The federal public housing program had its origin in the United States Housing Act of 1937. At present there are more than one million public housing units occupied by more than three million people. Despite the age and size of this program, there are no published estimates of its effect on consumption patterns. Henry Aaron, Robert Bish, and Eugene Smolensky and J. Douglas Gomery have measured the benefit to a family in public housing as the difference between the market rent of its dwelling and the rent paid by the family and have made rough estimates of market rent in order to determine the distribution of direct benefits from the program. The primary purposes of this paper are to estimate the effect of the federal public housing program on the consumption patterns of its occupants and to determine the distribution of direct benefits from this program.

NOTE: The data underlying this study were collected in 1973 for the Department of Housing and Urban Development Housing Policy Review Task Force. We are grateful to the many people involved in their collection. The results reported here differ somewhat from those in the task force report because additional time has allowed us to make some improvements. We are also grateful to Michael Murray and participants in a seminar at the University of Michigan for critical comments and to Raymond Yacouby and Robert Berry for computational assistance.
I. THEORETICAL FRAMEWORK

Assume that there are two goods, housing service and nonhousing goods, and that the markets for these goods are perfectly competitive and in long-run equilibrium. Assume that the long-run supply curves in both markets are perfectly elastic. These assumptions imply that the market prices of the two goods are unaffected by the public housing program.

Before proceeding, it is desirable to define the phrase housing service with some care. Housing service is a good provided in some unobservable quantity by each dwelling unit during each period of time. It is the one and only thing in a dwelling unit to which consumers attach value. More concretely, the quantity of housing service provided by a dwelling unit can be thought of as an index of all its attributes. If the private housing market is perfectly competitive, then in long-run equilibrium, each unit of housing service sold in one market sells for the same price. Hence, if we observe one apartment renting for $200 per month and another for $100 per month in the same market, then we say that the first apartment provides twice the quantity of housing service per time period that the second does.

Now let us use the assumptions made to derive formulas for calculating the effect of public housing on the consumption patterns of its occupants and the value of this program to these families. Figure 1 contains several indifference curves of one family living in public housing. In the absence of the program, this family would have some income $Y$ and could buy as much of each good as it could pay for at prices $P_n$ and $P^*$. It would select some combination $(Q^n, Q^*)$ of the two goods. Under the public housing program, the family has been offered and has accepted a particular dwelling unit providing some quantity of housing service $Q^*_f$. In order to occupy this dwelling, the family must pay a certain rent $P^*_f Q^*_f$ per time period. After paying this rent, the family has enough money left to buy $Q^*_n$ units of nonhousing goods. It is important to recognize that public housing does not change an eligible family’s situation by rotating its budget line. In the two-good case, it simply adds one point to the family’s budget space. Since the public housing authority could offer a family a dwelling worse than it would otherwise occupy (i.e., $Q^*_f < Q^*_n$) and charge a rent such that the family is able to increase its nonhousing consumption.
by more than enough to compensate for its decrease in housing consumption, the basic assumptions of the theory of consumer choice, together with the possible changes in budget spaces under the public housing program, do not imply that public housing tenants consume more housing service than they would in the absence of the program.

In the case depicted in Figure 1, the family consumes

$$100\left(\frac{P^*Q^* - P^mQ^m}{P^mQ^m}\right)\text{ percent}$$

**FIGURE 1 The Effect of Public Housing on a Family's Consumption Pattern and Well-Being**

NOTE: $Y$ is the family’s income, $P^n$ and $P^m$ are the market prices per unit of housing service and nonhousing goods, $Q^m$ and $Q^n$ are the quantities of housing service and nonhousing goods consumed by the family in the absence of the public housing program, $Q^f$ and $Q^n$ are the quantities of housing service and nonhousing goods consumed by the family under the program, $B$ is the value of the program to the family, $S$ is the difference between the market value of the goods consumed by the family under the program and the family’s income. $Q^s$ and $Q^n$ are the quantities of housing service and nonhousing goods that the family would consume were it given an unrestricted cash grant equal to $S$ instead of its eligibility for public housing.
more housing service than it would have consumed in the absence of the public housing program. In addition the family spends
\[ P^*_n - P^*_i Q^*_i \]
less on housing. As a result, it consumes a greater quantity of nonhousing goods. To be precise, it consumes
\[ 100\left(\frac{P^*_n - P^*_i Q^*_i}{P^*_n Q^*_n}\right) \text{ percent} \]
more nonhousing goods. Since there are only two goods, everything that the consumer does not spend on housing service is spent on nonhousing goods. Hence,
\[ P^*_n Q^*_n = Y - P^*_i Q^*_i \]
and
\[ P^*_n Q^*_n = Y - P^*_n Q^*_n \]

The proportional change in the total quantities of housing service and nonhousing goods for a set of families living in different cities can be calculated by formulas 1 and 2 where the subscript \( i \) indicates the \( i \)th family and the subscript \( j \) indicates the \( j \)th city. We write each formula in two ways to point out that our quantity indexes are arrived at by dividing expenditure figures by price indexes.

