Comment

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1 Overview

This paper provides a very nice expository framework to study and analyze the literature on financial market imperfections and aggregate economic activity. In particular, the paper works through a sequence of models with interesting applications that nicely capture the following key ideas:

1. Agency/enforcement problems imply that borrowers must pay a premium for external finance.
2. In general, this premium depends on:
   (a) Borrower balance sheet positions (i.e., financial structure matters!) and
   (b) Institutions (i.e., monitoring and enforcement capabilities).
3. Given points 1 and 2, feedback can arise between the financial and real sectors, with implications for allocations and dynamics.

The various applications illustrate how financial factors might influence business cycles, growth, inequality, and international capital flows.

While this paper primarily summarizes where the literature has been, I would like to focus my comments on where I think it should be heading. At this juncture I think there are two key issues. First, to assess which of the qualitative predictions of these models can be taken seriously, it is ultimately necessary to evaluate them against the data. The simple overlapping generations structure, while very useful for qualitative analysis, it is not well suited for doing a quantitative evaluation. Second, it is also important to understand whether restrictions on the contracting structure may be sweeping important considerations under
the rug. This paper, as well as many in the literature, restricts attention to static one period contracts in the form of noncontingent debt. There is a strong payoff in terms of tractability from going this route and a compelling justification from the standpoint of realism. At the same time, one has to wonder whether permitting a richer contracting structure might mitigate the incentive problems in a way that significantly alters the predictions. To be sure, often we do not see these richer structures in practice, but that begs the question of why we do not.

In my discussion I will address these two issues by sketching an extension of Matsuyama’s model that: (a) moves toward being suitable for quantitative analysis; and (b) allows for a richer contract structure. Further, the model retains the virtue of tractability that is present in his examples. In particular, it is possible to solve the steady state by hand. At the same time, it is possible to employ this framework for simple quantitative analysis. In particular, I will present a simple numerical experiment that analyzes the link between financial structure and development.

Before proceeding, I note that there is now a literature underway that is developing quantitative frameworks for the analysis of credit market frictions. Examples include Bernanke, Gertler, and Gilchrist (1999), Carlstrom and Fuerst (1997), Christiano, Motto, and Rostagno (2006), Beura and Shinn (2007), Greenwood, Sanchez, and Wang (2007), and Quadrini and Jermann (2007). The example I develop builds directly off of Matsuyama’s model but also captures the spirit of these other frameworks.

2 A Model of Financial Frictions with Infinitely Lived Agents

The model has the following features. The frictionless benchmark is a standard neoclassical growth model. Suppose that there is a representative family with a continuum of members of measure unity that share consumption equally. Each period a fraction $1 - f$ family members supply one unit of labor inelastically in a perfectly competitive market. In addition, each period a fraction $f$ are entrepreneurs that manage firms. Entrepreneurs acquire capital each period and hire labor in order to produce output using a standard Cobb-Douglas technology. Capital is perfectly mobile but the process of financing it is imperfect, as we discuss shortly.

In addition, I assume there is random turnover between entrepreneurs and workers: in any period $t$ an entrepreneur has a probability $\theta$ of surviving until the next period, which means that $(1 - \theta)f$ exit at $t + 1$. 
Entrepreneurs that exit at \( t + 1 \) pay out any retained earnings to their respective families and immediately resume their careers as workers. At the same time, with probability \( (1 - \theta)f/(1 - f) \), a worker at \( t \) becomes an entrepreneur at \( t + 1 \). This implies that there is a total of \( (1 - \theta)f \) of new entrepreneurs at \( t + 1 \), exactly offsetting the number that exit. In addition, the family gives each new entrepreneur a start-up transfer of \( d \) (in units of consumption). As will become clear shortly, I introduce turnover as a device to ensure that entrepreneurs do not indefinitely retain earnings to save their way out of the financial constraint.

