Volume Title: The Seasonal Variation of Interest Rates

Volume Author/Editor: Stanley Diller

Volume Publisher: NBER

Volume ISBN: 0-87014-205-A

Volume URL: http://www.nber.org/books/dill70-1

Publication Date: 1970

Chapter Title: Determinants of Seasonal Amplitude

Chapter Author: Stanley Diller

Chapter URL: http://www.nber.org/chapters/c3373

Chapter pages in book: (p. 76 - 103)

CHAPTER 4

Determinants of Seasonal Amplitude

INTRODUCTION

THIS CHAPTER CONSIDERS some topics related to the extent and variation of seasonal amplitudes. One of the topics it deals with is the profitability of arbitrage between periods of seasonally high and low yields on long-term securities. It was found that, taking account of direct transactions and opportunity costs, a relatively small seasonal would elicit arbitrage—the more so the longer is the term to maturity, the smaller the margin requirements, and the greater the stability of the seasonal. The breakeven point for profitable arbitrage, estimated through use of hypothetical though plausible data, is below the seasonal amplitude that actually persisted through most of the study period. The result implies that a relatively high risk premium is attached to the uncertainty with which the seasonal is regarded, as well as the expected dominance of the cyclical and irregular components. The breakeven seasonal adjusted for risk would therefore be much larger. Consideration of the business and bother costs of an arbitrage operation would also raise the breakeven point.

The greater importance of term structure on short-term yield differentials complicates the question of arbitrage in the short-term segment of the securities market. Several authors have observed that the seasonal patterns of some short-term securities lead those of shorter term securities. Since these leads are evidence of investors' awareness of the seasonal movement and more generally of their attempt to forecast short-term rates, they are also evidence of what this study is calling arbitrage. Arbitrage generally implies bridging a known discrepancy between two situations, usually between two markets at a particular time; but when the two situations occur in different periods the knowledge of the later period is at best a good forecast. The unstable character of the seasonal influence on interest rates revealed in Chapter 3 implies that using the term arbitrage in connection with seasonals is somewhat misleading. Certainly, arbitrage shades into speculation as the seasonal movement becomes more problematic.

The last section of this chapter relates changes in the seasonal amplitude of Treasury bill rates with corresponding changes in the seasonal amplitude of other series.[1] The seasonal amplitude of Treasury bill rates is inversely related to the seasonal amplitude of the stock of money and directly related to that of total bills outstanding. This finding, relevant in its own right as a description of events, serves also to illustrate the usefulness of decomposing a series to help explain its behavior. For example, when the aggregate series of interest rates and money supply are correlated, the expected inverse relation is usually obscured by the common effect of economic activity on the cyclical components of both series; although the series frequently turn at different stages of the cycle, there are many periods during which the series are moving in the same direction. However, when the cyclical component is filtered out, the expected inverse relation materializes. By replacing the original series with their seasonal factors, this study estimates the elasticity of short-term demand for credit with respect to interest rates to be a very small but statistically significant −.0237.[2]

[1] "Seasonal amplitude" refers to the average departure of the monthly factors from 100.0. When the pattern of factors is approximately constant from year to year the change of a given month's factor from one year to the next is a measure of the change in seasonal amplitude.

[2] The appropriateness of the concept of elasticity in the present context is evaluated in a brief appendix to this chapter.

ARBITRAGE

LONG-TERM SECURITIES

To determine the opportunity for arbitrage between seasonal phases, this study computed the effect on buying and selling prices, under a set of assumed conditions, of various seasonal amplitudes. Consider a twenty-year, 4 per cent bond with semiannual coupons whose average price over the year is the $1000 par value. Assume an investor purchases the bond at its seasonally peak yield in September and sells it, now with nineteen and one half years to maturity, the following March when yields are at their seasonal trough. The present value formula computes the prices in September and March for any desired seasonal change in yield in the following way:

$$\text{September price} = \frac{20}{(1.02\ P.F.)} + \frac{20}{(1.02\ P.F.)^2} + \cdots + \frac{20. + 1000}{(1.02\ P.F.)^{40}}$$

where $P.F.$ = peak seasonal factor; e.g., 101.0; $T.F.$ = trough seasonal factor; e.g., 99.0. Semiannual coupon payment is $20; principal is $1000; and average yield is 2 per cent per half-year.

The computation leads to the result that for each one-tenth of 1 per cent in the seasonal amplitude on either side of the average value (in other words, factors equal to 100.1 and 99.9 for September and March, respectively) the price differential for a thousand dollar bond comes to approximately $10.70. For an amplitude of two-tenths of a per cent (i.e., 100.2 and 99.8) the price differential is approximately $21.40.

To estimate the seasonal amplitude necessary to encourage arbitrage this study estimated the costs and returns to this activity in two hypothetical situations: $5000 is invested with a 5 per cent and with a 25 per cent margin requirement. In the first case the investor borrows $95,000 and purchases $100,000 worth of bonds; and in the second case $15,000 and purchases $20,000 worth of bonds. Table 14 lists the estimated costs in the two situations under the following assumptions:

—Foregone interest on $5000 at annual rate of 5 per cent
—Transaction cost is $5 per $1000 bond for combined buy and sell

TABLE 14

Costs and Returns of Arbitrage for Twenty-Year Securities Between Seasonal Peaks and Troughs Under Certain Hypothetical Conditions

Part A: $5000 Invested With 5 Per Cent Margin Requirement

Costs

Foregone interest[a] on $5000 for six months at assumed annual rate of 5 per cent	.025 x 5,000	=	$125.
Transactions cost (buy and sell at assumed $5 per $1000 bond	5 x 100	=	500.
Interest cost of borrowed money for six months at assumed rate of 1 percentage point above bond yield	.5 x .01 x 95,000	=	475.
Total Cost		=	$1,100.

Returns

Price differential (between September and March) at assumed $10.70 per $1000 bond for each .1 per cent of seasonal factor	100 x 10.70	=	$1070.
Required seasonal factors to cover cost	$100 + \frac{1100}{1070} \times .1$	=	100.10%
	$100 - \frac{1100}{1070} \times .1$	=	99.90

(continued)

TABLE 14 (concluded)

Part B: $5000 Invested With 25 Per Cent Margin Requirement

Costs			
Foregone interest[a] on $5000 for six months at assumed annual rate of 5 per cent	.025 x 5,000	=	$125.
Transactions cost (buy and sell) at assumed $5 per $1000 bond	5 x 20	=	100.
Interest cost of borrowed money for six months at assumed rate of 1 percentage point above bond yield	.5 x .01 x 15,000	=	75.
Total Cost		=	$300.
Returns			
Price differential (between September and March) at assumed $10.70 per $1000 bond for each .1 per cent of seasonal factor	20 x 10.70	=	$214.00
Required seasonal factors to cover cost	$100 + \frac{300}{214} \times .1$	=	100.14
	$100 - \frac{300}{214} \times .1$	=	99.86

[a]The capital on which the foregone interest is computed should include half the transactions cost and approximately half the borrowing cost. The greater accuracy, however, will not significantly improve the estimates.

