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# A SURVEY AND COMPARATIVE ANALYSIS OF VARIOUS METHODS OF SEASONAL ADJUSTMENT

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## HISTORICAL DEVELOPMENT OF SEASONAL ADJUSTMENT TECHNIQUES

The unobserved component model assumes that a time series $Y_{ij}$ consists of three parts: A trend-cycle component $T_{ij}$, a seasonal component $S_{ij}$, and an irregular component $I_{ij}$. If this relationship is assumed to be additive, the statistical model may be written as

$$Y_{ij}=T_{ij}+S_{ij}+I_{ij}$$

where $i=1, \ldots, 12$ indicates the months and $j=1, \ldots, n$, the years. (For more details, see [51].)

The methods used for seasonal adjustment by earlier research workers were adaptations of this model. For example, the filters of Buys Ballot [9] were used in early studies. (See [17; 18].)

The ratio-to-moving-average method was developed during the 1920's by Frederick R. Macaulay at the National Bureau of Economic Research. It was found that, for most economic time series, a multiplicative relationship between the three components holds. (See [25].)

Abraham Wald [50] developed the moving-amplitude method to overcome shortcomings of the then current methods. His method requires that the seasonal pattern, i.e., the proportionality relationship between the seasonal factors for the months within a year, remains stable. However, he allows for relatively rapid changes in the seasonal amplitude. Wald's method is described in detail by Godfrey and Karreman [16]. The method was generalised by Zaycoff [53] Tinbergen [43] applied these methods to time series for the Netherlands and found that they both gave superior results relative to then common methods. Mendershausen [29] gives a detailed description of work on seasonal adjustment, especially Wald's method.

An extensive survey of the historical development of seasonal adjustment methods is presented in BarOn [1].

## SEASONAL ADJUSTMENT METHODS STUDIED

From the outset, it was decided to limit this study to methods currently used by government agencies.

The methods which were analyzed are described in the following sections. They are the census X-11 method; Statistics Canada X-11; the Burman method, used by the Bank of England; the method of the European Economic Communities; the Berlin method, used by the DIW (Deutsches Institut für Wirtschaftsforschung) and the Statistisches Bundesamt in Wiesbaden; and the method of the Dutch Central Planning Bureau.

The seasonal factor method of the Bureau of Labor Statistics is quite similar to the X-11 method. It was excluded from this study because BLS now uses the X-11 method. It is described in [47].

Regression methods were used in the early 1960's by the Deutsche Bundesbank. They have recently been used experimentally at the Board of Governors of the Federal Reserve System. (See [42].) The British Central Statistical Office developed a regression technique that was used by the Department of Employment from 1969 to 1972. (See [6; 13].) This method allows for mixed adjustment, i.e., a combination of additive and multiplicative adjustment.

Box and Jenkins [4] have applied time series methods to seasonal adjustment. The Box-Jenkins approach has been used extensively by Brewer. (See [5]. Also see [10; 11; 44].)

Both the regression and the Box-Jenkins approach require careful analysis of each individual series. These methods are, thus, not suitable for use as the standard

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method of adjustment for a government statistical agency, but they may be useful for the adjustment of a limited number of important series.

De Vos [49] has applied ARIMA models to estimate the trend cycle component of the series of Dutch male unemployment. In his adjustment method, he also includes the number of days with a maximum temperature below zero.

## X–11 Method

Census method II was developed by the U.S. Bureau of the Census. It is based on the ratio-to-moving-average method. By switching to a computer program in 1954, large-scale adjustment of time-series became practicable. Census method II differs from the earlier method, because it estimates the trend-cycle, seasonal, and irregular components using several iterations, an adjustment for trading-day variations, and an adjustment for extreme values. Both an additive and a multiplicative version are available.

The census method II programme became stable by the end of 1961. The X–11 variant of the programme has been in current use since 1965. It is a modification of earlier Bureau of the Census programmes and is described in [46]. Also see [28; 52].)

Because this program is well-known, only a brief description will be given. The trend cycle component of the series is removed with a 9-, 13-, or 23-term Henderson moving average. The 9-term average is used for smooth series and the 23-term filter for highly irregular series (those with a preliminary I/C ratio of 3.5 and over). The seasonal factors are calculated with a [3, 3] filter in the first round and with a [3, 5] filter in the second round.

The $\sigma$ limit, used to eliminate extreme values completely, is 2.5. Between 1.5$\sigma$ and 2.5$\sigma$ graduated weights are used.

## Statistics Canada X–11

The moving-average methods employed by the X–11 programme require data for up to 3 additional years before the symmetric filters may be applied. This necessitates the use of asymmetric filters for the last observations of a time series.

In January 1975, Statistics Canada introduced a modification of the X–11 method that is used for the seasonal adjustment of about 60 of the more important Canadian Labor Force Survey series. It consists of enlarging the original time series by 1 additional year, with forecasts from ARIMA models. The method is described in [12]. This modification resulted in improved estimates of the current seasonal factors, based on a comparison with the stable factors, calculated when 3 years of additional observations have become available.

## Burman Method

This method was developed by J. P. Burman at the Bank of England. It is fully described in [7]. Both an additive and a multiplicative option are available. The multiplicative option is based on converting the series to natural logarithms.

A 13-term weighted average is subtracted from the series to eliminate the trend. This filter was developed by Burman and has weights (−0.0331, −0.0208, 0.0152, 0.0755, 0.1462, 0.2039, *0.2262*). Harmonic analysis on successive blocks of 12 terms of the SI series is then performed. The amplitudes are smoothed. Linear combinations of these smoothed amplitudes give the preliminary seasonal factors. Extreme trend values may be replaced by a weighted average of the neighbouring terms (six at each side). The preliminary adjusted series is extrapolated by a Box-Jenkins approach (0, 1, 1) or (0, 2, 2) model to give six more terms at the end and six more terms at the beginning. Using these terms, one obtains the trend and seasonal factors for the end terms. These terms are then used to extend the original unadjusted series, and this series is used to determine the final seasonal pattern in a second iteration.

The Burman method replaces extremes above 2.5$\sigma$ and uses graduated weights between 2.0$\sigma$ and 2.5$\sigma$. An option is available to omit the replacement of extremes and the extrapolation of the original series. Extrapolation is suppressed when a large number of extremes have been identified in the end terms.

## The Method of the European Communities

The EEC method, also known as the SEABIRD method, was developed by Bongard and Mesnage at the Statistical Office of the European Communities. As a supra national agency, the EEC obtains most of its statistical series from member countries, the data may have been adjusted, using different procedures, or may still be in an unadjusted form. They require a method which has to be universal, i.e., capable of effectively adjusting the widest possible variety of economic time series, and robust, i.e., manual intervention, either before or after adjustment, should not be required. Furthermore, because the primary use of the adjusted series is for economic analysis, it emphasizes the adjustment of the most recent data. A detailed description is given in [30].

As a first step, extreme values are eliminated. The second step applies the Bongard 19-term filter [2] to the raw data, modified for extreme values. The method assumes that the seasonal pattern (PSN) is relatively stable in time, but the magnitude of the seasonal fluctuation, measured by the coefficient of expansion $\delta$, may change rapidly. The seasonal component is, thus, written as

$$S = \delta.\text{PSN}$$

A first estimate of PSN is obtained by eliminating the irregular component from the SI series. The value of $\delta$ is then calculated as

$$\delta = \frac{\Sigma \text{PSN.SI}}{\Sigma \text{PSN.PSN}}$$

The values of PSN for the end terms are obtained by repeating the last value calculated. The values of $\delta$ are extrapolated by regressing $\delta$ on T.

The calculations are repeated once with the restriction that no new estimate of the seasonal pattern is made.

Recently, the EEC has developed a new method called DAINTIES, which will replace SEABIRD. (See [3; 31].)

## The Berlin Method

This method is, like the EEC method, primarily used for current analysis. Thus, revisions to the current adjusted observations are minimized. It is described in [36]. The version presently in use is ASA III. (See [34; 35].)

The Berlin method assumes an additive relationship between the trend-cycle, seasonal, and irregular component. Observations identified as extremes are first eliminated. An observation is considered to be extreme if it exceeds $3\sigma$, based on the 24 previous observations, and is replaced by the value of $2\sigma$. This implies that once an extreme has been replaced, the new observation will remain unchanged.

The trend and the seasonal component are obtained using filters which have been estimated in such a way that their transfer functions have, as far as possible, optimal spectral properties. The transfer function for the trend filter should be near one for low frequencies $(\lambda < \frac{\pi}{6})$ and should be approximately zero for the other frequencies $(\frac{\pi}{6} \leqq \lambda \leqq \pi)$, while the transfer function for the seasonal component should equal to one at seasonal frequencies $(\lambda = j\pi/6,\ j = 1, 2, \ldots 6)$ and be approximately equal to zero at all other frequencies.

The trend filters consist of polynomials of up to degree 3, together, in some cases, with trigonometric functions of a length of 36 or 60 months to represent a cyclical pattern. The seasonal filters are estimated with harmonic analysis using 11 trigonometric base functions. The trend filters use between 34 and 39 observations (27 to 34 for end terms), while the seasonal filters use either 47 or 59 observations (36 or 48 for end terms). The filters are applied asymmetrically over the full length of the time series. For example, several trend filters of length 36 are used that estimate the 25th, 27th, or 29th observation. The seasonal component filter with length 47 estimates the 24th observation, and the one with length 59, the 36th observation.

The trend-cycle component is estimated from the original series, corrected for extreme values. The seasonal component is estimated after the trend-cycle has been identified.

Berlin ASA-III is now used for about 300 monthly series which require a total of 47 different combinations of filters.

The filter selected is the one that minimizes the sum of squared derivations between the spectrum of the original series and that of the adjusted series at all but the seasonal frequencies.

## Central Planning Bureau Method

The Central Planning Bureau method allows for both additive and multiplicative seasonal influences. The method is an adaptation of a hand method which was used for many years by the Central Planning Bureau. It is used mainly for adjusting quarterly series but is now also used by the Bureau of Statistics for the monthly unemployment series. The procedure is related to the Wald method [50]. (For a detailed description, see [19].)

The CPB method assumes that the trend-cycle, seasonal and irregular components are related additively and that the seasonal component ($S_{ij}$) may be represented as the product of a seasonal factor ($s_{ij}$) and a multiplicator ($m_{ij}$), i.e.,

$$S_{ij} = s_{ij} \cdot m_{ij}$$

The seasonal factor represents exogenous influences (such as the weather, holidays, etc.) which have an effect on both the direction and size of the seasonal component, while the multiplicator represents endogenous influences affecting the size of the seasonal component only.

The seasonal factors are measured separately for each month and are fitted to a second degree polynomial, i.e.)

$$s_{ij} = a_{0i} + a_{1i}t + a_{2i}t^2$$

For most time series, the multiplicator may be estimated as a linear relationship

$$m_{ij} = (1 - \lambda) + \lambda(T_{ij}/\overline{T})$$

where $\overline{T}$ is the average trend-cycle component for the mid-year (i.e., when $t$ is 0). The value of $\lambda$ lies in the interval $0 \leqq \lambda \leqq 1$.

The seasonal component will, therefore, be independent of the level of the series when $\lambda = 0$ (i.e., complete additivity), while with $\lambda=1$, a multiplicative relationship holds.

