails to hold at some $(K,B)$ can be interpreted as a case in which this situation arises. Thus, the rational expectations interpretation just presented is not sufficient to justify the assumption that (20) holds everywhere.

There is a stronger form of the rational expectations hypothesis which can be used to justify the assumption that (20) holds everywhere. In particular, we could interpret rationality to require that the equilibrium expectation $N(K,B)$ actually be confirmed, or at least confirmable in some sense, at all $(K,B)$ levels. We could, for example, argue that individuals who understand the economy and its operation will, in essence, know proposition 2; i.e., they will know that if a $(K,B)$ is going to be observed in any equilibrium, it must be true that (20) holds at $(K,B)$. Sophisticated individuals will thus know that the only price expectations $N(K,B)$ that can ever be confirmed by observation are $N(K,B) = N_C(K,B)$ and that for other expectations the most that can be said is that they can never be refuted by observation. If we ask that rational expectations have this potential for confirmation, then $N(K,B) = N_C(K,B)$ is the unique rational price expectations function.

## 5.4 Properties of the Equilibria

The purpose of this section is to describe the properties of the entrepreneurial stock market equilibria and show that such equilibria exist. As mentioned above, this is done under the assumption that (20) holds at all $(K,B)$ levels. In this case, (13) implies that any entrepreneur $\alpha$ can be regarded as maximizing the special case of (7) in which $M(\beta) = 1$ and $K(\beta) = B(\beta)$ for all $\beta$, including $\alpha$, in $E$. When (20) holds for all $(K,B)$, (13) also implies that capitalists maximize the special case of (10) which is also obtained by setting $M(\beta) = 1$ and $K(\beta) = B(\beta)$ for all $\beta \in E$. Thus, entrepreneur $\alpha$ chooses $K(\alpha) \geq 0$ and $\Gamma(\alpha) \geq 0$ subject to (8) (with $B(\beta) = K(\beta)$ for all $\beta \in E$) so as to maximize

(21) $Eu\left(A + r[1 - \int_E [1 + K(\beta)]\Gamma(\alpha, d\beta)] + \int_E g(K(\beta),\tilde{x})\Gamma(\alpha, d\beta), \alpha\right)$.

If $\alpha$ is a capitalist, he takes $K(\beta)$ as given for all $\beta \in E$ and chooses $\Gamma(\alpha) \geqq 0$ to maximize (21) subject to (11) (with $B(\beta) = K(\beta)$ for all $\beta \in E$).

Using this simplification, we first show that the decision problems of capitalists and entrepreneurs can be substantially simplified. Specifically we show that no entrepreneur $\alpha$ will ever hold shares in a firm which employs capital in an amount which differs from his own optimal capital demand $K(\alpha)$. Furthermore, because the $x$'s are the same for all firms, each entrepreneur considers a partial share in his own firm as a perfect substitute for a partial share in any other firm which employs capital at the level which is optimal for him. As a consequence of these results, we will be able to assume that the entrepreneur's problem is to choose $K(\alpha)$ and $\gamma(\alpha)$ to maximize the expected utility of his income which is related to these decisions by the special case of (5) in which $B(\alpha) = K(\alpha)$. Formally, entrepreneur $\alpha$'s problem is then to choose $K(\alpha) \geq 0$ and $\gamma(\alpha) \geq 0$ so as to maximize

$$\begin{aligned} (22) \qquad H^{\alpha}(K(\alpha),\gamma(\alpha)) = Eu(A &+ r[1 - \gamma(\alpha)] \\ &+ \gamma(\alpha)[g(K(\alpha),\tilde{x}) - rK(\alpha)], \alpha). \end{aligned}$$

The special case of (8) which restricts the entrepreneur's $(K(\alpha),\gamma(\alpha))$ choice is

$$(23) \qquad A + r[1 - \gamma(\alpha)(1 + K(\alpha))] \geq 0.$$

We will denote the solution to this problem by $[\hat{K}(\alpha), \hat{\gamma}(\alpha)]$.

By similar arguments, analogous results can be established for the capitalist. That is, we can show that if there is an existing firm which is operated in the same way that a particular capitalist $\alpha$ would run it—i.e., which chooses to employ the same amount of capital $\alpha$ would employ if he were to become an entrepreneur—then $\alpha$ will hold shares only in that firm and others run the same way. Furthermore, for $\alpha$, shares in all firms operated in this way are perfect substitutes. Because of these results, we will be able to assume that the capitalist's problem is to choose $\gamma(\alpha)$ so as to maximize his expected utility when his income is related to this decision by the special case of (5) in which $B(\alpha) = K(\alpha)$ and $K(\alpha)$ is the capital level which he would choose if he were an entrepreneur. Formally, then, the capitalist chooses $\gamma(\alpha) \geq 0$ to maximize (22) where $K(\alpha) = \hat{K}(\alpha)$, and this choice is made subject to the constraint (23).

*Lemma 1. Assume that N satisfies* (20). *If* $K(\alpha) = K(\alpha')$, *then any investor* $\beta$, *whether he is an entrepreneur or a capitalist, is indifferent when choosing between portfolios* $\Gamma(\beta)$ *for which* $\Gamma(\beta,\{\alpha'\}) + \Gamma(\beta,\{\alpha\}) = \delta$, *where* $\delta$ *is some positive constant.*

*Proof.* When $K(\alpha) = K(\alpha') = K$ and $\Gamma(\beta,\{\alpha'\}) + \Gamma(\beta,\{\alpha\}) = \delta$,

$$\Gamma(\beta,\{\alpha\})[g(K(\alpha),\tilde{x}) - rK(\alpha)] + \Gamma(\beta,\{\alpha'\})[g(K(\alpha'),\tilde{x}) - rK(\alpha')]$$
$$= \delta(g(K,\tilde{x}) - rK].$$

Thus, each portfolio satisfying the hypothesis of the proposition yields a distribution of returns which is the same as the distribution of returns obtained by holding $\delta$ shares in either firm $\alpha$ or firm $\alpha'$. QED.

*Proposition 3. Assume that N satisfies* (20), *and that* A.1 *holds. Let* $(\hat{\hat{K}}(\alpha),\hat{\Gamma}(\alpha))$ *maximize* (7) *subject to* $K(\alpha) \geq 0, \Gamma(\alpha) \geq 0$, *and* (8). *Let* $I_\alpha = \{\alpha' : K(\alpha') = \hat{\hat{K}}(\alpha)\}$. *Then* $\hat{\hat{K}}(\alpha) = \hat{K}(\alpha)$, $\hat{\hat{\Gamma}}(\alpha, I_\alpha) = \hat{\gamma}(\alpha)$, *and* $\hat{\hat{\Gamma}}(\alpha, E \sim I_\alpha) = 0$ *where* $(\hat{K}(\alpha), \hat{\gamma}(\alpha))$ *maximizes* (22) *subject to* $K(\alpha) \geq 0, \gamma(\alpha) \geq 0$, *and* (23). *Thus, an entrepreneur invests only in firms which are run as he runs his own firm and he chooses* $\gamma(\alpha) \geqq 0$ *and* $K(\alpha) \geqq 0$ *subject to* (23) *to maximize* (22). *(If in equilibrium* $\hat{\gamma}(\alpha) > 1$, *he must be interpreted as investing in at least one other firm which employs* $\hat{K}(\alpha)$.)

*For any* $\alpha \in C$, *let* $I_\alpha = \{\alpha' \in E\colon K(\alpha') = \hat{K}(\alpha)\}$, *where* $\hat{K}(\alpha)$ *and* $\hat{\gamma}(\alpha)$ *are assumed to maximize* (22) *subject to* (23) *and the nonnegativity constraints. In equilibrium,* $I_\alpha$ *is not empty; i.e., there is some entrepreneur who runs his firm as* $\alpha$ *would if* $\alpha$ *were an entrepreneur*; $\Gamma(\alpha, I_\alpha) = \hat{\gamma}(\alpha)$ *and* $\Gamma(\alpha, E \sim I_\alpha) = 0$.

*Remarks.* Proposition 3 can be interpreted as asserting that the assumption of a concave production function and the assumption that $\tilde{x}$ is the same for all firms together imply that all of the possibilities for diversification available in the stock market are dominated by the investment opportunities available through the production function. However, in order to exploit these opportunities, it may be necessary to retain only a partial share of the output from the production function or, if $\gamma$ exceeds one, to receive a multiple of the output from the production function. The latter can be achieved by replication of a firm.

The mathematical result which is given a stock market interpretation in proposition 3 was obtained earlier and given a closely related sharecropping interpretation by Newbery (1977).

We are indebted to Sanford Grossman for suggesting the idea of the simple proof which appears below. Note that this proof does not require differentiability of $u$ or $g$. Our original proof was substantially longer and did use arguments which depended on differentiability.

It should also be emphasized that the assumption that $\tilde{x}$ is the same for every firm is crucial to the proof of this result. If $\tilde{x}$ were a different random variable for each firm, individual investors would find it advantageous to hold diversified portfolios; i.e., the stock market would augment the diversification possibilities available with the production function.

*Proof.* Jensen's inequality and the strict concavity of $g$ for each $x \neq \underline{x}$ imply that for any portfolio $\Gamma(\alpha)$ and for each $x \neq \underline{x}$,

$$(24)\quad \Gamma(\alpha,E)\left[g\left(\frac{\int_E K(\beta)\Gamma(\alpha,d\beta)}{\Gamma(\alpha,E)},x\right)-r\left(\frac{\int_E K(\beta)\Gamma(\alpha,d\beta)}{\Gamma(\alpha,E)}\right)\right]$$
$$>\int_E [g(K(\beta),x)-rK(\beta)]\Gamma(\alpha,d\beta).$$

Thus, any entrepreneur or capitalist can obtain higher returns in each state $x$ by becoming an entrepreneur employing

$$\frac{\int_E K(\beta)\,\Gamma(\alpha,d\beta)}{\Gamma(\alpha,E)}$$

units of operating capital and holding only $\Gamma(\alpha,E)$ shares in his own firm than he can by holding any diversified portfolio $\Gamma(\alpha)$. Note that the cost of $\Gamma(\alpha,E)$ shares in his own firm is the same as the cost of the portfolio $\Gamma(\alpha)$. QED.

Proposition 3 reduced the entrepreneur's problem to one of maximizing, by his $(K(\alpha),\gamma(\alpha))$ choice, (22). It also showed that the capitalist is faced with a simple choice of $\gamma(\alpha)$ for a fixed $K(\alpha)$ level. The next lemma shows that if solutions to both of these problems exist, they are unique.

*Lemma 2. Assume that N satisfies* (20) *and that u is strictly concave. If there exists a* $(\hat{K}(\alpha),\hat{\gamma}(\alpha))$ *choice which maximizes* (22) *and for which* $\hat{K}(\alpha)>0$, *it is unique.*

*Remark.* In standard proofs, uniqueness follows from strict concavity of the criterion function. In the present context, this approach is inapplicable because $H^\alpha$ is not concave in $(K(\alpha),\gamma(\alpha))$. It is, however, strictly concave in $K(\alpha)$ and in $\gamma(\alpha)$. These facts can be easily verified by differentiation when $u$ and $g$ are twice differentiable and when $u$ is strictly concave. If $u''<0$, the uniqueness of $(\hat{K}(\alpha),\hat{\gamma}(\alpha))$ can nevertheless be demonstrated by showing that, in spite of the nonconcavity of $H^\alpha$, the second-order sufficient conditions for a maximum are satisfied at any $(\hat{K}(\alpha),\hat{\gamma}(\alpha))$ which satisfy the first-order conditions. Functions which have this property are referred to as strictly pseudoconcave and have unique maxima. The proof of pseudoconcavity of $H^\alpha$ requires that $H^\alpha$ be twice differentiable. It is, however, possible, using the proof which follows, to prove uniqueness of $(\hat{K}(\alpha),\hat{\gamma}(\alpha))$ directly and simply without assuming differentiability.

When $u$ is concave but not necessarily strictly concave, a similar argument implies that $\hat{K}(\alpha)$ is unique. In this case, $\hat{\gamma}(\alpha)$ is not necessarily unique.

*Proof.* Suppose that there exist two maximizing choices $(\hat{K}_1(\alpha),\hat{\gamma}_1(\alpha))$ and $\left(\hat{K}_2(\alpha),\hat{\gamma}_2(\alpha)\right)$ with, say, $\hat{K}_1(\alpha)>0$. Then, as in the proof of proposition 2, the strict concavity of $g$ for each $x\neq \underline{x}$, implies that for any $t\in(0,1)$ and for any $x\neq\underline{x}$,

$$\gamma_t(\alpha)\left[g(K_t(\alpha),x)-rK_t(\alpha)\right]>t\hat{\gamma}_1(\alpha)[g(\hat{K}_1(\alpha),x)-r\hat{K}_1(\alpha)]$$
$$+(1-t)\hat{\gamma}_2(\alpha)[g(\hat{K}_2(\alpha),x)-r\hat{K}_2(\alpha)],$$

where

$$\gamma_t(\alpha) = t\hat{\gamma}_1(\alpha) + (1-t)\hat{\gamma}_2(\alpha)$$

and

$$K_t(\alpha) = \frac{t\hat{\gamma}_1(\alpha)\hat{K}_1(\alpha) + (1-t)\hat{\gamma}_2(\alpha)\hat{K}_2(\alpha)}{t\hat{\gamma}_1(\alpha) + (1-t)\hat{\gamma}_2(\alpha)}.$$

Thus,

$$\begin{aligned}
&Eu\big(A + r[1-\gamma_t(\alpha)] + \gamma_t(\alpha)[g(K_t(\alpha),\tilde{x}) - rK_t(\alpha)],\alpha\big) \\
(25)\qquad &\quad > Eu\big(A + r[1 - t\hat{\gamma}_1(\alpha) - (1-t)\hat{\gamma}_2(\alpha)] \\
&+ t\hat{\gamma}_1(\alpha)[g(\hat{K}_1(\alpha),\tilde{x}) - r\hat{K}_1(\alpha)] + (1-t)\hat{\gamma}_2(\alpha)[g(\hat{K}_2(\alpha),\tilde{x}) - r\hat{K}_2(\alpha)],\alpha\big)
\end{aligned}$$

because of the monotonicity of $u$. Now the strict concavity of $u$ implies that

$$\begin{aligned}
&Eu\big(A + r[1 - t\hat{\gamma}_1(\alpha) - (1-t)\hat{\gamma}_2(\alpha)] + t\hat{\gamma}_1(\alpha)[g(\hat{K}_1(\alpha),\tilde{x}) - r\hat{K}_1(\alpha)] \\
&\qquad + (1-t)\hat{\gamma}_2(\alpha)[g(\hat{K}_2(\alpha),\tilde{x}) - r\hat{K}_2(\alpha)],\alpha\big) \\
(26)\quad &> tEu\big(A + r[1-\hat{\gamma}_1(\alpha)] + \hat{\gamma}_1(\alpha)[g(\hat{K}_1(\alpha),\tilde{x}) - r\hat{K}_1(\alpha)],\alpha\big) \\
&\quad + (1-t)Eu\big(A + r[1-\hat{\gamma}_2(\alpha)] + \hat{\gamma}_2(\alpha)[g(\hat{K}_2(\alpha),\tilde{x}) - r\hat{K}_2(\alpha)],\alpha\big) \\
&\quad = H^\alpha(\hat{K}_1(\alpha),\hat{\gamma}_1(\alpha)) = H^\alpha(\hat{K}_2(\alpha),\hat{\gamma}_2(\alpha)).
\end{aligned}$$

Combining (25) and (26) implies that

$$H^\alpha(K_t(\alpha),\gamma_t(\alpha)) > H^\alpha(\hat{K}_1(\alpha),\hat{\gamma}_1(\alpha)) = H^\alpha(\hat{K}_2(\alpha),\hat{\gamma}_2(\alpha)).$$

Thus, neither $(\hat{K}_1(\alpha),\hat{\gamma}_1(\alpha))$ nor $(\hat{K}_2(\alpha),\hat{\gamma}_2(\alpha))$ can maximize $H^\alpha$, a contradiction. QED.

