Macroeconometric Models

### Part 2

#### FORECASTS AND ERROR DECOMPOSITION

E 3.27

Rs in Period $t + 1$ for GNP58 Components

<table>
<thead>
<tr>
<th>Period</th>
<th>Cross Product</th>
<th>Total</th>
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<tr>
<td>Constant Adjustment. 1st Q '53–4th Q '64</td>
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<tr>
<td>$R_8$</td>
<td>-0.60</td>
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<td>$R_8$</td>
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</tr>
<tr>
<td>$R_9$</td>
<td>-0.69</td>
<td>13.40</td>
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4.1 INTRODUCTION

The forecasting errors of the Wh... their sources via individual structural... Here we explain our procedure, forecasting errors into the following... attributable to the structural equation (b) the part attributable to the rest... due to the error in (a) after its reve... the error attributable to the forecast... equations fully by adjusting for these... the forecaster’s incorrect guesses... exogenous variable in the model: endogenous variables in multiperio... The breakdown of forecasting... answer a number of questions. We... primarily responsible for the errors in... of the errors systematically tend to... much does the simultaneous mani...
The Decomposition of Forecasting Error: Methodology

4.1 INTRODUCTION

The forecasting errors of the Wharton and OBE models are traced to their sources via individual structural equations in the next two chapters. Here we explain our procedure, which permits a decomposition of forecasting errors into the following components of error: (a) the part attributable to the structural equation explaining the variable in question; (b) the part attributable to the rest of the system, including the portion due to the error in (a) after its reverberation throughout the system; (c) the error attributable to the forecaster’s failure to correct the stochastic equations fully by adjusting for these problems; (d) the error caused by the forecaster’s incorrect guesses as to the future values of the exogenous variable in the model; and (e) the error caused by lagged endogenous variables in multiperiod forecasts.

The breakdown of forecasting errors along these lines enables us to answer a number of questions. Which sector or which specification is primarily responsible for the errors in the model? To what extent do some of the errors systematically tend to cancel or intensify each other? How much does the simultaneous manipulation of both the constant adjust-
ments and the exogenous values improve or hurt the forecasts? To what degree can error be attributed to lags in multiperiod forecasts?

In the presentation that follows we feature a decomposition of GNP, since the GNP series may be considered as an overall single measure of the forecasting performance of a model. The forecasting error in GNP is decomposed into the errors originating in the structural equations describing the endogenous determinants of GNP (these include its demand components), the components of disposable income (evaluated from the supply side), and the effect of the price level forecast. This procedure allows us to break down the observed forecast error into the five components listed above. We illustrate our arguments with a simple linear model. Nonlinearities in the solution of the model will be dealt with only in a heuristic way. It will be shown later that the effect of nonlinearity within the range of our interest appears to be inconsequential.

Our procedure uses the values of coefficients as estimated by the model builders. Thus, we trace the effects of observed error in individual equations on forecast error, using the specification of the model and the estimates of the structural parameters used for the forecast in question. Since the values of the true parameters for the equations are unknowable, they cannot be used. Furthermore, the adjustments of the individual equations by the econometric forecasters were based on the estimated parameters of the model. It should be noted that, while our procedure is appropriate for the systems forecasts we analyze, other procedures should be used to estimate what portion of forecast error is attributable to the inherent need for estimating structural parameters derived from a short sample period rather than using the true values of these parameters.¹

4.2 ILLUSTRATION WITH A SIMPLE LINEAR MODEL²

The first step is a condensed version of the model presented in Chapter 3, with the following structural equations:


² For material similar to some of the ideas in this section, see also P. Paulopoulou, A
aggregate econometric Models

prove or hurt the forecasts? To what
ags in multiperiod forecasts?