\[
\begin{align*}
\sum_{i=1}^{k} \sum_{j=1}^{\gamma} (Q_{ij} - Q_{ij}^*) & = \frac{\sum_{i=1}^{k} \sum_{j=1}^{\gamma} [(P^*_n Q_{ij}^*)/P^*_n] - (P^*_n Q_{ij}^*)}{\sum_{i=1}^{k} \sum_{j=1}^{\gamma} (P^*_n Q_{ij}^*/P^*_n)} \\
\sum_{i=1}^{k} \sum_{j=1}^{\gamma} Q_{ij}^* & = \sum_{i=1}^{k} \sum_{j=1}^{\gamma} (P^*_n Q_{ij}^*/P^*_n)
\end{align*}
\]

\[
\begin{align*}
\sum_{i=1}^{k} \sum_{j=1}^{\gamma} (Q_{ij} - Q_{ij}^*) & = \frac{\sum_{i=1}^{k} \sum_{j=1}^{\gamma} [(P^*_n Q_{ij}^*)/P^*_n] - (P^*_n Q_{ij}^*/P^*_n)]}{\sum_{i=1}^{k} \sum_{j=1}^{\gamma} (P^*_n Q_{ij}^*/P^*_n)} \\
\sum_{i=1}^{k} \sum_{j=1}^{\gamma} Q_{ij}^* & = \sum_{i=1}^{k} \sum_{j=1}^{\gamma} (P^*_n Q_{ij}^*/P^*_n)
\end{align*}
\]

Notice that these formulas incorporate an implication of our assumptions that all markets are perfectly competitive and in long-run equilibrium, namely, that within each city all consumers pay the same price per unit of housing service and the same price per unit of nonhousing goods. However, it is not presumed that either price is the same in all cities.

There is some unrestricted cash grant \( B \) which, if given to this family in place of its eligibility for public housing, would make the family as well off as it is under the public housing program. This is what we mean by the benefit (or value) of the program to the family. Obviously, the value of the program to this family depends on its indifference map. In order to estimate the benefits of the program to families living in public housing,
we assume that all such families have a utility function of the form

\[ U = Q^a Q^b - a \]

For a family having this utility function, Joseph De Salvo has shown that

\[ B = (P^n Q^n / a)^{1 - a} \]

The mean benefit to a set of families occupying public housing is obtained by computing the benefit to each family and calculating the mean of these numbers. Mean benefit cannot be obtained by substituting the mean values of \( a, Y, P^n Q^n \) into (4).

Under the assumptions made in this section, the effect of the program on aggregate consumption of housing service and non-housing goods by the public housing tenants in our sample and the benefit to each family can be calculated from a knowledge of each family's income, its expenditure on housing under the public housing program and in its absence, the market rent of its public housing unit, the parameter of its indifference map, and the differences in the market prices of housing service and non-housing goods in different cities. We know the income of, and the rent paid by, a sample of public housing tenants. The U.S. Department of Labor produces cross-sectional indexes of housing and non-housing prices. We must predict how much each family would spend on housing in the absence of the program, the parameter of its indifference map, and the market rent of its public housing unit.

II. PREDICTION OF THE PARAMETER OF A FAMILY'S INDIFFERENCE MAP AND ITS HOUSING EXPENDITURE IN THE ABSENCE OF THE PUBLIC HOUSING PROGRAM

The "parameter" \( a \) is simply the proportion of income spent on housing by a consumer with the preferences (3) facing the budget constraint

\[ P_n Q_n + P_o Q_o = Y \]

We assume that \( a \) depends in part on certain family characteristics but is not identical for all families which are the same with respect to these characteristics. Specifically, we assume

\[ a = b_0 + b_1 R + b_2 S + b_3 N + b_4 A + u \]

where \( R = 1 \) if the head of household is nonwhite and 0 if white, \( S = 1 \) if the head of the household is female and 0 if male, \( N = \) number of persons in the household, \( A = \) age of the head of the household, and \( u = \) a random
variable with mean zero and variance independent of these family characteristics. Therefore, the best linear unbiased predictor of \( a \) for a family selected at random is

\[
\hat{a} = \hat{b}_0 + \hat{b}_1 R + \hat{b}_2 S + \hat{b}_3 N + \hat{b}_4 A
\]

where the \( \hat{b} \)'s are the least-squares estimators of the \( b \)'s in (5), based on a sample of families who buy both goods in the private market.

