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# The Allocation of Goods Over the Life Cycle

## 2.1 PRELIMINARIES

The purpose of the empirical work reported in this chapter is to explain consumption behavior over the life cycle. The model developed in Chapter 1 predicted some systematic relationships between a household's consumption of goods (and time) and the wage rates of its earners over their lifetime. My task now will be to develop an empirical methodology capable of capturing these life cycle effects.

In Chapter 1 it was shown in particular that the change in the demand for goods at age t is:

$$\tilde{X}_{t} = b_{1} \tilde{w}_{1t}^{\star} + b_{2} \tilde{w}_{2t}^{\star} + b_{z} \tilde{Z}_{t} + b_{t}; \qquad (2.1)$$

with

$$egin{aligned} b_1 &= (\sigma_{x1} - \sigma_c) s_1; \ b_2 &= (\sigma_{x2} - \sigma_c) s_2; \ b_z &= (\sigma_c - 1) \epsilon; \ b_t &= \sigma_c (r_t^\star - 
ho_t); \end{aligned}$$

NOTE: Ghez is solely responsible for this chapter.

1. I am grateful to Barry Geller for helpful computational assistance.

where

- t = age of household head;
- $\tilde{X}_t$  = percentage change in the consumption of market goods at age *t*;
- $\tilde{w}_{1t}^{*}, \tilde{w}_{2t}^{*}$  = percentage change in the real wage rate of husband (subscript 1) and wife (subscript 2) at age *t*;
  - $\tilde{Z}_t$  = percentage change in some characteristic or environmental variable Z that governs nonmarket productivity;
  - $r_t^{\star}$  = real rate of interest at age *t*;
  - $\rho_t = \text{rate of time preference at age } t;$
- $\sigma_{x_1}$ ,  $\sigma_{x_2}$  = partial elasticities of substitution between goods and husband's and wife's home time;
  - $\sigma_c$  = elasticity of substitution between commodities in period t and commodities in period t + 1;
  - $s_1$ ,  $s_2$  = shares of husband's and wife's time in total costs of commodities at age t;
    - $\epsilon$  = elasticity of response of nonmarket productivity to a 1 per cent change in the environmental variable Z.

Equation (2.1) could be used directly as an estimating equation if the following four conditions were met: (i) if household expectations were in fact fulfilled; (ii) if complete life histories of households were available; (iii) if changes in nonmarket productivity could be related to some observable determinants called Z; (iv) if the elasticities  $b_1$ ,  $b_2$ ,  $b_z$ , and  $b_t$  were constant.

These conditions are hard to meet. Since perfect foresight, for one, is an unreasonable assumption, I formulate a more plausible expectations model in the next section.

In the second place, reinterview data are scarce and incomplete. In section 3, therefore, I develop a procedure for testing life cycle behavior with cross-sectional data.

Third, observable determinants of nonmarket productivity are difficult to come by with existing data. I shall assume henceforth that increases in family size are either the source or are highly correlated with factors that raise productivity in the home.

In the development of an estimating equation, I shall also assume that the elasticities  $b_1$ ,  $b_2$ ,  $b_z$ , and  $b_t$  in equation (2.1) are all constant. In other words, I assume that the rate of interest net of time prefer-

ence, the elasticities of substitution in production  $^2$  and in consumption, and factor shares are all constant. The validity of the assumption of constant factor shares is tested in Chapter 4.<sup>3</sup>

### 2.2 AN EXPECTATIONS MODEL

Suppose that household anticipations were not fully realized. What would be the effect of these errors on the consumption path? Consider first the substitution of goods for home time. Since we assumed that the production function is homogeneous of the first degree, factor proportions in production at any age depend only on the real wage rates and on the shape of the production function at that age. Goods intensity of production is independent of commodity output and therefore of all future variables. Since actual changes in factor proportions at any age depend on the actual change in factor prices at that age, they are independent of whether expectations are fulfilled or not.<sup>4</sup>

By contrast, the absolute level of consumption of goods at any age t depends on expectations held at that age, because the absolute amount of commodity consumption at t depends on estimated full wealth at t and on anticipated prices.

Thus, if price or income expectations were not fulfilled, the actual change in goods and commodity consumption would be governed not only by substitution effects, as explained in Chapter 1, but also by wealth effects.

To give a more formal presentation, I shall suppose that elas-

<sup>2.</sup> Subject to a minor qualification, if partial elasticities of substitution are constant they are necessarily equal, i.e.,  $\sigma_{x1} = \sigma_{x2} = \sigma_{12}$ . See Hirofumi Uzawa, "Production Functions with Constant Elasticities of Substitution," *Review of Economic Studies* (October 1962), pp. 291–299.

<sup>3.</sup> The factor shares  $s_1$  and  $s_2$  would not in general be constant if the elasticities of substitution in production were not unity.

<sup>4.</sup> This implication is no longer true if there are costs of adjustment associated with inputs. In the short run the stock of durable goods is fixed for each household. Therefore, unanticipated changes in real income give rise to diminishing returns in the use of household time, and generate induced substitution effects away from commodities produced with fixed stocks of durable goods. Further developments are contained in Gilbert R. Ghez, "Life Cycle Demand for Consumer Durable Goods" (unpublished, 1968).

ticities of substitution in consumption are constant and equal.<sup>5</sup> The actual change in commodity consumption of a given household at age t of the head is

$$\tilde{C}_t = (\hat{W}_t^* - \hat{P}_t^*) - \sigma_c(\tilde{\pi}_t^* - \hat{P}_t^*) + \sigma_c(r_t^* - \rho_t); \qquad (2.2)$$

where

- $\tilde{\pi}_{t}^{*}$  = the actual percentage change in the price of commodities at age t;
- $\hat{W}_t^{\star}$  = the unanticipated percentage change in full wealth at t;
- $\hat{P}_t^{\star}$  = the unanticipated percentage change at t in a price index of current and future commodities.

If all future prices and incomes were prefectly predicted, the actual change in commodity consumption at age *t* would be simply  $\tilde{C}_t = -\sigma_c \tilde{\pi}_t^* + \sigma_c (r^* - \rho)$ , as developed in Chapter 1. The introduction of unexpected changes has in general two effects:

i. It introduces wealth effects on consumption. The unexpected change in real wealth is  $\hat{W}^* - \hat{P}^*$ , and this creates a percentage change of  $\hat{W}^* - \hat{P}^*$  in commodity consumption, since the wealth elasticity of consumption has been assumed to equal unity, as explained in note 5.

ii. It affects the actual amount of substitution in consumption, since the relevant change in relative prices is  $\tilde{\pi}^* - \hat{P}^*$  rather than  $\tilde{\pi}^*$  alone.

For instance, an unexpectedly high wage rate at age t, whether or not it is accompanied by an upward revision of future wage expectations, raises full wealth. It also raises the value of the price index. As long as hours of work are positive, full wealth would rise more than the price index, and therefore real wealth would increase.<sup>6</sup>

6. We would have

$$\hat{W}_{t}^{\star} = \sum_{t'=t}^{T} (\alpha_{1t't} \hat{w}_{1t't}^{\star} + \alpha_{2t't} \hat{w}_{2t't}^{\star});$$
$$\hat{P}_{t}^{\star} = \sum_{t'=t}^{T} k_{t't} (s_{1t't} \hat{w}_{1t't}^{\star} + s_{2t't} \hat{w}_{2t't}^{\star});$$

where

<sup>5.</sup> Notice that if the utility function is additively separable and has constant elasticities of substitution, it is necessarily homogeneous. Wealth elasticities of consumption are equal to unity. For a proof, see Daniel McFadden, "Constant Elasticity of Substitution Production Functions," *Review of Economic Studies* (June 1963), pp. 73–83.

Over its life cycle, a household could be underestimating its real wealth at certain times and overestimating it at others. Therefore, if households were taken as units of observation, wealth and substitution effects could be sorted out only if a direct measure of estimated wealth were available or if a relationship could be established between expectations and past realizations.

Yet, if we consider a group of households whose permanent characteristics are the same, it is plausible that some households overestimate, while other households underestimate, their future incomes, their future market and nonmarket efficiency, and their life span. I shall suppose that on the average expectations are fulfilled. In other words, if all households that are homogeneous in such permanent characteristics as schooling and race are grouped by year of age, the average unexpected change in full wealth and in the price index is assumed to equal zero. Let there be *n* homogeneous households; then

$$\sum_{i=1}^{n} \hat{W}_{il}^{\star} = 0; \qquad t = 1, 2, \ldots, T.$$

$$\sum_{i=1}^{n} \hat{P}_{it}^{\star} = 0. \qquad t = 1, 2, \ldots, T.$$

- $\hat{w}^{*}_{il'l}$  = percentage revision, at age t of the household, of the *i*th member's wage rate expectation at age t';
- $\alpha_{it't}$  = ratio of the *i*th member's (estimated) discounted full earnings at age t' to household full wealth (estimated) at age t;
- $k_{t't}$  = discounted estimated share of commodities at age t' in family full wealth (estimated) at age t.

Therefore,

$$\alpha_{ii't} - k_{i't} \mathbf{s}_{ii't} = \frac{R_{i't}^* W_{ii't} N_{ii't}}{W_{t-1}^*} \cdot$$

Actually equation (2.2) is sufficiently general to accommodate the effects of mistaken expectations and revisions of future expectations not only about wage rates, but also about nonwage income, nonmarket efficiency, interest rates, and the life span. An unexpectedly high nonwage income, a windfall, raises real wealth by raising full wealth, while an unexpected increase in nonmarket efficiency raises real wealth by reducing the value of the price index. An unexpectedly low rate of interest at time *t* raises or reduces real wealth according as the household is a net borrower or a net lender at that particular time. An upward revision in life expectancy raises full wealth as long as the extra years are not all spent in retirement; since it also increases the number of periods over which wealth is to be allocated, current consumption would rise or fall according as the extra earnings were greater or smaller than the consumption of goods during the extra years of life. Since within-cohort expectations are unbiased, changes in cohort consumption are governed by substitution effects alone. For the *h*th homogeneous group we have:

$$\tilde{C}_{ht} = -\sigma_c \tilde{\pi}_{ht}^* + \sigma_c (\mathbf{r}^* - \rho)$$
  
=  $-\sigma_c (\mathbf{s}_1 \tilde{w}_{1ht}^* + \mathbf{s}_2 \tilde{w}_{2ht}^*) + \sigma_c \epsilon \tilde{Z}_{ht} + \sigma_c (\mathbf{r}^* - \rho);$  (2.4)

and

$$\tilde{X}_{ht} = (\sigma_{x1} - \sigma_c) s_1 \tilde{w}_{1ht}^* + (\sigma_{x2} - \sigma_c) s_2 \tilde{w}_{2ht}^* + (\sigma_c - 1) \epsilon \tilde{Z}_{ht} + \sigma_c (r^* - \rho);$$
(2.5)

where the subscripts ht denote the geometric mean of a variable over all households within group h at age t of the household head.

