Volume Title: A Theory of the Consumption Function

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Volume Publisher: Princeton University Press

Volume ISBN: 0-691-04182-2

Volume URL: http://www.nber.org/books/frie57-1

Publication Date: 1957

Chapter Title: Consistency of the Permanent Income Hypothesis with Existing Evidence on the Relation between Consumption and Income: Time Series Data

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Chapter URL: http://www.nber.org/chapters/c4407

Chapter pages in book: (p. 115 - 156)

CHAPTER V

Consistency of the Permanent Income Hypothesis with Existing Evidence on the Relation between Consumption and Income: Time Series Data

IN Chapter II, we saw that the permanent income hypothesis implied, under certain conditions, a relation between the aggregate permanent components for a group of consumer units of the same form as the relation between the permanent components for a single consumer unit. That is,

$$(2.10) \qquad c_p^* = k^*(\qquad)\cdot y_p^*,$$

where c_p^* and y_p^* are aggregate, or per capita, permanent consumption and income, respectively, for a group of consumer units, and k^* depends on the form of the function k for a single consumer unit as well as on the distribution of consumer units by the variables entering into k, in particular, i, w, and u.

Suppose that the conditions required to justify (2.10) can be regarded as satisfied for a particular set of data on the aggregate or per capita measured income and consumption of a group of consumer units in each of a series of time units. Suppose, further, that the numerical value of k^* can be taken as roughly the same for the different years or other time units covered, so that the variables that belong in the empty parenthesis of (2.10) need not be specified and can be neglected. Under these conditions, the applications of our hypothesis to hypothetical family budget data that were made in Chapter III, and to actual family budget data in Chapter IV, carry over directly to time series data. It is only necessary to reword the results to allow for the fact that the individual observation is on aggregate or per capita measured consumption and measured income rather than on measured consumption and measured income of a single consumer unit. In particular, the regression of measured consumption on measured income computed from time series will yield a marginal propensity less than the average propensity and an income elasticity of consumption approximately equal to P_{y^*}, where this symbol as before means the fraction of the variance of measured income attributable to variation in the permanent component of income.

This chapter examines the consistency of this simple model with existing time series data. The first topic is the general characteristics of the long-period savings estimates for the United States recently constructed by Raymond Goldsmith. These are the main justification for regarding k^* as numerically constant over this period. Section 2 covers more detailed features of regressions of consumption on contemporaneous incomes—in particular, the effect of the period covered and the form of data on observed elasticities, and the differences between time series and budget study elasticities. Section 3 interprets a number of regressions computed by other writers in which consumption is expressed as a function not only of contemporaneous but also of past income, and reports on a consumption function of a similar kind suggested by our hypothesis. While the data considered cover a fair span of time and a variety of features of consumption behavior, they are, like the budget data, much more limited geographically than would be desirable, being almost entirely for the United States.

1. *Recent Long-period Estimates of Aggregate Savings for the United States*

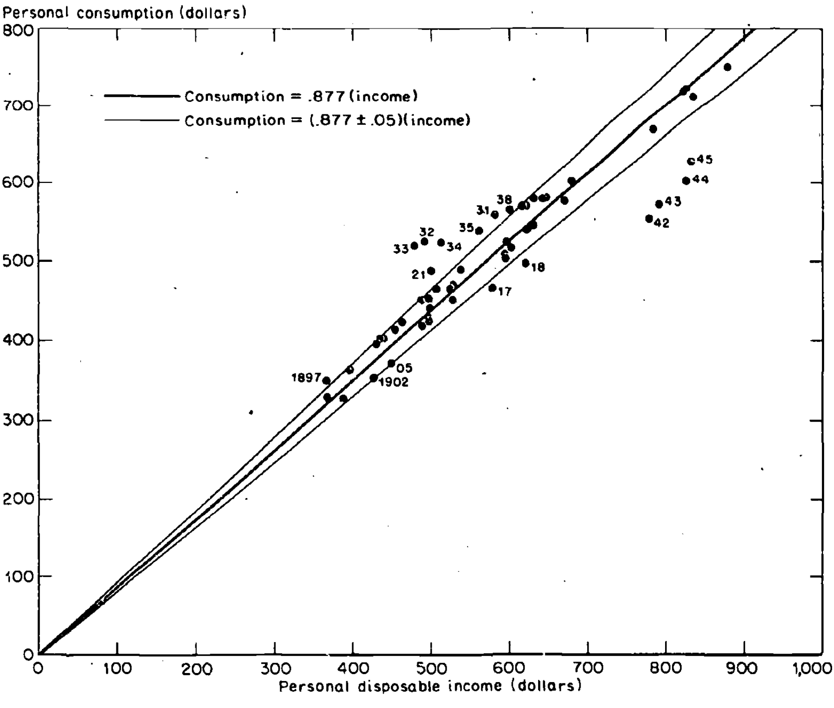
a. THEIR GENERAL PATTERN

Figure 13 plots per capita personal consumption expenditures in the United States against per capita personal disposable income, both expressed in 1929 prices, for the period 1897–1949; it is based on Raymond Goldsmith's comprehensive study of savings, the first study to estimate savings directly year by year for so long a period. The figures plotted treat expenditures on major consumer durable goods in excess of the use value of services rendered by them as savings, and include in consumption only the estimated use value of their services. Personal disposable income measures income received by consumer units after the payment of direct taxes; the variant used here includes increases in government pension and retirement funds, that is, treats social security on an accrued rather than cost basis; it excludes undistributed income of corporations as well as increases in private pension rights and similar income items. It is by no means clear that this concept of income is best for our purposes, since consumer units may take account of accrued but not distributed changes in their capital position other than social security. However, it is certainly fairly close to the desired concept.

Even a cursory examination shows that the general pattern of these data fits our hypothesis remarkably well. On our hypothesis, the ratio of planned or permanent consumption to permanent income depends on factors other than the level of income. If these other factors had been roughly constant, or offsetting, for the period 1897–1949, the ratio of permanent consumption to permanent income would have

FIGURE 13

Relation of Personal Consumption Expenditures per Capita to Personal Disposable Income per Capita, 1897–1949
(1929 prices)



Source: Data made available by Raymond Goldsmith.

been a constant during this period, which means that points in Figure 13 representing permanent consumption and permanent income would all be on a single straight line through the origin. The heavy line on the diagram is such a line, drawn for a consumption ratio of .877, the ratio of average consumption for 1897–1949 to average income for the same period (see line 14 of Table 12). The two lighter lines on either side of the heavy line are for consumption ratios 5 percentage points higher and lower.[1] The fan of three lines

[1] The figure of 5 percentage points was chosen rather arbitrarily, so no special significance should be attached to it. See the next footnote.

contains more than two-thirds of the points, and describes the pattern of the points reasonably well. More detailed study of the dates of the plotted points indicates no marked temporal pattern of the deviations from the central line. Some of the early years lie below, some above, and so on, except for the 1930's, which lie mostly above the line, and the 1940's, which lie mostly below, a phenomenon that is commented on further below. Clearly, on this superficial level, the scatter is not inconsistent with the hypothesis that the observed points were generated from points on the central line by the addition of transitory components of both income and consumption.

The identity of the points that lie outside the lighter lines is even more illuminating than the general consistency of the fan of lines with the points. The points below the lines are for 1942, 1943, 1944, 1945, 1918, 1917, and 1905, in that order from the lowest to the highest fraction of income consumed, and 1902 is only a trifle inside the line.[2] The first six points are all years of wartime inflation; and every year of wartime inflation is included among them. These are years in which one might expect both a positive mean transitory component of income and a negative mean transitory component of consumption: the former, because the wartime incomes were regarded as abnormally and temporarily high; the latter, because of unavailability of goods and patriotic drives to restrict consumption. Both would make for an abnormally low ratio of measured consumption to measured income. The other two points listed, 1902 and 1905, are for years of relatively high prosperity. Business annals record one of the deepest depressions on record in the 1890's, with a business cycle trough in 1894, an incomplete recovery to a submerged peak in 1895, and a relapse to another trough in 1897. A vigorous recovery, one of the sharpest on record, then occurred to a peak in 1899, followed by a mild and brief decline to 1900, and renewed but mild expansion to a peak in 1902. The subsequent decline from 1902 to 1904 is one of the mildest on record and was followed from 1904 to 1907 by a period of continued expansion. All told, the period from 1897 to 1907 was a period of expansion, punctuated by only brief and mild recessions. The year 1902 was a relative peak year in this expansion, the year 1905 a good year, though not a relative peak. In both years it is not

[2] There is a decided break between 1902 and the next higher observation (1899) in the array of points by fraction of income consumed. The fraction is .829 for 1902 and .843 for 1899, a difference of .014. So large a difference does not appear again as we proceed upwards in the array until we come to 1938, the first point above the upper line. The fraction for 1938 is .942, for 1911, the next lower observation, it is .924, a difference of .018. The largest difference between any two neighboring observations falling within the two light lines is .007. The existence of these gaps in the array was a major reason for choosing to locate the light lines ± 5 percentage points from the center line.

unreasonable to suppose that the transitory component of income was positive and that this is the reason why recorded consumption was an abnormally low ratio of measured income.

The points above the upper light line are for 1933, 1932, 1934, 1921, 1931, 1935, 1897, and 1938, in that order from the highest ratio of consumption to income to the lowest. Every year is a year of deep depression; and there is no year that clearly deserves to be so designated that is excluded from the list. These are all, therefore, years in which the transitory component of income was negative: income was lower than it could be expected to be over the long pull. It is not surprising that consumption, being on our hypothesis adjusted to permanent income, was an abnormally high ratio of measured income.

As noted above, all points for the decade of the 1930's are above the heavy line, all but one of the points for the decade of the 1940's below the line. Considerations like those adduced in the preceding two paragraphs make it highly plausible that the transitory component of income was generally negative for the 1930's and positive for the 1940's. This seems a more satisfactory explanation than the sudden emergence of a secular trend.

The consistency of our hypothesis with the general pattern of Figure 13 is not, of course, very strong evidence for the hypothesis. All that has so far been shown is that the hypothesis can explain some of the more striking features of the behavior of savings in this country over the past half-century or so. The later sections of this chapter submit the hypothesis to the much more stringent test of its ability to explain detailed quantitative features of the behavior of these and similar data. But before proceeding to this test, it is interesting to speculate on possible reasons for the rather surprising secular constancy of the ratio of consumption to income revealed by these data, as well as by the various budget studies presented in the preceding chapter.

b. THE CONSTANCY OF k^*

The secular constancy of k^*, while consistent with the permanent income hypothesis, is not in any way required by it. Our hypothesis only says that k for the single consumer unit is a function of variables other than the current level of income; it does not say that k is a numerical constant for each unit, let alone the same constant for different units. Even if the function k were the same over time for each unit or each relevant kind of unit—to avoid the problem of aging—separately, its numerical value could change because of changes in the variables determining k; and even if its numerical value were the same, either because these variables were unchanged or offset one another or because the function was insensitive to their values, the value of k^* for the aggregate could vary because of changes in the relative number of units of various kinds. So the observed rough constancy of k^* is about as much of a puzzle as substantial variations in it would be.