—Interest cost of borrowed money exceeds bond yield by 1 percentage point per year.[3]

The required peak and trough seasonal factors are computed for breakeven, account being taken of the opportunity costs of the investment.

While the estimated returns obviously depend on the conditions assumed to prevail, the orders of magnitude are established. The smaller the margin requirements, or obversely, the greater the amount borrowed relative to capital, the smaller is the seasonal amplitude required to produce the breakeven price differential. In the example, with a 5 per cent margin requirement the seasonal factors must exceed 0.10 per cent on either side of the level of rates to just cover the arbitrageur's costs. (While the example does not illustrate the point, it is also true that the greater the term to maturity, the greater the effect on the price differential of any given differential in yield.) When the margin requirements rise to 25 per cent, the required seasonal factors are 0.14 per cent on either side of the level of rates to just cover costs.

The long-term series considered in this study do not differentiate among terms to maturity; therefore, this study did not estimate the relation between term to maturity and the breakeven seasonal factors. However, one certainly expects the seasonal amplitude to diminish with an increasing term to maturity, since the longer the term the sooner will a given seasonal amplitude invite arbitrage. This point may explain the greater amplitude in three- to five-year Treasury securities than in the equivalent long-term securities and in the Treasury bill rates than in the nine- to twelve-month securities. It may also help explain the smaller amplitude of commercial paper rates, which are often six months to maturity, than in the short-term yields on bankers' acceptances and on 91-day Treasury bills.

While margin requirements vary over time and among borrowers, they are always lower for Treasury securities than for private and state-local issues. This point may account for the difference in the longevity of the seasonal amplitude in the two sets of securities, but the study makes only a *prima facie* case for the issue.

[3] The greater the difference between the long and short rate the greater is the incentive to arbitrage.

The illustrative example implies that, contrary to common opinion, there is nothing in investment behavior to preclude a seasonal influence on long-term rates provided its amplitude is sufficiently low. Since several years are required before investors perceive a seasonal, virtually any amplitude is possible for a limited period. The low breakeven points for eliciting arbitrage, 0.10 per cent and 0.14 per cent, for 5 per cent and 25 per cent margin requirements respectively, computed in the example no doubt understate the true values because of additional business costs not incorporated in the example, as well as the point, noted in the introduction, that arbitrage is in effect at one extreme—speculation is at the other—of a continuum as the certainty of the differential between two situations becomes more remote.[4]

Perhaps the greater seasonal amplitude of municipal securities is explained by the greater prominence of their irregular components. It is tempting to generalize this point—the direct relation between seasonal amplitude and relative importance of the irregular component—into an hypothesis. Other influences on these variables, combined with the fact that the seasonal and irregular components are not independently estimated,[5] would tend to obscure the relation, however. Yet we do find that the rank correlations between the ratio of the variance of the irregular component to that of the whole series (Table 4, column 1) and the seasonal amplitude as measured by the variance of the factors (Table 5) increases from the earliest to the latest years.[6] The increasing correlations imply a movement toward an equilibrium trade-off between yield and certainty of principal. In the absence of other causes of seasonal differences among long-term securities, the observed differentials in seasonal amplitude combined with the observed differences in the relative importance of the irregular component would produce a measure of the rate of trade-off between

[4] In this regard it would be better to replace at least in principle the seasonal factors in the illustrative example with confidence intervals or perhaps some form of certainty equivalents.

[5] In fact, the bias in the computation is toward an inverse relation, which strengthens the conclusion.

[6] For the years 1953, 1957, 1963, and 1965, the rank correlation coefficients are .30, .59, .76, and .82, respectively. These figures exclude the two long-term Treasury securities, although there was no attempt to determine how their inclusion would affect the results.

the two or, in other words, a measure of risk premium.[7] While this suggestion helps to illustrate the potential uses of time series decomposition, it suffers from the same problem this study emphasized throughout: the difficulty in distinguishing variations in seasonal amplitude from irregular movements.

SHORT-TERM SECURITIES

While the cost-return analysis of seasonal arbitrage applies equally well to short- as to long-term securities, the calculation of the cost component is complicated by the greater yield differentiation among proximate maturities at the short-end of the yield curve. Whereas the nonseasonal components of the yields on twenty- and nineteen and a half-year securities are approximately the same and therefore do not affect the arbitrage, a substantial differential between a one-month and a two-month or a nine-month and a three-month security may offset any seasonal differential. The calculated costs of arbitrage must, therefore, take account of the former differential.

The tendency for yield curves, the curves relating yield to maturity with maturity, to incline at a diminishing rate is a widely observed phenomenon and its explanation a subject of considerable dispute. Some writers attribute the phenomenon to the greater number of investors with short-term liabilities who prefer to match the maturities of their assets and liabilities than investors with long-term liabilities having similar preferences—the so-called hedging theory. Others emphasize investors' preference for short-term securities to minimize their vulnerability to capital losses—the liquidity preference theory. In either case yields increase with maturity to equilibrate supply and demand. Finally, the expectations hypothesis associates the yield structure with investors' expectations of future interest rates. While this theory does not account for the observed average incline in the yield curve, it can account for the greater differentiation among shorter term yields by recognizing the greater differentiation of investors

[7] In this context there is no need to consider differences in cyclical variation, which could further account for *aggregate* yield differentials among the various groups of securities, since the seasonal and irregular components abstract alike from the cyclical components of all the series. In principle, this measure of risk premium captures the true relation between the relative *dispersion* of yields and the yield differential of competitive securities as distinct from differences in the expected yields of the securities.

among their shorter span forecasts, for which they have more information, than their longer span forecasts, for which they are likely to rely more on extrapolations.[8]

