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THE PRODUCTION OF HUMAN CAPITAL OVER TIME

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THIS paper is a continuation of an earlier study[1] of the time path of investment in human capital over the life cycle. The theoretical model draws on, and was stimulated by, the earlier work of Becker and Mincer on investment in man,[2] and by the view of the household as a unit engaged in production.[3,4]

NOTE: I thank my colleagues, Michael Bruno, Reuben Gronau, and Gur Ofer for making valuable comments on an earlier draft. In rewriting the paper I benefited from comments by Mary Jean Bowman, Ruth Klinov, and Zvi Griliches and particularly from the valuable discussions of the paper by Jacob Mincer in the conference.

[1] Yoram Ben-Porath, "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, August 1967, as well as other parts of "Some Aspects of the Life Cycle of Earnings" (unpublished Ph.D. dissertation. Harvard University, February 1967).

[2] Gary S. Becker, "Investment in Human Capital: A Theoretical Analysis," Jacob Mincer, "On-the-Job Training: Costs, Returns, and Some Implications," *Journal of Political Economy*, Supplement, October 1962, as well as the earlier work by Mincer, "A Study of Personal Income Distribution" (unpublished Ph.D. dissertation, Columbia University, 1957) which is being currently revised.

[3] See in particular Gary S. Becker, "A Theory of the Allocation of Time," *Economic Journal*, September 1965, and Kelvin Lancaster, "A New Approach to Consumer Theory," *Journal of Political Economy*, April 1966, as well as the applications by Jacob Mincer and others. In particular, there is much in common between our model and Gary S. Becker, "Human Capital and the Personal Distribution of Income: An Analytical Approach," New York, NBER, October 1966 (mimeo.).

[4] The problem of the time path of investment in human capital was tackled from a different angle in Giora Hanoch, "Personal Earnings and Investment in Schooling" (unpublished Ph.D. dissertation, University of Chicago, August 1965), chapter II. An approach similar to ours was recently utilized in a more general model, systematically analyzed by Hajime Oniki in his doctoral dissertation entitled "A Theoretical Study on the Demand for Education," 1968 (mimeo). See also C. C. Weizsäcker, "Training Policy under Conditions of Technical Progress: A Theoretical Treatment," in *Mathematical Models in Educational Planning*, Paris, Office of Economic Cooperation and Development, 1967.

The purpose of our model is to introduce explicitly considerations related to the production side of human capital into the analysis of investment in man. This is done by writing down a production function of human capital of the individual. By letting this function generate rising marginal costs of human capital production a mechanism is provided for regulating investment over life.

The main purpose of this paper, aside from a slightly more general formulation of the model, is to use this model as a basis for dealing with a specific question concerning the nature of human capital, namely—to what extent can the accumulation of human capital be described as a neutral augmentation of human productive capacity, where neutrality is considered in reference to two types of activity: market activity, i.e., providing the labor input into the general economy; and the production of human capital, the building up of human productive capacity. While it is customary to emphasize the former role, there is no doubt about the existence of the latter. The question is examined by incorporating the "neutrality hypothesis" into the model. On the basis of estimates of investment in human capital prepared by Jacob Mincer, the hypothesis is tested and tentatively rejected. It is also shown that where the hypothesis is accepted, the key parameter in the production function of human capital can be estimated.

I

CONSIDER an individual, who expects with complete certainty to live for T years, and is faced with a given interest rate r, at which he can borrow and lend indefinitely, and a rental per unit of time of the services of a unit of human capital, w, both of which he expects to remain constant through life. In a classical fashion, define $K(t)$ to be an index of the homogeneous stock of human capital of the individual. It is a measure of labor in standard units and its size, together with the market-determined rental, defines earning capacity, $Y(t)$.

$$Y(t) = wK(t). \tag{1}$$

There are opportunities for increasing the stock of human capital by allocating some services of human capital and purchased inputs D into producing human capital at the rate Q. These opportunities are summarized in a production function:

$$Q(t) = F[s(t)K(t), D(t)]. \tag{2}$$

The volume of services of human capital depends on the size of the available stock K, which is given at any period but can be changed over time, and the fraction of time devoted to human capital production, s, which can be varied between the limits 0 and 1. (We proceed as if the length of each period is 1, yet still use continuous time for convenience.) Time is assumed to be allocated only between the labor market and the production of human capital, and other uses of time are ignored. Thus, the allocation of time between leisure and the two types of production activities is ignored here, a complete separation between consumption and production decisions is maintained, and hence, some potentially interesting implications for the problem at hand are excluded.

Let the cost-minimizing factor combination of producing one unit of human capital be $(s\tilde{K}, \tilde{D})$, where we assume that K is large enough for $s < 1$; unit cost is:

$$\tilde{I} = w\, s\tilde{K} + P_d\tilde{D}. \tag{3}$$

$\tilde{I}$ is composed of a "foregone earnings" element $(w\ s\tilde{K})$ and a direct costs element $(P_d\tilde{D})$, where P_d is the unit price of the composite purchased input.

Although in reality the production function may be shifting with age, probably first upward and later downward, the desire to focus on specific aspects of the problem leads us to assume F independent of age.

Assume that F is homogeneous of degree μ in sK and D, with $\mu < 1$. Total costs of producing Q at any period are thus given by:

$$I(Q) = \tilde{I}Q^{\frac{1}{\mu}} \tag{4}$$

and marginal costs are given by:

$$MC(Q) = \frac{1}{\mu}\tilde{I}Q^{\frac{1}{\mu}-1}. \tag{5}$$

By the assumption $\mu < 1$, $MC'' > 0$, i.e., marginal costs rise with the rate of production.

If the individual is assumed to behave as if maximizing the discounted value of his net earnings (earning capacity minus all investment costs), then:

$$W(t) = \int_t^T e^{-r(z-t)}[Y(z) - I(z)]\, dz = \int_t^T e^{-r(z-t)}E(z)\, dz. \tag{6}$$

Thus, the services of human capital as a durable consumption good are ignored, and so is the utility or disutility that may be associated with its production.

