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# Valuing Government Guarantees Fannie and Freddie Revisited

Deborah Lucas and Robert McDonald

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## 6.1 Introduction

The federal government explicitly guarantees a portion of deposit obligations of commercial banks and thrifts through deposit insurance, and is thought to provide protection beyond this legal obligation for institutions considered “too big to fail.” Although not explicitly guaranteed until recently, Fannie Mae and Freddie Mac also have been longtime beneficiaries of similar federal protection of their debt securities against default.

Despite the perception that Fannie and Freddie derive value from the implicit guarantee and pose significant risk to the government, quantifying the federal exposure is difficult and there is substantial disagreement in the literature about magnitudes. In general, spread-based estimates of guarantee value for Fannie and Freddie are significantly higher than options-based estimates. Spread-based estimates capitalize the difference between the interest expense of Fannie and Freddie and that of similarly rated financial institutions.<sup>1</sup> Using this approach, Passmore (2005) reports a present value over twenty-five years in the range of \$122 to \$182 billion as the subsidy to Fannie and Freddie. At the other extreme, in a study commissioned by Fannie

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1. In CBO (2001), based on the analysis of Ambrose and Warga (2002), the comparison is made using a “stand-alone” rating for Fannie and Freddie, which reflects their risk to the government. As of April 2008, the GSEs had a stand-alone rating of AA– from S&P. See also Nothaft et al. (2002).

Mae using an options pricing approach, Stiglitz, Orszag, and Orszag (2002) conclude that the cost of an implicit guarantee to the government does not exceed \$200 million. In a recent paper also using an options pricing approach (Lucas and McDonald 2006), we estimate a present value cost over twenty-five years of \$28 billion for the two enterprises, still an order of magnitude lower than in Passmore (2005).<sup>2</sup>

There are several possible explanations for the higher subsidy values generally implied by spread-based analyses. One is that the guarantee may be valued by investors in government-sponsored enterprises (GSE) securities not just because of the direct value of protection from default risk, but also because of other benefits such as increased liquidity, or because they satisfy regulatory restrictions. Thus, the reduction in the GSEs' borrowing costs may exceed the cost of expected defaults to the government. Whether these other benefits to GSE stakeholders should be included in a calculation of government cost depends on the question at hand. From a broad opportunity cost perspective, since other financial institutions would pay to obtain the same privileges, they are part of the cost. To answer the narrower question of the expected cost of defaults, it is probably appropriate to exclude the value of these sorts of additional benefits.

The theoretical model developed here suggests another reason that spread-based models overestimate guarantee values: they do not correct for the more conservative optimal default policy of an insured firm. To preserve the ability to borrow at a risk-free rate in the future, we show that a guaranteed firm will choose to make debt payments in some states of the world where an otherwise identical uninsured firm would default, lowering the cost to the government relative to what a spread-based estimate would imply. This finding is related to a large body of earlier work on risk taking, charter value, and bank regulation (see, e.g., Demsetz, Saidenberg, and Strahan [1996] and the references therein). As far as we know, however, this analysis is the first to highlight the implications for credit spreads as potentially biased estimators of subsidy value.

A further possibility is that simple options-based models fail to capture important dimensions of risk, and thereby underestimate the cost and risk to the government of providing insurance. To explore this possibility, we consider several possibilities that have not been taken into account in past options-based estimates for Fannie and Freddie. First, we develop a theoretical model to examine whether and how the presence of a guarantee may affect the statistical relation between equity and asset value, and hence affect the imputation of asset value and volatility. We then calibrate and simulate a generalized version of the model to consider its quantitative implications, and to incorporate a process for the evolution of assets that includes a jump as well as a diffusion component. In light of episodes such as Fannie's

2. See also CBO (2004), Feldman (1999), and Hubbard (2004).

accounting restatements and subsequent fall in share price and the spike in credit losses following the wave of subprime defaults, we also explore the sensitivity of options-based estimates to initial conditions for equity value and volatility. In all variations, we report insurance value in terms of an annual premium as well as reporting a present value, making costs easier to interpret and normalizing for the estimation horizon.

The simulation results suggest that an insurance premium of 20 to 30 basis points (bps) would have been fair compensation for the default risk assumed by the government at year-end 2005. Cost estimates of this magnitude are still smaller than from some spread-based analyses, but they are in line with others—for instance, the Congressional Budget Office (CBO) (2001) reports a GSE borrowing advantage of 41 bps over comparable nonguaranteed financial institutions. The results also show that the fair premium rate increases rapidly with the leverage ratio, suggesting a much higher fair rate following the decline in asset values starting in late 2007.

The remainder of the chapter is organized as follows. Section 6.2 provides a brief description of Fannie and Freddie, their risk exposure, and the regulatory environment. In section 6.3 we present the valuation model and discuss the effect of the government guarantee on the dynamic relation between the underlying assets and the value of equity. Section 6.4 describes the calibration used to quantify the value of the guarantee, and reports the results of sensitivity analysis. Section 6.5 concludes.

## 6.2 Background

Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs) that were created by Congress to provide liquidity and stability in the home mortgage market. They also are required to meet modest goals for low-income lending. The GSEs are hybrids of private corporations and federal entities. Although their debt securities explicitly state that they do not bear a government guarantee, their many federal ties and critical role in the housing and financial markets suggest otherwise. As a consequence, the GSEs raise capital through debt financing at a narrower spread over Treasury rates than similarly rated financial institutions, an advantage that is generally viewed as an unbooked federal subsidy.

Fannie and Freddie participate in the mortgage market in two distinct ways. One is by buying mortgages and financing the purchases with debt issues. Those on-balance-sheet holdings expose the enterprises to default, interest rate, and prepayment risk. The interest rate and prepayment risk is partially hedged with the use of derivatives and dynamic hedging strategies (see Jaffee [2003]). They also securitize mortgages, an off-balance-sheet activity in which Fannie and Freddie assume default risk by issuing a credit guarantee.

