APPENDIX I

ON SOME RECENT STUDIES OF INDUSTRY OUTPUT-ORDER-INVENTORY RELATIONS

Cost Functions and Production Planning

The current aggregative models of the economy either do not deal with the short-term changes in production schedules at all or deal with them indirectly via the inventory equations. The production functions they include are of the conventional type, relating output to inputs of labor and capital and to trends in technology; they are much more likely to reflect long-term tendencies than short-term adjustments. Such models are not focused on how previous commitments, expectations, and cost considerations influence the short-term production decisions. They are not constructed to handle the relationships in question, which require considerable disaggregation, such as the distinction between production to stock and to order. Yet a complete model which employs reasonably short unit periods such as quarters and includes equations for components of effective demand, output, and inventories must have some implications about the over-all mechanism underlying short-term production decisions.

These decisions, having the objective of minimizing cost over a time horizon, must involve the parameters of the relevant cost functions. This theme is developed in recent studies of production planning, which are largely microeconomic and normative.¹ This work applies

mainly to firms that produce to stock according to sales expectations derived in some assumed manner. Unfilled orders are treated as if they were equivalent to negative inventories. This approach may be appropriate in some individual cases, but it is not generally applicable and it is surely incorrect where aggregates, e.g., industry data, are concerned. As was shown in Chapter 2, some products are typically produced to stock, others to order. The aggregates of inventories and of unfilled orders refer in large measure to different products. Hence the concept of "net inventory," taken to mean the value of product inventory minus the value of order backlogs, may be meaningless even for a single firm. For an industry, which usually means a group of multi-product firms, the aggregates of stocks of goods and backlogs of orders are likely to be still more heterogeneous, since their composition would differ not only in terms of the "product mix" but also in terms of the "company mix."

The production studies work with quadratic functions for several cost categories: (a) costs of hiring and layoffs, overtime, machine setups, etc., which are incurred when the production rates are changed to absorb the fluctuations in sales; (b) costs of holding inventory and of unmet orders (stock-outs), which are incurred when the fluctuations in sales are absorbed by variations in inventories and in unfilled orders. Given the parameters of the cost functions, a linear "decision rule" is derived which links the scheduled production rate to forecasts of orders or sales, to the actual rate of output or labor input in the preceding period, and to the inventory situation at the time. Forecasts of sales in several future periods may be averaged, with the largest weight being given to the nearest and the smallest weight to the distant future. This would have the effect of smoothing output relative to the expected demand in production to stock (though smoothing relative to actual sales need not be assured because of possible errors of forecasts). The scheduled output rate is positively associated with the initial rate of production and size of the employed work force, since cutbacks in operations, either through layoffs or through underutilization of labor, are costly. Finally, the larger the product inventory on hand, the smaller is the rate of output needed to meet the given sales expectations (and the greater the need to lower the inventory for cost reasons). Hence the association between scheduled output and finished unsold inventory is, ceteris paribus, a negative one.
In production to order, the situation is in large part different. Other things being equal, output is positively related to the backlog of unfilled orders, just as it is negatively related to finished inventory in production to stock. It is also plausible that the costs of holding unfilled orders are quadratic. When the backlog becomes small, costly production cutbacks may have to be made and when the backlog becomes large, sales may be discouraged by the lengthening of the delivery periods. But one must not expect the effect of unfilled orders on production to be a stable function of some cost parameters. This effect reflects in a summary fashion the relationship between orders received in the past and current output resulting from the processing of some of these orders; and, as suggested by earlier analysis, this relation involves distributed and probably variable lags. Also, where output depends in a large measure on prior orders, it is correspondingly less closely guided by sales forecasts or expectations.

