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Chapter Two

Interdependence between Public-Sector Decisions and the Urban Housing Market

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INTRODUCTION

In the typical urban area, decisions by local governments, households, and housing suppliers are highly interrelated. The reason for much of this interdependence is that space is the principal basis for defining local public-sector jurisdictions. Most metropolitan areas are composed of many jurisdictions, each of which provides services and levies taxes on its own inhabitants. Local jurisdictions also typically control zoning and the issuance of building permits. These public-sector decisions affect the prices of land and housing and are a factor in determining who will choose to move into the zone. Housing and land prices together with zoning rules influence the nature of new development. At the same time, the price and quality of housing available in a jurisdiction will be factors in determining who will choose to reside there and, hence, the vote on public-sector decisions. Housing market outcomes and local public-sector decisions are thus highly interdependent.

In any model of public-sector decisions, housing stocks must play a prominent role. Housing stocks differ substantially across jurisdictions, and the time lags involved in changing the existing stock are relatively long. In jurisdictions with little vacant land for development, an area's housing stock may undergo relatively little change over time as long as prices remain above the level required to encourage normal maintenance. Even relative to other areas, the age and quality of a zone's housing may change only very slowly. Given the preferences of certain types of households for particular kinds of housing, variations in housing stocks among zones can be a useful predictor of the income and life cycle of the neighborhood's residents (Straszheim 1975). The limited empirical evidence on consumer preferences for public-sector outputs indicates that preferences are related to incomes and life cycle as well (Hirsch 1970; Borcherding and Deacon 1972). This suggests that differences in public sector actions from zone to zone and over time are likely closely related to the types of housing stocks available and their changes.

The theoretical literature has stressed the advantages of a system of multiple jurisdictions with different tax and service levels in each. To the extent that there are differences among households in income or tastes which affect the demand for government services, there is an obvious incentive for households to stratify into "homogeneous" jurisdictions according to their preferences for a smaller or greater role for the public sector. It has been shown that an efficient outcome will require that there be such differences (Tiebout 1956; McGuire 1974). Specifying voting procedures, zoning rules, and taxing instruments that will secure an efficient solution is more complex; it seems likely that customary voting and taxing instruments will not produce such a result (Buchanan and Goetz 1972).

In the typical metropolitan area there will not likely be sufficient jurisdictional fragmentation so that all residents in a zone will be in common agreement on the public sector budget and associated taxes. First, the number of jurisdictions is typically limited. In some urban areas there may exist only a few large jurisdictions, while in others the number may be substantially larger. Scale economies limit the extent to which jurisdictions can be reduced in size. In addition, externalities increase with reduction in size, much complicating the determination of efficient public-sector actions and limiting the opportunity to decentralize decision making. Certain income redistribution objectives can also be achieved by limiting the number of jurisdictions. (And state legislatures may consider other objectives as well.)

Housing stocks also typically vary within jurisdictions because housing units are often constructed over an extended period of time during which prices for housing, land, and construction vary. Thus, while variation in tastes for public services and housing can likely best be described by a continuous probability distribution with a positive covariance between housing and public-service preferences, the latter must be matched against a finite set of residential zones, each of which contains a fixed and heterogeneous housing stock. Under these conditions, it becomes increasingly unlikely that there is a spatial configuration of households matching the existing set of housing stocks which will yield unanimity on public-sector voting in each jurisdiction.

There may also be variation in preferences for public services within a zone that cannot be traced to differences in housing consumption. Households may have moved into a zone on the basis of faulty expectations about public-sector outputs or tax burdens. Also, because of relocation costs and other factors—for example, job location—that restrict residential mobility, some households may find themselves in a jurisdiction in which their preferences for public-sector outputs differ from those of their neighbors, though they all have identical housing units.

Relatively little attention has been devoted in the literature to the specification of difference among voters. In this study I discuss procedures for explicitly modeling the relationships between fixed housing stocks, variations in income and preferences across households, and local-government decisions. The principal issue addressed is the decision-making process of jurisdictions with heterogeneous interests, in large part arising from a fixed stock of housing which itself is hetereogeneous.

The general procedure outlined below is to represent differences in preferences parametrically, relating differences in how households in any jurisdiction vote on public-sector decisions to differences in housing stocks or underlying utility functions. This uses the technique of change of variable, in which probability density functions on housing stocks or tastes are translated into a density function describing voting preferences. The housing stock is regarded as fixed at a point in time. Decisions on current-period services and taxes and on zoning are analyzed; the latter affect future votes, tax burdens, and housing prices, all of which are important to the welfare of existing residents.

The analytic procedure using change of variable is quite general. For a variety of assumptions regarding initial probability distributions and change of variable, the resultant density function describing voting outcomes will be continuous. However, as in most change-of-variable problems, the class of problems that can be solved analytically is relatively small. Further development of these models is likely to involve numerical evaluation. The discussion in this chapter is focused primarily on the issues involved in the specification of the models rather than in their numerical solution and is intended to highlight the advantages and disadvantages of this general approach. Illustrative examples are presented.

In several important respects the models are partial equilibrium ones. The analysis focuses only on how decisions are made in one zone, taking all events "outside" the zone as given. "Competitive" behavior or the perceived "mutual dependence" between jurisdictions and its effects on decisions by each jurisdiction are not addressed.

Household locations are exogenous to the models. The models describing preferences of existing residents imply a variety of different assumptions regarding how the household's locational decision might be formulated. These are spelled out below. Neither the relocation decision nor the problem of matching households to available housing stocks in particular jurisdictions is addressed in this study.

