A Theory of Urban Housing Markets and Spatial Structure

In this and the next chapter, we develop an analytic framework for the examination of urban housing markets, the residential decisions of urban households, and the determinants of urban spatial structures. This framework is solidly in the tradition of earlier theoretical analyses. In the manner of traditional urban economic theory, we postulate that the residential-location and housing-consumption decisions of urban households are based on a utility-maximizing calculus in which households attempt to maximize their real incomes.

The way in which these housing- and transportation-cost tradeoffs are represented in the revised theory, however, differs significantly from the traditional analyses.

Traditional theories are concerned almost exclusively with the housing and location decisions of Central Business District (CBD) workers. Yet the overwhelming majority of workers in the United States metropolitan areas are employed at locations other than the CBD, and this fraction is rapidly increasing. Because they postulate a single workplace, traditional theories do not consider the effect of specific workplace location on the housing and locational choices of households. In our revised theory, in contrast, workplace location assumes a central position in predicting both the residential-location and housing-consumption decisions of urban households.

Our view of urban housing markets deviates from traditional theories in other important respects as well. Perhaps the most significant difference is our abandonment of the long-run equilibrium framework employed in traditional theories. Although it is a powerful theoretical simplification, the long-run equilibrium assumption, which requires a full adjustment of the nonresidential and residential capital stocks in response to any change in technology, incomes, or preferences, is widely at variance with reality.

What we term the "traditional theories" are derived largely from
von Thünen’s seminal theoretical analysis of the relationships between site rentals and the location of economic activity in an agrarian society characterized by a single marketplace (or port from which goods are exported).¹ The best known and most highly developed of these traditional theories are by William Alonso and Richard Muth.² Although the authors of traditional theories are careful to include certain disclaimers and caveats, there is little doubt that their theories are considered as serious explanations of the residential-location decisions of urban households, the spatial pattern of housing densities in urban areas, and the pattern of urban housing prices.

Traditional theories employ a more or less common set of assumptions in deriving their conclusions about urban housing markets. They typically assume that the city has a single workplace, at which all productive activity is located. In addition, they assume that the city is located on a featureless plain, that transportation costs are proportional to distance, and that all land used to produce housing is identical. Traditional theories further assume that “housing” is a single good which enters as a single argument in households’ preference functions along with other goods (and, in some models, transportation time). Households choose an amount of housing and a location (distance from the single workplace) to maximize their utilities subject to a budget constraint that includes the cost of housing, the costs of the work trip to the central place, and the costs of other goods.

Traditional theories, moreover, assume that the housing good is produced by perfectly competitive suppliers who combine two inputs, land and “nonland,” to maximize profits at given factor prices. In addition, they assume that the price of “nonland” inputs is constant throughout the city and that the size of the city is fixed (or that the opportunity cost of land is fixed and known). Finally and most critically, traditional theories assume that the market is in long-run equilibrium.³

¹John Heinrich von Thünen, Der Isolirt Staat in Beziehung auf Nationalökonomie und Landwirtschaft, 1828.


³This discussion is a more faithful description of Muth’s treatment of these questions than Alonso’s, since Alonso obtains most of the same results by postulating a household preference for residential space. In Muth’s theory, in contrast, variations in residential density are unrelated to preferences and instead are derived from input substitution by housing suppliers.
Under these assumptions, it can be shown that the competition for central locations will bid up the price of sites located closer to the single workplace. Since more centrally located land is relatively more expensive than land located further from the center, housing suppliers will use less land per unit of housing output at more central locations to produce the homogeneous good "housing." The prices of "nonland" inputs are assumed to be constant throughout the city and an all-round convex production function is assumed, typically Cobb-Douglas.

Using these assumptions, traditional theories obtain the result that the price of land declines with distance from the central workplace and that it declines at a diminishing rate. Further, these theories conclude that the gradient of land prices is steeper than the gradient of housing prices, as long as the elasticity of substitution between land and "nonland" in the production function for housing is not zero.

Given these equilibrium conditions for the housing market as a whole and the spatial pattern of housing prices, traditional theories imply that households of the same income will be indifferent among all residential locations in the city. At each distance from the single workplace, the incremental savings in housing expenditures associated with an increase in distance (evaluated at the household's optimum consumption of housing) will be exactly offset by the incremental transportation costs to the city center. Finally, if appropriate assumptions are made about the income elasticity of demand for housing and about the income elasticity of marginal transportation costs, it can be shown that higher-income persons will live further from the central workplace.

Casual empiricism, as well as more sophisticated quantitative analysis, indicates that on the average, housing prices and residential densities tend to decline with distance from the center of American cities and that, on the average, higher-income households live further from downtown. These aggregate empirical regularities are often cited as confirmation of the traditional theories. Still, it should be emphasized that these tests have little power to discriminate among alternative explanations of these phenomena and that these same empirical regularities are consistent with theories of urban spatial structure that employ radically different assumptions.4

Our own evaluation of these traditional theories leads us to question their usefulness as explanations of the structure and behavior of urban housing markets and of the housing choices of urban households. Although their elegance and simplicity have strong appeal, we find their

4See, for example, David Harrison and John F. Kain, "Cumulative Urban Growth and Urban Density Functions," *Urban Economics* 1, no. 1 (January 1974): 48–60; and John M. Quigley, "Residential Location With Multiple Workplaces and a Heterogeneous Housing Stock" (Ph.D. diss., Harvard University, 1972).
underlying assumptions and theoretical structure more appropriate to the rural society of von Thünen's day than to modern urban areas.

The analytical framework developed in this chapter retains the central behavioral relation used in these traditional theories—the opportunity available to urban households to substitute between transport costs and location rents. The way in which these housing and transport costs tradeoffs are represented in our revised theory differs significantly from their representation in traditional theories. For example, traditional theories are concerned almost exclusively with the housing and location decisions of Central Business District (CBD) workers. Yet the overwhelming majority of workers in the United States metropolitan areas are employed at locations other than the CBD; in fact, in 1970 more than half of all jobs in metropolitan areas were located outside the central cities, and this fraction is rapidly increasing. Because they postulate a single workplace, traditional theories do not consider the effect of specific workplace location on the housing and locational choices of households. In our revised theory, in contrast, workplace location assumes a central position in predicting both the residential-location and housing-consumption decisions of urban households.

Our view of urban housing markets deviates from traditional theories in other important respects as well. Perhaps the most significant difference is our abandonment of the long-run equilibrium framework employed in traditional theories. Although it is a powerful theoretical simplification, the long-run equilibrium assumption, which requires a full adjustment of the nonresidential and residential capital stocks in response to any change in technology, incomes, or preferences, is widely at variance with reality.

Abandoning the long-run equilibrium assumption makes it possible, indeed imperative, to address directly the implications of the heterogeneity and durability of the housing stock and of the irregular spatial distribution of specific housing capital for the consumption of housing by various types of households, for the residential-location decisions of these households, and for spatial location of specific kinds of housing investment.

Our approach has still another advantage. It enables us to consider explicitly the effect on housing markets of the nonmarket production of a number of important housing attributes, such as the quality of local government services and the physical characteristics, amenity, and socioeconomic composition of the neighborhood. Finally, within this theoretical framework, we are able to analyze many of the effects of racial discrimination, which is clearly the most serious market imperfection affecting urban housing markets. Our revised theory, then, differs from traditional theories in three major respects. These are: (1) emphasis on the substantial effect of alternative workplace locations on the hous-
ing and location decisions of urban households; (2) abandonment of the
confining long-run equilibrium assumption; and (3) explicit attention to
the role of various kinds of externalities and interdependence, particu-
larly racial discrimination. We now turn to a fuller discussion of the
theoretical implications of each of these differences in perspective,
beginning with the long-run equilibrium assumption.

THE NATURE OF HOUSING OUTPUT

The long-run equilibrium assumption, used in traditional theories,
assumes that all housing inputs except land are variable and therefore
implies that geographic variations in housing prices within urban areas
depend only on differences in land prices. This assumption, moreover,
permits the theorist to ignore the effects of durable nonresidential and
residential capital stocks and to define housing output as a single-valued,
homogeneous good, "housing services," measured by a household's
total expenditure for housing. 5

This approach suffers from a number of serious conceptual difficul-
ties. First, differences in housing expenditures can reflect either differ-
ences in quantity or differences in price. This fact has been recognized
by previous authors, and the most careful studies attempt to use price
indexes to correct for price changes. In every instance, however, the
indexes used fail to incorporate the effects of price changes attributable
to neighborhood change, to spatially distributed quasi rents, or even to
raw land prices. For example, Richard Muth's study of housing demand
deflates expenditures by the Boeckh construction-cost index for single-
family units. However, the Boeckh index represents only changes in the
prices of labor and materials used to construct a new dwelling unit. 6

De Leeuw, in his studies of rental housing, uses BLS surveys of the
monthly rent for a standard bundle of housing services to measure inter-
city variations in the price of rental housing. 7 The BLS surveys are
designed to obtain rents for comparable dwelling units in different cities
and, to some degree, they define the price of standard housing-services
bundles. The BLS procedures explicitly consider only a few of the many

5The most explicit statement of this approach is contained in Edgar O. Olsen, "A

6Richard Muth, "The Demand for Nonfarm Housing," in The Demand for Durable
Goods, A. Harberger, ed. (Chicago: University of Chicago Press, 1960); Frank de Leeuw,
"The Demand for Housing: A Review of Cross-Section Evidence," Review of Economics

Housing."
dimensions of the bundle of residential services, however, and as a result, they fail to provide uniform definitions of rental housing among cities. As a result, we suspect that the rental price measures used by de Leeuw in his studies embody both price and quantity variation among cities. If our suspicions are correct, this would provide an alternative explanation for the positive relationship he obtains between SMSA rent levels and SMSA incomes in his study of the supply of rental housing and would imply that de Leeuw's principal conclusion—that the supply of rental housing services is quite inelastic—is incorrect.

The use of housing expenditures to measure housing output has an even more fundamental weakness, however. When households buy or rent a residence, they simultaneously choose a large number of specific and identifiable attributes. These include the number of rooms, a structure type, a neighborhood environment, a set of neighbors, a diverse collection of public services, and a particular journey to work. In principle, as Equation 2-1 illustrates, there is a market price associated with each of these attributes.

\[
H_i = P_1 X_{1i} + P_2 X_{2i} + \ldots + P_n X_{ni}
\]

where \(H_i\) are the expenditures for the \(i\)th bundle of residential services, \((X_{1i}, X_{2i}, \ldots, X_{ni})\) are the quantities of different attributes comprising the \(i\)th bundle of residential services, and \((P_1, P_2, \ldots, P_n)\) are the unit prices of these attributes.

