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Appendix A

1. MAXIMIZING THE HOUSEHOLD'S UTILITY

Given the utility function (1.1) and the production function (1.2), we may write

$$U = u[f_1(x_1, t_1; H), \ldots, f_n(x_n, t_n; H)], \tag{A.1}$$

and the time and money income constraints (1.3) and (1.4) can be combined as

$$\begin{aligned} Y_m &= w(t - \sum_i t_i) + v = \sum x_i p_{x_i} \\ &= wt + v - w \sum_i t_i, \end{aligned}$$

with

$$\begin{aligned} Y_c &= w \sum_i t_i = \sum_i t_i p_{t_i} \\ Y &= Y_m + Y_c = wt + v = \sum_i (x_i p_{x_i} + t_i p_{t_i}). \end{aligned} \tag{A.2}$$

To maximize (A.1) subject to (A.2), the Lagrangian

$$L = u[f_1(x_1, t_1; H), \ldots, f_n(x_n, t_n; H)] - \lambda[\sum_i (x_i p_{x_i} - t_i p_{t_i}) - Y] \tag{A.3}$$

is differentiated with respect to each factor x_i and t_i:

$$\frac{\partial L}{\partial x_i} = \frac{\partial u}{\partial f_i} \frac{\partial f_i}{\partial x_i} - \lambda p_{x_i} = 0,$$

which may be written as

$$MU_{Z_i} \cdot MP_{x_i} - \lambda p_{x_i} = 0,$$

or the marginal utility of income, λ, is

$$\lambda = \frac{MU_{Z_i} MP_{x_i}}{p_{x_i}}. \tag{A.4}$$

Since (A.4) holds for all factors, the usual equilibrium condition emerges:

$$\frac{MP_{x_i}}{p_{x_i}} = \frac{MP_{t_i}}{p_{t_i}} = \frac{\lambda}{MU_{Z_i}} = \frac{1}{MC_{Z_i}}. \tag{A.5}$$

2. THE COMMODITY PRICE

Defining the average price of a specific commodity Z as $\Pi = (p_x x + p_t t)/Z$ and substituting from (A.4) for p_x and p_t,

$$\Pi = \left[\left(\frac{MU_Z MP_x}{\lambda}\right)x + \left(\frac{MU_Z MP_t}{\lambda}\right)t\right]\Big/ Z$$

$$= \left(\frac{MU_Z}{\lambda}\right)\left(\frac{MP_x x + MP_t t}{Z}\right).$$

If the production functions are homogeneous of degree n, from Euler's theorem and (A.5)

$$\Pi = MC\left(\frac{nZ}{Z}\right),$$

so in equilibrium for commodity Z_i

$$\Pi_i = MC_i n. \tag{A.6}$$

This development of (A.6) also implies

$$\lambda = \frac{MU_Z n}{\Pi}, \tag{A.7}$$

and combining (A.4) with (A.7),

$$\frac{\lambda}{MU_Z} = \frac{MP_x}{p_x} = \frac{n}{\Pi}$$

or

$$p_x = \frac{MP_x \Pi}{n}. \tag{A.8}$$

From (A.8) it is clear that production shares, $w_x = (MP_x x/nZ)$, and expenditure shares, $s_x = (p_x x/\Pi Z)$, are equivalent:

$$\frac{p_x x}{\Pi Z} = \frac{(MP_x \Pi/n)x}{\Pi Z} = \frac{MP_x x}{nZ}. \tag{A.9}$$

3. THE EFFECT OF H ON PRODUCTIVITY AND THE COMMODITY PRICE

The effect of H on the output of Z_i, with the level of the inputs held fixed, would be some average of its effect on the productivity of the factors. For a homogeneous production function:

$$Z = (xMP_x + tMP_t)/n$$

$$\left.\frac{dZ}{dH}\right|_{x,\,t} = MP_Z{}^H = \left(x\frac{\partial MP_x}{\partial H} + t\frac{\partial MP_t}{\partial H}\right)\bigg/n$$

