The demand for transportation

Transportation services are market inputs that combine with the traveler's time to produce a trip to a certain destination. An air trip \((Z_A)\), for example, is a combination of the air-carriers' services \((X_A)\) and of elapsed time \((T_A)\)

\[
Z_A = f(X_A, T_A).
\]  

Similarly, for a bus trip

\[
Z_B = f(X_B, T_B).
\]

The demand for the various transportation services \((X_A, X_B)\) is a derived demand depending on the demand for air and bus trips. The demand for trips depends, in turn, on the direct utility they yield and on their contribution to the production of a third activity—a visit to the point of destination \((Z_v)\),

\[
Z_v = f(X_v, T_v, Z_A, Z_B),
\]

where \(X_v\) is other market inputs involved in the production of the visit (e.g., hotel, restaurant, and transportation services at the point of destination), and \(T_v\) is other time inputs involved (i.e., the length of stay).

The activity "visit" is produced for one of two reasons: to yield direct utilities when the visit is for personal purposes, or to serve as an input in the production of market goods when the visit is for business purposes. The demand for personal visits and the demand for business
visits reflect the marginal utility and the marginal productivity of the visit, respectively, which depend in turn on the "attractiveness" of the point of destination, i.e., the size, the level of economic activity, and the scenery of the place. Given personal taste, the demand for personal visits is a function of the visit's price, the price of related activities (e.g., visits to other places, other recreation activities, and other forms of communication), and the household's income.

A business trip can be regarded as a short-run migration movement. Ignoring interest rates, the incentive to migrate is inversely related to the costs of migration and directly related to the immigrant's marginal product differential. The demand for business visits depends, therefore, on price factors similar to those affecting the demand for personal visits, and on the difference between the passenger's marginal product at the point of origin and his marginal product at the point of destination. This difference seems, as in other cases of migration, to increase with the passenger's skills and, hence, with his income.

The demand elasticity of a factor of production is directly related to the demand elasticity of the product, the elasticity of substitution in production between the factor and other inputs, and the share of the factor in total costs. Therefore, the demand elasticity of a trip depends, aside from the demand of visits, on the elasticity of substitution between trips and other inputs used in the production of the visit, and on the share of the trip in the visit's costs. The optimal allocation of inputs in the production of a visit calls for

$$\frac{\partial Z_v}{\partial Z_i} = \frac{x_i}{Z_i} = \frac{\Pi_i - (u_i/\lambda)}{P_v} = \frac{\Pi'_i}{P_v} \quad \text{i = A, B}$$

$$\frac{\partial Z_v}{\partial T_v} = \frac{t_v}{Z_v} = \frac{\Pi_i - (u_i/\lambda)}{K} = \frac{\Pi'_i}{K}$$

where $z_i = \frac{\partial Z_i}{\partial Z_v}$ is the marginal input of the trip in the production of a visit, $t_v$ and $x_v$ are the marginal inputs of other time and market goods, respectively, $P_v$ is the price of the other market goods, and $\Pi'_i$ is the trip's total price. The price of the trip consists of the time and money inputs required to produce the trip ($\Pi'_i$) minus the money equivalent of the marginal utility derived from the trip. An increase in the price of the trip
results in a substitution toward the other time and market inputs. Most of these inputs vary directly with the length of the stay. Hence, the higher the price of the trip, the greater is a passenger's tendency to prolong his stay and to cut the number of his trips. On the other hand, the higher the costs of hotels and restaurants and the lower the costs of the trip, the greater is the passenger's tendency to return home quickly. The price of the trip is directly related to its distance, and, thus, one expects the length of stay and the trip's distance to be positively correlated. The ease with which the passenger can increase his length of stay at the expense of the number of trips, and vice versa, directly affects the elasticity of substitution between the trip and the other factors. The share of the trip's costs in the total cost decreases with the length of the stay when this elasticity exceeds unity, and increases when the elasticity is smaller than unity.