The discounted value to the individual of an additional unit of human capacity that earns w, at each point in time from time t to T, is given by:

$$P(t) = \frac{w}{r}[1 - e^{-r(T-t)}]. \tag{7}$$

The quantity produced, Q^*, will be that quantity for which $MC(Q)^*$ equals $P(t)$. Alternatively, the *marginal* rate of return, i.e., the return $w(1 - e^{-r(T-t)})$ on the marginal dollar $\frac{1}{MC}$ will be equated to the rate of interest r. Thus, the optimum rate of production is given by:

$$Q^*(t) = \left[\frac{\mu}{r}\frac{w}{\tilde{I}}(1 - e^{-r(T-t)})\right]^{\frac{\mu}{1-\mu}} =$$

$$\left[\frac{\mu}{r}\frac{1}{\left(s\tilde{K} + \frac{\tilde{P}_d\tilde{D}}{w}\right)}(1 - e^{-r(T-t)})\right]^{\frac{\mu}{1-\mu}}. \tag{8}$$

Substitution of (8) into (4) gives the time profile of investment outlays:

$$I(t) = \tilde{I}\left[\frac{\mu}{r}\frac{w}{\tilde{I}}(1 - e^{-r(T-t)})\right]^{\frac{1}{1-\mu}} \tag{9}$$

Both Q and I decline with t. The relative rate of decline of investment is given by:

$$\frac{\dot{I}}{I} = \frac{-r}{1-\mu}\frac{e^{-r(T-t)}}{(1 - e^{-r(T-t)})} < 0. \tag{10}$$

The reason for spreading investment over many periods is that at any particular period opportunities are limited and one has to forgo more and more in the present in order to acquire additional future streams of earnings. Postponement of investment entails greater savings in the production costs of human capital, the smaller is μ; the postponement of associated benefits is less costly, the smaller is the rate of interest. It is therefore, reasonable to expect, as in (10), that the smaller are r and μ, the more evenly spread and more moderately declining is investment. Because of the assumption of homogeneity, direct costs ($P_d D$) and forgone earnings ($w\ sK$) remain in fixed proportion, both decreasing at the rate $\frac{\dot{I}}{I}$ (10). Time input into the production of human capital declines at a quicker rate—the human capital input is sK and if it changes at the same rate as I does, then it follows:

$$\frac{\dot{s}}{s} = \frac{\dot{I}}{I} - \frac{\dot{K}}{K} \tag{11}$$

It is easy to derive from this behavior the implied pattern of change of earnings over life. Net earnings as defined in (12) rise with age (13):

$$E(t) = Y(t) - I(t) = wK(t) - I(t) \tag{12}$$

$$\dot{E}(t) = wQ(t) - \dot{I}(t) > 0 \tag{13}$$

owing both to the increase in the stock of human capital and the decline in investment.

We spare the reader the laborious derivation of differentiating $\dot{E}(t)$ with respect to time (14). The result means that earnings rise at a declining rate:

$$\ddot{E}(t) = \dot{I}\left(\frac{w}{MC} + \frac{\mu - r}{1 - \mu}\right) < 0. \tag{14}$$

Forgone earnings are a constant fraction of investment outlays, so that a pattern like the one described by (13) and (14) holds for observed earnings, i.e., earnings realized in the labor market before deduction of direct costs. If a depreciation factor is introduced for the stock of human capital, the earnings profile will have a declining portion toward the end of life.

All the preceding analysis is confined to the phase where $s < 1$.

This is a situation where only part of the available stock of human capital is allocated to the production of human capital, the implication being that at the ruling prices, factors can be obtained without effective limitation and Q is produced at minimum cost. It is possible for K to be small enough relative to the desired Q so that the quantity demanded of services of human capital for the production of Q will exceed K, so that there is no labor force participation, $s = 1$, and the size of the available stock K is an effective constraint forcing the production of human capital at less than optimum factor combination. It is possible, and likely, to have in this phase periods in which investment rises over time. This is a description fitting early life, when the stock of human capital is still small, and the desired rate of adding to it is very high, so that all time is allocated to the formation of human capital.

II

THE model can be used for analyzing qualitatively the effects on behavior of two kinds of parametric disturbances: changes in market prices and variations in the technical parameters.

To the first group belong changes in r, P_d, and w. There are no surprises here—a rise in the rate of interest reduces the production of human capital and investment outlays and increases the rate at which investment declines. An equiproportionate rise in w and P_d leaves the quantity of human capital produced, Q, unaffected, but causes a rise of the same proportion in earning capacity, in investment outlays, and in net earnings of every period, leaving unchanged the (log) earning profile. If w rises more than P_d, Q will rise (see equation 8). The use of time for the production of human capital per unit produced will decrease, but the proportion of forgone earnings in investment costs will rise or fall depending on the elasticity of substitution between the two inputs.

The length of life T is treated as given and affects behavior through the demand price. The higher T is, the larger the quantity of human capital produced, approaching a constant rate

$$\left[\frac{\mu}{r}\Big/\left(s\widetilde{K}+\frac{P_d}{w}\widetilde{D}\right)\right]^{\frac{\mu}{1-\mu}} \text{ as } T \rightarrow \infty. \tag{15}$$

The other group of parameters consists of those appearing in the production function. The production function is a summary expression for the opportunities for increasing human capital, opportunities that depend both on the abilities of the individual and the structure of the market. Together with the market prices w and P_d, the technical parameters determine unit costs and structure of costs of human capital. The higher $\tilde{I}$, the less investment will be undertaken. For a given pair of prices (w, P_d), $\tilde{I}$ will be an index for ranking one dimension of the ability to produce human capital; ranking may be different with other sets of prices. An important technical parameter already discussed is μ; the smaller μ is, the quicker the ability of producing human capital declines with the rate of production—the faster the decline in the slope of the "learning curve."