$$\widetilde{MP}_i = \frac{MP_Z{}^H}{Z} = \left(\frac{xMP_x}{nZ}\right)\widetilde{MP}_x + \left(\frac{tMP_t}{nZ}\right)\widetilde{MP}_t$$

for commodity Z_i, hence

$$\widetilde{MP}_i = \sum_f w_f \widetilde{MP}_f, \tag{A.10}$$

where f is an index over the factors of production. If, instead, we allow the quantities of the inputs to change, from (1.2),

$$dZ = MP_x dx + MP_t dt,$$

and dividing by dH and Z

$$\tilde{Z} = \left(\frac{MP_x x}{Z}\right)\tilde{x} + \left(\frac{MP_t t}{Z}\right)\tilde{t}$$

$$\tilde{Z} = (w_x\tilde{x} + w_t\tilde{t})\, n, \tag{A.11}$$

i.e., a one per cent increase in x and t leads to an n per cent increase in Z. Equation (A.10) shows the direct effect of H on Z through its effect on the marginal products; equation (A.11) shows the indirect effect of H on Z through the induced changes in the quantities of the inputs.

The effect of H on the price of the commodity, Π, may be evaluated as follows. With the factor prices and the level of output held fixed, the differential is

$$d\Pi \bigg|_{p_x,\, p_t,\, Z} = \left(\frac{p_x}{Z}\right) dx + \left(\frac{p_t}{Z}\right) dt,$$

so

$$\frac{d\Pi}{dH} \bigg|_{p_x,\, p_t,\, Z} = \frac{p_x}{Z}\frac{dx}{dH} + \frac{p_t}{Z}\frac{dt}{dH}.$$

To evaluate dx/dH, sum (A.10) and (A.11) and set the sum equal to zero, which then holds the level of output constant, and solve for $\tilde{x}$:

$$\widetilde{MP}_i + (w_x\tilde{x} + w_t\tilde{t})n = 0$$

$$\tilde{x} = \frac{-\widetilde{MP}_i}{nw_x} - \frac{w_t}{w_x}\tilde{t}$$

or

$$\frac{dx}{dH} = -\frac{\widetilde{MP}_i x}{nw_x} - \frac{w_t x}{w_x t}\frac{dt}{dH}.$$

Substituting for dx/dH in $(d\Pi/dH)|p_x, p_t, Z$ for commodity i,

$$\frac{d\Pi_i}{dH} = \frac{p_x}{Z_i}\left(-\frac{\widetilde{MP}_i x}{w_x n} - \frac{w_t x}{w_x t}\frac{dt}{dH}\right) + \frac{p_t}{Z_i}\frac{dt}{dH}$$

and from (A.9)[1]

$$\tilde{\Pi}_i = \frac{-\widetilde{MP}_i}{n}. \qquad \text{(A.12)}$$

From (A.6)

$$d\Pi_i = (dMC_i)\, n$$

$$\frac{d\Pi_i}{\Pi_i} = \frac{dMC_i}{\Pi_i}n = \frac{dMC_i}{MC_i},$$

so

[1] If $\tilde{\Pi}$ is, instead, evaluated holding the level of the inputs and the factor prices and the degree of homogeneity fixed, n drops out of the expression:

$$\tilde{\Pi}_i = \widetilde{MC}_i. \tag{A.13}$$

4. THE PRICE LEVEL

Define the price level Π as

$$\Pi = \Pi_1^{s_1} \Pi_2^{s_2} \Pi_3^{s_3} \ldots \Pi_n^{s_n}$$

where the weights are the expenditure shares, then

$$\ln \Pi = \sum s_i \ln \Pi_i$$

or[2]

$$\tilde{\Pi} = \frac{d \ln \Pi}{dH} = \sum_i s_i \tilde{\Pi}_i. \tag{A.14}$$

5. THE DEMAND FOR THE COMMODITY AND THE INPUTS

If the demand for the commodity Z_i is written

$$Z_i^d = d_i\left(\frac{Y}{\Pi}, \frac{\Pi_i}{\Pi}\right) \tag{A.15}$$

$$\Pi = (p_x x + p_t t)/Z$$

$$\left.\frac{d\Pi}{dH}\right|_{p_x, p_t, x, t} = -\left(\frac{\Pi Z}{Z^2}\right) \left.\frac{dZ}{dH}\right|_{x, t},$$