Another factor affecting the demand for trips by a given mode is the elasticity of substitution among trips by different modes. The optimum conditions call for the marginal rate of technical substitution between trips by various modes to be equal to their relative prices

$$\frac{\partial Z_A}{\partial Z_B} = \frac{z_B}{z_A} = \frac{P_A x_A + K t_A - (u_A/\lambda)}{P_B x_B + K t_B - (u_B/\lambda)}$$

(3.5)

The activities “air-trip” and “bus-trip” are assumed to be perfect substitutes in the production of the visit. Both activities are equally efficient in conveying the passenger from one place to the other; hence, $z_B/z_A = 1$. The modal split is determined, therefore, exclusively by the shape of the price line. This shape reflects the differences among the various modes in the direct utilities $(u_i)$, and in the time intensities $[t_i/(P x_i + K t_i)]$. The direct utilities vary among the modes as a function of the convenience, prestige, and risk involved. In a special case, when the marginal utility of one of the modes is sufficiently large, the money equivalent of the marginal utility may exceed the money and time outlays involved in producing the trip, the price of the trip becomes negative, and the passenger will specialize, using only one mode. In general, the price line slopes downward. A stable interior equilibrium is attained at the point of tangency between the concave isocost and the straight isoquant curves. The traveler produces his visits using $Z_{A0}$ air trips and $Z_{B0}$ bus trips (Figure 1).

Changes in the traveler's income and in the distance of the trip affect
the relative prices and the modal choice by affecting both the trip’s costs of production and the money equivalent of the direct utility the trip generates.

The theory of discomfort tries to connect the choice of mode with the direct utilities involved in traveling by the various modes. To isolate the effect of the marginal direct utilities, let us assume that the passenger’s price of time equals zero. The relative price of an air trip compared with a bus trip is, in this case,

$$\frac{\Pi_A'}{\Pi_B'} = \frac{P_Ax_A - (u_A/\lambda)}{P_Bx_B - (u_B/\lambda)},$$  \hspace{1cm} (3.6)$$

The necessary condition for an interior equilibrium is

$$\frac{\Pi_A'}{\Pi_B'} = 1 = u_A - u_B = \lambda(P_Ax_A - P_Bx_B),$$  \hspace{1cm} (3.7)$$
i.e., the money equivalent of the marginal utility differential equals the difference in marginal money outlays per trip. A change in income, which changes the marginal utility of income ($\lambda$), affects the slope of the price line and the modal choice. Moreover, the difference between the marginal utilities and the difference between the money outlays may vary with the distance of the trip, resulting in a change in the optimal combination of modes as the distance increases.
Unfortunately, one cannot predict the effect of changes in income and the distance of the trip on the modal split without explicitly specifying (a) the effect of changes in income on the marginal utility of income, and (b) the functional form \( g \) relating utility with the number of trips and their distance \( M \)

\[
U_i = g(Z_i, M) \quad i = A, B. 
\]  

(3.8)

These two requirements cannot be satisfied unless one is ready to adopt a cardinal concept of utility. Given the conventional ordinal utility approach, the theory of discomfort can be used only as an *ad hoc* explanation of the transportation market; it is incapable of providing an operative tool of prediction.

To circumvent the difficulties inherent in the theory of discomfort, one has to assume that the trip does not convey any direct marginal utility \( u_i = 0 \). This assumption is followed throughout the rest of this study.

The price of a trip was defined in equation (3.4) as the sum of the time and money inputs required to produce the trip \( \Pi_i \) plus the money equivalent of the direct marginal utility. Given our simplifying assumption, this last term equals zero. Moreover, when the production of a trip by mode \( i \) requires a fixed money input \( P_i \) and a fixed time component \( T_i \) (i.e., when the marginal inputs of market goods and time are constant and equal \( x_i = 1 \) and \( t_i = T_i \), respectively), the price of the trip is

\[
\Pi_i = P_i + KT_i. 
\]  

(3.9)

This price is invariant to the level of activity \( Z_i \). The concave isocost curve in Figure 1 can, therefore, be replaced by a straight price line with a slope \( \Pi_A/\Pi_B \). Since the isoquant curve is also a straight line (with a slope of unity) the minimization of production costs leads to a corner solution. The traveler always chooses the cheapest mode. He prefers mode \( A \) to mode \( B \) when \( \Pi_A < \Pi_B \), otherwise he chooses mode \( B \).

