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Appendix C

DERIVATION OF CONSUMPTION MODEL FORMULAS

1. LIFE CYCLE PATTERNS

Let the utility function be

$$U = (\Sigma m^i h_i^{-B})^{-1/B} J(Z_i). \tag{C-1}$$

This is a constant elasticity of substitution function in terms of healthy time. If the flow of healthy time per unit of health capital were independent of the stock, then

$$\frac{UH_i}{UH_1} = (1 + r)^{1-i}\left(\frac{r + \delta_i}{r + \delta_1}\right) = m^{i-1}\left(\frac{H_1}{H_i}\right)^{B+1}. \tag{C-2}$$

Solving (C-2) for H_i and taking natural logarithms of the resulting expression, one gets

$$\begin{aligned} \ln H_i = \ln H_1 &+ \sigma(i - 1)\ln m + \sigma(i - 1)\ln(1 + r) \\ &+ \sigma[\ln(r + \delta_1) - \ln(r + \delta_i)], \end{aligned} \tag{C-3}$$

where $\sigma = 1/(1 + B)$. The derivative of $\ln H_i$ with respect to i is

$$\tilde{H}_i = \sigma[\ln m + \ln(1 + r) - s_i\tilde{\delta}]. \tag{C-4}$$

Note also that

$$\tilde{H}_{ii} = -s_i(1 - s_i)\sigma\tilde{\delta}^2. \tag{C-5}$$

It was shown in Chapter II that

$$\tilde{I}_i = \frac{\tilde{H}_i^2 + \tilde{H}_{ii} + \delta_i(\tilde{H}_i + \tilde{\delta})}{\tilde{H}_i + \delta_i}.$$

Substituting (C-4) and (C-5) into the last equation and assuming no time preference, one gets

$$\tilde{I}_i = \frac{\tilde{\delta}(1 - s_i\sigma)(\delta_i - s_i\sigma\tilde{\delta}) + s_i^2\sigma\tilde{\delta}^2 + r^2\sigma^2 - 2rs_i\sigma^2\tilde{\delta}^2 + \delta_i\sigma r}{\tilde{H}_i + \delta_i}. \tag{C-6}$$

If $r = 0$, equation (C-6) reduces to

$$\tilde{I}_i = \frac{\tilde{\delta}(1 - \sigma)(\delta_i - \sigma\tilde{\delta}) + \sigma\tilde{\delta}^2}{\delta_i - \sigma\tilde{\delta}}.$$

Since gross investment cannot be negative, $\delta_i > \sigma\tilde{\delta}$. Therefore, given a zero rate of interest, a sufficient condition for gross investment to be positively correlated with age is $\sigma < 1$. If r exceeds zero, it becomes somewhat more difficult to evaluate the sign of $\tilde{I}_i$. Suppose this sign is evaluated when $\tilde{H}_i$ is negative. Then the condition for positive gross investment requires that $\delta_i > s_i\sigma\tilde{\delta} - r$. This condition could hold even if $\delta_i < s_i\sigma\tilde{\delta}$. But provided the rate of interest is relatively small, it is not likely to be satisfied unless $\delta_i > s_i\sigma\tilde{\delta}$. In this situation, an elasticity of substitution less than unity would make all terms in the numerator of (C-6) positive except $-2rs_i\sigma^2\tilde{\delta}^2$. Thus, it is extremely likely that $\tilde{I}_i$ would be positive.

2. MARKET AND NONMARKET EFFICIENCY

To analyze the effects of variations in the shadow price of health among individuals of the same age, let the cross-sectional demand curve be

$$H = H(R^*, Q^*) \tag{C-7}$$

where $R^* = R/Q$ and $Q^* = (r + \delta)\pi/Q$. Differentiation of (C-7) with respect to the wage rate holding R^* fixed yields

$$\frac{dH}{dW}\frac{W}{H} = \frac{\partial H}{\partial Q^*}\frac{Q^*}{H}\frac{dQ^*}{dW}\frac{W}{Q^*}$$

$$e_{H,W} = -e_H\frac{d\ln Q^*}{d\ln W}.$$

An evaluation of the elasticity of Q^* with respect to W indicates

$$\frac{d\ln Q^*}{d\ln W} = K - \frac{d\ln Q}{d\ln W}.$$

Since $\ln Q = w\ln(r + \delta)\pi + (1 - w)\ln q$,

$$\frac{d\ln Q}{d\ln W} = wK + (1 - w)\frac{WT}{qZ} = \bar{K}.$$

Therefore,

$$\eta_{Q^*,W} = K - \bar{K}$$

and

$$e_{H,W} = -e_H(K - \bar{K}). \tag{C-8}$$

To compute the wage elasticity of medical care, note that

$$I(M, T) = (\tilde{H} + \delta)H.$$

The wage derivative in this equation is

$$Ie_H\frac{\mathrm{d}\pi}{\mathrm{d}W} + W\frac{\mathrm{d}TH}{\mathrm{d}W} + P\frac{\mathrm{d}M}{\mathrm{d}W} = Ie_H\bar{K}\frac{\pi}{W}.$$

This becomes the first equation in (B-13). The second and third equations remain the same, and the solution of the system is

$$e_{M,W} = K\sigma_p - (K - \bar{K})e_H. \tag{C-9}$$

By differentiating the demand function (C-7) with money full wealth and the wage rate fixed, the human capital parameter in the demand curve for health is calculated:

$$\frac{\mathrm{d}H}{\mathrm{d}E}\frac{1}{H} = \frac{\partial H}{\partial R^*}\frac{R^*}{H}\frac{\mathrm{d}R^*}{\mathrm{d}E}\frac{1}{R^*} + \frac{\partial H}{\partial Q^*}\frac{Q^*}{H}\frac{\mathrm{d}Q^*}{\mathrm{d}E}\frac{1}{Q^*}$$

$$\hat{H} = \eta_H\frac{\mathrm{d}\ln R^*}{\mathrm{d}E} - e_H\frac{\mathrm{d}\ln Q^*}{\mathrm{d}E}.$$

Since $\ln R^* = \ln R - \ln Q$ and since R is fixed,

$$\frac{\mathrm{d}\ln R^*}{\mathrm{d}E} = -\frac{\mathrm{d}\ln Q}{\mathrm{d}E} = r_E$$

$$\frac{\mathrm{d}\ln Q^*}{\mathrm{d}E} = \frac{\mathrm{d}\ln \pi}{\mathrm{d}E} - \frac{\mathrm{d}\ln Q}{\mathrm{d}E} = -r_H + r_E.$$

Hence,

$$\hat{H} = r_E\eta_H + e_H(r_H - r_E). \tag{C-10}$$

Since $\hat{M} = \hat{H} - r_H$, the human capital parameter in the demand curve for medical care would be

$$\hat{M} = r_E(\eta_H - 1) + (r_H - r_E)(e_H - 1). \tag{C-11}$$