APPENDIX D

A SUMMARY OF MAJOR VARIABLES AND PROGRAM OPERATIONS

Employment Location Submodel

\[ MAN(J, HY, HED) = JOB(J, IND) \times SICMAN(IND, HY, HED); \]  

(6.1)

where:

\[ MAN(J, HY, HED) = \] manpower requirements by workplace, income class, and equation class;

\[ JOB(J, IND) = \] number of jobs of primary workers by workplace and industry;

\[ SICMAN(IND, HY, HED) = \] worker characteristics matrix by industry, income class, and education class.

\[ PROVIS(J, H) = RATE(H) \times F(J, H); \]  

(6.2)

where:

\[ PROVIS(J, H) = \] basic mobility forecast—provisional estimate of number of movers (no employment change);

\[ RATE(H) = \] relocating rates for each household class \( H \);

\[ F(J, H) = \] the number of households at each work zone \( J \) in each household class \( H \).

\[ PMOVE(H, J) = PROVIS(H, J) + ADDITIONS(H, J) - LOSSES(H, J); \]  

(6.3)
where:

\[ PMOVE(H, J) = \text{number of demanders by household type and workplace location for use in the demand and allocation models}; \]

\[ ADDITIONS(H, J) = \text{additional demanders by household class and workplace resulting from workplace-specific employment increases by income and education classes}; \]

\[ LOSSES(H, J) = \text{reduction in demanders by household class and workplace resulting from workplace declines by income and education classes}. \]

\[ ADDITIONS(H, J) = ADD RATE(H) \quad \text{* EMPLOY INCREASE(HED, HY, J)}; \quad (6.4) \]

where:

\[ ADD RATE(H) = \text{weights for converting projected increases in employment by income and education class into increases by income, education, family size, and age classes}; \]

\[ EMPLOY INCREASES \quad (HED, HY, J) = \text{projected increases in employment by education, and income classes and workplace}. \]

\[ \sum_{HAG,HFS} \text{PROVIS(H, J)} \geq \text{EMPLOY DECLINE(HED, HY, J)}; \quad (6.5) \]

where:

\[ HAG, HFS = \text{age and family size categories}; \]

\[ \text{EMPLOY DECLINE(HED, HY, J)} = \text{employment declines by income-education category and workplace}. \]

\[ ELIGIBLE(J, H) = PROVIS(J, H) \quad \text{* LOSS RATE(HAG)}; \quad (6.6) \]
where:

\[ \text{LOSS RATE}(HAG) = \text{weights shown in Table 6.5}; \]
\[ \text{ELIGIBLE}(J, H) = \text{households eligible for losing jobs this period}. \]

**Vacancy Submodel**

\[ \text{PAVAIL}(J, K, HY) = \text{OCC RATE 2}(J, K, HAG, HFS) \]
\[ \quad \times \text{OCC RATE 1}(J, K, HY) \]
\[ \quad \times \text{PROVIS}(J, H); \]  

(6.7)

where:

\[ \text{OCC RATE 1}(J, K, HY) = \text{proportion of each income class } HY \]
\[ \text{residing in housing type } K \text{ at workplace } J \text{ at the end of the previous period}; \]

\[ \text{OCC RATE 2} \]
\[ (J, K, HAG, HFS) = \text{proportion of each family size, HFS, and age class, HAG, residing in housing type } K \text{ at each workplace } J \text{ at the end of the previous period}. \]

\[ \text{AVAIL}(K, I) = \frac{\text{PAVAIL}(J, K, HY)}{\text{POTENTIAL}(J, K, HY)} \times \text{TRIP}(I, J, HY) \]
\[ \quad \times \text{STOCK}(K, I) + \text{VACANT}(K, I); \]  

(6.8)

where:

\[ \text{AVAIL}(K, I) = \text{number of available units by housing type and residence zone}; \]
\[ \text{STOCK}(K, I) = \text{number of occupied units by each type in each residence zone}; \]
\[ \text{TRIP}(I, J, HY) = \text{number of trips from workplace } J \text{ to zone } I \text{ for income class } HY; \]
\[ \text{VACANT}(K, I) = \text{number of units available but not occupied in previous period}; \]
\[ \text{POTENTIAL}(J, K, HY) = \sum \text{TRIPS}(I, J, HY) \times \text{STOCK}(K, I) \].
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Other Bookkeeping

\[ RMOVE(H, J) = CHAR(H) \times PMOVE(H, J); \]  \hspace{1cm} (6.9)

where:

\[ RMOVE(H, J) = \text{housing demanders by characteristics at time of move;} \]
\[ PMOVE(H, J) = \text{housing demanders by old characteristics;} \]
\[ CHAR(H) = \text{demographic change matrix, modifies age and family size of demanders.} \]

\[ REV \ TRIPS(I, J, HY) = TRIP(I, J, HY) - PROVIS \ TRIPS(I, J, HY); \]  \hspace{1cm} (6.10)

where:

\[ REV \ TRIPS(I, J, HY) = \text{revised trip pattern by residence, workplace, and income class; includes only nonmoving households;} \]
\[ TRIP(I, J, HY) = \text{trip pattern of all households at beginning of period;} \]
\[ PROVIS \ TRIPS(I, J, HY) = \text{trips of households that vacate units this period.} \]