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The Decomposition of Forecasting Error: Methodology

Aggregate consumption:
\[ C_t = \alpha + \beta D_t + U_t \] (4.1)

Aggregate investment:
\[ I_t = \gamma Y_t + W_t \] (4.2)

Net of transfers and retained earnings:
\[ D_t = \xi Y_t + V_t \] (4.3)

National income identity:
\[ Y_t = C_t + I_t + G_t \] (4.4)

Disposable income identity:
\[ DI_t = Y_t + D_t - T_t \] (4.5)

Government expenditure:
\[ G_t = \text{Exogenous} \] (4.6)

Tax revenues:
\[ T_t = \text{Exogenous} \] (4.7)

From these one can derive the reduced form:

\[ C_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ \alpha(1 - \gamma) - \beta(1 - \gamma)(T_t + V_t) \right. \]
\[ + \beta(1 - \xi)(G_t + W_t) + (1 - \gamma)U_t \] (4.8)

\[ I_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ \alpha \gamma - \beta \gamma(T_t + V_t) + \gamma G_t \right. \]
\[ + [1 - \beta(1 - \xi)W_t + \gamma U_t] \] (4.9)

\[ Y_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ \alpha - \beta(T_t + V_t) + G_t + W_t + U_t \right] \] (4.10)

\[ D_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ \alpha \xi - \beta \xi T_t + (1 - \gamma - \beta)V_t \right. \]
\[ + \xi G_t + \xi W_t + \xi U_t \] (4.11)

APLE LINEAR MODEL

version of the model presented in
ural equations:

d Errors of Forecast of a Complete Econometric
78–92. J. W. Hooper and A. Zeilner, "The Error
timates of the Variance Covariance Matrix of
The Stochastic Simulation Technique," Ph.D.
as in this section, see also P. Pauloupolus. A

Statistical Model for the Greek Economy 1949–1959. Amsterdam, North-Holland Publishing
Let us compare four types of forecasts, which will differ from each other in two ways: (a) with respect to assumptions regarding knowledge of the values of the exogenous variables essential for the forecasts; and (b) with respect to the ad hoc adjustments made on the structural equations in specific forecasts.

In the first case we shall distinguish between ex post forecasts, where it is assumed that the exogenous values are known, and ex ante forecasts, where the forecaster provides his best guesses about future exogenous values. In the second case, we will show different adjustments made on the structural equations. These usually take the form of an additive disturbance inserted into the single equations in order to account for either the development of exogenous factors not included in the model in accordance with the forecaster’s judgments, or the patterns in the residuals of the particular equation and any shifts the forecaster may observe in the latest observable periods. This adjustment is usually termed “constant adjustment” because it is accomplished by changing the constant term (intercept) in the structural equations. Thus, if we consider two types of constant adjustments, we have four combinations of both types of forecasts:

1. Ex post plus constant adjustment type I;
2. Ex post plus constant adjustment type II;
3. Ex ante plus constant adjustment type I; and
4. Ex ante plus constant adjustment type II.

Let the ex post exogenous values be denoted by the superscript $p$, the ex ante guesses of the exogenous values by the superscript $a$, and their difference by $\delta$. Thus,

\[ T^a - T^p = \delta T, \quad T^p - T^a = -\delta T, \]
\[ G^a - G^p = \delta G, \quad G^p - G^a = -\delta G, \]

and, similarly,

\[ \delta W = W^I - W^II, \quad \delta U = U^I - U^II, \quad \delta V = V^I - V^II \]

for the constant adjustments types I and II.

Thus, the difference between these forecasts is obtained, with the appropriate change of the sign, by

\[
\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta(1 - \gamma)(\delta T + \delta V) + \beta(1 - \xi)(\delta G + \delta W) + (1 - \gamma)\delta U \right]
\]

The system of the last four equations, tracing the forecasting errors of the linear systems. In particular, it can be between the ex ante and ex post (without lags) for different constant adjustments. To calculate the difference between type I and ex ante with constant adjustment $-\delta T$ and $-\delta G$ (the ex post-ex ante discrepancies in constant adjustments of the estimated parameters $\beta$, $\gamma$, $\xi$, also investigate the pure effect of ex post-ex ante or differences in the constant adjustment $O$ for the former and $\delta T = \delta G = 0$ linear systems is that the total effect discrepancies and the constant adjustment sum of the pure effects. For instance, attributable solely to the ex ante exogenous variables in the system,

\[
\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta\xi \delta T \right]
\]

where $\delta T$ and $\delta G$ are the errors respectively, the tax revenues $T$. Similarly, if we want to investi...
The Decomposition of Forecasting Error: Methodology

\[ \delta l = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta\gamma\delta T + \delta V \right] + \gamma\delta G \\
+ \left[ 1 - \beta(1 - \xi) \right] \delta W + \gamma\delta U \]  
(4.13)

\[ \delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta\delta T + \delta V \right] + \delta G + \delta W + \delta U \]  
(4.14)

and

\[ \delta RE = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta\xi\delta T + (1 - \gamma - \beta)\delta V \right] + \xi\delta G + \xi\delta W + \xi\delta U \].

