Volume Title: NBER Macroeconomics Annual 2007, Volume 22

Volume Author/Editor: Daron Acemoglu, Kenneth Rogoff and Michael Woodford, editors

Volume Publisher: University of Chicago Press

Volume ISBN: 978-0-226-00202-6

Volume URL: http://www.nber.org/books/acem07-1

Conference Date: March 30-31, 2007

Publication Date: June 2008

Chapter Title: Comment on "Aggregate Implications of Credit Market Imperfections"

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Chapter URL: http://www.nber.org/chapters/c4080

Chapter pages in book: (p. 71 - 77)

Comment

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1 Introduction

What are the implications of borrowing constraint for aggregate production and income distribution? Kiminori Matsuyama's answer has two aspects. First, concerning the behavior of an individual producer, the borrowing constraint has a fairly robust prediction—that the investment of each producer depends upon its net worth, in addition to the cost of capital and the expected marginal product of capital. Secondly, concerning the aggregate outcomes, the author argues that there is no simple prediction, and that the borrowing constraint leads to a variety of aggregate predictions, depending on the underlying environment (in addition to the borrowing constraint). The borrowing constraint may generate persistent movement of aggregate quantities, or may cause volatility. The inefficient production may expand during recessions or may be aggravated during booms. The improvement of the financial system may first increase the volatility of aggregate production before reducing it.

At one level, this is an impressive piece of work—a world according to Matsuyama. Like a screwdriver with exchangeable heads, one mathematically simple framework with borrowing constraint brings in varieties of results in business cycles, development, and capital flows, depending on the attachments.

At another level, I find that the model is complicated in terms of theory, even if it may be simple in terms of mathematics. Hereafter I am going to be as critical as possible in order to highlight the differences between the author and me, even though I agree to many of his arguments.

Let me first lay out the key assumptions of his basic model:

1. Agents are homogeneous.
2. Projects are indivisible and there is no lottery.
3. Agents face borrowing constraints.
4. Agents receive credit by the rule of the first-come-first-serve.
5. Overlapping generations model with each agent living for two periods.
6. Agents choose one project from a menu of projects.
   (a) Productive versus unproductive;
   (b) Pledgeable versus not-so-pledgeable;
   (c) Produce capital versus produce consumption goods.

Do we really need all six of these assumptions? Assumptions of (1) homogeneous agent, (2) indivisible project, and (4) credit rationing are not essential. Here, because the agents are homogeneous, there are neither natural lenders nor borrowers. The only reason for borrowing is that the minimum scale of investment is larger than the saving of an individual agent and there is no lottery. (If there were a lottery so that the winner could get enough funds to finance the project, there would be no need for borrowing.) Because people earn higher returns if they receive credit to finance the project when the borrowing constraint is binding, credit rationing is needed in the equilibrium. Thus, the role of finance in this paper is to transfer the purchasing power from those who are back in the queue (unlucky agents) to those who are in front of the queue (lucky ones). This is a complicated theory of credit, because there are many auxiliary assumptions.[1] I think it is more natural to consider that the financial system transfers funds from those who have them to those who have investment opportunity, because the producers and consumers are genuinely heterogeneous.

Assumption of (5) overlapping generations model is more substantial than the assumptions of (1), (2), and (4). But, because the overlapping-generations model is rich by itself, it is difficult to clearly understand how many of the results are due to the borrowing constraint instead of the overlapping-generations model.

Can we develop an argument similar to the author's, only with the assumption of (3) borrowing constraint with heterogeneous agents? Here, we can use the insight that the overlapping-generations model is isomorphic to some credit-constrained economy. (See, e.g., Woodford [1986, 1990] and Aiyagari [1989, 1992]). Then the overlapping generations feature would be a result of the borrowing constraint, not the assumption.

2 Alternative Model

We consider an infinite horizon economy with homogeneous output, capital and labor at each date.[2] There are two (even and odd) types of a continuum of infinitely lived agents. The population size of each type of agents is normalized as unity. Each even-type agent supplies labor inelastically and has technology to invest goods in every even number of date as:

$$y_t = f(k_t) = Ak_t^{\alpha}, 0 < A, 0 < \alpha < 1. \tag{1}$$

where y_t is gross output at date t and k_t is gross investment on capital at date $t-1$, where $t-1$ is an even number.[3] Each odd-type agent supplies labor and invests on the same technology in every odd number of date. Both types have the preference as

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \ln c_s, 0 < \beta < 1 \tag{2}$$

where c_s is consumption at date s and $\ln c$ is natural log of c. Goods are storable between periods with the gross rate of return $B < 1/\beta$.