When the trip does not convey any direct utility, the utility derived from a visit is independent of the mode used. The visit’s production function [equation (3.3)] can, therefore, be reformulated

\[
Z_v = f(X_v, T_v, Z_T), 
\]  

(3.10)
where \( Z_T \) denotes the activity, "trip." The distinction between air trip (\( Z_A \)) and bus trip (\( Z_B \)) can be relaxed and the various modes are regarded merely as different combinations of the time and money inputs required to produce a trip (\( Z_T \)). Expanding the model to four modes of transportation, and arranging the various combinations by the increasing order of their time intensity, one can draw the trip's isoquant \( ABCD \) (Figure 2), where \( P_i \) and \( T_i \) denote the money and time inputs required to produce one trip (i.e., one unit of \( Z_T \)) by mode \( i, i = A, B, C, D \). The slope of any segment of the curve \( ABCD \) represents the marginal rate of technical substitution \( K^* \) (i.e., the ratio of the

\[ 1 \text{ A, B, C, and D may represent four different modes or four different kinds of equipment (e.g., jet and piston planes) that involve a trade-off between time and money. We do not discuss the choice among different kinds of service (e.g., local vs. express, or coach vs. first class), a choice which is primarily affected by availability and the demand for frills, and only very rarely involves a trade-off between time and money.} \]
marginal products of time and money in the production of $Z_T$). The traveler, aiming at the minimization of costs, chooses mode $i$ if

$$
\frac{P_i - P_{i+1}}{T_{i+1} - T_i} = K^*_{i,i+1} < K < K^*_{i-1,i} = \frac{P_{i-1} - P_i}{T_i - T_{i-1}} \tag{3.11}
$$

where $T_i > T_{i-1}$ and $P_i < P_{i-1}$, $i = A, B, C, D$. In particular, he prefers mode $A$ to $B$ if

$$
K > \frac{P_A - P_B}{T_B - T_A} = K^*_A - B \quad (3.12)
$$

Given the time intensities of the various modes, the choice of mode depends solely on the price of time, and given the price of time, the choice is a sole function of the time intensities.²

An increase in the price of time increases the price of the trip regardless of mode. The rate of price increase is directly related to the time intensities of the different modes. The relative price of the faster mode ($\Pi_A/\Pi_B$) decreases, and the passenger's tendency to switch to the faster mode increases.³ The price of time increases with income; hence, one would expect income and the choice of air transportation to be strongly related. Moreover, in the short run, the household may not be able to substitute its free time for work, resulting in a price of time which is higher for business trips than for personal trips. The passenger may, therefore, use air transportation on a business trip but ground transportation on a personal trip of equal length.

The traveler's tendency to switch to air transportation is stronger the smaller the marginal rate of technical substitution $K^*$, i.e., the smaller the slope of the isoquant in the interval $AB$. The slope of the isoquant depends on the time intensities of the various modes, and these change with the distance of the trip. Both time and money outlays are

² Note that this model does not rule out the possibility that the same individual will use two different modes on two different trips to the same destination, as long as his price of time differs on the two occasions. The passenger may, for example, go by air during the day when his price of time is high, and travel by train at night when his price of time is lower.