We had data on the housing expenditure, income, number of persons, and the age, race, and sex of the head of 168 households living in Boston, St. Louis, San Francisco, and Washington just prior to their admission to public housing. These data were used to estimate a relationship of the form (5) for each city separately and for all cities combined. The

| TABLE 1 Estimated Relationships between Rent–Income Ratio and Family Characteristics |
|----------------------------------------|--------|--------|--------|--------|--------|
|                                       | Boston | St. Louis | San Francisco | Washington | Combined |
| Constant                               | 0.3726 | 0.2830   | 0.3959  | 0.4788  | 0.4364  |
| (2.89)                                 | (2.16) | (2.39)   | (4.57)  | (5.89)  |
| Race of head of household              | -0.0688| -        | -0.0598 | -       | -0.0668 |
| (-0.97)                                | -      | (-0.72)  | -       | (-1.61) |
| Sex of head of household               | 0.0886 | 0.1758   | 0.0172  | 0.1311  | 0.0979  |
| (1.47)                                 | (2.30) | (0.26)   | (2.49)  | (3.19)  |
| Numbers of persons in household        | -0.0193| -0.0080  | 0.0109  | -0.0307 | -0.0164 |
| (-2.18)                                | (-0.69)| (.54)    | (-2.95) | (-2.95) |
| Age of head of household               | 0.0013 | -0.0016  | -0.0010 | -0.0023 | -0.0009 |
| (0.76)                                 | (-0.74)| (-0.49)  | (-1.39) | (-1.06) |
| Mean rent–income ratio                 | 0.35   | 0.34     | 0.35    | 0.39    | 0.36    |
| Number of observations                 | 40     | 48       | 40      | 40      | 168     |
| \( R^2 \) (adjusted)                   | 0.26   | 0.14     | -0.08   | 0.22    | 0.10    |
| Standard error                         | 0.15   | 0.17     | 0.19    | 0.14    | 0.17    |

NOTE: All families in the samples for St. Louis and Washington were nonwhite.
estimated coefficients, t-scores, and other statistics are reported in Table 1. We also had data on the characteristics of 333 families living in public housing in these four cities and Pittsburgh. The characteristics of the public housing tenants living in Boston, St. Louis, San Francisco, and Washington were substituted into the equations for these cities to predict the proportions of their incomes that they would devote to housing in the absence of the public housing program. Since we had no data on families living in private housing in Pittsburgh, we used the relationship based on data from all four cities to make predictions for public housing tenants in this city.

Each family's predicted rent-income ratio was multiplied by its income to predict how much the family would have spent on housing in the absence of the program.

III. PREDICTION OF THE MARKET RENT OF A PUBLIC HOUSING UNIT

Robert Gillingham has estimated relationships between market rent and housing characteristics in each of the five cities, using data for 1960. The housing characteristics included were age of structure; number of rooms; number of bathrooms; condition of unit; inclusion in rent of furnishings, refrigerator, air conditioning, and stove; the presence of hot running water, central heat, covered parking, and elevator; number of persons in unit; and race of the head of the household. We tried to obtain data on these characteristics for the 333 public housing units in our sample. There were some gaps in the data. Most importantly, year built was not reported for the 120 leased existing units in the sample. We assumed that the age of each of these units was equal to the median age of private housing in the same city. In all, somewhat less than 5 percent of the desired information was missing and was filled in with well-educated guesses. The information on the characteristics of the public housing units in the sample was substituted into Gillingham's equations to predict the market rents of these units in 1960. Bureau of Labor Statistics (BLS) time-series housing price indexes were used to adjust these predictions to 1972.

IV. EMPIRICAL RESULTS

We know the income of, and the rent paid by, a sample of 333 families living in public housing. There are cross-sectional indexes of housing and
nonhousing prices. The parameter of each family's indifference map, its housing expenditure in the absence of the program, and the market rent of its public housing unit have been predicted. The sample means of $P_0^xQ_0^x$, $P_0^nQ_0^n$, $P_n^xQ_n^x$, and $Y$ are $74$, $132$, $148$, and $450$ per month. The sample mean of $a$ is $.32$.

Using equations 1 and 2, we estimate that public housing tenants in this sample consumed 11 percent more housing service and 18 percent more nonhousing goods than they would have consumed in the absence of this program.\(^5\) Of course, different families experienced different percentage changes in their consumption of the two goods. Indeed, we estimate that 33 percent of these families occupied worse housing and 9 percent spent less on other goods.

The usual arguments for housing subsidies imply that the attainment of an efficient allocation of resources, preferred by everyone, to the allocation in the absence of subsidies requires that recipients consume more housing service and less nonhousing goods than they would consume were they allowed to choose any combination of goods with the same market value as the combination consumed under the housing subsidy program. Therefore, it is important to compare $Q_0^x$ and $Q_0^n$ in Figure 1. Table 2 presents the distribution of the variable

$$100(Q_0^x - Q_0^n)/Q_0^n = 100(P_n^xQ_n^x - P_0^xQ_0^x)/P_0^xQ_0^x$$

for the families in this sample. We estimate that public housing is more stimulative of housing consumption than cash grants for a bare majority (54 percent) of these families. Therefore, the effect of public housing on consumption patterns appears to be inconsistent with the rationale of this program for a substantial minority of its direct beneficiaries.

Formula 4 was used to estimate the value of the public housing program to each family in the sample. The mean of these numbers is $54$ per month.\(^6\) Had each of these families been allowed to consume any combination of goods with the same market value as the combination that it consumed under the program, the mean benefit would have been $74$ per month.\(^7\) The difference is a manifestation of the tremendous distortions in consumption patterns under the program from the viewpoint of public housing tenants. These distortions are evident in Table 2.

