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## Appendix A

### UTILITY MAXIMIZATIONS

#### 1. DISCRETE TIME

To maximize utility subject to the full wealth and production function constraints, form the Lagrangian expression

$$L = U(\phi_0 H_0, \dots, \phi_n H_n, Z_0, \dots, Z_n) + \lambda \left[ R - \sum \frac{C_i + C_{1i} + W_i T L_i}{(1+r)^i} \right], \quad (\text{A-1})$$

where  $C_i = P_i M_i + W_i T H_i$  and  $C_{1i} = F_i X_i + W_i T_i$ . Differentiating  $L$  with respect to gross investment in period  $i - 1$  and setting the partial derivative equal to zero, one obtains

$$\begin{aligned} & U h_i \frac{\partial h_i}{\partial H_i} \frac{\partial H_i}{\partial I_{i-1}} + U h_{i+1} \frac{\partial h_{i+1}}{\partial H_{i+1}} \frac{\partial H_{i+1}}{\partial I_{i-1}} + \dots + U h_n \frac{\partial h_n}{\partial H_n} \frac{\partial H_n}{\partial I_{i-1}} \\ &= \lambda \left[ \frac{(dC_{i-1}/dI_{i-1})}{(1+r)^{i-1}} + \frac{W_i (\partial T L_i / \partial H_i) (\partial H_i / \partial I_{i-1})}{(1+r)^i} \right. \\ &\quad + \frac{W_{i+1} (\partial T L_{i+1} / \partial H_{i+1}) (\partial H_{i+1} / \partial I_{i-1})}{(1+r)^{i+1}} + \dots \\ &\quad \left. + \frac{W_n (\partial T L_n / \partial H_n) (\partial H_n / \partial I_{i-1})}{(1+r)^n} \right]. \quad (\text{A-2}) \end{aligned}$$

But  $\partial h_i / \partial H_i = G_i$ ,  $\partial H_i / \partial I_{i-1} = 1$ ,  $\partial H_{i+1} / \partial I_{i-1} = (1 - \delta_i)$ ,  $\partial H_n / \partial I_{i-1} = (1 - \delta_i) \dots (1 - \delta_{n-1})$ ,  $dC_{i-1} / dI_{i-1} = \pi_{i-1}$ , and  $\partial T L_i / \partial H_i = -G_i$ . Therefore,

$$\begin{aligned} \frac{\pi_{i-1}}{(1+r)^{i-1}} &= \frac{W_i G_i}{(1+r)^i} + \frac{(1-\delta_i) W_{i+1} G_{i+1}}{(1+r)^{i+1}} + \dots \\ &\quad + \frac{(1-\delta_i) \dots (1-\delta_{n-1}) W_n G_n}{(1+r)^n} + \frac{U h_i G_i}{\lambda} \end{aligned}$$

$$\begin{aligned}
 &+ (1 - \delta_i) \frac{U h_{i+1}}{\lambda} G_{i+1} + \dots \\
 &+ (1 - \delta_i) \dots (1 - \delta_{n-1}) \frac{U h_n}{\lambda} G_n.
 \end{aligned} \tag{A-3}$$

## 2. CONTINUOUS TIME

Let the utility function be

$$U = \int m_i f(\phi_i H_i, Z_i) di, \tag{A-4}$$

where  $m_i$  is the weight attached to utility in period  $i$ . Equation (A-4) defines an additive utility function, but any monotonic transformation of this function could be employed.<sup>1</sup> Let all household production functions be homogeneous of degree one. Then  $C_i = \pi_i I_i$ ,  $C_{1i} = q_i Z_i$ , and full wealth can be written as

$$R = \int e^{-ri} (\pi_i I_i + q_i Z_i + W_i T L_i) di. \tag{A-5}$$

By definition,

$$I_i = \dot{H}_i + \delta_i H_i, \tag{A-6}$$

where  $\dot{H}_i$  is the instantaneous rate of change of capital stock. Substitution of (A-6) into (A-5) yields

$$R = \int e^{-ri} (\pi_i \delta_i H_i + \pi_i \dot{H}_i + q_i Z_i + W_i T L_i) di. \tag{A-7}$$

To maximize the utility function, form the Lagrangian

$$L - \lambda R = \int [m_i f(\phi_i H_i, Z_i) - \lambda e^{-ri} (\pi_i \delta_i H_i + \pi_i \dot{H}_i + q_i Z_i + W_i T L_i)] di, \tag{A-8}$$

or

$$L - \lambda R = \int J(H_i, \dot{H}_i, Z_i, i) di, \tag{A-9}$$

<sup>1</sup> Robert H. Strotz has shown, however, that certain restrictions must be placed on the  $m_i$ . In particular, the initial consumption plan will be fulfilled if and only if  $m_i = (m_0)^i$ . See "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 23, No. 62 (1955-56).

where

$$J = m_i f(\phi_i H_i, Z_i) - \lambda e^{-ri} (\pi_i \delta_i H_i + \pi_i \dot{H}_i + q_i Z_i + W_i T L_i). \quad (\text{A-10})$$

Euler's equation for the optimal path of  $H_i$  is

$$\frac{\partial J}{\partial H_i} = \frac{d}{di} \frac{\partial J}{\partial \dot{H}_i} \quad (\text{A-11})$$

In the present context,

$$\frac{\partial J}{\partial H_i} = U h_i G_i - \lambda e^{-ri} \pi_i \delta_i + \lambda e^{-ri} W_i G_i,$$

$$\frac{\partial J}{\partial \dot{H}_i} = -\lambda e^{-ri} \pi_i,$$

and

$$\frac{d}{di} \frac{\partial J}{\partial \dot{H}_i} = -\lambda e^{-ri} \dot{\pi}_i + \lambda e^{-ri} r \pi_i. \quad (\text{A-12})$$

Consequently,

$$G_i [W_i + (U h_i / \lambda) e^{ri}] = \pi_i (r - \dot{\pi}_i + \delta_i), \quad (\text{A-13})$$

which is the continuous time analogue of equation (1-13).