Short-Run Behavior of Production

If only because of grave aggregation problems, it is difficult to apply the lessons from the literature on quadratic cost functions and the associated linear decision rules—essentially a normative microanalysis—to comprehensive industry data. Moreover, unfilled orders and finished-goods inventories are in large measure determined by the demand forces and are only in part controllable "decision variables." Nevertheless, a few ambitious efforts were made recently to apply models similar to those proposed by Holt et al. in Planning Production to the current Census data for the major manufacturing industries. The study by David A. Belsley\(^2\) makes a clear distinction between production to stock and production to order. Belsley's results consist of a large number of direct and indirect estimates derived from several sets of regressions. Here it will only be possible to consider the primary direct estimates from his basic regression output.\(^3\)

Belsley derived series for gross value of production (call it \(Z'_t\)) from

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\(^3\) Belsley attempted in ingenious ways to develop separate estimates of reaction coefficients for the production-to-stock and production-to-order components of each of the major durable goods industries that report unfilled orders. But this can only be done indirectly, through transformations based on a number of particular assumptions of varying degrees of plausibility. The resulting estimates of the "structural" model, which contains unobserved variables, are difficult to evaluate. In any event their consideration seems logically posterior to the task of interpreting the underlying regressions whose coefficients furnished the inputs for the transformations.
the monthly Census data on shipments and finished inventory change ($S_t$ and $\Delta Q_t$) according to the identity $Z_t' = S_t + \Delta Q_t$. He then computed regressions of $Z_t'$ on $Q_{t-1}$, $\Delta Q_{t-1}$, $U_{t-1}$, $N_t$, and $Z_{t-1}'$, using the monthly, seasonally adjusted Census series on finished inventories and unfilled and new orders for the period from January 1953 to November 1964. In two other sets of estimates, $S_t$ and $\Delta Q_t$ were cast in the role of the dependent variable, while the above five series were used in each case as the explanatory variables. Finally, all these computations were performed again on series deflated by wholesale price indexes, 1957 = 1.00. Data for twelve industries reporting unfilled orders were thus processed.

As would be expected, the correlations obtained for the equations with either $Z_t'$ or $S_t$ as the dependent variable are generally very high (the $R^2$ coefficients exceed .9 in all but a few cases and often exceed .95), while the correlations for the $\Delta Q$ equations are very low and frequently insignificant. It is disturbing that some of the equations contain many identity elements. Of more interest are the regression estimates. For example, the undeflated value-of-output equations for three large durable goods industries are as follows:

**Primary metals**

$$Z_t' = -56.10 + .268Q_{t-1} + 1.706\Delta Q_{t-1}$$

$$+ .051U_{t-1} + .302N_t + .476Z_{t-1}'; \quad \bar{R}^2 = .880$$

**Machinery except electrical**

$$Z_t' = 6.03 + .119Q_{t-1} - .315\Delta Q_{t-1}$$

$$+ .011U_{t-1} + .218N_t + .651Z_{t-1}'; \quad \bar{R}^2 = .991$$

**Transportation equipment**

$$Z_t' = 319.27 + .384Q_{t-1} + 3.688\Delta Q_{t-1}$$

$$+ .002U_{t-1} + .135N_t + .717Z_{t-1}'; \quad \bar{R}^2 = .957$$

\(^4\)Thus $Z_t'$ is related to $Z_{t-1}'$, $Q_{t-1}$, and $N_t$ for industries without unfilled orders, where $N_t = S_t$; yet here $Z_t' = N_t + Q_t - Q_{t-1}$. More generally, consider also that $Z_t' = N_t + \Delta Q_t - \Delta U_t$ and that the series on the value of output, shipments, inventories, and unfilled orders typically show high serial correlations.
Since many orders require little time for production and many are sold and shipped from stock, $N_t$ should have a substantial effect on $Z_t^*$ and does. But in the above industries, manufacture to order with longer production and delivery periods is important, and hence earlier orders also influence current output. These distributed-lag relations presumably account in large part for the autoregressive properties of the output series as reflected in the major importance of the term $Z_{t-1}^*$. Simple aggregates of unfilled orders would not be expected to influence output as effectively as do the recent values of new orders taken with regression-determined weights (as in the equations of Chapter 5, for example). In any event, the combination of $N_t$ and $Z_{t-1}^*$ works to suppress the effect of the backlog factor, $U_{t-1}$. Indeed, the observed residual effects of $U_{t-1}$ are generally small and in some instances not significantly different from zero.