If there had been no cost of administering the public housing program and no inefficiency in producing housing service under the program, then $74$ would have been the mean cost incurred by taxpayers on behalf of the families in this sample. All existing estimates (Smolensky 1968, pp. 94–101; Olsen 1968, pp. 69–78; Muth 1973, pp. 7–20; Kraft and Olsen 1973, pp. 11–38) suggest that it costs more than a dollar to produce a dollar's worth of housing service under the conventional and turnkey variants of public housing. Therefore, even if the cost of administering a

---

58 | Kraft and Olsen
TABLE 2  Distribution of Deviations of Housing Consumption under the Public Housing Program from Housing Consumption with Unrestricted Cash Grants

<table>
<thead>
<tr>
<th>Percentage Deviations</th>
<th>Percentage of Families in Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than —49 to —40</td>
<td>6.6</td>
</tr>
<tr>
<td>—39 to —30</td>
<td>6.0</td>
</tr>
<tr>
<td>—29 to —20</td>
<td>6.6</td>
</tr>
<tr>
<td>—19 to —10</td>
<td>8.4</td>
</tr>
<tr>
<td>—9 to 0</td>
<td>6.9</td>
</tr>
<tr>
<td>0 to 9</td>
<td>11.4</td>
</tr>
<tr>
<td>10 to 19</td>
<td>12.0</td>
</tr>
<tr>
<td>20 to 29</td>
<td>8.7</td>
</tr>
<tr>
<td>30 to 39</td>
<td>6.9</td>
</tr>
<tr>
<td>40 to 49</td>
<td>7.8</td>
</tr>
<tr>
<td>More than 49</td>
<td>6.3</td>
</tr>
</tbody>
</table>

program of unrestricted cash grants was as great as the cost of administering the public housing program, the mean cost incurred by taxpayers on behalf of the families in this sample was probably greater than $74 per month. By giving these families unrestricted cash grants, it would have been possible to induce each family to consume a combination of goods that it considered to be as satisfactory as the combination consumed under the public housing program, while reducing the cost to taxpayers by at least 27 percent.

The preceding results are not based on a random sample from the population of families in public housing. Furthermore, the characteristics of the families in this sample differ significantly from the characteristics of all public housing families. Some of these differences are reported in Table 3.

In order to get better estimates of mean tenant benefit and proportional changes in consumption of housing service and nonhousing goods for the entire program, we regressed each of these variables on family characteristics, using data in the sample, and substituted the mean characteristics of all public housing families into these estimated
<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Sample</th>
<th>Universe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean monthly income</td>
<td>$450</td>
<td>$289</td>
</tr>
<tr>
<td>Mean number of persons</td>
<td>4.8</td>
<td>3.3</td>
</tr>
<tr>
<td>Mean age of head of household</td>
<td>39</td>
<td>49</td>
</tr>
<tr>
<td>Percentage of households headed by nonwhite</td>
<td>86</td>
<td>60</td>
</tr>
<tr>
<td>Percentage of households headed by female</td>
<td>60</td>
<td>72</td>
</tr>
</tbody>
</table>

**SOURCE:** The mean age of head of household for the entire population of households in public housing was estimated from the age distributions in Tables 108 and 113 of the 1971 HUD Statistical Abstract. The other means and percentages for the universe of households in public housing were estimated by Susan Kete of the U.S. Department of Housing and Urban Development.

relationships. In effect, we argue that, although the characteristics of the families in the sample differ significantly from the characteristics of all families in public housing, the relationships between the variables of interest and family characteristics in the sample are good estimates of the relationships for the population. The estimated coefficients, t scores, and other statistics are reported in Table 4. Substituting the mean characteristics of all families in public housing into the first two equations yields estimates of 33 and 14 percent for the changes in aggregate consumption of housing service and nonhousing goods. This compares with estimates of 11 and 18 percent based on the sample alone. These estimates lead us to believe that the increase in aggregate consumption of housing service is greater for all families in public housing than for the families in our sample and that the program distorts consumption towards housing for substantially more than 54 percent of all public-housing tenants. Substituting the mean characteristics of all families in public housing into the third equation leads us to conclude that mean tenant benefit for the entire program is $81 per month.

An estimated relationship between net tenant benefit and family characteristics is also useful for analyzing the distribution of benefits among families in public housing. For this purpose, we estimated the following relationship:

\[
B = 130.19 - 1.49Y - 0.000079Y^2 + 16.74N - 0.910N^2 \\
(4.05) \quad (-3.55) \quad (-2.75) \quad (3.38) \quad (-2.22) \\
-0.305A - 0.010A^2 - 15.11S - 22.53R \\
(-2.1) \quad (-0.6) \quad (-2.21) \quad (-2.46)
\]

\[R^2 (adj.) = 0.52; \quad S = 56\]

The numbers in parentheses are t scores.
Since we were able to reject at the 5 percent level of significance the hypothesis that the coefficients of the squared terms are all zero, we conclude that there are nonlinearities in the relationship between mean tenant benefit and family characteristics. Equation 6 indicates that mean benefit varies inversely with income and directly with family size for families of less than nine persons. We can be moderately confident that households headed by females receive smaller benefits than households headed by males and that nonwhite households receive smaller benefits than white households. The hypothesis that net tenant benefit varies with the age of the head of the household can be rejected at the 5 percent level of significance. Table 5 shows that if we do not hold constant other characteristics, mean benefit increases with income at the lowest income levels and decreases with income at higher levels.