## 2.3 TRENDS IN REAL WEALTH

As I mentioned in section 2.1, no systematic reinterview data are available. I proceed now to show how the model can be tested with cross-sectional data.

In a cross section, households differ in their real wealth. If, for a given cohort, all households that are homogeneous in permanent characteristics such as schooling and race are grouped by age of head, average real wealth would be independent of age, as indicated in the expectations model described in section 2.2. But even for homogeneous groups, average real wealth varies *across* cohorts. Hence, in a cross section, average real wealth is expected to vary with age. Since real wage rates are growing over time, younger schooling-and race-specific cohorts have higher real wealth than comparable older ones. Similarly, if average household productivity is rising over time, the absolute price of commodities will fall over time and real wealth will rise.

More formally, we have

$$\dot{C}_{ht} = (\dot{W}_{ht}^* - \dot{P}_{ht}^*) - \sigma_c(\dot{\pi}_{ht}^* - \dot{P}_{ht}^*) + \sigma_c(r^* - \rho); \qquad (2.6)$$

where the dot above a variable denotes the percentage difference in that variable for a one-year difference in age in the cross section, and

$$\dot{X}_{ht} = \dot{W}_{ht} - (1 - \sigma_c)\dot{P}_{ht} + (\sigma_{x1} - \sigma_c)s_1\dot{w}_{1ht} + (\sigma_{x2} - \sigma_c)s_2\dot{w}_{2ht} + (\sigma_c - 1)\epsilon\dot{Z}_{ht} + \sigma_c(r^* - \rho).$$
(2.7)

If growth in market productivity is age-neutral and occurs at a constant rate  $g_w$  for both husband and wife, in the sense that it raises the wages of both at a constant rate  $g_w$  regardless of age, and if nonmarket productivity growth occurs at the constant rate of  $g_f$ , then <sup>7</sup>  $-\dot{W}_{ht} = g_w$ ; and  $-\dot{P}_{ht} = (s_1 + s_2)g_w - g_f$ . Hence,

$$\dot{X}_{ht} = b_1 \dot{w}_{1ht} + b_2 \dot{w}_{2ht} + b_2 \dot{Z}_{ht} + b_t;$$
 (2.8)

where  $b_1$ ,  $b_2$ , and  $b_z$  are defined as for cohort behavior [see equation (2.1)], but where  $b_t$  now includes not only the effects of the interest rate and of time preference but also the effects of trends in market and nonmarket efficiency: <sup>8</sup>

$$b_t = \sigma_c(r^* - \rho) - [(1 - s_1 - s_2) + \sigma_c(s_1 + s_2)]g_w - (1 - \sigma_c)g_f.$$

If we integrate equation (2.8) we get the appropriate consumption function for the cross section:

$$\log X_{ht} = b_0 + b_1 \log w_{1ht} + b_2 \log w_{2ht} + b_z \log Z_{ht} + b_t t; \quad (2.9)$$

where  $b_0$  is an indicator of real wealth of the youngest cohort in the cross section. In principle, life cycle consumption behavior could be estimated from a single cross section by using equation (2.8) or (2.9). From a relatively complex model rather simple estimating equations have been generated.

For the actual empirical computations reported in the next several sections, these equations had to be modified somewhat, because of data limitations, which I discuss in more detail below. In particular, I use annual earnings to measure wage rates, and I assume that there are no differences among cohorts in nonmarket efficiency.<sup>9</sup>

$$\dot{L}_{iht} = a_{i1}\dot{w}_{1ht} + a_{i2}\dot{w}_{2ht} + a_{iz}\dot{Z}_{ht} + a_{it}. \qquad i = 1, 2.$$

Since the production function is homogeneous,

$$a_{it} = b_t; i = 1, 2.$$

In Chapter 3, Becker uses this methodology to estimate the demand for time by men.

<sup>7.</sup> If factor shares were not constant, P would also depend on the covariance between the budget shares, k, of consumption in full income and the combined share of husband's and wife's time,  $s_1 + s_2$ .

<sup>8.</sup> In the same manner, one can derive equations for consumption time appropriate for the cross section:

<sup>9.</sup> A simultaneous equation bias is introduced because earnings and consumption are jointly determined in the model. Instrumental variables estimation techniques would have been more appropriate than ordinary least squares.

#### 2.4 THE DATA

The main data source <sup>10</sup> used is the Survey of Consumer Expenditures for 1960–61, conducted by the Bureau of Labor Statistics.<sup>11</sup> This is a nationwide survey of family expenditure, income, and several personal characteristics of 13,728 households. The survey covers the two years 1960 and 1961. Observations from both periods were retained in order not to reduce the sample size inordinately.

Households were cross-classified in the following ways:12

i. By age of household head: 44 age groups by single year of age of the head, ranging from age 22 up to age 65.

ii. By education of the household head: level I: 0-8 years of schooling; level II: 9-12 years of schooling; level III: 13 or more years of schooling.

Age 22 was chosen as the lower bound because below that age the cells were often very small in size and because it is appropriate to delete those years of age in which many household heads (and their spouses) are pursuing full-time schooling.<sup>13</sup> Age 65 was chosen as the upper bound because in many households earners beyond that age are retired.<sup>14</sup>

Households were classified by schooling in order to verify whether or not life cycle patterns differ among households that differ in some permanent characteristic.<sup>15</sup> Households with a higher level

<sup>10.</sup> An auxiliary source used is the 1/1,000 sample of the U.S. population in 1960 (see Chapter 3, note 36, below). Some of the results which I obtained from pooling the BLS Survey with the 1/1,000 sample are reported in Chapter 4.

<sup>11.</sup> For a detailed description, see Consumer Expenditures and Income: Survey Guidelines, BLS Bull. 1684, 1971. See also Helen H. Lamale, Methodology of the Survey of Consumer Expenditures in 1950 (Philadelphia: University of Pennsylvania, 1959).

<sup>12.</sup> The General Purpose Tapes created by the BLS were used, rather than the tabulated data. The latter give mean expenditures and income by age of family head, but with intervals of ten years of age. Moreover, family income by source is not classified by age.

<sup>13.</sup> Indeed, under those conditions, the price of time would be measured by the discounted value of marginal returns to schooling, rather than by the potential wage rate. See Chapter 1.

<sup>14.</sup> In the BLS Survey of Consumer Expenditures, the proportion of retired persons rises from 10 per cent at age 62, to 21 per cent at age 65, 37 per cent at age 66, 46 per cent at age 67, and exceeds 50 per cent starting with age 69.

<sup>15.</sup> A sorting of households by race as well as schooling would have been undertaken had nonwhite cells been sufficiently large. But the mean cell size for nonwhites between ages 22 and 65 is 16.6 for education level I, 12.1 for level II, and 2.9 for level III, with many empty or small cells.

of education are expected to have a very different consumption pattern than households with less education. In the first place, after completion of school the potential wage rate of the former group is higher, thereby inducing substitution toward the consumption of goods. Second, higher-educated people may have a higher level of real wealth because they are more able or because they have easier access to funds to finance their investments. This difference in lifetime real income would imply that the higher the level of education. the higher the consumption profile. According to the model developed in Chapter 1, the shape of the consumption profile would depend largely on the shape of the wage profile. The responsiveness of consumption to the wage rate over the life cycle would depend on (i) the importance of time in household production and (ii) substitution elasticities. This responsiveness would be the same at all levels of education if factor shares and elasticities of substitution did not differ by level of education. The higher wage rates of the more educated imply a substitution toward goods, but the factor shares would still be the same for all levels of education if the elasticity of substitution in production was equal to unity. In other words, the responsiveness of consumption to changes in the wage rate over the life cycle need not differ by level of schooling. This is a question that empirical estimation can resolve.

As shown in Table 2.1, the mean cell size is largest for education level II, i.e., for those households whose heads have completed nine to 12 years of schooling. The range of cell size across observations is wide: cell sizes are largest during the central years of life and taper off at the extremes.

Within each cell, I constructed the arithmetic means of certain

|                        | Number of Households |         |  |  |
|------------------------|----------------------|---------|--|--|
| Education Level        | Mean                 | Range   |  |  |
|                        | 256.6                | 112-337 |  |  |
| I (0–8 years)          | 83.7                 | 11–145  |  |  |
| II (9-12 years)        | 118.2                | 50–145  |  |  |
| III (13 years or more) | 54.7                 | 19–89   |  |  |

TABLE 2.1

Cell Sizes in the BLS Survey of Consumer Expenditures, 1960–61, FOR HOUSEHOLD HEAD OF AGES 22 TO 65 variables.<sup>16</sup> Arithmetic means were used rather than the more appropriate geometric ones because the latter involve an obvious computational defect when zero values are encountered. In addition, the use of arithmetic means facilitates comparisons with other studies. Presumably this change of variable is not a source of much bias: under fairly general conditions the results are not affected at all.<sup>17</sup>

As my basic measure of consumption I took the sum of: (i) expenditures on nondurable goods and services;<sup>18</sup> (ii) the imputed value of housing services plus expenses for automobile operation; (iii) gifts.

For the model developed in Chapter 1, it was assumed that all goods are nondurable. To incorporate durable goods into the model, I suppose that their services enter the production function for commodities, along with nondurables and home time. As long as the price of the services of durable goods relative to the price of nondurables is constant over time, the sum of the value of the services of durables and nondurables would form a composite good whose behavior would depend on the real wage.

The BLS survey data are not adequate for calculating the value of the services of many durable goods. For housing, I estimated the average implicit rental of owners within each cell by the average rent paid by renters in the corresponding cell. This procedure is based on the assumption that capital markets are perfect, that there are no transactions costs associated with rentals or purchases, and that holders of durable goods are perfectly indifferent between ownership and rental of these goods. For automobiles, I included expenses for automobile operation in my measure of consumption on the assumption that these are proportional to the services of automobiles.

For other durable goods, such as house furnishings and durable recreation goods, no adjustment was possible. Hence, my measure of consumption understates true consumption. The underestimate

<sup>16.</sup> Since the Survey of Consumer Expenditures is a stratified sample, I used weighted means, using the reported survey weights. For the small-city stratum the weights for individual households were not reported by the BLS in order to preserve the anonymity of the respondents. Households in that stratum were weighted by the average weight of the stratum.

<sup>17.</sup> See the appendix to this chapter.