I also describe the basic change-of-variable approach. Applications of the model are based on particular assumptions regarding the factors that generate the underlying distribution of residents' preferences for public-sector decisions. In conclusion I review the insights derived from the models in this study as they relate to the more general problem of modeling public-sector-housing market interactions.

REPRESENTING VARIATIONS IN VOTER PREFERENCES BY CHANGE OF VARIABLE

In most general terms, variation in housing consumed and utility functions might be represented by a probability density function, $f(h, \lambda)$, where h denotes housing; and λ, a vector of parameters describing preferences. For example, λ might include the number and age of children in the household, the age of head of household, etc. Let λ have $n-1$ elements. Assume that households' preferences for public-sector outputs, g_i, are related to h and λ as follows:

$$g_i = d_i(h, \lambda);\ i = 1, \ldots, n \tag{2-1}$$

The elements g_i are themselves random variables, a transformation of h and λ, whose density function is given by change of variable. Assume the inverse functions $c_1, \ldots, c_n$ exist:

$$\begin{aligned} h &= c_1(g_1, g_2, \ldots, g_n) \\ \lambda_1 &= c_2(g_1, g_2, \ldots, g_n) \\ &\vdots \\ \lambda_{n-1} &= c_n(g_1, g_2, \ldots, g_n) \end{aligned} \tag{2-2}$$

and that the transformation is one to one. (This is guaranteed locally if the partial derivatives are continuous at the point of interest and if the Jacobian $|\partial c/\partial g|$ does not change sign.) The density function describing households' preferences for g is given by:

$$G(g_1, \ldots, g_n) = f[c_1(h, \lambda), c_2(h, \lambda), \ldots, c_n(h, \lambda)] \cdot \frac{\partial(c_1, \ldots, c_n)}{\partial(g_1, \ldots, g_n)} \tag{2-3}$$

In the models analyzed below, the public-sector decisions being voted upon are assumed to be one-dimensional. To derive the density function for g (where g is now assumed to be one-dimensional) on the basis of the transformation from (h, λ) to g, $n-1$ "dummy" variables must be defined; the latter are then integrated out to get the marginal density function on g. Let z_i denote $n-1$ dummy variables, related to h and λ as follows:

$$z_i = d_i(h, \lambda); i = 2, \ldots n \tag{2-4}$$

The specification of z_i is dictated solely by analytic convenience. Integrating out the z_i yields the density function for g:

$$G(g) = \int \ldots \int f[c_1(h, \lambda), c_2(h, \lambda), \ldots, c_n(h, \lambda)] \cdot \frac{\partial(c_1, \ldots, c_n)}{\partial(g,z)} \, dz_2, \ldots, dz_n \tag{2-5}$$

This provides a flexible analytic procedure for representing variation in tastes.

The density function $G(g)$ describes the degree of heterogeneity among voters. A variety of assumptions might be employed to determine how these differences in opinion over g are reconciled. For example, the probability that anyone votes might be related to preferences for g and could be readily incorporated once the density function is known. In most of the models below it is assumed that voting outcomes reflect preferences of the "median" vote. This simple rule has considerable appeal when a simple issue is being voted on, or preferences are single-peaked (Barr and Davis 1966; Bowen 1943; Comanor 1974). (The complexities of multi-dimensional public-sector decisions would be treated by a straightforward extension of the change-in-variable model describing preferences. However, the interpretation of voting results is much more complex when several issues are being voted on in a single election.)

The nature of available housing in any jurisdiction can be easily determined (e.g., from census data or household interview surveys taken in transportation planning studies). Specifying models that describe the variation in tastes of residents at any point in time and the relationships between h, λ, and voter preferences for g is more complex.

The simplest approach would be based on customary demand functions, and would assume a direct relationship for g in terms of housing consumed, tastes, and the price of public services, without tracing the relationship between g and those parameters to underlying utility functions or to the way households form expectations when they chose to reside in any zone. This has the advantage that existing empirical studies may be useful in specifying the relevant parameters.

In the models below it is assumed that the public-sector budget must be balanced in each jurisdiction. While there are many possible taxing policies, two important special cases are a head tax (a constant charge per household) and a property tax. The latter is assumed to be assessed on housing values in nondiscriminatory fashion, with each household's tax bill directly related to the size of h. Assume housing prices are proportional to h and that public services are provided by a linear cost function:

$$C(g, N) = N\beta g \qquad (2\text{-}6)$$

where $C(g, N)$ = cost of providing public sector output g to the N citizens in the jurisdiction, and β = cost per unit of g per person.

Balancing the public-sector budget implies that the tax bill for public services will vary with house size:

$$\beta g N = tP_h \int_0^\infty hf(h)dh \qquad (2\text{-}7)$$

where t = tax rate, and P_h = price of housing. Thus

$$\beta g N = tP_h\, \mu N, \text{ or } t\frac{\beta g}{\mu P_h} \qquad (2\text{-}8)$$

where μ = mean house size in jurisdiction. Hence, the unit price of public services is a linear function of house size: $P_g(h) = h\beta/\mu$.

The simplest demand model is based on the assumption of a linear relationship between housing, public service prices, and households' preferred choice of public-sector output. Assume the demand equation as follows:

$$g^0 = a_0 + a_1 h + a_2 P_g(h) \tag{2-9}$$

where $a_1 > 0$, $a_2 < 0$. If $P_g(h)$ is based on a property tax, Equation (2-9) will be linear in g and h:

$$g^0 = a_0 + h(a_1 + a_2 \frac{\beta}{\mu}) \tag{2-10}$$

Depending on the form of $f(h)$, a linear transformation from h to g may much simplify the determination of $G(g^0)$. If $f(h)$ is normal, $G(g^0)$ will also be normal, the mean and median will coincide, and the voting outcome can be easily solved analytically. [Since h cannot be negative, the assumption that h is normal is only an approximation; however, normality would provide a good approximation to the shape of $f(h)$ for most jurisdictions.]