In addition to the abilities associated with the production of human capital there is an initial endowment of human capital $K(0)$. The larger $K(0)$, the earlier will be the date of entry into the labor force and the longer the time spent in it.

III

THE model incorporates many assumptions that have become standard in human capital literature and their limitations are fairly well known. It contains, however, further restrictions, incorporated in the production function. One is the assumption of homogeneity, which is technical; the other, to which we refer as the "neutrality hypothesis," is substantive.

In what we called the production function of human capital (2), one of the inputs is $[sK(t)]$, which represents the services of human capital. At any period t, $K(t)$ is given so that variation in the input of human capital comes from varying the time allocated to the production of human capital. When K increases, as a result of engaging in an investment activity, the services of human capital carried by a unit of time likewise increase regardless of whether time is allocated to the market or to the production of human capital. Investment in human capital is thus treated as an augmentation of the human factor, neutral with respect to its use.

Because Q depends on (sK) the investment cost function (4) is

TABLE 1

On-the-Job Training by Age and Level of Education–Estimated Costs and Annual Rates of Decline, U.S. Males, 1958

	Total Costs ($)			Percentage Annual Rate of Decline of Investment[a]				Percentage Change in the Relative Decline of Investment[b]			
						College				College	
Age (1)	Elementary School (2)	High School (3)	College (4)	Elementary School (5)	High School (6)	Original (7)	Smoothed[c] (7)	Elementary School (8)	High School (9)	Original (10)	Smoothed (10)
14	296										
15	314			+5.9							
16	303			−3.6							
17	300			−1.0				112.5			
18	297	522		−1.0				0.9			
19	293	517		−1.4	−1.0			29.5			
20	289	512		−1.4	−1.0			0.7	1.0		
21	284	506		−1.8	−1.2			24.4	18.7		
22	278	498	2,588	−2.1	−1.6			20.1	30.4		
23	271	488	2,489	−2.5	−2.0	−3.9	−1.6	17.5	23.8		
24	262	476	2,378	−3.4	−2.5	−4.6	−1.8	28.0	20.4	16.5	7.0
25	251	462	2,353	−4.3	−3.0	−1.1	−2.0	24.0	18.3	−122.8	9.8
26	237	445	2,325	−5.7	−3.7	−1.2	−2.2	28.7	22.6	0.9	12.0
27	221	425	2,085	−7.0	−4.6	−10.9	−2.5	19.8	20.4	160.7	11.9
28	202	402	1,930	−9.0	−5.6	−7.7	−2.6	25.0	19.1	−34.4	6.6

29	180	375	1,752	−11.5	−6.9	−9.7	−3.0	24.8	22.1	23.0	12.4
30	161	350	1,547	−11.1	−6.9	−13.2	−3.5	−3.3	−0.7	30.6	15.4
31	153	336	1,485	−5.0	−4.1	−4.1	−4.1	−75.9	−51.2	−111.9	15.5
32	144	319	1,415	−6.1	−5.2	−4.8	−4.8	18.8	23.9	16.1	16.1
33	133	298	1,335	−7.9	−6.8	−5.8	−5.8	26.8	26.8	18.6	18.6
34	120	274	1,245	−10.3	−8.4	−7.0	−7.0	25.6	20.9	18.2	18.2
35	104	245	1,143	−14.3	−11.2	−8.5	−8.5	32.7	28.4	20.2	20.3
36	85	211	1,026	−20.1	−14.9	−10.8	−10.8	33.8	28.7	23.2	23.2
37	63	172	991	−29.7	−20.4	−3.5	−12.0	38.6	30.9	−102.0	10.7
38	37	126	742	−52.0	−30.9	−12.9	−14.6	90.0	41.0	114.7	19.5
39	6	73	574	−144.2	−53.3	−25.5	−17.1	94.0	53.2	65.6	15.8
40		43	466		−51.7	−20.8	−20.8		−2.9	−20.4	19.3
41		16	355		−91.5	−27.0	−27.0		55.5	25.9	26.3
42			246			−36.3	−36.3			29.4	29.2
43			144			−52.3					

Source: Columns 2–4, J. Mincer "On the Job Training: Costs, Returns, and Some Implications," *Journal of Political Economy*, Supp., October 1962, Table A7, (cols. 4–6).

[a]Calculated from columns 2–4 respectively by dividing differences of adjacent years by this average.

[b]Calculated from columns 5–7 by same method as columns 2–4.

[c]Figures obtained by reading from a curve fitted with a free hand to the original data (see Figure 1).

independent of the stock of human capital K, provided s is smaller than 1. If investment in human capital is biased towards the market, i.e., if people become comparatively more productive in the market than in the production of human capital, then any hour taken away from the market becomes more expensive and this makes the nonmarket good—in this case human capital—more costly. This provides another reason for the decline of investment over time.

The neutrality hypothesis is a strong one, and cannot be expected to hold strictly. It is strong enough to impart testability, and it is probably useful to know whether it is roughly consistent or grossly inconsistent with reality.

Several implications of the model can be tested in principle, but not in practice. Thus, there are implications for the allocation of time between "pure work" and the production of human capital, although in fact the two activities are intermingled and no meaningful separation is possible. Similarly, it is difficult to distinguish direct outlays on investment in human capital from other household expenditure.

It is clear, however, that if a series on investment in human capital is available some implications of the model can be tested. A series on investment in "on-the-job training" has been constructed by Jacob Mincer.[5] The series is not, of course, a result of direct measurement of investment outlays but is derived from the life cycle of earnings on the basis of the theoretical framework of the human-capital approach.[6] Thus, it is not suitable for a test of the general approach and could be viewed as a test of the specification of the production function, while the general principle is maintained.

