Volume Title: The Value of Time in Passenger Transportation: The Demand for Air Travel

Volume Author/Editor: Reuben Gronau

Volume Publisher: NBER

Volume ISBN: 0-87014-219-4

Volume URL: http://www.nber.org/books/gron70-1

Publication Date: 1970

Chapter Title: The Value of Time

Chapter Author: Reuben Gronau

Chapter URL: http://www.nber.org/chapters/c3388

Chapter pages in book: (p. 7 - 11)

TWO

THE VALUE OF TIME

"TIME IS MONEY" is a frequently used adage. Economists, recognizing the effect of time on the production process, have incorporated this effect in the distinction between a firm's short-run and long-run costs. It can be shown, however, that time exerts a somewhat similar effect on household consumption decisions. In effect, a household can be regarded as a producer of activities, combining its own time with market goods and services:

$$Z_i = f_i(X_i, T_i), \qquad i = 1, \ldots, n, \tag{2.1}$$

where Z_i denotes the ith activity, X_i the market inputs, and T_i the time involved. The activity, a "visit," for example, is produced by combining transportation, hotel and restaurant services, traveling time, and time spent at the point of destination.

Different units of time may vary in their productive capacity. For example, daytime may be more efficient for the production of a business visit, while nighttime may be more efficient for producing a visit to friends and relatives; wintertime may be more efficient for producing a visit to Florida, and summertime may be more efficient for producing a visit to Yellowstone National Park.

The production process is subject to two constraints: the budget constraint determines the total expenditures on the market inputs; and the time constraint determines the total expenditures of time inputs. Specifically,

$$\sum_{i=1}^{n} P_i X_i = Y, \qquad \sum_{i=1}^{n} T_i = T_0, \tag{2.2}$$

where P_i denotes the price of the market input in activity i, Y is income, and T_0 is the total time available for consumption activities. The household's aim is to maximize its utility (U),

$$U = U(Z_1, \ldots, Z_n), \tag{2.3}$$

subject to the above constraints.

The necessary conditions for an optimum are satisfied when

$$u_i = \lambda(P_i x_i + \frac{\mu}{\lambda} t_i) \qquad i = 1, \ldots, n, \tag{2.4}$$

where $u_i = \frac{\partial U}{\partial Z_i}$ denotes the marginal utility of activity i, λ is the marginal utility of income, μ is the marginal utility of time, and $x_i = \frac{\partial X_i}{\partial Z_i}$ and $t_i = \frac{\partial T_i}{\partial Z_i}$ are the marginal inputs of market goods and time, respectively.[1] The optimal combination of inputs in the production of any activity is attained when the ratio of the marginal products of time and market goods equals their relative prices,

$$\frac{\partial Z_i/\partial T_i}{\partial Z_i/\partial X_i} = \frac{x_i}{t_i} = \frac{\mu/\lambda}{P_i} = \frac{K}{P_i}, \tag{2.5}$$

where $K = \frac{\mu}{\lambda}$ is the (shadow) price of time.[2] The total price of activity i

[1]The equilibrium condition (2.4) is obtained by maximizing equation (2.3) subject to the constraints of equation (2.2). Using the Lagrange method,

$$L = U(Z_1, \ldots, Z_n) + \lambda(Y - \Sigma P_i X_i) + \mu(T_0 - \Sigma T_i). \tag{1}$$

The necessary conditions for a maximum are

$$\frac{\partial L}{\partial Z_i} = \frac{\partial U}{\partial Z_i} - \lambda P_i \frac{\partial X_i}{\partial Z_i} - \mu \frac{\partial T_i}{\partial Z_i} = u_i - \lambda P_i x_i - \mu t_i = 0, \tag{2}$$

that is, $u_i = \lambda P_i x_i + \mu t_i$.

[2]Maximizing equation (1) from note 1 with respect to X_i and T_i yields

$$\frac{\partial L}{\partial X_i} = \frac{\partial U}{\partial Z_i}\frac{\partial Z_i}{\partial X_i} - \lambda P_i = \frac{u_i}{x_i} - \lambda P_i = 0, \text{ that is, } u_i = \lambda P_i x_i, \tag{3}$$

and

$$\frac{\partial L}{\partial T_i} = \frac{\partial U}{\partial Z_i}\frac{\partial Z_i}{\partial T_i} - \mu = \frac{u_i}{t_i} - \mu = 0, \text{ that is, } u_i = \mu t_i. \tag{4}$$

Dividing (4) by (3) yields the equilibrium condition (2.5).