Only limited consideration of Equation 2-1 is needed to identify the serious problems inherent in relying on housing expenditures to measure housing output. Any number of combinations of attributes and quantities may be obtained for the same total outlay. Yet bundles with the same cost may be considered completely different by both demanders and suppliers of housing. Indeed, both groups may regard them as much less similar than other bundles that differ substantially in price. Price changes only compound the problem. Without identifying the individual attributes, their quantities, and their prices, it is virtually impossible to interpret changes in housing expenditures.

The apparent solution to these problems is clear. All that is required is to obtain and analyze information on the prices and quantities of the attributes included in each bundle. In spite of the obvious advantages of this approach and its beguiling simplicity, little progress has been made in its empirical implementation, and even less in considering the theoretical implications of this broader view of housing services.

There are several reasons why empirical and theoretical studies of the housing market have not viewed housing services in this way. Major theoretical and empirical difficulties arise from the fact that the individual attributes \((X_1, X_2, \ldots, X_n)\) cannot be purchased individually. Instead, they must be purchased as part of an entire bundle of residential
services. As a result, the individual prices \( (P_1, P_2, \ldots, P_n) \) are never observed directly; they must be imputed from the differences in the cost of "otherwise identical" bundles.

An analogy to studies of the demand for food may help clarify the special characteristics of housing markets. Empirical analyses of the demand for foodstuffs are probably more detailed and more highly developed than any other area of applied econometrics. These studies have examined household consumption of foods at various levels of aggregation; for example, all foods, meats, and beef. In each case, the aggregate prices and quantities are built up from microeconomic data on price and quantity. Why have similar procedures not been followed in housing markets, where the bundle of residential services is at least as heterogeneous as the market basket of consumer food purchases?

The explanation is to be found in the differences in the manner in which foods and housing are produced, marketed, and consumed. When shoppers go to the supermarket, each constructs his own market basket by combining quantities of individual types of food products in any way he wishes. In choosing how much of each item to include, the household compares the given price of that item to the given price of other items. The \( i \)th household's weekly food consumption \( F_i \) then is the simple sum of the amounts spent for each item (price times quantity). This expression, illustrated by Equation 2-2, appears identical to the definition of housing expenditures depicted by Equation 2-1:

\[
F_i = P_1 X_{1i} + P_2 X_{2i} + \ldots + P_n X_{ni}
\]  

(2-2)

The analogy breaks down at this point, however.

In supermarkets, the price of each item is clearly marked, and shoppers may take as many or as few of each item as they like. Housing consumers never directly observe the prices of individual attributes. Therefore, to make the supermarket analogy comparable, we must require shoppers to choose from among a finite number of grocery carts, each filled with some combination of items and selling at a fixed price. Equation 2-3 illustrates the supermarket problem, where \( F_i \) is the market price of the market basket purchased by the \( i \)th household, \( (X_1, X_2, \ldots, X_n) \) are the amounts of each grocery item in the basket, and the prices of individual items \( (P_1, \ldots, P_n) \) either are not known or are not provided by the supermarket. We suggest that had the grocery shopping problem been presented in this manner, it would have affected the behavior of both consumers and econometricians studying the demand for foodstuffs.

\[
F_i = g(X_{1i}, X_{2i}, \ldots, X_{ni})
\]  

(2-3)

Under favorable circumstances, it may be possible for either shoppers or econometricians to impute a set of prices to the individual items
in either the hypothetical market basket or the bundle of residential services. If there is a large enough variety of market baskets (bundles) and if the suppliers have priced the attributes identically in all bundles, a unique set of prices may be imputed. Unfortunately, these ideal circumstances do not exist in the housing market. As a consequence, the price relationship is unlikely to be a simple additive one and, worse still, a unique set of prices for housing attributes may not exist.

In addition, it is unclear how many attributes should be included in the bundle of residential services or how they should be defined and measured. The heterogeneity of dwelling units, structures, neighborhoods, and other dimensions is so great that a large number of plausible output definitions exist. Indeed, since one attribute of the bundle of residential services is a specific location, no two bundles can be precisely alike. Clearly, if the concept is to be operationally useful, housing bundles must include far fewer than the theoretically possible number of dimensions. A principal objective of the analysis presented in subsequent chapters is to identify, define, measure, and price the "important" dimensions of the bundle of residential services. Much of Chapter 4 deals with the problems of defining housing output and with the efforts to develop workable definitions of individual housing attributes. Chapter 8 describes efforts to impute prices to individual housing attributes.

Beyond the problems of defining and measuring the bundle, attempts to estimate attribute prices are hampered by the fact that many of the possible X vectors are never produced. In addition, the durability and the locational specificity of many dimensions of residential services make the interpretation of the estimated attribute prices obtained in subsequent chapters for heterogeneous stocks of residential capital and neighborhood attributes somewhat unclear. Both kinds of housing attributes have variable and uncertain lives. Because of these characteristics of housing bundles, the imputed market values of housing attributes ($P_1$, $P_2$, ..., $P_n$) include quasi rents. Moreover, the market prices of many housing attributes may be below their reproduction cost (supply price in long-run equilibrium).

Although it is admittedly somewhat arbitrary, we treat location differently from the remaining attributes. This convention permits us to define a spatial quasi rent for each attribute or collection of attributes, even those that are not being produced currently. The simple additive relationship depicted by Equation 2-1, though certainly oversimplified, can be extended to illustrate the concept of spatial quasi rents for individual housing attributes, or for bundles of attributes. Specifically the market value of each attribute in Equation 2-1 consists of two components: (1) the production cost (long-run equilibrium price) of the $k$th attribute, $p_k$, which does not vary spatially, and (2) the spatial quasi
rent for the kth attribute at the ith location, r_{ki}. Equation 2-4 depicts the housing price at the ith location.

\[
H_i = (p_1 + r_{1i})X_{1i} + (p_2 + r_{2i})X_{2i} + \ldots + (p_n + r_{ni})X_{ni}
\]

where

- \(p_k\) = supply price of the kth attribute if it is currently being produced or its market price at the least-cost location if it is not currently produced;
- \(r_{ki}\) = quasi rent for the kth attribute at the ith location;
- \(X_{ki}\) = quantity of the kth attribute at the ith location.

When a housing attribute is currently being produced, the market price of the housing attribute \((p_k + r_{ki})\) in Equation 2-4 has a straightforward interpretation, i.e., \(p_k\) is the unit cost of producing the attribute. At those locations where the attribute is being produced currently, the market value of the attribute in the competitive stock must equal or exceed the current cost of production. This inequality need not hold for other locations, however. In particular, as we illustrate in the appendix to this chapter, it is easy to imagine situations in which the market value of the attribute would be less than the reproduction cost at locations where the attribute is not being produced. At these locations, the quasi rent, \(r_{ki}\), is negative. These negative quasi rents will arise in circumstances where there is a decline in demand for the attribute and the stock cannot be profitably transformed.

For positive quasi rents to persist at a particular location or residence zone, there must be an effective supply constraint or spatial difference in production cost. When the supply costs \(p_k\) do vary by location, these spatial production-cost differences are capitalized as quasi rents. This phenomenon can arise from obstructed land that raises the effective land cost at particular locations or from the presence of housing attributes that are not supplied by competitive firms.

It is also possible to make some statements about the maximum values of positive quasi rents for individual attributes. In particular, drawing on the analytical insight provided by monocentric theories of residential location, it is clear that if housing attributes are produced independently, differences in quasi rents for a particular attribute \(k\) between any two zones \(i\) and \(j\) cannot exceed the difference in transport costs between the two zones for the marginal consumer of that attribute,

\[
r_{ki} - r_{kj} \leq t_i - t_j
\]

where \(t_i\) is the capitalized transport (journey-to-work) cost associated with location \(i\).

Even the foregoing discussion is oversimplified. The production of
housing attributes is often characterized by joint costs and, in practice, it may be impossible to separate individual housing-stock attributes and their prices. In principle, this jointness can be recognized by introducing interactions in attribute prices and quasi rents. Thus, Equation 2-4 can be expanded to include joint-price effects between housing attributes and quasi rents at location $i$, $P_{lm}$.

$$H_i = (P_1 + r_{1i}) X_{1i} + \cdots + (P_n + r_{ni}) X_{ni} + (P_{12} + r_{12i}) + X_{1i}X_{2i} + \cdots + (P_{1n} + r_{1ni}) X_{1i}X_{ni} + \cdots + (P_{n-1,n} + r_{n-1,n,i}) X_{n-1,i}X_{ni}$$

Important interactions may be numerous, however, and, at the limit, this procedure leads to the definition of discrete housing types defined by specified levels of several attributes. These discrete types, defined by particular levels of various attributes, are then considered to be homogeneous housing goods. At any location, the prices of these bundles can be expressed in terms of two components: a constant supply price of the housing bundle for the metropolitan area, $P_k$, and a spatial quasi rent $r_{ik}$.

While this approach has obvious advantages, the number of housing types quickly becomes very large if many dimensions are considered. Of course, it is not necessary to stratify the housing market by every housing attribute. A combination of discrete and parametric variables may be used in the analysis; that is, equations containing continuous variables describing housing attributes of the form (2-1) can be estimated for discrete categories of housing. This procedure permits a full interaction between those variables included in the equation and the housing attributes used to stratify the equation. Stratifications of this kind are performed in Chapter 8 for (1) rental and owner-occupied housing, (2) ghetto and nonghetto rental and owner-occupied properties, and (3) room size categories for rental and owner-occupied properties located outside the ghetto.

The best way of representing the complex and multidimensional housing surface of housing prices is ultimately an empirical issue. Theoretical considerations may suggest where to look for significant departures from long-run equilibrium and where jointness in the production and pricing of attributes is likely to occur. But, if useful descriptions of metropolitan surfaces of housing prices are to be obtained, it will be necessary both to develop large samples of price information and housing characteristics and to carry out careful econometric estimates of alternative models.

Although we remain uncertain about how best to represent housing prices in urban housing markets, one fact is clear from our research. The result obtained from traditional models, i.e., a single location rent gradient which declines with distance from the center, does not conform
well to reality. Instead, distinct and quite different rent surfaces exist for
the various housing attributes. Recent econometric studies of the San
Francisco-Oakland and Pittsburgh housing markets have obtained
empirical results that support this important conclusion.

