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# 1

## Human Capital and Consumption: The Theoretical Framework

THIS CHAPTER PRESENTS a conceptual framework in which the effects of human capital on nonmarket or consumption efficiency can be analyzed. The model is developed in the context of a general human capital variable,  $H$ , with the household as the unit of analysis. The effect of  $H$  on the productivity of the various production functions, on the relative prices of commodities, and on real full income is discussed in the first part of the chapter. The second part suggests some implications regarding the demand for commodities, for factors of production, and, briefly, for  $H$  itself.

### THE ROLE OF AN ENVIRONMENTAL VARIABLE

Suppose the household has some utility function

$$U = u(Z_1, Z_2, \dots, Z_n), \quad (1.1)$$

where the  $Z$ 's are various commodities produced by the household according to the production functions

$$Z_i = f_i(x_i, t_i; H). \quad (1.2)$$

The  $x_i$  is a vector of market goods,  $t_i$  is the household's own time input in the production of commodity  $i$ , and  $H$  represents the household's available quantity of some environmental variable. The household has some money income

$$Y_m = wt_w + V = \sum_{i=1}^n x_i p_{x_i}, \quad (1.3)$$

where  $w$  is its wage rate in the labor market,  $t_w$  is the time spent in the labor market, and  $V$  is its nonwage income for the period.<sup>1</sup> Given an appropriately inclusive definition of market goods, the money income is equal to the total money expenditure. The household also has some fixed amount of time at its disposal,  $t$ , which it uses in the production of the commodities and in the labor market:

$$t = \sum_{i=1}^n t_i + t_w. \quad (1.4)$$

Equations (1.3) and (1.4) can be combined into one full income constraint on the household's utility maximization:<sup>2</sup>

$$Y = wt + V = \sum_{i=1}^n (x_i p_{x_i} + wt_i). \quad (1.5)$$

In this framework the household is viewed as a small firm producing many products, called commodities, from which it derives utility. The environmental variable,  $H$ , affects production by influencing the conditions in which production takes place, the nature of the productive processes, or the effective quantities of the direct inputs of time and market goods. The environmental variable is distinguished from the direct inputs by the fact that production of a commodity uses up some of the household's available time and market goods but does not, in general, affect the quantity of the environmental variable.<sup>3</sup>

The variable  $H$  was defined as a human capital variable, but it may be interpreted more broadly to encompass any factor affecting the household's nonmarket productivity, excluding the direct factors of production themselves. Certain environmental variables—for example, the degree of political stability, the level of hygiene and sanitation, the rate of literacy, the climate—might affect all households in a particular geographical location in a similar manner, and their effects would be studied in interregional or international comparisons. On the other hand, the influence of variables like the family's size, its age structure, and its stock of some form of human capital' (such as formal schooling

<sup>1</sup> While the wage earnings of each family member might be treated separately, for simplicity's sake this will not be done here.

<sup>2</sup> In order to make this chapter more readable, the mathematical proofs of most of the statements are relegated to an appendix. Consequently, Appendix A roughly parallels the development in this chapter, but includes very little verbal description of the material.

<sup>3</sup> While the opportunity cost of using the environmental variable in producing

or the level of health), which affect each household individually, would be observed in an interhousehold comparison. While the model is developed here with the latter comparison in mind, the general framework is clearly applicable to interregional differences as well.

To analyze the effect of  $H$  on the production of a given commodity, we differentiate the production function with respect to  $H$ , holding the levels of  $x_i$  and  $t_i$  constant, which yields the marginal product of  $H$  in the production of  $Z_i$ . Assuming the production function to be homogeneous of the first degree in the direct factors  $x_i$  and  $t_i$ ,

$$\left. \frac{dZ_i}{dH} \right|_{x_i, t_i} = MP_i = \left( x_i \frac{\partial MP_{x_i}}{\partial H} + t_i \frac{\partial MP_{t_i}}{\partial H} \right), \quad (1.6)$$

or, using the convention that a tilde indicates a percentage change per unit change in  $H$ ,

$$\tilde{M}P_i = W_x \tilde{M}P_{x_i} + W_t \tilde{M}P_{t_i}, \quad (1.7)$$

where the weights are the production shares and the sum of the weights is unity. Equation (1.7) states that the environmental variable's marginal product in the production of  $Z_i$  is a weighted average of its effect on the productivity of the direct inputs. Since in general the effect on the marginal products of the factors may be positive or negative, so, too, may  $\tilde{M}P_i$ ; that is, the environmental variable may increase or decrease the productivity of the factors.

As a complement to (1.7), one might ask what effect  $H$  has on the price of the commodity  $Z_i$ . Defining  $\Pi_i$  as the average price of  $Z_i$ , the effect of  $H$  on  $\Pi_i$ , evaluated at a fixed level of  $Z_i$ , is simply

$$\tilde{\Pi}_i = -\tilde{M}P_i, \quad (1.8)$$

which suggests, for example, that a 3 per cent increase in the productivity of a linear homogeneous production function lowers the price of the commodity by 3 per cent.