Let c_t^i be consumption of the agent who is investing (even type if $t =$ even, odd type if $t =$ odd) and let c_t^h be consumption of the agent who is harvesting (odd type if $t =$ even, even type if $t =$ odd). The resource constraint of the economy can be written as:

$$c_t^i + c_t^h + k_{t+1} + z_{t+1} = Ak_t^{\alpha} + Bz_t, \tag{3}$$

where z_t is storage between date $t-1$ and date t.

We can immediately see the first best allocation starting from the initial capital stock, which is not very different from the steady state level, should satisfy the conditions:

$$z_{t+1} = 0, \tag{4}$$

$$k_{t+1} = \alpha\beta Ak_t^{\alpha}, \tag{5}$$

$$c_t^i + c_t^h = (1 - \alpha\beta)Ak_t^{\alpha}. \tag{6}$$

Equation (4) implies there should be no storage in the first best allocation, because the rate of returns is less than the time preference rate. Equation (5) implies the gross saving rate is constant and equal to $\alpha\beta$ in the efficient allocation, which is similar to the model of Brock and Mirman (1972).

In the market economy, in contrast to the first best, each agent faces the borrowing constraint. Following the author, we assume that the borrower can borrow as long as the debt repayment of the next period d_{t+1} does not exceed the constant fraction λ of returns from the present investment k_{t+1}:

$$d_{t+1} \leq \lambda R_{t+1} k_{t+1}, \tag{7}$$

where R_{t+1} is the gross rate of returns on the capital investment from date t to $t + 1$. The flow-of-fund constraint of the agent who is investing at date t implies

$$c_t^i + k_{t+1} = w_t + l_t + Bz_t + \frac{d_{t+1}}{r_t}. \tag{8}$$

Here, his expenditure on consumption and investment in the left-hand side (LHS) is financed by the wage income w_t, the returns from loan l_t, and storage, in addition to the borrowing d_{t+1}/r_t (where r_t is the gross real interest rate) in the right-hand side (RHS). We conjecture the investing agent does not store, which can be verified later.

For the agent who is harvesting, the flow-of-fund constraint implies

$$c_t^h + \frac{l_{t+1}}{r_t} + z_{t+1} = R_t k_t - d_t. \tag{9}$$

His returns on the investment of the previous period after repaying the debt in the RHS is used to finance consumption, loan l_{t+1}/r_t and storage in the LHS.

The competitive-market equilibrium implies the factor prices are equal to the marginal products:

$$R_t = f'(\bar{k}_t) = \alpha A \bar{k}_t^{\alpha-1}, \tag{10}$$

$$w_t = (1 - \alpha) A \bar{k}_t^{\alpha}, \tag{11}$$

where $\bar{k}_t$ is aggregate capital stock, which is equal to k_t in equilibrium. The credit-market clearing condition is:

$$l_{t+1} = d_{t+1}. \tag{12}$$

Although the goods-market clearing is given by equation (3), one of the market-clearing conditions is not independent by Walras' law. The competitive equilibrium is defined as a set of prices (R_t, w_t, r_t) and quantities

$(c_t^i, c_t^h, k_{t+1}, z_{t+1}, d_{t+1}, l_{t+1})$ as a function of the state variables (k_t, z_t, d_t, l_t), which satisfies the utility maximization of each agent and the market-clearing conditions.

We can show that, if the borrowing constraint is not tight, that is, λ is larger than the threshold, then the borrowing constraint is not binding and the allocation is the first best.

We can also show that if the borrowing constraint is tight (λ is small), then the economy is credit constrained. The credit-constrained equilibrium exhibits many features that the author emphasizes in his paper.