$$\tilde{\Pi} = -\widetilde{MP}_i .$$

[2] An alternative derivation of (A.14) uses an arithmetic price index. Let 0 represent a base level of H and 1 a unit increase in H. Defining each price relative to its base price and using base expenditure weights, the price index is

$$\pi^* = \sum_i s_i \frac{\pi_i^1}{\pi_i^0};\ s_i = \frac{\pi_i^0 Z_i^0}{Y^0};\ \sum_i s_i = 1.$$

Then the price index for the base level is unity

$$1 = \pi_0^* = \sum_i s_i \frac{\pi_i^0}{\pi_i^0}$$

$$d\pi^* = \pi_1^* - \pi_0^* = \sum_i s_i \frac{\pi_i^1 - \pi_i^0}{\pi_i^0} = \sum_1 s_i \frac{d\pi_i}{\pi_i^0}$$

so

$$\frac{d\pi^*}{dH} = \sum_i s_i \left.\frac{d\pi_i}{dH}\right/ \pi_i^0 = \sum_i s_i \tilde{\pi}_i$$

then the effect of an increase in H on Z_i^d can be derived from the total differential[3]

$$dZ_i^d = \frac{\partial Z}{(\partial Y/\Pi)} d(Y/\Pi) + \frac{\partial Z}{\partial(\Pi_i/\Pi)} d(\Pi_i/\Pi)$$

$$\widetilde{Z}_i^d = \frac{dZ_i^d}{dH}\Big/ Z_i^d = \left(\frac{\partial Z_i}{\partial(Y/\Pi)} \frac{(Y/\Pi)}{Z}\right)\left(\frac{d(Y/\Pi)}{dH} \frac{1}{(Y/\Pi)}\right) + \left(\frac{\partial Z}{\partial(\Pi_i/\Pi)} \frac{d(\Pi_i/\Pi)}{dH} \frac{1}{(\Pi_i/\Pi)}\right)$$

$$\widetilde{Z}_i^d = \eta_i\left(\frac{\widetilde{Y}}{\Pi}\right) + \epsilon_i\left(\frac{\widetilde{\Pi}_i}{\Pi}\right), \tag{A.16}$$

where η_i and ϵ_i are the commodity's income and own-price elasticity and

$$(\widetilde{Y/\Pi}) = \widetilde{Y} - \widetilde{\Pi};\ (\widetilde{\Pi_i/\Pi}) = \widetilde{\Pi}_i - \widetilde{\Pi}.^4$$

Substituting from (1.13), the equation (A.16) can be written as

$$\widetilde{Z}_i^d = \eta_i(\widetilde{Y} + \widetilde{Y}_e) + \epsilon_i(\widetilde{\Pi}_i - \widetilde{\Pi}). \tag{A.17}$$

For the factor x, used in the production of a commodity,

$$Z_i = f_i(x_i, t_i; H),$$

so

$$\frac{dZ}{dH} = \frac{\partial Z}{\partial x}\frac{dx}{dH} + \frac{\partial Z}{\partial t}\frac{dt}{dH} + \left.\frac{dZ}{dH}\right|_{x,\,t},$$

where the left-hand term is the gross or total effect of H on Z de-

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or

$$\widetilde{\pi}^* = \sum_i s_i \widetilde{\pi}_i.$$

[3] More generally, if $Z_i^d = d_i(Y/\Pi, \Pi_j/\Pi)$ for j from 1 to n,

$$\widetilde{Z}_i^d = \eta_i\left(\frac{\widetilde{Y}}{\Pi}\right) + \sum_j \epsilon_{ij}\left(\frac{\widetilde{\Pi}_j}{\Pi}\right)$$

where ϵ_{ij} represents own- and cross-price elasticities.