Lemmas 1 and 2 and propositions 1–3 can now be used with assumption A1 and the assumption that $u$ is concave to simplify substantially the structure of a CSMEE. The simplifications are described in proposition 4, which follows. Before stating the proposition, it is useful to recall that $\mu_i$ represents the measure of the set of individuals of type $i$ who have the common utility function $u(\bullet,i)$. If $i = i(\alpha)$, we define $H^i = H^\alpha$.

*Proposition 4. Suppose that* (20) *holds at all* $(K,B)$ *and that assumption* A1 *holds. Then in a* CSMEE *there are*

$$\nu_i = \gamma_i\mu_i \tag{27}$$

*entrepreneurs who operate firms which employ* $K_i$ *units of capital, where* $K_i$ *is the capital demand which together with* $\gamma_i$ *maximizes the function* $H^i$ *defined by* (22) *for type i individuals. Since* $\gamma_i$ *is the demand for shares by each of the* $\mu_i$ *type i individuals,* (27) *expresses the equality of supply and demand as required by* (iv) *of definition 1. If* $\gamma_i < 1$, *then* $\nu_i < \mu_i$ *and there are fewer firms of type i than individuals of type i. All individuals of type i* *are indifferent between being capitalists and entrepreneurs. Thus, any $\nu_i$ of the $\mu_i$ type i individuals will be entrepreneurs. The remaining $\mu_i - \nu_i$ type i individuals will be capitalists. If $\gamma_i > 1$, then $\nu_i > \mu_i$ and there must be more firms of type i than there are individuals of type i. The entrepreneurs for $\nu_i - \mu_i$ of these firms can be of some type $j \neq i$. These individuals of type j can become entrepreneurs, choose $K_i$ as the capital level for their firm, and sell their one unit share in the firm. If they set $B_i = K_i$, then they will receive* 1 *capital unit for the firm and they will be in the same position as a capitalist of type j. They will then hold $\gamma_j$ shares in a firm run by some entrepreneur of type j.*

*The condition that capital supply equals capital demand is*

$$\sum_{i=1}^{n} \nu_i K_i = 1 - \sum_{i=1}^{n} \nu_i \tag{28}$$

*or*

$$\sum_{i=1}^{n} \nu_i(1 + K_i) = 1, \tag{29}$$

*which becomes*

$$\sum_{i=1}^{n} \mu_i \gamma_i (1 + K_i) = 1 \tag{30}$$

*when* (27) *is substituted. Because of* (30), $\gamma_i$ *cannot exceed one for all i.*

The proposition follows immediately from lemmas 1 and 2 and propositions 1–3. The following corollary is also immediate.

*Corollary to proposition 4. If* (20) *holds at all* $(K,B)$ *and assumption* A1 *holds with* $n = 1$, *then in a* CSMEE,

$$\nu_1 = \hat{\gamma}_1 = \frac{1}{\hat{K}_1 + 1}. \tag{31}$$

Because of proposition 4, a vector $\langle\{(\hat{K}_i, \hat{\gamma}_i)\}_{i=1}^{n}, r\rangle$ can be identified with a CSMEE if, for each $i$, $(\hat{K}_i, \hat{\gamma}_i)$ maximizes $H^i$ and if $\hat{\gamma}_i$ and $\hat{K}_i$ satisfy (30).

This simplification will prove to be essential in the demonstration that an equilibrium exists. Another essential step in this demonstration is a transformation of each individual's maximization problem. This transformation serves two purposes. On one hand, it reduces the individual maximization problem to a form in which the existence of a solution is more easily obtained. On the other, it makes it possible to reduce the problem of finding a CSMEE to one of finding a competitive equilibrium in a simple two-good pure trade economy. These reductions in turn permit the application of familiar arguments to obtain an equilibrium.

An appealing feature of the transformation of the individual maximization problem and of the equilibrium is the alternative interpretation of the CSMEE which results. In the present paper we will pause only briefly to discuss this alternative interpretation. An extended discussion is contained in a subsequent paper (Kihlstrom and Laffont 1980).

The first part of the transformation referred to is accomplished by letting

$$C_i = \gamma_i[1 + K_i] \tag{32}$$

and defining a function $F^i$ by

$$F^i(K_i, C_i) \equiv Eu\left(A + r[1 - C_i] + C_i \frac{g(K_i,\tilde{x})}{[1 + K_i]}, i\right). \tag{33}$$

It is easily shown that if $(\hat{K}_i, \hat{\gamma}_i)$ maximizes $H^i$ subject to $K_i \geq 0$, $\gamma_i \geq 0$, and (23), then $\hat{K}_i$ and

$$\hat{C}_i = \hat{\gamma}_i[1 + \hat{K}_i] \tag{34}$$

maximize $F^i$ subject to $K_i \geq 0$, $C_i \geq 0$, and

$$A + r[1 - C_i] \geq 0. \tag{35}$$

Similarly, if $(\hat{K}_i, \hat{C}_i)$ maximize $F^i$ subject to $K_i \geq 0$, $C_i \geq 0$, and (35), then $\hat{K}_i$ and

$$\hat{\gamma}_i = \frac{\hat{C}_i}{[1 + \hat{K}_i]} \tag{36}$$

maximize $H^i$ subject to $K_i \geq 0$, $\gamma_i \geq 0$, and (23). It is also easily shown that the uniqueness of $(\hat{K}_i, \hat{C}_i)$ is equivalent to the uniqueness of $(\hat{K}_i, \hat{\gamma}_i)$.

The function $F^i$ and the variable $C_i$ can be interpreted by assuming that $B_i = 0$, i.e., by assuming that all capital is equity capital and that equity capital can be supplied by either the entrepreneur or others. In that case, all of the firm's capital, $[1 + K_i]$, is obtained in return for equity. The amount $C_i$ can be interpreted as the equity capital supplied by each individual of type $i$ to firms of type $i$, i.e., those employing $K_i$. When $C_i$ is related to $\gamma_i$ and $K_i$ by (32), $i$ will receive a share of $g(K_i,\tilde{x})$ which equals

$$\gamma_i = \frac{C_i}{1 + K_i}. \tag{37}$$

Thus, the income $i$ receives from firms of type $i$ in state $x$ is

$$C_i \frac{g(K_i,x)}{[1 + K_i]}. \tag{38}$$

Now if individuals of type $i$ decide to supply an amount $C_i$ of equity capital which is less than the one unit of capital with which they are endowed, they will have $(1 - C_i)r$ capital units that can be sold at the price $r$ to individuals of other types. If $C_i$ is chosen to exceed the one-unit capital endowment, then $(C_i - 1)$ capital units will have to be purchased at the price $r$ from individuals of other types. In either case, $(1 - C_i)r$ units of income will be added to the share (38) received from firms of type $i$. Of course, when $C_i$ exceeds one, this addition results in a reduction of $r(C_i - 1)$ in the income consumed.

Notice that when (32) is substituted in (30), the result is

$$\sum_{i=1}^{n} \mu_i C_i = 1, \tag{39}$$

which asserts that the total equity capital supplied is equal to the economy's total supply of capital. Equation (38) can also be written as

$$\sum_{i=1}^{n} \mu_i(1 - C_i) = 0. \tag{40}$$

Note also that (32) and (27) imply

$$\nu_i(1 + K_i) = \mu_i C_i. \tag{41}$$

Since $\nu_i(1 + K_i)$ is the total capital demand by all firms of type $i$ and $\mu_i C_i$ is the total equity capital supplied by type $i$ individuals, (41) expresses the equality of supply and demand for equity capital to type $i$ firms.

The transformation just described permits us to interpret a CSMEE as the equilibrium of an economy in which capital can be either sold or supplied to firms for equity. The share of output received from a firm is proportional to the share of equity capital supplied to that firm. For each type $i$ there are firms which employ the $K_i$ desired by type $i$ individuals. Specifically, the type $i$ firm employs $K_i$, which together with $C_i$ maximizes $F^i$ defined by (33). The number of firms of type $i$ is $\nu_i$. Since $\nu_i$ satisfies (41), there is an equality of the supply of and the demand for equity capital for type $i$ firms. Because of (40), or equivalently (39), there is also an equality of supply of and demand for capital across types. Note in fact that (39) can, using (41), be written as (29), which asserts that total firm demand for capital equals supply.

Note that if there were no uncertainty, each firm would maximize output per unit of capital in order to maximize $F^i$. This criterion is analogous to the criterion employed in models of labor management. In that literature, labor managed firms are assumed to maximize output per worker. Because of this analogy, we use the term *capital management equilibrium* (CME) to refer to a vector $\langle\{(\hat{K}_i, \hat{C}_i)\}_{i=1}^{n}, r\rangle$ if, for each $i$, $(\hat{K}_i, \hat{C}_i)$ maximizes $F^i$ and if (39) holds.

Having defined $F^i$, $C^i$ and a CME, we have only completed the first step in the transformation used to obtain the existence of $(\hat{K}_i, \hat{\gamma}_i)$ and of a CSMEE. The second step is to solve the problem of maximizing $F^i$ sequentially. We first demonstrate in lemma 3 below that for each $C_i$ there exists a $\bar{K}(C_i)$ which maximizes $F^i(K_i, C_i)$. It is then shown in lemma

4 that for certain values of $r$, there exists a $\bar{C}_i$ which maximizes $F^i(\bar{K}_i(\cdot),\cdot)$. This accomplished, it is easily seen that $\hat{C}_i = \bar{C}_i$ and $\hat{K}_i = \bar{K}_i(\bar{C}_i)$.

Before stating these lemmas, we describe the third and final step in the process by which the problem of proving the existence of a CSMEE is reduced to one of finding a competitive equilibrium. In this step we demonstrate how the CME can be reinterpreted as a competitive equilibrium in a two-good pure trade economy. In order to define this simple economy, we first define a new "good" and let $G_i$ denote the quantity of this good consumed by individuals of type $i$. The quantity $G_i$ is assumed to be related to the quantity $C_i$ by the budget constraint

$$G_i = A + r(1 - C_i). \tag{42}$$

Note that when (35) holds, the consumption of $G_i$ will always be nonnegative. Individual $i$'s preferences for alternative $(G_i, C_i)$ bundles are represented by the "utility function"

$$V^i(G_i, C_i) \equiv \max_{K_i} Eu(G_i + C_i \frac{g(K_i,\tilde{x})}{[1 + K_i]}, i). \tag{43}$$

This utility function is well defined when $G_i$ satisfies (42) because of the existence (yet to be demonstrated in lemma 3) of $\bar{K}_i(C_i)$ for each $C_i$. In this two-good economy $r$ will be the price of "good" $C$ while the price of good $G$ will be one. Each individual will begin with an endowment vector

$$\omega_i = (\omega_{G_i}, \omega_{C_i}) = (A, 1) \tag{44}$$

of these two goods. Note that with this endowment vector and with prices so defined, equation (42) becomes the budget constraint faced by all individuals. A vector $\langle\{(\hat{G}_i, \hat{C}_i)\}_{i=1}^n, r\rangle$ is a competitive equilibrium for this economy if, for each $i$, $(\hat{G}_i, \hat{C}_i)$ maximizes $V^i(G_i, C_i)$ subject to (42) and the nonnegativity constraints $G_i \geq 0$ and $C_i \geq 0$, and if supply equals demand in both markets, i.e., if

$$\sum_{i=1}^{n} \mu_i \hat{C}_i = 1 \tag{45}$$

and

$$\sum_{i=1}^{n} \mu_i \hat{G}_i = A. \tag{46}$$

Note that (45) is the same as (39) and that, as usual, (46) is implied by Walras's law and by (45). Walras's law is also true for the usual reason; i.e., it follows from the budget constraint (42).

It is now a simple matter to use the observation just made to establish in proposition 5 the relationship between a CSMEE and the competitive equilibrium just defined.

*Proposition 5. Assume that* (20) *holds for all* $(K, B)$, *and that assumption* A1 *holds. Then* $\langle\{(\hat{\gamma}_i, \hat{K}_i)\}_{i=1}^n, r\rangle$ *is a* CSMEE *if and only if* (i) *the allocation* $\langle\{(\hat{G}_i, \hat{C}_i)\}_{i=1}^n, r\rangle$ *is a competitive equilibrium of the economy in which there are* $\mu_i$ *consumers with utility functions* $V^i$ *defined by* (43) *and every consumer has the initial endowment* $\omega$ *defined by* (44) *and* (ii) $\hat{K}_i = \bar{K}_i(\hat{C}_i)$; *i.e.,* $\hat{K}_i$ *maximizes*

$$Eu\left(\hat{G}_i + \hat{C}_i \frac{g(K_i,\tilde{x})}{[1 + K_i]}, i\right)$$

Proposition 5 will make it possible for us to prove in theorem 1 below that a CSMEE exists. This will be done by demonstrating the existence of a competitive equilibrium in the economy just described. This demonstration can be accomplished with standard arguments if the choice $(\hat{C}_i, \hat{G}_i)$ that maximizes $V^i$ subject to (42) can be shown to be unique for each $r$ and to vary continuously with $r$. The uniqueness has already been established by lemma 2 for the case in which $u(\bullet, i)$ is strictly concave. But the existence and continuity in $r$ of an optimizing choice have yet to be demonstrated. The proof of existence is implied by lemmas 3 and 4, which we now state. As mentioned earlier, lemma 3 asserts that for each $C$ in the interval $(0,(A/r) + 1]$, there exists a $\bar{K}_i(C)$ which maximizes $F^i(K,C)$. Lemma 4 demonstrates the existence of $\hat{C}_i$ as the $C$ value which maximizes $F^i(\bar{K}_i(C),C) = V^i(A + r(1-C),C)$.

The proofs of these lemmas, which are given in appendix 2, use the second differentiability of $u$ and $g$. This is, in fact, the first point at which differentiability has been used. It will be noted, however, that in stating and proving these lemmas, $u$ is not assumed to be *strictly* concave.