<sup>18.</sup> This is the BLS category "expenditures on current consumption" minus purchases of durable goods reported in the survey (purchases of furnishings and equipment, automobiles, TVs, radios, and musical instruments) minus expenses on owned dwelling.

would be relatively small at early years of age, if at the outset of the life cycle, initial stocks of durable goods were below their optimum levels. Some evidence of this condition is seen in the pattern of net investment in durable goods in the BLS Survey of Consumer Expenditures: net investment in durable goods rises at early ages, reaching a peak in the late twenties, then gradually declining and turning negative in the late sixties. Therefore, total true consumption would rise more rapidly at least initially than a measure of consumption that excluded the services of durable goods.

Gifts are included in my measure of consumption since these are a form of expenditure and the model of derived demand developed in Chapter 1 can be presumed to apply to them as well. Expenditures on property insurance are included, but not expenditures on personal insurance.

The Survey of Consumer Expenditures covers annual earnings of each household during the survey year, but does not contain information on hours of work and wage rates of each family member separately. As a measure of the price of time, I used family earnings plus self-employment income.<sup>19</sup> A full justification for the use of family earnings as a measure of the price of time is given in the appendix to this chapter. In brief, if the wife's wage rate is relatively steady over the life cycle (barring growth in real wages), as in fact it seems to be,<sup>20</sup> variations in her yearly earnings will basically reflect variations in her hours of work resulting from changes in the husband's wage rate (and from changes in nonmarket productivity). Her earnings will rise as his wage rate rises if his and her time are either complementary or not very strong substitutes. Moreover, his yearly earnings will rise as his wage rate rises.<sup>21</sup> Hence, family earnings will be positively related to his wage rate, as long as the correlation between his and her earnings is positive or not strongly negative.22

<sup>19.</sup> These are before-tax earnings. No attempt was made to correct for the progressivity of the income tax.

<sup>20.</sup> See in particular James Smith, "The Life Cycle Allocation of Time in a Family Context" (University of Chicago: Ph.D. diss., 1972). In his data from the Survey of Economic Opportunity, 1967, men's wages fluctuate over a much wider range than women's.

<sup>21.</sup> In this discussion I exclude interest rate and time preference effects.

<sup>22.</sup> One difficulty is that changes in wife's earnings reflect also changes in her (and his) nonmarket productivity. Therefore, changes in family earnings would also reflect these life cycle changes. For instance, if nonmarket effects were neutral across

No data exist on interest rates by age of head. However, as explained above, as long as interest rates are the same at all years of age, their effect on consumption is incorporated in the effect of age of head.

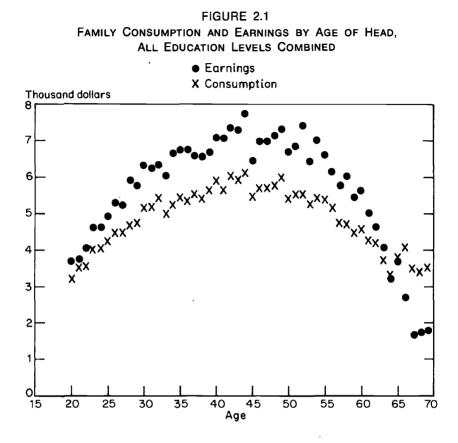
No direct measures of nonmarket productivity exist. I shall suppose that within each education group, production functions are the same for all cohorts in the cross section. In other words, at any particular time, the benefits of technological change in the household sector are spread over all households having the same level of education.

Other variables also retained from the BLS survey were family size and total family income.

The cross-sectional patterns of mean consumption and mean earnings by age of head are displayed in Figures 2.1, 2.2, and 2.3.

The curves in Figure 2.1 portray over-all mean earnings and consumption by age of head for all education levels combined. As predicted by the theory, earnings tend to rise initially, reach a peak in the mid- or late forties, and then decline. The rise and subsequent decline in earnings is presumably due not only to the rise and fall in wage rates, but also to the rise and fall in hours of work. Consumption distinctly follows the same path as earnings, rising initially, peaking at about age 45, and falling thereafter. It is important to note, however, that the consumption profile is less steep than the earnings profile: its initial rise is gentler, and its fall less rapid, than that of earnings, with earnings falling below consumption at about age 65. This smoothing of the income stream is an implication of the model developed in Chapter 1, although the point was not discussed there. If substitution in production is easier than in consumption, and the rate of interest net of time preference is close to zero, consumption will rise as wage rates and earnings rise, and fall as they fall. When consumption equals annual earnings its rate of change must be smaller (in absolute value) than that of earnings because if commodity output were constant (say because  $\sigma_c = 0$ ), consumption of goods would rise more gently than earnings when the wage rate rose; a fortiori, this would occur if commodity output fell when the

factors, and if the elasticity of substitution in consumption was smaller than unity, improvements in home productivity would induce all family members to increase their hours of work and thus family earnings.



wage rate rose because of commodity substitution.<sup>23</sup> As noted above, this smoothing of income is observed in the data, which thereby provide some support to the model.

Households in each successively higher level of schooling have earnings profiles higher than those in the levels below (Figure 2.2). Their earnings tend to rise more rapidly and for a longer period of time. This is precisely what one would expect if on-the-job training were positively related to schooling. The consumption of households

<sup>23.</sup> For proof, notice that earnings *E* net of consumption *X* are equal to full earnings net of expenditures on household commodities:  $E - X = wT - \pi C$ . The rate of change of expenditures on commodities is (neglecting effects of interest rates and time preferences)  $(1 - \sigma_c)s\tilde{w} < \tilde{w}$  for all values of  $\sigma_c \ge 0$ , since *s* (the combined share of household labor) is smaller than unity.

A slight modification in the argument is required if male and female wage rates grow at different rates, and if the production of human capital is considered.

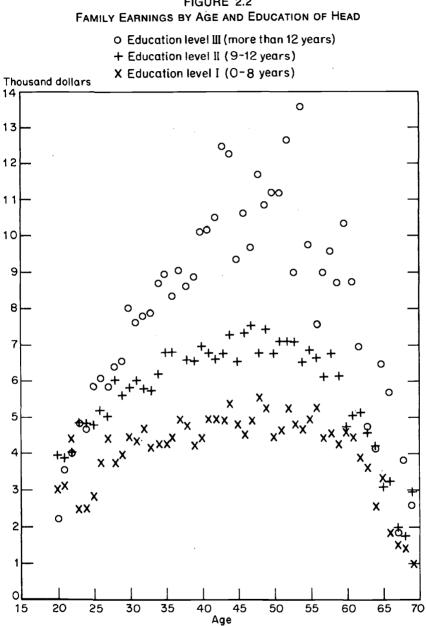
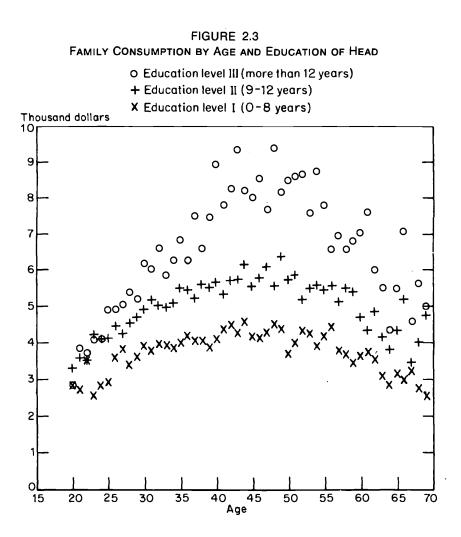


FIGURE 2.2

with more education also rises much more rapidly up to the midforties (Figure 2.3). Peak consumption is about two and a half times consumption at early years of age for households in which the head is at least a high school graduate, and less than twice that amount for households with the least-educated heads. This apparent parallel arching of the consumption profiles with the earnings profiles is one of the striking features revealed by the data, and predicted by the theory. Also, as one would expect, the consumption profile of households with more education tends to lie uniformly above that of house-



#### TABLE 2.2

|                       | Ear     | nings                 | Consumption |                       |  |
|-----------------------|---------|-----------------------|-------------|-----------------------|--|
| Education Level       | Mean    | Standard<br>Deviation | Mean        | Standard<br>Deviation |  |
| All                   | \$6,248 | \$1,033               | \$5,149     | \$ 681                |  |
| l (0–8 yrs.)          | 4,496   | 678                   | 3,925       | 449                   |  |
| ll (9–12 yrs.)        | 6,231   | 927                   | 5,222       | 627                   |  |
| III (13 yrs. or more) | 8,755   | 2,183                 | 6,870       | 1,440                 |  |

# MEAN AND STANDARD DEVIATION OF FAMILY EARNINGS AND CONSUMPTION BY EDUCATION OF HOUSEHOLD HEAD IN 1960-61

SOURCE: BLS Survey of Consumer Expenditures, 1960-61. Weighted statistics over age cells ranging from age 22 to 65 (weight = cell size).

holds with less education, because real wealth is greater for those having more schooling. This may be seen also in Table 2.2, which gives means and standard deviations of earnings and consumption by level of schooling.

For all heads combined and within each education group the consumption profile lies below the earnings profile except during old age. Part of the explanation for this is that the services of some durable goods are not included in my measure of consumption. Secondly, each cohort is transferring more assets to future generations than it received from previous ones.

#### 2.5 RESULTS

With observations ordered by age of head, I ran linear regressions of the logarithm of mean consumption of goods on the logarithm of mean earnings, the logarithm of mean family size, and age.<sup>24</sup> The standard format is:

$$\log X_t = B_0 + B_e \log E_t + B_{fs} \log FS_t + B_tt; \qquad (2.10)$$

where

<sup>24.</sup> Similar regressions are presented in Chapter 3 on the life cycle allocation of time. Becker and I recognize that simultaneous equation estimates would have been preferable, but feel strongly that the approach taken here is a useful first step.

 $X_t$  = mean family consumption of goods at age t of the head;

- $E_t$  = mean family earnings at age t;
- $FS_t$  = mean family size at age  $t.^{25}$

Regressions are weighted by the square root of cell size in order to reduce heteroscedasticity arising from differences in cell sizes within the sample.<sup>26</sup> Results are presented in Table 2.3 on the lines labeled X.

The coefficients of all the independent variables are positive in all cases. The *t* values for earnings are high: over 16.0 for all house-holds combined, and ranging from 7.7 to 13.0 within education groups.<sup>27</sup> For family size the *t* ratio is about 7.8 for the total sample, and ranges from about 3.0 to 6.0 within education groups. For age of head, the *t* value is 7.0 for the over-all sample and varies from 3.7 to 7.2 in the subsamples.