Anything like an exhaustive investigation into this phenomenon is a study in itself; my purpose here is only to speculate about some of the major factors such a study would have to examine in detail.[3] Of the variables entering into k, two—the rate of interest and the ratio of wealth to income, or the ratio of nonhuman to human wealth—have probably affected savings in opposite directions. The rate of interest—i.e. some kind of an average rate of interest—apparently rose from the turn of the century to about 1920 and then declined over the next three decades, ending at a lower level than that at which it began.[4] However, the interpretation of these changes, recorded most fully in bond yields, is clouded by changes in price levels which make the nominal interest rates imperfect measures of "real" rates of return and by frequently divergent movements of rates of return on fixed dollar obligations and on equities. Taken at face value, the initial rise in the rate of return would have tended to lower k^* and the subsequent fall to raise it by an even larger amount. On the other hand, to judge by the fraction of the national income estimated to have been derived from property, the ratio of nonhuman to human wealth appears to have declined, though again the evidence is by no means unambiguous. Any such decline would, other things the same, tend to reduce k^*.

I turn from these movements to three others that are plainer to the naked eye and that, perhaps for that reason alone, I am inclined to regard as of more consequence for the behavior of k^*. These are (1) the sharp reduction in the fraction of the population on farms, (2) the changing distribution of consumer units by size, and (3) the altered role of the state in the provision of security.

Since the turn of the century, there has been a sharp decline in the fraction of consumer units deriving most of their income from the operation of a farm. There seems to have been no comparable change in either direction in the fraction engaged in other entrepreneurial activities. We saw in the preceding chapter that entrepreneurs, including farmers, tend to save a relatively high percentage of permanent

[3] Compare Goldsmith, *A Study of Saving*, I, pp. 6–8, 11–19.

[4] See, for example, W. Braddock Hickman, *The Volume of Corporate Bond Financing since 1900* (Princeton University Press for National Bureau of Economic Research, 1953), p. 129.

income. In consequence, the reduction in the relative number of farm families is a factor making for a reduction in the fraction of income saved or an increase in k^*. Very rough calculations suffice to give an idea of the possible order of magnitude of this effect. Farm operator families apparently receive currently something in excess of one-tenth of total personal disposable income.[5] To judge from changes in the number of farm families, the corresponding fraction could hardly have exceeded one-third in 1900. Suppose farmers then and now saved on the average 20 per cent of their income, a figure that if anything seems too high on the evidence of the preceding chapter; and nonfarmers, 11 per cent. These numbers would imply average savings for both groups combined of 14 per cent in 1900, and of 12 per cent (approximately the observed percentage) currently, or a decline of 2 percentage points in the fraction of income saved. This is surely the maximum possible effect that can be attributed to this factor.

Since the turn of the century, there has been a sharp decline in the average size of the family, from nearly 5 persons per census family to approximately 3.5 persons, or a decline of about 30 per cent. In addition, there has been a change in the distribution of families by size; the extremely large families have become relatively less numerous so that the reduction in average size has been associated with, and in a measure produced by, a greater homogeneity of families by size. It has frequently been argued that the ratio of savings to income decreases as size of family increases. Unfortunately, the statistical evidence for this proposition is marred by the use of an inappropriate technique in deriving it. The studies that I know about have all examined the influence of size of family while holding measured income constant; they have used, that is, the partial correlation technique discussed above in section 2f of Chapter IV. But average income, and presumably average permanent income, tend to increase with size of family. In consequence, even though permanent consumption were the same fraction of permanent income for families of different size, measured consumption for a given measured income would tend to increase with size of family, and so produce the observed statistical results.

It is my impression from rather unsystematic but somewhat more than casual examination of the evidence that this deficiency of analysis does not account for the whole of the observed effect of size of family, that even if comparisons were made at the mean incomes of the several sizes of family, consumption would be found to be a larger

[5] See, for example, Department of Commerce, Office of Business Economics, *Income Distribution in the United States by Size, 1944–1950*, (Washington, 1953), pp. 8–11.

fraction of income, the larger the family.[6] And this statistical impression is consistent with a priori expectations. Children are, after all, a way of achieving security for old age; indeed in many cultures, the primary way. The raising of children can be viewed as a form of capital accumulation, only of human rather than nonhuman capital.[7] One might expect a reduction of savings in this form to be accompanied by an increase in other forms, and our statistics treat as savings only these other forms, so such a shift of form would show up in our data as an increase in savings. At any given time, those families that have fewer children than their neighbors and so are not providing as fully for their security in this form, might be expected to provide more fully in other ways. Over time, the changes in customs that are reducing the extent of reliance on one's children for security in old age—changes which are themselves both a cause and a consequence of the changing size of family—tend to promote accumulation of nonhuman capital. On both grounds, the reduction in the average size of family in the United States is a factor that, by itself, would have produced an increase in the fraction of income recorded as saved and hence a decrease in the observed value of k.

A rough idea of the possible order of magnitude of this effect can be obtained by disregarding the bias in the statistical measurement of the size of family effect. Dorothy Brady has estimated that, for a given measured income, consumption expenditures are proportional to the sixth root of the number of members of the family.[8] This implies that a family of 5 members spends 6 per cent more on consumption than a family of 3.5, or that if consumption of a family of 3.5 is 88 per cent of its income (the approximate average propensity according to Goldsmith's data), the consumption of a family of 5 would be almost 93 per cent of its income. Now this doubtless overestimates the effect of the change in family size, both because of the statistical bias that we have neglected and because it disregards the increased homogeneity of families by size, allowance for which would reduce the amount of correction called for.[9] Yet it makes clear that

[6] This statement oversimplifies the comparison required, since account would have to be taken of any factors that might make average measured income and consumption for a given family size differ from average permanent income and consumption.

[7] Size of family as measured by the Census at any given time is not the same thing as size of family in the sense relevant to this argument, for which the number of children who survive, whether or not they live with their parents, would perhaps be the best measure. A census family may be small because the children have set up separate families. My language is therefore inexact since I use size of families in these two different senses. I believe, however, that this inexactness is not a source of error; there tends to be a high correlation between size of family in the two senses.

[8] See Dorothy Brady, in Goldsmith, Brady, and Mendershausen, *A Study of Saving in the United States*, III, p. 211.

[9] Strictly speaking, the adjustment should be computed not for the arithmetic means

this effect is potentially of substantial magnitude. Perhaps it would not, by itself, have produced the rise of 5 percentage points in the fraction of income saved that these calculations suggest; but it might easily have offset or more than offset the decline of 2 percentage points that we estimated as the maximum possible effect of the decline in the relative number of farmers. This effect of changing size of family is hardly ever mentioned in discussions of the secular trend of savings;[10] yet it may be one of the major factors at work.

Over the period covered by these data, a drastic change has occurred in the responsibilities undertaken by the state to provide assistance to the aged, unemployed and otherwise dependent. This change has had divergent results on the particular data under discussion. The availability of assistance from the state would clearly tend to reduce the need for private reserves and so to reduce private saving—it is equivalent, in terms of our hypothesis, to a reduction in the variance of transitory components. However, the data under discussion include as personal savings the increases in government pension and retirement funds. If these fully matched the corresponding increase in the present value of accumulated benefits, the combined result might be expected to be an increase in recorded savings: a dollar in the form of a reserve held by the government and available to the individual only under narrowly specified circumstances is worth less to him than a dollar in privately held reserves that he can dispose of at will; in consequence, each dollar increase in government held reserve would tend to produce less than a dollar decrease in private savings. In fact, however, social security obligations are not fully funded; the increase in accumulated benefits exceeds the increase in government pension and retirement funds. It may well be, therefore that the increase in these funds has been less than the decrease in private savings that the existence of the corresponding benefit programs has produced. The conclusion is that, without much more detailed analysis, it is not possible to say whether the net effect of governmental social security and other programs has been to increase or to decrease recorded savings as a fraction of income, let alone by how much.[11]

These speculations are highly inconclusive, and a fuller and more

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alone but for the whole distribution, or, what comes to the same thing, for the geometric means of the distribution.

[10] The discussion by Brady, *ibid.*, is a notable exception.

[11] Another factor similar in kind to the emergence of savings through government is saving through corporations, either in the form of undistributed corporate profits or pension rights. I have neglected these; the first, because Goldsmith's figures show it to have been a roughly constant fraction of savings over the period covered; the second, because it has started to become quantitatively important after the period in question.

satisfactory analysis is much to be desired, but perhaps they suffice to show that there have been offsetting forces at work on the ratio of consumption to income. If these forces had all been in one direction, particularly if they had all been working in the direction of reduced saving, the observed constancy would tell against our hypothesis and for the more usual absolute income hypothesis, since it would be tempting to call in the rising average real income as a counterweight to the other factors. The fact that the forces here mentioned have been in different directions is hardly strong evidence for our hypothesis, but at least it raises no disturbing questions about the consistency of the hypothesis with the observed constancy of k^*.

We noted in the preceding chapter that the average propensity computed from United States budget studies was remarkably constant over a period of some six decades. Three major points have to be allowed for before the apparent agreement between this finding and the constancy shown by Goldsmith's data can be regarded as additional evidence that k^* has been roughly constant. (1) As noted in section 2d of Chapter IV, the earlier budget studies were for wage earners alone, the later studies, for a broader group. The use of withdrawals as a measure of income for entrepreneurial groups lessens the resulting noncomparability of the figures but probably does not eliminate it entirely. If the figures were corrected for this difference in coverage, presumably they would show a secular increase in the ratio of consumption to income or decrease in the savings ratio. (2) Goldsmith's savings figures include the value of the increase in the stock of durable goods; the budget data do not. Goldsmith points out that consumer durable goods account for an increasing fraction of savings and that the ratio of savings excluding consumer durable goods to income has declined over the period covered.[12] The decline is of the order of 2 percentage points. Adjustment of the budget study data to exclude the value of the increase in the stock of consumer durable goods from consumption and to include it in savings would therefore make for a secular decrease in the ratio of consumption to income or increase in the savings ratio. (3) The consumer budget data do not include the increase in government pension and retirement funds as savings. Inclusion of these items would make for a secular decrease in the ratio of consumption to income or increase in the savings ratio.

The consistency of the budget data and the time series data presumably means that points (2) and (3) just about offset point (1). In view of the likely size of these effects, such a result seems not at all implausible.

[12] See Goldsmith, *A Study of Saving in the United States*, I, p. 7.

2. *Regressions of Consumption on Current Income*

Table 12 summarizes some of the consumption-income relations that have been computed for the United States both by Goldsmith, from the data considered in the preceding section, and by Ferber, from related data compiled by him. As for the budget studies summarized in Table 1 of the preceding chapter, the computed marginal propensity is below the computed average propensity for every relation in Table 12, so that the elasticity of consumption with respect to measured income is uniformly less than unity. We have noted repeatedly how this is required by our hypothesis and how, in light of the highly stable average propensity, it makes it impossible to regard the computed functions as stable relations between consumption and income.

The marginal propensities to consume recorded in Table 12 vary much more widely than the average propensities; in consequence, so do the income elasticities of consumption, which are the ratios of the marginal propensities to the average propensities. The recorded marginal propensities vary from .45 to .93; the recorded elasticities, from .48 to 1.00. In small part, these differences reflect differences in the definition of consumption and in the basic data (section b below). For the most part, however, they reflect differences in the periods covered by the series from which they were computed. The marginal propensity and elasticity tend to be relatively low when the period covered is short, and especially when it includes the Great Depression; they tend to be higher, the longer the period covered. How this result fits our hypothesis is considered in section a below. These marginal propensities and income elasticities vary much more widely than the corresponding values for the United States computed from budget studies and recorded in Table 1. Why this should be so is considered in section c below.

a. EFFECT OF PERIOD COVERED

On the simple model under consideration, two features of the period covered can be expected to affect the observed income elasticity: its length, and its particular historical characteristics.