It is possible to interpret these relations differently, stressing that $N_t$ stands for sales expectations and that the coefficient of $Z_{t-1}^*$ reflects the costs of changing the rates of production. These conceptions, which correspond approximately to some of the ideas underlying Belsley's analysis, are probably partly valid, but on the basis of the available evidence, I believe them to be secondary to the aspects noted in the preceding paragraph.

Given the sales expectations that govern production to stock, the possession of large quantities of unsold finished inventory would tend to inhibit a company in its production of those goods that are made in anticipation of market sales. Hence, as already noted, a negative influence on output of finished stocks on hand is expected. But anticipations of high sales may stimulate output sufficiently to result also in additions to inventory. However, adjustments of inventory by means of output changes are not necessarily timely or efficient. They are subject to forecast errors and may be impeded by lack of flexibility on the

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8 The ratios of regression coefficients to their standard errors tend to be very large (exceeding 7.0) for both $N_t$ and $Z_{t-1}^*$; the corresponding $t$ ratios for the other variables are generally much smaller (in the 1 to 5 range). For industries in which production to order is particularly important, such as those making machinery, equipment, and instruments, the $t$ values for $Z_{t-1}^*$ exceed 10.0 and are much larger than the corresponding statistics for $N_t$. In contrast, the $t$ ratios are typically greater for $N_t$ than for $Z_{t-1}^*$ in the equations for industries in which production to stock and short delivery periods are characteristic, e.g., stone, clay, and glass; furniture; paper; and printing. These results are all consistent with our explanations. They are reported in Belsley, *Industry Production Behavior*, App. D ("The Basic Regression Output"). (We are referring to the regressions based on monthly, seasonally adjusted data. The use of seasonally unadjusted data naturally results in giving relatively more importance to $N_t$ and less to $Z_{t-1}^*$.)
input side due to fixed commitments, etc. Also some inventories result from production to order and consist of sold stocks in transit. These factors can produce elements of a positive association between output and finished inventory, which complicates the situation. In Belsley’s equations, the coefficients of $Q_{t-1}$ (call them $\lambda_1$) are typically positive but those of $\Delta Q_{t-1}(\lambda_2)$ are negative and considerably larger. Such a combination implies a negative net effect of $Q_{t-1}$ (equal to $\lambda_1 - \lambda_2$) and a larger positive effect of $Q_{t-2}$ (equal to $-\lambda_2$).\(^6\)

Define the average production period for the made-to-order (oth) part of an industry’s output as $x = \bar{U}/\bar{Z}^o$ and the average inventory-sales ratio for the made-to-stock (sth) part as $y = \bar{Q}/\bar{S}^s$. Then the relative importance of production to stock versus production to order can be expressed as:

$$\frac{\bar{Z}^s}{\bar{Z}^o} = \frac{\bar{Q}/y}{\bar{U}/x} = \frac{\bar{Q}x}{\bar{U}y},$$

on the assumptions that in these steady-state values $\bar{Q}$ appears solely in production to stock and $\bar{U}$ in production to order and that $\bar{S}^s = \bar{Z}^s$.

In Chapter 2 of this book, the $\bar{Q}/\bar{U}$ ratios were used to rank the industries and products according to the relative importance of production to stock versus production to order. But, if the above formulation is correct, $\bar{Q}/\bar{U}$ may not be an appropriate means of such ranking. Conceivably, the $x/y$ ratios could so vary among the industries as to make the ranking of the latter by $\bar{Z}^s/\bar{Z}^o$ differ significantly from the ranking by $\bar{Q}/\bar{U}$. Actually, however, this argument, made by Belsley,\(^7\) is unconvincing for several reasons. It depends itself on the implicit premise that $\bar{Q}$ is to be assigned to production to stock only and $\bar{U}$ to production to order only, as already noted and also, basically, on the stability of the $x$ and $y$ values and on the not necessarily plausible assumption that $\bar{S}^s = \bar{Z}^s$. Moreover, as shown by John A. Carlson,\(^8\) the $U/S$ ratio used in this book and elsewhere as an indicator of an industry’s average delivery period, is in a sense independent of the relative importance of production to order. If $r = S^o/S$ is the proportion of total shipments