*Lemma 3. For each* $r \geq 0$ *and* $C_i \in (0,(A/r) + 1]$, $F^i$ *is a strictly pseudoconcave function of* $K_i$ *and there exists a unique* $\bar{K}_i(C_i)$ *which maximizes* $F^i(K_i, C_i)$ *subject to* $K_i \geq 0$. *Furthermore, when* $C_i \in (0,(A/r) + 1]$, $\bar{K}_i(C_i)$ *is a differentiable function of* $C_i$ *and of* $r$. *If the individuals of type i are risk neutral, i.e., if* $u(\bullet, i)$ *is linear, or if* $g(K,x)$ *satisfies Diamond's assumption of stochastic constant returns to scale, i.e., if* $X$ *is a subset of real numbers and*

$$g(K,x) = h(K)x, \tag{47}$$

*then for all* $C_i \in (0,(A/r) + 1]$, $\bar{K}_i(C_i) = K^*$, *where* $K^*$ *is the unique* $K$ *which maximizes expected output per capital unit,*

$$\frac{Eg(K,\tilde{x})}{[1 + K]}, \tag{48}$$

*and which is the unique solution to*

$$\frac{Eg(K,\tilde{x})}{[1 + K]} = Eg_K(K,\tilde{x}). \tag{49}$$

*Finally, when the investors in i are risk averse,*

(50) $$\lim_{C_i \to 0} \bar{K}_i(C_i) = K^*.$$

Figure 5.2 uses equation (49) to describe $K^*$. Also note that when $g$ satisfies (47), (49) is equivalent to

(51) $$h(K) - h'(K)(1 + K) = 0.$$

In this case figure 5.2 can be reinterpreted to obtain figure 5.3.

*Lemma 4. First suppose that*

(52) $$r = \frac{Eg(K^*, \tilde{x})}{[1 + K^*]}.$$

*If i is a risk neutral type, then $\hat{K}_i = K^*$ and $\hat{C}_i$ can be chosen arbitrarily. If, however, i is a risk averse type, then $\hat{K}_i = 0 = \hat{C}_i$.*

*Next, suppose that*

(53) $$r < \frac{Eg(K^*, \tilde{x})}{[1 + K^*]}.$$

*If, in this case, i is a risk neutral type, then $(\hat{K}_i, \hat{C}_i) = (K^*, (A/r) + 1)$. If type i individuals are risk averse, then $\hat{K}_i > 0$ and $\hat{C}_i > 0$.*

*Finally, assume that*

(54) $$r > \frac{Eg(K^*, \tilde{x})}{[1 + K^*]}.$$

*In this case $\hat{K}_i = \hat{C}_i = 0$ for risk averse as well as for risk neutral types.*

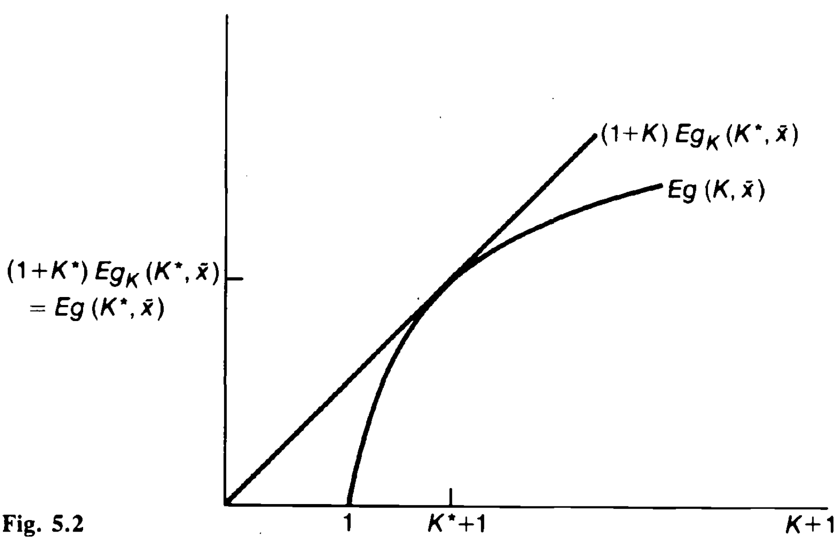


**Fig. 5.2**

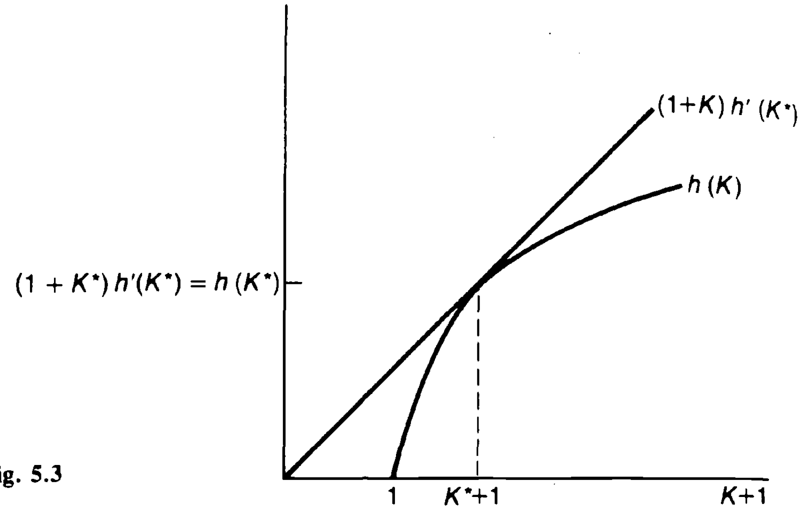


**Fig. 5.3**

In interpreting (52), (53), and (54) and subsequent references to these equations and inequalities, it should be recalled that $K^*$ is the unique operating capital level at which (49) holds.

These lemmas can now be used to prove that a competitive equilibrium exists in the two-commodity pure trade economy. As noted, this implies the existence of a CSMEE.

*Proposition 6. If assumption* A1 *is satisfied, then there exists a competitive equilibrium* $\langle\{(\hat{G}_i, \hat{C}_i)\}_{i=1}^n, r\rangle$ *such that, for each i,* $(\hat{G}_i, \hat{C}_i)$ *maximizes* $V^i(G_i, C_i)$ *subject to* (42) *and the nonnegativity constraints are such that* (45) *and* (46) *hold. If individuals of one type, say, type* 1, *are risk neutral and if*

$$\mu_1 \geq \frac{1}{[A/Eg_K(K^*, \tilde{x})] + 1}, \tag{55}$$

*then* (52) *holds,*

$$\hat{C}_1 = \frac{1}{\mu_1},$$

*and* $\hat{C}_i = 0$ *for* $i = 2, \ldots, n$. *If all individuals are risk averse or if individuals of some type, say, type* 1, *are risk neutral but* (55) *fails, then* (53) *holds. In the second of these two cases*

$$\hat{C}_1 = \frac{A}{r} + 1. \tag{56}$$

*Proof.* The assumptions made about $u$ and $g$ imply that $V^i$ is a continuous function. When no individuals are risk neutral, this continuity together with the uniqueness of $\hat{C}_i$ implies that $\hat{C}_i$ is a continuous function of $r$. Using this continuity, standard proofs of existence yield an equilibrium.

If individuals of some type, say, type 1, are risk neutral, the choice $(\hat{K}_1, \hat{C}_1)$ may not be unique. In particular, if (52) holds, then $\hat{K}_1 = K^*$ and $\hat{C}_1$ can be chosen arbitrarily. This nonuniqueness causes no problem in the risk neutral case since the linearity of $u$ and the fact that $K_1(C_1) = K^*$ for all $C_1$ imply that $V^1$ is linear; i.e.,

$$\text{(57)} \qquad V^1(G_1, C_1) = G_1 + C_1 \frac{Eg(K^*, \tilde{x})}{[1 + K^*]}.$$

Thus, for risk neutral individuals, preferences are convex and the resulting demand correspondence is upper semicontinuous. Again, standard existence proofs are applicable.

If $\mu_1$, the measure of the set of risk neutral types, is sufficiently large in the sense that (55) holds, then the equilibrium is easily exhibited. In this case $r$ can be equated to

$$Eg_K(K^*, \tilde{x}) = \frac{Eg(K^*, \tilde{x})}{[1 + K^*]}.$$

Lemma 4 then implies that for the risk averse types, i.e., for $i = 2, \ldots, n$, $\hat{C}_i = 0$. Thus, the equilibrium condition (45) reduces to

$$\text{(58)} \qquad \hat{C}_1 = \frac{1}{\mu_1}.$$

When (55) holds, $\hat{C}_1 = (1/\mu_1)$ will be less than $(A/r) + 1$, the bound imposed on $C_1$ by the condition (35). Lemma 4 implies that $\hat{C}_1$ solves the maximization problem of the type 1 individuals when $r$ satisfies (52). Taken together, these remarks imply that when type 1 individuals are risk neutral and (55) holds,

$$\langle (A + r(1 - \frac{1}{\mu_1}), \frac{1}{\mu_1}), (A + r, 0) \ldots (A + r, 0), r \rangle$$

is an equilibrium if $r$ is given by (52).

Suppose now that type 1 individuals are risk neutral but that (55) fails or that all individuals are risk averse. We want to show first that (53) holds, i.e., that neither (52) nor (54) holds. Lemma 4 implies immediately that (54) is inconsistent with equilibrium, since in that case

$$\text{(59)} \qquad \sum_{i=1}^{n} \mu_i \hat{C}_i = 0,$$

so that supply cannot equal demand in the $C$ market. The same situation occurs (i.e., (59) also holds) if $r$ satisfies (52) and if all individuals are risk averse. When individuals of type 1 are the only risk neutral individuals but (55) fails, lemma 4 implies

$$\text{(60)} \qquad \sum_{i=1}^{n} \mu_i \hat{C}_i = \mu_1 \hat{C}_1 \leq \mu_1 \left[ \frac{A}{Eg_K(K^*, \tilde{x})} + 1 \right].$$

if $r$ satisfies (52). If (55) fails, then (60) implies that equation (45) must also fail and again (52) is inconsistent with equilibrium.

When $r$ satisfies (53), (56) follows from lemma 4. QED.

The existence of a CSMEE when (20) holds is established in the theorem which follows. The theorem also describes the influence of attitudes toward risk and of certain important technological assumptions on the equilibrium allocations and on the price of capital. The theorem follows immediately from the results obtained above. Together with the interpretative remarks that follow, it can be viewed as a summary of the results established up to this point.

*Theorem 1. Suppose that assumption* A1 *is satisfied and that* (20) *holds for all* $(K,B)$. *Then there exists a* CSMEE $\langle\{(\hat{\gamma}_i,\hat{K}_i)\}_{i=1}^n, r\rangle$. *If all individuals are risk averse or if one type of individual, say, type* 1, *is risk neutral but the number of risk neutral individuals is small in the sense that* (55) *fails, then* $\hat{\gamma}_i > 0$, $\hat{K}_i > 0$, $\nu_i = \hat{\gamma}_i\mu_i > 0$, *r satisfies* (53) *and for any* $i > 1$,

$$
\begin{aligned}
r &= \frac{Eu'(A + r[1-\hat{\gamma}_i] + \hat{\gamma}_i[g(\hat{K}_i,\tilde{x}) - r\hat{K}_i], i)g_K(\hat{K}_i,\tilde{x})}{Eu'(A + r[1-\hat{\gamma}_i] + \hat{\gamma}_i[g(\hat{K}_i,\tilde{x}) - r\hat{K}_i], i)} \\
&= \frac{Eu'(A + r[1-\hat{\gamma}_i] + \hat{\gamma}_i[g(\hat{K}_i,\tilde{x}) - r\hat{K}_i], i)\dfrac{g(\hat{K}_i,\tilde{x})}{[1+\hat{K}_i]}}{Eu'(A + r[1-\hat{\gamma}_i] + \hat{\gamma}_i[g(\hat{K}_i,\tilde{x}) - r\hat{K}_i], i)} \qquad (61) \\
&< \frac{Eg(\hat{K}_i,\tilde{x})}{[1+\hat{K}_i]} < \frac{Eg(K^*,\tilde{x})}{[1+K^*]}
\end{aligned}
$$

*In the latter of these two cases, we will have, in addition,* $K_1 = K^*$ *and*

$$
\hat{\gamma}_1 = \frac{(A/r)+1}{[1+K^*]}. \qquad (62)
$$

*If type* 1 *individuals are risk neutral and there are enough individuals of this type, in the sense that* (55) *holds, then r satisfies* (52), $\hat{\gamma}_i = 0$, $\hat{K}_i = 0$, *and* $\nu_i = \hat{\gamma}_i\mu_i = 0$ *for* $i = 2,\ldots,n$ *while* $\hat{K}_1 = K^*$,

$$
\hat{\gamma}_1 = \frac{1}{\mu_1[1+K^*]},
$$

*and*

$$
\nu_1 = \hat{\gamma}_1\mu_1 = \frac{1}{[1+K^*]}.
$$

*Thus, if there are enough risk neutral individuals, all firms maximize expected profits and are held only by risk neutral individuals. The price of capital is the expected marginal product of capital in this case.*

*If g satisfies Diamond's assumption of stochastic constant returns to scale (i.e., if (47) holds), then $\hat{K}_i = K^*$ for all types i, even those which are risk averse. In this case, (29), which expresses the equality of capital supply and demand, can be reinterpreted to obtain the expression*

$$\sum_{i=1}^{n} \hat{\nu}_i = \frac{1}{1+K^*} \tag{63}$$

*for the number of firms.*

*If n = 1 (i.e., if all individuals are the same), then*

$$\nu_1 = \hat{\gamma}_1 = \frac{1}{\hat{K}_1 + 1}$$

*and $\hat{K}_1$ will be determined by*

$$\begin{aligned} Eu'(A + \left[\frac{1}{\hat{K}_1+1}\right] g(\hat{K}_1,\tilde{x}),1) g_K(\hat{K}_1,\tilde{x}) \\ = Eu'(A + \left[\frac{1}{\hat{K}_1+1}\right] g(\hat{K}_1,\tilde{x}),1) \frac{g(\hat{K}_1,\tilde{x})}{[1+\hat{K}_1]}. \end{aligned} \tag{64}$$

*In this case*

$$r = \frac{Eu'\left(A + \left[\frac{1}{\hat{K}_1+1}\right] g(\hat{K}_1,\tilde{x}),1\right) g_K(\hat{K}_1,\tilde{x})}{Eu'\left(A + \left[\frac{1}{\hat{K}_1+1}\right] g(\hat{K}_1,\tilde{x}),1\right)}.$$

*Remarks*. We are able to achieve an equilibrium because of a large-number hypothesis. Specifically, an equilibrium exists because there are many individuals of each type. To understand why this large-number hypothesis is necessary for equilibrium, note that in our model any individual will exercise his option to become an entrepreneur if the market fails to supply him with the investment opportunity, i.e., the firm, that he would find it optimal to create for himself as an entrepreneur. As a result, an equilibrium must supply the optimal investment opportunity for essentially every individual. However, because of the fixed costs, not all individuals will be able to be entrepreneurs; some will have to be satisfied with the investment opportunities provided for them by other entrepreneurs. Now capitalists of type *i* will of course be willing to invest in shares of firms created by type *i* entrepreneurs. In order to achieve an equilibrium in the market for shares to firms of type *i*, it must be possible to adjust the number of type *i* firms continuously until the supply of shares of these firms equals the demand, i.e., until equation (27) holds. If the supply of type *i* shares provided by an entering or exiting firm is a significant part of the total, variations in the number of firms do not result in continuous variations in the supply of type $i$ shares. When, as we assume, there is a large number (continuum) of possible firms of any type, each firm's supply of shares is insignificant and (27) can always be obtained by continuous variations in the number of firms.