Demand Allocation Submodel

\[ TCOST(I, J, HY, M) = OPC(I, J, M) + 0.4 \times WAGE(HY) \times HRS(I, J, M); \]  \hspace{1cm} (7.1)

where:

\[ TCOST(I, J, HY, M) = \text{the travel cost from residence zone I to workplace J for income class HY and mode M;} \]
\[ OPC(I, J, M) = \text{out-of-pocket costs for mode M;} \]
\[ WAGE(HY) = \text{implicit wage rate of income class HY;} \]
\[ HRS(I, J, M) = \text{interzonal travel time for mode M.} \]
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\[ RES(I, J, K, HY) = TCOST(I, J, HY, MIN) + P(K, I); \]  
(7.2)

where:

\[ RES(I, J, K, HY) = \text{array of gross price surfaces over residence zones } I, \text{ for each workplace } J, \text{ housing type } K, \text{ and income class } HY; \]

\[ TCOST(I, J, HY, MIN) = \text{travel cost for the cheapest mode for trips from residence zone to work zone by income class}; \]

\[ P(K, I) = \text{array of expected prices by housing type and residence zone}. \]

\[ WT(I, J, K, HY) = \frac{AVAIL(K, I) \times TRIP(I, J, HY)}{\sum_I[AVAIL(K, I) \times TRIP(I, J, HY)]}; \]  
(7.3)

where:

\[ WT(I, J, K, HY) = \text{weight applied to gross price surfaces by residence zone } I, \text{ workplace } J, \text{ housing type } K, \text{ and income class } HY; \]

\[ AVAIL(K, I) = \text{number of units available for occupancy by type and location}; \]

\[ TRIP(I, J, HY) = \text{work trips made by income class}. \]

\[ R(J, K, HY) = \sum_I[WT(I, J, K, HY) \times RES(I, J, K, HY)]; \]  
(7.4)

where \( R(J, K, HY) = \text{expected gross housing price by workplace, housing type, and income class}. \)

\[ PCT(H, J, K) = A(H, K) + B1 \times REL(J, HY, 1) \]
\[ + B2 \times REL(J, HY, 2) + \ldots ; \]  
(7.5)

where:

\[ PCT(H, J, K) = \text{the proportion of housing demanders of class } H \text{ at workplace } J \text{ that chooses housing type } K; \]

\[ A, B1, B2, \ldots = \text{estimated parameters of the demand equation}; \]

\[ REL(J, HY, 1) = \text{the expected gross price of unit 1 divided by the expected gross price of unit 10, e.g., } \]
\[ R(J, 1, HY)/R(J, 10, HY), \text{ and so on for each housing type}. \]
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\[ XMOV(J, K, H) = PCT(H, J, K) \times RMOVE(H, J); \]  

(7.6)

where:

\[ XMOV(J, K, H) = \text{number of housing demanders by workplace} \]

\[ J \text{ and household class } H \text{ who choose housing type } K. \]

\[ DEMAND(K) = DMND(K) \times VRATE(K); \]  

(7.7)

where:

\[ DEMAND(K) = \text{total expected demand for each housing type } K; \]

\[ DMND(K) = \text{demand by households for each housing type} \]

\[ = \sum_{J, HY} AMOV(J, K, HY); \]

\[ VRATE(K) = 1 + \text{normal vacancy rate for each housing type.} \]

Filtering Submodel

\[ PQ(KS, Q1, Q2, I) = P(KS, Q1, I) - P(KS, Q2, I); \]  

(7.8)

where:

\[ PQ(KS, Q1, Q2, I) = \text{the quality premium between quality} \]

\[ \text{levels } Q1 \text{ and } Q2 \text{ for structural type } KS \]

\[ \text{in zone } I; \]

\[ P(KS, Q1, I) = \text{the expected price for a unit of structural} \]

\[ \text{type } KS, \text{ and quality level } Q1 \text{ in zone } I; \]

\[ P(KS, Q2, I) = \text{expected price of similar units except of} \]

\[ \text{quality level } Q2. \]

\[ RATIO(KS, Q1, Q2, I) = PQ(KS, Q1, Q2, I)/COSTF(KS, Q1, Q2); \]  

(7.9)

where:

\[ RATIO(KS, Q1, Q2, I) = \text{a profitability measure of transforming} \]

\[ \text{a unit of structural type } KS \text{ from} \]

\[ \text{quality level } Q2 \text{ to quality level } Q1 \]

\[ \text{in zone } I; \]

\[ COSTF(KS, Q1, Q2) = \text{cost of upgrading a unit of structural} \]

\[ \text{type } KS \text{ from quality level } Q2 \text{ to} \]

\[ \text{quality level } Q1. \]
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\[ \text{AVAIL}_{F}(K, I) = \text{FILTER} \left[ \frac{KS}{Q1, Q2, I} \right] \ast \text{AVAIL}(K, I); \]  