### TABLE 4 Regressions of Weighted Proportional Changes in Consumption and Tenant Benefit on Family Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Weighted Proportional Change in Housing Consumption</th>
<th>Weighted Proportional Change in Nonhousing Consumption</th>
<th>Net Tenant Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>.991 (.987)</td>
<td>.0539 (1.55)</td>
<td>179.39</td>
</tr>
<tr>
<td>Monthly income</td>
<td>-.00232 (-25.72)</td>
<td>.00058 (18.48)</td>
<td>-.252 (-17.33)</td>
</tr>
<tr>
<td>Race of head of household</td>
<td>.0372 (.65)</td>
<td>-.0963 (-4.85)</td>
<td>-.2017 (-2.19)</td>
</tr>
<tr>
<td>Age of head of household</td>
<td>.00145 (.90)</td>
<td>-.0022 (-3.88)</td>
<td>-.541 (-2.09)</td>
</tr>
<tr>
<td>Number of persons in household</td>
<td>.0755 (8.53)</td>
<td>-.0189 (-6.15)</td>
<td>7.62 (5.34)</td>
</tr>
<tr>
<td>Sex of head of household</td>
<td>-.465 (-10.93)</td>
<td>.206 (13.92)</td>
<td>-16.86 (-2.46)</td>
</tr>
<tr>
<td>Mean of regressand</td>
<td>.11 .18</td>
<td>.54</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>333 333</td>
<td>333</td>
<td></td>
</tr>
<tr>
<td>$R^2$ (adjusted)</td>
<td>.67 .57</td>
<td>.50</td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>.35 .12</td>
<td>.57</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** $r$-statistics are in parentheses.
<table>
<thead>
<tr>
<th>Income Class</th>
<th>Mean Income of Households in Sample (1)</th>
<th>Mean Benefits to Households in Sample (2)</th>
<th>Standard Deviation of Benefits to Households in Sample (3)</th>
<th>Number of Households in Public Housing (4)</th>
<th>Number of Households in U.S. (5)</th>
<th>Percentage of Households Served by Program (6)</th>
<th>Mean Benefits to All Households in U.S. (7)</th>
<th>Standard Deviation of Benefits to All Households in U.S. (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0—999</td>
<td>$ 960</td>
<td>$ 478</td>
<td>$336</td>
<td>26,000</td>
<td>1,758,000</td>
<td>1.5</td>
<td>$ 7</td>
<td>$ 71</td>
</tr>
<tr>
<td>1,000—1,999</td>
<td>1,655</td>
<td>885</td>
<td>412</td>
<td>283,000</td>
<td>3,812,000</td>
<td>7.4</td>
<td>66</td>
<td>257</td>
</tr>
<tr>
<td>2,000—2,999</td>
<td>2,569</td>
<td>1,095</td>
<td>380</td>
<td>249,000</td>
<td>4,320,000</td>
<td>5.8</td>
<td>64</td>
<td>272</td>
</tr>
<tr>
<td>3,000—3,999</td>
<td>3,491</td>
<td>1,170</td>
<td>432</td>
<td>184,000</td>
<td>3,958,000</td>
<td>4.7</td>
<td>55</td>
<td>265</td>
</tr>
<tr>
<td>4,000—4,999</td>
<td>4,347</td>
<td>1,070</td>
<td>614</td>
<td>124,000</td>
<td>3,844,000</td>
<td>3.2</td>
<td>34</td>
<td>218</td>
</tr>
<tr>
<td>5,000—5,999</td>
<td>5,318</td>
<td>781</td>
<td>634</td>
<td>73,000</td>
<td>3,759,000</td>
<td>2.0</td>
<td>16</td>
<td>141</td>
</tr>
<tr>
<td>6,000—6,999</td>
<td>6,408</td>
<td>421</td>
<td>609</td>
<td>46,000</td>
<td>3,624,000</td>
<td>1.2</td>
<td>5</td>
<td>81</td>
</tr>
<tr>
<td>7,000—7,999</td>
<td>7,467</td>
<td>141</td>
<td>563</td>
<td>28,000</td>
<td>3,846,000</td>
<td>0.7</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1,055,000</td>
<td>68,537,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>5,397</td>
<td>649</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Columns (5), (6), and (7) are taken from Table 30 of Chapter 4 of Housing in the Seventies. All dollar amounts in this table are annual.
Not only does mean benefit vary inversely with income among families living in public housing, after taking account of the effects of other characteristics, but also such families are on the average poorer than those not served by the program. In 1972, the median annual income of all families in the United States was more than $10,000, whereas it was less than $3,000 for families in public housing.