In evaluating the empirical basis for the expectation hypothesis, Macaulay found evidence of market forecasting in the fact that the seasonal peak in yields on time loans preceded the peak in call loans.[9] Banks, he said, aware of the seasonal peak in call money rates during December, would not tie up money in, say, November, without insuring a return comparable to the average return on call money during the two months. The yield on time money would therefore peak earlier. With respect to this phenomenon, its amplitude would have to be smaller, as well. Consider the same phenomenon from the borrower's point of view: To avoid the December rush he can borrow in November for two months, perhaps lend the money for one month, and in effect acquire a forward loan for December at the lower November rate. These transactions would have the effect Macaulay observed, in addition to smoothing the one-month seasonal. But the borrower's ability to avoid the peak rate depends on the nonseasonal relation between the two-month and one-month rates in November. If the former were much greater than the latter, both rates adjusted for seasonality, what the borrower gains by avoiding the seasonal he loses in the term structure differential. Since this differential is known in November, it in part determines the extent of the seasonal arbitrage and, therefore, of the seasonal amplitude itself. This analysis, thus, suggests that a relation exists between the slope of the yield curve and the seasonal amplitude.

Testing for this relation obviously requires data for different but proximate maturities. The *Treasury Bulletin* publishes series on one-, two-, and three-month Treasury bills, but these data record the yields on the last trading day of the month instead of weekly averages, as in the *Federal Reserve Bulletin's* series on 91-day Treasury bills. The

[8] There are, in addition, various eclectic theories of the term structure. The literature on this subject has grown in recent years—much more, unfortunately, than our knowledge. Two standard works are: David Meiselmen, *The Term Structure of Interest Rates,* Englewood Cliffs, N.J., 1961; and Reuben Kessel, *The Cyclical Behavior of the Term Structure of Interest Rates,* New York, NBER, 1965.

[9] *Op. cit.,* p. 36. Kemmerer, *op. cit.,* p. 18, observed the same phenomenon and had the same explanation for it.

Treasury Bulletin's series therefore have a considerably greater random component distorting the estimated seasonal patterns. This study has therefore avoided the *Treasury Bulletin* data. The data do, however, illustrate this section's argument. A direct test for the relation between the seasonal amplitude of one-month rates in December and the nonseasonal yield differential between two- and one-month rates in November is available, simply, in a regression of the SI ratios for one-month bills in December on the differential in the trend-cycle components of two- and one-month rates in November. The correlation coefficient of this regression is .56; the regression coefficient, 53.56; and its *t*-value, 2.68. In general, the variance of the SI ratios for a given year (that is, the extent of seasonal amplitude) is directly related to the slope of the yield curve.[10] The converse is also true: the unadjusted term-structure data partly reflect the seasonal pattern—which was Macaulay's point.

The point is again manifest in the nonseasonal differential between nine- to twelve-month Treasury securities and 91-day Treasury bills. The average differential in July [11] is considerably greater during the period of peak seasonality, 1955–61, than during the earlier or later periods. In the period 1948–54, the mean differential in the trend cycle values of nine- to twelve-month and 91-day Treasury securities

[10] It is obviously necessary to work with the SI ratios, preferably modified for extremes, instead of the seasonal factors themselves since the factors are designed to smooth out the effects of year-to-year changes in seasonal amplitude. In other words, to the extent the above analysis is relevant the X-11 method of seasonal adjustment is inappropriate. There is nothing sacred about the December figures. In fact, the SI ratios of all seasonally high months are positively related to the slope of the yield curve, and those of all seasonably low months negatively related. In other words, the seasonal amplitude as a whole is positively related to the slope of the yield curve. While the relation for the seasonally high months is understandable, its application to the low months is less clear. Even if the principle stated in the text applied only to the high months, the observed effect on the low months would obtain due to the effect of the high months on the trend-cycle curve. This point is considered in Chapter 2.

[11] July replaces November in this calculation because of the difference in maturities involved. Here the borrower, say the U.S. Treasury, avoids the three-month peak rate in December by borrowing for nine months in July instead of for three months in December, perhaps simultaneously purchasing a six-month security to effect the forward loan. Curiously, the point in the text is most true for July, when the combination of nine- and three-month securities is appropriate to the December peak; although, to a lesser extent it applies to all the months.

was 6 basis points; in the period 1955–61, 45 basis points; and in 1962–65, 9 basis points (all figures are for July). The figures for each year for the months June through November are given in Table 15.

As much as this analysis accurately depicts one aspect of the seasonal problem it implies still another. Typically, though not always,

TABLE 15

The Differential in the Trend-Cycle Values Between Nine- to Twelve-Month and 91-Day Treasury Securities for June Through November, 1948–65[a]

(in basis points)

	June	July	August	September	October	November
1948	6	10	13	14	12	11
1949	-4	12	8	6	5	4
1950	0	3	8	4	5	5
1951	24	10	10	9	10	11
1952	-1	-7	5	17	13	4
1953	11	18	23	24	42	17
1954	1	-7	-13	-12	-2	12
1955	30	14	15	11	4	13
1956	11	27	25	26	25	23
1957	13	29	38	43	31	23
1958	44	54	19	-5	2	30
1959	74	78	93	67	66	70
1960	71	62	65	51	53	55
1961	44	53	50	71	64	55
1962	25	0	16	22	19	11
1963	14	9	6	13	14	23
1964	15	16	22	23	23	22
1965	11	9	16	14	14	23

[a]The figures are trend-cycle values for nine- to twelve-month securities minus trend-cycle values for 91-day bills. The figures indicate the slope of the yield curve in the designated range independently of the seasonal and irregular movements. For the present purposes the differential in the seasonally adjusted rates (that is, including the irregular component) is a relevant alternative to the figures presented here.

the slope of the yield curve is greatest when the level of rates is low.[12] Since seasonal amplitudes should depend inversely on the slope, they should also be inversely related to the level of rates, or at least the level in relation to that of adjacent years. Chapter 3 found no such relation in the data themselves. It may be, however, that this proposition works to offset a tendency in the opposite direction: In periods of tight money, during cyclical highs, it is harder or more costly to borrow money in order to arbitrage the seasonal movement. Consideration of this point concludes this section of the study.