For certain volatile stock variables, such as unemployed, this linear relationship may not be sufficiently general to describe the series adequately. In these cases, the multiplicator is assumed to be a function of both the level and the relative change in the trend. The relaship estimated, after transformation, is

$$m_{ij}=\exp\left[b_1\left\{ln\frac{T_{ij}/\overline{T}+b_2}{1+b_2}-\frac{T_{ij}/\overline{T}-1}{T_{\max}/\overline{T}+b_2}\right\}\right]$$

where $T_{max}$ is the trend value for which $m_{ij}$ reaches its maximum.

The trend and seasonal component are estimated iteratively. In the first round, a 12-term moving average, is used to determine the trend that is replaced during four later iterations by the 15-term Spencer.

Before each iteration, irregulars above $1.5\sigma$ are considered as extremes and eliminated.

## TIME SERIES USED

The results obtained from seasonally adjusting two series, U.S. total employed and U.S. total unemployed, will be reported. The period covered is from January 1953 to December 1975.

Total employed is a relatively smooth series with a 1-percent absolute month-to-month variation, while total unemployed is highly volatile, with a 9-percent absolute month-to-month variation.

In addition to these two series, fifteen other series were analyzed. Because it was found that the analytical results for these series were very similar to those obtained for total employed and total unemployed, the relevant summary measures have not been included.

## COMPARISONS OF SEASONAL ADJUSTMENT METHODS

The major difficulty in making comparisons between adjustment methods is that actual series have unknown composition. An experimental technique of generating artificial series and analysing the decomposition has been adopted in several studies. (See [16; 18].)

A difficulty with this approach is that the choice of generating mechanism of the components may in itself favor certain methods, because the underlying model used in the adjustment process was also used in generating the artificial series. In this study, only published series were used.

Fase, Koning, and Volgenant [15] have compared the seasonal adjustment for four published Dutch series and one generated series, using nine seasonal adjustment methods, including the X–11, Burman, CPB, and EEC methods. Fry [48] has compared the results from several seasonal adjustment methods on one series, the monthly money stock (M1).

### Choice Between Additive and Multiplicative Adjustment

The X–11 and Burman methods require that either additive or multiplicative adjustment be selected. Because most series, especially for the post war period, are better represented by a multiplicative pattern, there has been a tendency to use this method without checking if the additive option might be more appropriate.

Recently, model selection routines have been suggested that test both options. (For a description of work in Great Britain, see [13; 22].) Model selection routines are also used at Statistics Canada and the Bureau of Labor Statistics.

The following model test was applied. A SI series was obtained by subtracting a [12, 2] trend from the original series. A trend was then fitted to 7-year spans, i.e.,

$$SI=a+bt+e$$

If only the slope coefficient is significant, multiplicative adjustment is suggested, and additive adjustment in the reverse case. If both the intercept and slope coefficient are significant a mixed adjustment technique would be appropriate.

The $t$-values for the period July 1953–June 1975 are presented in table 1. The results show that multiplicative adjustment would have been appropriate for the unemployment series up to 1970, with a shift to mixed adjustment after 1970.

The employment series might have been adjusted with a mixed model during the full period.

Because no clear pattern of either additivity or multiplicativity was shown, it was decided to include both options for the X–11 and Burman programme in the comparison of seasonal adjustment methods for these series.

It should be mentioned that the BLS obtains the seasonally adjusted total employed and total unemployed by aggregating component series. The additive option of X–11 method is used for males 16–19 years old and for females 16–19 years old, while the multiplicative option is used for the other components.

### Summary Measures Used

The summary measures presented in this section are used to determine the extent to which there are significant differences between the various methods of seasonal adjustment analyzed.

They were calculated for the full period (1953–75); for the historical period (1956–72), for which the seasonal factors may be considered to be final in a statisti-

Table 1. t-VALUES FOR MODEL TEST FOR 7-YEAR SPANS

| Years | Employed | | Unemployed | |
|---|---|---|---|---|
| | Intercept | Slope | Intercept | Slope |
| 1953-59 | 2.0 | 3.1 | 1.2 | 0.8 |
| 1954-60 | 3.8 | 5.2 | .0 | 2.4 |
| 1955-61 | .3 | 1.3 | .1 | 2.8 |
| 1956-62 | .5 | .3 | .4 | 1.9 |
| 1957-63 | 1.1 | .2 | .5 | 1.1 |
| 1958-64 | 1.9 | .8 | 2.0 | 3.6 |
| 1959-65 | 3.1 | 1.7 | 1.5 | 5.2 |
| 1960-66 | 4.4 | 2.7 | 1.5 | 6.8 |
| 1961-67 | 8.3 | 3.4 | 2.0 | 7.9 |
| 1962-68 | 8.8 | 3.7 | .5 | 3.4 |
| 1963-69 | 8.2 | 4.3 | .3 | 3.6 |
| 1964-70 | 5.3 | 3.3 | 1.9 | .3 |
| 1965-71 | 6.3 | 4.4 | 2.4 | 1.4 |
| 1966-72 | 4.6 | 3.1 | 2.4 | 2.0 |
| 1967-73 | .9 | .0 | 2.5 | 1.9 |
| 1968-74 | 1.2 | 2.1 | .7 | 2.0 |
| 1969-75 | 2.0 | 2.4 | 1.6 | 5.1 |

Note: Additional tabular materials and computer printouts are available from the author upon request.

Table 2. AVERAGE ABSOLUTE MONTH-TO-MONTH PERCENTAGE CHANGE

| Method | 1953-75 | 1956-72 | 1973-75 | 1975 |
|---|---|---|---|---|
| | Employed | | | |
| X-11 Additive | 0.32 | 0.31 | 0.27 | 0.25 |
| X-11 Arima Additive | .32 | .31 | .25 | .20 |
| Burman Additive | .33 | .32 | .26 | .20 |
| Berlin | .30 | .29 | .27 | .24 |
| CPB | .32 | .32 | .27 | .24 |
| EEC | .32 | .32 | .29 | .34 |
| X-11 Multiplicative | .32 | .31 | .27 | .24 |
| X-11 Arima Multiplicative | .32 | .31 | .25 | .18 |
| Burman Multiplicative | .33 | .32 | .26 | .21 |
| Raw series | .92 | .93 | .91 | .75 |
| | Unemployed | | | |
| X-11 Additive | 3.56 | 3.34 | 2.60 | 1.40 |
| X-11 Arima Additive | 3.55 | 3.34 | 2.48 | 1.18 |
| Burman Additive | 3.83 | 3.37 | 2.77 | 1.19 |
| Berlin | 3.55 | 3.31 | 2.96 | 1.05 |
| CPB | 3.60 | 3.38 | 2.84 | 1.90 |
| EEC | 3.58 | 3.20 | 3.50 | 3.26 |
| X-11 Multiplicative | 3.66 | 3.38 | 3.29 | 3.06 |
| X-11 Arima Multiplicative | 3.66 | 3.38 | 3.25 | 2.99 |
| Burman Multiplicative | 3.87 | 3.51 | 3.39 | 3.47 |
| Raw series | 9.00 | 9.15 | 7.91 | 3.70 |

cal sense; and for the current period (1973–75). Where applicable, the calculations were also performed for individual years of the current period (1973, 1974, and 1975).

**Average absolute month-to-month percentage change**—The average absolute month-to-month percentage change indicates to what extent the series has been smoothed by seasonal adjustment.

These percentages changes are presented in table 2 for the periods 1953–75, 1956–72, 1973–75, and separately for 1975.

The reduction in the average absolute month-to-month change is about the same for all methods for the historical period (1956–72), but significant differences are shown for the current period (1973–75). The differences for 1975 are the most striking.

**Correlation coefficient**—The correlation coefficients between the adjusted series for the historical period, the current period, and for 1975 are presented in table 3 for employed and in table 4 for unemployed.

For the historical period, the correlation coefficients are very close to one, while, for the current period, divergences occur, especially in the series of unemployed.

**Inequality coefficient**—A second measure which was used to describe the differences between seasonal adjustment methods is the inequality coefficient. This coefficient was used by Fase, Koning, and Volgenant [15]. It quantifies differences between the estimated seasonal components. It is defined as the ratio of the average absolute difference between two methods of adjustment to the original series, multiplied by 100, i.e.,

$$IC = \Sigma \frac{|S1_t - S2_t|}{R_t} * 100$$

The statistic has a minimum value of zero when two adjustment procedures provide an identical seasonally adjusted series.

Inequality coefficients for all nine procedures are presented in table 5 for employed and table 6 for unemployed for the periods 1956–72, 1973–75 and for 1975. The results show very clearly that the differences between the seasonally adjusted series are relatively small for the historical period but that there are significant differences for the current period and especially for 1975.

**Relative contribution of components to variance in original series**—The relative contribution of the irregular, trend cycle and seasonal, component to the variance in the original series is widely used for analysis. This table is included in the summary measures table (pt. F) of the X–11 programme.

The contributions to between month variance are presented in table 7 for the historical period (1956–72) and the current period (1973–75).

This table shows that the relative contribution of each component is very similar in the historical period but that divergences occur during the current period.

**Average duration of run**—The average duration of run was used to check that the irregulars may be considered to have been generated by a random process. (See [21].) It should be noted that an oscillatory series may be generated when a moving average of a random series is taken (the Slutsky-Yule effect). (See [41].)

This measure was calculated for the irregulars (after extremes were removed). Table 8 gives a summary for 1953–75, 1956–72, and 1973–75. The expected value of this statistic for the full period is 1.50 with 95 percent confidence limits of 1.40 and 1.61. For a 3-year period, the expected value is 1.48, with confidence limits of 1.23 and 1.86.

The Berlin method is the only procedure which falls outside these limits, except for the employed during the historical period.

**Optimal properties of seasonal adjustment**—As an alternative to the summary measures used in this section, one might determine to what extent optimal properties of seasonal adjustment are met. (For this approach, see [26; 27].)

## Spectral Analysis

In many studies, an evaluation of seasonal adjustment procedures has been undertaken in spectral terms. (See [20; 24; 32; 33; 39; 40; 45]; for a discussion of spectral analysis in terms of the time domain, see [14].)

A spectral evaluation of the seasonally adjusted series was performed for the methods analyzed. One of the spectral requirements is that seasonal peaks, in the spectrum of the original series, should be removed in the spectrum of the seasonally adjusted series. Also, phase shifts should not occur. The power density spectra for the adjusted series were quite similar. All exhibited, to some extent, dips at the seasonal frequencies. Therefore, it was not practicable to discriminate between the procedures on the basis of the spectral criteria.

## Stability of the Seasonal Component

One of the important properties of a seasonal adjustment procedure is that the preliminary estimate of the seasonal factors be relatively close to the final estimate.

To test the extent to which the seasonal component changes when additional observations become available, the seasonal factors for the current year were compared with the seasonal factors calculated when 3 additional years of data have become available. At that time, the seasonal factors may be considered final.

Table 3. CORRELATION COEFFICIENTS—EMPLOYED

| Method | X-11 A | X-11 AA | BUR A | Berlin | CPB | EEC | X-11 M | X-11 AM | BUR M |
|---|---|---|---|---|---|---|---|---|---|
| Period 1956–72 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | 1.000 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | 1.000 | 1.000 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | 1.000 | 1.000 | 1.000 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | (X) |
| Burman Multiplicative | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Period 1973–75 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .997 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .990 | .996 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .995 | .992 | .984 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .999 | .997 | .989 | .995 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | .978 | .980 | .983 | .977 | .975 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | .999 | .998 | .993 | .993 | .997 | .982 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | .995 | .999 | .997 | .989 | .995 | .982 | .997 | 1.000 | (X) |
| Burman Multiplicative | .988 | .995 | .999 | .983 | .988 | .982 | .993 | .997 | 1.000 |
| Year 1975 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .995 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .948 | .967 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .993 | .991 | .962 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .993 | .993 | .942 | .985 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | .780 | .797 | .860 | .824 | .752 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | .998 | .996 | .959 | .996 | .991 | .812 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | .992 | .999 | .977 | .993 | .988 | .818 | .994 | 1.000 | (X) |
| Burman Multiplicative | .948 | .967 | .997 | .966 | .944 | .865 | .959 | .978 | 1.000 |

X Not applicable.