The contrast between the case in which (55) holds and the case when it fails should be emphasized. When (55) holds, there is a relatively large number of risk neutral individuals. In equilibrium, this group holds all shares to all firms. Because all final owners of firms are risk neutral, the firms are operated at the $K$ level which maximizes expected profits when $r$, the price of capital, equals its expected marginal product. In order to guarantee that risk neutral investors are indifferent between capital sales for the fixed return $r$ and the purchase of equity shares, the price of capital must also equal expected output per unit of capital. When this is true, all *risk averse* individuals actually prefer capital sales at $r$ to the purchase of equity shares. The condition (55) guarantees that when $r$ equals expected output per capital unit, risk neutral individuals possess enough initial wealth to avoid a positive probability of bankruptcy when they buy all shares to the $1/(1+K^*)$ firms. To understand why this is so, suppose that these firms have no debt, so that the value of their shares is $1 + K^*$. At this price the capital value of all $1/(1+K^*)$ shares is 1. When $r$ equals both the expected output per capital unit and the expected marginal product of capital, the capital value of the initial endowment (A,1) is

$$[A/Eg_K(K^*,\tilde{x})] + 1 .$$

Thus, the total capital value of the endowment of the entire set of risk neutral individuals is

$$\mu_1\{[A/Eg_K(K^*,\tilde{x})] + 1\}. \tag{65}$$

When this exceeds or equals 1, the capital value of all shares, the probability of bankruptcy is zero if all $1/(1+K^*)$ debtless firms are held by the $\mu_1$ risk neutral individuals. As asserted, the case in which the expression in (65) exceeds or equals one is exactly the case in which (55) holds.

When (55) fails, risk neutral investors do not possess enough resources to buy all $1/(1+K^*)$ firms. In this case, risk averse buyers must be induced to hold shares in firms. The inducement is provided for *all* types when $r$ falls below expected output per capital unit. Thus, individuals of all types hold shares to firms. The firms held by these risk averse individuals will not, in general, maximize expected profits unless there is stochastic constant returns to scale. And, in fact, there will in general be a different type of firm, i.e., a firm with a different $K$ level, for each type of individual.

When $r$ is exceeded by expected output per capital unit, risk neutral individuals have a real preference for equity shares over capital sales at $r$

and this preference is never eliminated until the maximum number of shares are purchased. This, of course, occurs when (62) holds.

Note that the extreme case in which there are no risk neutral individuals can also be interpreted as a case in which (55) fails because $\mu_1 = 0$. In the other extreme case which occurs when all individuals are risk neutral, (55) must hold because $\mu_1 = 1$ and

$$[A/Eg_K(K^*,\tilde{x})] + 1 > 1 .$$

When there are stochastic constant returns to scale, all firms choose to operate at the output level which maximizes expected profits if $r$ equals the expected marginal product of capital. In this case $r$ will actually equal the marginal product of capital if and only if (55) holds. In other cases, $r$ will be lower than the expected marginal product of capital since, in these cases, $r$ satisfies (61).

The fact that (47) implies that individuals of all types choose to operate their firms at $K^*$ is a manifestation of the results obtained by Ekern and Wilson (1974), Leland (1974), and Radner (1974). They showed that such unanimity would be achieved when a condition they called *spanning* was satisfied. Stochastic constant returns to scale are a special case of spanning.

*Proof*. Using (36) and (42), the equilibrium $\langle\{(\hat{G}_i, \hat{C}_i)\}_{i=1}^n, r\rangle$ which was shown to exist in proposition 6 can be translated into a CSMEE. For the cases in which (55) does and does not hold when type 1 individuals are risk neutral and for the case in which no individuals are risk neutral, the same equations can be used to translate, in an obvious way, the descriptions of $\langle\{(\hat{G}_i, \hat{C}_1)\}_{i=1}^n, r\rangle$ obtained in proposition 6 into descriptions of the CSMEE.

The results for the case in which (47) holds follow from the results obtained for this case in lemma 3.

Finally, the expressions for $r$ provided by the first two equalities in (61) are obtained from the first-order conditions for $\hat{K}_i$ and $\hat{\gamma}_i$, respectively. The first inequality in (61) follows from the fact that $u'' < 0$, while the second inequality follows from the definition of $K^*$.

The results for the case $n = 1$ also follow immediately from the first-order conditions. QED.

### 5.5 Efficiency

The present section asks whether and in what sense a CSMEE is efficient. Again, the discussion is limited to the case in which expectations are classical in the sense that $N = N_C$. The well-known theorem that a competitive equilibrium is efficient and the fact that a CSMEE is such an equilibrium in a suitably defined two-good pure trade economy combine to imply that a CSMEE is efficient relative to the class of basic allocations which are representable as allocations of the two goods in this pure trade economy. This fact will make it possible to establish that a CSMEE is efficient in the restricted sense of the term employed by Diamond (1967).

The proof is not immediate only because there are some allocations that are feasible in the restricted sense of Diamond which are not representable as feasible allocations in the two-good pure trade economy. Unrepresentable Diamond restricted feasible allocations are those in which individuals hold diversified portfolios not satisfying $\Gamma(\alpha, E \sim I_\alpha) = 0$. It is easily seen, however, that, for reasons identical to those underlying proposition 3, allocations involving diversified portfolios can be Pareto dominated by undiversified portfolios which are representable in the two-good pure trade economy. The formalization of this argument is the proof of theorem 2 below.

In the unrestricted sense of the term associated with Arrow and Debreu, a CSMEE is not in general efficient. This situation can occur because there are allocations of the basic economy which are not achievable as Diamond feasible allocations in the stock market economy and are also not representable in the above described two-good pure trade economy. Indeed, some of these unrepresentable allocations can be shown to Pareto dominate the CSMEE. This will be done after theorem 2. At that point we will also give conditions under which the CSMEE is unrestricted efficient. Briefly, this occurs when all individuals are alike or if the set of risk neutral individuals, which we again identify with type 1 individuals, is sufficiently large to imply that (55) holds. These results will follow from an application of the marginal conditions for unrestricted efficiency, obtained in Kihlstrom and Laffont (1979). These marginal conditions can also be applied to demonstrate that when Diamond's assumption of stochastic constant returns to scale is satisfied, all but one of the manifestations of "first-best" inefficiency disappear. Specifically, in this case all firms' production decisions are at efficient levels and the number of firms is efficient. The allocation of risk remains inefficient because the opportunities for exchanging risks in the market for firm shares are not sufficiently rich.

Before stating theorem 2, it is necessary to describe the set of allocations which are feasible in the restricted sense of Diamond. Diamond efficient allocations can then be formally discussed as the set of Diamond feasible allocations which are not Pareto dominated by other Diamond feasible allocations. The discussion of Diamond restricted feasible and Diamond efficient allocations is naturally preceded by some consideration of unrestricted feasibility and efficiency. Thus, we begin by defining the set of unrestricted efficient allocations. The definitions of unrestricted feasibility and unrestricted or "first-best" efficiency are adaptations to the present context of the familiar definitions of feasibility and efficiency applied in the Arrow-Debreu analysis of markets for contingent claims.

*Definition 2.* A contingent claims allocation *is a specification of a partition* $\{E, C\}$ *of* $[0,1]$ *and of two Lebesgue measurable functions*

$$K: E \to [0,\infty)$$

*and*

$$y: [0,1] \times X \to [0,\infty)$$

*such that*

$$\int_E K(\alpha)\mu(d\alpha) = \mu(C) \tag{66}$$

*and for each* $x \in X$

$$\int_0^1 y(\alpha,x)d\alpha = \int_E g(K(\alpha),x)d\alpha + A. \tag{67}$$

As usual, the set of feasible allocations satisfying definition 2 can be interpreted as the set of choices available to a central planner. Such a planner can, by assigning individuals to their respective roles in the economy, choose the set $E$ of entrepreneurs and $C$, the set of capitalists. He can also assign a capital allocation $K(\alpha)$ to each entrepreneur $\alpha$ in the chosen set $E$. Finally, he can distribute to each individual $\alpha$ in $[0,1]$ a contingent claims vector $\langle y(\alpha,x)\rangle_{x \in X}$. Of course, the allocation he chooses is necessarily restricted by the limited availability of income in each state $x$ and of capital. He does, however, have some control over the availability of income in state $x$. This control is exercised when he chooses $E$, $C$, and $K(\bullet)$. His choice of $E$ and $C$ determines the number of firms. The amount produced by each of these firms is determined by the capital allocation $K(\bullet)$. Once these choices have been made, the availability restriction imposed on the allocation of contingent claims to wealth in state $x$ is the supply equal demand condition (67). The choice of $E$, $C$, and $K(\bullet)$ is constrained by the availability of capital. This constraint is imposed in the capital supply equal capital demand equation (66).

In making choices, a planner is assumed to be guided by the usual principle of efficiency which is embodied in the following definition.

*Definition 3. A contingent claims allocation is* efficient *if* (i) *it is feasible in the sense that it satisfies definition* 2 *and* (ii) *there is no other feasible allocation which Pareto dominates it, i.e., which makes a significant subset of individuals better off while making almost no one worse off.*

The concept of efficiency introduced by Diamond demands less than definition 3. Specifically, it imposes restrictions, beyond those imposed in (66) and (67) by availability, on the set of permissible contingent claims allocations against which a potentially Diamond efficient allocation is compared. Of course, any Diamond efficient allocation must also satisfy these added restrictions. These new restrictions take the form of additional constraints imposed on the relationship between $y(\alpha,x)$, individual $\alpha$'s claim to consumption in state $x$, and $x$, the state on which $y(\alpha,x)$ is contingent. In particular, individuals are permitted to receive income in two forms. First, they can be paid or pay fixed amounts not contingent on the random variable $\tilde{x}$. Second, they can receive a proportional share, fixed in advance, of the output of any existing firm. The share is fixed in the sense that the proportion received from any firm is independent of $x$, the outcome of $\tilde{x}$. The contingent claims allocations obtainable are the same as those achievable when the only institution for the exchange of risk is a stock market in which firm shares are traded for fixed payments. The imposition of these restrictions on $y(\alpha,x)$ has the effect of constraining the planner to use the same risk trading institutions as those used by the market. If a planner cannot, using only the market institutions, Pareto dominate a market allocation, then that allocation is Diamond efficient.

The following definition describes the set of Diamond feasible allocations.

*Definition 4. A contingent claims allocation is* feasible in the sense of Diamond *if there exist two functions*

$\Gamma: [0,1] \rightarrow$ *the set of nonnegative measures on the Lebesgue measurable subsets of E*

*and*

$$f: [0,1] \rightarrow (-\infty,+\infty)$$

*such that* (i) for each $x \in X$ *and* $\alpha \in [0,1]$

$$y(\alpha,x) = \int_E g(K(\beta),x)\Gamma(\alpha,d\beta) + f(\alpha), \tag{68}$$

(ii) *for each* $E' \subseteq E$

$$\int_0^1 \Gamma(\alpha,E')\mu(d\alpha) = \mu(E'), \tag{69}$$

*and* (iii)

$$\int_0^1 f(\alpha)d\alpha = A. \tag{70}$$

*A specification of* $\{E,C\}, K, y, \Gamma$, *and* $f$ *satisfying (66), (68), (69), and (70) will be referred to as a* Diamond feasible allocation.

Definition 4 describes the set of contingent claims allocations achievable when the institutional constraints imposed by the presence of a stock market in firm shares are added to the availability constraints of definition 2. The added restrictions on $y(\alpha,x)$ are imposed by equation (68) in which the fixed payment received or made by $\alpha$ is $f(\alpha)$ and his noncontingent portfolio of shares to firm output is $\Gamma(\alpha)$. The income availability restriction (67) imposed in definition 2 is replaced, in definition 4, by the availability constraint (69) of shares to firms' output and by the constraint (70) on fixed payments. The number of available shares to firms operated by entrepreneurs in some subset $E'$ of $E$ is $\mu(E')$. When $\Gamma(\cdot)$ is the allocation of firm share portfolios, the number of outstanding shares to the firms of the entrepreneurs in $E'$ is the left side of (69). As (69) asserts, these must be equal in any feasible allocation. When $f(\cdot)$ is the allocation of noncontingent payments, the left side of (70) represents the total of these payments. The amount $A$ represents the economy's total initial allocation of nonrandom income. As (70) asserts, this is the amount available for the purpose of making nonrandom payments. Note that (68)–(70) imply (67) so that this condition does not have to be explicitly introduced when $y(\alpha,x)$ satisfies (68) for each $\alpha$ and when $\Gamma(\cdot)$ and $f(\cdot)$ satisfy (69) and (70), respectively.

A planner who is restricted to the choice of a contingent claims allocation which satisfies the stock market institutional constraints added in definition 4 can apply the usual Pareto criteria in the modified or second-best sense introduced by Diamond.

*Definition 5. A contingent claims allocation is* Diamond efficient *if* (i) *it is Diamond feasible and* (ii) *there is no other Diamond feasible allocation which Pareto dominates it, i.e., which makes a significant subset of individuals better off while making almost no one worse off.*

It can now be shown that a planner with Diamond efficiency as his goal cannot make better use of market institutions than the market does on its own, if individual price expectations are rational in the sense that $N = N_C$. In interpreting the rationality of the expectations, the reader should recall the discussion of rational expectations in section 5.3.