There are as many ways to arbitrage the seasonal influence in short-term rates as there are combinations of relevant maturities. The following discussion arbitrarily selects nine- and three-month securities and deals only with the peak-to-trough and trough-to-peak relationships; although, in principle, arbitrage is feasible between any pair of months.[13] The rule in seasonal arbitrage is simply to borrow cheaply and lend dearly; that is, borrow in July and lend in December. There are two ways to effect the transaction: Borrow and sell a nine-month security in July at high prices and cover the short sale in December at low prices. Alternatively, buy a nine-month security in December and sell it, now a three-month security, the following June. The term structure would work against the arbitraguer in the first alternative and for him in the second.[14]

[12] In the jargon of the expectations hypothesis, when current rates are below their normal or typical values they are expected to rise. Longer term lenders require a higher yield in compensation for the expected capital loss. The concept is analogous to the one underlying the Keynsian liquidity preference function. Admittedly, the figures in Table 15 do not cast a very favorable light on this hypothesis; although, most sets of term structure data support it. Reuben Kessel, *op. cit.,* argues that the very short-term part of the yield curve is dominated by liquidity premia which, he argues, are positively related to the level of rates.

[13] In this connection the smoothness of the seasonal patterns of interest rates, i.e., the absence of abrupt changes between adjacent months, is understandable. The major cost of seasonal arbitrage is borrowing cost, which is, of course, a linear function of the length of the loan. It is cheaper to arbitrage between adjacent months than across a six-month period, but the smoothness of the pattern reduces the opportunity. Whether the opportunity decreases at a faster rate than the costs as the span of the transaction decreases is determinable for any specific case.

[14] In addition, short positions are more costly to finance than long positions. The borrower must pay ½ of 1 per cent of the value of the security (annual rate) plus the interest that accrues to the security. See *A Study of the Dealer Market for Federal Securities, op. cit.,* p. 20. Moreover, the margin require-

Given the seasonal spread, the incentive to arbitrage is determined by the cost of borrowing. In periods near cyclical peaks the incentive to purchase nine-month securities in December in order to sell them the following June is limited by the higher borrowing cost. On this ground one might expect a greater seasonal amplitude, because of the reduced arbitrage, at cyclically high rates—a consideration that is apparently offset by the one noted earlier.

Aside from the cyclical effect, however, the incentive to arbitrage is influenced by the Federal Reserve's policy toward seasonal changes in the demand for credit. The Federal Reserve's failure to meet the peak demand for credit in December would by itself produce a rise in the bill rate both directly through its own operations and indirectly through the effect on the borrowing costs of arbitrageurs. Similarly, its failure to absorb redundant credit in June and July would prevent the arbitraguers' sales (to effect their capital gain) from driving the rates up. Admittedly, the failure to contract the credit supply, or more generally to diminish its rate of increase, during June and July would increase the incentive to arbitrage. This policy combined with an easy credit policy in December would, of course, lower the level of rates; but it would not eliminate the seasonal variance. Alternatively, the attempt to keep short rates high by contracting the credit supply in July and taking no action at other times would simply diminish the incentive to arbitrage and raise the December peak. In effect, it is a tight credit policy, which affects the level of rates but not the seasonality.[15] To counter the seasonal movement in interest rates, as distinct from the level of rates, requires, therefore, a relatively easy policy in December and a relatively tight one in July. The seasonal pattern in the money supply should therefore correspond with the pattern in short-term interest rates, as in fact it does. But the seasonal amplitude in money supply (that is, the extent to which the Federal Reserve alternates the relative tightness and ease) should

ment is much greater on a short position than on a long position: about 2½ per cent compared with ½ of 1 per cent on a long position in certificates. (*Ibid.*, p. 92.) It is unlikely, therefore, that this method of arbitrage would recommend itself for smoothing seasonal differentials.

[15] In both cases the change is in the over-all level of rates but not the intramonth relations. By allowing the rates in December and July to fall by the same amount, the moving average is lowered and the December peak maintained. Chapter 2 discussed this point in a related issue.

be *inversely* related to the seasonal amplitude in short-term interest rates: The more the Federal Reserve equilibrates the supply with the demand for short-term credit, the less will interest rates vary. The final section of this report investigates this relationship.

THE EFFECT OF THE SEASONAL AMPLITUDE OF THE MONEY SUPPLY

Chart 22 plots time series of the seasonal factors for Treasury bills and money supply, and total bills outstanding. The relative lows in June and July and the highs in the fall months are clear evidence of the Federal Reserve's policy of adjusting the supply to the seasonal changes in the demand for short-term credit.[16] The series on total bills outstanding is discussed later. However, if this policy were completely successful,[17] there would be no seasonal in interest rates. Figure 2 hypothetically depicts the situation. The demand for and supply of short-term credit in November results in a given interest rate, R_n. In December there is an increase in the demand, depicted by the outward shift in the demand curve. If the Federal Reserve did not increase the supply at all—that is, in the present context, if there were no seasonal movement in the money supply—the rate of interest would rise to R_P. At the other extreme, if the Federal Reserve had fully anticipated the rise in demand and increased the supply of money correspondingly (to S_0), the rate of interest would remain at R_N. Again, in the present context that would imply a relatively greater seasonal amplitude in the supply of money. Finally, if the Federal Reserve anticipated part but not all of the increase in demand,

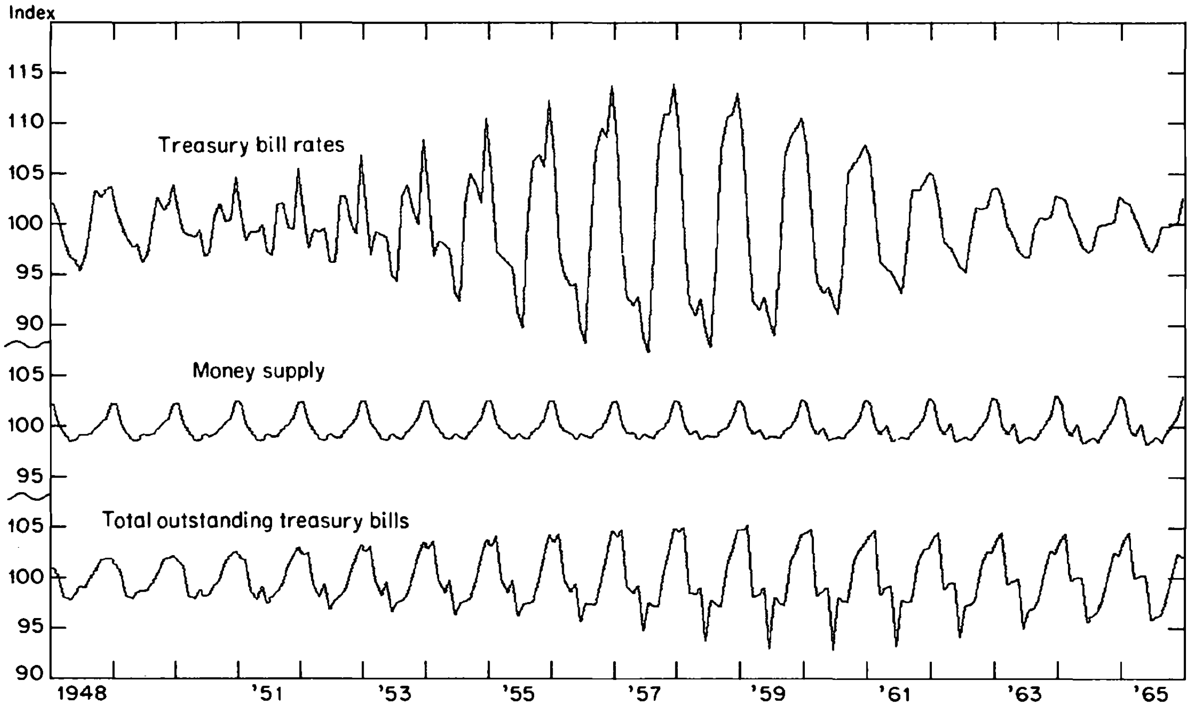
[16] The present section considers the monthly changes in the money supply synonymous with changes in the supply of short-term credit. The appendix to this chapter deals briefly with this subject to help evaluate the findings of this section.

