I. NATIONAL INCOME AS A WELFARE INDEX

The per capita net national income used as a measure of the level of welfare is a typical index number. The computation of an index number involves application of some well-defined but essentially arbitrary conventional procedures to direct or indirect measurements of observed, or at least in principle observable, phenomena.

The conventional interpretation of net national income valued in some constant prices can be conveniently rationalized in terms of the ad hoc assumption that preferences of a representative average consumer can be described by a social utility function or a fixed set of well-behaving social indifference curves.

At this point observed or at least observable facts come in. The bundle of goods actually consumed by a representative individual has been obviously preferred—so goes the argument—to all the other alternative bundles that were accessible to him.

Under the special conditions of a market economy the set of all alternative bundles accessible to a representative consumer is uniquely determined by (a) the amounts of various goods that he has actually consumed and (b) the relative prices of these goods. The relative prices represent the marginal opportunity costs of each good in terms of every other good as seen from the point of their actual or potential consumer.

This factual information, combined with the before-mentioned ad hoc assumption concerning the existence of a "well-behaved" set of collective indifference lines, permits us to identify some of the bundles of goods that the representative consumer apparently judges to be less desirable than the particular bundle that he actually chose to use.
This analytical proposition constitutes the basic, not to say the sole, theoretical justification for interpreting the differences in per capita net national income—valued in fixed prices—as an index of changes in the level of average per capita welfare attained by a particular society in different years.

Goods acquired through other means than purchases at given prices on a free market can still be taken into account in computation of the conventional welfare index provided their opportunities costs—as perceived by the representative consumer—can be ascertained in some other way.

Much of the work aimed at inclusion of various nonmarketable components into the measure of natural income is centered on devising plausible methods of determining the imputed prices or more generally the opportunity costs of such goods.

In the light of what has been said above, the inclusion of pollutants and other kinds of environmental repercussions of economic activities into the measurement of the per capita national income as a welfare index requires answers to two sets of questions. One concerns the establishment of acceptable conventions pertaining to the inclusion of environmental repercussions into the conceptual framework of an all-embracing social utility function and a corresponding set of representative indifference curves. The other pertains to the actual physical description and measurement of the generation and elimination of pollutants by the economic system and the empirical determination of their opportunity costs in terms of ordinary goods and of each other.

The answer that one can give to these questions is critically influenced by the typically external nature of most environmental repercussions of economic activities and also by the fact that because of that measures aimed at abatement of their undesirable effects must in most instances be promulgated by the government.

Speaking in this context of collective indifference lines or preferences of a representative individual one must interpret such preference—at least so far as the environmental effects of economic activities are concerned—as being revealed not through private but rather through collective choice reflected in specific actions of the government.

Moreover, in case the conjectured opportunity costs reflected in the level of antipollution actions actually observed differ from the true opportunity costs, it is the former rather than the latter that would have to provide the base for proper weighing of pollution components.
to be included in a revised, more comprehensive, national income index.

Who would pretend to know what opportunity costs (if any) are being taken into account in the design of antipollution measures now actually being carried out in the United States?

Many of the contributors to the present symposium when touching upon problems of social valuation abandon the difficult revealed preference criteria in favor of a strictly axiomatic approach.

That solves the problem of welfare measurement as simply as Columbus solved his problem with the egg. One chooses ad hoc a social utility function which for some ethical or mathematical reason is appealing, inserts into it the levels of consumption of ordinary goods and net output of pollutants as they actually are and then compares the index of welfare thus attained with the highest number of points that could be reached if the society were to move to the optimal point along the empirically given opportunity costs frontier.

Who can decide, however, what social utility function one should finally choose? Certainly not the economists in their professional capacity!

II. ENLARGED INPUT-OUTPUT TABLE, STRUCTURAL COEFFICIENT, AND INTERSECTORAL DEPENDENCE

Exhibit 1 presents a schematic outline of an expanded input-output table that traces not only the intersectoral flows of ordinary commodities and services, but also the generation and elimination of pollutants. The conventional classification of economic activities and goods is accordingly expanded to include the names of various pollutants and activities aimed at their elimination.