The test proposed here is based on the prediction of the model with respect to the time path of investment outlays. Equation 10 predicts a decline of investment with age. Let us now take the derivative of (10) with respect to time and divide it by (10), i.e., obtain the relative rate of change of the relative rate of change of investment. As shown in (16) it turns out to be a very simple expression—it is the rate of interest cor-

[5] Mincer, "On-the-Job Training."

[6] For a discussion of the procedure used by Mincer and the relation between our model and this procedure see Appendix.

rected for final life T, and this is the rate at which the decline in investment over the life cycle should accelerate:

$$\tilde{\tilde{r}} \equiv \frac{\partial}{\partial_t}\left(\frac{\dot{I}}{I}\right) \Big/ \frac{\dot{I}}{I} = \frac{r}{1 - e^{-r(T-t)}}. \tag{16}$$

We have here several elements of a test:

a. does $\tilde{\tilde{r}}$, as measured, *change* over time only to the very limited extent suggested by the correction for final life in (16)?

b. is the *level* of $\tilde{\tilde{r}}$ close enough to what we think is the relevant rate of interest?

c. or, is it close to the level of the (incremental) rate of return used in the calculation of the investment series?

Also note that: $\tilde{\tilde{r}}$ together with $\frac{\dot{I}}{I}$ provide, through equation 10 an estimate of μ, the degree of homogeneity in the production function of human capital; $\frac{r}{\mu}$ is then the average rate of return.

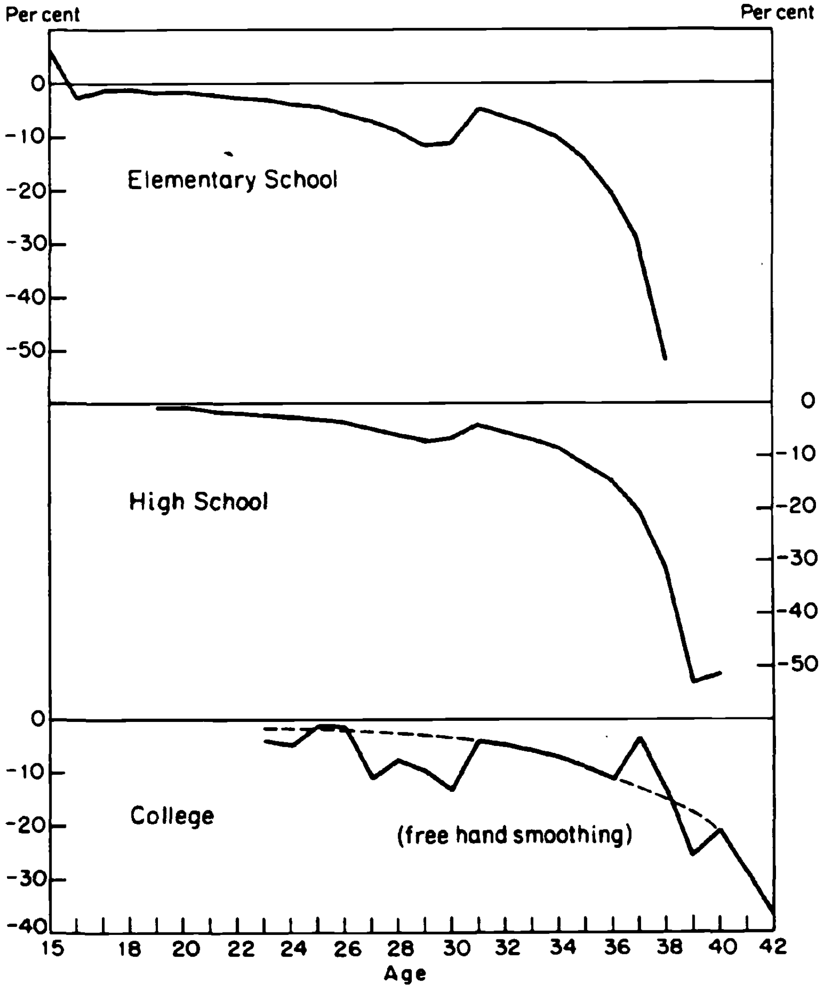
Confronting the data one should remember that Jacob Mincer did not prepare the estimates with an eye towards this type of "second derivative" experimentation. The limitations of the estimates, discussed in detail by Mincer, may be responsible for spuriousness in the result and the treatment of the following as an exercise is certainly not overcautious.

In Table 1 (columns 2–4) we have reproduced Mincer's original investment estimates for 1958, which constitute the better-behaved series in his study. The estimates for elementary school are really increments over the 0–4 years of schooling group. The total costs for the group with more schooling are sums of increments up to the respective levels. The rates of change of investment costs (columns 5–7) are piecewise well behaved for the elementary- and high-school levels and much less so for the college level (see Figure 1). The calculated $\tilde{\tilde{r}}$'s based on these series are presented in columns 8–10. Because of the irregularities in the original rates of decline, we took the liberty of smoothing the college series with a free hand. The nature of the adjustment is shown in Figure 1.

The estimates of $\tilde{\tilde{r}}$ hover in the high twenties for elementary school, in the lower twenties for the high-school level, and in the high teens for the college level. There is a sharp rise in $\tilde{\tilde{r}}$, i.e., an acceleration of the rate of decline of investment, as people pass the mid-thirties.

In comparing the different groups of investors, one notes that the level of $\tilde{\tilde{r}}$ tends to be lower for the bigger investors—for college compared to high school, high school compared to elementary school (note—this is intergroup and not intertemporal comparison). The levels of inter-

Figure 1. Percentage Annual Rates of Decline in Cost of On-the-Job Training



Source: Table 1, columns 5–7.

est rate implied are perhaps not unreasonably high for this type of decision, but are higher than the rates of return calculated by Mincer (19.3 per cent, 15.1 per cent, and 11.5 per cent for the elementary-school, high-school, and college groups respectively) and used in the construction of the investment series.