The supply series used in this study conforms to the narrow definition of publicly held currency and demand deposits. Since time deposits do not have a significant seasonal, the broad definition of money should yield similar results; the seasonal components of both series are very similar.

[17] The word "success" is artificially vital in the current context, since the Federal Reserve did not necessarily intend to smooth out the seasonal variance in short-term rates. The desirability of eliminating the seasonal influence on interest rate is discussed in Friedman and Schwartz, *op. ct.,* pp. 292–296.

CHART 22

Seasonal Factors for Treasury Bill Rates, Money Supply, and Total Bills Outstanding, 1948–65



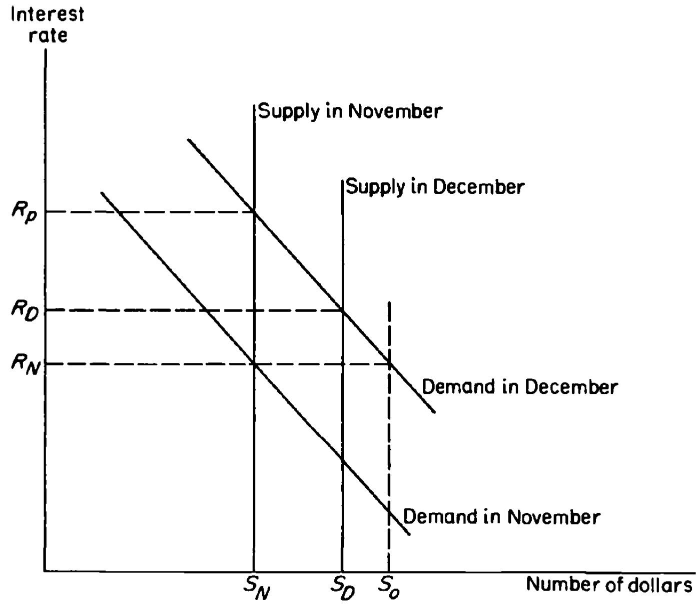


FIGURE 2

the rate would move to some intermediate position—say, R_D. The necessary seasonal amplitude in the supply of money to effect any given interest rate clearly would depend on the elasticity of demand for short-term credit: The more elastic the demand for short-term credit is with respect to the interest rate, the greater must be the seasonal amplitude in money supply necessary to prevent the seasonal increases in demand from imparting a seasonal variation to interest rates.

But how sensitive is the demand for short-term credit to the interest rate? Conversely, how sensitive is the interest rate to variations in the supply of money? Discussions of these questions typically bog down in the identification problem—that of distinguishing shifts in the demand curve from movements along it. In practice one can only observe the change in the interest rate and the change in the money supply as of a given time. Since the demand curve is itself varying, there is no sure way to associate the given readings of interest rate and money supply with a particular demand curve and, therefore, to ascertain the elasticity of the curve. The problem is soluble insofar as it is feasible to specify the variables that determine the demand curve and to fix these variables while allowing the variables affecting the supply curve [18] to move freely. In this situation, since the variables (such as income and expected changes in the price level) that determine demand are fixed, the demand curve is itself fixed, and the observed changes in interest rates and money supply may be read as points along a given demand curve. The method breaks down, however, when the same variables influence both the demand and the supply curves (such as, the preponderance of the common cyclical component in the variables affecting both curves).[19]

The study of seasonal behavior in the money market partly alleviates this problem for two reasons: (1) The use of the ratios-to-moving average of the relevant variables, or the smoothed seasonal factors, abstracts from the common cyclical component. This method is, of course, not peculiar to seasonal analysis. More importantly, (2) seasonal fluctuations in the demand for money are probably fairly stable over time; so that the seasonal shift in demand relative to the cyclical component of the shift from, say, November to December is relatively stable from year to year.[20] These seasonal

[18] Since the supply of new money is largely at the discretion of the Federal Reserve, to this extent the variables that affect the supply curve are those that affect the Federal Reserve's decision. This analysis presumes an autonomously determined supply of money. To the extent that the supply of money responds to interest rates apart from Federal Reserve activity, the purported separation in the determinants of supply and demand breaks down. While this point may weaken the analysis, the elasticity of supply with respect to interest rates is not likely to be sufficient to negate the substance of the analysis. In either case, the seasonal in interest rates depends on the seasonal in demand relative to supply. An endogenous supply would lessen the importance of Federal Reserve discretion in the matter of seasonality and would bias the estimated elasticity of demand for credit, since a simultaneous solution would be required. I am indebted to Walter Fisher for this point.

[19] Using averages for cyclical stages Cagan is able to show an inverse relationship between interest rates and *changes* in the money supply. (See his *Changes in the Cyclical Behavior of Interest Rates,* Occasional Paper 100, New York, NBER, 1966.) Note he relates *changes* in money supply to levels of interest rates; whereas this study deals with *seasonal* changes in both series.