The system of the last four equations provides the framework for tracing the forecasting errors of the different forecasting methods in linear systems. In particular, it can be used to explain the differences between the ex ante and ex post single-period forecasts (in systems without lags) for different constant adjustments. For instance, if we want to calculate the difference between ex post with constant adjustments type I and ex ante with constant adjustments type II, we need to insert \(-\delta T\) and \(-\delta G\) (the ex post-ex ante discrepancies) and \(\delta U, \delta W,\) and \(\delta V\) (the discrepancies in constant adjustments) and then, knowing the values of the estimated parameters \(\beta, \gamma, \xi\), compute \(\delta C, \delta l, \delta Y\) and \(\delta RE\). We can also investigate the pure effect of either ex post versus ex ante forecast, or differences in the constant adjustments, by setting \(\delta U = \delta W = \delta V = 0\) for the former and \(\delta T = \delta G = 0\) for the latter. The special feature of linear systems is that the total effect of the ex post versus ex ante discrepancies and the constant adjustment differences is made up of the sum of the pure effects. For instance, the forecasting error in \(C\) attributable solely to the ex ante-ex post discrepancies in the two exogenous variables in the system, \(G\) and \(T\), is given by

\[ \delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left[ -\beta(1 - \xi)\delta T + \beta(1 - \xi)\delta G \right] \]  
(4.12a)

where \(\delta T\) and \(\delta G\) are the errors the forecaster made in guessing, respectively, the tax revenues \(T\) and government expenditures \(G\). Similarly, if we want to investigate the additional forecasting error...
The residuals on the left hand side of equations (4.16)—(4.18) are called "structural equation residuals." They are obtained by substituting the realized values for the endogenous variables appearing on the right hand side of structural equations (4.1)—(4.3), rather than the model solution values used in both ex post and ex ante forecasts. The concept of "structural equation residuals" (henceforth denoted by SER) has the advantage of isolating forecasting errors due to the specification of the equation under investigation from the error due to the simultaneity of the model. Furthermore, the SERs can be incorporated easily into the general framework developed above. For this, an alternative interpretation for the SERs should be adopted—namely, that if the SERs were simultaneously employed as constant adjustments for all stochastic equations in the model, the ex post forecasting errors would vanish. Thus, $\delta C$, $\delta D$, and $\delta Y$ now become the forecasting errors of the forecasts with the constant adjustments under investigation. That is, viewed here as the sum of the discrepancies between the forecasts $U_t^*$, $W_t^*$, $V_t^*$ and the constant adjustments $C_t$, $\alpha$, $\beta$, $\gamma$, $\delta W + (1 - \gamma)\delta U$. 

The combined effect of the forecasting error attributable to both ex ante-ex post discrepancies and to the two types of constant adjustments is obtained by adding equations (4.12a) and (4.12b). If, on the other hand, the model were nonlinear, the two additive effects would account only approximately for the combined effect, depending upon the degree of nonlinearity.

A special kind of constant adjustment, an artificial one designed to facilitate the forecasting error analysis, is obtained by computing for a particular $t$:

$$U_t^* = C_t - \alpha - \beta(Y_t - D_t - T_t)$$

$$W_t^* = I - \gamma T_t$$

and

$$V_t^* = D_t - \xi Y_t.$$  

(4.18)