To motivate a friction in the capital market, we assume—following Matsuyama—that there is a costly enforcement problem: Only a fraction \( \lambda \) of the firm’s gross return is pledgeable. Because the borrower’s future earnings affect current incentives, however, the financial contract depends on intertemporal considerations. At the same time, however, the framework nests Matsuyama’s static contracting problem.

Let \( k_t \) denote the firm’s capital stock, \( R_{t+1} \) the firm’s return to capital from \( t \) to \( t + 1 \), \( r_{t+i} \) the frictionless borrowing rate (equal to the household return on saving), \( \beta \) the households’ subjective discount factor, and \( \Lambda_{t,t+i} \) the ratio of the household’s marginal utility of consumption at \( t + 1 \) to consumption at \( t \). We can then express the value \( V_t \) of an entrepreneur’s firm as follows:

\[
V_t = \max \sum_i (\theta\beta)^i \Lambda_{t,t+i}(R_{t+1+i} - r_{t+i+i})k_{t+i} \\
= \max[(R_t - r_t)k_t + \theta \beta V_{t+1}]
\]

The entrepreneur takes \( R_{t+1+i} \) and \( r_{t+i+i} \), as given. (Given constant returns to scale and perfectly mobile capital, \( R_{t+1+i} \) is independent of the firm’s size).

Intuitively, the entrepreneur maximizes the discounted value of earnings that he or she will eventually pay out to the family. For this reason, earnings each period \( t + i \) are weighted by the probability of survival \( \theta^i \). Note also that with frictionless capital markets, \( R_{t+1} = r_{t+1} \). In this instance, competitive pressures ensure that in equilibrium, the return of capital equals the frictionless borrowing rate. With capital market frictions, however, agency/enforcement problems add to the effective cost of external finance, driving a wedge between \( R_{t+1} \) and \( r_{t+1} \). Bernanke and Gertler (1989) term this difference, \( R_{t+1} - r_{t+1} \), as the premium for external finance. Generally speaking, this premium may be thought of as the measure of the degree of financial market frictions.

At the end of \( t - 1 \), the firm decides its capital stock for \( t \). It finances this
capital partly by issuing bonds, \( b_i \), and partly through retained earnings, \( w_i \).

\[ k_i = b_i + w_i \]  

(2)

Net worth, in turn, depends on the gross return to capital net the obligations to bondholders. It thus evolves as follows:

\[ w_{t+1} = R_{t+1}k_t - r_{t+1}b_i \]

\[ = R_{t+1}k_t - r_{t+1}(k_t - w_i) \]

\[ = (R_{t+1} - r_{t+1})k_t + r_{t+1}w_i \]

(3)

As noted earlier, we motivate the financial friction by assuming a costly enforcement problem that is a simple multiperiod generalization of the static problem in Matsuyama. In particular, the entrepreneur has the option of walking away with the fraction \((1 - \lambda)\) assets (and giving them to his family). The cost is that he or she forfeits the discounted proceeds from operating the firm, \( V_t \), as well as his or her equity stake, \( r_{t+1}w_i \).

For lenders willing to supply funds to the borrower, the following incentive constraint must be satisfied:

\[ r_{t+1}w_i + V_t \geq (1 - \lambda)R_{t+1}k_t \]

or equivalently,

\[ r_{t+1}w_i + (R_{t+1} - r_{t+1})k_t + \beta \theta V_{t+1} \geq (1 - \lambda)R_{t+1}k_t. \]  

(4)

If the incentive constraint is binding in equilibrium,

\[ k_t = \frac{1}{r_{t+1} - \lambda R_{t+1}}(r_{t+1}w_i + \beta \theta V_{t+1}). \]  

(5)

As in Matsuyama, for the incentive constraint to bind it must be the case that \( r_{t+1} > \lambda R_{t+1} \): otherwise the firm can pledge sufficient assets to eliminate any incentive to renege on borrowers.