[4] Since $\widetilde{\Pi}$ is defined holding p_i fixed (see the development of (A.12)), to be consistent:

manded and the final term is the change in the Z produced, with the factors held constant. Dividing by Z and letting $n = 1$,

$$\tilde{Z}^d = \left(\frac{MP_x x}{Z}\right)\tilde{x} + w_t\tilde{t} + \widetilde{MP}_i,$$

since $\partial Z/\partial x = MP_x$, and where $\widetilde{MP}_i$ is defined in (A.10). Since $w_x = (1 - w_t)$,

$$\tilde{Z}^d = \tilde{x} - w_t(\tilde{x} - \tilde{t}) + \widetilde{MP}_i\,,$$

so

$$\tilde{x} = \tilde{Z}^d - \widetilde{MP}_i + w_t(\tilde{x} - \tilde{t}). \tag{A.18}$$

Now, making use of the assumption of linear homogeneity of the production function and permitting factor nonneutrality,

$$MP_r = \phi(f_r, H),$$

where MP_r is the ratio of marginal products (MP_x/MP_t) and f_r is the ratio of factors of production (x/t). Then

$$\frac{dMP_r}{dH} = \frac{\partial MP_r}{\partial f_r}\frac{df_r}{dH} + \frac{\partial MP_r}{\partial H}$$

or

$$\tilde{Y} = \frac{d(wt_m + V)}{dH}\Big/ Y = \frac{t_m w}{Y}\tilde{w} + \frac{V}{Y}\tilde{V},$$

i.e., Y is the effect of H on real full income through market earnings and property income. If, instead, $\tilde{\Pi}^*$ allowed p_t to vary,

$$\tilde{\Pi}^* = \tilde{\Pi} + \sum_i \left(\frac{t_i p_{t_i}}{Y}\right)\tilde{p}_{t_i},$$

and since $p_{t_i} = w$,

$$\tilde{\Pi}^* = \tilde{\Pi} + \frac{(\Sigma t_i)w}{Y}\tilde{w} = \tilde{\Pi} + \frac{t_c w}{Y}\tilde{w}.$$

In this case $\tilde{Y}^* = (tw/Y)\tilde{w} + (V/Y)\tilde{V}$ and the term $(t_c w/Y)\tilde{w}$ will net itself out of $(\tilde{Y}^* - \tilde{\Pi}^*)$.

Notice, too, that if Y were to include a term $(wt_m/Y)\tilde{t}_m$ (i.e., a shift of hours into or from the market), the consumption income term would also include a term $(wt_c/Y)\,\tilde{t}_c$, and since $(dt_c/dH) = -\,dt_m/dH)$, these two terms will also net themselves out.

$$\frac{dMP_r}{dH}\frac{1}{MP_r} = \frac{-1}{\sigma}(\tilde{x} - \tilde{t}) + (\widetilde{MP}_x - \widetilde{MP}_t),$$

where the left-hand side shows the gross or total effect of H on the ratio of marginal products after the factor substitution, and σ is the elasticity of substitution in production, defined to be positive. Since in equilibrium $MP_r = p_x/p_t$,

$$dMP_r/MP_r = d(p_x/p_t)/(p_x/p_t).$$

So

$$\tilde{p}_x - \tilde{p}_t = \frac{-1}{\sigma}(\tilde{x} - \tilde{t}) + (\widetilde{MP}_x - \widetilde{MP}_t). \qquad (A.19)$$

Substituting $(\tilde{x} - \tilde{t})$ from (A.19) and substituting for $\tilde{Z}_i^d$ from (A.17) into (A.18) and rearranging,

$$\tilde{x} = \eta_i(\tilde{Y} + \tilde{Y}_c) - \widetilde{MP}_i + \epsilon_i(\tilde{\Pi}_i - \tilde{\Pi}) \\ + w_t\sigma(\widetilde{MP}_x - \widetilde{MP}_t) - w_t\sigma(\tilde{p}_x - \tilde{p}_t). \qquad (A.20)$$