The residuals on the left hand side of equations (4.16)—(4.18) are called "structural equation residuals." They are obtained by substituting the realized values for the endogenous variables appearing on the right hand side of structural equations (4.1)—(4.3), rather than the model solution values used in both ex post and ex ante forecasts. The concept of "structural equation residuals" (henceforth denoted by SER) has the advantage of isolating forecasting errors due to the specification of the equation under investigation from the error due to the simultaneity of the model. Furthermore, the SERs can be incorporated easily into the general framework developed above. For this, an alternative interpretation for the SERs should be adopted—namely, that if the SERs were simultaneously employed as constant adjustments for all stochastic equations in the model, the ex post forecasting errors would vanish. Thus, $\delta C$, $\delta D$, and $\delta Y$ now become the forecasting errors of the forecasts with the constant

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3 See Chapter 1, p. 9.
Macroeconometric Models

Beyond that of AR casts, we merely interpret type I and AR and "no constant adjustments," obtaining

\[(1 - \xi)\delta W + (1 - \gamma)\delta U.\]

The forecasting error attributable to both ex
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would vanish. Thus, \(\delta C, \delta l, \delta D\), and

4.3 A NONLINEAR EXAMPLE

The additive feature just described is lost in nonlinear systems. In these systems there are additional terms of interaction between exogenous variable discrepancies and constant adjustment differences, as well as interactions within these two groups.

One simple example of a nonlinear model will serve to illustrate the problems involved. Despite its simplicity it is typical of the nonlinearities that are usually present in macroeconometric models. The nonlinearities in the endogenous variables are introduced via a "money illusion" effect in the consumption function

\[C_t = \alpha^* + \beta^*D_t + \mu(1/P_t) + U_t.\]  
(4.19)

where \(P_t = \mu'[Y_t/(Y_t - Y_{MAX})]\) is the price level and \(Y_{MAX}\) is the maximum attainable gross national product in real terms, which is assumed to be exogenous in our simple model. That is, the price level is inversely related to the percentage that real GNP falls short of its highest attainable level at a particular time period.

If we solve now for real GNP we obtain:

\[Y_t = [1 - \beta^*(1 - \xi) - \gamma] - \gamma[\alpha^* - \beta^*(V_t + T_t)] + \mu^* + U_t + W_t + G_t + \mu^*Y_{MAX} = 0.\]  
(4.20)

where \(\mu^* = \mu'/\mu'\). and, thus.\(^*\)

\[Y_t = \frac{1}{1 - \beta^*(1 - \xi) - \gamma} \left(\alpha^* - \beta^*(V_t + T_t)\right) + \mu^* + U_t + W_t + G_t + \sqrt{[\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t]^2} - 4[1 - \beta^*(1 - \xi) - \gamma]\mu^*Y_{MAX} = 0.\]  
(4.21)

Another solution exists, too, with a negative sign preceding the square root, but usually only the one presented here will be economically feasible.
Forecasts with Quarterly Macroeconometric Models

The differences operators can be applied to the nonlinear model. Ignoring second order differencing and using the above relationship (4.21), we arrive at an expression of the discrepancy between two predicted values of real GNP with two different constant adjustments and different guesses about the values of the exogenous variables (YMAX was treated here as a definition in which no error can occur):

\[ \delta Y \approx \frac{-\beta^* (\delta V + \delta T) + \delta U + \delta W + \delta G}{2[1 - \beta^*(1 - \xi) - \gamma][1 - 1/(1 + \sqrt{1 - D})]} \]  

(4.22)

where \( D = \frac{4[1 - \beta^*(1 - \xi) - \gamma]\mu^* YMAX_t}{[\alpha^* + \mu^* - \beta^*(V_t + T_t) + W_t + G_t + U_t]^2} \).

Notice that by setting \( \mu^* = 0 \) (or \( YMAX_t = 0 \)) we eliminate the nonlinearity in the model. Indeed, equation (4.22) can be reduced to the expression for the corresponding linear system, i.e., to equation (4.14). More important, it shows that, although the multiplier, being a function of the random variables \( V, T, W, G, U, \) and \( UMAX \), is itself a random variable, it will hardly change with small variations in those variables. The next two chapters will demonstrate that the ex post-ex ante discrepancies varied only slightly when different constant adjustments were made. This last point is important because it allows us to decompose the total forecasting error into its additive components, and to ignore the nonlinear effect of the slight difference in “initial conditions” for alternative forecasts of the same equation.