This simple demand model formulation implies a common utility function with variation in preferences for g traceable only to variations in housing. Introduction of variation in tastes complicates the specifications. The simplest way is to assume that a_0 is a random variable described by a probability distribution, $f(a_0)$. If a_0 and h are normally and independently distributed, the density function for g will be normal, and can be readily described. In the more general case in which a_0 and h are independent (continuous) random variables, the density of their sum can also be relatively easily determined. In the case above, let $f_1(a_0)$ and $f_2(h)$ be the density functions. Define a dummy variable $z = [a_1 + a_2(\beta/\mu)]h$. The Jacobian ($J$) of the transformation from (a_0, h) to (g_1, Z) is given by:

$$J = \begin{vmatrix} \frac{\partial a_0}{\partial g} & \frac{\partial a_0}{\partial z} \\ \frac{\partial h}{\partial g} & \frac{\partial h}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & \frac{1}{a_1 + a_2 \frac{\beta}{\mu}} \end{vmatrix} = \frac{1}{a_1 + a_2 \frac{\beta}{\mu}} \tag{2-11}$$

and hence the density of g_0 is given by integrating out the dummy variable z.

$$G(g^0) = \int_{\infty}^{\infty} f(g-z) \frac{1}{a_1 + (a_2 \frac{\beta}{\mu})} f_2[z/a_1 + a_2 \frac{\beta}{\mu})]dz \tag{2-12}$$

In the case where other terms in the formula for g are random variables, e.g., a_1 or a_2, obtaining the density function for g is slightly more complex. For example, suppose a_1, the coefficient on h, is also a random variable, and the joint distribution of (h, a_1) can be described by the density function $f(h, a_1)$. In this case let the dummy variable z be defined equal to a_1. The Jacobian of the transformation from (h, a_1) to (g, z) is therefore:

$$J = \begin{vmatrix} 1 & 0 \\ -\dfrac{(g-a_0)}{z + a_2 \dfrac{\beta}{\mu}} & \dfrac{1}{z + a_2 \dfrac{\beta}{\mu}} \end{vmatrix} = \frac{1}{z + a_2 \dfrac{\beta}{\mu}}$$

and the density function for g_0 is given by integrating out z:

$$G(g^0) = \int_{\infty}^{\infty} f[(g - a_0)/(z + a_2 \frac{\beta}{\mu}), z]\ (z + a_2 \frac{\beta}{\mu})^{-1} dz \qquad (2\text{-}13)$$

This will likely require numerical evaluation for virtually any specification of $f(h, a_1)$ that is not a uniform distribution.

These voting models based on customary demand functions are flexible and may approach the limits of our knowledge about preferences for public-sector outputs. The more challenging specification is to relate current residents' preferences to underlying utility functions, and to the reasons why households chose any given zone (and have chosen to remain). This approach implicitly introduces a host of questions as to how households form expectations regarding the level of public services and taxes in the chosen zone relative to other options at the time of their move. In the next section, several classes of models are developed in which voter preferences for g are related to underlying utility functions and different assumptions regarding expectations.

SPECIFICATION OF THE SOURCES OF VARIATION IN PREFERENCES FOR PUBLIC SERVICES

Perfect Foresight: Variation in Utility Functions

In one class of models it is assumed that households have perfect foresight, correctly anticipating the level of g voted in the zone. If it is further assumed that there is continuous variation in g across zones, so that all households can pick a zone with just the g they prefer, all residents in a zone will be in unanimous support of the prevailing g. (This also assumes costless relocation.) This model yields unanimity in a vote on g in any jurisdiction. Since households in a given zone all receive the same g, the variation in utility functions among residents (necessary to yield unanimity on g) can be derived from the observed variation in the existing housing stock.

Describing this variation in preferences is most easily handled by assuming a common utility function, with certain parameters allowed to vary across households. Let the utility function for households be denoted by $U(h, g, x, \lambda)$, where g = public-sector output, h = housing, x = all other goods, and λ = parameter denoting household tastes, which vary across households according to a known probability distribution (λ is assumed to be one-dimensional, for ease of exposition). Utility maximization yields three equations for h, g, and x in income, Y a vector of prices, p, and λ.

$$h = f_1(Y, p, \lambda) \tag{2-14}$$

$$g = f_2(Y, p, \lambda)$$

$$x = f_3(Y, p, \lambda)$$

The solution of the demand equation for housing in terms of income is $Y = d(h, p, \lambda)$; substituting the latter into the demand equation for g yields an expression for g in terms of h, p, and λ:

$$g^0 = f_2[d(h, p, \lambda), p, \lambda] \tag{2-15}$$

Since all households locating in a zone correctly anticipate the prevailing g and face the same prices, expression (2-15) amounts to a one-to-one transformation from h into λ. This transformation is the basis for deriving the variation in utility functions for the density function for h. The usefulness of this class of models is in describing how the vote on g varies among jurisdictions with different housing stocks.