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\(^6\)Note that $\lambda_1Q_{t-1} - \lambda_2(Q_{t-1} - Q_{t-2}) = (\lambda_1 - \lambda_2)Q_{t-1} + \lambda_2Q_{t-2}$. For primary metals and transportation equipment, $\lambda_2$ is positive and very large, but for each of the other industries, $\lambda_2$ is negative. These exceptions are difficult to understand, but they refer to particularly recalcitrant cases. The relations for primary metals are disturbed by the effects of the major steel strikes, and transportation equipment is an exceedingly heterogeneous industry.

\(^7\)Industry Production Behavior, pp. 149–51.

\(^8\)“The Production Lag,” preliminary draft, July 1970.
that goes to fill backlog orders, then \( U/rS \) would be the average delivery period for items produced to order. The average delivery period on the rest of shipments is approximately zero, neglecting the short response time in filling orders from stock. Combining \( U/rS \) and zero with weights of \( r \) and \( (1 - r) \), respectively, gives \( U/S \) as the average delivery period for all shipments. Finally and most importantly, the evidence of the \( \bar{Q}/\bar{U} \) ratios is generally sensible and consistent with other information, as demonstrated in this study. No contrary evidence is presented by Belsley, who reports a "frustrated attempt to rank industries" without trying to implement the proposed \( \bar{Q}_x/\bar{U}_y \) measures (which, of course, are not directly observable).

Another interesting study of manufacturers' short-term production decisions\(^9\) focuses on how they vary between cyclical expansions and contractions and employs the stock-adjustment model of inventory investment. Recognizing that this model applies to production to stock and accepting the evidence of the \( \bar{Q}/\bar{U} \) ratios for determining the prevalence of that type of manufacture versus production to order, Moriguchi limits his statistical work to a few products made primarily to stock in the cement, paper, and lumber industries. Monthly data, 1949-60, for current production are regressed on current sales (shipments), alternative variants of sales anticipations, and lagged finished stock. Dummy variables are used to study separately the seasonal influences, the role of changes in capacity utilization, and the other effects of the distinction between business cycle prosperity and recession. Moriguchi's results suggest, among others, that the speed with which inventories are adjusted to the desired levels is reduced in recession because of manufacturers' skepticism about anticipated sales, even though lower capacity utilization in the same phase would counsel the opposite reaction, i.e., faster adjustments of the rate of production for stock.

Unfilled Orders and Finished Stocks

A study by Gerald Childs\(^{10}\) contains estimates of manufacturers' unfilled orders and finished-goods inventories based on monthly regres-

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\(^{10}\) *Unfilled Orders and Inventories: A Structural Analysis*, Amsterdam, 1967. (This monograph, as well as those by Belsley and Moriguchi, is published in the North-Holland Publishing Company series, "Contributions to Economic Analysis.")
8ions for 1953–64, each of which includes as the independent variables
\( Z_{t-1}, Q_{t-1}, \) and \( U_{t-1} \) as well as certain lagged or future values of new
orders. The latter are used to represent forecasts of demand (new or-
ders), which are either assumed perfect (with \( N_{t+i}, i = 0, 1, 2, \) included
in the equations) or alternatively are taken to be autoregressive (\( N_{t-i},
i = 1, 2, 3 \)). The desired level of inventories is assumed to depend
linearly on the current value of either new orders or shipments only,
although no rationale is provided for these seemingly arbitrary and
quite restrictive specifications. Combinations of these alternatives de-
fine several variants of the model.11

Two of the variants use the lagged “net inventory,” \( Q_{t-1} - U_{t-1} \), as
one of the determinants. As argued before, this is not likely to be a
meaningful concept. The coefficients of this stock-backlog difference
factor seem to reflect mainly the high positive autocorrelations that
are characteristic of both \( U \) and \( Q \).12 By the same token, in the other
two variants, where lagged backlogs and finished stock are included as
separate independent variables, \( U_{t-1} \) dominates the equations for \( U_t \),
and \( Q_{t-1} \) dominates the equations for \( Q_t \).