*Theorem 2. Assume that N satisfies* (20) *and that assumption* A1 *holds. Then a competitive stock market entrepreneurial equilibrium is Diamond efficient.*

*Proof.* Suppose that $A^* = \langle\{E^*,C^*\}, K^*, y^*, \Gamma^*, f^*\rangle$ is a Diamond feasible allocation which Pareto dominates an equilibrium $\hat{A} = \langle\{\hat{E},\hat{C}\}, \hat{K}, \hat{\Gamma}\rangle$. By what we have shown, for each individual $\alpha$ of type $i$ who is in $\hat{E}$, $\hat{K}(\alpha) = \hat{K}_i$, $\hat{\Gamma}(\alpha, I_\alpha) = \hat{\gamma}_i$, and $\hat{\Gamma}(\alpha, E \sim I_\alpha) = 0$. Similarly, each individual of type $i$ would also demand $\hat{\gamma}_i$ shares in firms employing $\hat{K}_i$ and would also demand no shares in other firms. Thus, in particular, for every $i$

$$\text{(71)} \qquad Eu\big(A + \hat{r}(1-\hat{\gamma}_i) + \hat{\gamma}_i[g(\hat{K}_i,\tilde{x}) - \hat{r}\hat{K}_i], i\big)$$
$$\geq Eu\big(A + \hat{r}[1 - \textstyle\int_{E^*}\Gamma^*(\alpha,d\beta)] + \int_{E^*}[g(K^*(\beta),\tilde{x}) - \hat{r}K^*(\beta)]\Gamma^*(\alpha,d\beta), i\big).$$

In (71), $\hat{r}$ is the equilibrium price of capital. This inequality holds because of proposition 3. (Strictly speaking, proposition 3 implies (71) when $E^*$ is replaced by $\hat{E}$ and $\Gamma^*$ is replaced by some portfolio of firms which exist in equilibrium. But the same argument as that used to establish proposition

3 applies for any set of entrepreneurs and any portfolio. In particular, it holds for $E^*$ and $\Gamma^*$.)

Now if $A^*$ Pareto dominates $\hat{A}$, then for all $\alpha$

$$Eu\left(\int_{E^*} g(K^*(\beta),\tilde{x})\Gamma^*(\alpha,d\beta)+f^*(\alpha),i(\alpha)\right) \tag{72}$$
$$\geq Eu\left(A+\hat{r}(1-\hat{\gamma}_{i(\alpha)})+\hat{\gamma}_{i(\alpha)}[g(K_{i(\alpha)},\tilde{x})-\hat{r}\hat{K}_{i(\alpha)}],i(\alpha)\right),$$

and the strict inequality must hold for a set of $\alpha$'s of positive $\mu$-measure. In (72), $i(\alpha)$ is $\alpha$'s type. By combining (71) and (72), we can conclude that, for all $\alpha$,

$$Eu\left(\int_{E^*} g(K^*(\beta),\tilde{x})\Gamma^*(\alpha,d\beta)+f^*(\alpha),i(\alpha)\right) \tag{73}$$
$$\geq Eu\left(A+\hat{r}[1-\int_{E^*}\Gamma^*(\alpha,d\beta)]+\int_{E^*}[g(K^*(\beta),\tilde{x})-\hat{r}K^*(\beta)]\Gamma^*(\alpha,d\beta),i(\alpha)\right)$$

and that (73) holds with a strict inequality for a set of $\alpha$'s of positive measure. Now (73) implies that, for all $\alpha$,

$$f^*(\alpha)\geq A+\hat{r}[1-\int_{E^*}(1+K^*(\beta))\Gamma^*(\alpha,d\beta)] \tag{74}$$

and that (74) holds with a strict inequality for a set of $\alpha$ of positive $\mu$-measure. Thus, integrating (74) over $[0,1]$, we get

$$\int_0^1 f^*(\alpha)\mu(d\alpha)>A+r[1-\int_0^1\int_{E^*}\Gamma(\alpha,d\beta)\mu(d\alpha) \tag{75}$$
$$-\int_0^1\int_{E^*}K^*(\beta)\Gamma(\alpha,d\beta)\mu(d\alpha)].$$

Because (69) holds for all $E'\subseteq E$, we can interchange the order of integration in (75) to obtain

$$\int_0^1\int_{E^*}\Gamma(\alpha,d\beta)\mu(d\alpha)=\Gamma(E^*) \tag{76}$$

and

$$\int_0^1\int_{E^*}K^*(\beta)\Gamma(\alpha,d\beta)\mu(d\alpha)=\int_{E^*}K(\beta)\mu(d\alpha). \tag{77}$$

Substituting (76) and (77) in (75) and using condition (66) that supply equals demand in the capital market, we obtain

$$\int_0^1 f^*(\alpha)\mu(d\alpha)>A. \tag{78}$$

Thus, (70) and (66) cannot hold simultaneously for $A^*$, the allocation which Pareto dominates the equilibrium $\hat{A}$. Thus, $\hat{A}$ must be efficient in the sense of Diamond. QED.

Theorem 2 asserts that a planner who is restricted to using the same institutions as the market cannot Pareto dominate the market allocation. In this appropriate second-best sense, the market is efficient. Suppose, however, that the planner is liberated from the constraints imposed by market institutions and required only to satisfy the availability constraints imposed in definition 2. In that case, he can choose allocations which are not obtainable in a market restricted by the institutional constraints (68)–(70) of definition 4. With these additional choices available the planner can indeed improve on a market allocation chosen subject to the institutional constraints. Thus, as will be shown below, the market allocation is, in general, inefficient in the first-best sense of Arrow-Debreu. We will also show, however, that there are important special cases in which the allocation achieved in a stock market for firm shares is efficient in the "first-best" sense. In these cases, the stock market institutional constraints are not binding. Their relaxation is therefore of no value to a planner. In this case, the enlarged class of feasible allocations satisfying only the availability constraints of definition 2 cannot improve on a market allocation satisfying the more stringent institutional constraints of definition 4.

As an introduction to the discussion of "first-best" efficiency, we recall, in the following proposition, the results obtained in Kihlstrom and Laffont (1979).

*Proposition 7. If* $\langle\{E,C\},K(\bullet),y(\bullet,\bullet)\rangle$ *is efficient in the sense of definition* 3, *then*

$$\frac{u'(y(\alpha,x),\alpha)}{u'(y(\alpha,x'),\alpha)}=\frac{u'(y(\beta,x),\beta)}{u'(y(\beta,x'),\beta)} \tag{79}$$

*for almost all* $\alpha$ *and* $\beta$ *in* $[0,1]$ *and for all* $x$ *and* $x'$ *in* $X$. *Furthermore, for almost all* $\alpha$ *and* $\beta$ *in* $E$,

$$K(\alpha)=K(\beta)=K^0, \tag{80}$$

*where* $K^0$ *is determined by*

$$Eu'(y(\alpha,\tilde{x}),\alpha)g_K(K^0,\tilde{x})[K^0+1]=\mathrm{E}u'(y(\alpha,\tilde{x}),\alpha)g(K^0,\tilde{x}) \tag{81}$$

*for almost any* $\alpha$. *Conversely, if an allocation satisfies (79), (80), and (81), it is efficient in the sense of definition* 3.

The proof of this proposition is contained in Kihlstrom and Laffont (1979). It will not be reproduced here. We will, however, discuss the intuition underlying these results as well as their interpretation. The derivation of these conditions clearly requires the differentiability assumed throughout this paper. For the proof that (79)–(81) are sufficient for efficiency, the assumptions that $u$ and $g$ are concave are also required.

It is well known that in any "first-best" efficient allocation of contingent claims, individual marginal rates of substitution must be equated. Condition (79) is simply an expression of this familiar condition.

The equality of the capital allocation received by each firm expressed in (80) is a less familiar condition. It is closely related to the individual nondiversification result obtained in proposition 3. It asserts that, for the economy, as for any individual, there are no gains to diversification. Economy-wide and individual nondiversification are optimal for the same two reasons: the concavity of the production function in $K$ and the fact that $\tilde{x}$ is the same random variable for all firms. When these two conditions hold, all of the vectors of state-contingent output achievable through diversification across a number of firms can be dominated by some state-contingent output vector obtained by having these same firms produce at a common level. In effect, the possibility of replicating the concave technology makes diversification unnecessary. Suppose, for example, that two firms $\alpha$ and $\beta$ did receive different capital allocations $K(\alpha) \neq K(\beta)$. The total output of the two firms could be increased in all states $x$ if they each received $\frac{1}{2}[K(\alpha) + K(\beta)]$. This is possible because the strict concavity of $g$ implies that, for all $x$,

$$2g(\tfrac{1}{2}[K(\alpha) + K(\beta)],x) > g(K(\alpha),x) + g(K(\beta),x).$$

This improvement would not be possible, however, if the $x$ value in $\alpha$'s production function were ever different from the $x$ value in $\beta$'s production function.

It should be remarked that the $K^0$ which solves (81) is independent of $\alpha$ because (79) holds for all $\alpha$ and $\beta$. It is this independence which permits economy-wide nondiversification to be optimal, i.e., which permits all firms to produce at the same capital level in an efficient allocation.

Since each firm is operated at $K^0$, the capital supply equal capital demand condition implies that the total number of firms created when each employs $[1 + K^0]$ total capital units is

$$\nu^0 = \left[\frac{1}{1 + K_0}\right]. \tag{82}$$

Thus, (82) can be interpreted as an equation which determines $\nu^0$, the number of firms. In fact, (82) can be inverted to obtain

$$K^0 = \frac{1}{\nu^0} - 1. \tag{83}$$

When (83) is substituted in (81), the result is

$$Eu'(y(\alpha,\tilde{x}),\alpha)g_K(\frac{1}{\nu^0} - 1,\tilde{x})\frac{1}{\nu^0} = Eu'(y(\alpha,\tilde{x}),\alpha)g(\frac{1}{\nu^0} - 1,\tilde{x}), \tag{84}$$

which is the same as equation (39) in Kihlstrom and Laffont (1979). Equation (84) directly determines $\nu^0$.

For the purpose of interpreting (81) and (84), we can follow common practice and use $g_K(K^0,x) = g_k((1/\nu^0) - 1,x)$ to measure the income value of capital in state $x$. The term $g_K(K^0,x)[K^0 + 1] = (1/\nu^0)\, g_K((1/\nu^0) - 1,x)$ then becomes the income value, in state $x$, of the $[1 + K^0] = 1/\nu^0$

total capital units required to operate a firm at $K^0$. The left sides of (81) and (84) can therefore be interpreted as the expected marginal utility of these $[1 + K^0] = 1/\nu^0$ capital units. The right sides of (81) and (84) represent the expected marginal utility of the output produced by a firm operated at $K^0$. Thus, (81) and (84) assert that new firms are created up to the point at which the expected marginal utility of the income value of the capital used to create a new firm and operate it at $K^0$ is just equated to the expected marginal utility of output produced by the firm.

Equation (81) can also be rewritten to obtain the equation

$$(85) \qquad Eu'(y(\alpha,\tilde{x}),\alpha)g_K(K^0,\tilde{x}) = Eu'(y(\alpha,\tilde{x}),\alpha)\frac{g(K^0,\tilde{x})}{[1 + K^0]},$$

which, using (82) and (83), is, in turn, equivalent to

$$(86) \quad Eu'(y(\alpha,\tilde{x}),\alpha)g_K(\frac{1}{\nu^0} - 1,\tilde{x}) = Eu'(y(\alpha,\tilde{x}),\alpha)g(\frac{1}{\nu^0} - 1,\tilde{x})\nu^0.$$

The expressions in (85) and (86) assert that capital is efficiently divided between its alternative uses as an operating and entrepreneurial input when the expected marginal utility of the marginal product of capital equals the expected marginal utility of the output per unit of capital. The left sides of (85) and (86) clearly measure the marginal utility of an additional unit of operating capital. It will now be argued that the right sides of (85) and (86) measure the marginal utility of an additional unit of entrepreneurial capital. Then (85) and (86) can be interpreted as expressions of a conventional wisdom; viz., an input, in this case capital, is efficiently allocated when it yields the same marginal utility in all alternative uses, in this case as entrepreneurial and as operating capital. To interpret the right sides of (85) and (86), note that when one new firm operating at $K^0$ is created, it produces $g(K^0,x)$ income units in state $x$. The proportion of entrepreneurial to total capital used to obtain this output is $1/(1 + K^0)$. Thus,

$$\left[\frac{1}{1 + K^0}\right]g(K^0,x)$$

is the share of additional output attributable to the one unit of entrepreneurial capital used to create the firm. As a consequence, the right sides of (85) and (86) measure the expected marginal utility of an additional unit of entrepreneurial capital.

From proposition 7 it is clear why a CSMEE is inefficient in the unrestricted Arrow-Debreu sense. First, there is no mechanism to guarantee that risk is efficiently allocated. Thus, in general, (79) fails to hold. In addition, there is, in the general case, a misallocation of capital to firms. This occurs because entrepreneurs of different types choose to operate their firms at different $K$ levels. Since $\hat{K}_i$ is, except in special circumstances, different from $\hat{K}_j$ when $i \neq j$, the equality required by (80) fails to hold.

Note that in spite of the fact that (79) and (80) fail to hold in a CSMEE, equation (61) implies that (81) does hold for all $i$. As noted, however, the $\hat{K}_i$ at which (81) is satisfied is different for each $i$. This can happen because risk is misallocated and (79) fails. The fact that $\hat{K}_i$ is different for different types makes it impossible to relate the number of firms to the firms' capital demand by a simple equation such as (83). Thus, the efficient number of firms can no longer be deduced from (84), which was obtained from (81) and (83).

Although a CSMEE is not, in general, efficient in the first-best sense, there are special circumstances of some importance in which first-best efficiency is achieved. The proposition which follows describes these cases.

*Proposition 8. Assume that* $N = N_C$. *If individuals of one type, say, type* 1, *are risk neutral and their number is sufficiently large to imply that* (55) *holds, then the equilibrium is efficient in the sense of definition* 3. *The equilibrium is also efficient in this sense if all individuals are alike.*

*Proof.* When type 1 individuals are risk neutral and (55) holds, theorem 1 asserts that all firms are completely owned by risk neutral individuals. Risk averse individuals receive the sure return $r$ for their capital. Thus, all risks are borne by the risk neutral individuals, as they must be if (79) is to hold in this case. Since all firms are operated at $K^*$, the $K$ level which maximizes expected profits, (80) holds and capital is efficiently allocated across firms. With risk neutral individuals, (81) reduces to (49), which is the equation defining $K^*$. Thus, the equilibrium $K$, $K^*$, is the efficient level at which to operate each firm and the equilibrium number of firms $1/(1 + K^*)$ is also the efficient number.

The proof that the equilibrium is efficient when there is only one type also follows from the marginal conditions derived in proposition 7 and from the properties of the equilibrium established for this case in theorem 1. Again, the important fact is that all firms are operated at the same level $\hat{K}_i$. Because of (61), this is the $K$ level which satisfies (81). The equality of marginal rates of substitution required in (79) is a consequence of the fact that all individuals are identical and hold identical portfolios. QED.

## 5.6 Relationship of a CSMEE to the Diamond Equilibrium and Long-Run Competitive Equilibrium without Uncertainty

The objective of this brief section is to argue that the equilibrium concept proposed in this paper can be viewed as a generalization of two important concepts of economic equilibrium.

The first of these concepts is Diamond's stock market equilibrium. The second is the classical long-run competitive equilibrium in which production takes place at minimum average cost.