Estimates obtained using the BLS category "expenditures on current consumption" (*ECC*) as the dependent variable are also shown in Table 2.3. *ECC* includes the purchase of all durable goods except dwellings, and underestimates the value of owned housing.<sup>28</sup> It is therefore a hybrid, closer in spirit to a measure of expenditure than to a measure of use of goods. The single most important difference between the *ECC* series and X stems from the treatment of durable goods. Direct evidence shows that the elasticity of purchases of major durable goods with respect to earnings exceeds the earnings elasticity of demand for many nondurables.<sup>29</sup> The coefficients I obtained for earnings using *ECC* are positive and slightly larger than those obtained using my constructed measure of consumption (X) (except in the college group). Moreover, the age coefficients are

29. Using the same basic double log format as in (2.10), I found the coefficient of earnings for all heads combined to be 0.83 for furnishings (net of insurance), 1.22 for automobile purchases, 0.69 for the purchase of durable recreation goods (radio, TV, musical instruments), but only 0.35 for food and 0.29 for adjusted rents. For clothing it is 1.01. All *t* values exceed 2.

<sup>25.</sup> On the use of arithmetic means instead of geometric ones, see the appendix to this chapter.

<sup>26.</sup> For a discussion of weighted regressions when the sample is stratified, see Lawrence Klein, A Textbook of Econometrics (Evanston, III.: Row, Peterson, 1953).

<sup>27.</sup> Standard errors have not been adjusted for possible nonindependence of the "time series."

<sup>28.</sup> For owners of dwellings the BLS survey records interest on mortgages, taxes, and repairs. This is an underestimate of the value of owned housing because mortgage interest is less than total interest forgone, and depreciation is not fully recorded.

smaller and consistently not significant when *ECC* is the dependent variable. The reason is that *ECC* understates the cost of home ownership, since it includes interest on mortgages, but not on equity. Since equity rises with age, *ECC* understates the cost of home ownership especially at older ages.

A nonzero coefficient for earnings is predicted by the theory developed in Chapter 1. An increase in the price of time raises the demand for goods relative to time and reduces the demand for future goods relative to present ones. The coefficient for the price of time would be zero, sampling and measurement errors aside, only in the

TABLE 2.3 REGRESSIONS FOR CONSUMPTION OF GOODS: ALTERNATIVE MEASURES

|                       |        | OF                  | CONSUM                          | PTION                           |                   |       |        |
|-----------------------|--------|---------------------|---------------------------------|---------------------------------|-------------------|-------|--------|
| Dependent             | Inter- | Indepe<br>(t value  | Mult.                           |                                 |                   |       |        |
| Variable<br>(in logs) |        |                     | Corr.<br>Coeff.                 | Adj.<br><i>R</i> ²              | Durbin-<br>Watson |       |        |
|                       |        | All Educat          | ion Levels                      | ; Ages 22-                      | ·65               |       |        |
| x                     | 3.4835 | 0.5253<br>(16.3577) | 0.2593<br>(7.7945)              | 0.0035<br>(7.2497)              | .9904             | .9794 | 1.7386 |
| ECC                   | 3.4388 | 0.5580<br>(16.2703) | 0.2450<br>(6.8961)              | 0.0000<br>(0.0017)              | .9909             | .9804 | 1.6328 |
|                       |        | Grade               | School; A                       | ges 22-65                       |                   |       |        |
| x                     | 3.6870 | 0.4859<br>(13.2405) | 0.2586<br>(5.9641)              | 0.0038<br>(3.7202)              | .9717             | .9401 | 2.3476 |
| ECC                   | 3.7020 | 0.5105<br>(12.0483) | 0.2612 (5.2166)                 | 0.0006 (0.5110)                 | .9727             | .9422 | 2.7890 |
|                       | 1      | High                | School; Ag                      | jes 22-65                       |                   |       |        |
| x                     | 4.2127 | 0.4219<br>(7.7016)  | 0.2932 (5.5886)                 | 0.0071<br>(7.1918)              | .9577             | .9109 | 2.0118 |
| ECC                   | 4.0093 | 0.4748<br>(9.0884)  | 0.2789 <sup>´</sup><br>(5.5751) | 0.0031 <sup>′</sup><br>(3.2708) | .9592             | .9140 | 2.1920 |
|                       |        | Coll                | ege; Ages                       | 22-65                           |                   |       |        |
| x                     | 2.9662 | 0.6001<br>(9.8524)  | 0.1746<br>(2.9582)              | 0.0051<br>(3.8379)              | .9659             | .9279 | 2.2035 |
| ECC                   | 3.3090 | 0.5889<br>(10.4940) | 0.1592 (2.9283)                 | 0.0011<br>(0.8740)              | .9620             | .9199 | 2.1413 |

NOTE: X = family consumption (see text for items included), ECC = BLS concept of expenditures on current consumption, E = family earnings, FS = family size.

singular case in which substitution in consumption was equal in magnitude to substitution in production. If we interpret family earnings as a measure of the price of time, the positive coefficients for earnings are consistent with the hypothesis that substitution in production is easier than in consumption.<sup>30</sup> The coefficients for earnings are in fact quite large. When all heads are combined it is equal to about 0.53: a 10 per cent rise in family earnings raises the demand for goods by more than 5 per cent.<sup>31</sup>

The test against the null hypothesis that earnings have no effect on consumption over the life cycle is a test against an alternative model of consumption behavior. Under that hypothesis goods provide utility directly, rather than through the production of commodities. Moreover, the allocation of time between work and other activities is determined exogenously, rather than within the model. With a lifetime horizon, perfect capital markets, and no unexpected changes in wealth, the consumption of each household will be the same at all ages if the rate of interest net of time preference is zero and family size is constant. Consumption rises with age if the rate of interest net of time preference is positive or if family size is rising. The rise or fall of consumption will be independent of the rise or fall of earnings. The earnings stream together with interest rates would determine the household's wealth and thus the level of the consumption stream, but not the rising or falling of consumption with age. If we assumed that income expectations were unbiased, that model would predict that changes in cohort consumption would be independent of changes in earnings with age. In other words, it predicts that  $B_e = 0$  in equation (2.10), sampling and measurement errors

<sup>30.</sup> In the appendix to this chapter, I show that  $B_e \gtrless 0$  as  $\sigma_f \gtrless \sigma_c$ . Strictly speaking, the double logarithmic format of equation (2.10) is not completely appropriate when earnings are used as a measure of the price of time, because the elasticity of earnings with respect to the wage rate is not constant.

<sup>31.</sup> These coefficients are smaller than the usual estimates of the elasticity of consumption with respect to permanent income, but somewhat larger than those of the elasticity of consumption with respect to transitory income. See for instance, Ronald Bodkin, "Windfall Income and Consumption" and Milton Friedman, Comments, in Irving Friend and Robert Jones, eds., *Proceedings of Conference on Consumption and Saving*, vol. 2 (Philadelphia, University of Pennsylvania Press, 1960), pp. 175–187 and 191–212. Also see Jacob Mincer, "Employment and Consumption," *Review of Economics and Statistics* (February 1960), pp. 20–26.

aside. This hypothesis is close in spirit to the Modigliani-Brumberg model of consumption planning,<sup>32</sup> if to their lifetime hypothesis we append the assumption that income expectations are unbiased.

The regressions reported in Table 2.3 provide strong evidence against the simple alternative hypothesis. The coefficient for earnings is well above zero, and the estimates are statistically significant.

Family size exerts a positive effect on consumption. For all heads combined the coefficient of family size is about 0.26: a doubling of family size raises consumption by a little more than 25 per cent. The effect of family size on consumption is in the same direction as the effect of earnings, but it is considerably weaker. A 5 per cent rise in consumption is elicited by a 10 per cent rise in earnings or by a 20 per cent rise in family size over the life cycle.

Family size is included in the regressions to control for some factors which influence the demand for goods (and time).<sup>33</sup> An increase in the number of children per family would raise the demand for the wife's time. To increase her time in the production of child services she would reduce her time spent at work. Eventually, with a sufficiently large family, she would cease working altogether. She would meet any further increase in family size by reducing her time in other, presumably less time-intensive, home activities. Predictions about the effect of changes in family size on goods consumed by the household are not so clear-cut. An increase in the number of children would raise the demand for goods used in other nonmarket activities, but it would raise the demand for current ones, as long as older children were more goods-intensive than

<sup>32.</sup> See the references listed in Chapter 1, note 25. See also Franco Modigliani and Albert Ando, "Tests of the Life Cycle Hypothesis of Saving," *Bulletin of the Oxford University Institute of Statistics* (May 1957), p. 105; and Modigliani and Ando, "The Life Cycle Hypothesis of Saving," *American Economic Review* (March 1963), p. 56.

<sup>33.</sup> A nonzero coefficient for family size is consistent with a wide variety of naive models. If utility depended on goods per unit of family size (or better, per unit of family equivalent), the effect of an increase in family size on goods over the life cycle would be positive or negative according as the elasticity of substitution in consumption was smaller than or greater than unity. Another naive decision rule would be that changes in goods per unit family size depend only on changes in earnings per unit family over the life cycle. This hypothesis is rejected, since the sum of the coefficients for earnings and for family size,  $B_e + B_{fs}$ , differs significantly from unity.

young ones. In principle, therefore, a control for family age composition as well as size would have been preferable.<sup>34</sup> Moreover, if births and their timing were endogenous, the demand for goods (and time) and the demand for children would be simultaneously determined. A full system to explain all these remains a subject for future research.<sup>35</sup>

The coefficient for age is 0.0035 when all households are combined: in the absence of differences in earnings or in family size, consumption would rise with age in the cross section at a rate of less than one-half of 1 per cent per year of age. Age is a variable often included in regression analyses of consumption behavior. Sometimes the only interpretation offered for the observed effect of age is that tastes may shift with age. More often it is discussed in relation to the changes in income expectations and family size that may occur over the life cycle, and to the importance of credit ceilings.<sup>36</sup> In the context of life cycle behavior, the interpretation of the coefficient of age is clear: it measures the combined effect of interest rates net of time preference and of trends in productivity.

Over all, the basic model seems consistent with the data. The regressions for each level of education are not very different from the over-all regression. Moreover, the  $R^2$  are high, and there is no evidence of serial correlation of the residuals as measured by the Durbin-Watson *d* statistic.

In order to test the robustness of the findings, I ran the same regressions in first-difference form:

$$\Delta \log X_t = B_e \Delta \log E_t + B_{fs} \Delta \log FS_t + B_t.$$
(2.11)

The estimates are reported on the first line of Table 2.9, below. Again, the coefficients of all the variables are found to be positive and of the

<sup>34.</sup> See, for instance, Michael Lansberger, "An Integrated Model of Consumption and Market Activity: The Children Effect" (mimeo., 1971). At any rate, instrumental variables estimation techniques would have been more appropriate than ordinary least squares, to account for the simultaneous determination of consumption and family size.