Behind these measures of central tendency lie some disturbing variances which reflect unfavorably upon the equity of the public housing program. The estimate of the standard deviation of the error term in regression 6 suggests that we should expect roughly one-third of a set of families that are the same with respect to the five characteristics included in the regression to receive benefits deviating from the mean by at least $56 per month. This enormous variance in benefits to similarly situated families in public housing can also be seen in column 4 of Table 5. In part, these differences in benefits are due to the great variance in the desirability of different public housing units. Among families that are the same with respect to the five characteristics, some occupy new units in excellent condition while others live in deteriorating older units.

As important as the large variance in benefits to similarly situated families who get into public housing is the fact that most families at the lowest income levels receive no benefits. Indeed, most of the neediest families are not served, while many less needy families are served. Table 5 indicates that about 50 percent of the families in public housing have annual incomes in excess of $3,000, while 95 percent of all families in the United States with annual incomes less than $3,000 are not served by the program.

Combining data in columns 3 through 7, we are able to estimate the mean and standard deviation of benefits to all households in each income class. This mean benefit first rises and then falls with income. While these standard deviations are smaller than the standard deviations of benefits among families who get into public housing, the coefficients of variation are much larger.

NOTES

1. This assumption is consistent with the finding of Richard Muth (1960, pp. 42-46), but Frank de Leeuw and Nkanta Ekanem find price elasticities of long-run supply between 0.3 and 0.7.

2. The Cobb-Douglas indifference map is appealing because it yields an explicit formula for calculating benefits. Other widely used indifference maps do not yield such a formula. In these cases, it is necessary to solve a different nonlinear programming problem for each family in order to estimate its benefit. The Cobb-Douglas indifference map is roughly consistent with empirical evidence on the demand for housing service which suggests that the price elasticity of demand is approximately equal to minus one and the permanent-income elasticity is about equal to one. Unfortunately, our data
include current income but do not contain a good proxy for permanent income, and
estimates of the current-income elasticity are substantially less than one. In order to test
our specification (3), we estimated the parameters of a displaced Cobb-Douglas
indifference map,
\[ U = (Q_n - b_2)^{\alpha} (Q_n - b_3)^{1-\alpha} \]
of which equation 3 is a special case. We rejected at the 5 percent level of significance
the hypothesis that both displacement parameters are zero. Michael Murray has
estimated the parameters of a generalized Constant Elasticity of Substitution (CES)
indifference map using current income. The Cobb-Douglas indifference map is also a
special case of the generalized CES. Murray rejected at the 5 percent level of
significance the Cobb-Douglas specification. Clearly, we have sacrificed accuracy for
simplicity in estimating benefits.

3. We tested for nonlinearities by estimating a relationship including age of head of
household and number of persons squared. For all four cities we could not reject at the 5
percent level of significance the hypothesis that the coefficients of these two variables
are both equal to zero.

4. It should be noted that the coefficients of determination range in value from 0.55 to
0.75. The regressand in these regressions is the natural logarithm of gross rent, and data
on individual dwelling units are used.

5. In response to Henry Aaron's closing remark, it should be noted that the accuracy of
these estimates depends on the accuracy with which the mean housing expenditure in
the absence of the program and the mean market rent have been estimated. This
depends upon, but is not the same thing as, the accuracy with which the housing
expenditures of individual families and the market rents of individual dwellings can be
predicted. With 333 observations, the means can be estimated with much greater
accuracy than the values for individuals.

6. It is common to use aggregate data to estimate the mean benefit of a government
program to a set of families. Substituting the sample means of \( a, P^t Q^t, P^o Q^o, \) and \( Y \)
into formula 4 yields $72 per month. Obviously, the results of this procedure can be
very misleading.

7. The most frequently used measure of the benefit of a government program to a family is
the excess of the market value of the goods consumed under the program over the
market value of the goods that would have been consumed in its absence. Seventy-four
dollars is the mean of these differences for our sample. Since this measure assumes that
an individual is indifferent between all combinations of goods with the same market
value, it is clearly cruder than the measure used in this paper which only assumes that an
individual is indifferent between all dwelling units with the same market value.

8. The proportional changes in aggregate consumption of housing service and nonhousing
goods are equal to the means of the weighted proportional changes for individual
families, where the weights are the ratio of each family's consumption of the good to
mean consumption. That is
\[ \sum_{i=1}^{n} (Q_i^t - Q_i^o) / \sum_{i=1}^{n} Q_i^o = (1/n) \sum_{i=1}^{n} \left[ Q_i^o / \left( \sum_{i=1}^{n} Q_i^o / n \right) \right] ((Q_i^t - Q_i^o) / Q_i^o) \]
These weighted proportional changes in consumption are the dependent variables in
the regressions reported in Table 4. Substituting the mean characteristics of the
households in our sample into these estimated relationships yields the estimates of the
proportional changes in aggregate consumption already reported.