The length of the period is important because, other things the same, P_{y^*}, and so the observed income elasticity, can be expected to be higher, the longer the period covered, provided that the society in question is undergoing a systematic secular change in income. The total variance of income equals the variance contributed by the transitory component plus the variance contributed by the permanent component, given our assumption that the two components are

TABLE 12

Relation between Consumption and Income Based on Time Series Data for the United States, for Different Periods and Concepts of Consumption

Period Covered	*Concept of Consumption Expenditure*	*Average Disposable Income per Capita (1929 prices)*	*Average Propensity to Consume*[a]	*Marginal Propensity to Consume*[b]	*Income Elasticity of Consumption*[c]
		A. Based on Data and Computations of Robert Ferber			
1. 1929 through 1940	D	$489	.97	.78	.80
2. 1923 through 1940	D	490	.97	.79	.82
3. 1923 through 1930, 1935 through 1940	D	510	.96	.93	.97
		B. Based on Data and Computations of Raymond Goldsmith			
4. 1897 through 1949, excl. 1917, 1918, 1930 through 1933, 1942 through 1945	D	$559	.91	.91	.996
5. 1897 through 1949	D	578	.89	.74	.83
6. 1897 through 1906	ND	420	.89	.72	.81
7. 1907 through 1916	ND	495	.89	.65	.73
8. 1919 through 1929	ND	591	.88	.60	.68
9. 1929 through 1941	ND	607	.94	.45	.48
10. 1897 through 1914	ND	451	.89	.87	.97
11. 1915 through 1929	ND	581	.87	.69	.80
12. 1930 through 1949	ND	691	.87	.46	.53
13. 1897 through 1949, excl. 1917, 1918, 1942 through 1945	ND	558	.90	.82	.91
14. 1897 through 1949	ND	578	.88	.70	.80

D = Consumption includes expenditure on consumer durable goods.

ND = Consumption excludes expenditure on consumer durable goods; includes estimated value of services rendered by durable goods.

[a] Ratio of average consumption expenditure for indicated period to average income for same period.

[b] Throughout the value of b in a regression of the form $c = a + by$, where c = personal consumption expenditure per capita in constant prices, y = personal disposable income per capita in constant prices.

[c] Ratio of the marginal propensity to consume to the average propensity to consume, as defined in notes a and b. It is therefore the elasticity of the regression at the point corresponding to mean income and mean consumption.

Source:

Part A, Robert Ferber, *A Study of Aggregate Consumption Functions*, National Bureau of Economic Research, Technical Paper 8, 1953. Part B, average disposable income and average consumption computed from annual data for 1897 to 1949 made available by Raymond W. Goldsmith and based on his *A Study of Saving in the United States*, Princeton University Press, 1956, Vol. I, Table T-1, col. 2, and Table T-6, col. 1 minus col. 5; Vol. III, Part V, Table N-2, col. 5. Marginal propensity to consume equal to 1 minus marginal propensity to save in Vol. III, Part IV, Table Y-1, p. 393, and Table Y-4, p. 400.

uncorrelated. The variance contributed by the transitory component is not systematically affected by lengthening the period; by definition, the transitory components are largely random and short-lived. True, the variance may be larger at one time than another—this is why the historical characteristics of the period are important—but there is no reason why it should be systematically larger or smaller for a long than for a short period.[13] The variance contributed by the permanent component, on the other hand, tends to be systematically larger, the longer the period covered; the more widely separated two dates are, the larger the secular difference in income between them tends to be. As between two neighboring years, the change in the permanent component may well be small relative to the change in the transitory component. On the other hand, between 1900 and 1950, say, any transitory effect is almost certain to be swamped by the secular change in the permanent component. P_{y^*}, the ratio of the variance contributed by the permanent component to the total variance, will therefore tend to be higher, the longer the period, and to approach unity as the period is indefinitely lengthened. If secular change were the only source of variation in the permanent component, the lower limit of P_{y^*} would be zero and this limit would tend to be approached as the length of the period covered approached zero. Since there are other sources of variation in the permanent component, all one can say is that P_{y^*} tends to approach some lower limit greater than zero as the length of the period approaches zero.

The figures in Table 12 conform to this expectation very well indeed. In almost every case, the elasticity for a longer period is higher than for the shorter periods contained within it, if the data are otherwise comparable—note that lines 3, 4, and 13 do not refer to shorter periods than lines 2, 5, and 14, respectively, but to regressions computed on the basis of only some of the years within periods of the same length; these comparisons are considered separately below. For Ferber's data, only one comparison is possible, between lines 1 and 2. The elasticity for line 2, the longer period, is higher than for line 1. For Goldsmith's data, the calculations shown on page 128 summarize the results.

If a steady secular trend were the only factor producing differences in permanent income, it would be possible to predict the quantitative as well as the qualitative effect of lengthening the period. For example, if the income elasticity were .675 for each 11 year period, it would be .974 for a 47 year period; if it were .765 for each 18 year

[13] This statement should be taken as referring to the variance of logarithmic components, or the ratio of the variance to the square of the mean income, or else the mean income should be impounded in *caeteris paribus*.

Lines	*Average Length of Period (years)*	*Years Included*	*Average Income Elasticity*
6, 7, 8, 9	10.75	1897 through 1941, excl. 1917, 1918	.675
13	47	1897 through 1949, excl. 1917, 1918, 1942 through 1945	.912
10, 11, 12	17.67	1897 through 1949	.765
14	53	1897 through 1949	.798

period, it would be .965 for a 53 year period.[14] As these examples show, the effect of lengthening the period, when computed in this way, is uniformly greater than the effect revealed by Table 12. And this is as it should be. For factors other than secular trend produce differences in permanent income so that the computed figures are estimates of the maximum effect to be expected, on our hypothesis, from lengthening the period. It would be most disturbing if the observed effects exceeded these maxima; the fact that they do not lends some minor additional support to our hypothesis.

Comparison of line 2 with line 3, line 4 with line 5, and line 13 with line 14 testifies to the effect of the character of the period covered. In each case, the two regressions in a pair are for data covering the same time span; however, one is based on data for fewer years within that time span. In each case, the years excluded are not an arbitrarily chosen set of years at the beginning or end of the period but years regarded as "abnormal." These years are bunched and come somewhere inside the period, so they do not reduce the range of variation in the permanent component introduced by secular factors. Their exclusion may reduce the variance contributed by the permanent component, but if so, by a much smaller amount than if the corresponding number of years were taken from one end or the other of the period; it is even possible that their exclusion increases the variance

[14] Let $y_p = a + xt$ be the permanent component, where x is the constant increment per time unit and t stands for time. The variance of the permanent component for any period is then $x^2(n^2 - 1)/12$, where n is the number of time units in the period. Let σ^2 stand for the variance contributed by the transitory component. Then

$$P_y = \frac{x^2\left(\frac{n^2-1}{12}\right)}{x^2\left(\frac{n^2-1}{12}\right) + \sigma^2} = \frac{\frac{n^2-1}{12}}{\frac{n^2-1}{12} + \frac{\sigma^2}{x^2}}.$$

Given P_y and n, one can compute σ^2/x^2; given σ^2/x^2 and n, one can compute P_y, which is how the figures in the text were obtained.

contributed by the permanent component. On the other hand, the "abnormal" years are clearly characterized by relatively high transitory components of income, so their exclusion lowers the variance contributed by the transitory component much more than the exclusion of the same number of years chosen at random. The exclusion of "abnormal" years might therefore be expected to *raise* rather than lower P_{y*} and so to have an effect precisely the opposite of that produced by dropping years from the beginning or end of the period. The systematic increase in the computed elasticity in Table 12 when "abnormal" years are eliminated is therefore fully consistent with, and indeed predicted by, our hypothesis.

There is a strong tendency in Table 12 for elasticities for short periods that include the Great Depression to be low: the elasticity in line 9 is less than in lines 6, 7, or 8; in line 12, than in lines 10 or 11. The explanation is presumably the large variation in the transitory component of income during this period and the resultant relatively low value of P_{y*}.

b. EFFECT OF FORM OF DATA

The relations summarized in Table 12 are all between figures on consumption and income expressed per capita and deflated to correct for price changes. Similar relations have been computed between per capita figures in current prices and between aggregates, both deflated and in current prices. How, on our hypothesis, would one expect the form of the data to affect the results?

Consider, first, correction for population. Since the population of the United States has been growing secularly along with income, the secular rate of rise of aggregate real income has been decidedly higher than of per capita real income—approximately 3 per cent per year rather than 2 per cent. Along the lines of the analysis of the preceding section, this increases the variance in the permanent component contributed by secular factors. On the other hand, there seems little reason why the variance of the transitory component should be any larger, and some why it should be smaller, in aggregate figures than in per capita figures (provided, of course, allowance is made for the difference in the absolute level of the two series). Population change proceeds smoothly and so is not likely to be an important source of random or transitory movements in either aggregate or per capita income.

To put the point in another way, our aggregate function is the summation of functions for individual consumer units. The aggregate permanent component is the sum of the permanent components for the separate consumer units; the aggregate transitory component, the sum of the transitory components. The consumer unit can be viewed as reacting to a permanent component expressed either per capita or as a total for the consumer unit. Long-run changes in number or size of consumer units, or short-run changes that are anticipated, introduce transitory elements into neither total nor per capita income. They simply change either the number of units for which permanent components are aggregated or the relation between total and per capita components. Their effect on the variance of the permanent components depends on the facts of the situation.

If it so happened that population growth was accompanied by a decline in per capita income, aggregate income might vary less over time than per capita income. In fact, of course, in the United States population growth has been accompanied by a rise in per capita income, so the (relative) variance of the permanent component clearly tends to be greater for aggregate than for per capita income. Unanticipated short-run changes in either the number of consumer units or their size do not affect the permanent components; they do introduce transitory elements. Whether the effect is larger for aggregate or per capita income depends again on the particular circumstances. It may be conjectured that the most frequent source of such unanticipated changes is the birth of children, which might be expected in general to have no effect on the total income of the consumer unit while introducing a transitory component into per capita income. If this is so, unanticipated population changes would increase the (relative) variance of transitory components more for per capita than for aggregate income. The effects of population changes on the permanent and transitory components thus reinforce one another: both make for a larger value of P_y for aggregate than for per capita figures. On our hypothesis, we are therefore led to expect income elasticities to be higher when computed from aggregate than from per capita data.