As in Bernanke and Gertler (1989), Matsuyama, and elsewhere, the capital market friction makes the firm’s demand for capital depend on entrepreneurial net worth. The difference here from the earlier literature is that the concept of net worth depends not only on end-of-period retained earnings, \( r_{t+1}w_i + (R_{t+1} - r_{t+1})k_t \), but also the firm’s discounted future earnings \( \beta \theta V_{t+1} \). This extra component of net worth enters due to the multiperiod nature of the problem: lenders recognize that the borrower’s incentives depend not only on current liquid assets but also on
his or her prospective net earnings. Note that as $\theta$ goes to zero, the incentive constraint reduces to exactly the one in Matsuyama’s static framework. In this instance, firms only operate for one period (since $\theta$ is zero they exit immediately in the subsequent period), and hence only current liquid assets affect the demand for capital.

Though the complete model is very easy to analyze and solve numerically, in what follows I restrict attention to the steady state, which can be solved by hand. In particular, in the steady state both $R_{t+1}$ and $r_{t+1}$ are fixed. Given this restriction, it is possible to find the following reduced-form expression for the firm’s demand for capital:

$$k_t = \phi r w_t,$$

where $\phi > 1$ and solves

$$\frac{\beta \theta r}{1 - \beta \theta r}(1 - \lambda)R(R - r)\phi^2 + (\lambda R - r)\phi + 1 = 0.$$

Given asset returns, the demand for capital at the firm level is proportionate to the firm’s stock of liquid assets. Observe that in the steady state liquid assets and capital will be evolving at the individual firm level but constant in the steady state, due to the death-and-birth process of firms. It is accordingly straightforward to aggregate the individual demands for capital to obtain the following steady state relation:

$$K = \phi r W$$

where $K = \int_0^f k(i) \, di$ is the steady state aggregate capital stock and $W = \int_0^f w(i) \, di$ is the steady aggregate stock of firms’ liquid assets.

Note next that aggregating across entrepreneurs yields the following equation of motion for aggregate liquid assets:

$$\int_0^f w(i) \, di = \theta [(R - r) \int_0^f k(i) \, di + r \int_0^f w(i) \, di] + (1 - \theta) f d$$

where the first term on the right is the total assets of entrepreneurs that survive between $t$ and $t + 1$ and the second is that of new entrants. Given that aggregate entrepreneurial assets and aggregate capital are fixed in steady state, we may express this relation as

$$W = \theta [(R - r) K + r W] + (1 - \theta) D$$

with $D = f d$.

We can now compactly characterize the steady state equilibrium and, in doing so, highlight the joint interaction between real and financial
conditions. Let $\alpha$ denote the capital share coefficient in the Cobb-Douglas production function, $A$ a common productivity factor, $\delta$ the depreciation rate, and $N = 1 - f$ total labor, which as we noted earlier is in inelastic supply. Then the steady system determines $K, R, r$, and $W$ as follows:

\[
R = \alpha \frac{Y}{K} + 1 - \delta = \alpha A \left( \frac{K}{N} \right)^{\alpha - 1} + 1 - \delta \tag{7}
\]

\[
\beta r = 1 \tag{8}
\]

\[
K = \phi(R, r)W \tag{9}
\]

\[
W = \frac{1 - \theta}{1 - \theta[\phi(R - r) + 1]}D \tag{10}
\]

Observe first that in the benchmark model with frictionless capital markets, the wealth constraint on capital no longer applies. Firms adjust the demand for capital until the marginal return to capital equals the household return on saving:

\[
R = r \tag{11}
\]

Given this condition and steady state consumption Euler condition (8), the return to capital $R$ equals the inverse of the subjective discount factor. In turn, $K$ adjusts to ensure that the gross marginal product of capital equals $\beta^{-1}$. Thus, just as in the frictionless neoclassical model, $K$ is determined independently of financial factors.