9. Two possibly important objections to this assumption are that the rent schedules and
the age and, hence, quality distributions of public housing units are likely to be different
between local housing authorities in the sample and other local housing authorities.
REFERENCES


2 COMMENTS

Henry Aaron

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Economists have long argued that subsidies should be provided in cash rather than in kind if the objective is to maximize the increase in the recipient's

NOTE: The views expressed are those of the author, and not necessarily those of the officers, trustees, or other staff members of the Brookings Institution or the University of Maryland.
well-being. As a number of authors have recently pointed out, this conclusion does not follow if those who are paying for the subsidy care about the items which the recipient consumes as well as (or instead of) his general well-being. Others have argued that society may be a better custodian of the rights of children than are parents, and that restriction of subsidies to items consumed jointly by the household (e.g., housing) or that bulk larger in the utility functions of children than in those of adults (e.g., food) may be warranted. Still others have held that the decision about which goods should be allocated through the market, which through other means, and which through some combination is itself a political judgment, in the making of which no automatic presumption in favor of market allocation is warranted. Indeed, Schmundt, Stieffel, and Smolensky have shown that if consideration is accorded to opinions about the consumption mix of beneficiaries held by persons other than the recipient, then transfers that do not distort consumption (i.e., unrestricted income transfers) are, in general, suboptimal.

However these issues are resolved, it is important to know how much particular subsidies do distort consumption choices of recipients. In such calculations, it is necessary, in principle, to take account of all other subsidies in measuring distortions, although in practice this counsel of perfection is usually ignored.

The paper by John Kraft and Edgar Olsen is an important step in the effort to estimate the inefficiencies of in-kind subsidies viewed solely in terms of the tastes of recipients. This paper grew out of evaluations performed within the Department of Housing and Urban Development to determine whether federal housing subsidies should be extended, revised, or abolished. Kraft and Olsen proceed by noting that public housing enlarges the choice set of recipients by one point. Access to public housing enables recipients to buy a certain quantity of housing at a subsidized price. In principle, this quantity of housing may be greater or less than the recipient would consume, and the rent charged may be greater or less than recipients would pay, in the absence of public housing. All that is required for the tenant to accept public housing is that he enjoy increased utility from the quantity of public housing and the quantity of other goods that can be purchased after the subsidized rent has been paid.

To clarify what Kraft and Olsen have done, consider the following simple identities

\[ C = \text{total cost of supplying public housing} \]
\[ -N = \text{inefficiencies of supplying public housing (may be positive or negative)} \]
\[ M = \text{market value of public housing} \]
\[ -S = \text{subsidy to tenants} \]
\[ R = \text{rent paid by tenants} \]

Clearly the net financial cost of public housing to various governments is \( N + S \), but the welfare cost of public housing is less to the extent that \( S \) improves tenant utility. Some unrestricted cash transfer, \( T \), would put the potential public housing
tenant on the same indifference surface as does public housing. If \( S - T = Z \) measures the consumption inefficiency of public housing, \( N + Z \) measures its welfare cost. Kraft and Olsen measure \( Z \). They ignore \( N \), which was treated in other HUD studies.

Kraft and Olsen computed \( M \) by modifying and applying hedonic price indexes (estimated by Robert Gillingham) to the physical characteristics of 333 public housing units. Given \( R \) from public housing records, \( S \) then is a residual. To calculate \( Z \), \( T \) must be estimated, which requires that Kraft and Olsen obtain the household's utility function.

They do so by assuming a Cobb-Douglas utility function of two goods, housing and other goods. The exponent of housing in this function is also its budget share. Kraft and Olsen estimate the proportion of income public-housing tenants would spend on housing if they rented on the unsubsidized market from a regression of this share on race, sex, and age of household head, and household size of a sample of low-income households from whom data were collected just before their admission to public housing in Boston, St. Louis, San Francisco, and Washington. These functions are used to calculate how much residents of public housing with various characteristics would spend on housing if they did not live in public housing. Given this share, the Cobb-Douglas utility function is specified (since the budget share of housing equals its exponent). It is then easy to calculate the unrestricted transfer \( T \) that would be just as attractive as access to public housing. Kraft and Olsen estimate that \( S = 74 \), \( T = 54 \), and that therefore the consumption inefficiency of public housing, \( Z \), was about 27 percent among sampled households.

Kraft and Olsen next regress the estimated change in housing and other consumption and estimated net benefit of sampled households on various household characteristics. Based on mean values of these characteristics for all public-housing tenants, they conclude that the distortion of consumption is considerably greater for all households than for sampled households.

Kraft and Olsen also estimate that net tenant benefit \( T \) first rises then falls with tenant income, presumably because the distortive effects of public housing are greater and, hence, the additions to utility are smaller as incomes tend toward zero. As incomes rise beyond a certain point, so do rents, thus reducing net benefits.

Finally, they observe that these inefficiencies do not occur so much because public housing on the average increases housing consumption more than consumption if the government gives \( S \) in cash, but rather because some households are forced to consume too much and some to consume too little. In fact, public housing induces a bare majority, 51 percent, of sample households to consume more housing than they would if given \( S \) in cash.