[20] Since demand *per se* is not observable this proposition must be hypothesized rather than demonstrated. Some evidence in support of the proposition lies in the absence of any systematic variation in the seasonal amplitude of GNP within the study period. The implicit seasonal factors for the fourth quarter, the period of peak seasonality in the GNP, are given below. The raw

shifts are determined by economic forces outside the control of the monetary authorities. Shifts in supply, on the other hand, are subject to the discretion of the authorities. An estimate of the demand for short-term credit, therefore, is available from successive observations of the rate of interest necessary for people to hold the amount of money that is offered. More specifically, the analysis reveals the seasonal shift in the interest rate necessary for people to accept the seasonal shift in supply of money, given their *fixed* seasonal shift in demand.[21] The only fixity in this hypothetical system is in the demand for short-term credit; the interest rate varies as the supply varies—hence, the moving seasonal in interest rates.

This study's immediate purpose is not to estimate the demand for short-term credit for its own sake, but rather to investigate the causes of the changing seasonal amplitudes (i.e., from year to year) of the interest rates. Given the above analysis, the first step would be to regress the seasonal factors of Treasury bills on those of money supply, one month at a time across years. In other words, regress the January factor for bill rates on the January factor for money supply in 1948, 1949, . . . 1965: eighteen observations in each of twelve regressions. The regression coefficient, its *t*-value, and the adjusted

data, of adjusted and nonadjusted series, are given in *The National Income and Product Accounts of the United States, 1929–1965*, pp. 11 and 30.

Year	*Implicit Factors Fourth Quarter*	*Year*	*Implicit Factors Fourth Quarter*
1948	107.2	1957	106.0
1949	106.7	1958	105.9
1950	105.7	1959	105.9
1951	105.3	1960	105.8
1952	105.6	1961	105.8
1953	106.0	1962	105.9
1954	105.6	1963	105.8
1955	105.3	1964	105.6
1956	105.9	1965	106.6

[21] The terminology used here, admittedly awkward, does not imply that the demand is for money to hold as an asset; a demand for which there is no obvious reason for a seasonal increase in the autumn. In the present context the "demand for money" is only an abstraction that may help explain the inverse correlation between the seasonal amplitudes of money supply and Treasury bill rates. The appendix to this chapter considers this point in greater detail.

coefficient of determination are listed in Table 16. While the results are far from conclusive, in the five cases where the regression coefficients are statistically significant they reveal the expected inverse relation between the demand for money and the interest rate.

While the assumed stability in the seasonal demand for credit is plausible in the case of private demand, the government may have occasion to vary its demand both to meet changing fiscal requirements and, where possible, to take advantage of any seasonal in interest rates that may occur. Introduction of this factor, in the form of variation of total bills outstanding, leads to a considerable improvement in the estimates. The results, analogous to those in Table 16 but with the addition of total bills outstanding, are shown in Table 17. Eight instead of five of the money supply coefficients are significant,

TABLE 16

The Regression of Seasonal Factors of Treasury Bill Rates on the Seasonal Factors of Money Supply, 1948–65

	b	t	R^2(adj)[a]
January	3.9474	.8381	v.s.
February	-9.4012	-6.3215[b]	.69622
March	1.4503	.4107	v.s.
April	-.9290	-.8665	v.s.
May	-9.2617	-2.0270[b]	.15460
June	-4.4874	-1.6004	.08412
July	14.3076	.9028	v.s.
August	.0644	.0279	v.s.
September	.6536	1.3084	.04020
October	-20.6351	-4.0472[b]	.47498
November	-23.4467	-2.0905[b]	.16545
December	-8.4171	-2.0905[b]	.16545

NOTE: Each of the twelve regressions is specified as follows: seasonal factor (bill rate) = $a + b$ [seasonal factor (money supply)] + E. Each regression is run with eighteen observations.

[a]v.s. (very small) indicates that the estimated adjusted coefficients of determination are negative.

[b]Statistically significant at 5 per cent level.

TABLE 17

Regression of Seasonal Factors of Treasury Bill Rates on the Seasonal Factors of Money Supply and Total Bills Outstanding, 1948–65

	b_{MON}	t	b_{TOT}	t	R^2(adj)
January	-7.8027	-2.6531[a]	2.4944	6.8616[a]	.73768
February	-12.6356	-11.0352[a]	-.6275	-4.9796[a]	.87787
March	.1628	.0468	1.5159	1.5604	.03512
April	-7.7407	-5.6509[a]	6.7563	5.5998[a]	.64973
May	-9.9774	-1.7788	-.3225	-.2443	.10181
June	-9.0975	-4.9189[a]	1.3450	5.5671[a]	.68138
July	18.7018	1.0639	-.8688	-.6337	-.05029
August	-12.1313	-3.0136[a]	3.2720	3.3853[a]	.35756
September	.7938	1.7885	-1.2068	-2.3739[a]	.25581
October	-25.5305	-4.6300[a]	1.8186	1.7840	.53800
November	-39.3554	-4.8579[a]	10.4859	4.7151[a]	.64137
December	-2.6776	-3.1134[a]	3.8698	19.5437[a]	.96636

NOTE: Each of the twelve regressions is specified as follows: seasonal factor bill rate) = $a + b_1$ [seasonal factor (money supply)] + b_2 [seasonal factor (total bills outstanding)] + E. Each regression is run with eighteen observations.

[a]Statistically significant at 5 per cent level.

and each of the eight is negative. Eight of the coefficients of bills outstanding are positive, and six of these are significant. That is, in most, but not all, cases where the coefficients are statistically significant they have the expected sign: Increases in the money supply reduce the bill rate; while increases in bills outstanding increase the bill rate.

To obtain these results it is obviously necessary to run the regressions one month at a time across the years (or to use the equivalent dummy variable technique described below) since the month-to-month changes in the seasonal factors of Treasury bill rates and money supply have virtually the same directions and are positively correlated. During a cyclical upturn both the demand for money and the supply of money increase, but since the demand increases faster than the supply, the interest rate increases as well. In this situation, an increase in the supply of money coincides with an increase in interest rates, and the careless observer sees a positively sloped demand curve. Similarly, over the course of the year the demand for money changes in the same direction as the supply but faster, so that the interest rate varies in the same direction as the supply. However, working with deviations from the trend-cycle component isolates the common cyclical component in the demand and the supply; and estimating the relation between interest rates and money supply for one month at a time in effect exploits the relative constancy in the seasonal shifts in demand. It is then feasible to measure the points of intersection between the fixed demand curve and the varying supply curve and, therefore, to estimate the elasticity of demand with respect to the interest rate.