The present model can be related to Diamond's from either of two points of view which differ in their interpretation of the production technology and of Diamond's assumption of stochastic constant returns to scale. The first of these alternative interpretations to be considered is the one implicitly adopted throughout our exposition. Thus, we first identify the technology with the production function $g$ and interpret stochastic constant returns to scale to mean that $g(K,x) = h(K)x$. Using this interpretation and making appropriate adjustments for the fact that we do, but Diamond did not, consider the case of free entry and exit, we will now argue that the stock market equilibrium considered here is the same as that considered by Diamond. Diamond's discussion of this equilibrium concept is limited to the case in which the technology satisfied the assumption of stochastic constant returns to scale. This restriction was necessary to simplify the study of the firm's maximization problem. As Leland (1974), Ekern and Wilson (1974), and Radner (1974) have shown, this assumption implies that stockholders unanimously agree on the choice criterion to be used by the firm. In fact, all stockholders agree that the firm should maximize its stock market value. We replace this criterion for firm maximization by the assumption of entrepreneurial expected utility maximization. Because of the unanimity results just mentioned, expected utility maximization implies stock market value maximization when there are stochastic constant returns to scale. In the present paper, unanimity is manifested in the observation made in the statement of theorem 1 that, when there are stochastic constant returns to scale, all firms produce at $K^*$, where $K^*$ satisfies (51). Thus, the capital level chosen by all firms in a CSMEE is the same as that chosen in a Diamond equilibrium, whenever Diamond's firm maximization criterion is applicable, i.e., whenever there are stochastic constant returns to scale. Diamond showed that his equilibrium was efficient in a sense consistent with definition 5. His concept of efficiency differed from definition 5 because he did not explicitly consider firm entry and exit or the efficient number of firms. Theorem 2 generalizes this efficiency result of Diamond to the class of technologies $g$ not satisfying stochastic constant returns to scale but in which free entry is permitted. In the case of stochastic constant returns to scale, we obtain a somewhat stronger result than simply efficiency in the sense of definition 5. Specifically, the observation that all firms produce at $K^*$ implies that the CSMEE *allocation of capital* is the same in all respects as the first-best efficient allocation of capital. Since all firms produce at the same $K$ level, the equality (80) required for first-best efficiency is satisfied. This implies that the distribution of capital across firms is efficient. In order to prove that the efficient $K$ level at which all firms should produce is $K^*$ and that the efficient number of firms is $1/(1 + K^*)$, we note that (81) implies (51) when $g$ satisfies (47). In spite of the fact that capital is efficiently allocated, the allocation of risk remains inefficient in the Arrow-Debreu sense because (79) fails to hold.

By interpreting the technology differently, we can reveal the features of our model which permit the generalization of Diamond's result. Specifically, there is a sense in which the technology studied here exhibits not only stochastic constant returns to scale, but constant returns to scale. This property is introduced by our assumption that the technology described by $g$ can be replicated without limit at a cost of one unit of capital per replication. From this point of view, the technology is more accurately represented by the production set

$$(87)\quad \Big\{(y_1,\ldots,y_S,Z) = \Big(\int_0^\infty g(K,x_1)\eta(dK),\ldots,\int_0^\infty g(K,x_S)\eta(dK),$$

$$\int_0^\infty [1 + K]\eta(dK)\Big) : \eta \text{ is a nonnegative measure on the Lebesgue measurable subsets of } [0,\infty) \text{ with } \int_0^\infty [1 + K]\eta(dK) < \infty\Big\}.$$

Since this production set is a cone, it exhibits constant returns to scale and *a fortiori* stochastic constant returns to scale in a sense which is slightly more general than that considered by Diamond. The added generality arises because Diamond's production sets are one-dimensional cones.

It should be emphasized that it is precisely the replication possibilities embodied in the free entry assumption which imply the linearity of the production set and thereby lead to the fulfillment of the stochastic constant returns to scale hypothesis.

From this point of view, theorem 2 could be viewed as an extension of Diamond's results to the more general linear technology sets described by (87).

When there is no uncertainty, a CSMEE is the same as a long-run perfectly competitive equilibrium in which entry forces price to equal minimum average cost. To see this, we can view the case of no uncertainty as a special case in which (47) holds with $x = 1$. In this case, a CSMEE is again characterized by the fact that $K = K^*$ for all firms. As before, $K^*$ is determined by (51) and is as shown in figure 5.3. Furthermore, for any $r$, $K^*$ is the $K$ level which minimizes the average cost $(r(1 + K)/h(K))$.

## 5.7 CSMEE When $N \neq N_C$

As mentioned earlier, there may exist CSMEE for expectation functions $N$ different from $N_C$. If so, there should, in fact, be a profusion of such equilibria. Suppose, for example, that we choose an arbitrary $\bar{K}$. Assume now that, for any $B, N(\bar{K}B) = N_C(\bar{K}, B)$ but that $N(K,B) < N_C(K,B)$ if $K \neq \bar{K}$. We can then find an $\bar{r}$ and $(\hat{\gamma}_1, \ldots, \hat{\gamma}_n)$ such that, for every $i$, $\hat{\gamma}_i$ maximizes

$$Eu\big(A + \bar{r}[1 - \hat{\gamma}_i(1 + \bar{K})] + \hat{\gamma}_i g(\bar{K}\tilde{x}), i\big)$$

and such that

$$\nu = \sum_{i=1}^{n} \mu_i \hat{\gamma}_i = \frac{1}{1 + \bar{K}}.$$

The problem of finding such an equilibrium is simply one of finding a competitive market equilibrium when there is one sure asset and one risky asset with return vector $(g(\bar{K}, x_1), \ldots, g(\bar{K}, x_S))$. Every investor has $1/(1 + \bar{K})$ initial shares in the risky asset and $A$ units of the sure asset. If $N(K,B)$ is sufficiently small relative to $N_C(K,B)$ when $K \neq \bar{K}$, it may be possible to interpret this asset market equilibrium as a CSMEE with the given $N$ function. In this CSMEE, the only firms ever created will be those employing $\bar{K}$ units of operating capital. Because $N(\bar{K}, B) = N_C(\bar{K}, B)$, every individual will be indifferent between remaining a capitalist and creating a firm operating at $\bar{K}$.

To demonstrate that we have in fact a CSMEE, it remains to be shown that it is not to any agent's advantage to set up a firm (or several firms) with $K \neq \bar{K}$ and hold them completely. We conjecture that a way of constructing an equilibrium with this property is to take $\bar{K}$ close to a $K$ level obtained in a CSMEE with $N = N_C$. Then, the equilibrium interest rate $\bar{r}$ should be close to $r$, the interest rate associated with the CSMEE, with $N = N_C$, and we can be assured, by taking $\bar{K}$ close enough to $K$, of obtaining a utility level for $i$ which is higher than what $i$ can obtain by himself.

Finally, it is worth noting that there is no guarantee that a CSMEE with nonclassical expectations is Pareto superior to the equilibrium obtained without a stock market (Kihlstrom and Laffont 1979).

## 5.8 Fixed Costs, Uncertainty, and the Need for a Stock Market

In this section we will argue, using the present model, that the necessity for a stock market arises from the existence of the fixed costs incurred in setting up a firm and from the presence of uncertainty. As mentioned in the introduction, the fact that the stock market plays a nontrivial role in the economy studied here follows from a comparison of the equilibrium of the stock market economy with the equilibrium of the same economy without a stock market. In Kihlstrom and Laffont (1979), we studied the equilibrium achieved in the non–stock market economy. That equilibrium differs in several ways from the one achieved with a stock market. First, without a stock market, some individuals strictly prefer to be capitalists rather than entrepreneurs while others strictly prefer to be entrepreneurs. In contrast, in the stock market model of this paper, all individuals are indifferent about their role as entrepreneurs or nonentrepreneurs. This is true because without a stock market only entrepreneurs bear the risks associated with the firms they create. Nonentrepreneurs bear no risks. As a result of the lack of risk sharing opportunities, the marginal condition (61) does not hold in the non–stock market economy. Furthermore, in contrast to the stock market economy in which $N = N_C$, the non–stock market economy is inefficient in Diamond's second-best sense. If, in addition, there is only one type of individual, a stock market equilibrium in which $N = N_C$ is efficient in the unrestricted sense while the non–stock market equilibrium is not. These observations imply that the introduction of the stock market plays an essential role in improving the efficiency of the economy's operation.[1] As we shall now show, this is not true if there is neither uncertainty nor a fixed cost to setting up a firm.

In the model discussed above, the fixed cost is borne in the form of the one unit of entrepreneurial capital required to create a firm. In general, we could have assumed that the fixed cost was $c$ units of capital, where $c \neq 1$. In order to extend the analysis to this trivially different case, it would of course be necessary to assume that there exists a $K(x)$ at which

$$g(K(x),x) = g_K(K(x),x)[K(x) + c]. \tag{88}$$

Thus, figure 5.1 would be replaced by figure 5.4.

In this case $N_C(K,B)$ would, of course, equal $K + c - B$.

If $c = 0$, the assumption that a $K(x)$ exists which satisfies (88) implies that $g(K,x) = Kx$. Thus, $g$ not only must exhibit stochastic constant returns to scale, but must, in fact, be a constant returns to scale function. In this case figure 5.4 becomes figure 5.5, and $K(x)$ is not unique. As a result, the equilibrium $K^*$ will not be unique. There will also be no need for a stock market in which to sell firm shares. This is true since any $(\hat{\gamma}_i, \hat{K}_i)$ choice which is optimal for $i$ will be indifferent to some other choice $(\check{\gamma}_i, \check{K}_i)$ with $\check{\gamma}_i = 1$ and $\check{K}_i = \hat{\gamma}_i\hat{K}_i$. Specifically, $i$'s state $x$ wealth from $(\hat{\gamma}_i, \hat{K}_i)$ is $A + r[1 + \hat{\gamma}_i\hat{K}_i] + \hat{\gamma}_i\hat{K}_i x$ and this equals $A + r[1 + \check{K}_i] + \check{K}_i x$ if $\check{K}_i = \hat{\gamma}_i\hat{K}_i$.

When $\check{\gamma}_i = 1$, the stock market is unnecessary. Every individual can simply create his own firm and hold it. This is feasible since there are no fixed costs to setting up the firm. This point has also been made in the context of a sharecropping model by Stiglitz (1974) and Newbery (1977).

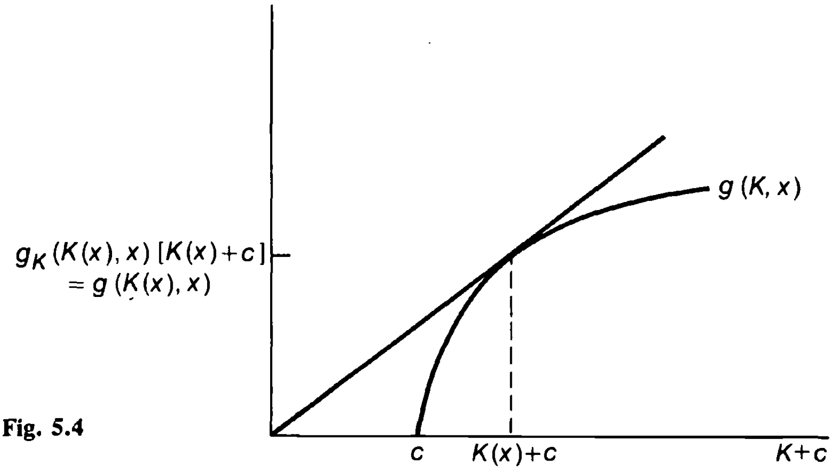


**Fig. 5.4**

With $c > 0$ and $N(K,B) = K + c - B$, $\hat{\gamma}_i$ will in general differ from one. Because fixed costs are positive, it is not feasible for every investor to set up his own firm. Thus, the stock market is essential for the exchange of shares between entrepreneurs and capitalists, i.e., between those who do not create firms but who want to hold firm shares.

It should also be added that the need for a stock market is eliminated when there is no uncertainty. This is true even if there are fixed costs, i.e., even if $c$ is positive.

Without stock trading, $1/(1 + K^*)$ individuals create firms and raise all operating capital by issuing debt. Their profits are

$$h(K^*) - rK^* = h(K^*) - \frac{h(K^*)}{1 + K^*}K^* = \frac{h(K^*)}{1 + K^*} = r.$$

The remaining $K^*/(1 + K^*)$ capitalists each receive their marginal product $h'(K^*) = r$ by selling their capital in the debt market. Thus, without a stock market all capitalists and all entrepreneurs have a final wealth which equals $A + r$.

This same result can also be obtained with a stock market, but it cannot be improved on. If there were a stock exchange in which some individual sold $(1 - \gamma)$ capital units in the debt market while investing $\gamma$ units in firm shares, his return would be

$$r(1 - \gamma) + \gamma[h(K^*) - rK^*] = r.$$

As noted, this is the same wealth he would obtain as an entrepreneur or as a capitalist when there is no stock market.

The need for a stock market is also eliminated if all type 1 individuals are risk neutral and if there are sufficiently many of these individuals to result in the satisfaction of (55). For the stock market to be superfluous in this case, however, it may be necessary to assume that risk neutral

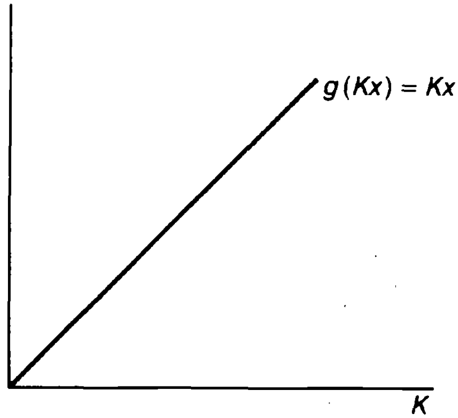


Fig. 5.5

individuals can set up more than one firm by buying the entrepreneurial capital in the debt market. If this is possible, the argument just given for the case of no uncertainty can be modified by replacing $h(K)$ with $Eg(K,\tilde{x})$. With this modification it can be shown that all risk neutral individuals can receive an expected wealth of $A + r$ by becoming entrepreneurs (possibly for more than one firm) and retaining all firm shares or by being capitalists. It can also be shown that all risk averse individuals receive a sure wealth of $A + r$ by remaining capitalists and holding no shares. Again, the introduction of a stock market fails to permit an improvement on this allocation of final wealth.

In Kihlstrom and Laffont (1979), entrepreneurs cannot set up more than one firm by buying entrepreneurial capital in the debt market. Thus, in the equilibrium of that paper it is possible that not all firms are held by the risk neutral individuals even though (55) holds.

## 5.9 Survey of Related Literature

The survey of this section describes the post-Diamond literature and relates it to the present paper. We first consider the literature on the possibility of stockholder unanimity.

As mentioned in the introduction, Leland (1974) used Diamond's framework to show that the production plan which maximizes stock market value receives unanimous shareholder approval. The same result was obtained for a slightly broader class of technologies—specifically, those which satisfy a condition referred to as spanning—by Ekern and Wilson (1974) and Radner (1974). Grossman and Stiglitz (1977) subsequently clarified the role of the competitivity assumption in the discussion of unanimity.