The rapid growth of on-balance-sheet holdings in the 1990s increasingly raised concerns about the government's risk exposure, specifically about

unhedged interest rate and prepayment risk (Frame and White 2005). Following the discovery of accounting irregularities at Fannie Mae,<sup>3</sup> its on-balance-sheet growth was temporarily slowed by a consent order from their regulator that limited its mortgage portfolio to \$727 billion, down from the \$904 billion it held at year-end 2004. Fannie's mortgage-backed security (MBS) outstanding continued to grow, and reached \$1.77 trillion as of November 2006. The consent decree was lifted in late 2007, after which growth in its on-balance-sheet obligations resumed. At year-end 2005, the time we focus on for the base case analysis of guarantee values, Freddie Mac had a comparable exposure to Fannie Mae, with \$710 billion of mortgages held on balance sheet, and \$1.34 trillion in MBS outstanding.

With the sharp downturn in the housing market that began in 2007, concerns about default risk—previously thought to be a minor concern—caused the stock price of both companies to plummet. Fair value estimates reported by the GSEs in early 2008 indicated that Freddie has negative equity value, and that Fannie was barely solvent. In July 2008, Congress granted Treasury the authority to infuse funds into the entities as needed over the next eighteen months, effectively making the implicit guarantee explicit, and incurring a present value cost to taxpayers estimated by the Congressional Budget Office (CBO) to be \$25 billion (CBO 2008). Treasury used this authority two months later to take both GSEs into federal conservatorship.

An independent regulator oversees the operations of the GSEs, but their activities are primarily constrained by statute.<sup>4</sup> By law, assets consist primarily of conforming mortgages, and the enterprises must meet minimum capital requirements.<sup>5</sup> Typically both firms maintain slightly more than the regulatory minimum capital, although capital on occasion has been a binding constraint. As for commercial banks with deposit insurance, economic theory predicts that to maximize the value of the implicit guarantee the enterprises would manage liabilities to keep capital close to the regulatory minimum.

Historically, the stock of both firms consistently outperformed the overall market. Even before the recent turmoil in the housing market, however, stock price volatility had increased and returns declined. Whether the historically high returns can be attributed to unanticipated growth in the implicit subsidy

3. Fannie Mae was found by the Securities and Exchange Commission (SEC) to have overstated profits by an estimated \$9 billion starting in the late 1990s.

4. Created in 1992, the Office of Federal Housing Enterprise Oversight (OFHEO), an independent entity within the Department of Housing and Urban Development, had limited regulatory authority over Fannie Mae and Freddie Mac. The Housing and Economic Recovery Act of 2008 created a new and stronger regulator to supersede OFHEO, the Federal Housing Finance Agency (FHFA). The FHFA now has oversight responsibility for the Federal Home Loan Banks as well as for Fannie and Freddie.

5. Current legislative proposals would increase the conforming mortgage limits in high-cost states, allowing GSE holdings of mortgages previously considered jumbo loans.

is a matter of some debate. Many observers contend that GSE stockholders benefit from their special status, but the enterprises counter that competitive pressure forces any cost advantage to be passed through to borrowers (Naranjo and Toevs 2002). To the extent that rents are captured by stockholders, returns should be affected by unanticipated changes in the value of the perceived guarantee, and Seiler (2003) presents some evidence of this effect. In any event the stock returns on the two firms are highly correlated, suggesting that they are affected by common risk factors including common regulatory risk.

### 6.3 Modeling Guarantee Value

We take an options pricing approach to modeling the dynamics of guarantee value and risk exposure. The model is based on the fundamental insight of Sharpe (1976) and Merton (1977), that insurance can be valued as a put option on the assets of the firm. To illustrate the basic idea of how the guarantee is valued, and to understand its effect on the relation between observed equity valuations and the unobserved value of operating assets, we begin by analyzing a simple closed-form model where debt is adjusted at fixed intervals as long as the firm remains solvent.

For a firm with guaranteed debt, equity value has two components. The first, analogous to the equity of a levered firm without a guarantee, is a call option on the operating assets of the firm. The second component is the value of the guarantee itself, which is the present value of the (uncertain) stream of savings from being able to borrow at the risk-free rate, rather than at a risk-adjusted rate. The theoretical model is used to explore how the presence of a guarantee affects the dynamics of equity returns and their relation to the dynamics of operating assets. Since the options pricing approach imputes the value and volatility of operating assets from the value and volatility of equity, understanding this relationship is critical to correctly imputing guarantee value.

To examine the value of the guarantees quantitatively, in section 6.3.3 we numerically implement a more complex version of the theoretical model using an approach similar to that of credit analysis firm KMV (as described in Crosbie and Bohn [2003]). It allows for externally financed asset growth, debt adjustment over time, a state-contingent bankruptcy trigger, and state-contingent conditional volatility. Expanding on the related analysis in Lucas and McDonald (2006), we incorporate a jump process, add new internal consistency checks motivated by the theoretical analysis, and investigate a wider range of parameter values, particularly the sensitivity to initial capital. The value of government insurance is calculated using a Monte Carlo simulation with risk-neutral probabilities. We also track the corresponding actual distribution of assets, liabilities, and defaults, in order to report the implied distribution of insurance payouts.

## 6.3.1 Single-Period Guarantee

We first consider the effect of a guarantee for a firm with a one-period debt contract, where a period has a length  $T$ . Consider two firms, one with insured debt and one with uninsured debt. Superscripts “ $I$ ” and “ $U$ ” denote quantities associated with the insured and uninsured firm, respectively; quantities without superscripts are the same for both. Suppose that at time 0 each promises the same debt payment at maturity  $T$ ,  $D_0(T)$ , and have the same initial value of operating assets,  $A_0(0)$ . For consistency with a multi-period model, we use the notation  $A_i(s)$  to denote the value of assets at time  $iT + s$ . In the single-period model,  $i = 0$ . The only source of uncertainty is the value of operating assets, which evolve stochastically over time.