4.4 STRUCTURAL EQUATION RESIDUALS VERSUS FORECASTING ERROR

In the second set of tables of Chapters 5 and 6, column I lists the SER minus the constant adjustments\(^5\) for the stochastic equations of all endogenous components of GNP and disposable income, respectively. In our notation these values can be properly expressed by \( \delta U \) and \( \delta W \) (the discrepancies between two adjustment procedures for the residuals of GNP’s endogenous components) and by \( \delta V \) (the discrepancies in the endogenous component of disposable income). To constant type I, and the constant adjustment type II in the above tables the values in the first column are forecasting errors, which are listed. In the second column we get

\[ \delta C - \delta U = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\ldots] \]

and

\[ \delta I = \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} \]

and the sum of both:

\[ \delta C + \delta I - \delta U - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\ldots] \]

Equations (4.23) to (4.25) show forecasting error into (a) the error of particular equations and (b) that of the former is the error attributable because it is calculated under the right-hand side of the indirect effect resulting from the constant adjustments, the reverberation throughout the system by the appropriate multipliers.

For instance, the indirect effect given by multiplying \( \delta U \) by \( \beta(1 - \xi) \) induced (indirect) effect of \( U \) on

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\(^5\) No constant adjustment is considered here a special case of constant adjustment, where the constant adjustment is equal to zero.
Macroeconometric Models

be applied to the nonlinear model, and using the above relationship of the discrepancy between two different constant adjustments values of the exogenous variables (in which no error can occur):

\[ + \delta U + \delta W + \delta G \]
\[ \frac{1}{1 - \beta(1 - \xi) - \gamma} \]
\[ - \gamma \mu \text{YMAX}_t \]
\[ + W_t + G_t + U_t \]
\[ \text{(or } \text{YMAX}_t = 0 \text{)} \]

we eliminate the equation (4.22) can be reduced to theear system, i.e., to equation (4.14), the multiplier, being a function of \( U \) and \( \text{UMAX} \), is itself a randomall variations in those variables. The that the ex post-ex ante discrepancy constant adjustments were made, it allows us to decompose the total components, and to ignore the erence in “initial conditions” foration.

RESIDUALS VERSUS FORE-

hapters 5 and 6, column I lists the 6 for the stochastic equations of all disposable income, respectively. In Early expressed by \( \delta U \) and \( \delta W \) (the int procedures for the residuals of by \( \delta V \) (the discrepancies in the is a special case of constant adjustment, where

The Decomposition of Forecasting Error: Methodology

endogenous component of disposable income). Now we assign the \( \text{SER} \) to constant type I, and the constant adjustment under consideration to constant adjustment type II in the definitions of the \( \delta \) operator. In our tables the values in the first column are subtracted from the corresponding forecasting errors, which are listed in the third column. Thus, in the second column we get

\[ \delta C - \delta U = \frac{1}{1 - \beta(1 - \xi) - \gamma} \]
\[ - \beta(1 - \gamma)\delta V \]
\[ + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta U \]  
(4.23)

and

\[ \delta I - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} \]
\[ - \beta\delta V + \delta W + \delta U \]  
(4.24)

and the sum of both:

\[ \delta C + \delta I - \delta U - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} \]
\[ - \beta\delta V + [\gamma + \beta(1 - \xi)](\delta W + \delta U) \]
\[ = - \beta\delta V + \delta W + \delta U \]
\[ 1 - \beta(1 - \xi) - \gamma = \delta W - \delta U. \]  
(4.25a)

Equations (4.23) to (4.25) show us how to decompose the ex post forecasting error into (a) the error due to the specifications of the particular equations and (b) that due to the simultaneity of the model. The former is the error attributable directly to the equation in question because it is calculated under the pretense that the “true” values of the variables on the right hand side of the equation are known. The latter is the indirect effect resulting from the reverberation of the \( \text{SER} \) adjusted by the constant adjustments, throughout the system. The effect of the reverberation throughout the system is given in equations (4.23)–(4.24) by the appropriate multipliers.