These terms represent, respectively, the gross income effect, the direct productivity effect, the substitution in consumption effect, and the substitution in production effect (through H's effect on relative marginal products and relative factor prices). If x is evaluated holding money income and factor prices fixed, equation (A.20) reduces to

$$\tilde{x} = \eta_i\tilde{Y}_c - \widetilde{MP}_i + \epsilon_i(\tilde{\Pi}_i - \tilde{\Pi}) + w_t\sigma(\widetilde{MP}_x - \widetilde{MP}_t). \qquad (A.21)$$

6. THE PRODUCTIVITY MODEL AND UTILITY

In this section the productivity model is couched in terms of utility, and it is shown that the relative increase in the demand for a commodity is greater the larger its relative utility elasticity. From the utility function (A.1), the marginal utility of the environmental variable H is simply:

$$\frac{\partial U}{\partial H} = \frac{\partial U}{\partial f_1}\frac{\partial f_1}{\partial H} + \frac{\partial U}{\partial f_2}\frac{\partial f_2}{\partial H} + \ldots + \frac{\partial U}{\partial f_n}\frac{\partial f_n}{\partial H} = \sum_i \frac{\partial U}{\partial f_i}\frac{\partial f_i}{\partial H} = \sum_i \frac{\tilde{Z}}{\mu_i}U,$$

where $\mu_i = (\partial Z_i/\partial U)U/Z_i$ is a utility elasticity of commodity Z_i.[5] Expressed in percentage terms per unit change in H,

$$\tilde{U} = \frac{\partial U}{\partial H}\frac{1}{U} = \sum \frac{\tilde{Z}_i}{\mu_i}. \tag{A.22}$$

Equation (A.22) will be positive if the environmental variable increases the output of all Z's (holding the direct inputs, x and t, constant) and if the commodities are "normal goods." H's effect on total utility is simply the sum of its indirect effect on utility through each commodity, measured in comparable units.

To determine how this might affect the quantities demanded of the Z's, rewrite (A.15) as a function of total utility (an alternative measure of the opportunity constraint) and relative prices,

$$Z_i^d = d_i\left(U, \frac{\Pi_i}{\Pi}\right), \tag{A.23}$$

or relative to some other commodity Z_j,

$$\left(\frac{Z_i}{Z_j}\right)^d = h\left(U, \frac{\Pi_i}{\Pi_j}\right). \tag{A.24}$$

From (A.24) the effect of H on the relative demand for the commodities is

$$\left(\frac{\tilde{Z}_i}{Z_j}\right)^d = (\mu_i - \mu_j)\tilde{U} - \sigma\left(\frac{\tilde{\Pi}_i}{\Pi_j}\right), \tag{A.25}$$

where σ is the elasticity of substitution in consumption

$$\sigma = \frac{-\partial Z_i/Z_j}{\partial \Pi_i/\Pi_j}\frac{\Pi_i Z_j}{\Pi_j Z_i}.$$

If H's productivity effect is biased toward Z_i relative to Z_j,

$$(\widetilde{\Pi_i/\Pi_j}) = \tilde{\Pi}_i - \tilde{\Pi}_j < 0,$$

[5] The utility elasticity of a commodity is identically equal to the ratio of its income elasticity to the elasticity of total utility with respect to income:

$$\mu_i = \frac{\partial Z_i}{\partial U}\frac{U}{Z_i} = \frac{\partial Z_i}{\partial U}\frac{U}{Z_i}\frac{\partial Y}{\partial Y}\frac{Y}{Y} = \left(\frac{\partial Z_i}{\partial Y}\frac{Y}{Z_i}\right)\bigg/\left(\frac{\partial U}{\partial Y}\frac{Y}{U}\right) = \frac{\eta_i}{\epsilon_{UY}}$$

where ϵ_{UY} is presumably positive. The ratio of utility elasticities of two items is then equal to the ratio of their income elasticities, and the absolute difference is:

$$(\mu_i - \mu_j) = (\eta_i - \eta_j)\frac{1}{\epsilon_{UY}}.$$

provided H reduces the prices of the commodities. Then the second term in (A.25) is positive $(-\sigma(\widetilde{\Pi_i/\Pi_i}) > 0)$ and tends to make $\tilde{Z}_i^d > \tilde{Z}_j^d$ (which is the usual case for a decline in the relative price of Z_i). If H is presumed to be commodity neutral, (A.25) reduces to:

$$(\widetilde{Z_i/Z_j})^d = (\mu_i - \mu_j)\tilde{U}. \tag{A.26}$$

Equation (A.26) suggests the effect on the relative demand for Z_i of the expansion in opportunities (or utility). If $\tilde{U}$ is positive, then

$$\tilde{Z}_i^d \gtreqless \tilde{Z}_j^d \quad \text{when} \quad \mu_i \gtreqless \mu_j.$$