Four studies—Mahlon Straszheim’s study of the San Francisco-
Oakland housing market, and separate studies of the Pittsburgh housing
market by Gregory K. Ingram, by John M. Quigley, and by William
Apgar and John F. Kain—all reveal rather large spatial variations in the
prices of housing attributes. Moreover, these analyses indicate that the
price surfaces differ substantially for the various housing attributes
considered, a finding that directly contradicts the long-run equilibrium
assumption of traditional models.

Suggestion of the results obtained from these studies is provided by
Table 2-1, which includes summaries of attribute-price estimates
obtained by Straszheim in his study of the housing choices of several
thousand San Francisco households. Straszheim obtained estimates of
the market prices of housing attributes for rental and owner-occupied
housing in the San Francisco-Oakland metropolitan area, using tech-
niques similar to those employed in Chapter 8 of this book. However, he
estimated equations of the form shown in Equation 2-1 for each of
seventy-three residence zones. This method allows the price of each
attribute included in the regressive equation, which in Straszheim’s
study included number of rooms, structure age, lot size, and structure
condition, to differ for each residence zone.

The first row in Table 2-1 gives the estimated price of a standard-
ized rental and owner-occupied unit near the CBD and in the suburbs.

Straszheim estimated that this standard unit would cost $57,150
adjacent to the San Francisco CBD but only $26,647 an hour’s commut-
ing time from downtown. The standardized rental unit cost an estimated
$186 per month adjacent to the CBD and $122 per month in the suburbs
an hour distant from downtown.

Table 2-1 illustrates that there is substantial variation among the
price gradients for housing attributes. All attributes shown in Table 2-1
are more expensive downtown than in the suburbs—but the variation is

8Mahlon R. Straszheim, An Econometric Analysis of the Urban Housing Market
(New York: National Bureau of Economic Research, 1975); Gregory K. Ingram, "A
Simulation Model of a Metropolitan Housing Market" (Ph.D. diss., Harvard University,
1971); John M. Quigley, "The Influence of Workplaces and Housing Stocks upon Resi-
dential Choice; A Crude Test of the 'Gross Price' Hypothesis," paper presented at the
Winter Meetings of the Econometric Society, Toronto, Ontario, Canada, December 28–30,
1972; William C. Apgar, Jr. and John F. Kain, "Neighborhood Attributes and the Resi-
dential Price Geography of Urban Areas," paper presented at the Winter Meetings of the

9Mahlon R. Straszheim, Econometric Analysis.
TABLE 2-1
Suburban–Central City Differences in Housing Prices in the San Francisco–Oakland SMSA by Characteristics of the Housing Bundle: Owner-Occupied and Rental Units

<table>
<thead>
<tr>
<th></th>
<th>Owners</th>
<th></th>
<th>Renters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CBD¹</td>
<td>Suburb²</td>
<td>CBD¹</td>
<td>Suburb²</td>
</tr>
<tr>
<td>Price of standardized unit³</td>
<td>$57,150</td>
<td>$26,647</td>
<td>$185.80</td>
<td>$122.12</td>
</tr>
<tr>
<td>Incremental cost per room</td>
<td>4,515</td>
<td>2,723</td>
<td>28.45</td>
<td>10.62</td>
</tr>
<tr>
<td>Incremental savings on standardized unit:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structure age 1950–65</td>
<td>15,151</td>
<td>3,640</td>
<td>34.16</td>
<td>15.41</td>
</tr>
<tr>
<td>Structure age 1940–50</td>
<td>26,277</td>
<td>3,795</td>
<td>42.05</td>
<td>22.79</td>
</tr>
<tr>
<td>Structure age pre-1940</td>
<td>32,485</td>
<td>5,911</td>
<td>51.01</td>
<td>30.15</td>
</tr>
<tr>
<td>Lot size &lt; .2 acres</td>
<td>5,357</td>
<td>6,100</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Lot size = .3–.5 acres</td>
<td>–8,605</td>
<td>–3,354</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Lot size &gt; .5 acres</td>
<td>–30,505</td>
<td>–16,116</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Unsound condition</td>
<td>–17,821</td>
<td>–14,251</td>
<td>38.38</td>
<td>–</td>
</tr>
</tbody>
</table>


¹Average for four zones located in Census Tracts B1–B10, J1–J20 in downtown San Francisco.
²Average for ten zones located .55–.65 hour commuting distance from CBD.
³Owner-occupied unit of 5.5 rooms, built since 1960, on .2–.3 acre lots in sound condition. Renter-occupied unit of 4 rooms, built since 1960, in sound condition.

great. For example, for owner-occupied units, an additional room costs $4,515 near the CBD and $2,723 in the suburbs. The price gradient for structure age is much steeper: the figures in Table 2-1 indicate that a household can expect to save $32,485 by purchasing a structure more than 30 years old near the CBD; however, the savings from purchasing a unit more than 30 years old in the suburbs are only $5,911. If traditional theories are interpreted literally, the price of attributes such as additional rooms or structure age—a proxy for quality features of the unit or its condition—should be identical for every zone. Variations in price by lot size are consistent with traditional theories, but not variations of the kind reported by Straszheim.

HOUSING PRODUCTION RELATIONSHIPS

An examination of the assumptions about the nature of housing production relationships assumed by traditional theories and by our revised theory of urban spatial structure may help clarify the distinction
between these two views of urban housing markets. Traditional theories of urban spatial structure assume a housing production function, similar to Equation 2-7, in which varying quantities of the homogeneous good, housing \( H \), at location \( i \) (measured as distance from the central place) may be produced by combining land and "nonland" factor inputs.\(^\text{10}\)

\[ H_i = f (L_i, C) \]

where

\[ L_i = \text{land input at distance } i \text{ from central place}; \]
\[ C = \text{"nonland" inputs}. \]

Traditional theories distinguish between land and nonland factors of production because they are concerned primarily with explaining variations in density, and because the price of nonland factors is assumed to be identical at all locations, whereas the price of land declines with distance from the central production center.

Land prices decline with distance from the center because of the accessibility advantages of residential sites near the single workplace. Since land is more expensive at central locations while nonland factors have the same prices at all locations, housing suppliers use less land to produce housing near the central location. In spite of these efforts to conserve its use, some land must be used to produce housing (a Cobb-Douglas production function is typically assumed). As a result, housing costs are higher near the central location, but the gradient of housing prices is less than the gradient of land prices. The higher cost of housing near the center encourages those households who consume large amounts of housing to commute farther. Housing consumption per household then increases with distance from the center, even though housing consumption per acre declines.

In contrast to the housing production function used in traditional theories, our revised theory asserts that the heterogeneous bundles of services sold in urban housing markets cannot be ordered uniquely along a single dimension. Each individual can rank all possible or available housing bundles, from those he prefers most to those that he prefers least, or from those which cost the least to him to those which cost the most to him, but it is impossible to aggregate these individual ordinal rankings into a unique ordering of bundles. Housing bundles can still be ranked cardinally by price, i.e., by monthly rent or market value, which is the convention employed in traditional theories. These rankings, however, are merely the current market value of bundles of heterogeneous attributes; they are not measures of the quantity of a homogeneous good. Rather than maintaining the fiction of a unidimensional

\(^{10}\)For example, see Muth, Cities and Housing, Chap. 3.
housing good, we contend that it is better to analyze housing in terms of consumption bundles consisting of combinations of housing attributes.

Instead of the quantity of a single good, measured by price, we describe housing output as a vector of housing attributes. Since housing services are normally consumed in more or less closely related "bundles" of individual attributes, due to the durability and lack of malleability of the housing stock, it is convenient to discuss these bundles rather than individual housing attributes. Equation 2-8 depicts a production function that embodies this view of the housing market and acknowledges the crucial roles of existing structures and of housing attributes that are not produced by competitive firms.

\[
H_i = f(L_i, C_i, S_i, N_i)
\]

where

\[
S_i = \text{sunk capital (existing structures and parcels at the } i\text{th location);}
\]
\[
N_i = \text{neighborhood attributes at the } i\text{th location.}
\]

Housing output \(H_i\) is a vector of \(K\) attributes or, for convenience, a composite housing bundle at residential location \(i\). The first two inputs in Equation 2-8, vacant land and nonland factors of production, are analogous to those in the production function used in most classical theories of urban spatial structure. The remaining inputs—sunk capital (existing structures and their parcels) and neighborhood attributes—are locationally given to housing suppliers in the short and medium run.

The addition of capital stocks to the housing production function is required by the abandonment of the long-run equilibrium assumption. The inclusion of neighborhood attributes in the production function simply acknowledges both the importance of various kinds of interdependence and the fact that many important housing attributes cannot be produced by individual property owners and are not supplied by competitive firms.\(^\text{11}\)

The production function depicted by Equation 2-8 represents several production relationships that have distinctly different implications for urban housing markets than those obtained from traditional models.

For analytical purposes, it is useful to distinguish among several types of production activities, or ways in which a particular housing bundle can be produced.

First, neighborhood attributes differ from the remaining inputs in Equation 2-8 in that they cannot be changed by the actions of individual homeowners or housing investors. Some scope exists for residents and property owners to modify the production and other decisions of local governments through various forms of political action. Housing suppliers and, to an even greater extent, homeowners devote considerable time to these activities. (Greater efforts by homeowners are presumably explained by the higher transactions and moving costs of homeowners, the psychic values many households come to attach to particular neighborhoods or communities, and the investment nature of these activities by owner-occupants.) Still, it is unlikely that many homeowners or housing investors make location or investment decisions in the expectation of obtaining major changes in government policies or of modifying the socioeconomic composition of their neighborhoods in significant ways.

While individual firms can do very little to modify neighborhood characteristics within built-up areas, they have somewhat more scope when they build new subdivisions on vacant land. Indeed to some extent, the developer of a large subdivision can determine the characteristics of his neighborhood at the time of its development, a consideration that may explain the homogeneity of most large subdivisions and the tendency to produce more high-quality than low-quality housing through new construction. But even large-scale subdividers have limited opportunities to produce neighborhood attributes. In particular, they are constrained by the norms and resources of the communities where they locate their subdivisions, and by the extent of the market for each kind of neighborhood in each part of the metropolitan area.

Within each type of neighborhood, several physical production possibilities exist. First, virtually any kind of housing bundle, consistent with that neighborhood location, can be produced by combining vacant land and various nonland factors of production. Moreover, since vacant land can always be produced through the demolition of existing structures, any kind of structure can be produced in this way. However, the cost of acquiring and demolishing existing structures (including the opportunity costs of existing structures and residential capital) makes demolition relatively infrequent.