The entries are organized in such a way as to have each column contain inputs and outputs controlled by the same autonomous set of structural relationships (i.e., by the same "cooking recipe"). The table is subdivided into rows and corresponding column strips. Each strip can be thought of as containing many rows and columns of figures not shown in this schematic presentation. Each of the rectangular intersections on a row and a column can be conveniently identified by two numbers.

All entries can be interpreted as representing physical quantities measured in appropriate physical units or indices of physical amounts. All dollar figures appearing in the table can be interpreted as such indices (with a defined or undefined base). Hence, the usual column sums are pointedly omitted.
EXHIBIT 1

Interindustrial Flows Expanded to Include the Generation and Elimination of Pollutants

<table>
<thead>
<tr>
<th></th>
<th>Industries 1</th>
<th>Pollution-eliminating Activities 2</th>
<th>Final Demand Sector 3</th>
<th>Totals 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>[s_I]</td>
<td>[s_g]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inputs of goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>into industries</td>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2)</td>
<td>[s_I]</td>
<td>[s_g]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inputs of goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>into pollution</td>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eliminating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>activities</td>
<td>(-)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delivery of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>goods to final</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>demand sector</td>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Empty)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total outputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>of goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>excluding the</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>amounts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>generated by the</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eliminating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>activities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1)</td>
<td>[s_g]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outputs of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollutants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>by industries (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td>[s_g]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elimination of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollutants by</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eliminating</td>
<td>(-)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>activities</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,3A)</td>
<td></td>
<td>[c_g]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outputs of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollutants by</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>final demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sector (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>connected with</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>of goods (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,3B)</td>
<td></td>
<td>[c_g]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outputs of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollutants by</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eliminating</td>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>activities (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,4)</td>
<td></td>
<td>[c_g]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net outputs of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollutants (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1)</td>
<td>[v_g]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inputs of primary</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>factors into</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>industries (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,2)</td>
<td>[v_g]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inputs of primary</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>factors into</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pollution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>eliminating</td>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>activities (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,3A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Empty)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,3B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delivery of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>primary factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>to final demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sector (+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total inputs of</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>primary factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(1,1) Inputs of (ordinary) goods into industries. Most of these goods are produced by industries listed on the left, but some might originate as the "by-product" in pollution-eliminating activities. See (1,3).

(1,2) Inputs of ordinary goods into various pollution-eliminating activities and outputs of ordinary goods (entered with a negative sign) generated as by-products of pollution-eliminating activities. Reprocessed materials, for example, are entered here.

(1,3A) Goods delivered to the final demand sector are entered along the main diagonal of this square. See (3,3B).

(1,4) These totals do not include amounts of ordinary goods (as their by-products) originating in the pollution-eliminating activities and thus represent the activity levels of ordinary industries.

(2,1) Each row shows the amounts of one particular pollutant generated by industries listed at the heads of different columns. In other words, pollutants are treated here the way by-products are treated in ordinary input-output tables.

(2,2) Along each row are entered—as negative numbers—the amounts of one particular pollutant eliminated by activities named at the heads of different columns. The amounts of a pollutant generated, as is often the case, in the process of elimination of some other pollutants are entered along its appropriate row as positive numbers.

(2,3A), (2,3B) For purely descriptive purposes the total amounts of various pollutants generated in the final demand sector can be presented in a single column. For purposes of structural analyses, however, these totals should be distributed among as many separate columns as there are different inputs, i.e., industrial product inputs and primary factor inputs, absorbed by the final demand sector. In the process of final consumption each of these inputs is liable to generate its own "column" of pollutants. The inputs of ordinary goods into the final demand sector are entered in rows along the main diagonal of the square formed by (1,2) and (2,2) considered together. It sounds rather complicated, but that is the price one has to pay for orderly bookkeeping.

(2,4) Each figure in this column is obtained by subtracting the sum of all negative from the sum of all positive entries appearing to the left along the entire length of the row. These are the undesirable net outputs of various pollutants delivered by the economic system to the final users alongside the desirable ordinary goods and primary factors entered in (1,3A) and (3,3B). Together they
Amenities and Disamenities of Economic Growth

make up the final results of economic activities upon which the welfare of the society supposedly depends.

(3,1), (3,2), (3,3B), (3,4) These contain a single row of aggregated value-added figures or several rows of physical or dollar figures depending on the amount of detail one wants to present.