That is, the change in the demand for Z_i is relatively great when Z_i's elasticity is relatively large. From footnote 5 above, (A.26) can also be expressed as

$$(\widetilde{Z_i/Z_j})^d = \tilde{Z}_i^d - \tilde{Z}_j^d = (\eta_i - \eta_j)\frac{\tilde{U}}{\epsilon_{UY}} \tag{A.27}$$

and since $(\tilde{U}/\epsilon_{UY}) > 0$, the relative demand for Z_i is greater the larger its (relative) income elasticity. Equation (A.27) again emphasizes the fact that, were the utility function homogeneous (i.e., were all income elasticities unity), there would be no effect on the relative demand for commodities resulting from a neutral productivity shift.

Equation (A.27) must imply the same relationship as was discussed previously, since, after all, it is simply a translation of that discussion. For example, if we write (A.16) for two commodities and consider their difference,

$$(\tilde{Z}_i^d - \tilde{Z}_j^d) = (\eta_i - \eta_j)\left(\frac{\tilde{Y}}{\Pi}\right) + (\epsilon_i\tilde{\Pi}_i - \epsilon_j\tilde{\Pi}_j) - \tilde{\Pi}(\epsilon_i - \epsilon_j), \tag{A.28}$$

and if $\tilde{\Pi}_i = \tilde{\Pi}_j = \tilde{\Pi}$,

$$(\tilde{Z}_i^d - \tilde{Z}_j^d) = (\eta_i - \eta_j)(\widetilde{Y/\Pi}), \tag{A.29}$$

which equated to (A.27) implies:

$$(\widetilde{Y/\Pi}) = (\tilde{U}/\epsilon_{UY}), \tag{A.30}$$

where the income term in ϵ_{UY} is understood to be in real terms.

7. A "CHANGE IN TASTES" INTERPRETATION

The previous section expressed the productivity model in terms of an effect on behavior through a change in the level of utility. But since the utility level was presumed to be affected by the increased non-market productivity, that section did not really offer an alternative way of viewing the effect of H on behavior. This section does so by suggesting that H affects the utility level directly, not through productivity but simply by changing the indifference map; that is, by changing tastes.

Specifically, consider the case of two commodities, Z_L, a luxury, and Z_N, a necessity, and suppose

$$MU_L/MU_N = g(Z_L/Z_N, U), \tag{A.31}$$

i.e., where the ratio of marginal utilities depends upon the level of total utility as well as upon the ratio of the commodities (or, the utility function is not homogeneous). Now, if H affects the total level of utility directly, then it indirectly affects (MU_L/MU_N) both through U and the induced change in (Z_L/Z_N). Letting A represent (MU_L/MU_N) and B represent (Z_L/Z_N):

$$\left.\frac{dA}{dH} = \frac{\partial A}{\partial B}\frac{dB}{dH} + \frac{\partial A}{\partial U}\frac{dU}{dH}\right|_{x,\, t} \tag{A.32}$$

or

$$\widetilde{A} = \frac{-1}{\sigma}(\widetilde{B}) + \epsilon_{AU}\widetilde{U}, \tag{A.33}$$

where σ is the elasticity of substitution in consumption ($\sigma > 0$) and ϵ_{AU} is the elasticity of (MU_L/MU_N) with respect to U.