One common type of stock transformation involves only incremental changes or modifications and results in improvements to existing structures and parcels that increase the desirability and market value of the bundle by the full amount of the incremental expenditure. Stock
transformations of this kind would include some kinds of redecoration, increased annual outlays for services, or the simple addition of a room or rooms that maintain the structural integrity of the unit. The essential characteristic of this example is that the original structure is fully utilized; something is merely added.

Other types of transformation involve the replacement of all or part of a structure by new physical capital. At one extreme, this transformation occurs when an entire structure (or several structures) is demolished and a new one is constructed. The combined outlays for the acquisition of the original structure(s) or parcel(s) and for demolition constitute the full cost of the vacant land created. As we have indicated previously, existing residential structures are demolished and replaced by new residential structures only infrequently. On the other hand, less extreme transformations of this kind are commonplace. The replacement of old kitchens or bathrooms, for example, may cost as much or more than constructing new facilities of equal quality. These transformations often involve the replacement of all fixtures, plumbing, and wiring. The essential difference between incremental changes and replacement is the significant demolition costs which must be borne before structural rearrangement may take place.

In the appendix to this chapter, we present some simple examples of how these different types of production relationships may affect the structure of housing prices and the spatial patterns of quasi rents for bundles of housing services. The analysis considers the implications of two polar types of structure transformations, using a simple housing-market model which analyzes price determination and supply responses for two types of residential structures (a "high-quality" and a "low-quality" structure) that may be consumed in a central built-up area and in a suburban area.

The examples illustrate that if heterogeneous and durable stocks exist, the spatial pattern of quasi rents will depend on the nature of structure transformation costs. The first example demonstrates that as long as the cost of transforming housing from one bundle type to another in the built-up central area is the same as the difference in production cost at the noncentral (suburban) area, the price differences between identical units at the two locations will reflect only accessibility differentials. The second example demonstrates that if the transformation costs exceed the difference in construction costs at the urban fringe (the suburban location), market prices at the central location may diverge from the long-run equilibrium prices.

In real-world housing markets, where there are large numbers of possible residence sites, workplaces, and housing bundles, the existence
of significant transformation cost differentials among housing submarkets is sufficient to cause substantial departures from long-run equilibrium prices without causing households to move or housing suppliers to transform the stock. This "equilibrium" may exist for long periods of time, but its implications may be totally different from the implications of the "equilibrium" analyzed by traditional theories.

HOUSING DEMAND AND RESIDENTIAL LOCATION

The difficulty and cost of making many kinds of physical transformations of residential structures, the inability of individual firms and households to change still other dimensions of housing bundles, and the interdependence of many housing attributes have strong implications for theories of housing demand and residential location.

There are at least two ways in which the demand for housing bundles can be analyzed. First, households can be depicted as having preferences for each housing attribute and for all other goods and services. Then, household demand for each attribute can be derived from these preferences, and from information on household income and prices—the prices of individual housing attributes and of all other goods and services. The bundle of housing consumed by each household thus becomes the simple sum of the household's consumption of each individual attribute. This view of the problem is convenient for statistical analysis, and although we consider it deficient in important respects, we employ it extensively in our analysis of the demand for both individual attributes and groups of housing attributes.

The most serious difficulty with viewing the demand for housing bundles as the simple sum of the demand for housing attributes is that households seldom have the opportunity of buying individual attributes. Instead, they usually must choose from among a large number of fixed bundles. Often, they are able to modify these bundles by making additional expenditures, but the scope for changing bundle characteristics in this way is definitely limited. Some types of changes are inordinately expensive when compared with the cost of new construction, and many important attributes cannot be produced by the actions of single housing suppliers.

These aspects of housing production functions insure that many types of bundles will seldom, perhaps never, be produced, and that the price of many attributes will be jointly determined. For these reasons, we believe it is more instructive to analyze housing markets in terms of the demand for, supply of, and production of housing bundles, rather
than employing either a single homogeneous good, housing, or housing attributes.

Viewing housing demand as the demand for specific housing bundles requires only slight modifications of traditional demand theory. The choice among quantities of homogeneous goods on the basis of preferences and prices, subject to a budget constraint, implies that consumers select goods in such a way that the satisfaction received from the last dollar's worth of each good is the same. The notion of housing bundles implies that each housing consumer chooses from a large number of differently priced bundles the one housing bundle which maximizes his real income. Corresponding to each possible binary choice is a different level of satisfaction or real income. As the appendix reveals, the discreteness of the housing choice implies that demand curves for housing types are not continuous; this, however, leads to no serious departure from traditional demand theory. The prices of housing bundles used in our revised theory, however, differ in an important respect from those employed in traditional demand theory and in traditional theories of urban spatial structure. The existence of spatial quasi rents in urban housing markets means that the price of a given housing bundle (defined to be homogeneous in all respects except location) may differ at each location. The spatial pattern of quasi rents can be expected to be quite complex, with the result that the ratios of the relative prices of housing bundles can be expected to vary considerably from one part of an urban area to another.

In addition, workers employed at different workplaces will view these prices quite differently. In traditional models, all workers are assumed to be employed at the same workplace; and as a result, they have the same view of alternative bundles and locations. Data on the location of employment in metropolitan areas clearly demonstrates the inappropriateness of this monocentric assumption. It is rare that as much as 10 percent of all employment is located in the core, or Central Business District; and central cities often contain less than half of all metropolitan employment. In 1963, 52 percent of all manufacturing

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12The authors of these theories acknowledge the lack of realism of this assumption, and all attempt to incorporate some noncentral employment into their models. The most common approach is to define a category of local workers, who presumably provide neighborhood services. The inclusion of these local workers in the models cannot be regarded as a meaningful departure from the monocentric assumption. Their behavior is never considered in any but the most trivial way, and their inclusion has little or no effect on the solutions obtained from the theories. For example, see Muth, Cities and Housing, pp. 42-45, 86-93.
employment and 29 percent of all wholesaling employment in forty of the largest metropolitan areas were found outside the central city.\(^\text{13}\)

The prevailing trend over the past half-century has been a relative, and often absolute, decline in central employment and a rapid growth of jobs in suburban areas.\(^\text{14}\) Theories that claim to explain the suburbanization of urban populations, changes in the length of the journey to work, and modifications of central and suburban densities without explicit references to these changes in the distribution of employment must be viewed with suspicion. Many of the past changes in urban structure, which monocentric theories attribute to increases in incomes and to declines in the real costs of transportation, may instead be the result of changes in the spatial distribution of employment. Existing empirical tests of the theories utterly fail to distinguish among these explanations.

Recognition of the polycentric nature of modern metropolitan areas makes the arithmetic somewhat more difficult, but it does not change the analytics of the household choice problem in any major way. It does, however, considerably increase the predictive power and usefulness of the theory. As in traditional theories, our theory recognizes that households incur transportation outlays in order to reside at a particular location. Moreover, as in traditional theories, the transportation expenditures that would be incurred at each possible residence location may differ by household because of differences in the number of trips made to actual or potential trip destinations, and because of differences in travel costs per mile.

To compute the transportation costs associated with each residence location for each household, it is necessary to make some fairly strong simplifying assumptions about the demand for trips. Specifically, we assume that all trips are made either to known and predetermined destinations, or to ubiquitous and substitutable ones.\(^\text{15}\) A second fairly


\(^{15}\) Even these assumptions could be relaxed in the name of theoretical elegance, and complications involving them could be incorporated into the theory. These complications would add little to the present analysis, however. Although our assumptions are fairly strong, they are much weaker than those employed in traditional theories, which generally assume an inelastic demand for trips to the single center.
strong assumption is that households place a monetary value on their travel time. Ample theoretical justification for this assumption is available, and a fairly large number of empirical studies have obtained surprisingly consistent quantitative estimates of the dollar-value urban households impute to their commuting time. Rather than explore these issues at this point, we shall simply assume that households place a monetary value on their travel time and that they use these costs in choosing housing bundles and residential locations.

The two assumptions: (1) that households have completely inelastic demands for trips to known and invariant destinations, and (2) that they place some monetary value on their travel time, enable us to compute accessibility costs for each household for each bundle type and for each residence location. These costs, which are used in determining the demand for housing bundles and the choice of residential locations, are part of gross prices.

Gross prices are the sum of three components: (1) the market price for a given bundle at a given location, including spatial quasi rents; (2) the out-of-pocket money transport costs associated with the given location; and (3) the cost of time incurred by choosing that location. It is clear that the gross prices of housing consumption may vary for different bundles of housing, as well as for different residential locations.

It should be noted, moreover, that the gross prices of housing may vary considerably for households with differing travel requirements. The usefulness of the model is enhanced by the fact that most of the variation in travel costs among alternative locations is attributable to a few kinds of trips, particularly trips to and from work. The latter account for 40 percent of all trips originating or ending at residences; and because they are longer on the average than other kinds of urban trips, they comprise an even larger percentage of miles of intraurban travel by American households. Many of the remaining trips, such as shopping trips, are made to highly ubiquitous locations.


17This estimate is based on origination data for thirty-eight metropolitan areas reported in Robert E. Schmidt and M. Earl Campbell, Highway Traffic Estimation
It is probably not overly unrealistic to treat the costs of these ubiquitous trips as invariant with location. The most important variation in the component of travel costs by bundle type probably exists between neighborhoods where car ownership is close to a necessity and those where it is merely a convenience. Whether a consumer who chooses the latter location also decides to buy a car will depend primarily on his trip demands, and on whether his trip destinations—particularly, his workplace—are well served by public transit.

The fact that the variation in gross prices for most households is primarily attributable to work trips allows a highly important analytical simplification of the theory, one which increases its usefulness and permits a variety of empirical tests. If only work trips are included in the definition of gross prices, the minimum gross price of each bundle will depend solely on workplace location and the value of travel time. This means that the consumption of housing bundles and locations should vary systematically by workplace.

The principal sources of variation in gross prices among households employed at the same workplace are caused by different travel demands and differences in the value that households assign to their travel time. The most important differences in travel demands probably arise from differences in family labor-force participation and in the frequency of social-recreational trips to urban centers. The largest sources of variation in the household valuation of travel time are probably differences in wage levels or earnings and in the ease with which the worker can vary his workweek. Families with two wage earners making daily work trips to centers located in the same general part of the region would significantly increase the travel-cost component of the gross prices. Households with no employed members would lack the largest, or at least most predictable, source of variation in gross prices. Though recognition of these differences in household travel demands greatly increases the difficulty of implementing the theory and applying it in an operational way, its essential character is unaffected.