## Table 4. CORRELATION COEFFICIENTS—UNEMPLOYED

| Method | X-11 A | X-11 AA | BUR A | Berlin | CPB | EEC | X-11 M | X-11 AM | BUR M |
|---|---|---|---|---|---|---|---|---|---|
| Period 1956–72 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | 1.000 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .998 | .998 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .996 | .996 | .996 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .998 | .998 | .998 | .995 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | .995 | .994 | .995 | .993 | .994 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | .997 | .997 | .997 | .995 | .998 | .996 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | .997 | .997 | .997 | .995 | .998 | .996 | 1.000 | 1.000 | (X) |
| Burman Multiplicative | .996 | .996 | .997 | .994 | .997 | .995 | .998 | .998 | 1.000 |
| Period 1973–75 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .999 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .999 | 1.000 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .997 | .994 | .994 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .999 | .998 | .997 | .998 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | .994 | .995 | .994 | .988 | .994 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | .995 | .996 | .995 | .991 | .996 | .998 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | .995 | .997 | .996 | .990 | .995 | .998 | 1.000 | 1.000 | (X) |
| Burman Multiplicative | .994 | .996 | .995 | .988 | .993 | .998 | .999 | .999 | 1.000 |
| Year 1975 | | | | | | | | | |
| X-11 Additive | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .885 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .880 | .958 | 1.000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .732 | .355 | .415 | 1.000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .927 | .764 | .718 | .717 | 1.000 | (X) | (X) | (X) | (X) |
| EEC | .542 | .673 | .614 | .069 | .626 | 1.000 | (X) | (X) | (X) |
| X-11 Multiplicative | .497 | .645 | .576 | .054 | .560 | .961 | 1.000 | (X) | (X) |
| X-11 Arima Multiplicative | .494 | .716 | .666 | -.020 | .494 | .922 | .966 | 1.000 | (X) |
| Burman Multiplicative | .395 | .684 | .617 | -.164 | .372 | .895 | .932 | .967 | 1.000 |

X Not applicable.

Table 5. INEQUALITY COEFFICIENTS—EMPLOYED

| Method | X-11 A | X-11 AA | BUR A | Berlin | CPB | EEC | X-11 M | X-11 AM | BUR M |
|---|---|---|---|---|---|---|---|---|---|
| Period 1956–72 | | | | | | | | | |
| X-11 Additive | 0.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .002 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .060 | .061 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .108 | .109 | .113 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .068 | .067 | .095 | .131 | .000 | (X) | (X) | (X) | (X) |
| EEC | .100 | .101 | .104 | .106 | .129 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | .016 | .018 | .065 | .108 | .069 | .100 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | .016 | .016 | .065 | .108 | .068 | .100 | .003 | .000 | (X) |
| Burman Multiplicative | .065 | .066 | .019 | .116 | .099 | .107 | .069 | .069 | .000 |
| Period 1973–75 | | | | | | | | | |
| X-11 Additive | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .066 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .123 | .079 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .079 | .109 | .150 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .039 | .070 | .130 | .089 | .000 | (X) | (X) | (X) | (X) |
| EEC | .144 | .143 | .140 | .162 | .157 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | .042 | .050 | .095 | .081 | .066 | .142 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | .089 | .034 | .062 | .130 | .097 | .139 | .066 | .000 | (X) |
| Burman Multiplicative | .127 | .084 | .030 | .147 | .133 | .145 | .097 | .062 | .000 |
| Year 1975 | | | | | | | | | |
| X-11 Additive | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .097 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .166 | .099 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | .052 | .101 | .153 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .048 | .089 | .170 | .076 | .000 | (X) | (X) | (X) | (X) |
| EEC | .282 | .249 | .215 | .254 | .298 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | .032 | .070 | .136 | .040 | .062 | .253 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | .133 | .038 | .074 | .128 | .127 | .238 | .105 | .000 | (X) |
| Burman Multiplicative | .163 | .099 | .026 | .145 | .163 | .217 | .132 | .072 | .000 |

X Not applicable.

Table 6. **INEQUALITY COEFFICIENTS–UNEMPLOYED**

| Method | X-11 A | X-11 AA | BUR A | Berlin | CPB | EEC | X-11 M | X-11 AM | BUR M |
|---|---|---|---|---|---|---|---|---|---|
| Period 1956–72 | | | | | | | | | |
| X-11 Additive | 0.000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .040 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | 1.007 | 1.023 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | 1.452 | 1.473 | 1.447 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | 1.126 | 1.128 | 1.172 | 1.690 | .000 | (X) | (X) | (X) | (X) |
| EEC | 1.719 | 1.735 | 1.652 | 1.764 | 1.876 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | 1.192 | 1.190 | 1.344 | 1.731 | 1.062 | 1.580 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | 1.187 | 1.183 | 1.347 | 1.739 | 1.070 | 1.585 | .016 | .000 | (X) |
| Burman Multiplicative | 1.604 | 1.617 | 1.256 | 1.723 | 1.355 | 1.715 | .910 | .919 | .000 |
| Period 1973–75 | | | | | | | | | |
| X-11 Additive | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .910 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .895 | .619 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | 1.642 | 2.467 | 2.361 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .966 | 1.602 | 1.800 | 1.263 | .000 | (X) | (X) | (X) | (X) |
| EEC | 2.308 | 2.124 | 2.333 | 2.995 | 2.303 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | 1.604 | 1.756 | 1.936 | 2.276 | 1.574 | 1.410 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | 1.561 | 1.559 | 1.766 | 2.477 | 1.789 | 1.456 | .591 | .000 | (X) |
| Burman Multiplicative | 1.756 | 1.665 | 1.844 | 2.801 | 1.993 | 1.538 | .876 | .644 | .000 |
| Year 1975 | | | | | | | | | |
| X-11 Additive | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| X-11 Arima Additive | .955 | .000 | (X) | (X) | (X) | (X) | (X) | (X) | (X) |
| Burman Additive | .928 | .539 | .000 | (X) | (X) | (X) | (X) | (X) | (X) |
| Berlin | 1.227 | 1.974 | 1.846 | .000 | (X) | (X) | (X) | (X) | (X) |
| CPB | .781 | 1.418 | 1.614 | 1.441 | .000 | (X) | (X) | (X) | (X) |
| EEC | 3.031 | 2.461 | 2.581 | 3.884 | 2.832 | .000 | (X) | (X) | (X) |
| X-11 Multiplicative | 2.813 | 2.192 | 2.303 | 3.830 | 2.762 | .858 | .000 | (X) | (X) |
| X-11 Arima Multiplicative | 2.888 | 2.174 | 2.216 | 3.885 | 2.970 | 1.047 | .841 | .000 | (X) |
| Burman Multiplicative | 3.386 | 2.692 | 2.734 | 4.403 | 3.494 | 1.593 | 1.303 | .867 | .000 |

X Not applicable.

Table 7. RELATIVE CONTRIBUTION OF COMPONENTS TO BETWEEN-MONTH VARIANCE IN ORIGINAL SERIES

| Method | 1956-72 | | | 1973-75 | | |
|---|---|---|---|---|---|---|
| | I | T | S | I | T | S |
| | Employed | | | | | |
| X-11 Additive | 7.9 | 3.6 | 88.5 | 4.3 | 6.3 | 89.4 |
| X-11 Arima Additive | 7.9 | 3.6 | 88.5 | 4.0 | 5.7 | 90.3 |
| Burman Additive | 8.4 | 3.4 | 88.2 | 4.9 | 5.8 | 89.3 |
| Berlin | 7.5 | 3.3 | 89.2 | 4.9 | 5.5 | 89.6 |
| CPB | 9.1 | 3.4 | 87.5 | 4.9 | 6.2 | 88.9 |
| EEC | 9.1 | 3.3 | 87.6 | 6.6 | 4.9 | 88.5 |
| X-11 Multiplicative | 8.2 | 3.6 | 88.2 | 4.3 | 5.9 | 89.8 |
| X-11 Arima Multiplicative | 8.2 | 3.6 | 88.2 | 3.7 | 5.6 | 90.7 |
| Burman Multiplicative | 8.9 | 3.3 | 87.9 | 4.8 | 5.5 | 89.7 |
| | Unemployed | | | | | |
| X-11 Additive | 7.8 | 4.5 | 87.7 | 6.6 | 10.0 | 83.4 |
| X-11 Arima Additive | 7.7 | 4.5 | 87.8 | 5.5 | 9.2 | 85.3 |
| Burman Additive | 8.7 | 3.8 | 87.4 | 6.7 | 9.8 | 83.5 |
| Berlin | 8.8 | 3.4 | 87.8 | 14.7 | 12.3 | 73.0 |
| CPB | 9.3 | 3.5 | 87.2 | 8.1 | 9.8 | 82.1 |
| EEC | 8.8 | 3.4 | 87.8 | 10.0 | 5.7 | 84.3 |
| X-11 Muliplicative | 8.4 | 3.9 | 87.7 | 9.0 | 6.0 | 85.0 |
| X-11 Arima Multiplicative | 8.4 | 3.9 | 87.7 | 8.2 | 6.2 | 85.6 |
| Burman Multiplicative | 10.4 | 3.2 | 86.3 | 8.5 | 5.5 | 86.0 |

## Table 8. AVERAGE DURATION OF RUN FOR IRREGULARS, EXCLUDING EXTREMES

| Method | 1953-75 | 1956-72 | 1973-75 |
|---|---|---|---|
| | Employed | | |
| X-11 Additive | 1.44 | 1.43 | 1.35 |
| X-11 Arima Additive | 1.46 | 1.43 | 1.46 |
| Burman Additive | 1.50 | 1.47 | 1.67 |
| Berlin | 1.61 | 1.52 | 2.33 |
| CPB | 1.57 | 1.54 | 1.84 |
| EEC | 1.52 | 1.47 | 1.84 |
| X-11 Multiplicative | 1.47 | 1.43 | 1.59 |
| X-11 Arima Multiplicative | 1.47 | 1.43 | 1.59 |
| Burman Multiplicative | 1.47 | 1.47 | 1.52 |
| | Unemployed | | |
| X-11 Additive | 1.38 | 1.36 | 1.52 |
| X-11 Arima Additive | 1.40 | 1.36 | 1.67 |
| Burman Additive | 1.57 | 1.57 | 1.46 |
| Berlin | 1.87 | 1.90 | 1.94 |
| CPB | 1.60 | 1.60 | 1.67 |
| EEC | 1.56 | 1.56 | 1.40 |
| X-11 Multiplicative | 1.34 | 1.36 | 1.25 |
| X-11 Arima Multiplicative | 1.36 | 1.36 | 1.40 |
| Burman Multiplicative | 1.56 | 1.52 | 1.52 |

## Table 9. STABILITY INDICATORS

| Method | Employed | | Unemployed | |
|---|---|---|---|---|
| | Mean algebraic difference | Mean absolute difference | Mean algebraic difference | Mean absolute difference |
| Seasonal factors | | | | |
| X-11 Additive | 0.060 | 0.102 | 0.031 | 0.062 |
| X-11 Arima Additive | .043 | .083 | .017 | .052 |
| Burman Additive | .064 | .104 | .040 | .070 |
| EEC | .091 | .129 | .046 | .083 |
| Seasonal ratios | | | | |
| X-11 Multiplicative | 0.114 | 0.160 | 0.927 | 1.692 |
| X-11 Arima Multiplicative | .072 | .121 | .491 | 1.387 |
| Burman Multiplicative | .097 | .142 | 1.164 | 1.812 |

The differences in the seasonal factors were averaged for each month separately for 10 years to exclude random fluctuations. The period covered was 1963 to 1972. The average deviation is a measure of the bias and the mean absolute deviation, a measure of the dispersion.