Differing Expectations: A Common Utility Function

An alternative class of models that do not yield unanimity on the level of g is based on the assumption that tastes are identical across households, but that households formed differing expectations on g when they chose to buy h in the zone. Assuming h and g are complements, those households that purchased large houses expected a large g to be voted. Observed differences in housing are therefore a proxy for differences in ex-ante expectations. The vote on g reveals the differences of opinion. In this case some households will be disappointed (and the question of whether this will encourage exit is raised—an issue not dealt with in this study). For present purposes it will be assumed that current residents are momentarily captive.

A special case of this approach is the assumption that households enter the zone expecting to get precisely the g they prefer when confronted with the prevailing prices for g and h. Under this assumption, demands for g and h can be represented by the standard first-order utility maximization conditions with respect to g, h, and x. In this case the only source of variation in preferences for g as a function of h, λ, and p (Equation (2-15)) in any jurisdiction can be related to differences among residents of a zone in their consumption of h. Variation in preferences for g depends only on the density function for h, $f(h)$, and the form of the utility function.

The following illustrates this class of models, assuming public services are financed by a "head" tax, defined as a constant charge on each dwelling unit regardless of its size. The budget constraint is linear with a head tax. The tractability of the results depends on the choice of a utility function and the nature of $f(h)$.

To illustrate, assume households have a Stone-Geary utility function:

$$U = a_1 \ln (g - b_1) + a_2 \ln (h - b_2) + a_3 \ln (x - b_3)$$

which is maximized subject to a budget constraint:

$$Y = P_x X + P_h h + P_g g$$

where P_g is the tax levied on each household, and is invariant to h. The first-order conditions are familiar and lead to linear expenditure relations:

$$g = b_1 + \frac{d_1}{P_g} (Y - b_1 P_x - b_2 P_h - b_3 P_g) \tag{2-16}$$

$$h = b_2 + \frac{d_2}{P_h} (Y - b_1 P_x - b_2 P_h - b_3 P_g)$$

$$x = b_3 + \frac{d_3}{P_x} (Y - b_1 P_x - b_2 P_h - b_3 P_g)$$

where $d_i = a_i / \Sigma_j a_j$. As noted earlier, it is assumed that households correctly anticipate the price of g, but choose an h in the expectation that the g they prefer will also result. Using the demand relationship between h and Y, a solution for Y in terms of h can be substituted into the demand equation for g, yielding an equation in h and prices. This indicates how households with different housing would vote for g:

$$g^0 = b_1 - \frac{d_1 P_h}{d_2 P_g} b_2 + \frac{d_1 P_h}{d_2 P_g} h \tag{2-17}$$

Since $h \geqslant b_2$, (2-17) is always positive. Since it is a linear expression in g and h, the density function for g^0, $G(g^0)$, is

$$G(g^0) = \frac{d_2 P_g}{d_1 P_h} f(h) \tag{2-18}$$

In the case where $f(h)$ is normal, g^0 will also be normal. The mean (and median) of g^0 is $b_1 - (d_1 P_h / d_2 P_g)\, b_2 + (d_1 P_h / d_2 P_g)\, \mu$. The variance of g^0 (as one possible measure of the variation in voter preferences) is $(d_1 P_h / d_2 P_g)\, \sigma_h^2$. (Again, this is only an approximation, since h can assume negative values if it is normal but is constrained to be $\geqslant b_2$ by the utility maximization model.) The voting outcome for g^0 is thus linear in μ. The elasticity of the g^0 chosen with respect to μ is $(d_1 P_h / d_2 P_g)\, (\mu / g^0)$, which exceeds 1.0 for small μ and tends toward 1.0 in the limit as μ rises. Interpretation of this elasticity is relatively straightforward: d_1 and d_2 are the marginal propensities to consume g and h respectively. As d_1/d_2 is increased, the elasticity is increased at each level of h. Comparisons across zones are straightforward, since the relationship between the median of g^0 and μ is linear.

Under the assumptions of a nondiscriminatory property tax, the results of a voting equilibrium with this type of model are much less tractable. Assume households maximize a utility function $U(g, h, x)$, subject to the budget constraint:

$$Y - P_x X - P_h h - \frac{h\beta}{\mu} g = 0$$

The first-order conditions are as follows, where subscripts on U denote partial derivatives:

$$\frac{U_g}{U_h} = \frac{\beta h}{P_h\mu + \beta g} \tag{2-19}$$

$$\frac{U_h}{U_x} = \frac{P_h h + \beta g}{\mu P_x}$$

$$Y - P_x X - P_h h - \frac{h}{\mu}\beta g = 0$$

The nonlinearity in the budget contraint arising from introduction of a property tax results in far moré complicated expressions for voting equilibrium.

To illustrate the simplest case, consider the utility function $U = ag + bh + c\ ln\ x$. (A linear function in all arguments is unsatisfactory since this results in corner solutions, with the household consuming all its income on one good.) In this case the expression for U_h/U_x becomes:

$$\frac{bx}{C} = \frac{P_h h + \beta g}{\mu P_x} \tag{2-20}$$

Substituting for x yields an expression with the product term hg:

$$Y\mu - P_h h\mu_h - \beta hg = \frac{c}{b}(P_h h + \beta g) \tag{2-21}$$

Since g is linear in h in this case, i.e., $g = (b/a)h - (P\mu/\beta)$, quadratic demand equations for h or g in terms of Y and prices result, as follows:

$$Y\mu - \frac{\beta b}{a}h^2 - (\frac{c}{b}P_h + \frac{c\beta}{a})h + \frac{c}{b}P_h\mu = 0 \tag{2-22}$$

$$Y\mu - (P_h\mu + \beta g - cP_h)(\frac{a}{b}g + \frac{a}{b}\frac{P_h\mu}{\beta}) \quad {}^{-c\beta g} = 0 \tag{2-23}$$

In each case the positive root gives the appropriate solution for h and g.