While the analysis is developed in terms of the commodity  $Z_i$ , the environmental variable  $H$  may affect the production of each  $Z$  in the household's commodity basket. If  $H$  has different effects on the various production functions, it will affect the relative prices of the commodities.

any commodity may be zero, this does not imply that changing the level of  $H$  is costless. Determining the optimal level of the environmental variable is discussed at the end of this chapter.

In the most general case,  $H$  may have a different effect on each factor's marginal product in each production function, and these may differ in sign as well as in magnitude. Even if all the absolute price changes have the same sign, relative prices may be affected.<sup>4</sup>

In order to analyze these relative price effects more explicitly, define a price level as

$$\Pi = \Pi_1^{s_1}, \Pi_2^{s_2}, \dots, \Pi_n^{s_n}, \quad (1.9)$$

where  $s_i$  is the expenditure share on commodity  $i$  and the sum of the shares equals unity.<sup>5</sup> Then the aggregate effect of an increase in  $H$  can be expressed as

$$\tilde{\Pi} = \sum_i s_i \tilde{\Pi}_i = - \sum_i s_i \tilde{M}P_i. \quad (1.10)$$

That is, the percentage change in the price level is a weighted average of the productivity effects of  $H$  on the various production functions, with the direction of the effect reversed. Since the relative price of  $Z_i$  is  $\Pi_i/\Pi$ , the change in the relative price resulting from an increase in  $H$  would be

$$\tilde{\Pi}_i/\Pi = \tilde{\Pi}_i - \tilde{\Pi}. \quad (1.11)$$

There is an alternative method of evaluating the effect of the environmental variable that involves computing the value of the marginal

<sup>4</sup> This can be illustrated with a very simple case. Suppose that for all commodities  $\tilde{M}P_{t_i} = \tilde{M}P_{t_j} \neq \tilde{M}P_{r_i} = \tilde{M}P_{r_j}$ ; that is,  $H$  has the same percentage effect on the productivity of time used in all activities and also a different but equal effect on the productivity of all goods used in all activities. Suppose, furthermore, that  $H$  affects the productivity of the time input more than the productivity of the goods input, i.e.,  $\tilde{M}P_t > \tilde{M}P_r$ . In such a case the effect of  $H$  on the productivity of the function  $i$  or on the price of  $Z_i$  would be greater the more time-intensive the production of  $Z_i$ . Since  $\tilde{\Pi}_i$  is simply a weighted average of the inputs' productivity effects and the weights are production shares, under these assumed conditions the ranking of the  $\tilde{\Pi}_i$  would be exactly the same as the ranking of the time intensities if the substitution elasticities did not differ greatly. The relative prices of those commodities that were relatively time-intensive would fall (if the  $\tilde{M}P$  were positive) and the relative prices of commodities that were relatively goods-intensive would rise.

<sup>5</sup> A geometric or logarithmic mean is used since our interest is in averaging percentage changes in these prices. Although (1.9) is called a price level, it uses fixed weights, and a price index would simply be the ratio of  $\Pi$  in one period or one situation to  $\Pi^0$ , the base period's price level.

product of  $H$  in each productive activity and summing these  $VMP_i$ . Intuitively, the sum of these dollar values is a measure of  $H$ 's effect on real income. Let

$$VMP_i = \Pi_i(MP_i) = \Pi_i Z_i(\tilde{M}P_i),$$

then, summing over all commodities,

$$\tilde{Y}_c = \sum_i (VMP_i)/Y = \sum_i s_i(\tilde{M}P_i). \quad (1.12)$$

$\tilde{Y}_c$  will be called the "change in consumption income."  $VMP_i$  is the value of the additional units of  $Z_i$  resulting from a unit increase in  $H$ ; the sum of the  $VMP_i$  is then the total value of  $H$ 's effect.  $\tilde{Y}_c$  expresses this total value as a percentage of full income. The term abstracts completely from any effects of  $H$  on market earnings, but incorporates the nonmarket effects. From equations (1.10) and (1.12)

$$\tilde{Y}_c = \sum_i s_i(\tilde{M}P_i) = -\tilde{\Pi}. \quad (1.13)$$

Thus, a change in  $H$  which affects nonmarket productivity may be evaluated as either a reduction in the cost of achieving a given output or as an increase in the output of commodities, holding the levels of the factors of production constant.

The household's full income,  $Y$ , defined by equation (1.5), combines price level,  $\Pi$ , reflects its capacity to convert these time and money re- its time and money resources into a single constraint. The household's sources into commodities. Reductions in market prices of goods or increases in the efficiency with which commodities are produced lower the household's price level. The household's "real full income" may be defined as  $(Y/\Pi)$ , which indicates its resource constraint expressed in real terms. So, for example, if  $H$  adversely affects nonmarket productivity, households with higher levels of  $H$  will have a higher price level and a lower real full income. Since the probable direction of  $H$ 's effect depends upon the nature of the environmental variable, no hypothesis has been formulated as yet. The model simply suggests a framework in which the influence of an environmental variable on the household's real income may be analyzed.