The mean absolute values for the 12 months are presented in table 9. The calculations were not available for the Berlin and the CPB methods.

The results shown that the X–11 ARIMA method performs best on the criterium of stability of the seasonal component. The percentage reduction in the bias (mean algebraic difference) was 33 percent for employed and 46 percent for unemployed, while the reduction in dispersion (mean absolute differences) was 21 percent for the employed and 19 percent for the unemployed. The Burman method performed slightly better than the X–11 for the employed but performed worse for the unemployed. The EEC method showed the largest revisions.

### Elimination of Extremes

As discussed in the section "Seasonal Adjustment Methods Studied," the various procedures determine extremes quite differently.

The number of extremes identified will be a function of the $\sigma$ limit above which an irregular is considered to be an outlier.

The method of replacement will have an effect on the irregulars after replacement of extremes. For example, the Berlin method replaces extremes by the value of $2\sigma$, while the Burman and X–11 methods replace the extreme by an average of those neighbouring irregulars that were not identified as extremes.

Table 10 gives a summary of the extreme adjustments made for the historical (1956–72) and the current period (1973–75). Table 11 compares the irregulars for both of these periods. In this table, the RMS for all irregulars, for those without extreme adjustment and those with extreme adjustment, is shown.

The results of this analysis cannot be used to rank the seasonal adjustment procedures studied. However, they give one of the reasons why adjustment procedures may differ substantially in some cases. When two seasonal adjustment methods use the same procedure to replace extremes, the one with the lower number of extremes will be more appropriate. This criterium may be used in the choice between multiplicative and additive adjustment.

### Method of Calculation

The analysis was performed using the MATOP software package. (See [23].) This package includes subroutines for all seasonal adjustment procedures used in this study with the exception of the Berlin method. The trend, seasonal and irregular components, as well as the extremes, for the adjusted series are retained in memory and, thus, remain available for subsequent analysis.

## CONCLUSION

It was found that the seasonal adjustment procedures studied tend to give a similar adjustment for the historical period (the part of the series excluding the first and last 3 years) but that the adjustment for the current period (the last 3 years) may be quite different.

Because the adjustment for the current period is the most important for policy analysis, it may be useful, when adjusting the more important series, to compare the results obtained with more than one method, especially when an additive or multiplicative option must be selected.

This is also suggested by Kendall and Stuart who stated, "Our general recommendation would be to try several methods and to choose the one which appears to give the most reasonable results" [21].

The extent to which preliminary seasonal factors are revised when new data becomes available is an important consideration for policy analysis. Using this criterium, the X–11 ARIMA method performed best. It should be noted that the procedure projects the unadjusted series for a period of 12 months. Therefore, it is possible to use the extrapolated series together with other seasonal adjustment procedures.

## Table 10. EXTREME VALUES

| Method | 1956-72 | | 1973-75 | |
|---|---|---|---|---|
| | Number | RMS | Number | RMS |
| | Employed | | | |
| X-11 Additive | 35 | 0.305 | 7 | 0.300 |
| X-11 Arima Additive | 32 | .319 | 9 | .255 |
| Burman Additive | 8 | .405 | 3 | .176 |
| Berlin | 0 | (X) | 0 | (X) |
| CPB | 24 | .146 | 5 | .156 |
| EEC | 2 | .461 | 0 | (X) |
| X-11 Multiplicative | 32 | .318 | 6 | .292 |
| X-11 Arima Multiplicative | 32 | .318 | 9 | .216 |
| Burman Multiplicative | 8 | .419 | 1 | .089 |
| | Unemployed | | | |
| X-11 Additive | 36 | 0.171 | 8 | 0.250 |
| X-11 Arima Additive | 37 | .169 | 7 | .216 |
| Burman Additive | 6 | .229 | 3 | .262 |
| Berlin | 1 | .871 | 2 | 1.345 |
| CPB | 24 | .087 | 5 | .245 |
| EEC | 2 | .262 | 0 | (X) |
| X-11 Multiplicative | 36 | .152 | 7 | .322 |
| X-11 Arima Multiplicative | 34 | .157 | 8 | .241 |
| Burman Multiplicative | 10 | .188 | 2 | .399 |

X Not applicable.

Table 11. ROOT MEAN-SQUARE ERROR OR IRREGULARS, EXCLUDING EXTREMES

| Method | 1956-72 | | | 1973-75 | | |
|---|---|---|---|---|---|---|
| | All observations | Observations without extremes | Observations with extremes | All observations | Observations without extremes | Observations with extremes |
| | | | Employed | | | |
| X-11 Additive | 0.076 | 0.072 | 0.092 | 0.076 | 0.075 | 0.079 |
| X-11 Arima Additive | .077 | .074 | .090 | .065 | .068 | .054 |
| Burman Additive | .128 | .126 | .169 | .117 | .109 | .178 |
| Berlin | .155 | .155 | (X) | .236 | .236 | (X) |
| CPB | .142 | .117 | .261 | .146 | .124 | .241 |
| EEC | .150 | .151 | .046 | .172 | .172 | (X) |
| X-11 Multiplicative | .077 | .074 | .089 | .074 | .077 | .059 |
| X-11 Arima Multiplicative | .078 | .075 | .090 | .066 | .065 | .068 |
| Burman Multiplicative | .132 | .129 | .186 | .125 | .120 | .242 |
| | | | Unemployed | | | |
| X-11 Additive | 0.039 | 0.037 | 0.044 | 0.038 | 0.036 | 0.042 |
| X-11 Arima Additive | .039 | .037 | .045 | .041 | .038 | .049 |
| Burman Additive | .077 | .076 | .087 | .070 | .069 | .079 |
| Berlin | .119 | .112 | .574 | .162 | .154 | .263 |
| CPB | .083 | .071 | .143 | .100 | .079 | .183 |
| EEC | .082 | .082 | .051 | .141 | .141 | (X) |
| X-11 Multiplicative | .044 | .042 | .056 | .047 | .047 | .043 |
| X-11 Arima Multiplicative | .044 | .042 | .054 | .043 | .041 | .049 |
| Burman Multiplicative | .081 | .079 | .102 | .093 | .095 | .053 |

X Not applicable.

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# COMMENTS ON "A SURVEY AND COMPARATIVE ANALYSIS OF VARIOUS METHODS OF SEASONAL ADJUSTMENT" BY JOHN KUIPER

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## INTRODUCTION

It might be thought somewhat embarrassing for me to act as a discussant of a paper which compares my own seasonal adjustment method with those of others. But my method has been around for 11 years, and I feel that it is now old enough to take care of itself. Changing the metaphor, the present state of the art may be compared with that of the early Christian Church: The dominant stream of orthodox theology—the X–11 method, of course—and a number of heresies. The heretics are each sure that they are right, and there is a dialogue of the deaf amongst them and between them and the orthodox theologians.

The deafness arises from the lack of agreement on objective criteria for judging the quality of the adjustments to a series, a lack which, I hope very strongly, that this conference will be able to remedy. A lot of effort has been spent on the development of new and complicated methods, which seem to have arbitrary elements in them, because the authors have not taken us step-by-step through the processes that led them to introduce these—I may have been guilty of this myself.

## FEATURES OF METHODS

The paper before us gives a brief description of five methods. I would like to concentrate on their important features and differences. Every procedure can be divided conceptually into trend-removal and smoothing—with an intermediate stage in the case of the Bank of England (Burman) method of choosing the smoothing average. Table 1 sets out the stages for four of the methods. Three of them use various symmetric averages for trend-removal, but, in contrast, the BERLIN method [6] has an asymmetric 23-term average. However, I do not believe that the differences in trend-removal are the major cause of differences in the final results: Any differences in the spectral properties of the filters at low frequencies will be attenuated at the smoothing stage.

What matters most are—

1. The choice of additive, multiplicative or mixed model;
2. The method of smoothing the SI series;
3. The treatment of extremes.

The mixed model of the Dutch Planning Bureau (CPB) avoids the choice in (1), but the large number of parameters causes problems. The CPB reduce these by assuming a fixed ratio between the additive and multiplicative components and a deterministic evolution of the seasonal pattern.

The manner of smoothing determines the flexibility of the method. X–11 method uses a {3} {5} year smoothing on the final round. EEC splits the seasonal into a normalised pattern (PSN), smoothed over 5 years, and a scaling factor, estimated over 1 year; thus, it is considerably more flexible than the X–11 method. Consequently, the seasonal adjustments for a given month are generally more erratic than those of the X–11 method, and they do not sum to approximately zero over a year. For the BERLIN method, the smoothing average covers 45 months (again asymmetric), which would make this method more flexible than the X–11 method. The CPB model, with its deterministic function for moving seasonality, is apparently fitted over periods of 8 years.

In contrast to the foregoing methods, the Burman method has variable smoothing. For each of the 11 harmonics, it selects from the following: Fixed, exponential smoothing with a ratio of 0.9, 0.8, or 0.7, {3} {5} and {5}. In practice, this method is usually less flexible than the X–11 method because most of the components are smoothed by longer averages than {3} {5}.

The treatment of extremes varies a good deal. For example,

X–11 treats all extremes in isolation, with tapering weights between 1.5 and 2.5 sigma (estimated iteratively). Burman tapers between 2.0 and 2.5 sigma, allowing for the effect of isolated extremes on the trend and for interaction between two adjacent extremes [4].

EEC deals with isolated extremes but has also an elaborate system of successive truncations (MOTARD).

## COMPARISON OF RESULTS

Professor Kuiper has obviously devoted a great deal of effort to developing a system of programs for carrying out simultaneous tests; but, I am sorry that he has not found time to do more tests or to analyse more than two series. I am sure he would agree that generalisations cannot be made from one or two series, and I understand he plans to add comparisons for a number of other series before publication.

The tables showing the variance contributions of T, S, and I and the MCD's tell us something about each series, but they provide no information to help in ranking the methods of seasonal adjustment. The inequality coefficients show the close relation between the X–11 additive and Burman additive and also between the corresponding multiplicative methods. CPB stands next closest to these two groups; BERLIN (being additive) is closer to the additives X–11 and Burman; and EEC is closer to their multiplicative versions (because of its scaling factor). But, what use are we to make of these family relationships?

A cursory look at the tables of the irregular component show runs of the same sign near the end of the unemployment series for the BERLIN method and, to a lesser extent, CPB. The BERLIN method has a distinctly high average length of run over the whole series (as Kuiper points out), suggesting something unsatisfactory in its trend filter.

One objective test is to use spectral analysis to detect residual seasonality in the adjusted series: The paper contains two examples, and I hope that more will be added. The difficulty is to devise a numerical test of whether the peaks have been removed and a measure of the loss of power at nonseasonal frequencies. (The discussions by Tukey on [5] and Wecker on [7] show that the latter cannot be completely avoided.) More simply, the sequence of irregulars for each month can be tested for nonrandomness. Stability can be measured by the extent of revisions in the latest year or two when the series is up-dated. Obviously, there is a trade-off between these tests: The more flexible the method, the more likely that it will pass the simple residual seasonality test, but the larger the revisions will tend to be.