(a) Investment is an increasing function of the net worth of the investing agents:

$$k_{t+1} = \frac{w_t + l_t + Bz_t - c_t^i}{1 - (\lambda R_{t+1}/r_t)}, \tag{13}$$

where the numerator is the gross saving of the investing agents—the net worth minus the consumption. The denominator is the down payment required for the investment of one unit—the gap between the cost of investment of one unit and the maximum amount of borrowing available. The investing agents use gross saving to finance the gap between the cost of investment and the amount of external finance.

(b) Net worth of the investing agents is an increasing function of investment and storage of the previous period in the equilibrium:

$$w_t + l_t + Bz_t = (1 - \alpha + \lambda\alpha)Ak_t^\alpha + Bz_t.$$

(c) Inefficient storage is used and the real interest rate to the savers is lower than the time-preference rate:

$$z_t > 0, \text{ and } r_t = B < \frac{1}{\beta}.$$

(d) Consumption of individual agents is larger at harvesting time than investing time:

$$\frac{c_t^h}{c_{t+1}^i} = \frac{1}{\beta B} > 1,$$

$$\frac{c_{t+1}^h}{c_t^i} = \beta\frac{(1-\lambda)R_{t+1}}{1 - (\lambda R_{t+1}/r_t)} > 1$$

Here, because the investing agents save more than harvesting agents, the aggregate saving is an increasing function of the ratio of the net worth of the investing agents and that of the harvesting agents. This association of income distribution and aggregate saving is common to the overlapping-generations model, even though it is not as extreme as the overlapping-generations model (in which the harvesting agents consume everything).

Here, as the author emphasizes, the present distribution of wealth affects the present investment, as seen in (a), and the present investment affects the future wealth distribution as seen (b). This contemporaneous and intertemporal interaction between the wealth distribution and the investment is the unique feature of the credit-constrained economy. Combining (a) and (b) with the other market clearing conditions, we have:

$$k_{t+1} = \frac{(1 - \alpha + \lambda\alpha)Ak_t^\alpha + Bz_t - c_t^i}{1 - (\lambda\alpha Ak_{t+1}^{\alpha-1}/B)}, \tag{14}$$

$$z_{t+1} = Ak_t^\alpha + Bz_t - k_{t+1} - c_t^i - c_t^h. \tag{15}$$

This dynamic interaction between the wealth distribution and investment may generate rich dynamics of aggregate economic activities. The movement of the aggregate quantities is persistent because the present investment depends on the net worth of the investing agents, which is a function of the previous investments. The aggregate quantities may be more volatile, because the net worth of the investing agents affect the compositions of the productive capital investment and the unproductive storage, which changes the total factor productivity (TFP) endogenously.

Although this alternative model shares many features of the author's model, this model, with infinitely lived agents, is more difficult to generate exotic nonlinear dynamics than the author's overlapping generations model with indivisible project. But I do not consider this is the shortcoming of the alternative model, because the income distribution tends to move more sluggishly than the growth rate of output during the business cycle.

3 Conclusion

Perhaps it may be a violation of etiquette to present an alternative model. I agree with the author that the borrowing constraint is important for understanding the various features of aggregate economic activities. I also consider that the borrowing constraint (or more broadly, the liquidity constraint) helps in understanding the various features of the monetary economy that the author did not address. Namely, money is used as the medium of exchange, as the means of short-term saving, and as the unit of contracts. Therefore, the researchers on the subject of liquidity constraints must communicate well with other researchers in order to have an impact on the way people understand the economy. Criteria for good communication include simplicity of theory and how well the simple theory empirically explains the observations. Finally, concerning simplicity of theory, it is important to explain the wide phenomena as clearly as possible with the minimum number of auxiliary assumptions.

Endnotes

1. In remark 3, the author argues that the credit rationing is not needed if the initial wealth of the investing agents are different and the difference is converging to zero in the limit. However, I still think it artificial that agents who have a penny more than the average invest, while agents who have a penny less do not invest. If agents have some flexibility in choosing the size of project, or if there is a lottery, then the borrowing would become negligible as the agents become essentially homogeneous.

2. This model is based on Woodford (1990) and Kiyotaki and Moore (2001, 2005).

3. Thus k_t is capital stock at the beginning of date t, which is invested at date $t - 1$. Here, I follow the author's notations of capital investment as closely as possible.

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