To determine how households with different housing stocks will vote, expression (2-22) for h is substituted for Y in the demand equation (2-23) for g. This yields an expression showing how voting preferences for g are related to housing stocks. Y appears in only one term in the expression for g^0.

$$g^0 = \frac{cb}{2a} + \frac{P_h \mu}{\beta} - \frac{cP_h}{\beta} \pm \Big\{ (c\beta + 2P_h \frac{a}{b} \mu - cP_h \frac{a}{b})^2 \qquad (2\text{-}24)$$

$$+ 4\beta \frac{a}{b} [Y\mu - (1 - c\mu) \frac{a}{b} P_h^2 \frac{\mu}{\beta}] \Big\}^{1/2}$$

Solving for Y in terms of h yields

$$Y = \frac{1}{\mu} [\beta \frac{b}{a} h^2 + (\frac{c}{b} P_h + \beta \frac{c}{a}) h - \frac{cP_h \mu}{b}] \qquad (2\text{-}25)$$

Substituting in (2-24) yields g^0 as a function of h in the following form:

$$g^0 = D_1 + (D_2 h^2 + D_3 h + D_4)^{1/2} \qquad (2\text{-}26)$$

where D_1, D_2, D_3, and D_4 are constants. Equation (2-26) may involve multiple solutions for g in terms of h, or negative solutions. The latter must be excluded; of the positive solutions, only the largest value is relevant to the household utility maximization problem. The appropriate transformation therefore must include only the positive value of the square root term defining g^0. This transformation is complex, as is the Jacobian appearing in $G(g^0)$, eliminating any hope of ready interpretation of the probability density function $G(g^0)$.

$$G(g^0) = \frac{8(g^0 - D_1)}{[D_3^2 + 4D_2(g^0 - D_1)]^2}{}^{-1/2} f(h) \qquad (2\text{-}27)$$

Since $G(g^0)$ is not a recognizable density function, its properties can only be examined numerically. However, it is possible to make certain inferences about the voting equilibrium. If the transformation from h to g^0 is unique and monotonic over the range of $f(h)$, the transformation is order preserving, and hence the median of $G(g^0)$ will be defined by the g^0 preferred by the household with median h. The voting equilibrium can therefore be determined by solving (2-14) for h equal to median h. Examination of (2-14) and (2-15) reveals that dg^0/dY and $dY/dh > 0$; hence dg^0/dh will be positive throughout the range of h, and the transformation will be order preserving. For the household with median $h = \mu$, the expression for Y in terms of h becomes:

$$Y = \beta\frac{b}{a}\mu + (\frac{c}{b}P_h + \beta\frac{c}{a}) - \frac{cP_h}{b} = \frac{\beta}{a}(c + b\mu) \qquad (2\text{-}28)$$

Substituting into (2-24) yields:

$$\mathrm{Med}(g^0) = (\frac{cb}{2a} + \frac{P_h\mu}{\beta} - \frac{cP_h}{\beta}) + \frac{b}{\beta a}\left\{(\beta + 2P_h\mu\frac{a}{b} - cP_h\frac{a}{b})^2 \right. \qquad (2\text{-}29)$$

$$\left. + 4\beta\frac{a}{b}[\frac{\beta}{a}(c + b\mu) - (1 - c\mu)\frac{a}{b}P_h^2\frac{\mu}{\beta}]\right\}^{1/2}$$

which can be solved.

The example above is the simplest case I have found. For example, the Stone-Geary utility function yields equations in higher powers of g when a property tax is employed. In general, models relating votes on g to housing stocks tend to involve rather complex transformations when utility functions are employed. These more complex density functions in g^0 will require numerical evaluation.

Other Sources of Variation in Tastes

In both of the above models, variation in preferences for g could be fully described by the marginal distributions for either h or λ. One broad class of models contains variation in residents' preferences independent of variation in available housing stocks. The discussion below indicates some of the issues in specifying such models. Often they implicitly involve the household location decision and why the reasons for the household's choice of zone.

Although preferences for g are likely to vary continuously over a wide range, only a relatively small number of jurisdictions are ordinarily available to choose from, and this restriction is an important source of variation in tastes independent of the amount of housing consumed. Under these circumstances, preferences in any one zone will not be completely homogeneous, even though households stratify according to their preferences for g, as Tiebout and McGuire outline. Under their assumption, differences in preferences for g would exist within each zone even if all housing in every zone were the same. As the number of available jurisdictions is increased (or the underlying variation in utility functions is reduced), heterogeneity of tastes in each jurisdiction will be reduced.

A second source of variation in tastes arises from relocation costs. As relocation costs increase, households are more likely to find themselves in a jurisdiction in which public-sector decisions are not those they prefer. As residential composition, incomes, and tastes change over time, voting outcomes will vary as well. At any point in time, higher relocation costs imply greater variation in tastes among residents.

A third case arises from differences among households in their expectations regarding g prior to their move into a zone. The model described in which variation in housing consumed, h, represents variation in preferences, was a special case. Clearly, there may be many different bases for households' formation of expectations regarding g.

ZONING DECISIONS

In addition to determining government services and tax levels in a jurisdiction, its members make important decisions that affect the type of development of the zone. Zoning rules on minimum sizes for lots and structures, permissible amounts of rental property, and the nature of nonresidential development all may have important effects on the type of new construction undertaken, and hence, on the socioeconomic characteristics of incoming residents.