III. STRUCTURAL RELATIONSHIPS AND OPPORTUNITY COSTS

The figures entered in each one of the separate columns of the first three vertical strips of the enlarged flow table can be interpreted as representing the inputs absorbed and outputs generated by one particular process carried on side by side with many other structurally different processes within the framework of the given economic system.

Assuming that the structure of each such process can be described in terms of a linear or at least linearized “cooking recipe,” the actual level of each output and each input as entered in the flow table can be interpreted as a product of two numbers: a technical coefficient and a number describing the level at which the process that absorbs that particular input or generates that particular output actually operates.

The levels of operation of ordinary industries are usually measured in terms of their principal output, while the level of operation of a pollution-eliminating activity can be conveniently described by the number of units of the specific pollutant that it eliminates. The levels of consumption activities that might generate pollution are described by the number of units of a particular good or primary factor delivered to the final demand sector.

The structural matrix of the economy—corresponding to the enlarged flow table described above—can be written in the following partitioned form:

\[
\begin{array}{c|ccc}
\hline
 & 1 & 2 & A \\
\hline
1 & [a_{ij}] & [a_{ij}] & \\
2 & [a_{ij}] & [a_{ij}] & [c_{ij}] \\
3 & & & [c_{ij}] \\
\hline
\end{array}
\]

The elements of each submatrix are technical input or output coefficients; they are defined concisely in the Mathematical Appendix, below.

While the input coefficient of ordinary goods can usually be derived
from the observed flows, information on the magnitude of the structural coefficient describing the generation and elimination of pollutants has in most instances to be obtained directly from technological sources. Combined with appropriate figures of the outputs of all pollution-generating activities, these coefficients provide a basis for estimation of the pollution flows.

In many, not to say in most, instances pollution is being combated not through the operation of separate elimination processes, but rather through the use of less polluting alternative techniques for production of ordinary goods. To incorporate such additional information the structural matrix would have to describe the input structure of some industrial and possibly even of some final demand sectors in terms of several alternative columns of input and output coefficients. The corresponding flow tables would, and actually do already in many instances, show for some sector two or more columns of input-output flows.

Without explaining in detail the mathematical formulation and solution of the system of input-output equations involved¹ it suffices here to say that on the basis of the information contained in an enlarged structural matrix of a given economy it would be possible to compute (and some such computations have already been made) the total factor inputs (measured in physical amounts or more or less aggregated "value added" dollars) required directly and indirectly: (a) to deliver to final users one additional unit of any particular good while keeping the deliveries of all the other goods and the net outputs of all pollutants constant; (b) to reduce by one unit the net output of any particular pollutant while keeping constant the net outputs of all the other pollutants and final deliveries of all goods.

This means that factual information contained in an enlarged structural matrix of a particular economy would permit us to compute in a rough and ready fashion the opportunity costs of an additional unit of any good and of an eliminated unit of the "net output" of each pollutant. The basic matrix of structural coefficients that governs the physical flows presented on the enlarged input-output table determines also a corresponding set of price-cost relationships.

The elimination of pollutants originating in various sectors can be paid for either directly by the final users or by the producing sectors

572 Amenities and Disamenities of Economic Growth

in which they are being generated. In the latter case the cost of doing so will obviously be included in the price of the finished product. I have explained elsewhere 2 how these institutionally determined parameters can be introduced in standard input-output formulation of balanced price-cost equations.

If the prices are expected to reflect the true opportunity costs of various goods (including the "products" of pollution-eliminating activities) to final users, they must cover the costs of eliminating all additional pollution generated in the process of their production. Otherwise in purchasing a useful good the consumer would receive, probably unwittingly, an additional delivery of undesirable pollutants. Hence, the system of prices to be used for purposes of welfare decisions should be computed on the assumption that each industry and each pollution-eliminating process bears the full cost of eliminating all pollutants generated by it. This of course does not imply that the actual institutional arrangement and consequently the actual pricing should necessarily be governed by the same principle, the more so that the distributional effect of such "pure" opportunity cost pricing might turn out to be undesirable.