The previous value of output, \( Z_{t-1} \), appears in all estimated relation-
ships, predominantly with a negative net effect on \( U_t \) and a positive
one on \( Q_t \). The explanation given is that production fills some of the
backlog orders and that some of the output is being added to the
finished-goods stock.

The coefficients of \( Q_{t-1} \) in the equations for unfilled orders are nega-
tive and at least twice as large as their standard errors, while the co-
efficients of \( U_{t-1} \) in the equations for finished-goods inventories are
positive and appear for the most part to be reasonably significant. But
in production to order, finished-goods inventories would not be ex-
pected to depend systematically on unfilled orders except perhaps in-
directly or as a reflection of common growth trends of the industry.13
Similarly, for items sold from stock, there is no relation to be expected
between the finished-goods inventories and the unfilled order backlogs.

11 There are four variants applied to \( U_t \) and then again to \( Q_t \), thus making a set of eight regressions
for each of the seven industries covered, including all manufacturing, total durables and four major
components, and total nondurables.

12 These coefficients are therefore always negative in the equations for \( U_t \), and positive in the equa-
tions for \( Q_t \). On these and other results discussed below in this section, see Childs, Unfilled Orders,
Tables 5-1 through 5-16, pp. 68–83.

13 The behavior of finished inventories that are already sold and held only transitorily before
delivery to the buyer is largely random (see Chapter 10). However, the greater the order backlog,
the higher the rates of production are likely to be, for they may be associated with shipping delays
and hence with larger stocks of finished goods in transit.
since there is no tendency for backlogs to accumulate in the first place. Logic and evidence indicate, as shown earlier, that $U$ and $Q$ typically refer to different goods. And the backlogs of goods made to order are likely to be essentially independent of the inventories of goods made to stock, unless the two categories of product are complementary or unless further expansion of production for one of them imposes a limitation on the other because of an effective capacity constraint.\(^\text{14}\) Childs notes that the signs of the cross-effects of $Q_{t-1}$ and $U_{t-1}$ are those that would be expected if the items produced to stock were used as inputs in production to order. It is not known to what extent such input-output relations actually exist within the industries concerned. The suggested explanation provides one interesting possibility but does not preclude others.\(^\text{15}\)

The impact on unfilled orders of past and current new orders ($N_{t-i}$, $i = 0, 1, 2, 3$) is positive and on the whole highly significant, as would be expected. It is also not surprising that future new orders, $N_{t+1}$ and $N_{t+2}$, have virtually no effect upon current backlogs, $U_t$. This merely shows that outputs manufactured to order are not typically based on forecasts of demand, an aspect that is largely ignored in Childs's specifications.

In contrast to their strong positive effects on $U_t$, the new-order variables are, with few exceptions, very weakly, if at all, related to the finished-goods inventories. However, the coefficients of $N_{t-1}$ in several regressions for $Q_t$ are significantly negative. Some of these new orders are no doubt filled from stock, which should be a partial reason for this relation. A capacity constraint could also contribute to this result, since an increase in the demand for goods produced to order would then be associated with a reduction in the output of goods produced to stock.

\(^{14}\) The assumption of independence in the absence of either or both of these two conditions (complementarity and capacity constraint can, of course, coexist) is made by Childs, *Unfilled Orders*, p. 42.

\(^{15}\) Under the input hypothesis, a rise in $Q_{t-i}$ enables production to order to be increased in period $t$, thereby reducing the end-of-period backlog, $U_t$. Also, a rise in $U_{t-i}$ makes it advisable to increase $Q_t$ so that back orders can be more efficiently filled in the near future (see ibid., p. 84). An alternative pair of hypotheses is: (a) that a rise (fall) in backlogs on the order-oriented side of the industry leads to the expectation of higher (lower) sales on the stock-oriented side; (b) that a rise of demand is met first by reductions in unsold stocks of some products and next by backlogging of orders for others (and analogously for a fall of demand).