At time 0, the equity value of the going concern is the present value of the expected payoff to equity holders. Let  $E_0[\cdot]$  denote the expectation conditional on time 0 information under the risk-neutral measure. Because both firms have the same physical assets and the same promised debt repayment, the market value of equity of both firms is:  $E_0(0) = e^{-rT} E_0[\max(0, A_0(T) - D_0(T))]$ , where  $r$  is the risk-free rate. Between times 0 and  $T$ , the equity values remain the same: both claims are a call option on the same underlying assets, with identical strike price and maturity.

Unlike for equity, the present value of the debt of the two firms prior to maturity is not equal. At any time  $t \leq T$ , the value of insured debt is simply the present value of the promised payment:  $D_0^I(t) = e^{-r(T-t)} D_0(T)$ . The realized payment on uninsured debt will be the promised amount,  $D_0(T)$ , or the asset value at time  $T$ ,  $A_0(T)$ , whichever is less. Hence the value of uninsured debt is the present value of the *expected* payment to debt holders:  $D_0^U(t) = e^{-r(T-t)} E_t[\min(A_0(T), D_0(T))]$ .

The value of the  $T$ -period guarantee made at time 0,  $G_0(0)$  is the difference between the initial value of the insured and uninsured debt:

$$(1) \quad G_0(0) = e^{-rT} D_0(T) - e^{-rT} E_0[\min(A_0(T), D_0(T))] \\ = e^{-rT} E_0[\max(D_0(T) - A_0(T), 0)].$$

The expression on the right-hand side of equation (1) is the value of a put option on the operating assets of the firm, where the strike price is the promised payment on debt. When assets are lognormally distributed, the value can be computed using the standard Black Scholes formula for a put option.

We assume that the guarantee value accrues to equity holders.<sup>6</sup> Thus at time 0, after the guarantee is announced but before debt is issued, the market

6. To the extent that Fannie and Freddie are able to act as duopolists rather than as competitors, we expect the guarantee value to accrue to their equity holders rather than to mortgage borrowers or other stakeholders. Some of the benefit may be passed to borrowers in the form of lower rates. As long as the pass-through is a constant proportion of guarantee value, the implications for imputing equity value are similar.

value of equity is  $G_0(0) + E_0(0)$ . Since we assume that the scale of operating assets is not affected by the presence of a guarantee,  $G_0(0)$  can be thought of as being immediately distributed either via a dividend, a share repurchase, or equivalently, as a reduction in the initial investment required from the original equity holders. Following the cash distribution, equity price dynamics, as described previously, are identical to that of the uninsured firm.

For the government—both from a production cost and opportunity cost perspective—the value of the guarantee is also  $G_0(0)$ . The guarantee is equivalent to the government writing a put option worth  $G_0(0)$ , and the firm would be willing to pay up to  $G_0(0)$  for the insurance.

### 6.3.2 Repeated Debt Guarantees

The debt guarantee as just modeled is static: the firm issues debt and then at time  $T$  either pays the debt in full or the government makes up the shortfall. This description of the guarantee is overly simplified along several dimensions. First, if the insured firm does not go bankrupt at time  $T$ , it will likely have the opportunity to issue additional guaranteed debt. Second, whether or not the insured firm will declare bankruptcy depends on the market value of assets, which is inclusive of current and anticipated future guarantees. Third, the insured firm may readjust its capital structure over time. For example, if assets appreciate the firm may issue more guaranteed debt, whereas if assets fall the firm may buy back some of the guaranteed debt. Such behavior will affect the value of the guarantee and its relation to the value of equity and operating assets. In this section we derive the value of a debt guarantee of an ongoing firm, taking into account these considerations.<sup>7</sup>

#### *Operating Asset Dynamics*

We distinguish between “operating assets,” which denote the financial and physical assets of a firm, and “market assets,” which in addition includes the value of credit guarantees. As in Merton (1977) and Merton (1976), we assume that the evolution of firm operating assets over time has three components: an expected return, a random component that is lognormally distributed, and (in the simulations only) a discrete jump in value. Specifically, under the risk-neutral distribution, the percentage change in assets over time is given by the process:

$$(2) \quad dA_t = (r - \lambda k - \delta) A_t dt + \sigma_A A_t dZ_t + A_t dq$$

where  $A_t$  is the asset value,  $\sigma$  is the volatility parameter,  $dZ_t$  is a Brownian motion,  $dq$  is a random variable that over the interval  $dt$  is zero with

7. For tractability, we take the risk of operating assets as exogenous, but the presence of a guarantee can also affect the characteristics and dynamics of operating assets (see Keely [1990]).



probability  $1 - \lambda dt$  and  $Y - 1$  with probability  $\lambda dt$  and  $k = E(Y - 1)$ . The  $dq$  term permits the value of assets to jump discretely with probability  $\lambda dt$  over an interval  $dt$ . The jump takes assets from  $A_t$  to  $YA_t$ , so the percentage change is  $(Y - 1)$ . Subtracting  $\lambda k$  from  $r$  corrects the drift for the average effect of jumps. Formulations like equation (2) appear regularly in the literature on debt valuation and bankruptcy.

### *Valuing a Repeated Guarantee*

Here we derive the value of a debt guarantee for a firm with a stationary target debt-to-operating asset ratio. The firm periodically issues debt to fixed, one-period maturity  $T$ , setting the amount of new debt to achieve its target debt ratio. Each period the firm also chooses whether or not to declare bankruptcy so as to maximize the value of equity. For tractability we take the target leverage ratio as given, but a similar policy could arise in response to a regulatory capital requirement, or as an optimal policy in a stationary environment in the presence of fixed adjustment costs. Also for simplicity we assume that the value of operating assets does not jump; that is,  $Y = 1$  in equation (2).