Long-run or anticipated short-run changes in prices have much the same effect as corresponding changes in population: they introduce no transitory elements into either current or deflated income. Again, their effect on the variance of the permanent component depends on the facts. If prices tended to be high when output was low, and low when output was high, current income would vary less than deflated income. Again, the facts are the reverse. Over the past fifty years, prices have on the average displayed a secular rise and so has output; within the period, prices and output have generally tended to move together during cyclical swings. The relation is much the same during the two wartime periods.[15]

[15] A minor qualification is required for these periods: prices and aggregate output

In consequence, the variance of the permanent component can clearly be expected to be greater for current than for deflated income. It is much more difficult to make a firm judgment about the effect of unanticipated short-run changes in prices. They introduce transitory elements into both current and deflated income; and, in the short run, movements in prices and output are more likely to be negatively related than over rather longer periods. Nonetheless, it seems not implausible that even in short-run periods, unanticipated changes in prices introduce larger transitory changes into current than into deflated income. On this analysis, the variance of both the permanent and transitory components is larger for current than for deflated income, so that an unambiguous conclusion cannot be reached about the size of P_{y^*}, though there is perhaps some presumption that the effect on the permanent components, being clearer, also tends to be larger and hence that P_{y^*} is generally larger for current than for deflated income. This very weak result can be sharpened by taking account of the length of the period. Along the lines of the preceding section, the effect on the transitory component is independent of the length of period covered; the effect on the permanent component increases with the length of the period. We are therefore led to expect, on our hypothesis, that income elasticities will generally tend to be higher for the United States when computed from current than from deflated income; that this tendency will be strongest when the period covered is fairly long, and that it may be weak or nonexistent for short periods.

These predictions about the effect of correcting for population and price changes correspond closely to the available evidence. Table 13 gives marginal propensities to consume computed by Ferber and Goldsmith from time series data in different forms and for various periods. Our predictions are, it is true, in terms of income elasticities rather than marginal propensities. However, since the average propensity to consume is likely to differ only negligibly with the form of the data, the marginal propensities on any one line are approximately in the same proportion as the elasticities, so it did not seem worth computing the elasticities.[16]

moved together, but at times prices and the fraction of output corresponding to disposable consumer income may have moved in opposite directions.

[16] For each year separately, of course, the ratio of consumption to income is identically the same regardless of the form of the data, provided the same population and price series are used to deflate both consumption and income. For any period of years, however, the ratio of average consumption to average income need not be the same, since this is a weighted average of the ratios for the individual years and the weights are different for the different forms of the data. However, these differences in weights are hardly likely to lead to sizable changes in the ratio of the averages.

TABLE 13

Marginal Propensities to Consume Computed from Four Different Forms of Time Series Data for the United States

		Marginal Propensity to Consume			
	Concept of Consumption Expenditure	*Current Prices*		*Deflated*	
Period Covered		*Aggregate*	*Per Capita*	*Aggregate*	*Per*
		A. Computed by Robert Ferber			
1. 1929 through 1940	D	.848	.853	.800	.7
2. 1923 through 1940	D	.864	.870	.858	.7
3. 1923 through 1930, 1935 through 1940	D	.965	.947	.964	.9
		B. Computed by Raymond Goldsmith			
4. 1897 through 1929	D	.89			.8
5. 1897 through 1949, excl. 1917, 1918, 1930 through 1933, 1942 through 1945	D	.913			.9
6. 1897 through 1949	D	.84			.7
7. 1897 through 1906	ND	.80	.78	.77	.7
8. 1907 through 1916	ND	.77	.72	.65	.6
9. 1919 through 1929	ND	.72	.67	.74	.6
10. 1929 through 1941	ND	.60	.60	.52	.4
11. 1897 through 1914	ND	.90	.89	.89	.8
12. 1915 through 1929	ND	.89	.89	.84	.6
13. 1930 through 1949	ND	.75	.72	.58	.4
14. 1923 through 1940	ND	.57	.60	.60	.3
15. 1915 through 1929, excl. 1917, 1918	ND	.87	.88	.82	.7
16. 1930 through 1949, excl. 1942 through 1945	ND	.81	.80	.69	.6
17. 1897 through 1929	ND	.86	.86	.84	.7
18. 1897 through 1941	ND	.88	.86	.89	.8
19. 1897 through 1949, excl. 1917, 1918, 1942 through 1945	ND	.86	.86	.86	.8
20. 1897 through 1949	ND	.81	.80	.79	.7

D = Consumption includes expenditure on consumer durable goods.

ND = Consumption excludes expenditure on consumer durable goods; includes estimated of services rendered by durable goods.

Source:

Lines 1 to 3, Robert Ferber, *A Study of Aggregate Consumption Functions*, National E of Economic Research, Technical Paper 8, 1953. Lines 4 to 20, Raymond W. Goldsmith, *A St Saving in the United States*, Princeton University Press, 1956, Vol. III, Table Y-1, p. 393, Tabl p. 400.

The effect of using per capita rather than aggregate data is as follows:

Marginal Propensity for Aggregate Data	Number of Entries for: Current Data	Deflated Data	Total
Greater than for per capita	9	16	25
Same	4	1	5
Less than for per capita	4	0	4
Total	17	17	34

There are only four clear exceptions to the predicted tendency for marginal propensities to be greater when computed from aggregate than from per capita data, and even these are illuminated by the preceding analysis. All four are for current data. Two are for the period 1923 through 1940, the other two for 1929 through 1940, and 1915 through 1929 (excluding 1917 and 1918). Because of the Great Depression, the correlation for 1923 through 1940 between changes in population and in real output is probably negative; and between changes in population and money income in current dollars—which is what is relevant—almost certainly negative; much the same is true for 1929 through 1940 though to a smaller extent; for 1915 through 1929, the real output correlation is positive, but the sharp drop in prices in 1921 and the generally lower level in the 1920's may well have made the money-income correlation negative. These cases are, therefore, exceptions to the explicit prediction but not to the analysis leading to it.

The effect of using deflated rather than current data is as follows:

Marginal Propensity for Current Data	Number of Entries for: Aggregate Data	Per Capita Data	Total
Greater than for deflated	13	17	30
Same	1	0	1
Less than for deflated	3	0	3
Total	17	17	34

The general tendency is again as predicted; indeed, even more clearly than for the preceding comparison. This result is a bit disturbing, since our analysis leads to a more unambiguous conclusion about the effect of adjusting for population change than for prices. Presumably, the explanation is to be found in a factor neglected in our earlier analysis: the relative size of the longer-term movements in population and prices. Population roughly doubled from 1900 to 1950; consumer prices considerably more than doubled from 1900 to 1920, fell by well over a third from 1920 to 1933, and then nearly doubled by 1948, so that by 1950, prices were about triple their level in 1900, and there had been a substantial additional movement within the period. Both the larger secular movement in prices than in population and the tendency for prices and output to move together during many of the shorter swings enhanced the effect of correcting for price changes on the variance of the permanent component.

The exceptions to the general tendency are less illuminating for prices than for population. They are for aggregate data and 1919 through 1929, 1923 through 1940, and 1897 through 1941. Only one is for the kind of brief period for which our analysis suggests exceptions, and there is not much all three have in common which distinguishes them from the rest of the observations.

The three comparisons in Table 13 not included in the preceding summaries, lines 4, 5, and 6, all conform to expectation: the marginal propensity is larger when computed from current aggregate data than from deflated per capita data, both adjustments working in the same direction.

The entries in Table 13 are by no means all independent. The Ferber and Goldsmith data have common roots; many of the relations computed by each are for periods that overlap or that differ only by the inclusion or exclusion of a few years. The number of comparisons listed in the preceding summary tables therefore greatly overstates the number of independent observations, a consideration that reduces the significance to be attached to this agreement between experience and the implications of our hypothesis.

C. THE RELATION BETWEEN TIME SERIES AND BUDGET ELASTICITIES

On our hypothesis, income elasticities of consumption computed from time series data and from budget data are estimates of different things. Neither tells anything directly about consumption behavior—or rather adds anything to what is incorporated in our hypothesis. Both measure instead a feature of the income structure, and they measure different features. The budget elasticity measures the fraction of the variance of the incomes of a group of consumer units *at a point in time* contributed by differences in permanent components. The time series elasticity measures the fraction of the variance of aggregate or per capita incomes of a *series of time units* contributed by differences in permanent components. These two features of the income structure are not entirely unrelated. If, for example, all differences among consumer units were attributable to permanent components, the transitory component, being zero for each unit, would be zero for all taken together. In consequence, all differences among years (or other time units) would also be attributable to differences in permanent components, so the income elasticity would, on our hypothesis, be unity whether computed from budget data or time series data. At the other extreme, the connection is much looser. Even if all differences among consumer units were transitory, so that all had the same permanent component, this common permanent component might, and presumably would, change from year to year; on our hypothesis, the elasticity computed from budget data would be zero; the elasticity computed from time series data would be greater than zero and, indeed, might be close to unity, since the transitory component could average out nearly to zero for each time unit separately. The relation is equally loose for the intermediate cases. The influence of the first extreme gives some reason to expect that if, say, the elasticity computed from budget data is systematically higher for comparable groups for country *A* than for country *B*, the elasticity computed from time series data for periods of equal length will also be higher for country *A*. But I have not been able to find any way to predict the quantitative relation between the two.

We have seen that time series elasticities depend critically on the length and character of the period covered: they tend to be low for short periods, and to increase with the length of the period covered, at least for communities experiencing a secular change in income. Budget elasticities depend primarily on the characteristics of the group covered. Consider a fairly broad group within which there are substantial differences in permanent income so that the elasticity is reasonably high—say urban or all families in the United States for whom the elasticity is about .8. The time series elasticity might then be expected to be less than this elasticity when computed from data for a short period, and to exceed it when computed from data for a long period. The length of period for which the two are equal cannot be expected to be a constant; it depends critically on the characteristics of the period, being relatively short for a period characterized by rapid and smooth secular progress, relatively long for a period characterized by stagnation and violent short-period movements, For example, for the 10 year period, 1897 through 1906, which was just such a period of rather smooth and rapid growth, the elasticity computed by Goldsmith (.81) about equals the budget elasticity; for the 12 year period, 1929 through 1941, which was a period of stagnation and sharp short-period movements, the elasticity computed by Goldsmith (.48) is decidedly less than the budget elasticity; for the whole 53 year period, 1897 through 1949, which is very much of a mixture of periods of smooth, rapid growth and periods of violent short-term movements, the elasticity (.80) is about equal to the budget elasticity. The relevant period is thus in the one case 10 years, in the other, over 50 years.

One other feature brought out in our earlier discussion of Table 12 deserves attention: the apparently greater variability among time series elasticities than among budget elasticities for similar groups. One reason why this is to be expected on our hypothesis is explicit in the preceding discussion: the effect of length of period covered means that the time series elasticities in Table 12 are estimates of different things, and consequently vary for a reason that does not apply to budget elasticities for similar groups. Another reason is implicit in the discussion of the character of the period covered: the time series elasticities are computed for very small effective samples—at most, 53 items; such samples can therefore be expected to differ widely among themselves, and so to yield widely differing estimates of the value of P_{y^*}, even if, in some sense, P_{y^*} is not subject to long-run secular change. The budget elasticities, on the other hand, are computed from very much larger samples, generally numbering in the thousands. Put differently, both types of elasticities vary foı two reasons: underlying differences from time to time in the characteristics of the income structure that they estimate, and sampling errors. Both sources of variation can be expected to be larger for time series elasticities: the first, because of the importance of length of period; the second, because of the drastically smaller size of sample.

A number of attempts have been made in recent years to combine budget and time series data in computing statistical demand functions for particular commodities.[17] The procedure is generally to compute from budget data an income elasticity of expenditures on the particular commodity or category of consumption for which the demand function is being computed. This income elasticity is taken to apply more or less directly to aggregate data reported in time series. The remaining parameters in the desired demand function are then estimated from time series data.