(7.10)

where:

\[ \text{AVAIL}_{F}(K, I) = \text{the stock of available dwelling units after some units have changed quality level}; \]
\[ \text{AVAIL}(K, I) = \text{number of available units supplied by the vacancy submodel}; \]
\[ \text{FILTER} = \text{the filtering rate response function}. \]

Supply Submodel

\[ \text{XDMND}(K) = \text{DEMAND}(K) - \sum_{I} \text{AVAIL}_{F}(K, I); \]  

(7.11)

where \( \text{XDMND}(K) \) = the excess demand including normal vacancies for each housing type \( K \) in the current period.

\[ \text{PSTRUT}(K, I) = P(K, I) \ast \text{AVGNO}(K); \]  

(7.12)

where:

\[ \text{PSTRUT}(K, I) = \text{expected structure price by housing type and zone}; \]
\[ \text{AVGNO}(K) = \text{average number of units per structure by housing type } K. \]

\[ \text{PINPUT}(K, KO) = \text{PSTRUT}(K, I) \ast \text{INPTNO}(K, KO) \ast \text{AGLOM}; \]  

(7.13)

where:

\[ \text{PINPUT}(K, KO) = \text{price of input } K \text{ when housing type } KO \text{ is output}; \]
\[ \text{INPTNO}(K, KO) = \text{number of structures of type } K \text{ required to produce a structure of type } KO; \]
\[ \text{AGLOM} = [1.0 + 0.025 \ast \text{INPTNO}(K, KO)] \text{ if INPTNO exceeds 1.0, and 1.0 otherwise.} \]

\[ \text{PROFIT}(I, K, KO) = \text{POUTPUT}(I, K, KO) - [\text{PINPUT}(I, K, KO) + \text{COST}(K, KO)]; \]  

(7.14)
where:

\[ \text{PROFIT}(I, K, KO) = \text{the expected profit of producing structures of type } KO \text{ from inputs of type } K \text{ in zone } I; \]
\[ \text{POUTPUT}(I, K, KO) = \text{the total expected price of output structures produced by the activity}; \]
\[ \text{COST}(K, KO) = \text{exogenously estimated cost of transforming } K \text{ to } KO. \]

\[ \text{RATE}(I, K, KO) = \frac{\text{PROFIT}(I, K, KO)}{\text{PINPUT}(I, K, KO) + \text{COST}(K, KO)}; \]

where \( \text{RATE}(I, K, KO) \) = gross profit rate of producing output \( KO \) from input \( K \).

\[ \text{AVAILS}(K, I) = \text{SUPPLY}\{\text{AVAILF}(K, I)\}; \] (7.16)

subject to:

a. Profit:

\[ \text{RATE}(I, K, KO) > 0; \]

b. Availability:

\[ \text{AVAILS}(KO, I) \leq \sum_{K} \left\{ \frac{\text{AVAILF}(K, I)}{\text{INPTNO}(K, KO)} \right\} \]
\[ + \frac{\text{VLAND}(I)}{\text{INPTNO}(28, KO)}; \]

c. Zoning:

\[ \text{AVAILS}(K, I) - \text{AVAILF}(K, I) \leq \text{ZONE}(K, I); \]

d. Forecast demand:

\[ \sum_{I} \text{AVAILS}(K, I) - \sum_{I} \text{AVAILF}(K, I) \leq \text{XDMND}(K); \]

where:

\[ \text{AVAILS}(K, I) = \text{the number of units available for occupancy this period after new construction and transformations}; \]
\[ \text{SUPPLY} = \text{algorithm used to assign levels to transformation activities; chooses activities according to profitability}; \]
\[ \text{VLAND}(I) = \text{the quantity of vacant land available in each zone during a period}; \]
\[ \text{ZONE}(K, I) = \text{the zoning constraint which limits the number of output structures of each type which can be produced in a zone}. \]
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Market Clearing Submodel

\[
\text{MIN } \sum_{I,J, HY} TCOST(I, J, HY) \times X(I, J, HY); \quad (7.17)
\]

for each separate \( K \) subject to:

\[
\sum_{I} X(I, J, HY) = AMOV(J, K, HY);
\]

\[
\sum_{J,HY} X(I, J, HY) = AVAILS(K, I);
\]

where \( X(I, J, HY) \) = households of income class \( HY \) employed at workplace \( J \) who locate in zone \( I \), given they have chosen housing type \( K \).

\[
PLAND(I) = \sum_{K} \left[ \frac{LRENT(K, I) \times STOCK(K, I)}{\sum_{K} STOCK(K, I)} \right]; \quad (7.18)
\]

where:

- \( PLAND(I) \) = the one-period equilibrium price of land in zone \( I \);
- \( LRENT(K, I) \) = the location rent of land in zone \( I \) under housing type \( K \);
- \( STOCK(K, I) \) = the number of units of type \( K \) in zone \( I \).