For instance, the indirect effect of \( \delta U \) on the consumption error is given by multiplying \( \delta U \) by \( \beta(1 - \xi)/[1 - \beta(1 - \xi) - \gamma] \); this is the induced (indirect) effect of \( U \) on consumption. When we add up all the
errors in the endogenous components of GNP (see equation 4.25a) we get the total (indirect plus direct) effect of the errors in these components less the direct effect, leaving only the indirect effect after their reverberation through the system. Moreover, the induced effect of the errors included in disposable income, \( -\delta V \) in our simple example, should be added. Therefore, the direct effects of the errors in the disposable income components are listed in the same set of tables. They are multiplied by the appropriate multipliers and then summed up.

### 4.5 THE PRICE EFFECT

Next, it is interesting to isolate the price effect from the real value effect on nominal GNP. To this end we use the formula

\[
GNP_t = P_t \cdot Y_t. \tag{4.26}
\]

and thus

\[
\delta GNP = (Y_t + \delta Y)\delta P + P\delta Y. \tag{4.27}
\]

We call the first term on the right hand side of (4.27) "error due to price." This error can be further decomposed into the errors in the components of real GNP and their corresponding prices. For this, let us denote the real GNP components by \( Y_t \) (i.e., \( Y = \sum Y_t \)) and their corresponding prices by \( P_t \). We have

\[
P = \frac{GNP}{Y} = \frac{\sum Y_t P_t}{\sum Y_t} \tag{4.28}
\]

and, applying the \( \delta \) operator, we get

\[
(P + \delta P) \left[ \sum (Y_t + \delta Y_t) \right] = \sum (Y_t + \delta Y_t)(P_t + \delta P_t). \tag{4.29}
\]

Subtracting (4.28) from (4.29), we get

\[
\delta P \sum (Y_t + \delta Y_t) = \sum \delta Y_t (P_t - P) + \sum (Y_t + \delta Y_t)\delta P_t. \tag{4.30}
\]

Unless we have peculiar situations in which large discrepancies in \( \delta Y_t \) are systematically associated with positive or negative discrepancies between the corresponding prices, \( P_t \) and \( P \), the first term on the right hand side of (4.30) can be ignored, and we finally get

\[
\delta P \sum (Y_t + \delta Y_t) \approx \sum (Y_t + \delta Y_t)\delta P_t. \tag{4.31}
\]

The left hand side is nothing more than the "error due to price." The expression on the right of (4.31) can be multiplied by the appropriate multipliers and then summed up.
The Decomposition of Forecasting Error: Methodology

The components of GNP (see equation 4.25a) we consider in our decompositions are the indirect effect of the errors in these components on the components of GNP. Moreover, the induced effect of the errors in the components of GNP is also considered. The errors in the prices of the components of GNP are also considered, and the error due to price is further decomposed into two sets of endogenous and exogenous prices. In the two sets of tables mentioned above we have further decomposed the "error due to price" into the exogenous and endogenous price effects, which serve as part of the ex post-ex ante error decomposition.

4.6 MULTIPERIOD FORECASTS

We have deliberately avoided lags in our simple model, but it is time now to introduce them. To illustrate the effect of lags on forecasting errors we resort to our simple linear models, but modify the consumption and investment equations to include lags:

\[ C_t = \alpha + \sum_{i=0}^{q} \beta_i D_i - \xi + U_t, \]
\[ I_t = \sum_{i=0}^{q} \gamma_i Y_{t-i} + W_t \]

where (say) \( q > p \) and \( \beta_{p+1} = \ldots = \beta_q = 0 \). The reduced form equation for \( Y \) becomes

\[ Y_t = \frac{1}{1 - \beta_0 (1 - \xi)} - \gamma_0 \left\{ \alpha + \sum_{i=1}^{q} \beta_i (1 - \xi) + \gamma_i \right\} \]
\[ - \sum_{i=0}^{q} \beta_i (V_{t-i} + T_{t-i}) + U_t + W_t + G_t \]

and, applying the differencing operator \( \delta \), we finally get

\[ \delta Y_t = \frac{1}{1 - \beta_0 (1 - \xi)} - \gamma_0 \left\{ \sum_{i=1}^{q} \delta Y_{t-i} [\beta_i (1 - \xi) + \gamma_i] \right\} \]
\[ - \sum_{i=0}^{q} \beta_i (\delta V_{t-i} + \delta T_{t-i}) + \delta U + \delta W + \delta G \]