We denote the value of a quantity  $X$  at time  $mT + t$  as  $X_m(t)$ . We also denote the risk-neutral expectation at time  $mT$  conditional on information at that time as  $E_m$ . We can then express the constant target debt ratio as  $\gamma e^{rT}$ , so that for  $m = 0, 1, \dots$ ,

$$(3) \quad D_m(T) = \gamma e^{rT} A_m(0).$$

The equity value and the default decision for the guaranteed firm will depend on the expected value of current and future credit guarantees. To calculate these quantities, we need to calculate expectations conditional on future solvency. Let  $p_m^j(0)$  denote the risk-neutral probability, conditional on information at time  $mT$ , that firm  $j = \{I, U\}$  is not bankrupt at time  $(m + 1)T$ . Further, let  $\lambda_m^j(0)$  be the expectation of the asset growth rate conditional on no bankruptcy at time  $(m + 1)T$ . Then define  $\psi_m^j(0) \equiv \lambda_m^j(0) \times p_m^j(0)$  and let  $\phi_m^j(0) = e^{-rT} \psi_m^j(0)$ . In the analysis of a stationary equilibrium we drop the time subscripts. These values will depend on the specific condition in any period that determines whether the insured firm declares bankruptcy.

As in the one-period case, we compare the value of the guaranteed firm with that of a similar uninsured firm, where both have the same operating assets and target debt ratio, given by equations (2) and (3). For the guaranteed firm, the guarantee remains in place as long as the firm does not experience a default. If the firm does default, we assume that the value of future debt guarantees is lost forever to current stakeholders. To maintain equivalence of operating assets, we assume that the guarantee value, which is realized through higher proceeds at the time of each debt issue, is paid out immediately as a dividend to the equity holders of the guaranteed firm.

We denote the cum dividend equity value at time  $mT$  as  $E_{m-1}(T)$ , and the ex-dividend equity value as  $E_m(0)$ .

The one-period guarantee value, and hence the incremental dividend received by the equity holders of the insured firm, is a constant proportion,  $g$ , of asset value. This follows from the assumption that the amount of newly issued debt is a constant fraction of current asset value, and that the value of a one-period guarantee depends only on the stationary default rule of the *uninsured* firm. Using equations (1) and (3), the proportional guarantee value at a debt reset time  $mT$  is

$$(4) \quad g = \frac{G_m(0)}{A_m(0)} = \frac{D_m^I(0) - D_m^U(0)}{A_m(0)} = E_m \left[ \max \left( \gamma - \frac{e^{-rT} A_m(T)}{A_m(0)}, 0 \right) \right].$$

This can be rewritten, using the Black Scholes formula for a put option and the notation defined in equation (4) as

$$(5) \quad g = (1 - p_m^U(0)) \gamma - (1 - \phi_m^U(0)).$$

Consider a guaranteed firm, which will continue to operate until it declares bankruptcy, at a debt reset date  $mT$ . If the firm is solvent, it will issue guaranteed debt maturing at  $(m + 1)T$ . What is the solvency condition at time  $mT$  that maximizes equity value? If the firm remains in business, equity holders will receive a call option on the operating assets, and a claim to the present value of current and future dividends generated by the guarantee. Thus, equity holders will pay off the debt coming due,  $D_{m-1}(T)$ , as long as the value of operating assets plus the guarantee value exceeds the promised debt payment.

Notice that for a comparable uninsured firm, the bankruptcy condition is  $A_m(T) > D_{m-1}(T)$ . The call option on the operating assets has the same value as for the insured firm, but there is no additional value from the ongoing guarantee. Thus, there are states of the world where an insured firm continues to operate to preserve future guarantee value, but an uninsured firm declares bankruptcy. The different solvency conditions imply that the value to the firm of the current one-period guarantee,  $gA_m(0)$ , is no longer equal to the one-period production cost for the government. The former depends on the default policy of the uninsured firm, whereas the latter depends on the more conservative default policy of the insured firm. The additional losses absorbed by the insured firm's equity holders generate a commensurate reduction in cost to the government of the guarantee.

These considerations suggest that to find the value of the guarantee to the insured firm, it is convenient to characterize it in terms of two components. The first is the present value of the incremental dividend stream generated by the guarantee,  $\Gamma A_m(0)$ . On average, operating assets will grow at their expected rate conditional on the insured firm remaining solvent. Thus, the value of the dividend stream associated with the perpetual guarantee, starting with current asset value  $A$ , is:

$$(6) \quad \Gamma A = gA \sum_{i=0}^{\infty} e^{-riT} [\psi^I]^i = \frac{gA}{(1 - \phi^I)}.$$

The second component,  $HA_m(0)$ , is the cost to equity holders of paying off the debt in states of the world where an uninsured firm would declare bankruptcy. At time  $mT$ , the expected difference between  $gA_m(0)$  and the one-period guarantee production cost of the government is:

$$(7) \quad \int_{A_m(0)(\gamma e^{-\Gamma+H})}^{\gamma e^{rT} A_m} [\gamma^{rT} A_m(0) - \alpha] f(\alpha | A_m(0)) d\alpha = \eta A_m(0)$$

where  $f(\alpha | A_m(0))$  is the probability density of firm asset value at time  $(m + 1)T$  conditional on asset value the previous period, and  $\eta$  denotes the cost differential as a fraction of asset value. Like guarantee value, the present value of the cost differential depends on the expected future growth rate of assets, conditional on the probability that the firm remains solvent:

$$(8) \quad HA = \eta A \sum_{i=0}^{\infty} e^{-riT} [\psi^I]^i = \frac{\eta A}{(1 - \phi^I)}.$$

Thus, at  $mT$ , if the insured firm is solvent, its equity value exceeds that of the uninsured firm by

$$(9) \quad A_m(0)[\Gamma - H].$$

It follows that one reason previous studies that estimated subsidy cost on the basis of interest rate spreads reported higher costs than derivative-based estimates is that they implicitly set  $H$  to 0 in equation (9). The size of the bias, however, is difficult to assess. To the extent that the comparison firms were banks with subsidized federal deposit insurance and access to FHLB advances, it is not clear whether the GSEs or banks have a greater incentive to default conservatively to preserve the value of subsidized insurance.