<sup>35.</sup> For instance, the timing of children is not independent of the timing of marriage.

<sup>36.</sup> See the rather brief discussions of the effects of age in Milton Friedman, A Theory of the Consumption Function (Princeton, N.J.: Princeton University Press for NBER, 1957) and Harold W. Watts, "Long-Run Income Expectations and Consumer Saving," in T. Dernberg et al., eds., Studies in Household Economic Behavior, vol. 9 (New Haven: Yale University Press, 1958).

same order of magnitude as in the level equations of (2.10). However, the *t* values are somewhat reduced.

If interest rates or time preference varied systematically with age, or if trends in productivity were not constant, age would not operate linearly on consumption. To test against this alternative, I introduced the square of age into the level regressions of equation (2.10). In Table 2.4, it can be seen that in all samples except that for collegeeducated heads, the coefficient of age squared is zero.

One effect of the introduction of age squared is to reduce the size of the coefficients for earnings. This suggests that a systematic relation may exist. Mincer has shown that earnings are well explained by years of schooling and a quadratic function of experience.<sup>37</sup> And indeed if years of experience <sup>38</sup> and its square are used, rather than age and its square, to explain consumption, the coefficients for earnings are reduced slightly more (see Table 2.4).

#### 2.6 FURTHER TESTS

In the previous section I showed that earnings exert a positive effect on consumption over the life cycle. This result is consistent with the model of consumption developed in Chapter 1. Yet this does not ensure that the model provides a complete description of behavior. Although I have produced high values of  $R^2$  with a limited set of variables, nevertheless, I may well have neglected some important determinants. Moreover, a positive relationship between consumption and earnings could be predicted under quite different hypotheses. A positive correlation between consumption and earnings is no proof that only substitution effects are at work over the life cycle. Indeed, in general, it may reflect both income and substitution effects and the appropriate task is to determine the relative strength of each. Consider the following model: <sup>39</sup>

$$X_t = b_0 + b_1 P_t + b_2 U_t + \epsilon_t; \qquad (2.12)$$

37. See Jacob Mincer, Schooling, Experience, and Earnings (New York: NBER, 1974).

<sup>38.</sup> Years of experience are defined as: Age  $-t_s$ , where  $t_s = 11$  years for the over-all sample, 7 years for the grade school group, 11 years for the high school group, and 15 years for the college group.

<sup>39.</sup> I am grateful to James Heckman for this formulation.

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#### TABLE 2.4

#### REGRESSIONS FOR CONSUMPTION OF GOODS: EFFECTS OF AGE AND POST-SCHOOL TRAINING (dependent variable is log of family consumption)

| Inter- | Inde                             | ependent V         | ariables | (t values in         | parenth            | eses)                | Mult.<br>Corr. | Adi.           | Durbin- |
|--------|----------------------------------|--------------------|----------|----------------------|--------------------|----------------------|----------------|----------------|---------|
| cept   | Log E                            | Log FS             | Age      | (Age) <sup>2</sup>   | Exp.               | (Exp.)²              | Coeff.         | R <sup>2</sup> | Watson  |
|        |                                  |                    | All Educ | ation Leve           | els; Ages          | 22-65                |                |                |         |
| 3.5207 | 0.5189<br>(10.1785)              | 0.2509<br>(4.1032) |          | -0.0000<br>(-0.1640) |                    |                      | .9904          | <u>,</u> 9789  | 1.7492  |
|        | All Education Levels; Ages 24–65 |                    |          |                      |                    |                      |                |                |         |
| 3.5682 | 0.5178<br>(9.8963)               | 0.2608<br>(3.9859) |          |                      |                    | -0.0000<br>(-0.1265) | .9894          | .9767          | 1.7633  |
|        |                                  |                    | Grad     | le School;           | Ages 22            | -65                  |                |                |         |
| 3.7148 | 0.4809<br>(10.8285)              | 0.2486<br>(3.7892) |          | -0.0000<br>(-0.2053) |                    |                      | .9718          | .9386          | 2.3277  |
|        |                                  |                    | Grad     | le School;           | Ages 20            | -65                  |                |                |         |
| 3.7503 | 0.4796<br>(11.0454)              | 0.2525<br>(3.9897) |          |                      |                    | -0.0000<br>(-0.2226) | .9724          | .9403          | 2.3474  |
|        |                                  |                    | Hig      | h School; /          | Ages 22–           | 65                   |                |                |         |
| 4.3760 | 0.3958<br>(4.6103)               | 0.2583<br>(2.5227) |          | -0.0001<br>(-0.3990) |                    |                      | .9578          | .9090          | 2.0478  |
|        |                                  |                    | Hig      | h.School;            | Ages 24-           | •65                  |                |                |         |
| 4.6027 | 0.3786<br>(4.2835)               | 0.2769<br>(2.5051) |          |                      |                    | -0.0001<br>(0.4817)  | .9502          | .8924          | 2.0678  |
|        |                                  |                    | с        | ollege; Ag           | es 22–65           | 5                    |                |                |         |
| 3.3282 | 0.4721                           | -0.0404            | 0.0588   | -0.0006              |                    |                      | .9708          | .9365          | 2.0963  |
|        | (6.1930)                         | (0.3992)           | (2.7734) | (-2.5389)            |                    |                      |                |                |         |
|        |                                  |                    | C        | ollege; Ag           | es 28-65           | 5                    |                |                |         |
| 3.9957 | 0.4699<br>(5.6297)               | 0.0159<br>(0.1282) |          | -                    | 0.0389<br>(2.5920) | -0.0006<br>(2.1849)  | .9460          | .8822          | 2.2341  |

NOTE: E = family earnings, FS = family size, Exp. = experience in the labor market.

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where

- $P_t$  = life cycle or permanent level of earnings appropriate to age group t;
- $U_t$  = deviation of measured earnings from their life cycle component, i.e., E = P + U;

 $\epsilon_t$  = disturbance term.

The relation as written in equation (2.10) is misspecified. To examine the bias, write equation (2.12) as

$$X_t = b_0 + b_1 E_t + (b_2 - b_1) U_t + \epsilon_t.$$
 (2.13)

The bias in the estimate of  $b_1$  from the omission of the "measurement error"  $U_t$  is  $E(\hat{b}_1) - b_1 = (b_2 - b_1)b_{eu}$ , where  $E(\hat{b}_1)$  is the expected value of the least squares estimate of  $b_1$ , and  $b_{eu}$  is the regression coefficient in a regression of the omitted variable U on the included variable E; in other words,

$$b_{eu} = \frac{\text{covariance } (E_t, U_t)}{\text{variance } (E_t)}$$

If we assume there to be no correlation in the sample between permanent levels  $P_t$  and "measurement error"  $U_t$ , then <sup>40</sup>

$$b_{eu} = \frac{\text{variance } (U_t)}{\text{variance } (P_t) + \text{variance } (U_t)}$$

If we approximate the numerator by the sampling variance of the mean for a typical age group, an estimate of  $b_{eu}$  can be obtained. Table 2.5 contains values of the variance of family earnings by selected years of age of the household head. This variance essentially rises with age and years of schooling. Using the variance at age 40 (which is close to the mean age in the sample), and the variance of mean earnings across age groups we get the estimates of  $b_{eu}$  in Table 2.6. On the face of it, the bias resulting from income effects is relatively small.

Regressions reported in Table 2.3 explained variations in consumption by variations in earnings, family size, and age. In order to treat the components of income more symmetrically, I introduce nonwage income, R, as an additional variable. It is computed as the dif-

<sup>40.</sup> In equation (2.12) and therefore in the expression for  $b_{eu}$ , all variables are in logarithms. The calculations of  $b_{eu}$  below take account of this fact.

| TABLE 2.5  |
|--|
| VARIANCE OF FAMILY EARNINGS BY SELECTED YEARS OF AGE |
| AND EDUCATION LEVEL OF THE HOUSEHOLD HEAD, 1960–61   |
| (millions of dollars)                                |

----

| Age of<br>Household |       | Educatio | on Level |       |
|---------------------|-------|----------|----------|-------|
| Head                | All   | 1        | II       | 111   |
| 25                  | 5.93  | 2.51     | 4.30     |       |
| 30                  | 13.92 | 7.07     | 9.28     | 18.88 |
| 35                  | 15.10 | 6.98     | 12.26    | 17.78 |
| 40                  | 18.76 | 10.42    | 12.23    | 23.60 |
| 45                  | 16.16 | 7.44     | 13.65    | 26.65 |
| 50                  | 24.37 | 10.16    | 16.11    | 39.46 |
| 55                  | 18.70 | 12.97    | 14.37    | 24.06 |
| 60                  | 27.71 | 13.54    | 16.90    | 64.66 |
| 65                  | 22.78 | 18.36    | 10.40    | 58.73 |

SOURCE: BLS Survey of Consumer Expenditures, 1960–61.

| TABLE 2.6                      |
|--------------------------------|
| COMPUTATION OF REGRESSION BIAS |

|  | Education Level |            |             |            |  |  |
|--|-----------------|------------|-------------|------------|--|--|
|  | All             | . 1        | II          | 111        |  |  |
| <ol> <li>Estimate of the variance<br/>of the log of earnings at<br/>age 40 <sup>a</sup></li> <li>Cell size at age 40</li> <li>Variance of the log of<br/>mean earnings across</li> </ol> | 0.37<br>337     | 0.53<br>88 | 0.25<br>169 | 0.23<br>80 |  |  |
| age groups   | 0.04            | 0.03       | 0.03        | 0.02       |  |  |
| b <sub>eu</sub>  | 0.03            | 0.17       | 0.05        | 0.10       |  |  |

NOTE:  $b_{eu} = (\text{line 1/line 2})/[\text{line 3} + (\text{line 1/line 2})].$ 

SOURCE: BLS Survey of Consumer Expenditures, 1960-61.

a. This estimate is a linear approximation (around mean earnings at age 40) to the variance of the logarithm of earnings at age 40: it is equal to the square of the coefficient of variation of earnings at age 40. For this computation, I used the variances given in Table 2.5.

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ference between total family income, Y, and family earnings, E, and is entered in the regression in logarithmic form. Results are given in Table 2.8, below (first line). Nonwage income exerts a positive effect on consumption over the life cycle. The *t* ratio is 3.8 when all households are combined, and ranges from 1.9 to 2.9 within education classes. In all cases the size of the coefficient is relatively small. It is about 0.066 for all households combined and is of the same order of magnitude within each education group: a 10 per cent rise in nonwage income raises family consumption by approximately two-thirds of 1 per cent.