The conclusion is clear though unstated. For the sampled households, public housing distorts consumption but increases it no more on the average than would an unrestricted transfer. Whatever might be said on behalf of a program that increased housing consumption of most families, little can be said for one that capriciously distorts it. Some other form of housing assistance must be better, and housing allowances spring to mind. While I agree with the conclusion, I have serious quibbles about the analysis.
My comments fall into two categories. The first concerns the possibility of bias in the point estimates Kraft and Olsen present. The second concerns the standard error that surrounds those estimates.

1. The fraction of income households would freely spend on housing is crucial to the analysis. This fraction is based on regressions using data for households just prior to their admission to public housing. The proportion of the variance in the housing income ratio explained by these regressions is remarkably low [one $R^2$ (adjusted) is negative, the others range from .14 to .26]. This means that most of the variance in the housing-income ratio is due to other factors. The method that Kraft and Olsen have used converts these unexplained variances into estimates of distortion by public housing.

An example will illustrate the problem. Assume that we estimate the rent-income ratio for a sample of households as a function of several variables, exactly as Kraft and Olsen did, and that the regression explains, say, 20 percent of the variance in the rent-income ratio. Assume now that this sample moves into public housing and that each family consumes exactly the same amount of housing as it would have consumed if it had been given $S$ in cash. By definition there is no distortion in housing consumption: $S = T$ and $Z = 0$. Yet if we estimate the distortion from the predicted rent-income ratio, in a manner analogous to that used by Kraft and Olsen, we will conclude that 80 percent of the variation in rent-income ratios is the result of distortion caused by public housing. This problem vanishes only if the equation predicting the rent-income ratio has an $R^2$ of 1.0, and it is directly proportional to the amount by which $R^2$ is less than 1.0. One must hold suspect calculations which are based on regressions that leave at least three-quarters of the variation in the rent-income ratio unexplained, and which, in effect, charge as distortions of public housing the difference between the actual rent-income ratio and such an estimated value.

There are two other reasons why, I think, Kraft and Olsen overestimate distortion. First, one would expect that families with above-average tastes for housing would show positive residuals in the equation that estimates rent-income ratios, and would be more likely than families with lower tastes to accept public housing only if it were better than average (and, of course, relatively cheap). The reverse argument suggests that households with below-average tastes for housing would differentially end up in lower-quality public housing units. By ignoring both such assortative effects, Kraft and Olsen tend to overestimate distortion.

But even if distortion is overestimated, any distortion is too much if public housing does not cause more housing to be consumed than would an unrestricted cash transfer. I think there is reason to suspect that Kraft and Olsen underestimate the impact of public housing on housing consumption.

The rent-income ratio of households just before admission to public housing is likely to be abnormally high. It is more likely that the current measured income of such households is below rather than above normal income. Kraft and Olsen acknowledge that using measured income may cause problems. In fact, they understate the increase in housing consumption to the extent that the ratio of housing consumption to current income exceeds the ratio of housing consumption to normal income.
Second, Kraft and Olsen use a convenient construct, developed, I think, by Muth, that defines a unit of housing service as whatever $1 buys. It is a very useful approach and I have used it, as has Olsen, in previous work. The rationale for using this construct is that under conditions of equilibrium, the marginal utility of $1 spent by the tenant on each feature of the house must be equal and must equal the cost of providing it. Those of us who feel that the housing industry is fairly competitive have no trouble accepting this assumption for aggregative analysis. However, I think that it is unacceptable at the micro level.

Unless all households are in equilibrium all the time, the marginal value to particular households of the various features of a housing unit may not equal the market value of these features. Even if we are willing to apply hedonic indexes estimated for market housing to public housing, there is no reason to think that public-housing tenants value the particular bundle of features public housing units contain in the same way as the market would. In other words, there is no way of knowing whether a public-housing tenant, who if given $S$ would have spent exactly $M$ on housing, would have chosen public housing—with its peculiar set of features often unavailable in the free market—or market housing if both had a market value of $M$ and rented for $M$. Once again, however, there is some reason to think that households that cared less than average for the features that public housing had in greatest supply would be less likely to end up in public housing. Conversely, one would expect that households that liked the features of public housing more than average would be more likely to end up in public housing.

2. The estimates presented in the Kraft-Olsen paper are consistently insignificant. Not only are the relationships between rent-income ratios very loose (8 coefficients out of 14 are smaller than their standard errors), but, more importantly, the estimates of the impact of public housing on consumption of housing and of other goods and the estimates of the net tenant benefit are so unreliable that one cannot reject any of the following hypotheses for particular households at even unimpressive levels of significance:

1. that housing consumption doubles;
2. that housing consumption declines 50 percent;
3. that housing consumption goes up more than nonhousing consumption;
4. that housing consumption goes up less than nonhousing consumption; or
5. that net benefits of public housing are negative.

Having worried about the same questions Kraft and Olsen address in their paper, and having gotten nowhere, I admire the ingenuity and the care that they have demonstrated. I also have preconceptions about the desirability of subsidies tied to housing demand rather than to supply, which I know they share. I look forward to evidence in support of our views that is free of the questionable elements of the current paper. In the meantime, we should all be grateful for the step forward that their effort represents.