The minimum gross price of each bundle for each household serves two functions in our revised theory. First, the gross prices of bundles are important determinants of the choice of housing bundle. Second, once the household has decided which bundle to consume, the gross prices determine where it will reside. The way in which gross prices affect

(Saugatuck, Conn.: The Eno Foundation for Highway Traffic Control, 1956) Table 11-4. The statistics for the remaining trip purposes are: (1) business, 7 percent; (2) social-recreation, 21 percent; (3) shopping, 12 percent; (4) school, 5 percent; and (5) all others, 11 percent.

18Moses, "Income, Leisure, and Wage Pressure."
these decisions, and particularly the effect of workplace location, can easily be shown by simple graphical analysis.

SOME SIMPLE ANALYTICS

The use of gross prices makes it relatively easy to examine the effect that alternative workplace locations and variations in travel costs have upon the type of housing consumed and its location. Some geometric examples are useful in illustrating the power of this simple theory of individual housing consumers.\(^{19}\)

Figure 2-1 depicts gradients of bundle prices for a hypothetical urban region comprised of many workplaces. The origin in Figure 2-1 refers to an employment centroid, at which, by coincidence, the prices of all bundles are highest. The housing-bundle price gradients shown in Figure 2-1 resemble the location rent gradients used in traditional demand analysis.

theories, in that they decline with distance from the employment centroid. (In most traditional theories, this corresponds to the single employment location.) It should be clearly understood that regular shapes depicted for the housing-bundle price gradients are not required by the theory and are used merely to simplify the diagrams. The framework can accommodate metropolitan surfaces of bundle prices of any shape or complexity.

The prices shown in Figure 2-1 are what might be termed net housing prices. To obtain gross prices, the household’s monthly travel costs must be added to the net housing prices. Figure 2-2 graphs monthly travel costs of two households employed at the employment centroid, drawn on the assumption that journey-to-work costs are the only element of monthly travel costs that vary with residence location, and that household A has a higher travel cost per mile (hour) than household B. This difference could arise if trip-makers in household A value their travel time at a higher rate, or if members of household A make more trips than members of household B. For example, household A might have two members employed at the centroid and household B only one.

In Figure 2-3, we add the bundle prices and transport costs to obtain gross price surfaces for households A and B. From Figure 2-3, it is apparent: (1) that there is a minimum cost location for each household type and each bundle, and (2) that within each bundle type, the household with higher monthly travel costs will face higher gross prices and
Figure 2-3
Gross Prices

Figure 2-4
House Prices and Travel Costs for Two Workplaces
live closer to work. This does not mean, however, that household B will in fact live further from work than household A. As is evident from Figure 2-3, this conclusion will hold only if households A and B choose the same housing bundle.

In Figures 2-4 and 2-5, we illustrate the gross-price computations for household A and for a third household, C, which has the same travel costs per mile as household A, but where the worker(s) is employed at an outlying workplace, W, rather than at the employment centroid. The curves shown in Figure 2-4 illustrate several important propositions: (1) the gross price of each bundle type will differ for otherwise identical households employed at different workplaces; (2) households A and C, if they consume the same housing bundle (which may be unlikely), will live at different residence locations and travel different distances to and from work; (3) relative gross prices will differ for households A and C; indeed the gross prices need not even have the same ordinal rankings.

The above analysis, simple though it may be, illustrates that a number of important and quite specific conclusions and predictions can be obtained from our model if a few parameters can be specified even in general terms. These parameters are: (1) preferences or demands of
households with different sociodemographic characteristics for different bundles; (2) the general shapes of bundle price surfaces within urban regions; and (3) the variation in monthly travel costs by households with differing socioeconomic and sociodemographic characteristics. If even crude answers can be obtained for these questions (and we believe reasonable answers can be provided), the theory of the individual housing consumer can provide useful, quite specific, and testable hypotheses about the behavior of housing consumers.

Quigley's analysis of the Pittsburgh housing market employs gross prices in precisely the way suggested by the preceding analysis. Using a sample of 3,000 households in the Pittsburgh region which have recently moved, Quigley computes the cost of work trips from each of 333 workplaces to each of 136 residential areas and infers the gross prices for 18 types of rental housing bundles in each of these residence zones. Then he obtains minimum gross prices for each of the 18 types of rental housing bundles for each household. These minimum gross prices and their spatial locations varied for housing bundle (18 types), for employment location (333 discrete zones), and by income class (6 categories). When the gross prices were used in a series of econometrically estimated demand equations, the demand for housing bundles was found to be significantly responsive to changes in the prices of that bundle and of possible substitutes.

Quigley also investigated the second role of these gross prices in our revised theory of urban spatial structure: their position as determinants of residential location. His results indicated that more than 40 percent of rental households and more than 50 percent of owner households selected a residential location that was within the lowest 10 percent of their gross-price distributions.

HOUSING INVESTMENT AND NEIGHBORHOOD CHANGE

Recognition of: (1) the importance of housing stocks, (2) accessibility to many workplaces, and (3) the role of goods not produced by

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20This entire analysis was reproduced for several alternative values of commuter time.

21The minimum gross price of each housing bundle was represented by the price required for each household to purchase any one of the units in the cheapest 5 percent of the gross-price distribution facing it.

22Quigley, "Housing Demand."

23Quigley, "Residential Location."
competitive markets, provides insight into the dynamics of neighborhood change.

Most neighborhood attributes are produced by the collective actions of consumers through local governments, by the aggregation of locational decisions by individual households, by the investment decisions of housing suppliers, or by the interaction of these several determinants. This does not mean, however, that neighborhood characteristics are immutable.

Rapid neighborhood change is common. Some neighborhoods are improved in quality by means of stock transformations and housing investment, while others are allowed to decline by undermaintenance. Similarly, levels of income and sociodemographic composition often change rapidly. Surprisingly little is known about these neighborhood dynamics, and the role of market forces is particularly unclear. Before a fully adequate theory of urban spatial structure can be worked out, a better understanding of these aspects of neighborhood dynamics is essential.

Although we cannot provide a unified theory of neighborhood dynamics, we can identify some important ingredients for such a theory. First, it is clear that the expectations of housing demanders and investors concerning future neighborhood characteristics are as important, if not more important, than current conditions. Home buyers are likely to be more strongly influenced by expectations than renters in choosing housing bundles, inasmuch as the former are also housing investors and have higher transactions and moving costs than the latter. Moreover, home ownership is most attractive to those households who expect to remain in the same residential area for several years and to households who strongly value this kind of stability.

While a variety of considerations affect expectations concerning residential neighborhoods, most households undoubtedly rely fairly heavily on simple extrapolations of recent trends. This suggests that analyses of housing demand should probably begin by projecting simple trends and by using such measurements in defining bundles. The amounts and types of new investment in each neighborhood may be the best indicators of these changes, both because they are visible and because they reflect the assessments of other owners and investors concerning the future of the area. Both the physical appearance of the

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neighborhood and what it suggests in terms of a pattern of neighborhood improvement, stability, or disinvestment are likely to have important effects on the views of housing consumers.

The sociodemographic structure may be an important factor in understanding neighborhood dynamics and change. In many neighborhoods, all or most structures were built within a few years of each other. Moreover, there is often a predominant type or style of structure. Often, these similar dwellings were first occupied at almost the same time by new households of similar age, income, and family composition. As a result, many neighborhoods were initially relatively uniform in terms of housing and household characteristics. This suggests that changes in the composition of resident populations by neighborhood may be quite "lumpy" and might be analyzed better as a cascade than as a steady-state process. Some neighborhoods, particularly owner-occupied ones, may appear quite stable for long periods of time. Then, in response to life-cycle influences, they may change completely within a few years as a generation of residents die, dissolve their households, or change residences. Once the process has begun, it reinforces itself as those left behind have less and less reason to remain. When this occurs, the neighborhood can change character and socioeconomic composition very rapidly.25

Similar socioeconomic composition is by no means the only kind of homogeneity found in urban neighborhoods. Many such neighborhoods are rather uniform in terms of the physical characteristics of the structures located there. These structural characteristics and neighborhood arrangements may determine which kinds of stock transformations are feasible or economic. If important differences exist in stocks of this kind—defined either in terms of characteristics of individual structures or neighborhood arrangements—these manifestations of housing-stock heterogeneity will act as a strong magnet for certain types of investment (or disinvestment) and will be an important force in creating particular types of neighborhoods.

Assume, for example, that because of changes in the composition of the urban labor force or in its aggregate distribution, a latent demand develops for three-bedroom, high-quality housing bundles at a central location. Assume further, that there are several neighborhoods now devoted to the production of three-bedroom low-quality housing bundles, but that the structures in one of these neighborhoods can be more cheaply transformed into high-quality bundles than those found elsewhere. These lower transformation costs will attract pioneers, who will

begin to transform three-bedroom units from low-quality to high-quality use. Once the neighborhood is well identified as \textit{the} transitional neighborhood, it is likely that part or even most of these savings will be captured by the existing property owners.

A similar result would be obtained with even more certainty if some characteristics of the neighborhood as a whole make it more suitable for the production of the high-quality bundles, i.e., its overall density of structures, the favorable layout of local streets, the presence of public open space or similar amenities. Since it would be expensive for individual housing suppliers to provide such amenities in other neighborhoods, the presence of these amenities can be an additional force in allocating housing investment among neighborhoods. Greater attention to the nature of these housing production relationships might improve our ability to describe different neighborhoods and to predict neighborhood change.

Local services and local accessibility considerations can similarly affect the likely futures of various, otherwise identical neighborhoods. The growth of high-income employment at a major subcenter will increase prices in residential neighborhoods in the immediate vicinity, will discourage many households from choosing these locations, and will attract households who value their time highly. Prosperous young singles and couples, older couples, and others with minimal demands for neighborhood services will typically predominate in the first wave of new residents. As these households become more numerous, they will begin to upgrade the neighborhood, and other types of households, which are more concerned about neighborhood attributes, will follow. Again, once the neighborhood becomes clearly identified as being on the upswing, this will provide a strong impetus to further development in the same direction.

**SOME KEY ASSUMPTIONS**

Although we believe that the revised theory outlined in this chapter represents a more useful and more correct view of the housing market than traditional theories, it does depend on several strong behavioral assumptions of its own. It is appropriate to consider them now.