One also notes that the acceleration in the rate of decline of investment over time beyond the age of thirty-five is higher than could be predicted from equation 15 and at the mid-forties investment stops completely. This may be partly a result of the early flattening of cross-section earning profiles in comparison to longitudinal earning curves.

As indicated, we could use equation 10 and the calculated $\tilde{\tilde{r}}$ to calculate the implied μ provided the hypothesis was not rejected. The values of μ that we would get in this way (assuming $T = 65$) are presented in Table 2 and are all in the range .90–.99. Theoretically, they should have been the same for all ages. They fall in a relatively narrow range and are not far from 1, i.e., in terms of the model there would be no great departure from constant returns to scale.

One could start from another end: assume r from the outside, using (10) solve for μ at a given age and then check for how well the pairing r and μ predict the rate of decline of investment at another age. What makes this test weak is that if the resultant μ happens to be close to 1, then very small changes in μ can bring very large changes in $\frac{\dot{I}}{I}$, because in (10) $1 - \mu$ appears in the denominator.[7] The test through the second derivative is thus more conclusive.

IV

ONE might say that the neutrality hypothesis incorporated into the model and discussed in the previous section relegates to the demand side most of the burden of explaining changing behavior over the life cycle, because movements of the demand curve along a stationary marginal-cost curve are responsible for the decline in investment, although the shape of the rising cost function has an effect, as indicated. Mincer, comment-

[7] This paragraph was written in response to comments by Jacob Mincer.

TABLE 2

Estimates of μ at Selected Ages

	$100\tilde{\tilde{r}}$ [a]			$100\frac{\dot{I}}{I}$			μ [c]		
Age	Elementary School	High School	College	Elementary School	High School	College	Elementary School	High School	College
25	26.9	20.5	9.5[b]	4.5	3.1	2.0[b]	.95	.94	.99[b]
29	24.9	20.6	11.5[b]	10.5	6.5	3.0[b]	.94	.97	.98[b]
35	30.8	26.0	20.6	14.9	11.5	8.8	.93	.93	.91

[a]Three year averages, centered on the specified age, taken from Table 1, columns 8–10.

[b]Based on the smoothed series.

[c]Caculated on the basis of equation 10: $\frac{\dot{I}}{I} = \frac{-r}{1-\mu} \quad \frac{e^{-r\,(T-t)}}{1-e^{-r(T-t)}}$ using three-year averages centered on the specific age.

ing on his own estimates, mentions only the demand side by saying that "the decline of training with age is consistent with *a priori* expectations about investment behavior: younger people have a greater incentive to invest in themselves than older ones because they can collect the returns for a longer time."[8] Becker tends to emphasize the role of changing cost conditions.[9]

We should recall that the properties attributed to the neutrality hypothesis hold only at the phase in which there is labor force participation ($s < 1$). In the earlier period, when there is complete specialization in human capital production, cost functions change with K (they actually decline). Most of our analysis thus refers to the period beyond full-time formal education.

We want to end this paper with a few comments on this assumption. Doubts about its validity on *a priori* grounds are linked to the heterogeneity of the ways in which people can invest in themselves. We know that many activities increase earning capacity but leave unaffected the ability to increase earning capacity further. On the other hand, there are activities that do not directly contribute much to earning capacity but increase the ability to produce the kind of human capital which, if produced, would increase earning capacity. Even if any particular type of school, occupation, or job offers a relatively rigid mix of these types of skill, it is possible to choose among activities and to decide on an educational and occupational career that will bring about a desirable mix of the different "capital goods." The market does not make it possible to get something for nothing, so that neutral improvement in human capacity costs more than specialized improvement, and at any period people will choose the optimum early in life. When there is still a large investment program ahead, it is advisable to emphasize devices that reduce future investment costs and make the individual a more efficient producer of human capital. Later in life, when future planned investment is smaller, the fraction of investment outlays devoted to skills that are for purposes of further investment will also be smaller. The formal schooling system, at least in the precollege levels, tends to emphasize skills that are tools for further learning (the more general skills are likely to be produced by

[8] Mincer, "On-the-Job Training," p. 55.
[9] Becker, "Human Capital and the Personal Distribution of Income."

"firms" specializing in the production of skills). On the other hand, certain types of higher education and a large part of on-the-job training impart directly productive skills,[10] building upon the tools previously acquired in school. One presumes, however, that even within the period of labor force participation there is enough variety of opportunity in the labor market to allow a change in the mix. Thus, the structure of the individual's stock of human capital can be expected to change over time and become relatively more market-oriented, reflecting a (planned) shift in comparative advantage. In comparison with the pattern described under the neutrality hypothesis there should be a steeper decline in the production of human capital and an even steeper decline in the time input, because of the substitution of purchased inputs for own time, as the price of the latter increases.

APPENDIX

ON THE ESTIMATION PROCEDURE OF INVESTMENT COSTS

THE purpose of this appendix is to relate the Becker-Mincer procedure of estimating costs of on-the-job training to the model presented in the text (for a more general critique of the procedure see Mary Jean Bowman "The Assessment of Human Investment as Growth Strategy" in the *Compendium on Human Resources* prepared for the Joint Economic Committee of the U. S. Congress, February 1968).