Once the prices of all outputs (including those of all antipollution activities) have been determined, all entries in the expanded tables of interindustrial flows can be valued in dollars. Marginal totals can be entered not only at the end of each row but also at the bottom of each column. The outputs of all pollutants will be represented by negative dollar figures; the amounts of pollutants eliminated by positive dollar figures. In particular the net outputs of pollutants delivered to final users (2,4) will add up to a negative dollar figure. It can be interpreted as representing the upper limit of the amount that would have to be spent (but in fact was not spent) for this particular purpose if the final users decided to eliminate all pollution actually delivered to them.

MATHEMATICAL APPENDIX

The Numbering of Goods, Pollutants, and Primary Factors

1, 2, ..., i, ..., j, ..., n
goods.
n + 1, n + 2, ..., g, ..., k, ..., n + m
pollutants.
n + m + 1, n + m + 2, ..., f, ..., n + m + h
primary factors.

2 Ibid.
Technical Coefficients

\( a_{ij} \) — input of good \( i \) per unit of output of good \( j \) (produced by industry \( i \)).

\( a_{ij} \) — if \( > 0 \), input of good \( i \) per unit of eliminated pollutant \( g \); if \( < 0 \), output of good \( i \) per unit of eliminated pollutant \( g \).

\( a_{ij} \) — if \( > 0 \), output of pollutant \( g \) per unit of output of good \( i \) (produced by industry \( i \)); if \( < 0 \), input (productive use) of pollutant \( g \) per unit of output of good \( i \) (produced by industry \( i \)).

\( a_{kg} \) — output of pollutant \( g \) per unit of eliminated pollutant \( k \).

\( c_{g} \) — output of pollutant \( g \) generated in the final demand sector in the process of consuming one unit of good \( i \).

\( c_{g} \) — output of pollutant \( g \) generated in the final demand sector in the process of consuming one unit of the primary factor \( f \).

\( \nu_{if} \) — input of factor \( f \) per unit output of good \( i \) (produced by industry \( i \)).

\( \nu_{if} \) — input of factor \( f \) per unit of eliminated pollutant \( g \).

\( v_{i} \) — "value added" paid out by industry \( i \) per unit of its output.

\( v_{i} \) — "value added" paid out by the pollution-eliminating sector \( g \) per unit of pollution eliminated.

Vectors of Technical Coefficients

\([a_{ij}], [a_{g}], \text{etc.}\)

Variables

\( x_{i} \) — total output of good \( i \) by industry \( i \).

\( x_{g} \) — total amount of pollutant \( g \) eliminated by pollutant-eliminating activity \( g \).

\( x_{f} \) — total amount of factor \( f \) used in all sectors.

\( y_{i} \) — total amount of good \( i \) delivered to final demand.

\( y_{g} \) — net output of pollutant (delivered to final demand).

\( y_{f} \) — total amount of factor \( f \) delivered to final demand.

\( p_{i} \) — price of one unit of good produced by industry \( i \).

\( p_{g} \) — price of eliminating one unit of pollution \( g \) by sector \( g \).
**Amenities and Disamenities of Economic Growth**

*Vectors of Variables*

\[
\begin{align*}
X_1 &= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_j \\ \vdots \\ x_n \end{bmatrix} \\
X_2 &= \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \vdots \\ x_j \\ \vdots \\ x_k \\ \vdots \\ x_{n+m} \end{bmatrix} \\
X_3 &= \begin{bmatrix} x_{n+m+1} \\ x_{n+m+2} \\ \vdots \\ x_j \\ \vdots \\ x_{n+m} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
Y_1 &= \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \\ \vdots \\ y_j \\ \vdots \\ y_n \end{bmatrix} \\
Y_2 &= \begin{bmatrix} y_{n+1} \\ y_{n+2} \\ \vdots \\ y_j \\ \vdots \\ y_{n+m} \end{bmatrix} \\
Y_3 &= \begin{bmatrix} y_{n+m+1} \\ y_{n+m+2} \\ \vdots \\ y_j \\ \vdots \\ y_{n+m} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
V_1 &= \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \\ \vdots \\ v_j \\ \vdots \\ v_n \end{bmatrix} \\
V_2 &= \begin{bmatrix} v_{n+1} \\ v_{n+2} \\ \vdots \\ v_j \\ \vdots \\ v_{n+m} \end{bmatrix} \\
P_1 &= \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_i \\ \vdots \\ p_j \end{bmatrix} \\
P_2 &= \begin{bmatrix} p_{n+1} \\ p_{n+2} \\ \vdots \\ p_j \end{bmatrix}
\end{align*}
\]
Balance Equations

Each of the following matrix equations describes the balance between the outputs and the inputs entered in one of the three row strips of the enlarged input-output table.

