Appendix 5-B

Derivation of Transfer Functions for Prices

The hypothesis of rational expectations in our model implies that the expected $p_t$ is computed as if the public attempted to obtain an optimal unbiased forecast of $p_t$ using Equation (5-6). Combining (5-14) and (5-6) we can write:

$$p_t = \left( \frac{1}{1 - b} \right) \sum_{j=0}^{\infty} \left( \frac{1}{1 - b^{-1}} \right)^j \left( E\phi m_{t+j} - y_{n,t+j} \right)$$  \hspace{1cm} (B5-1)

$$+ \left[ \frac{J_3}{1 - J_0} \right] \sum_{j=0}^{\infty} \left( \frac{\alpha}{1 - b^{-1}} \right)^j y_{c,t-1} + \gamma_0 + \gamma_4 t$$

In (B5-1) we have a term in $E\phi m_{t+j}$. Developing this term for $j = 0, 1, \ldots$ taking expectations and recalling that

$$\phi = \phi_0 + \phi_1 L + \phi_2 L^2 + \ldots,$$ 

we have

$$E\phi m_{t+j} = E\phi_0 m_t + \phi_1 m_{t-1} + \phi_2 m_{t-2} + \ldots \hspace{1cm} j = 0$$

$$E\phi m_{t+j} = E\phi_0 m_{t+1} + E\phi_1 m_t + \phi_2 m_{t-1} + \ldots \hspace{1cm} j = 1$$
Recall that the $E$ operator is conditional on the information in period $t-1$, so $Em_{t-1} = m_{t-1}$, and so on, for periods before period $t-1$. Now provided that we use the process (5-12) to obtain $Em_{t+j}, j = 1, 2, \ldots$, we notice that the forecasts of $m_t$ are obtained through linear combinations of $m_{t-1}, m_{t-2}, m_{t-3}, \ldots$. These linear combinations should be combined with the other terms in $m_{t-1}, m_{t-2}, m_{t-3}, \ldots$ that appear because of the lagged response of prices to changes in $m_t$, and with $y_{n,t+j}, j = 1, 2, \ldots$ and $y_{c,t-1}$ to forecast $p_t$. Then we can rearrange the terms in $m_{t-1}, m_{t-2}, m_{t-3}, \ldots$, and rewrite (B5-1) as

$$E \phi m_{t+j} = E \phi_0 m_{t+2} + E \phi_1 m_{t+1} + E \phi_2 m_{t-1} + \ldots$$

$$j = 2$$

The first difference form of this equation is:

$$p_t = v(L)Lm_t - (1/[1 - b]) \sum_{j=0}^{\infty} (1/[1 - b^{-1}])^j y_{n,t+j}$$

$$+ (J_0/[1 - J_0]) \sum_{j=0}^{\infty} (a/[1 - b^{-1}])^j y_{c,t-1} + c_0 + u_{4t}$$

Equation (B5-3) is reproduced in the text as equation (5-14-a), which in turn is parsimoniously (in terms of the number of parameters) represented by the transfer function (5-15).