The principle underlying the Becker-Mincer estimating procedure can be described as follows (using our notation): recall the identity $Y \equiv E + I$, i.e., earning capacity equals net earnings plus investment. Think of two (representative) individuals a and b for whom earning

[10] If this is indeed true and if these direct skills are of the general type (in Becker's terminology) the initial earning capacity or salary per unit of pure working time of, for example, a high-school graduate should not be higher than that of an elementary-school graduate. But, on the other hand, when on-the-job-training includes specific skills then the employer will, from the start, pay the rent on the superior learning tools that the high-school graduate possesses—he will pay him for his ability to produce efficiently specific skills on the job.

capacities are equal at time 0 and the values of the left-hand side of (1′) are known.

$$\begin{aligned}\Delta E(0) &\equiv E^a(0) - E^b(0)\\ \Delta I(0) &\equiv I^a(0) - I^b(0)\\ \Delta Y(0) &\equiv Y^a(0) - Y^b(0) = 0\end{aligned} \tag{1′}$$

Note that Δ refer always to differences between people and not between periods. Let r be the internal rate of return obtained by equating to zero the discounted differences in net earning between two individuals

$\int_0^T \frac{\Delta E(z)dz}{1 - e^{-\tilde{r}(T-t)}} = 0$ and assume that it is constant through life. It is then argued that the following holds true:

$$\Delta I(1) = \tilde{r}\Delta I(0) - \Delta E(1). \tag{2′}$$

The first term on the right-hand side of (2′) is the difference in earning capacities in year 1 that was created by the difference in investment in year 0; if, in addition, the bigger investor earns in year 1 less than the small investor does, the difference in their respective investments in year 1 is higher by this term.

This can be repeated and in general incremental costs can be calculated with the aid of (3′) which is Mincer's equation 1 (in a continuous form).

$$\Delta I(t) = \tilde{r}\int_0^t \frac{\Delta I(z)dz}{1 - e^{-r(T-s)}} - \Delta E(t) \tag{3′}$$

Rearranging (3′) we get:

$$\Delta Y(t) = \tilde{r}\int_0^t \frac{\Delta I(z)dz}{1 - e^{-r(T-s)}}, \tag{4′}$$

by which the difference in earning capacity at time t between two individuals starting with the same earning capacity at time 0 is equal to the internal rate of return times the cumulative difference in investment costs (corrected for final life T). If the increments in costs between the groups compared are small enough, $\tilde{r}$ can be regarded as the marginal rate of return. Sequences of increments in investment for groups facing smaller and smaller rates of interest can be accumulated to obtain total investment of each of the groups.

As far as one can see, the procedure does not depend on a particular form of the production or cost functions and can therefore be described as a general procedure. If, on the other hand, investment differentials, $\Delta I(t)$, are large, then the rates at which investment costs are converted to returns should be *average* rates, which may vary with the size of the investment differentials, and the use of $\tilde{r}$ as a constant through time is not generally justified.

Let us go in what looks like the same direction, using the model of Section I. The addition to earning capacity $Y(z)$ of each period from t to T resulting from the investment of $I(t)$ at time t, is $wQ(t)$, the quantity produced in time t, multiplied by the market rental. If we divide the latter by the former and use (4) and (8) we get:

$$\frac{wQ(t)}{I(t)} = \frac{r}{\mu} \frac{1}{[1 - e^{-r(T-t)}]} \qquad (5')$$

Transposing and accumulating from time 0 we get:

$$Y(t) - Y(0) = \int_0^t wQ(z)dz = \frac{r}{\mu} \int_0^t \frac{I(z)dz}{[1 - e^{-r(T-z)}]}. \qquad (6')$$

The difference between two individuals a and b who have the same earning capacities at time 0 is thus given by:

$$\Delta Y(t) = Y^a(t) - Y^b(t) = \frac{r}{\mu} \int_0^t \frac{\Delta I(z)dz}{1 - e^{-r(T-t)}}. \qquad (7')$$

Compare (7′) with (4′), which represents the Becker-Mincer procedure. There is a considerable similarity between the two, but they differ in the assumptions they carry. Equation (7′) does not depend for its validity on the size of the investment differentials, $\Delta I(t)$, but rests on the assumption that the two (representative) individuals compared face the same interest rates and have the same μ; in (5′) r is the rate of interest and the marginal rate of return is equated to it, and (5′) is the *average* rate of return—average over the dollars invested in a given period—the proper translator of total investment into returns.

The reason for the difference in investment that underlies the Becker-Mincer procedure is the difference in the interest rates that different investors face. On the other hand, the investment differentials that (7′) claims to measure stem from differences in the cost functions of human capital production, with the severe restriction that the individuals compared have the same elasticities of costs with respect to output Q, i.e., the same μ. The difference in their cost functions are only differences in levels, i.e., in $\tilde{T}^j$. In addition, the interest rates that they face are assumed to be the same.

One might ask whether the use of a constant average rate of return requires as much as is assumed in the model of Section I. When the quantities of human capital produced vary over time then, with a given marginal rate of return, the constancy of the average rate of return is linked to a constancy of the ratio of marginal to average costs of producing human capital, which in turn implies a constant elasticity of costs with respect to output—the counterpart of a homogeneous production function. It is also difficult to assume a constant average rate of return without accepting the neutrality hypothesis.

COMMENTS

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Ben-Porath's contribution to this conference contains a generalization of his recently published model of production of human capital, and an attempt at empirical testing of some of its features. The generalization consists of a weaker specification of the production function: homogeneity and diminishing returns to scale are maintained, but the specific Cobb-Douglass form is dropped. The empirical test refers to age profiles of postschool investment expenditures: According to equations 10 and 16 that profile is predicted to be declining with age at an accelerating rate with a relative acceleration roughly equal to the interest rate r. Given empirical estimates of such age profiles empirical tests are feasible and were carried out.

In my view the Ben-Porath models presented here and in his previous work represent a very useful step forward in the analysis of human capital. The particular specification of the HPF (human capital production function) is less important than the general approach, given the latter's potential flexibility.

As to the particular restrictions in the model, the most important one, as Ben-Porath rightly stresses, is the "neutrality hypothesis," according to which human capital increases productivities in the market at the same rate as it does in the production of additions to the stock of human capital. This is a rather attractive exception to the usual view of "household" (or "nonmarket") production functions. As economists, we tend to ascribe a market bias to most of the effects of human capital, but as educators we are inclined to exempt from the bias the effects of learning on further learning. Perhaps this is correct: Ben-Porath's point is that the issues can be tested, at least in principle.