Goods \([I - a_{0i}]X_1 - [a_{0j}]X_2 = Y_2\)

Pollutants \(-[a_{p1}]X_1 + [I - a_{p3}]X_2 = [c_{p1}]Y_1 - Y_2 + [c_{p7}]Y_3\) \((1)\)

Factors \(-[\nu_{p1}]X_1 - [\nu_{p2}]X_2 + X_3 = Y_3\)

The general solution of that system for the unknown \(x\)'s in terms of given \(y\)'s is

\[
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix} = \begin{bmatrix}
[I - a_{ij}]; & -[a_{0j}]; & 0 \\
-[a_{ip}]; & [I - a_{ip}]; & 0 \\
-\nu_{p1}; & -\nu_{p2}; & [I]
\end{bmatrix}^{-1}
\begin{bmatrix}
Y_1 \\
[c_{p1}]Y_1 - Y_2 + [c_{p7}]Y_3 \\
Y_3
\end{bmatrix}
\]

\((2)\)

Separating the effects of the three kinds of outputs delivered to the final demand sector and expressing the relationship \((2)\) in incremental terms:

\[
\begin{bmatrix}
\Delta X_1 \\
\Delta X_2 \\
\Delta X_3
\end{bmatrix} = \begin{bmatrix}
\Delta Y_1 \\
[c_{p1}]\Delta Y_1 \\
0
\end{bmatrix}^{-1}
\begin{bmatrix}
0 \\
-\Delta Y_2 \\
0
\end{bmatrix} + \begin{bmatrix}
M \\
M \\
M
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta Y_3
\end{bmatrix}
\]

\((3)\)

The inverse of the enlarged structural matrix of the economy appearing on the right-hand side is the same that appears in \((2)\) above.

The first and the third terms on the right-hand side describe the effect—on the output of goods \((\Delta X_1)\), the level of antipollution activities \((\Delta X_2)\) and on total factor inputs \((\Delta X_3)\)—of a given change in the final demand for goods \((\Delta Y_1)\) and, respectively, final demand for primary factors \((\Delta Y_2)\). These effects are computed on the assumption that the level of pollution-eliminating activities will be adjusted in such a way as to leave the net delivery of pollutants to final users unchanged (i.e., \(\Delta Y_2 = 0\)).

The second right-hand term shows what it would take—in total outputs of goods and total primary factor inputs—to reduce the delivery of (uneliminated) pollution to final users by the amount \(\Delta Y_2\), while holding the
Amenities and Disamenities of Economic Growth

deliveries of goods and factor services constant \((\Delta Y_1 = 0, \Delta Y_3 = 0)\).

For purposes of price-cost computations, all primary factor flows entered along the second row-strip of the expanded input-output table can be valued in dollars and consolidated into a single row of "value-added" figures. Accordingly the two coefficient matrices \([-v_{1i}]\) and \([v_{1P}]\) can be reduced to row vectors \(V_1\) and \(V_2\) of value-added coefficients.

If each industry and each antipollution activity were to pay—and include in the price of its product—the costs of eliminating all pollution directly generated by it, the balance between revenues and outlays in all goods-producing and pollution-eliminating sectors could be described by the following matrix equations.

\[
\begin{align*}
\text{Goods} & \quad [I - a'_{ij}]P_1 - [a'_{jk}]P_2 = V_1 \\
\text{Pollutant elimination} & \quad [a'_{jk}]P_1 + [I - a'_{jk}]P_2 = V_2 \\
\end{align*}
\]

(4)

The general solution of that system for unknown \(p's\) in terms of given \(v's\) is

\[
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix} = \begin{bmatrix}
[I - a'_{ij}] & -[a'_{jk}] \\
- & -
\end{bmatrix}^{-1} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

(5)

\(^3\) For price computations based on different assumptions see the article cited in notes 1 and 2.