If cohort expectations about nonwage income were unbiased, savings would be undertaken simply to make the consumption program feasible. After some initial period of indebtedness, assets would rise, reaching a peak well beyond the peak earnings age, and then contract as the household retires. The general life cycle path of assets is given in Table 2.7, as it appeared in a 1962 survey. The decline in assets sets in after age 65.

Nonwage income can be computed with the BLS data as the difference between income and earnings. One component is the yield on transferable assets; with a fixed rate of interest this component would be proportional to assets themselves. Other components are alimony, social security, and pension payments, much of which comes

| BY AGE OF HEAD, DECEMBER 31, 1962 |                |  |  |  |  |  |
|-----------------------------------|----------------|--|--|--|--|--|
| Age of Head                       | Mean Net Worth |  |  |  |  |  |
| Under 25                          | \$ 557         |  |  |  |  |  |
| 25-34                             | 4,831          |  |  |  |  |  |
| 35-44                             | 14,792         |  |  |  |  |  |

22.237

32,511

30,124

45-54

55 - 64

65 and over

TABLE 2.7 MEAN NET WORTH OF CONSUMER UNITS BY AGE OF HEAD, DECEMBER 31, 1962

SOURCE: Dorothy S. Projector, *Survey of Changes in Family Finances* (Board of Governors of the Federal Reserve System, 1968), Table S 17.

late in life. The over-all pattern of nonwage income in the BLS survey, as in other surveys,<sup>41</sup> is that it rises primarily with age.

If expectations about nonwage income were continuously fulfilled, the theory predicts that nonwage income would have no effect on consumption over the life cycle, sampling and measurement errors aside. A positive effect of nonwage income on consumption over the life cycle could be interpreted as resulting from incorrect expectations about nonwage income.

The coefficients for nonwage income in Table 2.8 (first line) are positive, but about one-tenth the size of those for earnings.<sup>42</sup> The modest size of the coefficients is consistent with the emphasis of my model on the greater importance of variations in earnings compared to variations in nonwage income in explaining life cycle consumption, which is based on the observation that the former give rise to substitution effects, while the latter do not. The positive signs of the coefficients of nonwage income lend some credibility to the notion that future income is not perfectly predicted. It is also noteworthy that the coefficients of age and the *t* values are reduced when nonwage income is included in the regression, owing to the positive correlation of the latter with age.

Another, quite different, interpretation of the positive coefficient for nonwage income is that households cannot borrow and lend at fixed rates of interest, but that the cost of transferring income over time largely depends on the household's net indebtedness.<sup>43</sup> An extreme version of credit rationing is one in which consumption is entirely constrained by current income. In order to test against this hypothesis, I calculated the regression of consumption on total income, nonwage income, family size, and age (all variables in logs except age). Results are presented in Table 2.8 (second line for each category). Total income has a positive effect on consumption: the coefficient is about 0.72 when all households are combined, and

<sup>41.</sup> This is true for instance in the 1/1,000 sample of U.S. population, 1960, and in the Survey of Economic Opportunity, 1967.

<sup>42.</sup> Becker also finds positive coefficients for other income in his regressions for male time, but his are smaller in magnitude (see Table 3.1, below). Barring measurement errors, these coefficients should be of the same size given constant returns to scale in production.

<sup>43.</sup> A discussion of this hypothesis is given in Gilbert R. Ghez, "Life Cycle Consumption in the Presence of Segmentation in the Capital Market" (unpublished, 1968).

# TABLE 2.8 REGRESSIONS FOR CONSUMPTION OF GOODS: EFFECTS OF NONWAGE INCOME (dependent variable is log of family consumption)

| Inter-                           | Independ            | lent Variabl                     | es (t value         | s in paren                     | theses)                        | Mult.<br>Corr. | Adj.           | Durbin- |  |  |
|----------------------------------|---------------------|----------------------------------|---------------------|--------------------------------|--------------------------------|----------------|----------------|---------|--|--|
| cept                             | Log E               | Log R                            | Log Y               | Log FS                         | Age                            | Coeff.         | R <sup>2</sup> | Watson  |  |  |
|                                  |                     | All E                            | ducation l          | .evels; Aq                     | ges 22–65                      |                |                |         |  |  |
| 2.0967                           | 0.5528<br>(19.2407) | 0.0656<br>(3.7914)               |                     | 0.2489<br>(8.6043)             | 0.0018<br>(3.0834)             | .9930          | .9845          | 2.0443  |  |  |
| 1.9663                           | (10.2401)           | -0.0027<br>(0.1570)              | 0.7157<br>(18.4428) | 0.1916 (5.8391)                | 0.0008 (1.2199)                | .9924          | .9833          | 1.9414  |  |  |
| All Education Levels; Ages 35–65 |                     |                                  |                     |                                |                                |                |                |         |  |  |
| 2.7970                           | 0.5267              | 0.0626                           |                     | 0.3886                         | 0.0057                         | .9948          | .9881          | 2.7354  |  |  |
| 2.0228                           | (17.1997)           | (2.9391)<br>-0.0145<br>(-0.7122) | 0.6753<br>(16.5907) | (6.8324)<br>0.3377<br>(5.5320) | (3.6753)<br>0.0049<br>(3.0286) | .9945          | .9873          | 2.6631  |  |  |
|                                  |                     | , ,                              | Grade Sch           | · · ·                          | . ,                            |                |                |         |  |  |
| 3.2739                           | 0.5106<br>(13.6623) | 0.0455<br>(2.0314)               |                     | 0.2458 (5.8192)                | 0.0025<br>(2.1385)             | .9745          | .9444          | 2.3735  |  |  |
| 2.5882                           | (                   | -0.0130<br>(-0.6225)             | 0.6310<br>(14.1268) | 0.2373<br>(5.7356)             | 0.0020<br>(1.7157)             | .9759          | .9474          | 2.2835  |  |  |
|                                  |                     | (                                | Grade Sch           | ool; Ages                      | 35-65                          |                |                |         |  |  |
| 3.2612                           | 0.4945              | 0.0553                           |                     | 0.2846                         | 0.0033                         | .9732          | .9389          | 2.4465  |  |  |
| 2.5977                           | (10.5647)           | (1.7835)<br>0.0147<br>(0.4982)   | 0.6218<br>(10.7868) | (3.5241)<br>0.2666<br>(3.3244) | (1.2827)<br>0.0029<br>(1.1377) | .9741          | .9409          | 2.3364  |  |  |
|                                  |                     |                                  | High Scho           | ol; Ages                       | 22-65                          |                |                |         |  |  |
| 3.3693                           | 0.4812              | 0.0677                           |                     | 0.2847                         | 0.0050                         | .9637          | .9215          | 1.8205  |  |  |
| 2.0412                           | (8.5113)            | (2.5258)<br>0.0049<br>(0.2152)   | 0.6921<br>(9.5792)  | (5.7658)<br>0.2095<br>(4.1441) | (3.9916)<br>0.0029<br>(2.2005) | .9692          | .9331          | 1.7918  |  |  |
|                                  |                     |                                  | High Scho           | ol; Ages                       | 35-65                          |                |                |         |  |  |
| 2.3956                           | 0.5370              | 0.1057                           | -                   | 0.3857                         | 0.0075                         | .9346          | .8539          | 1.8002  |  |  |
| 0.5297                           | (7.7549)            | (3.0787)<br>0.0423<br>(1.7134)   | 0.8103<br>(9.9672)  | (3.9677)<br>0.3038<br>(3.6588) | (2.6665)<br>0.0051<br>(2.1443) | .9555          | .8996          | 1.8375  |  |  |
|                                  |                     | ·                                | College             | ; Ages 22                      | -65                            |                |                |         |  |  |
| 2.6320                           | 0.6068<br>(10.5800) | 0.0542<br>(2.4925)               |                     | 0.1680<br>(3.0228)             | 0.0031<br>(2.1120)             | .9706          | .9362          | 2.3635  |  |  |
| 2.1255                           |                     | -0.0119<br>(-0.4985)             | 0.7087<br>(9.7013)  | 0.1599                         | 0.0022 (1.3385)                | .9666          | .9277          | 2.2982  |  |  |
|                                  |                     |                                  | College             | ; Ages 35                      | -65                            |                |                |         |  |  |
| 2.9065                           | 0.6013<br>(8.9805)  | 0.0521<br>(1.4604)               |                     | 0.1034<br>(0.7962)             | 0.0005<br>(0.1044)             | .9127          | .8074          | 2.2430  |  |  |
| 2.3753                           | (0.3000)            | -0.0355<br>(-0.9243)             | 0.7146<br>(8.4318)  | 0.1117<br>(0.8207)             | 0.0006<br>(0.1319)             | .9037          | .7884          | 2.2579  |  |  |

NOTE: E = family earnings, R = family nonwage income, Y = family income, FS = family size.

ranges from 0.63 to 0.71 within education classes; all these coefficients have high *t* values. Nonwage income has a very slight, usually negative, effect on consumption (when holding total income constant), but none of the estimates is statistically different from zero. The same results appear in the first-difference regressions given in Table 2.9.

| TABLE | 2.9 |
|-------|-----|
|-------|-----|

REGRESSIONS FOR CONSUMPTION OF GOODS: FIRST-DIFFERENCE EQUATIONS (dependent variable is  $\Delta$  log-family consumption)

| 1              |                     | Independer<br>(t values in p | nt Variables<br>parentheses |                      | Mult.<br>Corr.  | A                  | Durbin- |
|----------------|---------------------|------------------------------|-----------------------------|----------------------|-----------------|--------------------|---------|
| Inter-<br>cept | ΔLog E              | Δ Log R                      | Δ Log Y                     | Δ Log FS             | Corr.<br>Coeff. | Adj.<br><i>R</i> ² | Watson  |
|                |                     | All Educ                     | ation Level                 | s; Ages 22-          | 65              |                    |         |
| .0045          | 0.5377<br>(9.4550)  | •                            |                             | 0.2675<br>(3.1582)   | .8882           | .7783              | 3.0556  |
| .0028          | 0.5608<br>(10.4666) | 0.0507<br>(2.6925)           |                             | 0.2698<br>(3.4253)   | .9066           | .8083              | 3.0747  |
| .0011          |                     | -0.0061<br>(-0.3261)         | 0.6423<br>(0.4250)          | 0.2515<br>(3.1539)   | .9061           | .8072              | 3.0631  |
|                |                     | Grad                         | le School;                  | Ages 22–65           |                 |                    |         |
| .0026          | 0.4642<br>(8.6485)  |                              |                             | 0.1815<br>(2.3061)   | .8406           | .6919              | 2.8512  |
| .0003          | 0.4769<br>(9.1396)  | 0.0454<br>(1.9908)           |                             | 0.1759<br>(2.3138)   | .8565           | .7131              | 2.7757  |
| 0010           |                     | -0.0011<br>(-0.0495)         | 0.5685<br>(10.0180)         | 0.1687<br>(2.3653)   | .8751           | .7478              | 2.7063  |
|                |                     | Hig                          | h School; A                 | ges 22–65            |                 |                    |         |
| .0098          | 0.4041<br>(4.2662)  |                              |                             | 0.3464<br>(2.7413)   | .6771           | .4314              | 2.5312  |
| .0072          | 0.4076<br>(4.9316)  | 0.0889<br>(3.6782)           |                             | 0.3946<br>(3.5533)   | .7732           | .5670              | 2.4095  |
| .0044          |                     | 0.0343<br>(1.4495)           | 0.5794<br>(6.1248)          | 0.3290<br>(3.1861)   | .8169           | .6417              | 2.4700  |
|                |                     | с                            | ollege; Age                 | s 22-65              |                 |                    |         |
| .0074          | 0.5495<br>(6.7195)  |                              |                             | -0.1262<br>(-1.1860) | .7308           | .5108              | 2.9477  |
| .0057          | 0.5739<br>(7.1951)  | 0.0433<br>(2.0113)           |                             | -0.1353<br>(-1.3178) | .7602           | .5454              | 2.9197  |
| .0052          |                     | -0.0210<br>(-0.8846)         | 0.6477<br>(6.2977)          | -0.1172<br>(-1.0639) | .7162           | .4754              | 2.9722  |

NOTE:  $\Delta$  = first-difference operator taken over adjacent years of age, E = family earnings, R = family nonwage income, Y = family income, FS = family size.