Perhaps the most crucial of these assumptions is what might be termed the "workplace-dominance" assumption. We assume that intra-metropolitan variations in gross prices (housing cost plus transportation costs at the least-cost location for each housing bundle) result primarily from differences in workplace location and from spatially distributed housing-attribute prices. Identical workers employed at different work-
places are confronted with different location-rent—transportation-cost tradeoffs for each housing type. These prices determine both the household’s choice among housing types, which depends on its tastes, incomes, and relative gross prices; and the household’s residence location, which is dictated by the least-cost location for consuming its preferred housing type. However, in order for households to compute workplace-specific housing costs, they must have a predetermined workplace. Therefore, this alternative theory of urban spatial structure assumes that households decide their place of work before choosing the type and location of their housing. Because of its critical role in our theory, we must give serious consideration to the question of which way the causation runs between job and residence choices. Traditional theories avoid this knotty theoretical problem altogether by assuming that there is only one workplace.

The workplace-dominance assumption is hardly defensible on narrow theoretical grounds. Obviously, workers prefer some jobs to others and wages may differ from one workplace to another. The authors of traditional theories have pointed out that, in equilibrium, local workers must obtain lower money wages than centrally employed workers in order to compensate the centrally employed for the higher housing and/or travel costs which they incur.\(^{26}\)

These deficiencies of our revised theory can be quite easily corrected if we are prepared to make it sufficiently general and to proceed to a sufficiently high level of abstraction. The characteristics and money wage of each job can be specified, and households can be depicted as simultaneously choosing a job (a workplace location and a money wage), a housing type and residential location, a time spent in traveling, and all other goods so as to maximize their utility. It is not difficult to specify formally such a general utility-maximization problem and to describe the general properties of the solution. Unfortunately, the resulting solutions are so general as to provide little useful insight.

In our view, a more productive approach is to make the stronger behavioral assumption that workplaces for each household are predetermined. If this assumption can be empirically supported, it yields specific conclusions about urban growth and development, and the motivating philosophy behind it does not differ from that which led to the single-workplace and long-run equilibrium assumptions in traditional theories of urban spatial structure. The assumption that workplaces are predetermined is less restrictive than the single-workplace assumption of traditional theories.

Theoretical support for the proposition can be found in the notion

\(^{26}\)For example, Muth, Cities and Housing, p. 42.
that job choice and, hence, workplace location, depend upon particular training or aptitude and are most important to the household's economic well-being. Hence, particular job and work site choice predates residential-location choice. In addition, it can be shown that under many circumstances, identical logical and empirical results would be obtained if households reversed the process, i.e., if they had fixed residences and chose workplace locations so as to maximize their real incomes (money wage minus travel costs). Finally, empirical tests of the workplace-dominance assumption and of the theory of residential location derived from it provide considerable support for its validity.

Indirect empirical support for the workplace-dominance assumption is of two kinds: (1) evidence on the effect of changes in workplace location on household decisions to move; and (2) evidence on the effect of workplace-specific housing costs on the types and location of housing chosen by spending units.

Evidence of the first kind can be inferred from studies of household moving behavior. Several such studies have attempted to examine the effect of job changes on households' decisions to relocate. Most mobility studies are virtually unanimous in the conclusion that intrametropolitan changes in workplace locations have little or no effect on household moving decisions. For example, Goldstein and Mayer conclude that "intra-urban residential moves are not associated with changes in job location." Rossi is more cautious but emphasizes the life-cycle aspects of moving behavior to the virtual exclusion of employment location or job changes. The near universality of agreement on the question is illustrated by a 1968 review article by J. W. Simmons, who after a careful review of studies of mover behavior states, "all studies reject job location as an important reason for moving."

The unanimity of these views is disturbing; if the conclusions are correct, they undermine the workplace-dominance assumption. Of course, it would be possible for households to employ the calculus outlined above when house hunting initially, even if they do not move in response to job changes. But, unless households adjust their residence choices to significant changes in job location, the empirical and theoretical bases for the model presented above are seriously shaken. Households change jobs frequently, and any initial explanatory power provided by the model would be weakened over time if the households then

TABLE 2-2
Moving Rates by Job-Change Characteristics and by Kind of Move for San Francisco Area Households

<table>
<thead>
<tr>
<th>Job-Change Characteristics</th>
<th>All Moves</th>
<th>Within Tract</th>
<th>Outside Tract</th>
</tr>
</thead>
<tbody>
<tr>
<td>No change</td>
<td>.111</td>
<td>.015</td>
<td>.096</td>
</tr>
<tr>
<td>Change within zone</td>
<td>.170</td>
<td>.037</td>
<td>.133</td>
</tr>
<tr>
<td>Change outside zone</td>
<td>.280</td>
<td>.027</td>
<td>.253</td>
</tr>
</tbody>
</table>


fail to respond to significant changes in job location by choosing a more suitable residence location.

Careful examination of these studies of intrametropolitan mobility, however, reveals that most have considered job changes almost as an afterthought. As a result, the effects of job changes on residential location are difficult, if not impossible, to isolate. In contrast, recent NBER research by Brown and Kain, using more suitable data and methodology, strongly supports the workplace-dominance assumption. The employment and residence histories analyzed by Brown and Kain indicate that significant intrametropolitan workplace changes do cause households to change residential location.

Shown in Table 2-2 are mobility rates for San Francisco households from the Brown and Kain study. These mobility rates are the proportions of each of three types of households which moved during a single year. The three are: (1) those which had no job change within an eighteen-month period—twelve months prior to, and six months after, July 1 of each year, (2) those which changed jobs within this period but continued to be employed within the same workplace zone (these zones are quite small; there are 290 in the San Francisco region), and (3) those which changed jobs during the period and took a job in another workplace zone. The data further distinguish between short (within the same census tract) and long (outside of the census tract) residential moves.

The results indicate that job changes have little association with short residential moves: the rates are uniformly low, ranging from a moving probability of .02 for households with no job change to .04 for households which changed jobs within the same workplace zone. The association between job change and the rate of long-distance intrametro-
politan moving is, by comparison, striking. The probability of a house-
hold's moving from its census tract of residence is about .10 for house-
holds with no job change, about .13 for households who change to
nearby jobs, and .25 for households who take jobs in another workplace
zone.

Further evidence of the effect of job changes on household moving
behavior is provided by Table 2-3, which shows moving rates for house-
holds which changed jobs (both long- and short-distance job change) and
for those which did not change jobs. Mibility rates are presented by age
of head and tenure before the move, the two most important determin-
ants of moving identified by earlier studies. From Table 2-3, it appears
that job changes have a substantial effect on moving rates, even when
tenure and age of head are held constant. For example, the probability of
a young (less than thirty years of age) homeowner's moving in a particu-
lar year is nearly twice as large if he changes his job within the region as if
he does not, i.e., .140 versus .076.

Statistics on changes in travel time between home and work for
households with job changes provide further support for the hypothesis
that households do change their residences in response to job changes
that significantly alter their gross housing prices. Again, two groups can
be identified: (1) households which changed both their jobs and their
residences within the region, and (2) households which changed their
jobs but not their residences. For the first category, a comparison of
mean travel times between the old workplace and the old residence and
between the new workplace and the old residence indicates that, on the
average, changes in job location would have increased both the distance
and travel time between work and home if the households had not

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>Renters</th>
<th></th>
<th>Owners</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Job Change</td>
<td>Job Change</td>
<td>No Job Change</td>
<td>Job Change</td>
</tr>
<tr>
<td>30</td>
<td>.408</td>
<td>.526</td>
<td>.076</td>
<td>.140</td>
</tr>
<tr>
<td>30-60</td>
<td>.232</td>
<td>.423</td>
<td>.050</td>
<td>.089</td>
</tr>
<tr>
<td>60+</td>
<td>.102</td>
<td>.228</td>
<td>.025</td>
<td>.041</td>
</tr>
</tbody>
</table>

SOURCE: H. James Brown and John F. Kain, "Moving Behavior of
San Francisco Households" in "Supporting Empirical Studies" vol. 2 of
"The NBER Urban Simulation Model," John F. Kain, ed., processed
moved. For those who changed both their workplace and their residence, mean travel time between the old workplace and the old residence was 17.2 minutes; but between the new workplace and the old residence, the travel time was nearly 7 minutes longer. After change in residence location, however, the difference is less than 2 minutes. Households which did not change their residences in response to a workplace change lived, on the average, closer to their residences after the job change than before.

Evidence of the effect of workplace-specific housing expense on housing choices must also come primarily from NBER studies. However, the first systematic evidence that we are aware of was provided by John F. Kain in a series of papers based on analysis of origin and destination data from Detroit and Chicago. More rigorous tests of the hypothesis have been provided by a series of NBER studies by Dresch (Detroit), Straszheim (San Francisco), Brown-Kain (San Francisco-Oakland), Ingram (Pittsburgh), and Quigley (Pittsburgh). All of these studies find strong evidence that the location of the household's workplace systematically affects the choice of residence site and housing type but each represents housing expense differently and uses a somewhat different representation of the housing bundle.

Straszheim, for example, who estimates demand functions for several housing attributes, includes the minimum expected housing price (estimated from the attributive price equations which we referred to previously) 20 minutes from the workplace as the measure of workplace-specific housing prices. He also includes the head of household's actual commuting time in his demand functions.

Brown and Kain, because their sample is relatively small, use dummy variables for six large workplace zones to represent intrametropolitan variations in housing expense. In their formulation, discrete housing types (bundles) are used to represent housing services, and equations are estimated relating the probability of choice among housing types to household income, socioeconomic characteristics, and the proxies for housing-price variation. Ingram and Dresch also represent

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31 A fairly complete presentation of the analyses by Brown and Kain and by Dresch is presented in Chapter 8 and Appendix B of Gregory K. Ingram, John F. Kain, and J. Royce Ginn, *The Detroit Prototype of the NBER Urban Simulation Model* (New York, National Bureau of Economic Research, 1972); the remaining analyses are presented in Gregory K. Ingram, "A Simulation Model of a Metropolitan Housing Market" (Ph.D. diss., Harvard University, 1971); Quigley, "Residential Location"; and Straszheim, *Econometric Analysis*. 
housing services by discrete housing types in their individual analyses of the demand for housing in Pittsburgh and Detroit.

As has been discussed previously, Quigley defines gross price variables for individuals as the minimum total housing expenditure (housing plus commutation costs) which must be incurred to consume each of eighteen types of rental housing. The gross prices vary by workplace, reflecting accessibility to the stock of each housing type, and by income, reflecting the cost of commuting time. Quigley then includes these relative prices in attribute demand equations for rental units in Pittsburgh. The results suggest that demand for housing attributes is responsive to the intrametropolitan price variation resulting from workplace differences and their effects upon the gross price of housing consumption.