### THE IMPLIED EFFECTS ON THE HOUSEHOLD'S DEMAND

Through its effect on the marginal products of the direct inputs,  $H$  can change the relative prices of the commodities and can alter the real

income of the household. Thus, the usual analysis would suggest that  $H$  would create substitution and income effects on the demand for the commodities; that is,  $H$  would alter the optimal basket of commodities. If we write the demand function for commodity  $Z_i$  as

$$Z_i^d = d_i \left( \frac{Y}{\Pi}, \frac{\Pi_i}{\Pi} \right), \quad (1.14)$$

where the arguments are real income and the relative price of  $Z_i$ ,  $H$ 's effect on  $Z_i^d$ , abstracting from its effects on money income, would be<sup>9</sup>

$$\tilde{Z}_i^d = \eta_i(\tilde{Y}_c) + \epsilon_i(\tilde{\Pi}_i - \tilde{\Pi}) \quad (1.15)$$

where  $\tilde{Z}_i^d = (dZ_i^d/dH)/Z_i^d$  and  $\eta_i$  and  $\epsilon_i$  are the commodity's income and own price elasticity. The first term in (1.15) is the income effect, which will be positive for any "normal" good if  $H$ 's nonmarket effect on real income is positive; the second term is the substitution effect—if  $H$  is biased toward  $Z_i$ ,  $|\tilde{\Pi}_i| > |\tilde{\Pi}|$  and its relative price falls, and since  $\epsilon_i < 0$ , the effect on  $Z_i^d$  is positive. If the productivity effects on  $Z_i$  is equal to the average productivity effect,  $\tilde{\Pi}_i = \tilde{\Pi}$  and the substitution term drops out.

Similarly, if an increase in  $H$  affects the optimal quantity of the commodities, it would be expected to affect the derived demand for the factors  $x_i$  and  $t_i$ . From production function (1.2), the percentage change in the demand for factor  $x_i$  per unit change in  $H$  would be

$$\tilde{x}_i = (\tilde{Z}_i^d - \tilde{M}\tilde{P}_i) + w_i(\tilde{x}_i - \tilde{t}_i), \quad (1.16)$$

where  $w_i$  is the production share. If the production function is homogeneous, and  $H$  is assumed not to affect the ratio of factor prices,  $p_x/p_t$ , then the evaluation of  $(\tilde{x}_i - \tilde{t}_i)$  in equilibrium gives

$$\tilde{x}_i = (\tilde{Z}_i^d - \tilde{M}\tilde{P}_i) + w_i\sigma(\tilde{M}\tilde{P}_{x_i} - \tilde{M}\tilde{P}_{t_i}), \quad (1.17)$$

where  $\sigma$  is the elasticity of substitution in production ( $\sigma \geq 0$ ); substituting (1.15) and rearranging,

$$\tilde{x}_i = \eta_i\tilde{Y}_c - \tilde{M}\tilde{P}_i + \epsilon_i(\tilde{\Pi}_i - \tilde{\Pi}) + w_i\sigma(\tilde{M}\tilde{P}_{x_i} - \tilde{M}\tilde{P}_{t_i}). \quad (1.18)$$

The first term on the right indicates the gross increase in the demand

<sup>9</sup> See section 5 of Appendix A for the derivations of equations (1.15) through (1.18).

for  $Z_i$  resulting from the effect of  $H$  on real income; the second represents the change in the production of  $Z_i$  from the initial quantities of the inputs, and the third represents the influence of commodity bias. The sum of these three terms indicates the net change in  $Z_i$  demanded—a demand which is met by altering the quantities of the factors of production. The final term shows the effect of factor bias—if  $H$  enhances productivity and is biased toward  $x$ ,  $\tilde{M}P_x > \tilde{M}P_i > 0$ , and the final term is positive.

The human capital variable  $H$  has been viewed as an exogenous variable throughout this chapter. The model indicates the effects of  $H$  on utility maximization without developing the implications for the optimal stock of  $H$ . To the extent productivity in nonmarket activities is altered by  $H$ , the return on an investment in  $H$  is also affected. *Ceteris paribus*, if consumption efficiency is enhanced by human capital, the return will be larger, and if the marginal return is increased, the household will be induced to increase its stock of  $H$ .

Clearly, the human capital variable could be incorporated in the analysis as an endogenous variable, with consumption income treated as a return on the investment in  $H$ . In this way the effect of human capital on consumption—on the activities associated with nonmarket time—would be included in the evaluation of  $H$  as an investment prospect. In addition to investigating the return through earnings in the market sector, one would also investigate the return through productivity in the nonmarket sector. The sum of these pecuniary and consumption effects is a more adequate reflection of the full return on the investment. It is these consumption income effects that are the focus of this study.