To describe how residents make decisions about zoning, several components of the housing development process must be described. Zoning rules affect the financial returns to various types of developments; with some time lag, development will occur in response to profit opportunities. The type of new housing in turn will influence the preferences of future residents for public services, and will affect tax burdens for all residents.

In the model below, the relationship between particular zoning rules and the rate or type of new construction is assumed to be exogenous. It is assumed that current residents can predict how any zoning rule will alter the course of new development. Only two time periods are considered, the present and the future. Current residents choose a zoning strategy that maximizes their welfare by majority vote, and with no influence exerted by developers or other interested parties.

The utility maximization decision for current residents is to choose that z which leads to a future g and an associated tax bill which maximizes current residents' welfare. Formulation of the voting outcome on a zoning policy is again a problem in change of variable. However, in this case two transformations must be considered. Let $f_z(h')$ denote the future housing stock, conditional on zoning policy z; $g'(z)$ is the outcome of the vote on g in the future; and $g'P_g(h) = g'\beta h/\mu'(z)$ is the tax bill for future government service g' to a future resident household with housing h. (The primes on g and h denote future outcomes.) To describe current residents' preferences for zoning strategies, z, requires that current voters predict future votes on g. Those predicted future votes, conditional on the current zoning decision, determine the level of g and tax burdens in the future. Thus, describing the outcome of the vote of current residents on zoning requires that future voting outcomes be predicted for each possible zoning decision, each of which involves a change-of-variable problem.

Two financing alternatives were considered in the previous section: a head tax and a housing property tax. The former is uninteresting for present purposes since its use removes all considerations of differential tax burdens; the objective of current residents would be to select that zoning strategy which results in entry by households who will vote for a level of g in the future identical with the preferences of current residents. The median voter today would in essence control the zoning outcome, and entry over time would therefore result in increasingly greater neighborhood homogeneity.

The case of property tax financing is more interesting. Here, a zoning policy that alters the distribution of housing stocks has two consequences: If mean house size, μ, is reduced, tax bills for existing residents per unit of g must rise. In addition, if preferences for g and h are positively correlated, the effect of lowering μ is a vote for a lower g. A zoning rule that lowered μ could not receive support. Conversely, an increase in μ through zoning increases the tax base, but also increases the amount of g voters will support. The "income effect" of lower tax bills may offset the loss in utility that residents (preferring a lower g) will sustain if a higher level of g is voted. Current residents who fall below the median μ before the zoning change may accept a higher level of g in the future because of the tax benefits of having neighbors with larger h.

These consequences can be described formally. Assume each zoning choice, z, yields a unique mean housing size in the future, μ'. Current residents therefore maximize utility with respect to μ'.

$$\max U(g, h, x, \lambda)$$

$$\text{Subject to: } Y = P_h h + P_x x + g(\mu')\frac{h\beta}{P_h \mu'}$$

For existing resident households, with given h and λ, utility maximization with respect to μ' requires:

$$\frac{dU}{d\mu} = \frac{\partial U}{\partial g'}\frac{\partial g'}{\partial \mu'} + \frac{\partial U}{\partial x'}\frac{\partial x'}{\partial \mu'} = 0 \qquad (2\text{-}30)$$

(This requires that $\partial g'/\partial\mu'$ and $\partial x'/\partial\mu'$ exist and are continuous and that the future vote on g varies continuously with the mean level of housing.) Since

$$x' = \frac{1}{P_x}[Y - P_h h - g'(\mu')\frac{h\beta}{P_h\mu'}]$$

(2-30) becomes:

$$\frac{\partial U}{\partial g'}\frac{\partial g'}{\partial \mu'} + \frac{\partial U}{\partial x'}\left\{\frac{h\beta}{P_hP_x\mu'}\left[\frac{\partial g}{\partial \mu'} - \frac{g'(\mu')}{\mu'}\right]\right\} = 0$$

$$\frac{\partial U}{\partial g'}\Big/\frac{\partial U}{\partial x'} = -\frac{\partial x'}{\partial g'} = -\frac{h\beta}{P_hP_x\mu'}(1 - \frac{1}{\eta}) \qquad (2\text{-}31)$$

where η is the elasticity of g' with respect to μ'. The sign of the righthand side may be positive or negative, depending on whether η is less than or greater than unity. This result has a simple interpretation. If $\eta < 1$, an increase in μ' will allow current residents to acquire more x and more g. If both g and x have positive marginal utility, there would be unanimous support to increase μ as much as possible. (The amount by which μ could be increased would be determined by the amount of vacant land available and the willingness of those wanting large h to build in this jurisdiction.) Alternatively, if $\eta > 1$, the tax bill of current residents for any level of g will rise as μ rises, and less x can be purchased; current residents can enjoy more g only by giving up x. (It is possible that an increase in μ' actually leaves all current residents within their current budget opportunity locus. New residents contribute less to the tax bill facing current residents than they cost by voting a high g.)

While empirical results are far from conclusive on the nature of the relationship between μ and g, it is likely that at both extremes of the income distribution, the elasticity is less than one. In a neighborhood of low-income households, the entry of somewhat wealthier ones would not be likely to result in a vote for large increases in g. At the high end of the income distribution, private services are often a substitute for public services, and hence the elasticity of g with respect to μ is likely less than one for higher-income neighborhoods.