Instead of estimating twelve separate regressions (one for each month) of the seasonal factors for Treasury bill rates on those of the money supply and the total number of bills outstanding, it is preferable to pool all the observations and isolate the intrayear, month-to-month movements by means of dummy variables. Table 18 lists the results of this regression estimated both ways, with and without dummy variables. In regression A, without dummy variables, the common seasonal patterns dominate the relation between the seasonal factors of Treasury bill rates and money supply, and thus the regression coefficient is positive. In terms of the schematic representation, both the demand and the supply curves vary together, the demand varying more than the supply; therefore, the interest rate varies with the supply. An analogous result is frequently observed in the positive correlation between interest rates and money supply over the business cycle when no allowance is made for the joint movement of supply and demand.

In regression B, however, dummy variables for each month prevent the joint movement of supply and demand from month to month from obscuring the inverse relation between interest rates and the supply of money. The dummy variables, in effect, permit the substantive coefficients to summarize only the movement from, say, January 1956 to January 1957 and February 1951 to February 1952, instead of the movement from June 1958 to July 1958. In so doing, it allows the varying supply across all the Decembers to intersect the

TABLE 18

Multiple Regression Statistics for the Pooled Data of the Seasonal Factors of Treasury Bill Rates on Those of the Money Supply and the Total Number of Bills Outstanding, All Months, 1948–65

A Without Dummy Variables for the Months		B With Dummy Variables for the Months	
b_{MON} = 1.6002	$t_{b(MON)}$ = 4.9316	b_{MON} = −3.6817	$t_{b(MON)}$ = −4.3944
b_{TOT} = .5904	$t_{b(TOT)}$ = 3.7776	b_{TOT} = 1.0767	$t_{b(TOT)}$ = 5.3460
R = .6540		R = .8471	
R^2_{adj} = .4223		R^2_{adj} = .6993	

NOTE: The regressions are computed with time series of the seasonal factors of the three variables: Treasury bill rates, money supply, and total bills outstanding. The first observation is January, 1948; the second, February 1948; and the thirteenth, January 1949. Regression A looks as follows:

$$\text{Factor (bill rates)} = a + b_{MON}\ \text{Factor (Money)} + b_{TOT}\ \text{Factor (TOTAL)} + \epsilon$$

The constant term is not shown. Regression B looks as follows:

$$\text{Factor (bill rates)} = a + b_{MON}\ \text{Factor (Money)} + b_{TOT}\ \text{Factor (TOTAL)} + b_i D_i + \epsilon$$

where b_i is the regression coefficient of the dummy variable for the i^{th} month; eleven in all. These coefficients are not listed in the table.

seasonally fixed demand for December. In this way it traces out the demand curve.[22]

An alternative estimation form to depict the seasonal influences of money supply and government borrowing makes use of the variances of the seasonal factors described in Chapter 1. The variance of the monthly factors, computed for each year, measures the amplitude of the seasonal factors. Regressing the variance of Treasury bill rate

[22] The higher correlation coefficient in regression B is due to the introduction of the dummy variables. Not all of the variation of the seasonal factors of Treasury bill rates is due to the variation of the two independent variables. But since the seasonal factors for bill rates are not constant throughout the period, their average values, which are reflected in the regression coefficients attached to the dummy variables (not shown) do not explain all their variation.

TABLE 19

Multiple Regression Statistics for the Variance of the Seasonal Factors of Treasury Bill Rates on the Variances of the Seasonal Factors of Money Supply and Total Bills Outstanding, 1948–65

	b	t_b	Constant	Partial Correlation	R	R^2_{adj}
Money supply	-75.1311	-5.1253		-.7978		
Total bill			114.4613		.8969	.7783
Outstanding	6.7873	6.8102		.8693		

NOTE: The regressions are computed with time series of the variances of the monthly seasonal factors for each series. For a given year and series the variance is computed of the twelve factors from January through December. Since the mean factor is 1, a greater seasonal amplitude implies a greater dispersion around the mean and hence a greater variance. The form of the regression is as follows:

$$\text{Var. (Fact. Bill Rates)} = \text{CONST.} + b_{MON}\ \text{Var (Fact. Money Supply)} + b_{TOT}\ \text{Var (Fact. Bills Outstanding)} + \text{residual}$$

factors (eighteen observations) on the variance of money supply and total bills outstanding factors reveals the inverse and direct relationships, respectively, of the seasonal influence of these two series on the bill rate seasonal. The results of the regression are recorded in Table 19.

It is of course not possible to distinguish intentional changes in the seasonal variation of government borrowing to take advantage of the seasonal in interest rates from the unintentional responses to seasonal fiscal requirements.[23] The Treasury's ability to adjust the timing of its offerings to benefit from seasonal lows in interest rates is not unlimited. It is pointless to borrow merely because the rate is low. The problem here is analogous to the arbitrage issue discussed earlier in this chapter.

[23] In the case of the money supply, the Federal Reserve was merely assumed to have discretion over the supply. To the extent this assumption is unwarranted the distinction discussed in the text applies to the money supply as well. However, arguments against Federal Reserve control of the money supply rely to a large extent on the variability of time deposits, which, in the absence of a seasonal, are not germane to the present discussion.

TABLE 20

Multiple Regression Statistics for the Pooled Data of the Seasonal Factors for Money Supply on Those of Treasury Bill Rates and Total Bills Outstanding, All Months, 1948–65

b_{bill}	= −.0237	$t_{b(bill)}$	= −4.3944
b_{Tot}	= .0884	$t_{b(Tot)}$	= 5.4861
R	= .9849[a]		
R^2_{adj}	= .9681[a]		

NOTE: The regression is computed as follows:

$$\text{Factor (money)} = a + b_{bill}\ \text{Factor (bill rate)} + b_{Tot}\ \text{Factor (Tot)} + b_i D_i + \epsilon$$

where: Factor (money) = the seasonal factors of money supply; Factor (bill rate) = the seasonal factors of Treasury bill rates; Factor (total) = the seasonal factors of total bills outstanding; and b_i = regression coefficient of dummy variable for i^{th} month.

[a] The correlation coefficient is very high because the dummy variables explain a large part of the seasonal variation of money supply. The strength of this relationship is due to the relative stability of the seasonal factors of money supply and their susceptibility, therefore, to the dummy variable technique for seasonal adjustment. (This point is considered in Chapter 2.)