#### *Asset Value and Volatility*

We can observe the value and volatility of market equity, dividend policy, promised debt repayment, debt maturity, and the risk-free rate, but must infer the value and volatility of assets. The problem of finding the value and volatility of market assets is conceptually similar to that considered in Marcus and Shaked (1984), who modeled the value of Federal Deposit Insurance Corporation (FDIC) insurance in a one-period setting using an options pricing model. As discussed earlier, the value of equity for the guaranteed firm is a call option on market assets, which include both operating assets, with dynamics given by equation (2), and the value of future guarantees. Using equation (9), market assets on a debt reset date  $mT$  can be written as:

$$(10) \quad A_m(0)^* = A_m(0)[1 + \Gamma - H].$$

Looking forward to the next reset date, the volatility of market assets is proportional to that of operating assets:  $\sigma_{A^*} = \sigma_A[1 + \Gamma - H]$ . Further, the continuation condition that maximizes equity value for the insured firm at each debt reset date is:

$$(11) \quad A_m(0)[1 + \Gamma - H] \geq D_{m-1}(T).$$

Then the relation between the distribution of equity returns and asset returns can be found following Merton's approach as the simultaneous solution to two nonlinear equations, but with  $A_m(0)^*$  in place of  $A_m(0)$ , and with the dividend yield,  $\delta^*$ , expressed as a share of  $A_m(0)^*$  rather than as a share of operating assets. Let  $C(A, D, \sigma_A, \delta, T)$  denote the Black-Scholes value of a European call option with underlying assets  $A$ , promised debt payment  $D$ , asset volatility  $\sigma_A$ , dividend yield on market assets ( $\delta^*$ ), and time to maturity  $T$ . Then the value of equity for an insured firm is:

$$(12) \quad E_m(0) = C(A_m(0)^*, D_m(T), \sigma_{A^*}, \delta^*, T).$$

The value and volatility of market assets is found by solving equation (12) simultaneously with:

$$(13) \quad \sigma_{A^*} = \sigma_E / (N(d_1) A_m(0)^* e^{-\delta^* T} / E_m(0))$$

where

$$(14) \quad d_1 = [\ln(A_m(0)^* / D_m(T)) + (r - \delta^* + .5\sigma_{A^*}^2)T] / (\sigma_{A^*} T^{.5})$$

$$d_2 = d_1 - \sigma_{A^*} T^{.5}.$$

Equation (13) comes from the relation,  $\sigma_E = (\partial E / \partial A)(A/E) \sigma_A$ .

### Discussion

The preceding analysis is useful for understanding the relation between the value and volatility of operating assets, equity, and a government guarantee on debt. The most straightforward conclusion that emerges is that the market value of debt plus equity exceeds the value of operating assets by the value of the present value of expected guarantee payments. Expected recoveries in the event of default, which depend only on the value of operating assets, must be adjusted discretely downward for this effect. The bankruptcy trigger must also be adjusted to take into account the effect of guarantee value on behavior. However, inferences about the volatility of operating assets made on the basis of stock price volatility, using the framework of equations (12) and (13) and using observations of equity prices on debt reset dates, are basically the same for a firm with or without a guarantee.

This analysis abstracts from what happens between reset dates. As the value of operating assets evolves, so too does the probability of solvency and the expectation of asset value on the next reset date, and hence the

expectation of the present value of future guarantees. The fixed proportionality of guarantee value to asset value at the next reset date, however, implies that the dynamics between reset dates are also unaffected by the presence of the guarantee.

In fact, government policy may not be stationary, and the value of the guarantee may be perceived by the market as changing over time with economic and political events or as a function of the financial situation of the GSEs. Whether this would make equity value more or less volatile relative to operating assets is unclear, as it would depend on the correlation between the strength of the guarantee and the objective situation of the firm, among other things. Clearly the model can be modified to take other hypotheses into account, but in its stationary form provides a neutral starting point or “guarantee irrelevance theorem” for thinking about these effects.

### 6.3.3 Monte Carlo Valuation of the Guarantee

Here we employ a discrete time version of equation (2) that is suitable for simulation. The calibrated model accommodates more complex assumptions about liability management and default behavior, and allows us to explore the effect of a variety of regulatory policies on guarantee cost.

Under a risk-neutral representation in discrete time, operating assets evolve according to:

$$(15)A_{t+h} = (1 + I_{J,t}\omega)A_t \text{Exp} \left[ \left( r_f - p_j\omega + \theta_t - \delta \frac{E_o}{A_o} - .5\sigma_A^2 \right) h + \sigma_{A,t} \epsilon \sqrt{h} \right],$$

where  $h$  is the time step,  $t$  subscripts represent time,  $E$  is equity,  $r_f$  is the risk-free rate,  $\theta_t$  is externally financed firm asset growth,  $\delta$  is the dividend yield on equity (hence  $\delta E_o/A_o$  is taken to be the dividend yield on assets),  $\sigma_{A,t}$  is the possibly time-dependent volatility of operating assets,  $\epsilon$  is a draw from a standard normal distribution,  $\omega = Y - 1$  is the nonstochastic jump size,  $I_{J,t}$  is an indicator that a jump has occurred,  $p_j h$  is the probability of a jump over an interval of length  $h$ . The actual evolution of operating assets is identical except that  $r_f$  is replaced by the expected return on assets  $r_A$ .

Here  $A_t$  represents the value of all of the firm’s operating and investment activities, both on and off balance sheet. It includes the mortgage portfolio, the MBS business, derivative market activities, and so forth. Asset value is affected by a variety of factors, including interest rate, credit, and other risks. Unhedged interest rate risk on the retained portfolio, and the associated prepayment and extension risk that arise due to the prepayment option on residential mortgages, until recently has been considered the greatest source of risk. Credit risk arises both from mortgages held on balance sheet, and from the MBS they guarantee. This risk is mitigated by the collateral value of the underlying real estate. The remaining risks—political, accounting, fraud, liquidity, model, counterparty, and so forth—are potentially important but difficult to quantify. Political risks include the possibility of leg-

isolation that restricts growth or increases competition, reducing franchise value. Accounting misrepresentations or fraud may cause downward jumps in perceived asset value, and can prolong the time between when a problem arises and is recognized, increasing the severity of losses.

Importantly, this measure of operating assets represents the true financial condition of the company, and we take it to be the recovery value in bankruptcy. The market value of assets, however, also includes the value of current and future expected guarantees,  $G_t$ . As suggested by the analysis of section 6.3.2, we assume that the guarantee value is a constant proportion of the market value of assets:

$$(16) \quad A_t^* = (1 + \Gamma - H)A_t.$$

We do not, however, attempt to identify the two components of guarantee value separately.