According to the theory developed in Chapter 1, variations in consumption do not depend on variations in nonwage income over the life cycle. The coefficient for total income measures the effect of changes in earnings on changes in consumption. On the other hand, with total income held constant a rise in nonwage income must be accompanied by a fall in earnings. Hence the model in Chapter 1 would predict that with total income held constant the coefficient for nonwage income should have a sign opposite to that of total income. Moreover, since the regressions are logarithmic, the coefficient for nonwage income should equal in absolute value the product of the elasticity of consumption with respect to earnings multiplied by the ratio of nonwage income to earnings. In other words, the absolute value of the ratio of the coefficient for nonwage income to that of total income should equal the ratio of nonwage income to total income. While the prediction on the relative signs of the coefficients is borne out in the regressions, the prediction about their relative magnitude is not, since nonwage income accounts for about 25 per cent of total income.44

The absolute income hypothesis seems to explain this body of data remarkably well. It is well known, however, that this hypothesis has been rejected on many grounds and with much evidence.<sup>45</sup> In particular, it fails to reconcile the secular stability of the savings ratio with the declining average propensity to consume observed in cross sections.

One possible interpretation is that nonwage income is poorly measured. This would bias the coefficient of nonwage income downward and that of total income upward.<sup>46</sup> Another interpretation is that if credit rationing exists, it must surely operate more severely for borrowers than for lenders. It might be expected that if the years of age in which households are heavy borrowers are excluded, the pre-

<sup>44.</sup> This method of testing is by no means accurate, since the expected value of the ratio of two parameter estimates is not equal to the ratio of the expected values of these estimates. Approximate tests would have been more appropriate.

<sup>45.</sup> To name but a few pieces in this voluminous literature, see Friedman, *Theory* of the Consumption Function; Hendrik H. Houthakker, "The Permanent Income Hypothesis," *American Economic Review* (June 1958); Robert Eisner, Comment, *American Economic Review* (December 1958); Michael K. Evans, "The Importance of Wealth in the Consumption Function," *Journal of Political Economy* (August 1967), pp. 335–351.

<sup>46.</sup> One may also suggest that the BLS in its attempt to reconcile expenditures and incomes introduces a systematic positive association between consumption and income.

dictions of the theory developed in Chapter 1 would be more clearly borne out. For this purpose, I ran regressions similar to those in Table 2.8 (second line) but including only households in which the head was at least 35 years old. The results are also given in Table 2.8. They are somewhat better than the results obtained with the wider sample: the coefficients for nonwage income, still negative, are slightly increased in absolute value, both for all households combined and within education groups. Moreover, the significance levels of these estimates is slightly increased (except in the case of high school heads).

## 2.7 SUMMARY

In this chapter, an attempt was made to provide orders of magnitude of the responsiveness of the consumption of goods to its determinants over the life cycle. Using the Bureau of Labor Statistics Survey of Consumer Expenditures for 1960–61, the following main conclusions were drawn:

i. Consumption responds positively to earnings over the life cycle. A 10 per cent rise in earnings raises consumption by about 5 per cent.

ii. This positive response can result from three main sources: (a) substitution effects as described in Chapter 1, (b) income effects resulting from incorrect income and wage rate expectations, and (c) income effects resulting from credit rationing. While income effects are present in the estimates, they by no means account fully for the positive response of consumption to earnings. In other words, the substitution effects resulting from lifetime changes in the wage rate do play a role in determining life cycle consumption.

iii. Increases in family size tend to raise consumption, a finding consistent with that of many other studies.

iv. The consumption profile has a positive trend in the cross section. Put differently, age of head has an independent effect on consumption. This effect is the combined result of interest rate plus time preference effects and time series trends in productivity that are captured by drawing observations from different cohorts in a cross section.

v. In sum, from a relatively complicated model a rather simple

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estimating equation was developed. Results obtained when the equation was applied to observed cross-cohort consumption behavior are essentially consistent with the theory. Other interpretations are possible, but many of these lack a theoretical basis. The general theoretical framework, which stresses the importance of time in the home, is capable of generating many other hypotheses which also appear to be supported by existing evidence (see the references given in the Introduction to this volume). It is the wide applicability and broad explanatory power of the framework that is encouraging evidence of its usefulness. In Chapter 3 Becker provides yet another piece of evidence related to the life cycle model—the allocation of hours worked by men over their lifetime. In Chapter 4, the estimates obtained are used to interpret still other bodies of data.

#### APPENDIX

In this appendix, I examine the possible biases arising from (1) nonconstant factor shares, (2) the use of arithmetic means rather than geometric ones, (3) the use of earnings rather than wage rates as the price of time.

For simplicity of presentation, I assume that the family is composed of only one earner. I assume also that elasticities of substitution in consumption and in production are constant, and that technological change in the household is factor-neutral.

#### 1 ON THE CONSTANCY OF FACTOR SHARES

For any given individual the change in demand for goods and time at age t are 47

$$\tilde{X}_{t} = \hat{W}_{t} - (1 - \sigma_{c})\hat{P}_{t} + (\sigma_{f} - \sigma_{c})s\tilde{w}_{t} - (1 - \sigma_{c})\tilde{F}_{t} + \sigma_{c}(r - \rho); \quad (A2.1)$$
$$\tilde{L}_{t} = \hat{W}_{t} - (1 - \sigma_{c})\hat{P}_{t} - [\sigma_{f}(1 - s) + \sigma_{c}s]\tilde{w}_{t} - (1 - \sigma_{c})\tilde{F}_{t} + \sigma_{c}(r - \rho); \quad (A2.2)$$

where s is the share of time in the cost of commodities, and other variables are defined in the text. With factor-neutral technological changes in the household, the share s would rise, fall, or remain the same as the wage rate rose, depending on whether the elasticity of substitution in production was smaller than, greater than, or equal to unity. Suppose we approximated

<sup>47.</sup> All prices and incomes are in terms of goods. Asterisks were used in the text to distinguish them, but are omitted here to simplify the notation.

the share of time by 48

$$\mathbf{s}_t = \Phi_1 + \Phi_2 \log w_t, \tag{A2.3}$$

$$s_t \tilde{w}_t \cong s_t \frac{d \log w_t}{dt} = \Phi_1 \left( \frac{d \log w}{dt} \right) + \Phi_2 \log w \left( \frac{d \log w}{dt} \right),$$
 (A2.4)

its integral is

$$\Phi_1(\log w_l) + \frac{\Phi_2}{2} (\log w_l)^2 + \Phi_0. \tag{A2.5}$$

Let  $\overline{X}$  denote the geometric mean of the variable x. If we obtain the regression of the mean of consumption by age on age and on the mean wage rate by age, as in log  $\overline{X}_t = b_0 + b_w \log \overline{w}_t + b_t t$ , we would be omitting the quadratic term

$$\frac{1}{n_t}\sum_{i=1}^{n_t} (\log w_t)^2 \equiv Q_t,$$

where the summation runs over the  $n_t$  individuals of age t.<sup>49</sup> The bias in the estimate of  $b_w$  would be equal to the product of  $\Phi_2/2$  and the regression coefficient of the linear term log  $\overline{\overline{w}}$  in a regression of  $Q_t$  on log  $\overline{\overline{w}}_t$  and t. Since this regression coefficient is bound to be positive and

$$\Phi_2 \stackrel{\geq}{\underset{\sim}{\underset{\sim}{\underset{\sim}{\underset{\sim}}}}} 0 \text{ as } \sigma_f \stackrel{\leq}{\underset{\sim}{\underset{\sim}{\underset{\sim}{\underset{\sim}}}}} 1,$$

48. This is a first-order approximation to the share given by the production function with constant elasticity of substitution. Indeed, when

$$C = (\delta_1 L^{(\sigma-1)/\sigma} + \delta_x X^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)},$$

with  $\delta_1 + \delta_x = 1$ , the share of time is

$$\mathbf{s} = \frac{\delta_1^{\sigma} \mathbf{w}^{1-\sigma}}{d_1^{\sigma} \mathbf{w}^{1-\sigma} + \delta_x}.$$

A Taylor expansion of s around  $\sigma = 1$ , dropping second- and higher-order terms, is

 $s = \delta_1 + \delta_1 (\log \delta_1 - \delta_1 \log \delta_1 - \delta_x \log \delta_x)(\sigma - 1) - \delta_1 (1 - \delta_1) (\log w)(\sigma - 1),$ 

or  $s = \Phi_1 + \Phi_2 \log w$ , with

$$\begin{split} \Phi_1 &= \delta_1 + \delta_1(\sigma - 1)(\log \delta_1 - \delta_1 \log \delta_1 - \delta_x \log \delta_x); \\ \Phi_2 &= \delta_1(1 - \delta_1)(1 - \sigma), \end{split}$$

so that  $\Phi_1$  tends to  $\delta_a$  and  $\Phi_2$  tends to zero as  $\sigma$  tends to unity.

49. There is no bias from omission of a wealth variable, given the expectation model developed in section 2.2. Moreover, there is no bias from omission of a variable for nonmarket productivity, since technological change in the household sector is assumed to be disembodied.