Professor Kuiper kindly supplied me with the 15 series he obtained from the NBER, and we have done some comparisons between X–11 and Burman methods. Using a von Neumann test for residual seasonality on the 15 series, the Burman method had only 6 months out of 180 significantly nonrandom at the 5-percent level—less than the 9 to be expected by chance. The X–11 method should show even fewer on this test because of its greater flexibility. We also looked at the autocorrelation structure of the first differences of seven of the seasonally adjusted series—those with a relatively significant seasonal pattern and not too noisy. (Where the seasonal adjustments indicated a multiplicative model, a logarithmic transformation was made.) The results for $r_{12}$ are given in table 2. All but one of the series has a small negative autocorrelation, suggesting slight overadjustment, and in 5 out of 6 cases, the Burman method shows the smaller negative figure. The positive figure for retail sales is mentioned in the following section.

We also tested the size of revisions of the seasonals for the seven series. The adjusted figures for the penultimate year were compared with those obtained from running the series with the last year omitted. The results in table 3 slightly favour the Burman method in four of the seven series.

An extreme example of differences between the methods is in housing starts, a very noisy series: The Burman method finds only one of the eleven harmonics is moving, the first cosine term. Nevertheless, its adjusted series passes the residual seasonality test in all 12 months. This and similar results with the Consumer Price Index support findings [7] that a fixed deterministic seasonal pattern is sufficient for these two series.

It might be possible to estimate the trade-off between maximum flexibility and minimum revisions, using an objective forecasting technique such as the Box-Jenkins. The steps would be to—

1. Seasonally adjust and forecast the adjustments, for example, 6 months ahead.
2. Fit a Box-Jenkins nonseasonal model to the adjusted series and forecast it 6 months ahead.
3. Combine these two forecasts to project the original series and find the mean forecasting error.

If, with a wide range of series, one method of seasonal adjustment led to smaller mean forecasting errors than another method, the former could be said, unequivocally, to be superior. The thought behind this is that, if a method was too flexible, it would impart noise to the forecast seasonal adjustments; if not flexible enough, it would fail to pick up genuine changes of pattern.

For the seven series selected, we fitted Box-Jenkins models to the X–11 and Burman adjusted series. From a range of models fitted, we chose a common one for both verisions of each series. The results are in table 4.

For all except retail sales, the fit of the models is very poor (after differencing), so that both sets of forecast errors are large; but on the basis of this limited evidence, the X–11 method does better than the Burman method. However, earlier work on 60 simulated series, on which I reported at a seminar in Amsterdam in April 1976, suggested that on average the two methods performed equally well.

The good fit of the AR model for retail sales seems to result from a damped 3–month cycle in the adjusted series. Presumably, this is due to imperfect length-of-month adjustment of 4- and 5-week periods; this feature cannot be picked up by the seasonal adjustment as the 4- and 5-week periods shift around. The fact that $r_{12}>0$ for this series may be due to masking of negative autocorrelation by the approximate 3-monthly cycle.

The results of this section are conflicting: Two tests suggest that the Burman method is flexible enough and that the X–11 method is slightly over-adjusting. The third favours the X–11 method.

## SEASONAL ADJUSTMENT BASED ON MODELING

I now wish to discuss a very different method of seasonal adjustment using Box-Jenkins models, partly based on [2]. Assume that an ARIMA seasonal model can be fitted to the series

$$z_t=\frac{\Theta(B)}{\psi(B)\Phi(B^s)} \qquad (s=\text{period of seasonality})$$

Brewer showed that, if this is expressed as a polynomial (possibly zero) and partial fractions, the latter can be divided into trendlike and seasonallike terms. The real positive roots of $\Phi(x)=0$ generate sets of $s$ roots of $\Phi(B^s)=0$ : each set contains a real positive root and $(s-1)$ complex roots, corresponding to the seasonal frequencies. The former is naturally associated with the trend, and the latter, with the seasonal component. But, any negative or complex roots of $\Phi(x)=0$ produce sets of roots of $\Phi(B^s)=0$ that do not correspond to seasonal frequencies and, thus, would more naturally be associated with the trend.

The polynomial represents the transient or irregular component that can be combined with the trend to give an adjusted series. The partition into seasonal and nonseasonal components at time $t$ depends on that at previous times, but it becomes unique and obvious in the eventual forecast function. (See the app. for details.) Brewer, therefore, suggests that after model estimation, backcasting should be used to provide an EFF at the start of the series, and then forward forecasting provides estimates of the seasonal and nonseasonal components up to time $t$. But, this makes the seasonal adjustment filter entirely one-sided (except for the effect of the later terms in the backcasting; this is negligible for long series). Brewer, therefore, recommended a complex and somewhat arbitrary way of making the filter two-sided. I have shown (in unpublished correspondence) that his extension gives a filter that asymptotically has not the right properties for seasonal adjustment.

I digress for a moment to discuss a general property of seasonal adjustment filters. All of the methods considered earlier consist of a two-sided filter which is approximately linear. The filters are not symmetric, except for the middle terms of the series, but all except those for the BERLIN method have the following property: The application of the process to the reverse of a series produces the reverse of the original seasonally adjusted series. This may be called weak symmetry.

Returning to Brewer's one-sided filter, we see that, instead of backcasting and then forecasting, we could have reversed the order, i.e., finding an EFF at the end of the series instead of at the beginning. A weighted average of the two estimates of the adjusted series provides a filter with weak symmetry.

## CONCLUSION

Looking at the five methods compared in Kuiper's paper, it seems to me that—

1. The rationale of the EEC method is quite clear (apart from the truncation procedure for dealing with extremes called MOTARD), but it produces seasonal patterns considerably more erratic than the X–11 method.
2. The rationale of the CPB method is clear, but it assumes deterministic moving seasonality. It seems doubtful whether this can respond to rapid changes of pattern as well as the X–11 method.
3. I do not understand the rationale of the BERLIN method with its asymmetric filters.
4. There seems to be no evidence that the X–11 method is not flexible enough; if anything, it is sometimes too flexible. The problems that several countries have had recently with unemployment series stem mainly from the use of the *Multiplicative* X–11 method: This was indistinguishable from the additive model when unemployment was low, but the seasonality has shown itself at least partly additive with high unemployment. Therefore, I feel that there is little justification for methods more flexible than the X–11 method (e.g., the EEC and BERLIN methods).
5. The advantage claimed for the Burman method over the X–11 method is that it offers an admittedly crude and suboptimal, but automatic, way of choosing from a range of alternative models. The disadvantage is that, on occasion, the choice of model changes as a result of an annual up-date.
6. But, if direct use of Box-Jenkins models on a large scale proves feasible, the signal extraction methods described in [1; 7] might be better than any of the traditional moving average methods.
7. In any case, we need more comparative, objective tests on a large number of economic series.

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# APPENDIX

## BOX-JENKINS MODELS FOR SEASONAL ADJUSTMENT

Consider the IMA model

$$z_t = \frac{\Theta(B)}{\Phi(B)} \quad a_t$$

where

$$\Phi(B) = (1-B)^d(1-B^s)^D$$

As Brewer [2] has shown, this model can be split up uniquely into a polynomial (zero if it is bottom heavy) and two groups of partial fractions, i.e.,

$$z_t = \left\{ \text{pol}(B) + \frac{\Theta_T(B)}{d(B)} + \frac{\Theta_s(B)}{s(B)} \right\} a_t \qquad \text{(A-1)}$$

where

$$d(B) = (1-B)^{d+D}$$
$$s(B) = (1+\mathrm{B} \ldots + B^{s-1})^D$$

The first part is a transient or irregular component, the second represents the trend, and the third, the seasonal component. The first two parts can be combined to give the seasonally adjusted series

$$z_t = \left\{ \frac{\Theta_a(B)}{d(B)} + \frac{\Theta_s(B)}{s(B)} \right\} a_t \qquad \text{(A-2)}$$

This formulation in terms of the sum of three components $(T+S+I)$ can be translated into an equivalent filter for the adjustment process, i.e.,

$$z_t^a = \frac{\Theta_a(B)}{d(B)} a_t = \frac{\Theta_a(B)d(B)s(B)}{d(B)\Theta(B)} z_t$$

$$= \frac{\Theta_a(B)s(B)}{\Theta(B)} z_t \qquad \text{(A-3)}$$

Again following Brewer, the operators for the seasonal component and the adjusted series imply recurrence relations which need to be started up. If we estimate the parameters by least squares or ML, the eventual forecast function (EFF) satisfies

$$\phi(B)\hat{z}_{t+i} = O[i > q + Qs]$$

since

$$\phi(B)\hat{z}_{t+i} = \Theta(B)a_{t+i} \text{ and } a_{t+i} = O[i > o].$$

The EFF breaks down into a polynomial trend of degree $(d+D-1)$ and a set of $(s-1)$ seasonals that are polynomials of degree $(D-1)$. If $D=1$, the breakdown is unique, since the seasonal pattern of the EFF is fixed, and the sum of the $s$ elements is zero. If $D>1$, the breakdown is not unique but in a trivial way; e.g., if $D=2$, the seasonal pattern changes linearly, the sum of the changes over $s$ observations is zero, and the sum of the levels in some specific set of $s$ observations (e.g., calendar years) is zero. If we change the zero condition from the calendar year to some other year, the constant term in the trend EFF changes, but the total EFF remains the same.

Brewer's proposal is that, when the model parameters have been estimated, it should be used for backcasting from the end of the series to provide an EFF at the beginning.[1] This can then be partitioned into seasonal component and adjusted series and used as a starting point for the recurrence relations:

$$d(B)z_a(t) = \Theta_a(B)a_t$$
$$s(B)z_s(t) = \Theta_s(B)a_t$$

But, this approach gives a one-sided filter for seasonal adjustment, whereas nearly all of the classic methods have weak symmetry. (Here we depart from Brewer.) It seems logical to make another estimate of the adjusted series by using the forward EFF as the starting point for partitioning and backcasting to obtain another version of the adjusted series. The forward and backward versions can then be combined by a linearly weighted average to give a weakly symmetric estimate.

If $z_t^{af}$ and $z_t^{ab}$ are the forward and backward adjusted series, the former starts with the EFF at $\hat{z}^a_{-q^*}$ and the latter with the EFF at $\hat{z}^a_{N+q^*+1}$ (N = the number of observations, $q^* = q + sQ$). The final adjusted series is

$$\hat{z}_t^a = w_t z_t^{af} + (1-w_t) z_t^{ab}$$

where

$$w_t = \frac{q^* + t}{N + 2q^* + 1}$$

The parameters may be estimated by constrained least squares (setting $a_{-q^*+1} = a_{-q^*+2} \ldots = a_o$ and $e_{N+1} = e_{N+2} \ldots = e_{N+q^*}$, where $e_t$ is the backcasting error). Alternatively, they may be estimated by ML, which gives estimates $\hat{a}_{-q^*+1}, \hat{a}_{-q^*+2}, \ldots \hat{a}_o$; since these depend only on the $\Theta$ parameters and not on the observations, $\hat{e}_{N+1} = \hat{a}_o$, $\hat{e}_{N+2} = \hat{a}_{-1}$, etc. Thus, in both cases, weak symmetry is preserved.

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[1] That is, estimating $e_t$ from $\theta(F)e_t = \phi(F)z_t$.