(Π_i) equals its marginal cost of production and consists of two parts, the money component and the time component

$$\Pi_i = P_i x_i + K t_i. \tag{2.6}$$

Equation (2.4) can be written in the familiar form

$$u_i = \lambda \Pi_i. \tag{2.7}$$

The price of time (K) depends on the household's tastes, its production functions, its income, and the total consumption time that stands at its disposal. When the production functions are linear homogeneous, the price of time becomes a sole function of the household's tastes and factor scarcity (Y/T_0). An increase in income results in an increase in the intrinsic price of time, a shift in production toward goods, and a shift in consumption toward goods-intensive activities.[3] The share of goods-intensive activities in the household's optimal activity "basket" is bound to increase, unless these activities are associated with very low income elasticities.

Up to now we have assumed that the amount of work supplied by the household is exogenously determined. The number of daily working hours, the length of the work week, and the length of vacations are given at least in the short run. The amount of consumption time, this argument goes, is outside the household's realm of decisions. This assumption proves to be too restrictive when the household's long-run decisions are analyzed. Assuming that the household can change the amount of work it offers in the market (Z_w), the constraints confronting the production process and the household's objective function must be redefined. The household aims at maximizing its utility function,

$$U = U(Z_1, \ldots, Z_n, Z_w), \tag{2.8}$$

subject to an endogenous budget constraint,

$$\sum_{i=1}^{n} P_i X_i + P_w X_w = W(Z_w) + V = Y(Z_w), \tag{2.9}$$

and an exogenous time constraint,

[3] See T. M. Rybczynski, "Factor Endowment and Relative Commodity Prices," *Economica,* November 1955, for the application of a similar analysis to the field of international trade.

$$\sum_{i=1}^{n} T_i + T_w = T_0', \tag{2.10}$$

where $W(Z_w)$ denotes earning, V other income, and T_0' is the total time available (e.g., 24 hours a day, 168 hours a week). The optimal supply of work is determined by the equation

$$u_w = \mu t_w - \lambda w, \tag{2.11}$$

where $w = \frac{\partial W(Z_w)}{\partial Z_w} - P_w x_w$ is the net marginal wage rate (i.e., the household's remuneration for the marginal unit of work it sells in the market minus any money costs incurred).[4] Measuring the activity "work" (Z_w) in terms of its time inputs (e.g., hours of work) $t_w = 1$, the value of the marginal utility of time is $\mu = \lambda w + u_w$ and the price of time equals the sum of the marginal wage rate and the money equivalent of the marginal utility of work

$$K = \frac{\mu}{\lambda} = w + \frac{u_w}{\lambda}. \tag{2.12}$$

Equations (2.5), (2.6), and (2.7) describe the necessary optimum conditions for production and consumption. The price of time is determined by the net marginal wage rate, by the household's taste for work, and by the marginal utility of income. Only when work does not involve any utility or disutility ($u_w = 0$) does the price of time equal the marginal wage rate. In general, it may either exceed it, if the psychic income is sufficiently large ($u_w > 0$), or be smaller, when work involves marginal disutilities ($u_w < 0$). If $u_w \neq 0$, the price of time may change even if the marginal wage rate is constant, since a change in income or a change in the taste for work may change u_w/λ.

[4] Let

$$L = U(Z_1, \ldots, Z_n, Z_w) + \lambda\,[W(Z_w) + V - (\Sigma P_i X_i + P_w X_w)]$$
$$+ \mu[T_0' - (\Sigma T_i + T_w)]. \tag{5}$$

Differentiating with respect to Z_w,

$$\frac{\partial L}{\partial Z_w} = \frac{\partial U}{\partial Z_w} + \lambda\left(\frac{\partial W}{\partial Z_w} - P_w \frac{\partial X_w}{\partial Z_w}\right) - \mu \frac{\partial T_w}{\partial Z_w} = u_w + \lambda w - \mu t_w = 0. \tag{6}$$

Any increase in the price of time, whether as a result of a change in the wage rate or of a change in the money equivalent of the marginal utility of work, results in a substitution in production in favor of goods and a substitution in consumption in favor of the goods-intensive activities. The effect of this change on the relative share of the various activities depends on whether it is accompanied by any income effects, and on the direction and magnitude of these effects. An increase of wages, for example, tends to reduce the share of time-intensive activities in the optimal activity combination, unless their income elasticity is substantially greater than unity.