Figure 2-1 portrays the choices open to current residents before and after the zoning change. Several budget lines denoting choices between x and g for different households with different incomes are portrayed. The x intercepts depend on utility functions, incomes, and expectations, which together determine how large an h was initially purchased by each household. Property tax financing implies increasing tax burdens on households with larger h. The original problem of determining the vote on g amounts to analyzing the transformation from $f(h)$ to $G(g)$. For each zoning decision that increases μ, a higher g will be voted in the future. Each current resident therefore confronts a set of points above his current budget line. The locus of possible future voting outcomes on g, conditional on the zoning policy, are denoted by the BC curves. Each point on the BC curve for any current resident is derived from a prediction of the future vote under an alternative zoning policy. (It is assumed here that all can predict future votes.)

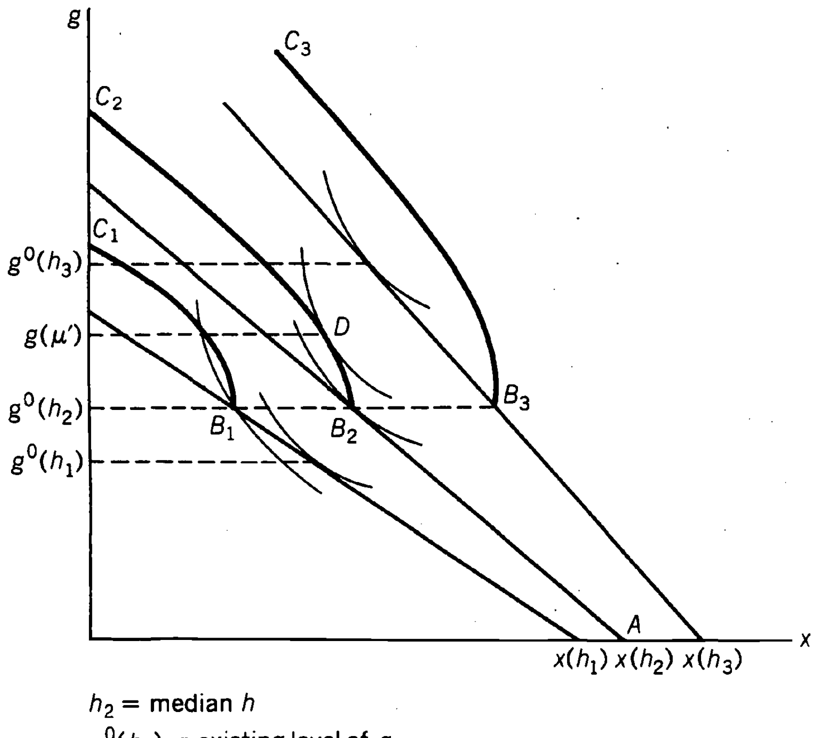

Figure 2-1. Variation in Voter Preferences for Zoning Alternatives.

The zoning strategy adopted will be given by finding the median of current residents' preferences for μ', where each resident is choosing a point along his BC locus. It will be shown below that under certain assumptions the vote by current residents on g and their vote on μ' for the future can be represented by two distributions, one of which is a monotonic transformation of the other. Since this implies no change in order, the zoning vote is therefore given by the outcome preferred by the median voter from the existing distribution of h. In this case, it is the tangent point D of the indifference curve of the household, with h = median of the existing distribution $f(h)$, to the locus AB_2C_2. It should be noted that there are some residents with h below that of the median voter who are better off after the zoning change than at present, though they would have preferred a different zoning strategy than that selected. The household with h_1 is just indifferent to the outcomes before and after the zoning change.

There are two steps in the analytic development of such a model of zoning decisions. The first is to predict future votes on g (and hence tax burdens) conditional on each $f_z(h)$. This is in essence what was done in the preceding two sections. The second is to determine how current residents, who control the zoning decision, rank these possible outcomes. This involves specifying a utility function (or functions if variation in tastes is to be included).

The principal difficulty in predicting future voting outcomes conditional on any z is that the density function describing housing stocks is altered by the zoning action. Thus, even if the original $f(h)$ has a simple analytic shape—for example, normal—the new density function will differ. Likewise, if the density function describing voter preferences for g before the zoning change was normal, or had some other known form, making determination of the median of $G(g)$ simple, it will not be so after the zoning change.

There is one special case in which the determination of g^0 conditional on z is relatively simple, namely, when all new entrants have h *above* the new median. In this instance the new median g is determined solely by the *number* of new entrants (but not the size of h they occupy), and the shape of the original probability density function $g(g^0)$. If N is the original community size and there are to be N' new entrants, the new median is given by g', where

$$\int_0^{g'} g(g^0)\,dg = 0.5\left(1 + \frac{N'}{N}\right) \tag{2-32}$$

In this circumstance, current residents' preferences determine g'.

The more general case is that in which some new entrants prefer a level of g *below* that which is ultimately voted. In this case preferences of entrants help determine the voting outcome, and there is no substitute for calculating the new density function describing preferences for g and finding its median. For virtually any interesting specification of $f(h)$ for current residents, and preferences for g by incoming residents as related to h, the density function density $G(g')$ conditional on any z must be examined numerically. If future votes on g can only be described numerically (or if g' and μ' are complex functions in z), the transformation from h to z involved in describing current residents' preferences for z will itself be complex and require numerical examination.