Table A-1. **COMPARISON OF MAIN STEPS IN FOUR SEASONAL ADJUSTMENT METHODS**

| Step | X-11 | EEC Seabird | Bank of England | Berlin |
|---|---|---|---|---|
| Model | Additive or multiplicative (ratios). | Additive only. Multiplicative assumed covered by rapid adjustment of scaling factor. (See below.) | Additive or multiplicative (log transform). | Additive. |
| Trend removal | 9-, 13-, or 23-term weighted average (at 2d iteration). Symmetric, end terms lost | 19-term weighted average. Symmetric in central part, skew for end terms. | 13-term weighted average (except 1st harmonic that uses 25-term weighted and 13-term unweighted averages). Symmetric, end terms lost. | Regression of cubic (plus harmonic variables) fitted to 23 observations, 12 before and 10 after the term being estimated. |
| Components of pattern | 12 months constrained to sum to zero. | 11 harmonics (excluding insignificant ones).[1] | 11 harmonics. | 11 harmonics (excluding insignificant ones).[1] |
| Choice of smoothing | (X) | (X) | Pooled Von Neumann ratio as criterion of moving pattern. | (X) |
| Smoothing seasonals | [3] [3] 1st round. [3] [5] 2d round. | [5] | Fixed or exponential ($\lambda$ = 0.9, 0.8, or 0.7) or [3] [5] or [5]. | Regression of 11 harmonic variables (plus 5th-degree polynomial) fitted to 45 observations, 23 before and 21 after the term being estimated. |
| Special feature | (X) | Seasonal split into pattern (moving 5-year average) and scaling factor (moving 12-month average).[1] | (X) | (X) |
| Extremes replaced | Graduated weights between 1.5 and 2.5 sigma.[1] | Effect of extremes muted by various steps with truncation.[1] Also, extremes identified and given zero weight.[1] | Graduated weights between 2.0 and 2.5 sigma.[1] | Identified by comparison with previous 24 terms. Replaced by upper/lower bound of selected confidence interval. |
| Quarterly version | Yes | No | Yes | Yes |
| Trading-day adjustment | Yes | No | Not included but has been used in separate program. | No |

X Not applicable.

[1] Steps which create nonlinearity in the filters.

## Table A-2. SEASONAL AUTOCORRELATION IN ADJUSTED SERIES

| Series | Model | X-11 | BUR |
|---|---|---|---|
| Unemployed | A | -0.22 | -0.05 |
| Retail sales | M | .03 | - |
| M1 | M | -.17 | -.10 |
| Shipments: | | | |
| All manufacturing | M | -.10 | -.10 |
| Durables | M | -.18 | -.07 |
| Nondurables | M | -.07 | -.02 |
| New orders | M | -.14 | -.11 |

– Entry represents zero.

## Table A-3. MEAN ABSOLUTE REVISIONS IN LATEST YEAR

| Series | X-11 | BUR |
|---|---|---|
| Unemployment | 0.097 | 0.093 |
| Retail sales | 155.5 | 161.8 |
| M1, deposits | 179.5 | 153.8 |
| Shipments: | | |
| Manufacturing | 195.9 | 181.9 |
| Durables | 126.8 | 174.0 |
| Nondurables | 93.2 | 105.3 |
| New orders | 246.8 | 237.1 |

## Table A-4. COMPARISON OF MODEL FORECASTING ERRORS

| Series | Box-Jenkins model | Percent variance[1] explained | | Mean absolute forecast error | |
|---|---|---|---|---|---|
| | | X-11 | BUR | X-11 | BUR |
| Unemployed | (2,1,0) | 12.5 | 8.4 | 1.10 | 1.26 |
| Retail sales | (2,1,0) | 47.8 | 44.1 | 823 | 840 |
| M1, deposits | (1,1,0) | 4.4 | 4.4 | 1376 | 1839 |
| Shipments: | | | | | |
| All manufacturing | (1,1,3) | 6.3 | 6.0 | 2636 | 4080 |
| Durables | (1,1,3) | 8.5 | 7.3 | 893 | 2017 |
| Nondurables | (1,1,3) | 10.4 | 11.0 | 1755 | 1903 |
| New orders | (2,1,1) | 11.1 | 13.4 | 1253 | 1371 |

[1] After differencing.

It might be thought that the forecasting and backcasting could be iterated, but a little thought shows that it is unnecessary for the IMA models. After forecasting and backcasting once, the first forward step (calculating $a_1$) does not depend on $\hat{z}_0$, $\hat{z}_{-1}$, etc., but only on $\hat{a}_0$, $\hat{a}_{-1}$, etc., which are the same as before.

## EXTENSION TO AR MODELS

The following changes are necessary:

1. The seasonal AR function in $\Phi(B)$ has to be partitioned between trend and seasonal: if it is of the form $(1-\Phi_1 B^s)$, the real positive root goes with the trend and the $(s-1)$ complex roots with the seasonal. If $P=2$, the two roots nearest unity go with the trend and the remaining $2(s-1)$ roots with the seasonal; but, if there are no roots very close to unity, it is not clear what to do.
2. The EFF of the two components are still $d(B)$ and $s(B)$, but these are only reached asymptotically as the AR portion dies away, instead of in a finite number of steps.
3. It is possible that iteration of forecasting and backcasting alternately will be useful, because $a_1$ depends on $\hat{z}_0$, $\hat{z}_{-1}$, etc.

# COMMENTS ON "A SURVEY AND COMPARATIVE ANALYSIS OF VARIOUS METHODS OF SEASONAL ADJUSTMENT" BY JOHN KUIPER

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In his thorough and interesting study on the comparison of various methods of seasonal adjustment officially adopted by Statistical Bureaux,[1] Professor Kuiper reaches the following conclusions:

1. There are no significant differences among the seasonally adjusted values obtained by each method for the total period of the series analysed (1953–75). This is shown in the corresponding tables of inequality coefficients, correlation coefficients, and summary measures.
2. There are significant differences in the current seasonally adjusted values (1975) produced by the various methods as shown in the tables of inequality coefficients and correlation coefficients.[2] To a lesser degree, this is also found to be true for the seasonally adjusted figures of the last 3 years of observations.
3. The smallest mean algebraic error and mean absolute error in the current seasonal factors is obtained by the X–11 ARIMA method[3] that I developed for statistics Canada, as shown in the table of stability indicators.

I will comment on these three points and show that they are not exclusive of the series considered but are the results of the underlying basic assumptions of the methods surveyed. These methods belong to the class that estimates the seasonal component by purely mechanical procedures and not on the basis of a causal explanation of the seasonal variation.

The time series probabilistic model of these methods is the classical one known in the theory of stochastic processes as error model. (see [1; 3].)

In an error model, the generating mechanism of a time series is assumed to be composed of a systematic component (sometimes called signal) that is a completely determined function of time $f(t)$ and a random component (the noise) $u_t$ that obeys a probability law. The random element is supposed to be purely random, i.e., identically distributed with constant mean, constant variance, and zero autocorrelation.

The signal of the observed time series, like the random element, is not observable, and assumptions must be made concerning its behaviour.

In general, two types of functions of time are assumed by these methods. One is a polynomial of fairly low degree which fulfills the assumption that the economic phenomenon moves slowly, smoothly, and progressively through time (the trend). The other is a linear combination of sines and cosines of different amplitudes and frequencies (representing cyclical oscillations), strictly periodic or not (the cycle and the seasonality).

When the systematic part is assumed to be approximated closely by simple functions of time over the entire range of the series, the statistical technique used is that of regression analysis (the classical least squares theory).

The methods surveyed, however, make the assumption that, although the signal is a smooth function of time, it cannot be approximated well by simple functions over the entire range. Therefore, they use the statistical technique of smoothing.

The general basis for most smoothing procedures is to fit a polynomial to $2n+1$ successive observations and use this fitted polynomial to estimate the trend cycle at the middle value. Since the estimates of the parameters

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[1] The methods analysed are: (1) the U.S. Bureau of the Census method II X–11 variant; (2) Statistics Canada X–11 ARIMA method; (3) Burman method of the Bank of England; (4) Berlin method, ASA–II; (5) the method of the Statistical Office of the European Economic Communities of Brussels, and (6) the method of the Dutch Central Planning Bureau.

[2] The current seasonally adjusted values were obtained by applying current seasonal factors from data to December 1975 and not seasonal factor forecasts.

[3] The total error is defined as the difference between the current seasonal factor $S_{t,j}^{t}$ and the estimate of the same seasonal factor when the series is enlarged with 3 more years of observations, $S_{t,j}^{t+3}$. The two statistics chosen to determine which of the methods generates better current seasonal factors are the mean algebraic error and the mean absolute error.

of the polynomial are linear in the observed values, say $X_{t+k}$, the smoothed series has the form,

$$(1)\ X_t^* = \sum_{k=-n}^{n} c_k X_{t+k},\ t=n+1,\ \ldots,\ T-n$$

The (1) is a moving weighted average of the observed values where the $c_k$'s are constant weights, $n$ is a positive integer, $2n+1$ is the span of the average. It is called moving, because the weights are moved one position to the right relative to the $X_t$ to obtain successive smoothed values.

The process of fitting a polynomial by the moving average technique consists of determining the weights $c$ that are functions of the length of the moving average, $2n+1$, and the degree of the polynomial to be fitted, e.g., $p$. For a given $p$, the variance of the smoothed series decreases with increasing $n$, and, for a given $n$, the variance goes up with increasing $p$ [1, p. 54]). The methods surveyed fix $p$ for each systematic component and let the $n$ vary. Therefore, depending on the $n$, some methods are more flexible than others. Although this does not affect their historical performance, it indeed introduces differences in their current performance.

The basic properties of moving averages are: (a) Scale preservation, (b) superposition principle, and (c) time invariance.

The property of scale preservation means that if the original series $X_t$ is amplified by a given constant, the smoothed series $X_t^*$ will be amplified by the same factor.

The superposition principle means that if two time series are added together and presented as the input to the given moving average, then the output will be the sum of the two smoothed time series that would have resulted from using the original series as inputs to the moving average separately. That is $(X_t+Y_t)^a = X_t^a + Y_t^a$, where the superscript $a$ indicates that a moving average has been applied to the original series.[4]

Properties (a) and (b) are a consequence of the fact that moving averages are linear transformations (often called smoothing linear filters).

The time invariant property means that if two inputs to the moving average are the same except for a relative time displacement then the outputs will also be the same except for the time displacement, i.e. if $(X_t)^a = Z_t$ then $(X_{t+h})^a = Z_{t+h}$. In other words, no matter what time in history a given input is presented to the filter, it will always respond in the same way. Its behaviour does not change with time.

The methods surveyed apply symmetric filters to estimate the components that fall in the middle of their span, e.g., $2n+1$, and asymmetric filters to the $n$– first and last observations.[5]

The sum of the weights of both kinds of filters equals one; therefore, the mean of the original series is unchanged in the filtering process.[6]

It is desirable in filter design that the filter does not displace, in time, the components of the output relative to the input, i.e., the filter should not introduce phase shifts.

The symmetric moving averages have a phase shift function that is equal to zero or $\pm\Pi$. A phase shift of $\pm\Pi$ is interpreted as a reversal of polarity of a sinusoid which means that its maxima are turned into minima and vice versa.

For practical purposes, however, symmetric moving averages act as though the phase shift is null. This is because the sinusoids that have a phase shift of $\pm 180°$ in the filtering process are cycles of short periodicities (annual or less) and moving averages tend to suppress or significantly reduce their presence in the output.

On the other hand, the asymmetric filters introduce phase shifts for most of the components of the original time series.[7]

Aside from the fact that the asymmetric filters of these methods are bound to introduce phase shifts, the functions not affected by these filters are different from those corresponding to the symmetric filters.