To summarize the different roles of operating assets and market value assets in the calibrations: operating assets are identified with the recovery value of the firm in bankruptcy. Market value assets determine the continuation condition for the firm. The procedure for setting the initial conditions identifies the initial market value of assets, as described in a later section.

### *Liabilities*

Representing debt as having a single fixed maturity, as we did in sections 6.3.1 and 6.3.2, abstracts from the possibility of more complex debt rebalancing strategies and future growth opportunities. Closed-form solutions for the value of debt under optimal or stationary debt policies have been derived for a few special cases (e.g., Leland 1994; Collin-Dufresne and Goldstein 2001), but those do not allow for state dependent changes in debt policy or continuation rules. To allow for more complicated patterns of behavior we choose instead to specify a liability process that allows for gradual adjustment of debt toward a target ratio, with asymmetry in the upward and downward speed of adjustment reflecting the relative difficulty of reducing debt when asset value falls. Book liabilities,  $L$ , evolve according to:

$$(17) \quad L_{t+h} = L_t e^{(r_d + \gamma\theta)h} + I_t \alpha_t h (\lambda^* - L_t e^{r_d h} / A_t) A_t,$$

where  $\alpha_t$  is the annual rate of adjustment, which may be state dependent,  $\lambda^*$  is the target liability to operating asset ratio, and  $I_t$  is an indicator variable that equals one in a period where liabilities are adjusted, and 0 otherwise. Liabilities grow at a rate  $r_d$  to cover promised interest.<sup>8</sup> In addition, a fraction  $\gamma$  of externally financed growth is supported by debt. This representation applies to both the actual and risk-neutral calculations, but the realized paths differ because the return on debt and externally financed growth take

8. An alternative would be to reduce assets by the amount of a periodic interest payment, which would reduce the scale of the enterprises over time relative to what is assumed here.

on different values in each instance, and the ratio of assets to liabilities displays different dynamics. Although computationally it would be straightforward to add volatility to liabilities, we assume instead that the estimated volatility of assets implicitly captures volatility arising from all sources including liabilities.

The promised interest rate,  $r_{db}$ , depends on what one assumes about the strength of the government guarantee. If it were completely firm, and abstracting from other differences between Treasuries and agency securities, then setting  $r_{db}$  equal to  $r_f$  would be appropriate. In the calibrations we assume a positive rate spread that is somewhat smaller than the average observed in the data. This is consistent with our view that the guarantee is not risky, but that there are some other features that make Treasury debt more valuable than agency securities.

### *Insolvency Trigger*

Consistent with the analysis in section 6.3.2, we assume that the solvency condition depends on the market value of assets relative to book liabilities. As in Merton (1977), we assume that bankruptcy only occurs during periodic audits. If the solvency condition is not met, the auditor closes the firm and makes a guarantee payment to debt holders.

Several insolvency triggers have been proposed in the literature. One that is roughly consistent with observed bankruptcy experience is to liquidate the firm when the market value of assets falls below the level of current liabilities, plus half of the book value of long-term liabilities. Another is that the market value of assets falls below a fraction of the total book value of liabilities. We use the latter type of rule, since distinguishing between the long- and short-term liabilities of Fannie and Freddie is complicated by the frequent maturity conversions taking place through derivatives market transactions. Given this rule, we are interested in finding the bankruptcy trigger value—the proportional gap between assets and liabilities—that maximizes the value of equity. In simulations we calculate the guarantee cost and the value of equity for a range of bankruptcy triggers and we report the results for which the value of equity is maximized.

In practice, frictions are likely to increase the guarantee cost. A drawn-out reorganization or closure process, or regulatory forbearance, can add to guarantee costs by allowing a failing firm to continue operating. This effect can be exacerbated if asset volatility increases with financial distress. This could occur, for instance, if there is a correlation between conditions that cause distress and overall market volatility; if distress raises the cost of hedging; or if management deliberately takes more risk to try to make up for past losses. This increase in volatility may not be easily discernable in historical data, both because its occurrence is a low probability event, and because it is likely to persist for relatively short periods of time when it does occur. In Lucas and McDonald (2006) we found this to be a significant potential driver of guarantee cost, and we also incorporate it into these estimates.

### *Equity*

Equations (15) and (17), which govern the evolution of firm operating assets and book liabilities, respectively, implicitly define the cash flows to equity. Those consist of the dividend payment each period, and cash raised from subsequent debt issues not used to finance exogenous asset growth. Exogenous asset growth not assumed to be debt financed further implies a negative cash flow to initial equity holders, or equivalently an equity issue.

The time 0 value of equity is the present value of all future cash flows to equity. That value is computed in the Monte Carlo simulations under the risk-neutral measure, by discounting cash flows at the risk-free rate. As a proxy for cash flows beyond the simulation horizon  $T$ , the terminal value of equity at time  $T$  is approximated by  $A_T - L_T$ . This neglects the value of the guarantee after time  $T$ , but that effect becomes small as  $T$  increases. Calculating the implied equity value using this approach provides a valuable check on the internal consistency of the model, since it can be compared to the observed equity value used to determine the initial value of assets and liabilities.

### *Deriving Initial Conditions and Accounting for Guarantee Value*

The initial market value and volatility of firm assets must be estimated since these quantities are not directly observable. The analysis of section 6.3.2 suggests that we can do this using Merton's framework, where equity can be valued as a call option on the firm's market assets. Specifically, we use equations (12) and (13), calibrated with market and balance sheet data from Fannie and Freddie, to estimate the initial market value of assets and their volatility.<sup>9</sup> What is tricky conceptually is to choose a horizon for debt, since liabilities follow equation (17) and there is no specific maturity date. We use the reported average effective maturity of debt as a proxy, and consider the sensitivity of the results to varying the assumed debt maturity.