Hart (1979) has shown that even without spanning, unanimous agreement on market value maximization can be achieved in economies with a large number of firms. A similar result obtains in our model. Even if the production function does not satisfy a spanning condition, all equilibrium shareholders of every firm are unanimous about the goals of the firm. This unanimity is achieved because of the clientele effect described in proposition 4. This effect arises because of a process of market self-selection that results in firms each of which are held by identical stockholders who therefore agree completely on the firm's operation. Except in special cases, which, of course, include spanning and stochastic constant returns to scale, there is not, however, unanimity across firms. This happens because firms held by individuals of different types, in general, choose different operating capital levels. One case, not mentioned earlier, in which there is always unanimity across firms occurs when the utility functions of all agents are from a class for which portfolio separation holds. If, for example, the utility functions $u(\cdot, i)$ are all risk averse and all exhibit constant absolute risk aversion, then they will all choose to hold firms operated at the same $K$ level which will in general differ from $K^*$.[2]

The post-Diamond stock market literature on efficiency includes contributions by Hart (1975, 1977, 1979), Drèze (1974), Grossman and Hart (1979), Helpman and Razin (1978), Stiglitz (1972, 1975), Jensen and Long (1972), and Merton and Subrahmanyam (1974). In addition, the literature on sharecropping contains the closely related work of Stiglitz (1974) and Newbery (1977) discussed in section 5.8.

Hart (1975) showed that Diamond's results could not be extended to the case in which there were several goods traded in spot markets which opened after the resolution of uncertainty.

Drèze (1974) proposed a criterion for firm behavior which was implementable even when the technology failed to satisfy stochastic constant returns to scale. Drèze's approach was suggested by his observation that the firm's choices of production plans are, in general, public goods for the shareholders. Firms, in effect, choose the assets available to investors. This observation had been made earlier by Smith (1970) and is also the basis for a later contribution by Helpman and Razin (1978). Drèze exploited the public good interpretation by defining an equilibrium in which the firm used the stockholder's "Lindahl prices" to compute a value of each production decision. It is this value which he assumed the firm maximized, *taking the distribution of ownership as given*. He observed a nonconvexity in the consumption space implied by the stock market model. Drèze's criterion for firm maximization avoided the difficulties usually created by nonconvexities precisely because firms treated ownership shares parametrically and because investors took production decisions as given when choosing a portfolio of ownership shares. The cost of decentralized decision making in Drèze's model is the possibility of inefficient equilibria.

The justification for Drèze's approach varies. Gevers (1974) provides a rationale based on majority voting without side-payments. Grossman and Hart (1979) give an argument in favor of a closely related criterion. Their argument is in the spirit of Hart's earlier paper (1977) and is based on the possibility of a takeover.

One important point to be observed is that the public good externality observed by Drèze fails to arise when there are stochastic constant returns to scale. In this case, the firms have no control over the assets which will be available to investors. An alternative method by which constrained efficiency can be restored is to devise a framework in which the separation of production and investment decisions is eliminated, i.e., in which the externality is internalized. In our model each individual can, if he finds it necessary, jointly make the production and investment decisions by becoming an entrepreneur. As pointed out earlier, not all individuals find it necessary to exercise this option in our equilibrium. For any given individual, there are many individuals who become entrepreneurs and run firms in accordance with the given individual's desires.

There is a potential problem which must be faced in any attempt to coordinate production and investment decisions. Specifically, such coordination reintroduces the nonconvexity Drèze avoided by decentralization. In our model, this nonconvexity enters in the objective function of entrepreneurs. Fortunately, the problem created by this nonconvexity is not fundamental. This is demonstrated in our lemma 2, which establishes the uniqueness of the solution to the entrepreneurs' maximization problem.

Hart (1977) shows that the inefficient Drèze equilibria can be eliminated in large economies by permitting takeovers. A situation qualifies as a takeover bid equilibrium if it is a Drèze equilibrium for which no takeover is possible. In Hart's model, the agent who takes over internalizes the externality by buying the firm, reorganizing production, and selling shares. This agent plays the role of our entrepreneur. The possibility of a takeover becomes a force which results in efficiency. In our model, this force is provided by the possibility of entry.

Helpman and Razin (1978) suppress the separation of decisions by setting up a (participation) noncooperative game in which each agent contributes a share of input. However, in this game the contribution of an agent creates an externality of the atmosphere type (since the production depends on the *sum* of individual inputs). Then a uniform subsidy on the input contribution activity financed by lump sum transfers helps restore efficiency. However, there seems to be no reason why an agent should take the value of the firm (i.e., here the value of future outputs) as independent of his actions (since he provides inputs); a large-number assumption seems to be required to justify this "competitive" behavior.

Sticking to the public good analogy, Hart's takeover bid equilibrium is analogous to a Foley politico-equilibrium, Helpman and Razin's participation equilibrium is analogous to an equilibrium with subscription made efficient by an appropriate tax system, and our entrepreneurial equilibrium is similar to a Tiebout equilibrium.

There exists, in financial economics, a closely related literature which uses a special model, the mean-variance model.

Two cases must be distinguished, the case of independently distributed production functions where a general technology is used and the case of partially correlated returns where the assumption of multiplicative uncertainty is made.

Let us first consider the case where some correlation exists. The mean-variance model permits the derivation of the equilibrium value of the firm as a function of its investment policy. Using such a model, Stiglitz (1972) showed that the stock market equilibrium was inefficient. The inefficiency can be attributed to noncompetitive behavior on the part of firms which take into account the nonproportional effect of their investment policy on their equilibrium value. The externality argument cannot be invoked here to explain the inefficiency since multiplicative uncertainty is assumed. Jensen and Long (1972) have shown that in this model the equilibrium converges to a Pareto optimum as the number of firms goes to infinity. Indeed, as the number of firms goes to infinity, the noncompetitive value maximization behavior is transformed into competitive behavior, so that in the limit we are in a special case of Diamond (1967).

Merton and Subrahmanyam (1974) have argued that perfect competition requires perfect correlation since all technologies should be available to all individuals. It is also the point of view we have taken in this paper. Allowing free entry at a zero cost, they show that Jensen and Long's model (with noncompetitive behavior) is unstable with respect to the number of firms. Free entry is shown to lead to an infinite number of firms and Pareto optimality. The case of positive set up costs was subsequently considered by Stiglitz (1975).

There remains the case of *completely independent* firms. Without (Stiglitz 1972) or with (Jensen and Long 1972) multiplicative uncertainty, one does not obtain efficiency in the limit. This shows that the difficulty comes from the fact that in this case noncompetitive behavior is not transformed into competitive behavior in the limit. In fact, Merton and Subrahmanyam have argued that with free entry the limit of these noncompetitive equilibria does not even exist. They also considered the case of competitive behavior and multiplicative uncertainty. In this case, they showed that the economy was Diamond efficient in the small-numbers case as well as in the limit. This result is to be expected from Diamond's analysis.

Finally, we mention the paper of Novshek and Sonnenschein (1978), who consider a model without uncertainty but with free entry, fixed costs, and noncompetitive behavior. They show that when fixed costs are small relative to demand and when demand curves slope down, the noncompetitive Cournot equilibrium exists and approximates the perfectly competitive equilibrium in which price equals minimum average cost. Thus, when there is no uncertainty, the Novshek-Sonnenschein equilibrium will approximate the equilibrium described in this paper if the demand curves slope down.

# Appendix 1

The proof of proposition 2 is based on the two simple lemmas which follow.

*Lemma A1. If* $\alpha \in E$ *and* $\alpha' \in C$, *if* $u(\bullet,\alpha) = u(\bullet,\alpha')$, *and if* (18) *holds, then* (17) *holds. If* $\alpha$ *and* $\alpha'$ *are in E, if* $u(\bullet,\alpha) = u(\bullet,\alpha')$, *and if* (19) *holds, then for some* $k$,

$$\text{(A1)} \qquad k = N(K(\alpha),B(\alpha)) + B(\alpha) - K(\alpha) = N(K(\alpha'),B(\alpha')) + B(\alpha') - K(\alpha').$$

*Proof.* Consider first the case in which $\alpha \in E$ and $\alpha' \in C$. The equilibrium choice, call it $(\hat{K}(\alpha),\hat{B}(\alpha))$, is $\alpha$'s maximizing choice for these values. Once $(K(\alpha),B(\alpha))$ is fixed at these values, the equilibrium $\Gamma(\alpha)$ is chosen to solve

$$\max_{\Gamma(\alpha)} Eu(\tilde{W}_E(\hat{K}(\alpha),\hat{B}(\alpha),\Gamma(\alpha)),\alpha)$$

and

$$\text{(A2)} \qquad \max_{\Gamma(\alpha)} Eu(\tilde{W}_E(\hat{K}(\alpha),\hat{B}(\alpha),\Gamma(\alpha)),\alpha) = \max_{\substack{\Gamma(\alpha)\\ K(\alpha)\\ B(\alpha)}} Eu(w_E(K(\alpha),B(\alpha),\Gamma(\alpha)),\alpha).$$

If now

$$\text{(A3)} \qquad N(\hat{K}(\alpha),\hat{B}(\alpha)) < 1 + \hat{K}(\alpha) - \hat{B}(\alpha),$$

then

$$\text{(A4)} \quad \max_{\Gamma(\alpha)} Eu(\tilde{W}_E(\hat{K}(\alpha),\hat{B}(\alpha),\Gamma(\alpha)),\alpha) < \max_{\Gamma(\alpha')} Eu(\tilde{W}_C(\Gamma(\alpha')),\alpha').$$

This is so as a result of two facts. First, each portfolio that $\alpha$ is able to choose can also be chosen by $\alpha'$. Second, for each portfolio $\Gamma(\alpha)$, (A3) will imply that

$$\tilde{W}_E(\hat{K}(\alpha),\hat{B}(\alpha),\Gamma(\alpha)) < \tilde{W}_C(\Gamma(\alpha))$$

with probability one. Taken together (A2) and (A4) assert that (18) fails. As a consequence (18) implies

$$N(K(\alpha),B(\alpha)) \geq 1 + K(\alpha) - B(\alpha),$$

which together with proposition 1 implies (17). A similar argument establishes (A1). QED.

*Lemma A2. Suppose that* $u(\bullet,\alpha) = u(\bullet,\alpha')$. *If* $\alpha' \in C$ *and* $\alpha \in E$ *and if* $\Gamma(\alpha',\{\alpha\}) = 0$, *then* (18) *holds. If* $\alpha$ *and* $\alpha'$ *are in* $E$ *and* $\Gamma(\alpha',\{\alpha\}) = 0$, *then*

$$\text{(A5)} \qquad \max_{\substack{\Gamma(\alpha') \\ K(\alpha') \\ B(\alpha')}} \mathrm{E}u(\tilde{W}_E(K(\alpha'),B(\alpha'),\Gamma(\alpha')),\alpha') \geq \max_{\substack{\Gamma(\alpha) \\ K(\alpha) \\ B(\alpha)}} Eu(\tilde{W}_E(K(\alpha),B(\alpha),\Gamma(\alpha)),\alpha).$$

*Proof.* If $\Gamma(\alpha',\{\alpha\}) = 0$, then $\alpha$'s firm is not significant in the portfolio of $\alpha'$. If the right side of (18) was exceeded by the left, $\alpha$ would prefer to switch his role from entrepreneur to capitalist. Since his firm is not significant in $\Gamma(\alpha')$, he can imitate $\alpha'$ and purchase this portfolio. This would yield him a higher utility than he has in the equilibrium. But then the equilibrium condition (12) must fail for $\alpha$, a contradiction. This contradiction implies that the right side of (18) must exceed or equal the left side. The opposite inequality is obtained by a similar argument as is the inequality (A5). QED.

*Proof of Proposition 2.* The proof proceeds by treating each type $i$ separately. We first consider those types in which there are capitalists. For such a type, we choose a specific capitalist $\alpha'$ and consider the set

$$P_{\alpha'} = \{\alpha : \alpha \text{ is an entrepreneur of type } i \text{ and } \Gamma(\alpha',\{\alpha\}) > 0\},$$

which is the set of entrepreneurs of type $i$ in whose firms $\alpha'$ makes a significant investment. This set is of Lebesgue measure zero. Thus, for almost all entrepreneurs of type $i$, $\Gamma(\alpha',\{\alpha\}) = 0$. Because of this, lemmas A1 and A2 imply that (17) and (18) hold for almost all entrepreneurs of type $i$.

If there are capitalists of every type, the proof is completed by the above argument. Suppose, however, that there is some type $i$ which contains no capitalists. If we choose an arbitrary entrepreneur $\alpha'$ of this type, it will again be true that $\mu(P_{\alpha'}) = 0$. Thus, again $\Gamma(\alpha',\{\alpha\}) = 0$ except for a set of type $i$ entrepreneurs of measure zero. Since $\alpha'$ was chosen arbitrarily, lemma A2 implies that for any $\alpha'$ of type $i$, (A5) holds for almost all entrepreneurs $\alpha$ of type $i$. But this can only happen if (19) holds for almost all $\alpha$ and $\alpha'$ of type $i$. Thus, almost all entrepreneurs of type $i$ must have the same expected utility in equilibrium. Then because of lemma A1 there must be a $k$ such that (A1) holds for almost all $\alpha$ and $\alpha'$ of type $i$.

It now remains to be shown that $k = 1$. To accomplish this, we will show that if $k < 1$, then almost any entrepreneur of type $i$ can raise his extended utility over that which he obtains in equilibrium by becoming a capitalist. The only problem that may arise in this argument is that by becoming capitalists and eliminating their firms, the type $i$ entrepreneurs might lose significant investment opportunities.

To see how this possibility can be avoided, we must again choose some representative type $i$ entrepreneur $\alpha'$. He can be chosen so that (19) and (A1) hold for almost all other $\alpha$ of type $i$. As before, $\Gamma(\alpha',\{\alpha\}) = 0$ for almost all type $i$ entrepreneurs $\alpha$. Thus, almost any type $i$ entrepreneur $\alpha$ can still buy the portfolio $\Gamma(\alpha')$ after exiting as an entrepreneur. If $\alpha$ does exit to become a capitalist and does buy $\Gamma(\alpha')$, his random income will be

$$A + r[1 - \int_E N(K(\beta),B(\beta))\Gamma(\alpha',d\beta)] + \int_E \tilde{\pi}(K(\beta),B(\beta))\Gamma(\alpha',d\beta).$$

With probability one this will exceed, by $1 - k$, the random equilibrium income of $\alpha'$, which is

$$A + r[k - \int_E N(K(\beta),B(\beta))\Gamma(\alpha',d\beta)] + \int_E \tilde{\pi}(K(\beta),B(\beta))\Gamma(\alpha',d\beta).$$

Thus, by becoming a capitalist and buying $\Gamma(\alpha')$, almost every entrepreneur $\alpha$ of type $i$ can obtain a higher expected utility than $\alpha'$ does. But recall that the equilibrium expected utility of $\alpha'$ is equal to that of almost all entrepreneurs $\alpha$ of type $i$. Thus, when $k < 1$, almost all entrepreneurs of type $i$ will prefer to be capitalists rather than entrepreneurs. Thus, $k < 1$ is not consistent with equilibrium. QED.