³ The change in relative prices does not assure the passenger's switch to the faster mode as the slower mode may still be (absolutely) cheaper ($\Pi_A > \Pi_B$). Put differently, the increase in the slope of the price line in Figure 2 does not necessarily lead to a shift from $B$ to $A$, because of the kink at point $B$. 
functions of the distance traveled. The production of a trip involves a fixed cost component of the time and money spent on the way or at the terminal, and a variable cost component related to distance. The relationship between the cost components and the distance \((M)\) can be approximated by a linear function

\[
T_i = \alpha_{0i} + \alpha_{1i}M \\
P_i = \beta_{0i} + \beta_{1i}M \\
\Pi_i = (\alpha_{0i}K + \beta_{0i}) + (\alpha_{1i}K + \beta_{1i})M.
\] (3.13)

The goods-intensive mode \(A\) is preferred to \(B\) when the price of time

\[
K > \frac{(\beta_{0A} - \beta_{0B}) + (\beta_{1A} - \beta_{1B})M}{(\alpha_{0B} - \alpha_{0A}) + (\alpha_{1B} - \alpha_{1A})M} = K^*.
\] (3.14)

No passenger will use mode \(A\) if \(T_A > T_B\), i.e., assuming \(\alpha_{1B} > \alpha_{1A}\)

\[
M < \frac{\alpha_{0A} - \alpha_{0B}}{\alpha_{1B} - \alpha_{1A}}
\] (3.15)

and every passenger will prefer the faster mode if \(P_B > P_A\), i.e., assuming \(\beta_{1A} > \beta_{1B}\)

\[
M < \frac{\beta_{0B} - \beta_{0A}}{\beta_{1A} - \beta_{1B}}
\] (3.16)

The marginal rate of technical substitution \(K^*\) is inversely related to the distance of the trip when an increase in the distance increases the time differential \((T_B - T_A)\) at a faster rate than the increase in the money differential \((P_A - P_B)\)

\[
\frac{\partial K^*}{\partial M} < 0 = \frac{\partial(T_B - T_A)/\partial M}{T_B - T_A} > \frac{\partial(P_A - P_B)/\partial M}{P_A - P_B}
\] (3.17)

i.e., when

\[
\frac{\beta_{0A} - \beta_{0B}}{\beta_{1A} - \beta_{1B}} > \frac{\alpha_{0B} - \alpha_{0A}}{\alpha_{1B} - \alpha_{1A}}
\] (3.18)

In this case the passenger does not use the faster mode unless the distance of the trip

\[
M > \frac{(\beta_{0A} - \beta_{0B}) + (\alpha_{0A} - \alpha_{0B})K}{(\beta_{1B} - \beta_{1A}) + (\alpha_{1B} - \alpha_{1A})K} = M^*.
\] (3.19)
The switching distance $M^*$ (the distance at which the passenger switches from slower to faster modes) is inversely related to the price of time. An increase in the price of time affects mode $A$ less than mode $B$, and cuts the switching distance from $M_0^*$ to $M_1^*$ (see Figure 3). High income passengers are expected to use airline transportation for shorter distances than low income passengers. The same holds for business vs. personal travelers. Only those travelers whose price of time is

$$K < \frac{\beta_{1A} - \beta_{1B}}{\alpha_{1B} - \alpha_{1A}}$$

will never use the faster mode.\(^4\)

An increase in income increases the demand for visits and trips, but increases also the price of time and, hence, the prices of these two activities. An increase in income from $Y_0$ to $Y_1$ (Figure 4) shifts the demand for “trips to point $i$” from $D_0$ to $D_1$. However, the accompany-

\(^4\) (3.20) and (3.15) are the asymptotes of the rectangular hyperbola (3.14).
ing change in the price of time raises the price of the trip from $\Pi_0$ to $\Pi_1$, and shifts the income-consumption curve from $C_0$ to $C_1$. The net income effect $X_0X_2$ is the difference between the income effect $X_0X_1$ and the price effect $X_1X_2$. The net income effect is directly related to the income elasticity of trips, and inversely related to the trip's price elasticity, the elasticity of the price of time with respect to income [$\eta_{KY} = (dK/dY) \cdot (Y/K)$], and the time intensity of the mode used. The passenger's tendency to use the faster (less time-intensive) mode increases with his income. Thus, other things being equal, the net income effect and income are positively correlated.

FIGURE 4