It is clear that, on our hypothesis, this procedure is erroneous. Though stated in terms of the elasticity of total expenditures, our conclusion that elasticities computed from budget data and from time

[17] See, for example, James Tobin, "A Statistical Demand Function for Food in the U.S.A.," *Journal of the Royal Statistical Society*, Series A, CXIII (1950), pp. 113–140; Herman Wold, *op. cit.*, pp. 228–234, and Richard Stone (assisted by D. A. Rowe and W. J. Cortlett, Renée Hurstfield, Muriel Potter), *The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom 1920–1938*, I (The University Press, Cambridge, 1954), pp. 275–278.

series are estimates of different magnitudes applies also to the elasticity for a particular category (for a fuller discussion, see Chapter VIII, section 2). The income elasticity computed from budget data cannot be expected to be the same (on the average) as that computed from time series data for a particular span of years unless transitory components of income have the same importance for the two bodies of data. There is no reason to expect the transitory components to have the same importance, and, as we have seen, if they do for one span of years, they will not for a longer or shorter span. (See Chapter VIII, section 2, for some suggestions about other ways of combining budget and time series data.)

3. *Regressions of Consumption on Current and Past Income*

The lack of success in predicting consumption by means of simple regressions of consumption on income like those considered in the preceding section led to experiments with more complicated functions. In connection with their emphasis on relative income position, Modigliani and Duesenberry expressed consumption as a function of the ratio of current income to the highest level of income previously experienced. Ruth Mack, in connection with her emphasis on changes in income, expressed consumption as a function of income in the current year and the change in income from the preceding year. These equations readily lend themselves to interpretation in terms of our hypothesis, and this interpretation in turn suggests an extension of them.

a. FUNCTIONS BY MODIGLIANI, DUESENBERRY, AND MACK

The relations computed by Modigliani and Duesenberry are of the form

$$\frac{c^*}{y^*} = f\left(\frac{y^*}{y_0^*}\right), \tag{5.1}$$

where y_0^* is the highest income experienced prior to the year in question, and all the variables are deflated and expressed per capita. If this relation is computed from a regression of the consumption ratio (or equally the savings ratio) on the income ratio,—or from the regression of consumption (or savings) on y^* and y_0^*,—and if the transitory component of consumption can be regarded as having a mean of zero, then, on our assumptions, c^* on the left hand side of the computed regression can be replaced by c_p^*, just as in the corresponding regressions from family budget data.

In our simple model,

$$c_p^* = k^* y_p^*, \tag{2.10}$$

or

$$\frac{c_p^*}{y^*} = k^* \frac{y_p^*}{y^*}, \tag{5.2}$$

so the righthand side of (5.1) is to be interpreted as an estimate of the righthand side of (5.2). A plausible way to do so is to regard the introduction of y_0^* as a means of estimating the permanent component. It hardly seems reasonable to regard y_0^* itself as an estimate of the permanent component since this would mean that the estimated permanent component would remain unchanged during a decline and subsequent recovery to a new peak. It seems more reasonable to regard a weighted average of y_0^* and y^* as an estimate of y_p^*, say:

$$\text{Estimate of } y_p^* = w_1 y_0^* + w_2 y^*, \tag{5.3}$$

where

$$w_1 + w_2 = 1. \tag{5.4}$$

In some of his regressions, Modigliani introduces the income of the preceding year, say y_{-1}^*, as a variable. In these cases we can expand (5.3) to

$$\text{Estimate of } y_p^* = w_1 y_0^* + w_2 y^* + w_3 y_{-1}^*, \tag{5.5}$$

where

$$w_1 + w_2 + w_3 = 1. \tag{5.6}$$

Inserting (5.5) in (5.2), and replacing c_p^* by c^*, gives

$$\begin{aligned} \frac{c^*}{y^*} &= \frac{k^*(w_1 y_0^* + w_2 y^* + w_3 y_{-1}^*)}{y^*} \\ &= k^* w_1 \frac{y_0^*}{y^*} + k^* w_2 + k^* w_3 \frac{y_{-1}^*}{y^*}. \end{aligned} \tag{5.7}$$

This is precisely the form of some of the regressions computed by Modigliani, though for the regressions for which he uses this form he omits y_{-1}^*, i.e. takes $w_3 = 0$. The form that Modigliani uses for the rest of his regressions can be obtained by multiplying both sides of (5.7) by y^* and adding a constant term to the righthand side to give:

$$c^* = a + k^* w_1 y_0^* + k^* w_2 y^* + k^* w_3 y_{-1}^*, \tag{5.8}$$

though of course in neither case does he use our notation. With one exception—his regressions for Sweden—Modigliani finds that the constant term is not statistically significant, so that, with this one exception, his numerical regressions are essentially in the form (5.7).

Duesenberry uses a form slightly different from (5.7), namely,

$$\frac{c^*}{y^*} = a + b\frac{y^*}{y_0^*}.$$

This can be converted into the form (5.7) by replacing y^*/y_0^* by its approximation by a Taylor series in y_0^*/y^*.

Mack uses the function

$$(5.9) \qquad c^* = a + by^* + c\,\Delta y^* = a + by^* + c(y^* - y^*_{-1}),$$

which requires only minor rearrangement to be put into the same form as (5.8), with $w_1 = 0$.

The values of k^* and w_1, w_2, and w_3 implied by the various functions calculated by Modigliani, Duesenberry, and Mack can be determined readily by using equations (5.6), (5.7), and (5.8); the results are summarized in the upper half of Table 14.

The differences among the values of k^* reflect at least in part, and perhaps in major part, differences in definition. The lowest value, for Canada, is for the ratio of consumption not to income but to gross national product, which is necessarily larger than income. For Sweden, the computed regression, which is of the form described by (5.8) has a significant constant term. This is consistent neither with our hypothesis nor with the results for the remaining regressions, and means that the unbracketed value of k^* in the table, computed by neglecting the constant term, understates the average ratio of consumption to income. Allowance for this understatement raises k^* to about .93, more nearly in line with the other values in the table.

There is some similarity, though it is not marked, in the weights assigned to the different incomes. In all cases, the highest previous income receives a weight decidedly less than one-half; it is as low as one-seventh in two cases. Considerably greater similarity emerges in the lower half of the table, which presents the recomputations of these relations by Ferber using the same data and comparable periods for all the relations. However, the appearance of increased homogeneity in the lower half of the table reflects mainly the exclusion of the Canadian and Swedish results rather than the greater comparability of the data and periods.

The practical identity of Ferber's recomputations of the Modigliani and Duesenberry equations is a purely arithmetic result: the two equations, as our earlier discussion shows, are algebraic transformations of one another and so can yield different results only because they imply different statistical methods of estimating the parameters

TABLE 14

Selected Measures Derived from Regressions of Consumption on Current and Past Income Computed by Modigliani, Duesenberry, and Mack, and Recomputed by Ferber

Country, Years Covered, Income Variable	Ratio of Permanent Consumption to Permanent Income (k^*)	Weight Attached, in Computing Permanent Income, to: Highest Previous Income (w_1)	Current Income (w_2)	Precedin[g] Year's Income (w_3)
Modigliani				
1. United States, 1921–40, disposable income	.90	.14	.86	
2. United States, 1921–40, income = disposable income plus corporate savings	.90	.14	.56	.30
3. Canada, 1923–39, gross national product	.79	.32	.17	.51
4. Sweden, 1896–1913, 1919–1934, national income	.85 (.93)[a]	.41	.59	
Duesenberry				
1. United States, 1929–40, disposable income	.95	.20	.80	
Mack				
1. United States, 1929–40, disposable income	.86(.97)[a]		.93	.07
Ferber Recomputations (All United States, disposable income)				
1. Following Modigliani				
a. 1923–1940	.96	.16	.84	
b. 1923–30, 1935–40	.96	.10	.90	
2. Following Duesenberry				
a. 1923–1940	.96	.16	.84	
b. 1923–30, 1935–40	.96	.10	.90	
3. Following Mack				
a. 1929–1940	.79(.97)[a]		.96	.04
b. 1923–1940	.82(.97)[a]		.90	.10
c. 1923–1930, 1935–1940	.96		.87	.13

[a] Value allowing for significant constant term.

Source:

Franco Modigliani, "Fluctuations in the Saving-Income Ratio: A Problem in Economic Forecasting," *Studies in Income and Wealth*, XI (New York: National Bureau of Economic Research, 1949).

1. Equation III-1, *ibid.*, p. 381. The dependent variable is the ratio of personal savings (or consump tion) to disposable income; the independent, the ratio of the difference between current and previou peak income to current income, with all variables deflated by a price index and expressed per capita

2. Equation XII-3, *ibid.*, p. 423. The dependent variable is individual plus corporate saving; th independent variables, disposable income plus corporate savings in the current year, the precedin year, and the preceding peak year, with all variables deflated by a price index and expressed per capita A constant term was computed but did not differ significantly from zero.

(cont. on next page)

TABLE 14 (cont.)

3. Equation VI-1a, *ibid.*, p. 394. Dependent variable is personal consumption, excluding expenditures on consumer durables except motor cars, plus government expenditure; independent variables, gross national product for current, preceding, and previous peak year, all variables deflated by a price index, but apparently expressed as aggregates rather than per capita. The computed constant term was not significantly different from zero.

4. Equation VI-2, *ibid.*, p. 396. Dependent variable, personal consumption plus government expenditures on goods and services; independent variables, national income (including taxes and corporate savings) for current and previous peak year, all variables deflated by a price index and expressed per consumption unit. The computed constant term was significantly different from zero, which throws some doubt on our interpretation, which treats it as zero.

James S. Duesenberry, *Income, Saving and the Theory of Consumer Behavior*, Harvard University Press, 1952, pp. 90–91.

(a) Dependent variable, ratio of personal savings to disposable income; independent variable, ratio of disposable income in current year to highest previous disposable income. All variables on a per capita basis and deflated by a price index. (b) Duesenberry gives the constant term of his linear equation as .196; this is presumably a typographical error, since internal evidence indicates that it is −.196. The slope is .25. In converting his equation to the form described by (5.7), his y^*/y_0^* was replaced by $1.754 - .769y_0^*/y^*$, which are the first two terms of a Taylor's expansion around a value of $y_0^*/y^* = 1.14$, the approximate average for the period covered.

Ruth P. Mack, "The Direction of Change in Income and the Consumption Function," *Review of Economics and Statistics*, XXX (1948), p. 256.

The dependent variable is consumption; the independent variables are disposable income and the change in disposable income. All variables are national aggregates in current dollars.

Robert Ferber, *A Study of Aggregate Consumption Functions*, National Bureau of Economic Research, Technical Paper 8, 1953.

1. a, b, *ibid.*, p. 69, equations (2.21b) and (2.21c) respectively. Dependent variable is ratio of personal savings to disposable income; the independent, the ratio of the difference between current and previous peak income to current income, with all variables deflated by a price index and expressed per capita.

2. a, b, *ibid.*, p. 69, equations (2.22b) and (2.22c) respectively. Dependent variable is ratio of personal savings to disposable income; the independent, the ratio of current income to previous peak income, with all variables deflated by a price index and expressed per capita. In converting these equations to the form described by (5.7), y^*/y_0^* was replaced by $1.84 - .846y_0^*/y^*$ for 2a, and by $1.963 - .963y_0^*/y^*$ for 2b. These are the first two terms of Taylor's expansion around values of y_0^*/y^* of 1.087 and 1.019 respectively, the means for the relevant periods.