We use the estimated asset volatility and asset value to compute the cost of the guarantee. As part of the estimation, we also compute the market value of equity for Fannie and Freddie. This later serves as an internal consistency check against the equity value derived from estimated discounted cash flows accruing to equity.

## **6.4 Calibration and Results**

The model in the base case is calibrated to year-end 2005, a time when the reported financial condition of both firms was strong. We will then look at how the estimated cost of the guarantee to the government changes as

9. We also used this approach for deriving initial conditions for asset value and volatility in Lucas and McDonald (2005). Marcus and Shaked (1984) show that the same equations can be used to estimate the value of the government guarantee, and use that insight to estimate the value of deposit insurance for US banks.



their financial condition deteriorates, and the sensitivity to other parametric assumptions and policy variables.

Three critical inputs for guarantee valuation are market value equity, equity volatility, and liabilities. Table 6.1 reports these statistics, along with the other parameters used for the base case. Data acquisition for 2005 was complicated because Fannie Mae delayed in filing financial reports since it had to restate its financial statements through 2004. As of December 2006 it had not filed any further financial reports. Fannie did, however, provide monthly information on the size of their mortgage portfolio and MBS outstanding. We have imputed some of the missing information for Fannie by relying on Freddie's disclosures. Specifically, we estimate book liabilities for Fannie and assume that the ratio of liabilities to retained mortgages is the same for both firms.

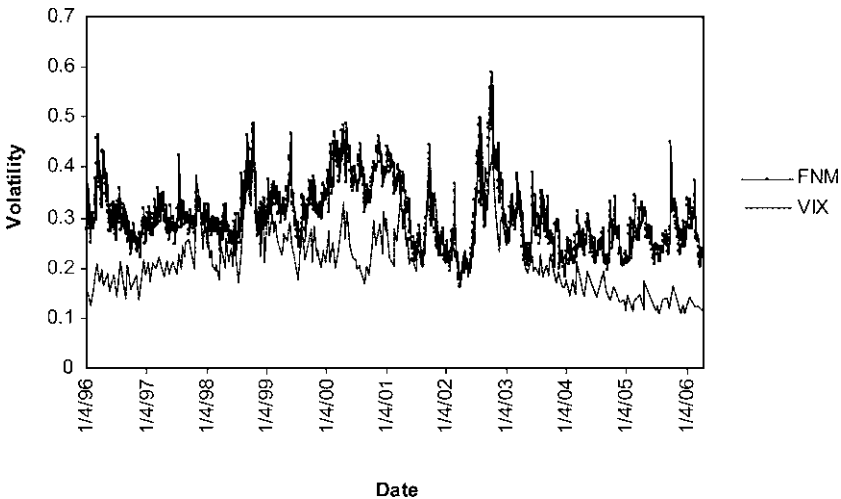
We infer base case equity volatility using historical implied annualized thirty-day volatility from option prices. The series are shown for Fannie Mae and Freddie Mac in figures 6.1 and 6.2. Both series are graphed against thirty-day implied volatility for the Standard and Poor (S&P) 500 index (the VIX index) in order to highlight changes in volatility, which are firm specific rather than market-wide. Implied volatility for both firms ranges from 20 percent to 60 percent, with an average of about 30 percent, our base estimate for both firms. Implied volatility in 2006 and 2007 remained at similar levels.

Estimates of guarantee value are based on 50,000 Monte Carlo runs, for ten- and twenty-year horizons. As in Lucas and McDonald (2006), asset volatility is assumed to increase to four times its normal level when assets fall to 101 percent of liabilities, representing increased volatility in periods of financial distress. Management and regulatory decisions (debt adjustment and solvency determination) are evaluated at a quarterly frequency, while assets returns are calculated at a monthly frequency. Several variables are parameterized differently than in our previous study. The ability to adjust down liabilities is more constrained, a change that achieves greater consistency between observed and computed equity values. We set exogenous asset growth to zero (in contrast to the 6 percent previously assumed), because it seemed in 2005 unlikely that future growth would match historical rates. Liabilities still grow on average at about 9 percent annually, however, because of the assumption that interest accumulates as increased debt and because the expected return on assets exceeds the dividend rate, creating growth from retained earnings that, on average, causes the target debt level to grow.

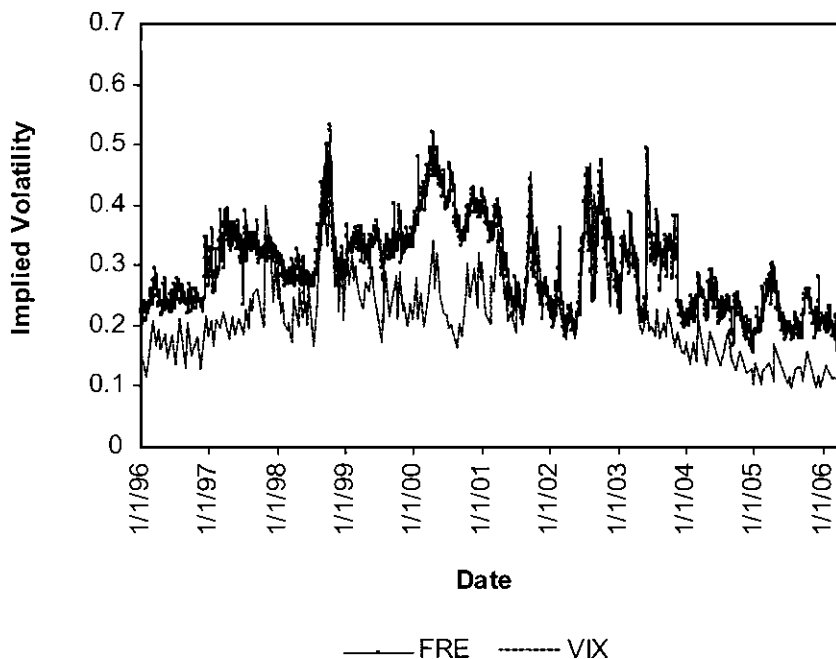
As discussed in the previous section, using equations (12) and (13) to estimate initial asset value and asset volatility is problematic because it requires a fixed debt maturity as an input. Nevertheless, it is a useful starting point for estimation. In 2004, the last year for which we have obtained average maturity data, Fannie's effective debt maturity was 2.65 years and Freddie's was 3.05 years. Since the agencies normally match the duration of assets and