The terms in (A.33) can be evaluated as follows. The effect on the ratio of the marginal utilities of an increase in (Z_L/Z_N), holding U constant, is:

$$\frac{\partial A}{\partial B} = \left(\frac{\partial MU_L}{\partial B}\frac{1}{MU_L} - \frac{\partial MU_N}{\partial B}\frac{1}{MU_N}\right)A$$

or

$$\frac{-1}{\sigma} = (\epsilon_{MU_LB} - \epsilon_{MU_NB}) < 0, \tag{A.34}$$

since the indifference curve is convex. Similarly, the effect on (MU_L/MU_N) of an increase in U, holding (Z_L/Z_N) fixed, would be:

$$\frac{\partial A}{\partial U} = \left(\frac{\partial MU_L}{\partial U}\frac{1}{MU_L} - \frac{\partial MU_N}{\partial U}\frac{1}{MU_N}\right) A > 0$$

or

$$\epsilon_{AU} = \frac{\partial A}{\partial U}\frac{U}{A} = (\epsilon_{MU_LU} - \epsilon_{MU_NU}) > 0. \tag{A.35}$$

Since Z_L is a luxury, along a ray the slope of the indifference curves (MU_L/MU_N) increases (the slope rises to the left and falls to the right of P, which is the locus of tangency points from a parallel shift in the budget constant in Figure A).

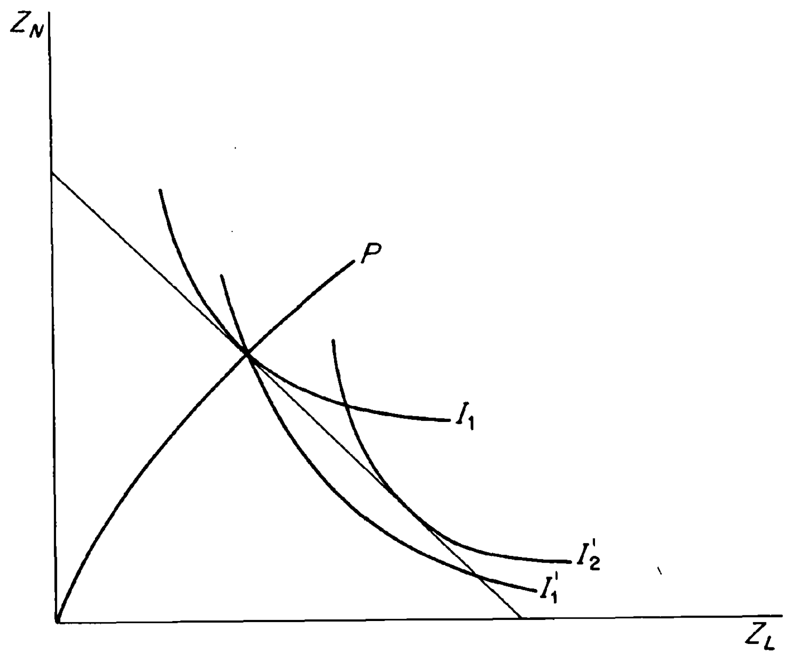

FIGURE A

Initially, as H rises and increases U (and before the induced change in (Z_L/Z_N) occurs), (MU_L/MU_N) rises, since $(\epsilon_{AU}\tilde{U})$ is positive. But since a change in H does not affect the price ratio (Π_L/Π_N), the equilibrium level of (MU_L/MU_N) must be unchanged. So, setting (A.33) to zero shows the induced effect on (Z_L/Z_N) of H's influence on U:

$$(\widetilde{Z_L/Z_N}) = \sigma\epsilon_{AU}\widetilde{U} > 0 \tag{A.36}$$

and

$$\sigma\epsilon_{AU} = \frac{\partial B}{\partial U}\frac{U}{B} = (\mu_L - \mu_N) > 0, \tag{A.37}$$

where μ_i is the utility elasticity defined in the previous section. Thus, we are left with

$$(\widetilde{Z_L/Z_N}) = (\mu_L - \mu_N)\widetilde{U} > 0,$$

as in the previous section. The interpretation here is that H raises the level of utility and in so doing raises the ratio (MU_L/MU_N), altering the indifference map from that represented by I_1 to that represented by I_1'. But since prices are unaffected, the initial combination of Z_L and Z_N is no longer an optimal one, and the new equilibrium contains relatively more of the luxury.