A second key behavioral assumption in our revised theory of urban spatial structure is that households, however implicitly, value their travel time at some dollar amount. This hourly travel-time cost is used by the households in computing their workplace-specific housing expenses. This assumption is not controversial; an extensive literature provides both theoretical and empirical justification. As is true of the workplace-dominance assumption, the assumption of predetermined value of travel time is a considerable simplification.

It is convenient to be able to speak of a fixed and predetermined unit value of travel time, but the underlying behavioral assumption needed is much weaker: all that is required is that households place some value on travel time and have some disutility of travel at all workplaces.

SUMMARY

The discussion presented in this chapter provides an alternative, more general, and, we believe, more useful view of urban housing markets. The revised theory which results from this alternative perspective has much in common with traditional theories of residential location. In particular, it starts with the assumption that a household chooses that quantity of housing services and that location which maximize its real income. One important difference between the revised and traditional theories is the former's conception of housing as a bundle of heterogeneous attributes, rather than as a homogeneous good. This

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distinction becomes important as soon as the possibility of significant departures from long-run equilibrium is acknowledged. In our revised theory, households base their decisions about which housing bundles to consume and where to reside on the combined housing and transport costs they would incur in consuming each housing bundle at its least-cost location to them. These combined costs, or gross prices, are the expected sum of outlays for the journey to work—including both money and time expenditures—and housing expenditures.

In some respects, this emphasis on tradeoffs between housing and commuting costs corresponds quite closely to traditional theories of residential location. Important differences arise, however, from our emphasis on the multiple perspectives of the housing market provided by multiple workplace locations and the heterogeneous nature of the housing stock. Each household employed at each workplace estimates the expected residential expenditure required to consume a particular housing bundle. Traditional theories of location assume that the prices of all housing inputs except land are the same everywhere in the urban area. Therefore, these theories obtain a single housing price gradient. Our alternative view of the housing market, which emphasizes the importance of stocks and the nonmarket production of many housing attributes, suggests, however, that many housing attributes are relatively fixed and earn quasi rents. Therefore, we expect the prices of housing attributes or bundles to exhibit irregular and quite complex patterns of spatial variation.

Housing capital is exceedingly durable, and many types of structures are all but impossible to transform. The market prices of these structures can deviate substantially from their cost of production and from one part of an urban area to another. Other housing attributes, such as police protection, garbage collection, and public schools are produced by an interaction between local decision-makers and neighborhood populations. Still others, such as the racial and socioeconomic characteristics of neighborhoods, are produced by the collective location decisions of individual households. A consequence of housing stock durability and the nonmarket provision of certain attributes is that there are generally different price surfaces for the various housing attributes or bundles.

Traditional theories, because they consider only long-run equilibrium solutions, provide no information about the process of stock adjustment or the time path of adjustment. This failure to consider dynamic adjustment mechanisms explicitly is a general weakness of economic theory and analysis. However, because capital stocks are especially important for urban housing markets, the weakness is particularly serious in the analysis of urban development.

Stocks of nonresidential and residential capital in cities are seldom
demolished and replaced by new structures. Furthermore, these stocks have a powerful effect both on the types of new investment and on its location. New construction is generally concentrated on those types of housing services that are not easily or cheaply produced from the existing stock of residential capital. Except where there are compelling locational advantages, new construction occurs on vacant land—most of which is found at the periphery of the built-up area. The result is that the spatial distribution of housing capital of different types depends on the timing of development and differs quite substantially from that which would occur if the city were built de novo each year, or if the full adjustment of the capital stock to changes in supply or demand conditions, as implied by the long-run equilibrium state, were observed.33

The empirical analyses presented in Chapters 5 through 10 represent a serious but modest attempt to address several of the issues raised by this alternative perspective on urban housing markets. Throughout the remaining chapters, we describe and quantify housing as a collection of particular residential attributes and analyze households' demand for housing attributes and the structure of housing prices in a single market which is not characterized by long-run equilibrium. However, our empirical analysis does not address the multiplicity of workplaces directly. This omission arises from the limited geographic coverage within the single metropolitan area implied by our sample.

APPENDIX TO CHAPTER 2:
THE EXISTENCE OF QUASI RENTS FOR HOUSING ATTRIBUTES

This appendix amplifies the discussion of spatial quasi rents presented in Chapter 2. Specifically, it provides a somewhat more rigorous demonstration of the simple proposition that when the housing stock is durable and heterogeneous, distinct surfaces of spatial quasi rents for individual housing attributes (or bundles of housing attributes) will typically exist, and these spatial quasi rents may persist for long periods of time.

To illustrate this important conclusion, we present a simple housing-market model which incorporates two possible residence locations ("city" and "suburb"), two possible workplaces ("city" and "suburb"), and two housing bundles ("high quality" and "low quality"). We assume moreover that: (1) housing capital is specific and can be used to

produce only one type of housing output, in our example, either high-quality or low-quality bundles; (2) it is possible to transform one kind of specific housing capital to another, but these transformations are relatively expensive; and (3) the housing capital has an infinite life and continues to produce a given level of housing output without any annual outlays.

The first two assumptions are fundamental to the general point that we are trying to illustrate. The third merely simplifies the analytics. Its relaxation would permit the "equilibrium" quasi rents indicated by the examples to erode over time. Obviously, the assumption that the specific housing-capital stocks have infinite lives does not conform to reality. However, it probably does less violence to reality than the opposite assumption of a full and instantaneous adjustment in specific and heterogeneous stocks of residential and nonresidential capital.

Using these three assumptions, it is possible to show that an increase in demand for one housing type can lead to several "equilibrium" price ratios. Moreover, generalization of the example to a larger number of residences and workplaces indicates that there could, in principle, exist a large number of "equilibrium" price ratios, if equilibrium is defined as a situation where no housing supplier has an incentive to convert units between submarkets and no household has an incentive to relocate.

The examples used assume that all individuals have identical incomes and preferences. Variations in incomes and preferences would produce still more stable "equilibrium" price ratios. The model assumes that the central neighborhoods are completely built up as a result of historical patterns of urban development. This is the case in American cities which, in general, have developed around a dominant center where most employment was located. During the nineteenth and early twentieth centuries, both residential and nonresidential development occurred in a more-or-less regular fashion around this dominant center.

The concentration of employment in central locations in the early periods insured that the prices of all housing bundles were higher in central than in suburban areas. Location rent differentials less than or equal to the savings in transportation costs for those residing in central locations could exist between the two localities.

Changes in production and transportation technology modified the location of employment, but the existing stock of residential capital was geographically fixed. Thus, due to substantial declines in central-area employment and to the growth of suburban employment, it is no longer possible to assert with certainty that the prices (quasi rents) of all housing bundles are higher in central than in suburban locations. In fact, if the difference in supply prices exceeds the difference in transport
costs, a market price differential may persist without causing households to move or housing suppliers to transform the stock.

Two simple examples illustrate these propositions. In the first, transformation costs for the specific housing capital equal the long-run equilibrium price differentials for high- and low-quality bundles. This transformation-cost assumption produces price ratios identical to those obtained in traditional, i.e., long-run equilibrium, theories. In the second, the cost of transforming specific initial capital stocks of one kind to another kind exceeds the initial differences in production cost. This assumption produces a variety of price ratios.

Case 1:

Suppose there are \( n \) city and \( m \) suburban workers with identical demand curves for high (\( H \)) and low (\( L \)) quality bundles. Travel cost between the city and suburbs is \( d \) dollars. The prices of \( H \) bundles and \( L \) bundles in the city are \( P_{HC} \) and \( P_{LC} \). The prices in the suburbs are \( P_{HS} \) and \( P_{LS} \).

If there are \( N \) residences in the city and \( n > N \), some city workers will be forced to live in the suburbs; equilibrium requires that each city worker be just as well off in either location. If city workers inhabit both types of housing at both locations, the relationship between city and suburban prices must be

\[
\begin{align*}
P_{HC} &= P_{HS} + d \\
P_{LC} &= P_{LS} + d
\end{align*}
\]

Assume, for example, that \( L \) bundles can be produced in the suburbs for 10 and \( H \) bundles can be produced for 20. Assume further that transport costs are 5 and that \( L \) bundles can be transformed to \( H \) bundles for 10 at either location, and from \( H \) bundles to \( L \) bundles for \(-10\) at either location. The costs of transformation are assumed equal to the difference in original production or supply costs.

In this case, the supply and demand relationships for city and suburban properties are shown in Figures 2-6a and 2-6b, respectively. The vertical axis measures the ratio of \( H \) to \( L \) bundle prices at either location; the percent of housing units that are of high quality in each zone is shown on the horizontal axis. A change in demand for \( H \) bundles (due, for example, to rise in income) from \( D_1 \) to \( D_2 \) will change the percentage of units of high quality at either location. In the city, housing units will be converted from \( L \) to \( H \) bundles. In the suburbs either new construction or conversion will take place. The relative price relationships will remain unchanged at both locations.
All \( m \) suburban workers will live in the suburbs. The city workers living in the suburbs will pay \( P_{HC} - d \) and \( P_{LC} - d \), respectively, for \( H \) and \( L \) bundles, and will be indifferent to location. Case 1 thus provides the long-run equilibrium view of the housing market; in Case 2, we consider what happens when initial specific capital stocks exist and conversion costs are large.
Case 2:

In Case 2, we assume that, once housing is in place at either location, it requires demolition and replacement to modify it.

High- and low-quality bundles can still be produced in the suburbs at $P_{HS} = 20$ and $P_{LS} = 10$, respectively. However, we assume that the cost of transforming $L$ bundles to $H$ bundles and $H$ bundles to $L$ bundles is described by Equations 3 and 4.

\begin{align*}
(3) \quad P_{HC} &= P_{LC} + 20 + 2 \\
(4) \quad P_{LC} &= P_{HC} + 10 + 2
\end{align*}

Both markets are initially in equilibrium, and the supply and demand relationships are those depicted in Figures 2-7a and 2-7b.

The lowest possible price that $L$ bundles in the city could sell for is 5+. If their price fell to 5 or less, suburban workers would move to the city. At a price greater than 5, the cost of producing $H$ bundles in the city would exceed 27. However, the selling price of $H$ bundles in the city could not rise above 25; otherwise city workers buying $H$ bundles would not be willing to live in the city.

In the city, where no vacant land exists, $H$ and $L$ bundles can only be produced by converting existing units. In the suburbs, either new construction or conversion is possible.