One way in which the determination of z can be simplified is in the specification of how existing voters form expectations regarding future voting outcomes. Instead of deriving a solution for g' for every μ' based on utility maximization voting behavior for all households who will occupy the zone in the future, existing households' expectations might be represented in a more ad hoc fashion. In essence this simplifies the specification of the transformation from z to g', i.e., the description of the BC loci in Figure 2-1. If the BC loci can be described analytically, it may be possible to get a reasonable tractable statement of the transformation from h to z. One way to achieve this type of simplification is to base expectations regarding the future vote on g on the mean rather than the median of the distribution describing preferences. Future tax burdens will also be related to mean h in the future. If, further, preferences for g are linearly related to h and $P_g(h)$, and it is assumed that the household with $h = \mu'$ determines the future vote on g, g' is a linear function in μ'.

$$\begin{aligned} g^0 &= a_0 + a_1 h + a_2\left(\frac{\beta h}{\mu}\right) \\ &= a_1 + \mu' \quad a_1 + \frac{a_2\beta}{\mu'} = (a_0 + a_2\beta) + a_1\mu' \qquad (2\text{-}33) \\ &= A_1 + A_2\mu' \end{aligned}$$

Existing households (with fixed h and λ) would maximize $U[g'(\mu'), x'(\mu'), h, \lambda]$ subject to μ', which would determine g and x in the future. Utility maximization would require:

$$\frac{U_g}{U_x} = -\frac{\partial x}{\partial \mu'} \Big/ \frac{\partial g}{\partial \mu'} = \frac{A_1 h\beta/P_x(\mu')^2}{A_2} = -\frac{A_1 h\beta}{A_2 P_x(\mu')^2} \qquad (2\text{-}34)$$

Depending on the choice of a utility function, Equation (2-34) may be complex, at least a quadratic in μ'. The simplest case is $U = a\ \ln g + bx$. In this case (2-34) becomes:

$$\frac{aA_2}{A_1 + A_2\mu'} = -\frac{bA_1 h\beta}{A_2 P_x(\mu')^2} \tag{2-35}$$

or

$$\mu' = \frac{A_1}{2aA_2P_x}[bh\beta + bh(g)^2 + 4ab\beta P_x h]^{1/2}$$

While this transformation from h to μ' yields a complex density function, it is monotonic since $d\mu'/dh > 0$ for all positive h. The solution to the zoning strategy is therefore given by solving (2-35) for the case where $h = \mu$.

CONCLUDING OBSERVATIONS: MODELING THE INTERACTIONS

In this study, I have taken up only one small part of the problem of the interactions between the housing market and the public sector, namely, ways of modeling the link from existing housing stocks in a jurisdiction to voting on public-sector decisions. The principal motivation for taking this partial-equilibrium approach is to abstract from the very many complexities of modeling all the interdependencies in the evolving process of housing market adjustments and household relocation over time. Large-scale simulation models are probably required to address many of the important general-equilibrium aspects of urban development processes.

Implicit in the approach of this paper is an endorsement of the importance of relatively fixed housing stocks and jurisdictional boundaries. The NBER urban simulation model (Ingram, Kain, Ginn 1972) is one large-scale model organized around the housing stock adaptation process. That model focuses on two sets of decisions over time; household relocation choices and incremental housing stock adaptations. In this study I suggest a third process to be modeled, again in incremental fashion: public-sector decision making in jurisdictions with essentially fixed boundaries and fixed stocks. Public-sector outcomes would in turn be one argument in the household relocation models, as well as affecting housing development processes via the zoning process.

Since the models above consider only how one jurisdiction behaves independent of events in all other jurisdictions, they are not well suited to addressing problems of tax capitalization. While there is a theoretical argument for public-sector decisions to be capitalized into property values, the nature of tax capitalization in any jurisdiction depends on outcomes in all other jurisdictions and on the distribution of household preferences. Depending on the degree of heterogeneity in tastes and households' willingness to move, housing and public-sector opportunities in one jurisdiction may or may not be regarded as a close substitute by residents of other jurisdictions. The extent to which alternative sites are regarded as substitutes will influence the extent to which particular decisions on public service and tax levels in any one area are capitalized into housing values there and in all other jurisdictions. An analytic solution of how public-sector decisions affect property values is, in short, a general-equilibrium problem as complex as that of determining how all households might arrange themselves among diverse housing units in different jurisdictions.

While the models in this chapter as an application of a standard change-of-variable problem are formally quite simple, unfortunately the transformation from a probability distribution describing housing stocks and tastes to one describing voter preferences generally proves to be quite complex analytically. Use of a property tax as a financing device results in a nonlinear budget constraint that virtually eliminates any hope of finding simple analytic results. The evaluation of density functions as a basis for predicting voting outcomes must be done numerically. However, if numerical evaluation is necessary, virtually any utility function and density functions can be employed.

There are other ingredients to a descriptive model of local-government decision making which should also be considered in extending these models. There will be other interested parties besides local residents who will affect outcomes. These include public-sector employees, merchants, developers, landholders, and (possibly) the outside sector. Objectives of these groups will differ. Public employees are likely to be concerned principally about employee compensation; for some, this will be wages, while those in a management role may be concerned about the size and growth of the public budget. Some merchants may favor a strategy that maximizes total community income, since this is likely to be a good proxy for retail sales. Other retailers may prefer a strategy that maximizes per capita income. Developers will prefer a strategy that maximizes the rate of growth and, probably, one (at least in most metropolitan areas) that allows more rather than fewer rental units, and high- rather than low-density single-family development.

The dynamics of the adjustment process of changing g are also important. Existing capital facilities, public budgets, and employee numbers are important constraints in the short run. For example, tenure and attrition rates in the labor force may be important constraints. Existing sewer and other public facilities may be long-lived capital stocks and a significant constraint on the types of additional capital that can be put in place in the future. A fully satisfactory model of local-government decisions must include these complexities.

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