From the above analysis it is a small step to compute the actual elasticity of demand for money with respect to the short-term interest rate. To do this, the variables in regression B of Table 18 have simply been rearranged.[24] Now the seasonal factors for money supply form the dependent variable, and the seasonal factors for Treasury bill rates one of the independent variables. Since these variables are already expressed as percentages of the moving average, the regression coefficients signify elasticities. The elasticity of demand for short-term credit with respect to the short-term interest rate according to this method of estimation is −.0237 (Table 20). The puniness of the estimated elasticity by no means implies its economic insignificance. On the contrary, it implies that a relatively small change in money

[24] The pooled data were used for this experiment since the regression coefficients computed with the variance data are further removed from the concept of elasticity.

supply will have a relatively large short-run impact on interest rates. Chart 22 foreshadowed this result in the association it showed between the relatively small changes in the seasonal amplitude of money and the relatively large changes in the opposite direction of the seasonal amplitude of Treasury bill rates.

Interpretation of this result, however, must take account of an important limitation of the estimation procedure used. By definition the seasonal factors for a given month are serially correlated, one year with the next. If the factor is high in December 1952, it will be high in December 1953 as well. This serial correlation in the observations severely limits the actual degrees of freedom as distinct from the nominal amount. In effect, the seasonal amplitude of Treasury bill rates is low, then high, then low; and that of the money stock high, then low, then high. In addition to these three points, there are smaller changes in between, especially with respect to the variation of the several months; but the total is not even near the nominal 202 degrees of freedom.[25] The uncertain degrees of freedom reduces the importance of the estimated test of significance of the estimated elasticities. The figures are therefore less reliable estimates, although there is no reason for thinking them biased. In any case, the relations described constitute an hypothesis that further work can corroborate or refute.

CONCLUSIONS

This chapter reached the following conclusions:

(1) A seasonal variation in long-term bonds can survive arbitrage so long as the amplitude does not exceed some specified amount. This amount will be greater the more important is the irregular component of the series, the shorter is the maturity of the bond, and the greater is the margin requirement for borrowing money to purchase bonds. There are, no doubt, other factors that this section did not consider.

[25] The number is computed as follows: 12 months in each of 18 years comes to 216. There are 2 independent variables, a constant term, and 11 dummy variables. 216 − 14 = 202. Substitution of the SI ratios for the factors will not solve this problem (though it would reduce it) because the presence of seasonality implies the serial correlation of the SI ratios.

(2) The analogous computation for short-term securities is complicated by the term structure of interest rates. Other things the same, the seasonal amplitude for a given year will be greater, the greater is the slope of the yield curve. Since the yield curve is typically steepest when the level of rates is low, the seasonal amplitude on this account should be greatest when the level of rates is low. This consideration is apparently offset by the higher borrowing costs to arbitrageurs, when the level of rates is high.

(3) The variation in the seasonal amplitude of the Treasury bill rate is closely related to the movement of the seasonal amplitudes of money supply and total bills outstanding. These relationships demonstrate the influence over the seasonal in the Treasury bill rate enjoyed by the Federal Reserve and the U.S. Treasury.

(4) There is an inverse relation between the seasonal amplitude of Treasury bill rates and that of money stock. This relationship implies a negatively sloped demand curve for money with respect to interest rates. The elasticity of this curve is very small.

APPENDIX

There are at least three interpretations of what the text calls the estimated elasticity of demand for money with respect to interest rates: the slope in the observed regression of the logarithms of money supply on interest rates; the elasticity with respect to interest rates of the demand for money to hold as an asset; and the elasticity with respect to interest rates of the demand for loanable funds.

The first simply describes an observed association and is noncontroversial. The second implies the interest rate is one determinant of the demand for money-as-an-asset. However, there is no reason for a seasonal in this demand; and since the method used to estimate the elasticity assumes a seasonal shift in demand, this interpretation is not appropriate. The third assumes that the only seasonally operative component of the change in the supply of loanable funds is the supply of new money so that, given the demand for loanable funds, the shift in supply due to the change in money supply would determine the interest rate. But with a lower interest rate the demand for money-as-an-asset would rise and offset—partially, totally, or more than offset, depending on the relevant elasticities—the new money component of loanable funds. Therefore, according to the third interpretation the estimate of the elasticity of demand for loanable funds is biased downward (in absolute magnitude) because the change in loanable funds is less than the change in money supply.

This problem is only one illustration of the difficulty in specifying the conditions for which a demand curve is drawn. As already noted, the method used here avoids the problem of a cyclical component common to both the supply of and demand for money. Its reference to month-to-month variation probably alleviates other difficulties encountered in demand studies.[26] Its short-term character obviates consideration of the effect of additional supplies of money combined with lower interest rates on nominal income and, through income, the increased demand for money for transactions purposes. Depending on the relevant elasticities and periods of adjustment, additional money could conceivably raise rather than lower the interest rate by increasing the demand for money both as an asset and as a medium of exchange. With the increased income the demand for loanable funds would rise. All these effects could offset the effect on interest rates of the increased supply of money. In addition, the short-run analysis obviates consideration of the effect of a change in interest rates on the proportion of income that is saved, which would, in turn, affect the supply of loanable funds. For the same reason any effect of the change in money supply on the price level and, through this effect, on interest rates is also outside the scope of this analysis.

These points have in common the difficulty of holding constant nominal (or real) income, fixing the demand curve for money while the supply of money is allowed to vary. Variation in money supply implies variation in income and that, in turn, implies shifts in the demand curve for money. The relationship among the three—money supply, income, and demand for money—is stronger the greater is the period allowed for adjustment. Choosing coeval observations of the relevant variables that span brief periods (months, for example, instead of years) limits the process of diffusion of the new money supply and alleviates the identification problem. To the extent, how-

[26] Some of these difficulties are noted by Milton Friedman and Anna Jacobson Schwartz in a preliminary draft of their study of monetary trends (forthcoming from the National Bureau).

ever, that the diffusion process is anticipated in the market, as, for example, when increases in money supply are taken to forebode inflation, estimated parameters based on short-period observations will suffer from the identification problem.

These problems in demand analysis are by no means peculiar to this study nor even to analyses of the demand for money, although the ubiquity of money may aggravate the problems of demand analysis. Ultimately, one is sure only of the first interpretation, namely that the estimated parameters described an observed association. Depending on how the problem is set up—how the demand curve is specified, what is the source of the observations and their time dimension—and what relationship among the variables is assumed, the writer can infer behavioral parameters from the observed association. It is then his responsibility to justify the inferences.