The frictionless allocation, however, is not feasible if the enforcement constraint is binding. In this instance, equations (9) and (10) determine $K$. As in the literature that Matsuyama surveys, $K$ is proportionate to the internal equity in the entrepreneurial sector, $W$. In general, when this balance sheet constraint is binding, $K$ lies below its value in the frictionless equilibrium and, in correspondence, $R$ lies above its frictionless counterpart. Put differently, the financial market friction gives rise to a positive relation between aggregate real activity and balance sheet strength. This is also manifested in an inverse relation between the premium for external finance, measured by $R - r$, an aggregate economic activity. This kind of behavior underlies most of the applications in Matsuyama’s paper’s feature. The main difference here is I jettison the overlapping generations setup and instead employ a framework that is a variation of the conventional infinite horizon/representative agent framework that is commonly used for quantitative analysis in macroeconomics.
3 A Simple Numerical Experiment

The virtue of the general approach, accordingly, is that one can get a sense of potential empirical relevance. I illustrate this with an example based on Greenwood, Sanchez, and Wang (2007). These authors present evidence of a positive relation between various measures of financial development and capital and output per capita. They proceed to develop a theoretical model of finance and development and then perform a calibration exercise to assess how well the model can characterize the facts. The model I present here is much simpler than theirs, but it is nonetheless sufficiently rich to permit a similar kind of exercise for illustrative purposes.

In particular, I present two different calibrations of the model. The first is a benchmark calibration meant to represent an economy with highly developed financial markets, such as the United States. The second calibration, in turn, is meant to capture an emerging-market economy that has relatively less-developed financial markets. I then explore the implications for capital intensity across the two countries. I should stress that this example is meant for illustrative purposes only.

I assume the real (i.e., nonfinancial) parameters are the same for both countries. Given the period length is one year, I set \( \beta = 0.96 \) and \( \delta = 0.1 \). I also set \( \alpha = 0.33 \). All these values are standard. I allow the parameters that govern the financial market friction, \( \lambda, \theta, \) and \( D \), to vary across the two countries. For the United States, I fix these variable to target a 2 percent external finance premium (based on evidence on the spread between the prime lending rate and the riskless rate), a debt-equity ratio of roughly unity (again roughly in line with the historical data) and also a firm survival rate to roughly match the evidence. This leads me to choose \( \lambda = 0.3, \theta = 0.895, \) and \( D \) to deliver a value of \( W = 0.861 \). For the emerging-market economy, I simply pick a set of parameters consistent with it having weaker financial institutions: \( \lambda = 0.15, \theta = 0.700, \) and \( D \) to deliver a value of \( W = 0.431 \).

The results are illustrated in figure 1C1.1. The horizontal line plots the frictionless interest rate. The downward sloping line portrays the marginal return to capital for each value of the capital stock. Point 0 defines the frictionless equilibrium, where the capital stock adjusts to the point where the marginal return to capital equals the frictionless interest rate. The point US denotes the equilibrium for the developed economy. The enforcement constraint, given by the vertical line, intersects the demand curve at the point where the capital stock is roughly 20 percent below its
frictionless equilibrium value. Even though the marginal return to capital exceeds the frictionless equilibrium interest rate, entrepreneurs do not have sufficient net worth to obtain additional funds to move beyond this point.

The distortion is much greater for the emerging market economy: the capital stock is about 45 percent below its frictionless equilibrium value. The premium for external finance, in turn, increases to roughly 7 percent. Though this example is very informal, it illustrates how in principle such a model could potentially explain the facts presented in Greenwood, Sanchez, and Wang (2007).

Again, I stress that the qualitative insights from this example are present in Matsuyama's analysis. The virtue of extending the analysis in the way I have, however, is that one can begin to think about quantitative relevance. Further, while I have just presented a steady state analysis, it is very easy to numerically solve for the dynamics. The simple structure of the model, further, makes the results easy to interpret.
One important caveat, though, is the potentially sensitivity of the financial contracts to aggregate uncertainty. There is a complex issue of how financial contracts might be conditioned on aggregate risk. In this instance, the dynamics can depend heavily on borrowers’ ability to insure against aggregate risks to their balance sheets.

4 Concluding Remarks

Overall, I think this is an excellent survey of the literature—definitely reading list material. The next step in the progress of this literature is to develop models for quantitative evaluation. Hopefully, five years from now Kiminori Matsuyama will survey this new literature!

References