3. a, b, c, *ibid.*, p. 66, equations (2.8a), (2.8b), and (2.8c) respectively. Dependent variable is personal savings; independent variables are current and previous year's disposable income, with all variables deflated by a price index and expressed per capita.

from the available data.[18] For the Mack recomputations, the constant term is significantly different from zero for two out of the three relations. Apparently, the inclusion of only the preceding year's income is not as successful as the inclusion of the highest previous income in rendering the equation homogeneous.

[18] The Modigliani function fitted by Ferber is

$$\frac{c^*}{y^*} = a + b\frac{y^* - y_0^*}{y^*} = (a + b) - b\frac{y_0^*}{y^*}.$$

Perhaps the most interesting result, common to all three sets of equations is the dominant weight attached to current income in computing permanent income. However, this result, as we shall see in the next section, is partly accounted for by the brevity of the time series from which the equations are computed.

b. ALTERNATIVE FUNCTIONS FITTED TO DATA FOR A LONG PERIOD

The computations summarized in Table 14 are all for the interwar period or parts thereof. It is possible to fit comparable functions to Goldsmith's data for a much longer period. This section records the results of doing so, and, in addition, of an experiment in fitting a related function suggested by our interpretation of the functions covered by Table 14.

On this interpretation, the incomes of prior years enter into the functions as a means of estimating permanent income. Judged from this point of view, the Modigliani-Duesenberry and Mack functions are questionable in several respects. In the first place, they estimate permanent income as the average of two or at most three years, yet it seems plausible that permanent income should be estimated from a longer period. More important, this is not an issue that should be decided a priori; the data themselves should dictate the appropriate number of years. In the second place, the use of the highest previous income seems rather arbitrary. For example, it might lead to use of a different year according to the form of the data—one year, say, for per capita deflated data, another for aggregates in current prices. It seems rather arbitrary, too, that the same weight should be attached to the highest previous income regardless of how many years separate it from the current year.

One alternative is to construct a weighted average of a longer series of years, allowing both the weights and the number of years to be determined by the data; the weights, by multiple correlation, the number of years, by adding years until an additional year produces no significant increase in the correlation. Unpleasantly complex in theory, this alternative also has the statistical defect that it uses up an undue number of degrees of freedom in application. But it does indicate a direction along which to proceed.

One way to proceed in this direction is to limit the characteristics of the weighting pattern to be determined from the data by expressing the weights as a function of the elapsed time between any given time

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The Duesenberry function is

$$\frac{c^*}{y^*} = a + b\frac{y^*}{y_0^*}.$$

The only difference is that the independent variable in the one equation is the reciprocal of the independent variable in the other.

unit and the time unit for which permanent income is being estimated. Given the relatively heavy weight of current income revealed by Table 14, it seems appropriate to use a weighting pattern that gives most weight to current income and successively declining weights to earlier incomes. To state the procedure in its most general form, free from an arbitrary time unit, let us regard measured income as a continuous function of time and denote it by

$$y^*(t)\,. \tag{5.10}$$

We might then construct an estimate of the permanent component at time T as

$$\text{Estimate of } y_p^*(T) = \int_{-\infty}^{T} w(t-T)y^*(t)\,dt\,, \tag{5.11}$$

where

$$\int_{-\infty}^{T} w(t-T)\,dt = 1\,. \tag{5.12}$$

One simple weighting pattern that has acceptable characteristics is an exponential, declining as one goes backward in time, say

$$w(t-T) = \beta e^{\beta(t-T)}\,. \tag{5.13}$$

This weighting pattern has been used for a rather similar problem by Phillip Cagan, namely, to estimate the expected rate of change of prices during hyper-inflations from the time series of past rates of change.[19] The model that led him to his weighting pattern can be readily adapted to the present problem and may perhaps make the use of this pattern seem somewhat less arbitrary than the strictly empirical approach that I have so far followed.

For this purpose, tentatively regard y_p^* as the "expected" or predicted value of current measured income. Suppose this expected value is revised over time at a rate that is proportional to the difference between expected and actual income, or

$$\frac{dy_p^*}{dT} = \beta[y^*(T) - y_p^*(T)]\,. \tag{5.14}$$

The solution of this differential equation with suitable initial conditions to make the constant term zero, is

$$y_p^*(T) = \beta\int_{-\infty}^{T} e^{\beta(t-T)}y^*(t)\,dt\,, \tag{5.15}$$

or the estimate stated earlier.[20]

[19] See Phillip Cagan, "The Monetary Dynamics of Hyperinflation," in Milton Friedman (ed.), *Studies in the Quantity Theory of Money* (University of Chicago Press, 1956), pp. 25–117.

[20] Note that, to first order terms, the same estimate is valid if the adjustment equation is expressed in logarithmic terms, or

$$\frac{dY_p^*}{dT} = \beta[Y^*(T) - Y_p^*(T)]\,. \tag{5.14'}$$

One obvious defect of this approach is that it does not allow for predicted secular growth. Being an average of earlier observations, the estimated y_p^* is necessarily between the lowest and the highest, so that this method of estimation applied to a steadily growing series yields estimated values systematically below the observed values. To allow for this, we can suppose y_p^* to be estimated in two parts: first, a trend value which is taken to grow at a constant percentage rate, and second, a weighted average of adjusted deviations of past values from the trend, the adjustment being made to allow for the trend change itself, and thus to put all deviations at the same level as the present deviation. This would give:

$$(5.16)\quad y_p^*(T) = y_0 e^{\alpha T} + \beta \int_{-\infty}^{T} e^{\beta(t-T)}[y^*(t) - y_0 e^{\alpha t}]e^{\alpha(T-t)}\, dt,$$

where α is the estimated rate of growth and y_0, the value of income at the time taken as zero. This expression reduces to the much simpler form:

$$(5.17)\qquad y_p^*(T) = \beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)} y^*(t)\, dt\,,$$

and this is the form that we shall use. If we combine (5.17) with our basic consumption equation (2.10), and recall that measured consumption on the average equals permanent consumption for any given value of measured income, we have as a consumption function to be fitted to aggregate data:[21]

$$(5.18)\qquad c^*(T) = k^*\beta \int_{-\infty}^{T} e^{(\beta-\alpha)(t-T)}\, y^*(t)\, dt\,.$$

This equation has three parameters: β, α, and k^*. One of these, however, which we take to be α, must be determined in some way other than by getting the best fitting approximation to (5.18), so in effect there are only two parameters to be determined from the set of data on measured consumption and measured income.[22] Yet, in principle, the equation estimates permanent income from the whole set of observed values of measured income.

In fact, of course, the earlier years get rapidly diminishing weights in determining y_p^*, so that beyond some point in time the observations have a negligible effect on the estimate. The span of time that matters depends on the size of β, the adjustment coefficient. The larger β, the larger the adaptation to any existing discrepancy between measured and expected income, and hence the more rapid the adjustment and

[21] It is interesting that Robert Solow suggested precisely this form of consumption function in "A Note on Dynamic Multipliers," *Econometrica*, XIX (July, 1951), p. 308.

[22] The fitting process makes it possible to determine only $(\beta - \alpha)$ and $k^*\beta$. Any triplet of values of β, α, and k^* which yield the same values of $\beta - \alpha$ and $k^*\beta$ will necessarily lead to the same prediction of consumption from a given series of observations on $y^*(t)$.

the shorter the retrospective time span that matters. One way of measuring the effective time span is by computing the weighted average time span between the observations that are weighted and the present or

$$(5.19) \qquad T - \bar{t} = \beta \int_{-\infty}^{T} e^{\beta(t-T)}(T - t)\, dt = \frac{1}{\beta}.$$

This is the average time lag between the estimated permanent income and the observations from which it is estimated; twice this time lag may be called the "effective weighting period."[23]

Of course, equation (5.18) cannot be fitted directly to data for discrete time units, such as years. For application, $y^*(t)$ is treated as a step function having the same value throughout each year. This is equivalent to converting the integral in (5.18) into a summation of annual terms, the weight for each year being the integral of the weight function over the corresponding period. Only a limited number of terms are retained, the number depending on the value of β and being determined so that the retained terms account for the great bulk of the weight. In this form, the equation has been fitted to the data in Figure 13, namely, real disposable income per capita and real consumption per capita based on Goldsmith's savings estimates. A number of details about the fitting process deserve explicit mention: (1) The method of fitting involved successive approximations and was worked out by Phillip Cagan in connection with the study mentioned earlier.[24] (2) The necessity of using incomes for prior years made it necessary to drop the earlier values of c^* available. The number that had to be dropped depends itself on the value of β. After preliminary experimentation the final function was fitted to data for 1905–51. This same period was used for comparability for the other functions covered in Table 15.[25] (3) In the final computation

[23] An alternative way of measuring the lag is to determine how far back one has to go to account for half the weights. This median time lag is $.69/\beta$.

[24] Cagan supervised the present statistical computations, and I am deeply indebted to him for his help, which has affected most of what follows in this section even where I make no explicit mention of his contribution. See Cagan, *op. cit.*, pp. 92–93.

[25] After this work was done and the description in the present section written, I discovered that the data used to fit the function were not comparable for the whole period. The data for the final two years, 1950 and 1951, are rough-and-ready extrapolations of estimates constructed in great detail for the period through 1949. Moreover, more extensive data that have become available since these extrapolations were made show them to be wide of the mark. It would have been better not to have used them. At the same time, since their omission could hardly affect the results substantially, I have not thought it worth the cost that would be involved to recompute the results, especially since it is likely that more nearly comparable figures for these and still later years will become available in the near future and so permit recomputation not only to correct this defect but also to cover a longer period. I have therefore contented myself with omitting 1950 and 1951 from Figure 14.

17 terms were retained in computing expected income; with this number, the weights, when rounded to three decimal places, sum to unity. The use of this number of terms for the earlier period made it necessary to extend the data back in time. This was done by extrapolating the 1897 figure backward along an exponential growth trend rising at the rate of 2 per cent per year. Since the sum of the weights applied to these hypothetical figures never exceeds .027, this expedient cannot introduce serious error and has the great virtue of enabling us to use a longer period for estimation.[26] (4) The value of α was taken as .02 on the basis of the secular rate of growth of c^*. This did not affect the fitting process but only the interpretation of the computed constants. (5) The war years, 1917, 1918, 1942 through 1945, were excluded on the grounds that special circumstances of those years made it rather absurd to use a formula like (5.15) to estimate permanent income and that the consumption data had abnormal transitory elements. For similar reasons, in computing permanent income in postwar years, the actual measured income in the war years was replaced by expected income in the last prewar year (1916 and 1941 respectively) plus 2 per cent per year to allow for secular growth.[27]

Table 15 summarizes the results of this computation and compares it with the results of fitting to the same data functions like those used by Modigliani and Duesenberry, and by Mack. The Modigliani-Duesenberry function used is

$$c^* = k^*(w_1 y_0^* + w_2 y^*)\,. \tag{5.20}$$

The Mack function used is

$$c^* = k^*(w_2 y^* + w_3 y_{-1}^*)\,. \tag{5.21}$$

All three functions involve determining two parameters from the set of data on consumption and income, and so they are all strictly comparable in this respect. In addition to the estimates in the table, based on fitting (5.18), (5.20), and (5.21) to the data, we also computed corresponding equations with a constant term added, in order to check the homogeneity of the equations. The constant term was smallest for the expected income equation and decidedly larger for the other two; in all three, however, forcing it to equal zero had relatively little effect on the estimates of the other parameters.[28]

[26] The retention of as many as 17 terms is doubtless an excess of precision. It is dubious that the results would be appreciably affected by retaining, say, only 9 terms and adjusting the weights for them to sum to unity.