The hypothesis is a strong one, and for that one must pay a price: It imparts a high degree of rigidity to the Ben-Porath model. Indeed, it

TABLE 1

Predicted and Observed Percentage Declines in Net Investment

	Age					Age of
	25	30	35	40	45	Termination
Elementary						
Predicted	.0 [a]	-.7	- 1.6	-3.8	-8.7	59
Observed	-4.5	-9.2	-14.9	– [b]	– [b]	40
High School						
Predicted	-.7	-1.5	-3.3	-7.2	-15.6	56
Observed	-3.1	-6.0	-11.5	-51.7	– [b]	42
College						
Predicted	-3.2	-4.6	-7.8	-14.7	-25.3	53
Observed	-2.0	-3.6	-8.8	-20.8	– [b]	45

[a] Almost zero.

[b] Decline to zero (-100 per cent).

neutralizes almost everything, allowing only one source of motion: the finiteness of human life. Unquestionably, this is a major fact. Interestingly, however, finiteness of life is usually of secondary importance in many aspects of the economics of human capital. This is because the discounting of values to be realized decades in the future reduces them to negligibility. The present model is no exception. The age profile of investment predicted by equation 10 declines *only because* of the finite life span T. But with T large the profile is practically horizontal, because the numerator of the second term on the right is almost zero.

The second derivative type of test that was performed indicates that the actual relative acceleration in the decline of "observed" investments is larger than that predicted by equation 16, but perhaps not enough to reject the null hypothesis—particularly in view of the few degrees of freedom. However, the first derivative—the relative speed of decline —is equally amenable to test by equation 10, and is worth considering. Table 1 gives a comparison of actual and predicted speeds of decline in investment at several ages. The predicted magnitudes were calculated by means of equation 10, using $T = 65$, $\mu = .95$, and the values of r for the three schooling groups as given in the original data.

There is a noticeable conformity with prediction in the college series almost up to age forty, and a widening divergence thereafter. In the two lower schooling groups, the divergence is much larger than any similarity: the predicted series is almost completely flat throughout the period when observed investments are positive, with the predicted series reaching zero almost two decades later than the actual.

Note that this test by equation 10 is not independent of the test by equation 16, except that it uses the discount rates from the original data which are produced by equation 16 rather than $\tilde{\tilde{r}}$. The same test can also be viewed, independently of 16, as follows: Given the original data for r and for the rate of decline in investments, what values of μ in equation 10 will produce the best fit? The answer is: μ must be extremely close to unity (.99 or so). This is too close for comfort: Unity would mean that all investments take place in the initial period—a clear contradiction; but anything significantly less than unity makes for too slow a decline in investments relative to the observed decline. The specific version of the model is, therefore, substantively fragile and statistically intractable.

If the comparisons of actual with predicted lead to a rejection rather than acceptance of the maintained hypothesis, one may be tempted to blame the results on, or to explain them by, the somewhat conjectural quality of the estimated series. As its producer, I can only say that the series reflects the state of the arts circa 1961, rather than the shoddiness of a surprisingly long-lived monopoly. As Ben-Porath correctly notes, my interest in that work was not in the age profile of postschooling investments, that is in the individual terms of the series, but in their sum, the total investment costs. However, a little reflection suggests that the particular expedients I used to convert the several income intervals into an annual series are not decisive. Shapes of earnings profiles are recognizable and a sizeable stock of them has accumulated in the available statistics. As Ben-Porath's equations 12 and 13 indicate, everything stated about investment profiles can be translated into statements about earnings profiles. Thus, the predicted magnitudes (rows a) imply that earnings profiles, at least below college, are almost exactly linear throughout the first half of working life, long *after* the observed concavity is most noticeable. The other features of the test, which can be rejected simply on the basis of even casual familiarity with earnings profiles, are: (1) the predicted ages of peak earnings (termination of net investment) are systematically later the lower the schooling group, and (2) in this model the slope of the earnings profile is a function of age, not of experience (i.e., length of stay in the labor force).

Incidentally, it is not clear why T is to be put at age sixty-five. If T denotes end of life span, rather than end of working span, it should be a more advanced age, and the predicted profile of investment would be shifted to the right by a corresponding interval, *increasing* the discrepancy between predicted and observed in all schooling groups. The question of the meaning of T and how it is to be brought into the analysis is very important.

One reason this question was left open is the absence of depreciation in the model, or the implicit assumption that it constitutes a fixed fraction of earning capacity throughout life. A depreciation rate rising with age during the second half of the working life seems more realistic and could explain much of the story.

Net investment, not gross, is the factor underlying the earnings profiles. After middle age, gross investment is progressively eroded and eventually outstripped by depreciation. Hence the more rapid decline in net investment and the earlier termination of it—relative to the patterns predicted by the model. The assumption of an exogenously fixed T, whether viewed as end of life or retirement age, may be dropped. A major objective of gross investment at the later stages of the life cycle is to extend the productive years and the life span. With larger gross investments T is postponed to later ages.

As for the earlier ages, other amendments might still be needed, including a "breach of neutrality," possibly along the lines expressed at the conclusion of the paper. This will complicate the model, but it will add life and robustness to the analysis. In the meantime, Ben-Porath is to be applauded for a valiant first step in the right direction.

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Professor Yoram Ben-Porath develops a theoretical model to explain lifetime investment in human capital. The model is then tested against Mincer's estimates of the actual lifetime distribution of human capital investment. Operationally the model must explain why human-capital investment is concentrated in the early years of a person's working life. There are two assumptions in Ben-Porath's model which lead to investment concentration. First, a man is assumed to have a finite working life. The earlier an asset is acquired the longer its working life. Second, the opportunity costs of investing increase as human capital is acquired. Human capital raises a man's marginal product and consequently the opportunity costs of time spent on human investment. Early assets make later assets more expensive.