In effect, the symmetric moving averages that are applied to estimate the trend-cycle component reproduce the middle observation of a third-degree polynomial within the span of the filter. The fact that the trend is assumed to follow a cubic over an interval of short duration (one or two years approximately) makes the assumptions of these methods quite adequate for the historical adjustment of a large class of economic time series.

The same conclusions are valid for the symmetric filters that estimate the seasonal component; they can fit closely a local linearly moving seasonality. These

[4] In practise, however, the equality is not fulfilled by the methods analysed because of nonlinearities introduced at different stages of the calculations, for example, in the replacement of the extreme values.

[5] The only exception being the Berlin method ASA-II that applies an asymmetric fiter for trend-cycle removal but with a weighting scheme based on a third degree polynomial.

[6] The sum of the weights of a filter determines the ratio of the mean of the smoothed series to the mean of the original series, assuming that these means are computed over periods long enough to insure stable results.

[7] The necessary and sufficient condition for a linear filter to have a phase shift function $\Phi(\lambda)=0$ for all $\lambda$ is that its transfer function be real valued and nonnegative definite for all $\lambda$. Symmetric filters have real valued, but not necessarily nonnegative, transfer functions which lead to the possibility that $\Phi(\lambda)=\pm\Pi$ for some frequencies. A digital filter with (real valued) weights $\{{}^{c}k : k=o, \pm 1, \ldots\}$ is said to be nonnegative definite if, for every positive integer $n$ and complex number $a_k$, $k=o, \pm 1, \ldots \pm n$, we have $\Sigma_n{}^{\nu}{}_{j=-n}\ \Sigma^{n}{}_{k=-n} a_j \bar{a}_k c_{jk} > o$ [4, pp. 206–207].

methods assume that the seasonal pattern changes gradually with only occasional reversals in direction.[8]

For current estimation, however, more rigid patterns of behaviour are assumed, namely, a straight line representing the trend cycle and a stable seasonality. These more restrictive assumptions generally produce systematic errors in the current seasonally adjusted values that are gradually corrected as the series is enlarged by inserting more years of observations.

Since the implicit functions calculated by the symmetric filters of these methods are different from those corresponding to the asymmetric filters, the historical seasonal adjustment will always differ significantly from the current one, except for the trivial (nonexistent) case of series with a constant trend cycle and a stable seasonality.

There will also be significant differences among the current estimates obtained by the various methods. These differences will be more apparent for those series that are highly irregular or have extreme values present in the most recent years. This is due to the short length of the asymmetric filters that does not allow a significant reduction in the variance of the smoothed series and to the fact that these methods use different procedures and sigma limits for the replacement of the outliers.

The third finding of Kuiper's study, i.e., the smallest total error in the current seasonal factors is produced by the X–11 ARIMA method, is also explainable by the basic properties of this method.

The X–11 ARIMA generates seasonal factor forecasts from the combination of two filters: (1) The filters of autoregressive integrated moving averages (ARIMA) models to forecast raw data and (2) the filters of census II X–11 variant to seasonally adjust current observations. (See [4].)

This procedure proved to be superior to the X–11 program in the sense that the size of the total error in the monthly forecasts and also in the current seasonal factors (measured by the monthly absolute means) was significantly smaller for the 12 months, and the same happened for the bias (measured by the monthly algebraic means).

The main reasons for significant reductions in the total error of the seasonal factor forecasts and current seasonal factors are that—

1. The seasonal factor forecasts of X–11 ARIMA are obtained from forecasted raw data, whereas the X–11 method forecasts from estimated seasonal factors; it is well known that the seasonal factors for the last 3 years are less reliable.
2. The forecasting filter of the X–11 method is the same for all series, while in the ARIMA models, the forecasting filters depend on the model chosen and the parameter estimates. The ARIMA filters are very flexible and are able to pick up the most recent movements of the series.
3. The trend-cycle estimate for the last observation is made with the central weights of the Henderson's moving averages (the same for the centered 12-term moving average) which are capable of reproducing a cubic in their time interval. This is very important for years with turning points, since the X–11 program applies the asymmetric weights of the Henderson method that only estimate well a linear trend cycle.
4. The replacement of the extreme values for the last 2 years of data is improved. In effect, by adding 1 more year of data (with no extremes, since they are forecasts), a better estimate of the variance of the irregulars is obtained.
5. The sets of weights applied to the seasonal irregular ratios (differences) are closer to the central weights, and, thus, the moving seasonality can be estimated with more accuracy.

Although Kuiper's analysis was made for the current seasonal factors, I obtained similar results for the seasonal factor forecasts. The mean algebraic error and the mean absolute error were reduced by approximately 40 percent and 20 percent with respect to those of X–11 method. Moreover, the Wilcoxon-signed-rank test indicated that the differences for each month were significant and in favor of X–11 ARIMA method.

This is a very important conclusion, especially if one takes into account that producers of current seasonally adjusted data tend to use the seasonal factor forecasts more often than to rerun the series each time that a new observation is added to it.

[8] Series with abrupt or rapid changes in the seasonal variation cannot be seasonally adjusted properly. Sudden changes in the seasonal amplitude can be found, for example, in agricultural series, where the level varies considerably from year to year, and in series such as unemployment, which undergo rapid changes in composition when the economy changes from expansion to recession and back to expansion.

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# APPENDIX

## STATISTICS CANADA X–11 ARIMA METHOD OF SEASONAL ADJUSTMENT

### Introduction

The Statistics Canada X–11 ARIMA method of seasonal adjustment is a modified version of the Bureau of the Census method II X–11 variant that consists of enlarging unadjusted series with 1 year of forecasted raw data and then seasonally adjusting the enlarged series with the X–11 program. (See [4].) The forecasts of the raw data are made by ARIMA (autoregressive integrated moving averages) models of the Box-Jenkins type that have been identified and fitted to the original series.

The seasonal factor forecasts are, thus, obtained from the forecasted raw data and their estimation results from the combination of two filters: (1) The filters of ARIMA models to forecast raw data and (2) the filters of the X–11 program to seasonally adjust current observations.

This new technique produces seasonal factor forecasts and current seasonal factor superior to those of the census method II X–11 in the sense that the mean absolute error and the mean algebraic error of the seasonal factors is significantly smaller for the 12 months.

When applied to Canadian and U.S. series, the reduction found was about 40 percent in the bias and 20 percent in the absolute value of the total error. Another advantage of the X–11 ARIMA is that if current seasonal factors are used to obtain current seasonally adjusted data, there is no need to revise the series more than twice. For many series, just one revision will produce seasonal factors that are final in a statistical sense.

The X–11 ARIMA also provides a univariate time series model that describes the behaviour of the unadjusted series. Confidence intervals can be constructed for the original observations, and, since the one-step forecast is an unbiased minimum mean square error forecast, it can be used by producers of raw data as a benchmark for the last available figure.

### The Forecasting Filters of ARIMA Models and Their Properties

The ARIMA models used for forecasting the unadjusted series are of the general multiplicative type [2], i.e.,

$$\phi_p(B)\Phi_P(B^s)\Delta^d\Delta^D_s Z_t=\Theta_q(B)\Theta_Q(B^s)a_t \qquad \text{(A-1)}$$

where $s$ denotes the periodicity of the seasonal component (equal to 12 for monthly series); $B$ denotes the backward operator, i.e., $BZ_t=Z_{t-1}$; $B^sZ_t=Z_{t-s}$; $\nabla^d=(1-B)^d$ is the ordinary difference operator of order $d$; $\nabla^D_s=(1-B^s)^D$ is the seasonal difference operator of order $D$; $\phi_p(B)$ and $\Phi_P(B^s)$ are stationary autoregressive operators (they are polynomials in $B$ of degree $p$ and in $B^s$ of degree $P$, respectively); $\Theta_q(B)$ and $\Theta_Q(B^s)$ are invertible moving-average operators (they are polynomials in $B$ of degree $q$ and in $B^s$ of degree $Q$, respectively); and $a_t$ is a purely random process.

The general multiplicative model (A-1) is said to be of order $(p,d,q)$ $(P,D,Q)_s$. Its forecasting function can be expressed in different forms. For computational purpose, the difference equation form is the most useful. Thus, at time $t+1$, the ARIMA model (1) may be written

$$Z_{t+l}=\psi_1Z_{t+l-1}+\dots+\psi_mZ_{t+l-m}-a_{t+l}-\pi_1a_{t+l-1}-\dots-\pi_na_{t+l-n} \qquad \text{(A-2)}$$

where $m=p+s.P+d+s.D$ and $n=q+s.Q$; $\psi(B)=\phi_p(B)\Phi_p(B^s)\nabla^d\nabla^D_s$ is the general autoregressive operator; and $\pi(B)=\Theta_q(B)\Theta_Q(B)$ is the general moving average operator. For example, if the ARIMA model is of order $(2,1,1)(0,1,1)_{12}$ the difference equation form that generates the observations $Z_{t+l}$ is

$$\begin{aligned}Z_{t+l}=&(1+\phi_1)Z_{t+l-1}+(\phi_2-\phi_1)Z_{t+l-2}-\phi_2Z_{t+l-3}\\&+Z_{t+l-12}-(1+\phi_1)Z_{t+l-13}+(\phi_1-\phi_2)\\&Z_{t+l-14}+\phi_2Z_{t+l-15}+a_{t+l}\\&-\Theta a_{t+l-1}-\Theta a_{t+l-12}+\Theta\Theta a_{t+l-13}\end{aligned} \qquad \text{(A-3)}$$

Standing at origin $t$, to make a forecast $Z_t(l)$ of $Z_{t+l}$, the conditional expectation of (A-2) is taken at time $t$ with the following assumptions:

$$E_t(Z_{t+l})=Z_{t+l},\ l\leq 0;\ E_t(Z_{t+l})=\hat{Z}_t(l),\ l>0 \qquad \text{(A-4)}$$

$$E_t(a_{t+l})=a_{t+l},\ l\leq 0;\ E_t(a_{t+l})=0,\ l>0 \qquad \text{(A-5)}$$

where $E_t(Z_{t+l})$ is the conditional expectation of $Z_{t+l}$ taken at origin $t$. Thus, the forecasts $\hat{Z}_t(l)$ for each lead time are computed from previous observed $Z$'s, previous forecasts of $Z$'s, and current and previous random shocks $a$'s. The unknown $a$'s are replaced by zeroes.

In general, if the moving average operator $\pi(B) = \Theta(B)\Theta(B^s)$ is of degree $q+s.Q$, the forecast equations for $\hat{Z}_t(1)$, $\hat{Z}_t(2), \ldots, \hat{Z}_t(q+s.Q)$ will depend directly on the $a$'s, but forecasts at longer lead times will not. The latter will receive indirectly the impact of the $a$'s by means of the previous forecasts. In effect, $\hat{Z}_t(q+s.Q+1)$ will depend on the $q+s.Q$ previous $\hat{Z}_t$ that, in turn, will depend on the $a$'s.