[27] It may be worth recording that this device was decided on before the computations were made and was in no way altered in light of the results.

[28] The numerical value of the constant term was -4.0 for the expected income equation, $+52.8$ for the preceding year's income equation, and $+98$ for the highest previous

TABLE 15

e Consumption Functions for the United States: Regressions of Consumption on Current and Past Incomes, Nonwar Years 1905 through 1951[a]

Regression	Ratio of Permanent Consumption to Permanent Income (k^*)	Weight Attached, in Computing Permanent Income, to: Highest Previous Income	Current Income	Preceding Year's Income	All Prior Years Combined	Square of Multiple Correlation Coefficient (R^2)	Standard Error of Estimate as a % of Average Value of Measured Consumption
iest previous income[b]	.88	.45	.55			.98	2.8
eding year's income[b]	.90		.64	.36		.94	5.0
cted income[c]	.88		.33	.22	.45	.96	4.0

Excluded years are 1917–18, 1942 through 1945.
Although the war years 1942 through 1945 were excluded from current income in computing these ssions, 1945 was used as the highest preceding income and as the preceding year's income for the current income observation, since 1941 was so far out of line. For World War I, since no break introduced, 1917–18 was omitted for the other variables as well.
The estimated value of β, on which the weights are based, is .4. The weights for 17 individual years ree decimals are as follows, starting with the current year and going backward in time: .330, .221, .099, .067, .045, .030, .020, .013, .009, .006, .004, .003, .002, .001, .001, .001.
ote: Consumption = real consumption per capita. Income = real disposable income per capita.

Figure 14 presents the results graphically, plotting the time series of measured income, measured consumption, and consumption as predicted by each of the three equations. As both the table and the figure show, all three of the equations fit the observed data extremely well: the squares of the multiple correlation coefficient range from .94 to .98; the standard errors of estimate, from about 3 to 5 per cent of the average level of consumption.

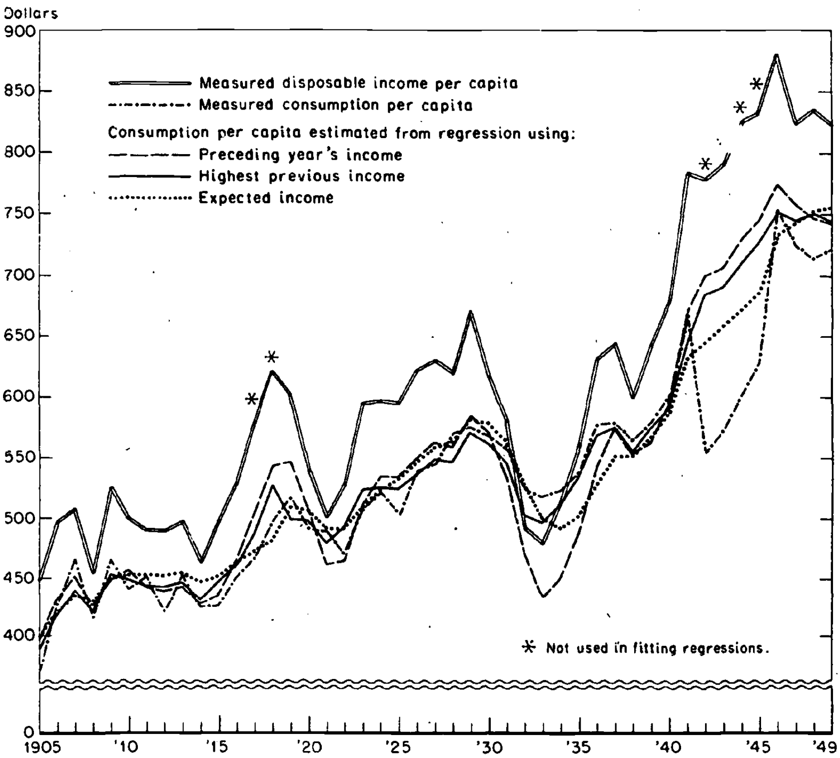
The squares of the multiple correlation coefficient and standard errors of estimate entered in Table 15 are, however, somewhat misleading for two rather different reasons. In the first place, as is evident from the graph, both measured income and measured consumption have common and fairly steady upward trends of about 2 per cent per year. This common trend accounts for a large part of the high multiple correlation. Predicting consumption from its own trend yields a standard error of estimate of 6.6 per cent, and a value comparable to the squares of the multiple correlation coefficient in Table 15 of .90, so that 90 per cent of the variance of consumption over the period in question is accounted for simply by its own trend,

year's income equation. These are respectively approximately .24, .6, and 2.7 times their approximate standard errors.

and hence by the similar trend of income. Of course, the common trend of income and consumption is itself evidence in favor of the hypothesis that the permanent components of income and consumption are proportional; however, it is evidence from one observation, as it were, not from 41, the number from which the estimates in the table are supposedly computed.

FIGURE 14

Measured Disposable Income per Capita, and Consumption per Capita Measured and as Estimated from Three Regressions, 1905–1949



Source: See Figure 13 and Table 15.

In light of this common trend, a more meaningful way to interpret the squares of the correlation coefficient may be to regard the preceding year's income equation as explaining 40 per cent of the variance not accounted for by the common trend; the expected income equation, as explaining 60 per cent; and the highest previous income equation, as explaining 80 per cent. Viewed in this way, the differences among the equations seem much larger and more important. But even these need further examination. Instead of predicting consumption from its own trend, suppose we were to predict it as a constant multiple of measured income in the same year. The result is a standard error of 5.7 per cent, and a value comparable to the square of the multiple correlation coefficient of .92. The terms other than current income can therefore be regarded as accounting for one-quarter, one-half, and three-quarters of the remaining variance for the preceding year's income, the expected income, and the highest previous income equation, respectively. Finally, we might predict consumption as a linear function of current income rather than a simple multiple of it—this is the absolute income hypothesis itself, and unlike the preceding comparison involves computing from the data the same number of constants as for the three equations in Table 15. The result is a standard error of estimate of 4.9 per cent and a squared correlation coefficient of .94. The preceding year's income equation makes no improvement compared with this alternative; the expected income equation accounts for one-third of the remaining variance; the highest previous income equation for two-thirds.

The importance of current income gives rise to the second reason why the standard errors of estimate and the squares of the multiple correlation coefficient in Table 15 are somewhat misleading, particularly in respect of the comparative success of the expected income and the highest previous income equations in predicting current consumption. The figures on consumption used in fitting these equations were computed as the difference between Goldsmith's estimates of savings and his separately derived estimates of disposable income. There is no reason to suppose that these two series have any important common sources of statistical error. But this means that the difference between them has the statistical errors of both. What is crucial for our present purpose, any statistical error in the estimate of disposable income means a statistical error of the same size and sign in the estimate of consumption. This common statistical error is a source of spurious correlation between measured consumption and measured current income which makes all of the correlation coefficients we have been citing too high.[29] To put it differently: given these statistical counterparts of our theoretical constructs, measured income can successfully predict in part the statistical errors in consumption. The result is the same as, and statistically indistinguishable from, a positive correlation between the transitory components of consumption and income. This bias in our estimated correlation coefficients is more important the higher the weight attached to current income. For this reason, the preceding year's income equation

[29] I owe this point to Phillip Cagan.

must be regarded as somewhat superior to the simple absolute income equation even though both yield the same correlation, because it gives a weight of 64 per cent rather than 100 per cent to current income. By the same token, both the other equations must be regarded as superior to the preceding year's income equation by an even wider margin than our earlier comparisons suggested.

The effect of the spurious correlation on the comparison between the expected income and highest previous income equations is much more difficult to evaluate. For the spurious correlation works in the same direction as the difference in the observed correlation. The highest previous income equation gives the higher correlation; it also gives greater weight to current income; the common error in consumption and income therefore raises its correlation by more than it raises the correlation for the expected income equation. Can the whole of the difference between the observed correlations be accounted for in this way? This question is examined in the Appendix to this Chapter. The conclusion reached is that it can be, though it is by no means clear that it is.

On statistical grounds alone, therefore, there is little basis for choosing between the highest previous income equation and the expected income equation. Despite what seem to me the theoretical defects of the highest previous income equation, it fits the data better than the aesthetically more appealing expected income equation, though both fit the data extremely well, and the difference in fit is of an order of magnitude that can be explained on purely statistical grounds of spurious correlation.

The estimates in Table 15 in all cases assign a much lower relative weight to current income than the estimates in Table 14 for equations fitted to shorter periods. In the highest previous income and preceding year's income equations, current income still gets more than half the weight; in the expected income equation, it gets only one-third the weight.

The expected income equation gives evidence on a feature of the consumption relation largely assumed in the other equations, namely, the average lag. The value of β turned out to be .4, implying an average lag of $2^1/_2$ years, or an "effective weighting period" of 5 years.[30] In terms of our hypothesis, this period is presumably related to the horizon implicit in judgments of permanent income by individual consumer units. It seems plausible that this period would be longer for aggregate data than the corresponding horizon for individual units, due to the averaging out of random factors.

In Cagan's study of hyper-inflations, he derived values of β strictly

[30] The "median" lag is 1.72 years.

comparable with ours, except that his relate to the lag in adjusting expected rates of price change to actual rates of price change. Under comparable circumstances, there seems no reason why men's horizons or speed of adjustment should be any different in adjusting expected income to measured income than in adjusting the expected rate of price change to the actual rate of price change. Indeed, insofar as we regard men as estimating both expected money and expected real income, expectations about price changes enter into expectations about income. Periods of rapid change might be expected to produce a shortening of horizons, or a speeding up of adjustment with respect to both variables. The average lag might therefore be expected to be shorter—that is, the value of β to be higher—for hyper-inflation periods like those studied by Cagan than for the less erratic peacetime period of our calculations. The results conform to expectation: Cagan finds values of β between .6 and 4.2 compared with our estimate of .4, or an average lag between $^1/_4$ of a year and $1^2/_3$ years compared with our $2^1/_2$ years.[31] The consistency of these estimates for different countries, periods, and phenomena is both striking and highly relevant to the plausibility of our procedure. Certainly, if the computed value of β had turned out to be smaller for hyper-inflations than for other periods, it would have been necessary to reject the interpretation offered for one or the other set of data.

A number of segments of Figure 14 sharpen the impression derived from the summary parameters in Table 15. One is for the two wars. For 1917 and 1918, neither of which was used in fitting the functions, the preceding year and previously highest year functions both decidedly overestimate measured consumption. The expected income function gives very close estimates. For the excluded years of the second world war, 1942 through 1945, all three functions decidedly overestimate consumption, though the expected income function does so less than the others. These results are consistent with the interpretation that measured income contains a significant positive transitory component in both wars, while measured consumption was largely free from any transitory component in World War I, and contained a sizable negative transitory component in World War II. The estimates for the expected income function, as they were constructed, are not affected at all by the transitory component of income, whereas the others are, and none of the estimates takes into account any transitory component of consumption. This interpretation is

[31] The numbers cited in Cagan are on a per month basis. I have multiplied them by 12 to make them comparable with the value of β for the annual consumption data. *Ibid.*, p. 43.

eminently plausible in light of other information for the two periods. Certainly, there was a decided negative transitory component in consumption during World War II, caused by the unavailability of some goods and the explicit rationing of others. It is very much less clear that a similar situation existed during World War I, which was shorter, which drew upon a smaller fraction of aggregate resources, and which involved no explicit rationing and little direct control over production.