Although it is interesting to see how these simple assumptions can produce investment concentration, the model is too simple to present a realistic explanation of lifetime human investment. In order to explain human investment, it is necessary to take into account the peculiarities of human capital. I will first outline some of the peculiarities and then indicate how they affect Professor Ben-Porath's results.

1. Human capital is not a negotiable asset. Since it cannot be separated from the person who possesses it, it cannot be sold. Illiquidity lowers its value in an uncertain, risky world, but more importantly it means that a man must accompany his human capital. Capitalists can be

viewed as profit maximizers, but most individual utility maximizers will not be earnings maximizers. Probably no one at this conference can claim that he chose the occupation that maximized his earnings stream.

2. Since a man must accompany his human capital, production and consumption cannot be easily separated. Consumption benefits (positive and negative) come from the process of production and investment. Ideally, these complementary goods should be included in the wage rate used to evaluate an investment. Since they are not, market prices are presumably less reflective of real prices in the human capital area than in the physical capital.

3. The problem goes beyond complementary consumption goods received in the process of production. Much of a man's consumption is self produced. Self-produced goods and services are never priced since they never enter the market place. Yet human capital will be acquired to produce these goods.

4. Human beings possess a collection of human capital assets. Some of these assets are complementary. Many are substitutes or at least cannot be used simultaneously. The worth of a human-capital asset depends on what other assets a man possesses or will possess. One occupational skill may dominate another skill and make it worthless. Physical-capital assets can be separated and used simultaneously. Human-capital assets cannot.

5. Human-capital investment may affect preferences. It may, in fact, be designed to affect preferences. Music appreciation courses are only the most obvious example. Preference functions are needed to make human-capital investment decisions, but they are in turn affected by the investments. Viewed retrospectively, an economically rational individual might say that going to college was a good decision if he had gone to college and that not going to college was a good decision if he had not gone to college. Stable preference functions make much more sense when viewing physical investment.

6. Production and investment costs are not easily separated from consumption costs. How should maintenance costs be treated? Man eats and sleeps to consume as well as to produce or invest. Consumption, production, and investment are joint products of human maintenance activities. There is no way in which to allocate these maintenance costs among different activities since they are joint products of the maintenance costs.

7. Human capital has some of the characteristics of physical capital and some of the characteristics of a natural resource. Some skills, talents, and knowledge are producible; some are not. Most human capital is arrayed between these extremes. It is producible but the costs differ markedly from one person to another. A man may have the ability to acquire one skill and not another. Thus, the human-capital production functions will differ for different individuals. One man cannot acquire another's production function or the most efficient production function. Perfect knowledge allows everyone to acquire the same production functions for physical investment, but not for human investment.

8. The efficiency of the human-capital production function may also change over a person's lifetime. Viewed as a learning machine, a man may become less and less productive as he grows older. Thus, the marginal cost of investment may rise over his lifetime even if it does not rise in any given year. Athletic skills are only the most obvious example. The productivity of physical investment presumably does not depend upon the age of the investor.

9. Human capital investments are lumpy. This is especially true of the major investments in occupational skills. If a man is to work as an electrician, he must acquire most of the skills of an electrician before he can begin to work. To learn to be an electrician efficiently may also require discrete lumps of time.

10. Many investment decisions must be made at an age when the investor is not making his own decisions. Parents make decisions for their children based on their own preferences and not those of their children. In addition, parents may be able to have a major impact on the efficiency of their children's human-capital production functions. Environment may make it impossible for them to acquire many skills.

If Professor Ben-Porath is trying to explain why human-capital investment rapidly decelerates and reaches zero in the middle forties, I suggest that items 8 and 9 cannot be ignored. The available production functions become less efficient with age. It takes longer periods of time to acquire some skills; others cannot be acquired at all. Physical energy for learning or any other activity diminishes as a man grows older. Human-capital investment is also lumpy. We continually learn new skills regardless of our occupation, but initially we must make a major investment to acquire the basic occupational skills. I cannot work as an electrician until I have become an electrician. I may become a better electrician with age, but I must acquire that initial lumpy bundle of skills.

In the model in Ben-Porath's paper, a person invests in each period of time until the rate of return on investment reaches the market interest rate. I would suggest that this definition of equilibrium is too limited for human-capital investment. Each investor is subject to two budget constraints. First, there is a financial or budgetary constraint on his investment behavior. Second, there is a time constraint on his investment behavior. Human-capital investment requires time. Each person possesses a limited amount of time. There is nothing in the human capital market that guarantees that both the financial and time constraints will be effective. The time constraint could easily be the effective one. In this case, markets will not proceed to financial equilibrium. The market rate of interest will not equal the marginal productivity of human capital. The problem is especially acute when we allow different people to have different human-capital production functions. Those with efficient production functions will not drive their rate of return on human-capital investment to market rates of interest. Since the time constraint is their effective constraint the problem is similar to that of imperfect financial markets. Time is rigidly divided among individuals and cannot be transferred to those whose marginal product is higher. Thus, the inability to sell time makes the human-capital market inherently imperfect. Ben-Porath mentions the possibility of boundary solutions while young, but I suggest they are much more extensive than he indicates.

Finally, the "facts" he is trying to explain might be a function of a limited view of human capital. Mincer's investment series is not observed, but calculated from lifetime earnings. Income increases decelerate, so investment must have decelerated. If self-produced goods and complementary consumption goods were measured, real incomes might not decelerate. If real incomes do not decelerate, investment does not decelerate in Mincer's estimation procedure. In any case, we know that wage rates do not flatten out at the same rate as explained by increases in leisure. Using wage-rate-age profiles we obtain different "facts" than earnings-age profiles. Presumably the wage-rate-age profile is the relevant profile. If leisure were not worth more than income, individuals would be working. Consequently, Mincer's data probably overestimates the concentration of human-capital investment.