Similarly, the price of $H$ bundles in the city could not fall below 15;
if they sold for a lower price, suburban workers would move to the city. If the price of $H$ bundles in the city was 15 or more, the cost of producing $L$ bundles in the city would be equal to or more than 27.

However, the price of $L$ bundles in the city cannot rise above 15; otherwise city workers buying $L$ bundles would not be willing to live in the city.

Thus any of the wide range of price ratios shown by Equation 5 are equilibrium values.

$$\frac{15}{15} < \frac{P_{HC}}{P_{LC}} < \frac{25}{5}$$

In this simple case, where all workers have identical tastes and incomes, only three relative prices are possible: 15/15, 25/15 (the long-run equilibrium price ratio), and 25/5.

Figures 2-8a and 2-8b illustrate the possible equilibrium relative prices associated with an increase in demand for $H$ bundles. In the city, where there is no vacant land, the supply curve is perfectly inelastic for relative prices within the range 15/27 to 27/5, reflecting the costs of converting housing bundles. In the suburbs, where new construction is possible, the supply curve is elastic.

If the demand for $H$ bundles increases from $D_1$ to $D_2$ on the diagrams, the relative price of $L$ bundles in the city may decline. If the price of $L$ bundles declines below 15, all of the city workers buying $L$ bundles in the suburbs would compete for the $L$ bundles in the city. If, after the change in the demand curves, the switch of all those city workers buying $L$ bundles in the suburbs to the purchase of $L$ bundles in the city were not enough to fill the existing number of available $L$ units in
the city, the price of $L$ bundles in the city would fall to 5, the level necessary to attract suburban workers who consume $L$ bundles to city properties. The price would not fall below 5, however; because at this price, city $L$ bundles would be just as attractive as suburban $L$ bundles to those suburban workers who consume $L$ bundles.

If the price of $L$ bundles in the city fell below 15, indicating that all city workers consuming $L$ bundles lived in the city, the price of $H$...
bundles in the city could not fall below 25. A price for $H$ bundles below 25 would indicate that all city workers were able to locate in the city. Since $n > N$, this outcome is impossible.

A decline in the price of $H$ bundles would shift the city demand curve for $H$ bundles further to $D_3$ and would shift the suburban demand curve for $H$ bundles back to $D_3$. The result would be an "equilibrium" price ratio of 25/5 in the city and 20/10 in the suburbs.

If, on the other hand, the decline in demand for $L$ bundles in the city was not large enough to create more $L$ bundles in the city than the number of city workers consuming them, the price of $L$ bundles in the city would remain at 15. Since the price of $H$ bundles cannot exceed 25 in the city and no suburban workers consume $L$ bundles in the city, the relative price ratio in the city cannot change.

A price ratio of 25/15 would shift the demand curves in Figures 2-8a and 2-8b to $D_4$. The new demand curve for city properties will be identical to the original, and all the increase in demand will be accommodated by construction of $H$ bundles in the suburbs. The result would be "equilibrium" price ratios of 25/15 in the city and 20/10 in the suburbs.

These shifts in the demand curves, from $D_2$ to $D_3$ or from $D_2$ to $D_4$ in Figures 2-8a and 2-8b arise because workers at either location compare the prices of consuming each type of housing at each location and switch the location for consuming their preferred type of housing in response to price changes.

The mechanism which shifts the demand curves can be illustrated simply in the case of two workplaces and two kinds of housing. If individuals have identical preferences, a linear demand curve for the $n$ workers employed in the city may be expressed as

$$P(H) = a + b \frac{\min (P_{HC}, P_{HS} + d)}{\min (P_{LC}, P_{LS} + d)}$$

For the $m$ suburban workers:

$$P(H) = a + b \frac{\min (P_{HS}, P_{HC} + d)}{\min (P_{LS}, P_{LC} + d)}$$

$P(H)$ is the individual's probability of consuming $H$ bundles (regardless of location) and $a$ and $b$ are parameters.

Table 2-4 illustrates the demands of city and suburban workers, for $H$ and $L$ bundles in the city ($HC$ and $LC$) and for $H$ and $L$ bundles in the suburbs ($HS$ and $LS$) at four relative prices. 0 indicates no demand; "—" indicates indifference in location at city or suburbs for consumption of this type of housing; 1 indicates all demands for this type of housing will be expressed at this location.

Because switching can occur in the neighborhood of these price
TABLE 2-4
Demands of City and Suburban Workers for High-Quality and Low-Quality Bundles in the City and in the Suburbs at Four Relative Prices

<table>
<thead>
<tr>
<th>Prices $P_{HC} =$</th>
<th>(A) $P_{HS} + d$</th>
<th>(B) $P_{HS} - d$</th>
<th>(C) $P_{HS} + d$</th>
<th>(D) $P_{HS} - d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices $P_{LC} =$</td>
<td>$P_{LS} + d$</td>
<td>$P_{LS} - d$</td>
<td>$P_{LS} + d$</td>
<td>$P_{LS} - d$</td>
</tr>
</tbody>
</table>

| Workplace | Housing | |
|-----------|---------|---
| City      | $HC$    | $1$ | $1$ | $1$ |
|           | $LC$    | $1$ | $1$ | $1$ |
|           | $HS$    | $0$ | $0$ | $0$ |
|           | $LS$    | $1$ | $1$ | $1$ |
| Suburb    | $HC$    | $0$ | $0$ | $0$ |
|           | $LC$    | $0$ | $0$ | $0$ |
|           | $HS$    | $1$ | $1$ | $1$ |
|           | $LS$    | $1$ | $1$ | $1$ |

ratios, and workers at a given location will switch the location for consuming their preferred housing bundles, there are in fact different market demand curves associated with these price ratios. For example, the market demand curves for high-quality city units associated with the relative prices A, B, and C shown in Table 2-4 are:

Price ratio A

$\%HC = n \left[ a + b \frac{P_{HS} + d}{P_{LS} + d} \right] \frac{1}{2}$

Price ratio B

$\%HC = n \left[ a + b \frac{P_{HS} - d}{P_{LS} + d} \right] \frac{1}{2} + m \left[ a + b \frac{P_{HS}}{P_{LS} - d} \right] \frac{1}{2}$

Price ratio C

$\%HC = n \left[ a + b \frac{P_{HS} + d}{P_{LS} - d} \right] \frac{1}{2}$

where $\%HC$ is the percent of all city housing comprised of high-quality bundles.

Similarly, the market demand curves for high-quality suburban bundles associated with relative prices A, B, and C are:

Price ratio A

$\%HS = n \left[ a + b \frac{P_{HS} + d}{P_{LS} + d} \right] \frac{1}{2} + m \left[ a + b \frac{P_{HS}}{P_{LS}} \right] \frac{1}{2}$
Price ratio B

\[
\%HS = m \left[ a + b \frac{P_{HS}}{P_{LS}} \right]^\frac{1}{2}
\]

Price ratio C

\[
\%HS = n \left[ a + b \frac{P_{HS} + d}{P_{LS} - d} \right]^\frac{1}{2} + m \left[ a + b \frac{P_{HS}}{P_{LS}} \right]^\frac{1}{2}
\]

where \(\%HS\) is the percent of all suburban housing comprised of high-quality bundles.

Starting from the equilibrium relative-price relationship A (where \(P_{HC} = P_{HS} + d\) and \(P_{LC} = P_{LS} + d\)), suppose that there is an identical increase in everyone's demand for \(H\) bundles as a result of rising incomes. Let \(a\) increase by \(\Delta\). The demand for \(HC\) increases by \(n\Delta/2\) and the demand for \(HS\) increases by \(n\Delta/2 + m\). If the supply curve in the city is highly inelastic (transformation costs are large), the price ratio in the city must rise. However, the price of \(HC\), \(P_{HC}\), cannot rise since there are city workers living in \(H\) bundles in the suburbs and we have just postulated that demand for \(H\) bundles has risen.

If the corresponding decline in demand for \(L\) bundles associated with \(\Delta\) is large enough so that all city workers who now demand \(L\) bundles can live in the city and there is at least 1 vacant unit, the price of \(L\) bundles in the city will decline from \(P_{LS} + d\) to \(P_{LS} - d\) and the price ratio in the city will increase.

If the decline in demand for \(L\) bundles associated with \(\Delta\) is not large enough to create an excess supply of \(L\) bundles in the city relative to the demand by city workers for \(L\) bundles, the price ratio will remain unchanged and all the additional demand will be channeled to the suburbs.

With regard to the four possible outcomes shown in Table 2-4:

1. Price ratio D is clearly impossible in \(N < n\), that is, by the statement of the problem (there are not enough city houses so that all city workers may live there).

2. The price ratios (and market demand curves) A (and equations 6 and 9), B (and equations 7 and 10) or C (and equations 8 and 11) may be stable “equilibrium” price relationships. If a change in demand produces an excess supply of one kind of city unit relative to the new number of city demanders, the price ratio will change. The price ratio may be shifted from A (the long-run equilibrium price relationship) to C if there is an increase in demand for \(H\)-bundles; the price ratio may be shifted from A to B if there is an increase in demand for \(L\) bundles. If the change in demand is not sufficient to produce this excess supply, the price will not change in the city, the additional demand will be satisfied in the suburbs, and the long-run equilibrium price ratios will prevail.
The analyses presented in Case 2 can easily be generalized to consider additional workplaces and residences. For example, if there are three zones, two of which have excess workers, an increase in price demand for $H$ bundles can lead to two stable "equilibrium" price ratios besides the long-run equilibrium prices (one stable price ratio if the decrease in demand for $L$ bundles creates an excess supply of $L$ bundles relative to the number working in the same zone demanding $L$ bundles, and one additional stable price ratio if the decrease creates an excess supply large enough to accommodate the second zone's demand). The generalization to additional housing types changes none of the foregoing analysis. Switching among submarkets and locations insures that the same type of relative-price relationships will exist.

If individuals are not assumed to have the same incomes and preferences, the possibilities of stable relative prices which differ from long-run supply prices are more numerous. In the simple example with two workplaces and housing types, all individuals with the same workplace will not switch between suburban and city locations at the same relative price. This indicates that in response to an increase in demand for $H$ bundles, a stable equilibrium may exist somewhere in the interval between relative price $A (P_{HS} + d/P_{LS} + d)$ and $C (P_{HS} + d/P_{LS} - d)$, and the equilibrium price of $L$ in the city, $P_{LC}$, will merely be bounded by $P_{LS} + d \geq P_{LC} \geq P_{LS} - d$. 