# Appendix 2

*Proof of Lemma 3.* Note first that

$$\text{(A6)} \qquad F^i_{K_i}(K_i,C_i) = Eu'(\tilde{t},i)\left[\frac{g_{K_i}(K_i,\tilde{x})}{(1+K_i)} - \frac{g(K_i,\tilde{x})}{(1+K_i)^2}\right]$$

and

$$\text{(A7)} \qquad F^i_{K_iK_i}(K_i,C_i) = Eu''(\tilde{t},i)\left[\frac{g_{K_i}(K_i,\tilde{x})}{(1+K_i)} - \frac{g(K_i,\tilde{x})}{(1+K_i)^2}\right]^2$$
$$+ Eu'(\tilde{t},i)\left[\frac{g_{K_iK_i}(K_i,\tilde{x})}{(1+K_i)} - 2\frac{g_{K_i}(K_i,\tilde{x})}{(1+K_i)^2} + 2\frac{g(K_i,\tilde{x})}{(1+K_i)^3}\right],$$

where

$$\tilde{t} = A + r[1 - C_i] + C_i \frac{g(K_i, \tilde{x})}{1 + K_i}.$$

Also, notice that (A6) and (A7) imply that

$$\text{(A8)} \qquad F^i_{K_iK_i}(K_i, C_i) = Eu''(\tilde{t}, i)\left[\frac{g_{K_i}(K_i, \tilde{x})}{(1 + K_i)} - \frac{g(K_i, \tilde{x})}{(1 + K_i)^2}\right]^2 + Eu'(\tilde{t}, i)\left[\frac{g_{K_iK_i}(K_i, \tilde{x})}{(1 + K_i)}\right] < 0,$$

if $F^i_{K_i}(K_i, C_i) = 0$. Since $g_{KK} < 0$ for all $x \neq \underline{x}$, the strict inequality in (A8) holds even if type $i$ individuals are risk neutral and $u''(\bullet, i) = 0$. Thus, if there exists $\bar{K}_i(C_i)$ such that

$$F^i_{K_i}(\bar{K}_i(C_i), C_i) = 0,$$

then $\bar{K}_i(C_i)$ must be a unique global maximizer of $F^i(K_i, C_i)$.

We now prove that for each $C_i$ and $r$, there exists a $K_i$ satisfying $F^i_{K_i} = 0$, which, because of (A6), is equivalent to

$$\text{(A9)} \qquad Eu'(\tilde{t}, i)[g_{K_i}(K_i, \tilde{x})(1 + K_i) - g(K_i, \tilde{x})] = 0.$$

Now under our assumptions

$$g_K(0, x)[1 + 0] - g(0, x) = g_K(0, x)\begin{cases} > 0 \text{ if } x \neq \underline{x} \\ = 0 \text{ if } x = \underline{x} \end{cases}.$$

By continuity, there exists a $\underline{K} > 0$ such that $K \leq \underline{K}$ implies

$$g_K(K, x)[1 + K] - g(K, x) > 0 \text{ if } x \neq \underline{x}.$$

Thus $K_i \leq \underline{K}$ implies

$$Eu'(\tilde{t}, i)[g_{K_i}(K_i, \tilde{x})[1 + K_i] - g(K_i, \tilde{x})] > 0.$$

In addition, there exists a $\hat{K}$ such that $K_i \geq \hat{K}$ implies that

$$\text{(A10)} \qquad g_{K_i}(K_i, x)[1 + K_i] - g(K_i, x) < 0$$

for all $x \neq \underline{x}$. If $x = \underline{x}$, the difference on the left side of (A10) is zero. Thus, $K_i \geq \hat{K}$ implies

$$\text{(A11)} \qquad Eu'(\tilde{t}, i)[g_{K_i}(K_i, \tilde{x})(1 + K_i) - g(K_i, \tilde{x})] < 0.$$

Now the continuity of $u$ and $g$ implies that there exists $\bar{K}_i(C_i) \in (\underline{K}, \hat{K})$ at which (A9) holds and $F^i(\bar{K}_i(C_i), C_i) = 0$. Implicit function theorem arguments guarantee that $\bar{K}_i(C_i)$ is a differentiable function of $C_i$ and $r$.

If $u(\bullet, i)$ is linear, $u'(t, i)$ is independent of the value $t$ taken by $\tilde{t}$. Thus, (A9) reduces to (49). If (47) holds, (A9) becomes

(A12) $$Eu'(\tilde{t},i)[h'(K_i)(1+K_i)-h(K_i)]\tilde{x}\,,$$

which is zero when (51) holds. As noted in the text, (51) and (49) are equivalent when $g$ satisfies (47).

To prove that

(A13) $$\lim_{C_i\to 0}\bar{K}_i(C_i)=K^*\,,$$

recall that $\bar{K}_i(C_i)$ must be in the interval $[\underline{K},\hat{K}]$. If (A13) fails, there will be a sequence $\{C_i^m\}$ converging to zero such that

$$\lim_{m\to\infty}\bar{K}_i(C_i^m)=K^{**}\neq K^*\,,$$

where $K^{**}\in[\underline{K},\hat{K}]$. Since $F^i_{K_i}$ is continuous in $K_i$ and $C_i$,

$$\lim_{m\to\infty}F_{K_i}(\bar{K}_i(C_i),C_i)=F_{K_i}(K^{**},0)$$

$$=\frac{1}{(1+K^{**})}u'(A+r,i)E[g_{K_i}(K^{**},\tilde{x})(1+K^{**})-g_{K_i}(K^{**},\tilde{x})]\neq 0\,.$$

But for each $m$,

$$F^i_{K_i}(\bar{K}_i(C_i^m),C_i^m)=0\,,$$

so that

$$\lim_{m\to\infty}F_{K_i}(\bar{K}_i(C_i^m),C_i^m)=0\,.$$

As a result of this contradiction, (A13) must hold. QED.

*Proof of Lemma 4*. We consider the risk neutral case first. In this case, lemma 3 and (52) imply

$$F^i(\bar{K}_i(C_i),C_i)=E[A+E\,\frac{g(K^*,\tilde{x})}{(1+K^*)}(1-C_i)+C_i\,\frac{g(K^*,\tilde{x})}{(1+K^*)}]$$

$$=A+E\,\frac{g(K^*,\tilde{x})}{(1+K^*)}\,,$$

which is independent of $C_i$. Thus, $C_i$ can be chosen arbitrarily. If (53) holds,

$$\frac{\partial}{\partial C_i}F^i(\bar{K}_i(C_i),C_i)=-r+\frac{Eg(K^*,\tilde{x})}{(1+K^*)}>0\,.$$

Since the marginal utility of $C_i$ is always positive for risk neutral individuals, they will choose $C_i$ to equal the upper bound, $(A/r)+1$, imposed by the constraint that the probability of bankruptcy must be zero. Finally, suppose that (54) holds. In this case,

$$\frac{\partial}{\partial C_i}F^i(\bar{K}_i(C_i),C_i)=-r+\frac{Eg(K^*,\tilde{x})}{(1+K^*)}<0$$

and the optimal $C_i$ is zero.

Now suppose that $i$ is risk averse and consider first the cases in which (52) and (54) hold. Fix $K_i$ at any level and note that

$$F^i_{C_i}(K_i,0) = u'(A + r,i)\left[-r + \frac{Eg(K_i,\tilde{x})}{(1 + K_i)}\right]. \tag{A14}$$

Since $u'(\bullet,i) > 0$ and $K^*$ maximizes $(Eg(K,\tilde{x})/(1 + K))$, the expression for the derivative in (A14) is nonpositive if

$$r \geq \frac{Eg(K^*,\tilde{x})}{(1 + K^*)},$$

i.e., if (52) or (54) hold. Since

$$F^i_{C_iC_i}(K_i, C_i) = Eu''(\tilde{t},i)\left[-r + \frac{Eg(K_i,\tilde{x})}{(1 + K_i)}\right]^2 < 0,$$

it will never be optimal to let $C_i$ be positive regardless of $K_i$. Thus, $\hat{C}_i = 0$ and $\hat{K}_i$ is arbitrary.

Finally, we consider the case in which (53) holds. First, recall that it was shown in lemma 3 that

$$\lim_{C_i\to 0} \bar{K}_i(C_i) = K^*.$$

Because of this and the continuity assumptions made about $u(\bullet,i)$ and $g$,

$$\lim_{C_i\to 0} F^i(\bar{K}_i(C_i), C_i) = F^i(K^*,0) = u(A + r,i).$$

For $C_i \in (0,(A/r) + 1]$, lemma 3 implies that $\bar{K}_i(C_i)$ is differentiable and therefore continuous. Thus, $F^i(\bar{K}_i(C_i), C_i)$ is continuous on the entire interval $[0, (A/r) + 1]$. It therefore attains a maximum $\bar{C}_i$ on this interval. We let $\hat{K}_i = \bar{K}_i(\bar{C}_i)$ and $\hat{C}_i = \bar{C}_i$.

It remains to be shown that $\hat{C}_i > 0$. If $(\partial/\partial C_i)\, F^i(\bar{K}_i(C_i), C_i)$ exists at $C_i = 0$, the positivity of $\hat{C}_i > 0$ can be established by proving

$$\frac{\partial}{\partial C_i} F^i(\bar{K}_i(0),0) > 0. \tag{A15}$$

Unfortunately, even the differentiability of $u(\bullet,i)$ and of $g$ together with (50) does not imply the existence of $(\partial/\partial C_i)\, F^i(\bar{K}_i(0),0)$. As a result, a more complicated proof is required.

First, recall that, because of lemma 3, $\bar{K}_i(C_i)$ is a differentiable function of $C_i$ on $(0,(A/r) + 1]$. Thus, the differentiability of $u(\bullet,i)$ and $g$ imply that $(\partial/\partial C_i)\, F^i(\bar{K}_i(C_i), C_i)$ exists on this interval. Furthermore, when this derivative exists, the envelope theorem implies that

$$\frac{\partial}{\partial C_i} F^i(\bar{K}_i(C_i), C_i) = Eu'(\tilde{t},i)\left[-r + \frac{g(\bar{K}_i(C_i),\tilde{x})}{(1 + \bar{K}_i(C_i))}\right]. \tag{A16}$$

The continuity of $u'(\bullet,i)$ and of $g$, the limiting result (50), and the expression (A16) imply that, for $C_i$ sufficiently small, $(\partial/\partial C_i)$ $F^i(\bar{K}_i(C_i), C_i)$ is approximately

$$\text{(A17)} \qquad u'(A + r, i)\left[ -r + \frac{Eg(K^*, \tilde{x})}{(1 + K^*)} \right],$$

which is positive because of (53). Thus, $F^i(K_i(C_i), C_i)$ is a strictly increasing function near zero. As a result, $\hat{C}_i$ cannot equal zero; i.e., $\hat{C}_i > 0$.

It should be noted that when $(\partial/\partial C_i)$ $F^i(\bar{K}_i(0), 0)$ exists, it equals the expression in (A17) and is therefore positive as required in (A15). QED.

# Notes

1. As mentioned in section 5.7, the introduction of a stock market may not improve efficiency if $N \neq N_C$. This can occur because the stock market may not be Diamond efficient in this case.

2. If, specifically, $u(I,i) = -\exp(-a_i I)$, then it is easily verified that for each $i$, $K_i = \bar{\bar{K}}$, where $\bar{\bar{K}}$ satisfies

$$E\left\{\left[g_K(\bar{\bar{K}},\tilde{x}) - \frac{g(\bar{\bar{K}},\tilde{x})}{(1+\bar{\bar{K}})}\right] \exp\left[\left(\sum_{i=1}^{n} \mu_i a_i^{-1}\right)^{-1} \frac{g(\bar{\bar{K}},\tilde{x})}{(1+\bar{\bar{K}})}\right]\right\} = 0.$$

The equilibrium is completely described if we now let $\nu = 1/(1 + \bar{\bar{K}})$,

$$\gamma_1 = \left[\sum_{i=1}^{n} \mu_i a_i^{-1}\right]^{-1} \frac{a_1^{-1}}{(1+\bar{\bar{K}})},$$

$$\gamma_i = \frac{a_1 \gamma_1}{a_i},$$

and

$$r = \frac{E\left\{g_K(\bar{\bar{K}},\tilde{x}) \exp\left[\left(\sum_{i=1}^{n} \mu_i a_i^{-1}\right)^{-1} \frac{g(\bar{\bar{K}},\tilde{x})}{(1+\bar{\bar{K}})}\right]\right\}}{E\left\{\exp\left[\left(\sum_{i=1}^{n} \mu_i a_i^{-1}\right)^{-1} \frac{g(\bar{\bar{K}},\tilde{x})}{(1+\bar{\bar{K}})}\right]\right\}}.$$

Note that $\bar{\bar{K}}$ is not equal to $K^*$.

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## Comment David Levhari

The paper by Kihlstrom and Laffont is another example of the efficient use of the continuum of traders model to show the optimality of competition. In this particular application it is shown that stock market equilibrium provides a constrained optimum in Diamond's sense without having to use the special assumptions on production and uncertainty that Diamond adopts in his paper. Kihlstrom and Laffont allow firms to possess a regular U-shaped cost function, and yet the equilibrium generated possesses the properties of social optimum in Diamond's sense. Somehow, the existence of a continuum of traders allows us to use some sort of a "law of large numbers" so that the ensuing equilibrium is socially efficient.

The assumptions of Kihlstrom and Laffont are also somewhat special. All individuals have identical abilities, and all of them face identical random variables. There are no learning possibilities, and no changing of subjective probability distributions is allowed. All individuals are similarly endowed. There is no distinction between control and ownership. Thus, there is no distinction between the entrepreneurs and the firms they establish in the aims of maximization.

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With respect to the Aumann-like continuum of traders model, one wonders whether there is also a limiting theorem that as the number of traders tends to infinity in a regular fashion, the market equilibrium becomes efficient. That is, the question is whether a Debreu-Scarf structure can be established to prove that as the number of traders grows in some regular fashion, the market equilibrium tends to an optimal allocation.

Some of the questions that come to mind are as follows: What are the essential simplifications in Diamond that allow him to obtain his results without invoking a continuum of traders? Is the assumption in Kihlstrom and Laffont that firms face identical random variables not oversimplistic? Is it possible to build a framework of similar nature in which firms' ownership and control are not identical and the attitudes of the firms toward risky ventures cannot be identified with those of the entrepreneurs?

The paper is thus an interesting use of the continuum of traders model to show equivalence, in the Aumann sense, between equilibrium and efficiency of allocation, and one may just wonder whether other realistic and possibly more complex assumptions can be incorporated in the present model.