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the least squares estimate of  $b_w$  would be biased upward, downward, or not biased according as  $\sigma_f$  is less than, more than, or equal to 1.

#### 2 ON THE USE OF ARITHMETIC MEANS

Suppose that factor shares were constant. We would then have the exact expressions appropriate for the cross section:

$$\log \overline{X}_t = b_0 + b_w \log \overline{\overline{w}}_t + b_t t; \qquad (A2.6)$$

$$\log \overline{\overline{L}}_t = a_0 + a_w \log \overline{\overline{w}}_t + a_t t; \qquad (A2.7)$$

with

$$b_w = (\sigma_f - \sigma_c)s;$$
  

$$a_w = -[\sigma_c s + \sigma_f(1 - s)];$$
  

$$b_t = a_t = \sigma_c(r - \rho) - [1 - s + \sigma_c s]g_w - (1 - \sigma_c)g_f.$$

I shall suppose that the elasticities  $b_w$ ,  $a_w$ ,  $b_t$ , and  $a_t$  are unchanged if we use arithmetic means rather than geometric ones; the only effect of the change of variable is to change the value of the intercept. We can then in principle write (leaving out the disturbance terms):

$$\log \overline{X}_t = b'_0 + b_w \log \overline{w}_t + b_t t; \qquad (A2.8)$$

$$\log \overline{L}_t = a_0' + a_w \log \overline{w}_t + a_t t; \qquad (A2.9)$$

where  $\overline{x}$  is the arithmetic mean of the variable x. The validity of this substitution has been given elsewhere.<sup>50</sup> In brief, if any variable x is log-normally distributed, then the difference between its mean logarithm and the logarithm of its mean is equal to (minus) one-half the variance of its logarithm:

$$\log \bar{\bar{x}} = \frac{1}{n} \sum \log x = \log \bar{x} - \frac{1}{2} \operatorname{var} (\log x), \qquad (A2.10)$$

where  $\bar{x}$  is the arithmetic mean and var denotes variance. Therefore, on the assumption that at any given year of age wage rates, real wealth, and goods are log-normally distributed, and that the variances of the logarithms of these variables are constant, the intercept  $b'_0$  would also be constant, and would be related to  $b_0$  in the following way:

$$b'_0 = b_0 + \frac{1}{2} \operatorname{var} (\log X; t) - \frac{1}{2} b_w \operatorname{var} (\log w; t).$$
 (A2.11)

If consumption time were also log-normally distributed, the intercept  $a'_0$  would be constant and related to  $a_0$  as follows:

$$a'_0 = a_0 + \frac{1}{2} \operatorname{var} (\log L; t) - \frac{1}{2} a_w \operatorname{var} (\log w; t).$$
 (A2.12)

<sup>50.</sup> See J. Aitchison and J. A. C. Brown, *The Lognormal Distribution* (Cambridge, Engl.: Cambridge University Press, 1963).

#### 3 ON THE USE OF EARNINGS AS THE PRICE OF TIME

I now show how the coefficients are affected if earnings rather than wages are used as a measure of the price of time.

By definition earnings per period, E, are the product of wage rates and hours of work, N. If no investment in human capital were made at a given age and if no other time were "lost," hours of work at that age would be the mere image of hours spent in consumption:

$$E_t = w_t N_t = w_t (\theta - L_t). \tag{A2.13}$$

From the definition of a covariance, mean earnings at age t,  $\overline{E}_t$ , are

$$\overline{E}_t = \operatorname{cov}(w_t, N_t; t) + \overline{w}_t \overline{N}_t; \qquad (A2.14)$$

where cov (w, N; t) is the covariance between wage rates and hours of work at age t. I shall suppose that this covariance is the same at all ages. The cross-sectional difference between mean earnings of households at age t + 1 of the head and at age t is then

$$\overline{E}_{t+1} - \overline{E}_t = \overline{w}_t (\overline{N}_{t+1} - \overline{N}_t) + \overline{N}_t (\overline{w}_{t+1} - \overline{w}_t);$$

and the percentage difference in mean earnings by age is

$$\dot{E}_{t} = \left(\frac{\bar{w}_{t}\bar{N}_{t}}{\bar{E}_{t}}\right)\dot{N}_{t} + \left(\frac{\bar{w}_{t}\bar{N}_{t}}{\bar{E}_{t}}\right)\dot{w}_{t}; \qquad (A2.15)$$

where  $\dot{x}_t = (\bar{x}_{t+1} - \bar{x}_t)/\bar{x}_t$  in the cross section. But since  $N = \theta - L$ , we can relate differences in earnings to differences in wage rates by substituting equation (A2.9) into equation (A2.15):

$$\begin{split} \bar{E}_{t} &= -\frac{\bar{w}_{t}\bar{L}_{t}}{\bar{E}_{t}} \ \bar{L}_{t} + \frac{\bar{w}_{t}\bar{N}_{t}}{\bar{E}_{t}} \ \bar{w}_{t} \\ &= -\frac{\bar{w}_{t}\bar{L}_{t}}{\bar{E}_{t}} \left[ a_{w}\bar{w} + a_{t} \right] + \frac{\bar{w}_{t}\bar{N}_{t}}{\bar{E}_{t}} \ \bar{w}_{t} \\ &= c_{w}\bar{w}_{t} + c_{t}a_{t}; \end{split}$$
(A2.16)

where  $c_w = (\bar{w}_t \bar{N}_t - \bar{w}_t \bar{L}_t a_w) / \bar{E}_t$  and  $c_t = -\bar{w}_t \bar{L}_t / \bar{E}_t$ . The elasticity  $c_w$  is necessarily positive since  $a_w < 0$  over the life cycle, according to the theory developed in Chapter 1. Changes in earnings are positively related to changes in wage rates, because changes in hours of work are positively related to changes in wage rates. The elasticity  $c_t$  is negative: hours of work and therefore earnings peak sooner than wage rates if interest rates net of time preference and growth effects are positive.

Notice also that if, at any given age, the covariance between wage rates and hours of work is small, hence  $\overline{E}_t \cong \overline{w}_t \overline{N}_t$ , then the elasticity  $c_w \cong 1 - (\overline{L}_t/\overline{N}_t)a_w$  and the elasticity  $c_t \cong - (\overline{L}_t/\overline{N}_t)$ . Therefore if cov (w, N; t) = 0,  $c_w$  is necessarily greater than unity. More generally APPENDIX

$$\mathbf{c}_{w} = \mathbf{1} - \frac{\bar{w}_{t}\bar{L}_{t}a_{w}}{\bar{E}_{t}} - \frac{\operatorname{cov}(w, N; t)}{\bar{E}_{t}};$$

hence  $c_w > 1$  unless the covariance between wages and hours of work at any given age is sufficiently positive. As long as differences in wage rates are accompanied by differences in real wealth among households and under the usual assumption that wealth effects dominate substitution effects on the supply of labor, we expect the covariance between wages and hours of work at any given age to be negative, and therefore,

$$c_w > 1 - \frac{\bar{w}_t \bar{L}_t}{\bar{E}_t} a_w > 1.$$
 (A2.17)

Now suppose that instead of taking regressions of changes in mean consumption of goods on changes in the price of time measured by mean wage rates, as in (A2.8),  $\dot{X}_t = \dot{p}_w \dot{w}_t + b_t$ , I used earnings as a measure of the price of time:

$$\overline{X}_t = B_e \overline{E}_t + B_t, \qquad (A2.18)$$

with

$$B_e = b_w / c_w, \tag{A2.19}$$

where  $c_w$ , defined in (A2.16), measures the effect on earnings of changes in wage rates. Since  $c_w > 0$ , and since  $b_w = (\sigma_f - \sigma_c)s$  with s > 0, we have

$$B_e \gtrless 0 \text{ as } \sigma_f \gtrless \sigma_c. \tag{A2.20}$$

Under the plausible assumption that  $c_w > 1$ ,

$$|B_e| < |b_w|. \tag{A2.21}$$

On the other hand, the relation between  $B_t$ , the effect on consumption of interest rates net of time preference and growth when earnings are used to measure the price of time, and  $b_t$ , the effect of these parameters when wage rates are used, is given by  $b_t = B_t + B_e c_t a_t$ , or

$$B_t = b_t (1 - B_e c_t),$$
 (A2.22)

since  $a_t = b_t$ . Since  $c_t$ , the effect of trends on earnings is negative,  $|B_t/b_t| \ge 1$  as  $B_e \ge 0$ , or using (A2.19):

$$|B_t/b_t| \stackrel{\geq}{\equiv} 1 \text{ as } \sigma_f \stackrel{\geq}{\equiv} \sigma_c.$$
 (A2.23)

The interpretation is clear: since earnings peak sooner than wage rates, wage rates will still be rising when earnings reach a peak. Hence, when earnings are at a peak, the consumption of goods will be rising or falling as  $\sigma_f$  is greater than or less than  $\sigma_c$ .

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Now consider the ratio of  $B_t$  to  $B_e$ :

$$\frac{B_t}{B_e} = \frac{b_t}{b_w} (c_w - c_t b_w)$$
$$= \frac{b_t}{b_w} \left[ \frac{\tilde{w}_t \bar{L}_t}{\bar{E}_t} (b_w - a_w) + \frac{\bar{w}_t \bar{N}_t}{\bar{E}_t} \right];$$
(A2.24)

with  $c_w$  and  $c_t$  defined as in (A2.16). But

$$b_w - a_w = \sigma_f. \tag{A2.25}$$

Therefore,

$$\frac{B_t}{B_c} = \frac{b_t}{b_w} \left( \frac{\bar{w}_t \bar{L}_t}{\bar{E}_t} \sigma_f + \frac{\bar{w}_t \bar{N}_t}{\bar{E}_t} \right)$$

or

$$\frac{B_{t}}{B_{e}} = \frac{b_{t}}{b_{w}} \left[ \frac{\overline{w}_{t} \overline{L}_{t}}{\overline{E}_{t}} \left( \sigma_{f} - 1 \right) + \frac{\overline{w}_{t} \theta}{\overline{E}_{t}} \right].$$
(A2.26)

In particular, if  $\sigma_f = 1$ ,

$$\frac{B_t}{B_e} = \frac{b_t}{b_w} \frac{\overline{w}_t \theta}{\overline{E}_t}$$

Since  $\bar{w}_t \theta > \bar{E}_t$ , we have

$$\frac{B_t}{B_e} > \frac{b_t}{b_w}.$$
 (A2.27)

The extension of these results to the case where both husband and wife are earners is straightforward. If for instance the wife's wage rate is approximately constant over time, the conditions (A2.20), (A2.21), (A2.23), and (A2.27) would still apply, with  $b_w$  now interpreted to be the coefficient of the wage of the husband.