From the point of view of studying the nature of the forecasts, it is important to consider the explicit form of the forecasting function. For $l>n=q+s.Q$, the conditional expectation of (A-2) at time $t$ is

$$\hat{Z}_t(l) - \psi_1\hat{Z}_t(l-1) - \ldots - \psi_m\hat{Z}_t(l-m) = 0 \quad l>m \quad \text{(A-6)}$$

and the solution of this difference equation is

$$\hat{Z}_t(l) = b_0^{(t)} f_0(l) + b_1^{(t)} f_1(l) + \ldots + b_{m-1}^{(t)} f_{m-1}^{(l)} \quad l>n-m \quad \text{(A-7)}$$

This function is called the eventual forecast function, eventual because, when $n>m$, it supplies the forecasts only for lead times $l>n-m$. In (A-7), $f_0(l)$, $f_1(l), \ldots f_{m-1}(l)$ are functions of the lead time $l$, and, in general, they include polynomials, exponentials, sines and cosines, and products of these functions. For a given origin $t$, the coefficients $b_j^{(t)}$ are constants applying for all lead time $l$, but they change from one origin to the next, adapting themselves to the particular part of the series being considered. It is important to point out that it is the general autoregressive operator $\psi(B)$ defined above that determines the mathematical form of the forecasts function, i.e., the nature of the $f$'s. In other words, it determines whether the forecasting function is to be a polynomial, a mixture of sines and cosines, a mixture of exponentials, or some combinations of these functions. The ARIMA forecasts are minimum mean square error forecasts and can be easily updated as new raw values become available.

In the context of the X-11 ARIMA, the forecasts should follow the general movement of the series.

In my experience with Canadian and U.S. economic time series, I found that the ARIMA models chosen must fit the data well and produce forecasts for each of the last 3 years with a mean absolute error smaller than 5 percent for well-behaved series (e.g., employed men, over 20 years old) and smaller than 10 percent for volatile series (e.g., unemployed women, 16–19 years old.) The smaller the forecasting error, the better. This is particularly true for the forecasting error of the first 6 months, given the way they will be treated by the X-11 filters.

Since ARIMA models are robust, the identification is often good for several years. However, they should be checked when an extra year of data becomes available to insure that the most recent movements of the series are properly followed by the model.

## The Seasonal Adjustment Filters of the U.S. Bureau of Census Method II X-11 Program

The Bureau of the Census program is summarized in [6] and described fully in [7].

The main steps of this method for obtaining the seasonally adjusted series are as follows:[1]

1. Compute the ratios between the original series and a centered 12-term moving average (2 × 12-term moving average, i.e., 2-term average of a 12-term average) as a first estimate of the seasonal and irregular components.
2. Apply a weighted 5-term moving average to each month separately (a 3 × 3-term moving average) to obtain an estimate of the seasonal factors.
3. Compute a centered 12-term moving average of the preliminary factors in (2) for the entire series. To obtain the six missing values at either end of this average, repeat the first (last) available moving average value six times. Adjust the factors to add to 12 (approximately) over any 12-month period by dividing the centered 12-term average into the factors.
4. Divide the seasonal factor estimates into the seasonal irregular (SI) ratios to obtain an estimate of the irregular component.
5. Compute a moving 5-year standard deviation ($\sigma$) of the estimates of the irregular component and test the irregulars in the central year of the 5-year period against $2.5\sigma$. Remove values beyond $2.5\sigma$ as extreme and recompute the moving 5-year $\sigma$.
   Assign a zero weight to irregulars beyond $2.5\sigma$ and a weight of 1 (full weight) to irregulars within $1.5\sigma$. Assign a linearly graduated weight between 0 and 1 to irregulars between $2.5\sigma$ and $1.5\sigma$.
6. For the first 2 years, the $\sigma$ limits computed for the third year are used; for the last 2 years, the $\sigma$ limits computed for the third-from-end year are used. To replace an extreme ratio in either of the two beginning or ending years, the average of the ratio times its weight and the three nearest full-weight ratios for that month is taken.
7. Apply a weighted 7-term moving average to the SI ratios with extreme values replaced for each month separately to estimate preliminary seasonal factors.
8. Repeat step (3).
9. To obtain a preliminary seasonally adjusted series divide (8) into the original series.

[1] It is assumed that the relationship among the time series components is multiplicative. For an additive model, the words "difference" and "subtracting" are substituted for "ratio" and "dividing."

10. Apply a 9-, 13-, or 23-term Henderson moving average to the seasonally adjusted series and divide the resulting trend cycle into the original series to give a second estimate of the SI ratios.
11. Apply a weighted 7-term moving average (3 × 5-term moving average) to each month separately, to obtain a second estimate of the seasonal component. Compute estimates of seasonal factors one year ahead by the formula

$$S_{j,t+1}=S_{j,t}+\tfrac{1}{2}(S_{j,t}-S_{j,t-1})$$

where $j=1, 2, \ldots, 12$ denotes the month and $t$, the year.
12. Repeat step (3).
13. Divide these final seasonal factors into the original series to obtain the seasonally adjusted series.

Allan Young [8], using a linear approximation of the census method II, arrives at the conclusion that a 145-term moving average is needed to estimate one seasonal factor with central weights if the trend-cycle component is adjusted with a 13-term Henderson moving average. The first and last 72 seasonal factors (6 years) are estimated using sets of asymmetrical end weights. It is important to point out, however, that the weights given to the more distant observations are very small, and, therefore, the moving average can be very well approximated by taking one-half of the total number of terms plus one. Thus, if a 145-term moving average is used to estimate the seasonal factor of the central observation, a good approximation is obtained with only 73 terms, i.e., 6 years of observations. This means that the seasonal factor estimates from unadjusted series that have observations ending at least 3 years later can be considered final in the sense that they will not change significantly when new observations are added to the raw data.

The forecasting function specified in step (11) for each monthly seasonal factor forecasts perfectly if the seasonal factors are relatively constant through the years (stable seasonal pattern). However, if the seasonal pattern is evolving through time, with a trend that is linear within the span of the moving average used to estimate the seasonal pattern, a bias is easily introduced. Months for which the seasonal factors tend to decrease will have a forecasted seasonal factor larger than expected. The opposite will happen for those months in which the seasonal factors tend to increase. Moreover, the size of the bias will be larger, the larger the slope of the line followed by the seasonal factors. It is evident then that in the case of linearly evolving seasonality, the seasonal factor forecasts for some months will have larger biases than others.

In the X–11 ARIMA, the unadjusted series is enlarged with 1 more year of data (forecasted values). Consequently, the X–11 program estimates the components with better filters.

The trend cycle is no longer estimated with the end weights of the centered 12-term and Henderson moving averages but with their central weights. This means that the Henderson filters will not miss a turning point at the end of series, since the central weights of these filters minimize the sum of the squares of the third difference of the trend-cycle curve.

The end weights applied to the seasonal factors are closer to the central weights and can reproduce a local linearly moving seasonality with less error. In effect, for the 3 × 3-term moving averages, its forecasting filter is now 0.185, 0.407, and 0.407 instead of −0.056, 0.148, 0.426, and 0.481. Similarly, the weights of 3 × 3-term moving average for the current seasonal factors are now only one step ahead of its central weights.

In the case of the 3 × 5-term moving average, its forecasting filter is 0.150, 0.283, 0.283, and 0.283 instead of −0.034, 0.134, 0.300, 0.300, and 0.300. Observe that the new forecasting filters are those that the X–11 program apply for current seasonal factors, and none of their weights is negative.

Because of their longer filtering intervals for given cutoff frequencies, smothing filters, having negative weights beyond the positive central values, tend to stretch too far the implicit assumption in filtering that the periodicities present at the time for which the filtered variable is estimated are unchanged in amplitude and phase during the filtering interval.

The replacement of the extreme values for the last year of observed data is also improved. In effect, by adding 1 more year of values with no extremes, since they are forecasts, a better estimate of the residuals is obtained.

## Design of the Experiment and Conclusions

The X–11 ARIMA has been tested with Canadian and U.S. economic time series. Two statistics were chosen to determine which of the two methods, X–11 ARIMA or X–11, generates better current seasonal factors and forecasts, namely—

1. The mean algebraic error of the current and forecasted seasonal factors for each month.
2. The mean absolute error of the current and forecasted seasonal factors for each month.

The method giving the lowest statistics is considered the best. The Wilcoxon-signed-rank test was applied to matched pairs of statistics (1) and (2), obtained from current and forecasted seasonal factors given by both procedures to determine whether the differences are due to chance variations or whether they are really significant. Since an improvement would mean low statistics (1) and (2), a one-sided test of the null hypothesis $H_0$ (zero difference) versus the alternative $H_1$ cutoff frequencies, smothing filters having negative (positive difference) was applied at a 5 percent level of significance.

To obtain statistics (1) and (2) corresponding to the seasonal factor forecasts given by each method for the series, I proceed as follows (the same was done for the current seasonal factors):

1. Estimate the one-step seasonal factor forecast $S_{ij}^{i-1}$ for each month $j=1, 2, \ldots, 12$ and year $i=1963, 1964, \ldots, 1975$. (The superscript denotes the last year available of an unadjusted series with a minimum of 7 years of monthly data. In my case, I used data from 1953.)
2. Estimate the seasonal factor $S_{ij}^{i+3}$ for each month $j=1, 2, \ldots, 12$ and year $i=1963, 1964, \ldots, 1972$ from an unadjusted series ending in year $i+3$. (According to the type of filter used by census method II X–11 variant, this seasonal factor can be considered final in the sense that it will not change significantly when more observations are added to the original series.)
3. Define $e_{ij}=S_{ij}^{i+3}-S_{ij}^{i-1}$ as the total error in the seasonal factor forecasts.
4. For each series, build a double entry table of the $e$'s defined in (3).
5. For each double entry table of the $e$'s, calculate—
   a. The mean algebraic error for each month, i.e., $1/n\sum_{i=1}^{n} e_{ij}, j=1, 2, \ldots, 12$.
   b. The mean absolute error for each month, i.e., $1/n\sum_{i=1}^{n} |e_{ij}|, j=1, 2, \ldots, 12$.

The results from the Wilcoxon-signed-rank test indicated that the seasonal factor forecasts obtained from X–11 ARIMA were superior to those produced by census method II X–11 variant [4]. The same conclusions applied to the current seasonal factors $S_{ij}^{i}$. Statistics Canada X–11 ARIMA was officially adopted by Statistics Canada in January 1975 for the seasonal adjustment of the main labour force series.

In its present versions, this new method is not fully mechanized. The user should be able to identify, for each series, an ARIMA model that fits the data well and produces reasonable forecasts according to the general guidelines mentioned in section (2). The identified model is checked only once a year and usually not changed for at least 3 years.

Since there is a class of simple ARIMA models that fit and forecast well a large number of series. I am presently working on the selections of a limited number of ARIMA models to be able to fully automate this procedure.

# RESPONSE TO DISCUSSANTS

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The main conclusion of my paper is that the seasonal adjustment methods studied tend to give similar results during the historical period but that significant differences may occur during the current period (the last 3 years) and especially during the last year.

Based on the performance for each criterion evaluated, one can, thus, rank the methods. However, I did not feel justified to make such a ranking, because it would have required an aggregation over the various measures used to evaluate the quality of seasonal adjustment. In any case, a ranking based on two series would be inappropriate but, in my opinion, so would a ranking based on the 15 series made available for this conference, as suggested by Burman.

One of the measures used to evaluate the quality of the seasonal factors for the last year (i.e., the preliminary factors) is the stability indicator. Farley and Zeller used the indicator to measure dispersion. The bias is measured by the statistic

$$\frac{1}{M}\Sigma\left[\frac{1}{K}\Sigma(S^{p}_{m,k}-S^{f}_{m,k})\right]$$

where $k$ indicates the year for which seasonal factor differences are taken and $m$ the month.

It appears that Fase took differences between the preliminary and first revised seasonal factors (calculated with 12 additional observations), while I took differences over 3 years, because the seasonal factors tend toward stability at that point. Also note that the bias will be relatively more significant when moving seasonality is present, which occurred in both series studied during the recent period.