Appendix C

DERIVATION OF CONSUMPTION MODEL FORMULAS

1. LIFE CYCLE PATTERNS

Let the utility function be

\[ U = (\Sigma m^i h_i^{-(b)})^{-1/b} J(Z_i) \]  \hspace{1cm} (C-1)

This is a constant elasticity of substitution function in terms of healthy time. If the flow of healthy time per unit of health capital were independent of the stock, then

\[ \frac{UH_i}{UH_1} = (1 + r)^{1 - i} \left( \frac{r + \delta_i}{r + \delta_i} \right) = m^i - 1 \left( \frac{H_1}{H_i} \right)^{b+1} \] \hspace{1cm} (C-2)

Solving (C-2) for \( H_i \) and taking natural logarithms of the resulting expression, one gets

\[ \ln H_i = \ln H_1 + \sigma(i - 1) \ln m + \sigma(i - 1) \ln (1 + r) \]
\[ + \sigma[\ln (r + \delta_i) - \ln (r + \delta_i)], \] \hspace{1cm} (C-3)

where \( \sigma = 1/(1 + B) \). The derivative of \( \ln H_i \) with respect to \( i \) is

\[ \tilde{H}_i = \sigma[\ln m + \ln(1 + r) - s_i \delta]. \] \hspace{1cm} (C-4)

Note also that

\[ \tilde{H}_{ii} = -s_i(1 - s_i)\sigma \delta^2. \] \hspace{1cm} (C-5)

It was shown in Chapter II that

\[ J_i = \frac{\tilde{H}_i^2 + \tilde{H}_{ii} + \delta(\tilde{H}_i + \delta)}{\tilde{H}_i + \delta}. \]

Substituting (C-4) and (C-5) into the last equation and assuming no time preference, one gets

\[ J_i = \frac{\delta(1 - s_i \sigma)(\delta_i - s_i \sigma \delta) + s_i^2 \sigma \delta^2 + r^2 \sigma^2 - 2r s_i \sigma^2 \delta^2 + \delta_i \sigma r}{\tilde{H}_i + \delta_i}. \] \hspace{1cm} (C-6)
Appendix C

If $r = 0$, equation (C-6) reduces to

$$I_i = \frac{\delta(1 - \sigma)(\delta_i - \sigma \delta) + \sigma \delta^2}{\delta_i - \sigma \delta}.$$  

Since gross investment cannot be negative, $\delta_i > \sigma \delta$. Therefore, given a zero rate of interest, a sufficient condition for gross investment to be positively correlated with age is $\sigma < 1$. If $r$ exceeds zero, it becomes somewhat more difficult to evaluate the sign of $I_i$. Suppose this sign is evaluated when $\bar{H}_i$ is negative. Then the condition for positive gross investment requires that $\delta_i > s_i \delta - r$. This condition could hold even if $\delta_i < s_i \sigma \delta$. But provided the rate of interest is relatively small, it is not likely to be satisfied unless $\delta_i > s_i \sigma \delta$. In this situation, an elasticity of substitution less than unity would make all terms in the numerator of (C-6) positive except $-2rs_i \sigma^2 \delta^2$. Thus, it is extremely likely that $I_i$ would be positive.

2. MARKET AND NONMARKET EFFICIENCY

To analyze the effects of variations in the shadow price of health among individuals of the same age, let the cross-sectional demand curve be

$$H = H(R^*, Q^*)$$  

(C-7)

where $R^* = R/Q$ and $Q^* = (r + \delta)\pi/Q$. Differentiation of (C-7) with respect to the wage rate holding $R^*$ fixed yields

$$\frac{dH}{dW} \frac{dW}{H} = \frac{\partial H}{\partial Q^*} \frac{dQ^*}{W} \frac{dW}{Q^*}$$

$$e_{H,w} = -e_{H} \frac{d \ln Q^*}{d \ln W}.$$  

An evaluation of the elasticity of $Q^*$ with respect to $W$ indicates

$$\frac{d \ln Q^*}{d \ln W} = K - \frac{d \ln Q}{d \ln W}.$$  

Since $\ln Q = w \ln (r + \delta) + (1 - w) \ln q$,

$$\frac{d \ln Q}{d \ln W} = wK + (1 - w) \frac{WT}{qZ} = \bar{K}.$$  

Therefore,

$$\eta_{Q^*,w} = K - \bar{K}.$$
Appendix C

and

\[ e_{H,w} = -e_H(K - \bar{K}). \]  \hspace{1cm} (C-8)

To compute the wage elasticity of medical care, note that

\[ I(M, T) = (\bar{H} + \delta)H. \]

The wage derivative in this equation is

\[ Ie_H \frac{d\pi}{dW} + W \frac{dTH}{dW} + P \frac{dM}{dW} = Ie_H \bar{K} \frac{\pi}{W}. \]

This becomes the first equation in (B-13). The second and third equations remain the same, and the solution of the system is

\[ e_{M,w} = K\sigma_p - (K - \bar{K})e_H. \]  \hspace{1cm} (C-9)

By differentiating the demand function (C-7) with money full wealth and the wage rate fixed, the human capital parameter in the demand curve for health is calculated:

\[ \frac{dH}{dE} H = \frac{\partial H}{\partial R^*} \frac{dR^*}{dE} \frac{1}{H} + \frac{\partial H}{\partial Q^*} \frac{dQ^*}{dE} \frac{1}{Q^*} \]

\[ \bar{H} = \eta_H \frac{d \ln R^*}{dE} - e_H \frac{d \ln Q^*}{dE}. \]

Since \( \ln R^* = \ln R - \ln Q \) and since \( R \) is fixed,

\[ \frac{d \ln R^*}{dE} = - \frac{d \ln Q}{dE} = r_{E} \]

\[ \frac{d \ln Q^*}{dE} = \frac{d \ln \pi}{dE} - \frac{d \ln Q}{dE} = -r_H + r_{E}. \]

Hence,

\[ \bar{H} = r_{E}\eta_H + e_H(r_H - r_{E}). \]  \hspace{1cm} (C-10)

Since \( \bar{M} = \bar{H} - r_H \), the human capital parameter in the demand curve for medical care would be

\[ \bar{M} = r_{E}(\eta_H - 1) + (r_H - r_{E})(e_H - 1). \]  \hspace{1cm} (C-11)