That is, the multiperiod forecasting error in period \( t \) is the sum of (a) the errors made in \( Y \) in the earlier periods, weighted by the respective marginal propensities to consume times the leakage in retained earnings and marginal propensities to invest appropriate to each period, (b) the errors in the retained earnings equation and the errors made by the forecasters in guessing the values of the exogenous variable \( T \) in their ex post model.
Forecasts with Quarterly Macroeconometric Models

ante forecasts, weighted by the marginal propensity to consume, and (c) the contemporaneous errors in consumption, investment, and the exogenous variable government expenditure (in ex ante forecasts)—all multiplied by the multiplier.

Alternatively, equation (4.35) can be rewritten as

\[ \delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left\{ \sum_{i=1}^{\xi} \gamma_i (\delta Y_{t-i}) - \beta(\delta V + \delta T) + \sum_{i=1}^{\xi} \beta_i (\delta DI_{t-i}) + \delta U + \delta W + \delta G \right\} \]  

(4.36)

This, again, is a function of the earlier errors in real income and disposable income and of the contemporaneous errors in the real income and disposable income components, all weighted by the appropriate marginal propensities and the multiplier.

Our analytical scheme can be used to detect the effect of lags on forecasting errors. This is done by comparing a long-span forecast aimed at a particular period with shorter-span forecasts made later, and aimed at the same period. This comparison is useful for the error decomposition because the SERs pertaining to the same period are the same, irrespective of the forecasting span they represent, and thus the remaining error is due to the different lags. In order to put this formally we write

\[ \delta Y_{t(a)} = \frac{1}{1 - \beta(1 - \xi) - \gamma} \left\{ \sum_{i=1}^{r} \delta Y_{t(a)} [\beta_i (1 - \xi) + \delta Y_i] - \sum_{s=0}^{r-1} \beta_i (\delta V_{t(s-1)} + \delta U_t + \delta W_t + \delta G_t) \right\} \]  

(4.37)

where \( r = \text{min}(s, q) \) and the subscript in parentheses denotes the forecasting span, while the subscript preceding it denotes the jump-off period (the latest period for which data were available). Thus, the period for which the forecast was made is given by adding up both subscripts. Notice that by definition \( \delta Y_{t(0)} = 0 \).

Now we may decrease the forecasting span by one period and move the jump-off period ahead, since we wish to compare forecasts made for the same period. We obtain
The Decomposition of Forecasting Error: Methodology

\[
\delta Y_{t+1(s-1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^{s-1} \delta Y_{t+1(s-1-i)} | \beta_i(1 - \xi) + \gamma_i | \\
- \sum_{i=0}^{r-1} \beta_i(\delta V_{t+1(s-1-i)} + \delta T_{t+1(s-1-i)} + \delta U_{t+1(s-1-i)}) \right\} + \delta W_{t+1(s-1)} + \delta G_{t+1(s-1)}
\]

The formula can be easily extended to compare, say, also a four-quarter forecast with a first-quarter forecast made three quarters later and pertaining to the same quarter that the four-quarter forecast
Forecasts with Quarterly Macroeconometric Models

was aiming at, i.e.:

$$\delta Y_{t(4)} - \delta Y_{t+3(1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \left[ \beta_1(1 - \xi) + \gamma_1 \right] \delta Y_{t(3)} + \beta_2 \delta Y_{t(2)} + \gamma_3 \delta Y_{t(1)} \right\}.$$  \hspace{1cm} (4.41)

Thus, to isolate the pure effect of lags, the forecaster's wrong guesses as to the effects of different constant adjustments, there need only subtract the ex post "no change" in a later period from one pertaining to this will be demonstrated in the last set.
Thus, to isolate the pure effect of lags in multiperiod forecasts, with the effects of different constant adjustments in the various forecasts and of the forecaster's wrong guesses as to exogenous values filtered off, one need only subtract the ex post "no constant adjustment" forecast made in a later period from one pertaining to the same period but made earlier. This will be demonstrated in the last set of tables in Chapters 5 and 6.