Another segment of Figure 14 worth attention is the Great Depression. In 1933, 1934, and 1935, consumption was higher than predicted by any of the functions and in 1931 and 1932, higher than predicted by either the preceding year or previously highest year function. Further, the expected income function continued to underestimate consumption substantially in 1936 and 1937. The interpretation seems reasonably straightforward. Human beings are more flexible than the particular mathematical equations we used to summarize their behavior; they recognized, as these equations could not, that the Great Depression was something exceptional and special, to be taken into account in a different way than the run-of-the-mill up and down of economic activity. Accordingly, they attributed a much larger part of the decline in income to a negative transitory component of income than do the various devices for estimating permanent income embodied in our equations. They therefore maintained a higher level of consumption than these equations predicted during much of the depression. The expected income function, having, as it were, the longest memory, went astray later than the others, but, having gone astray, stayed astray well after the others had come back into line.[32] This deviation from our functions during the Great Depression tells against the specific equations used to describe consumer behavior but seems, if anything, to support our general interpretation of consumer behavior.

Appendix to Section 3:

Effect on Multiple Correlation of Common Errors in Measured Consumption and Current Income

In what follows, I shall use the following notation:

[32] This interpretation of consumer behavior during the Great Depression, if accepted, has obvious and important implications for the cyclical interpretation of the period. It implies that expectations, far from being destabilizing as has so often been asserted, were, at least on the part of consumers, a stabilizing factor, which means that the extraordinary depth of the depression would have to be explained in some other way, as produced by some pressure on the system, such as the rapid decline in the stock of money from 1929 to 1933, particularly from 1931 to 1933.

	Symbol for the:		
Variable	*Measured Value*	*Error of Measurement*	*"Correct" Value*
Income	y	δy	η
Savings	s	δs	Σ
Consumption	c	δc	γ

In addition, let the subscript t denote a current value of a variable; the subscript l, a lagged value; the subscript p, a "permanent" value.

All variables are taken to have means of zero; that is, they are expressed as deviations from the means of the corresponding unadjusted variables.

Both the expected income and the highest previous income equation can be interpreted as setting

(5.22) $$y_p = \alpha y_l + \beta y_t\,,$$

(5.23) $$\eta_p = \alpha\eta_l + \beta\eta_t\,,$$

(5.24) $$\alpha + \beta = 1\,.$$

In the expected income equation, y_l is to be interpreted as a weighted average of incomes in all years preceding the current year; in the highest previous income equation, as the income of the highest previous year. So both equations can be interpreted as correlating c_t with y_p, and our problem reduces to determining the effect of the value of β on the correlation between c_t and $y_p(r_{c_t y_p})$ given common errors in c_t and y_t.

We take c to be computed as $y - s$, where y and s are independently measured, so

(5.25) $$\delta c = \delta y - \delta s\,.$$

Assume that

(5.26) $$r_{\delta y\delta s} = r_{\eta\delta y} = r_{\Sigma\delta s} = r_{\eta\delta s} = r_{\Sigma\delta y} = r_{\gamma\delta y} = r_{\gamma\delta s} = 0\,,$$

whether the variables refer to the same or different years, and also that

(5.27) $$r_{\delta y_t\delta y_l} = r_{\delta s_t\delta s_l} = 0\,.$$

The value of $r_{c_t y_p}$, the effect on which of the size of β is our goal, is

(5.28) $$r_{c_t y_p} = \frac{Ec_t(\alpha y_l + \beta y_t)}{[Ec_t^2]^{1/2}[E(\alpha y_l + \beta y_t)^2]^{1/2}}\,.$$

Now

(5.29) $$Ec_t(\alpha y_l + \beta y_t) = E(\gamma_t + \delta c_t)(\alpha\eta_l + \beta\eta_t + \alpha\delta y_l + \beta\delta y_t)$$

(5.30) $$= E\gamma_t\eta_p + \beta E\delta c_t\delta y_t\,,$$

all other cross-products being zero by virtue of (5.26) and (5.27),

$$= r_{\gamma_t\eta_p}\sigma_{\gamma_t}\sigma_{\eta_p} + \beta E(\delta y_t - \delta s_t)\delta y_t = r_{\gamma_t\eta_p}\sigma_{\gamma_t}\sigma_{\eta_p} + \beta\sigma^2_{\delta y_t}, \tag{5.31}$$

where σ stands for the standard deviation of the variable designated by its subscript.

$$Ec_t^2 = E(\gamma_t + \delta c_t)^2 = E(\gamma_t + \delta y_t - \delta s_t)^2 = \sigma^2_{\gamma_t} + \sigma^2_{\delta y_t} + \sigma^2_{\delta s_t}, \tag{5.32}$$

$$Ey_p^2 = E(\eta_p + \alpha\delta y_l + \beta\delta y_t)^2 = \sigma^2_{\eta_p} + \alpha^2\sigma^2_{\delta y_l} + \beta^2\sigma^2_{\delta y} . \tag{5.33}$$

Let us assume that

$$\sigma^2_{\delta y_l} = \sigma^2_{\delta y_t} = \sigma^2_{\delta y}, \tag{5.34}$$

so

$$\sigma^2_{y_p} = \sigma^2_{\eta_p} + (\alpha^2 + \beta^2)\sigma^2_{\delta y} . \tag{5.35}$$

Substituting in (5.28), we have

$$r_{c_t y_p} = \frac{r_{\gamma_t\eta_p}\sigma_{\gamma_t}\sigma_{\eta_p} + \beta\sigma^2_{\delta y}}{[\sigma^2_{\gamma_t} + \sigma^2_{\delta y} + \sigma^2_{\delta s_t}]^{1/2}[\sigma^2_{\eta_p} + (\alpha^2 + \beta^2)\sigma^2_{\delta y}]^{1/2}} \tag{5.36}$$

$$= \frac{r_{\gamma_t\eta_p} + \dfrac{\beta\sigma^2_{\delta y}}{\sigma_{\gamma_t}\sigma_{\eta_p}}}{\left[1 + \dfrac{\sigma^2_{\delta y} + \sigma^2_{\delta s_t}}{\sigma^2_{\gamma_t}}\right]^{1/2}\left[1 + (\alpha^2 + \beta^2)\dfrac{\sigma^2_{\delta y}}{\sigma^2_{\eta_p}}\right]^{1/2}} . \tag{5.37}$$

We have the following numerical values from our computations:

	Expected Income Equation	*Highest Previous Income Equation*
$r^2_{c_t y_p}$	·96	·98
$r_{c_t y_p}$	·98	·99
α	·67	·455
β	·33	·545
$\alpha^2+\beta^2$	·5578	·5040

In order to use formula (5.37) we must have estimates of

$$\frac{\sigma_{\delta y}}{\sigma_{\gamma_t}}, \quad \frac{\sigma_{\delta s}}{\sigma_{\gamma_t}}, \quad \frac{\sigma_{\delta y}}{\sigma_{\eta_p}} .$$

Call these R, S, and T respectively.

Now we know that, approximately,

$$\gamma = .9\eta_p \tag{5.38}$$

$$\text{so} \quad \sigma_\gamma = .9\sigma_{\eta_p}, \tag{5.39}$$

$$R = \frac{1}{.9}T = 1.11T . \tag{5.40}$$

To get an estimate of S, assume that

$$\frac{\sigma_{\delta s}}{\sigma_\Sigma} = \frac{\sigma_{\delta y}}{\sigma_y} \tag{5.41}$$

from which

$$S = \frac{\sigma_{\delta s}}{\sigma_Y} = \frac{\sigma_\Sigma}{\sigma_y} \cdot \frac{\sigma_{\delta Y}}{\sigma_Y} = \frac{\sigma_\Sigma}{\sigma_y} \cdot R\,. \tag{5.42}$$

Now σ_Σ/σ_Y is approximately equal to σ_s/σ_c which by computation from the data, equals .36. So that

$$\sigma_\Sigma = .36\sigma_Y = .36(.9)\sigma_{\eta_p}\,. \tag{5.43}$$

But, on our hypothesis, the elasticity of the regression of consumption on income is an estimate of the fraction of the variation in y that is attributable to η_p. For the period in question, the computed elasticity is .87. It follows that

$$\sigma^2_{\eta_p} = .87\sigma^2_y \tag{5.44}$$

so

$$\sigma_\Sigma = (.36)(.9)(.87)^{1/2}\sigma_y, \quad \text{or} \tag{5.45}$$

$$S = \frac{\sigma_\Sigma}{\sigma_y} \cdot R = (.36)(.9)(.87)^{1/2}(1.11)T = .33T\,. \tag{5.46}$$

Suppose, now, that we insert the expressions for R and S in terms of T from (5.40) and (5.46) into (5.37) as well as the computed value of $r_{c_t y_p}$ from the highest previous income equation, the corresponding value of $\beta(\beta = .545)$, and finally, assume that $r_{Y_t\eta_p} = 1$ so that only statistical measurement errors account for correlations less than unity. This gives an equation in T, from which we can compute its value as

$$T = (.0397)^{1/2}\,.$$

Let us now use this value of T, together with the values of R and S as computed from (5.40) and (5.46), a value of $\beta =$ 33, and again assume that $r_{Y_t\eta_p} = 1$, and compute the value of $r_{c_t y_p}$ implied by (5.37). This is then an estimate of the correlation coefficient for the expected income equation on the assumption that the only reason why it differs from that for the highest previous income equation is because of the smaller value of β. The resulting answer is

$$r^2_{c_t y_p} = .957\,,$$

or a number slightly less than the observed .960. It follows, that, under our assumptions, the whole of the difference between the two observed correlations can be accounted for by the difference in β.

One test of the reasonableness of our assumptions is to see whether they imply reasonable values for the various standard deviations. We have that

$$T^2 = .04 = \frac{\sigma^2_{\delta y}}{\sigma^2_{\eta_p}}$$

or

$$\sigma^2_{\delta y} = .04\sigma^2_{\eta_p} = (.04)(.87)\sigma^2_y = (.0348)\sigma^2_y\ .$$

But σ^2_y can be computed from the data. The resulting value of $\sigma_{\delta y}$ is about $25, or about 4 per cent of mean measured income. In similar fashion, the computed value of $\sigma_{\delta s}$ turns out to be about 13 per cent of mean saving. These figures seem not unreasonable. If anything, they imply rather smaller errors in saving and income than I would have expected.

As another check on the reasonableness of these results, I made a similar computation for what seem two extreme assumptions:

Assumption 1:	$R = S = T$
Assumption 1A:	$R = T;\ S = 0$.

It seems plausible that these would be extremes, because savings are so much smaller on the average than consumption or income that one might expect the measurement error in savings to be smaller in absolute size than in income, though very likely much larger as a percentage of the magnitude being estimated. The results were as follows:

	Hypothetical value of $r^2_{c_t v_p}$ for $\beta = .33$
Assumption 1:	.973
Assumption 1A:	.956

These two values are on either side of the observed value of .96. Similarly, the value earlier computed of .957 is between them as well.