**Table 6.1** Base case parameter values, year-end 2005

| Short name     | Value    | Description   |
|----------------|----------|---|
| Fannie Mae     |          |   |
| FLinit         | \$744    | Initial imputed book value of liabilities (\$ billions) |
| MVEquity       | \$48,750 | Initial market value of equity (\$ millions)            |
| Dividend yield | 0.028    |   |
| Freddie Mac    |          |   |
| FLinit         | \$727    | Initial book value of liabilities (\$ billions)         |
| MVEquity       | \$47,056 | Initial market value of equity (\$ millions)            |
| Dividend yield | 0.03     |   |
| Common values  |          |   |
| FAvol_h        | FAvol*4  | Firm asset volatility in high volatility state          |
| rf             | 0.045    | Risk free rate  |
| rd             | 0.0475   | Promised return on debt                                 |
| FAer_a         | 0.053    | Firm assets expected return (actual)                    |
| FAer           | 0.045    | Firm assets expected return (risk-neutral)              |
| FLrate_d       | 0.03 / 4 | Quarterly adjustment of liabilities to lower target     |
| FLrate_u       | 0.8 / 4  | Quarterly adjustment of liabilities to higher target    |
| growth         | 0.0      | Externally financed growth if enough capital            |
| growth_debt    | 1        | Proportion of external financing that is debt           |
| trig_volh      | 1.01     | Trigger of assets/liabilities for higher volatility     |
| look           | 4        | Frequency of checking bankruptcy trigger per year       |
| look_l         | 4        | Frequency of updating debt                              |
| FLFAtarget     | .93      | Target liability to asset ratio                         |
| newFLFA        | 1        | Proportion of debt financed exogenous asset growth      |
| nmonte         | 50,000   | Number of Monte Carlo simulations                       |
| nyear          | 10       | Number of years in each simulation run                  |
| nfreq          | 12       | Time steps per year                                     |



**Fig. 6.1** Implied volatility for Fannie Mae and for the S&P 500 (VIX), 1996 to 2006  
 Source: Optionmetrics and Yahoo.



**Fig. 6.2** Implied volatility for Freddie Mac and for the S&P 500 (VIX), 1996 to 2006

Source: Optionmetrics and Yahoo.

liabilities, it seems likely that their effective maturity of debt increased over the next year with the lengthening effective maturity of mortgages. Table 6.2 illustrates the effect of the debt maturity assumption on implied asset value and volatility, using the parameter assumptions in table 6.1, for maturities of 2.5, 5, and 7.5 years. Both implied asset value and volatility increases with assumed debt maturity. Although the increases appear small in percentage terms, model estimates are very sensitive to assumed asset volatility, and hence to the initial maturity assumption.

Table 6.3 reports the guarantee and equity values in the base case with no jumps in asset value, and using initial conditions assuming Fannie's (Freddie's) effective debt maturity is 2.65 (3.05) years. The default trigger is that which maximizes the value of equity. The combined guarantee value over twenty years is \$65 billion. The guarantee value expressed as a premium rate on liabilities is 23 to 27 bps. For both firms, the implied equity values are somewhat higher than the observed values used to estimate asset volatility and value, but small changes in parameters (e.g., volatility) can easily reconcile the equity values.

We also report the risk-neutral and actual probabilities of default over the indicated horizon. The risk-neutral probability is inferred from observed prices and model assumptions. If the assets have a positive risk premium,

**Table 6.2** Implied asset value and volatility as function of assumed debt maturity

|               | Fannie | Freddie | Fannie | Freddie | Fannie | Freddie |
|---------------|--------|---------|--------|---------|--------|---------|
| Horizon (yrs) | 2.5    | 2.5     | 5      | 5       | 7.5    | 7.5     |
| Asset value   | 797.0  | 778.4   | 800.4  | 781.2   | 803.1  | 780.9   |
| Asset vol     | .0208  | .0185   | .0225  | .0204   | .0238  | .0230   |

**Table 6.3** Base case 2005 guarantee value estimates

|                                    | Fannie | Freddie | Fannie | Freddie |
|------------------------------------|--------|---------|--------|---------|
| Horizon                            | 10     | 10      | 20     | 20      |
| Guarantee cost (\$ billions)       | 14.46  | 9.16    | 35.49  | 29.50   |
| Premium rate (bp)                  | 20.53  | 16.46   | 27.01  | 22.91   |
| Implied equity value (\$ billions) | 49.99  | 45.40   | 55.78  | 48.73   |
| Default prob. (risk-neutral)       | 0.19   | 0.18    | 0.34   | 0.34    |
| Default prob. (actual)             | 0.050  | 0.033   | 0.084  | 0.059   |
| Default trigger (L/A)              | 1.08   | 1.07    | 1.13   | 1.11    |

the risk-neutral probability is an upper bound on the physical probability of default. Identifying a physical probability of default requires making an additional assumption about the required rate of return on assets. We follow Lucas and McDonald (2006) and assume a required rate of return on assets 80 basis points greater than the risk free rate in the base case. Given the implied physical probabilities, we can also compute value at risk (VaR). Under the base case assumptions, we compute a VaR over twenty years at the 5 percent level for Fannie (Freddie) of \$165 billion (\$112 billion). At the 1 percent level the VaR increases to \$252 billion (\$201 billion).

Table 6.4 shows the effects of exogenously varying the default trigger rather than setting it at a value maximizing level as in the base case in table 6.3. As the default trigger increases from 1.0, there is a rapid increase in equity value and an increase in the premium rate. (With continuous monitoring and no bankruptcy costs, at a trigger of 1.0 there would be a zero default premium paid on bonds because bondholders would have 100 percent recovery. Because bankruptcy can only occur at discrete times, bondholders do on average suffer a loss when bankruptcy occurs.) As the trigger increases from 1.0, equity values increase, the premium rate increases, and the probability of bankruptcy declines. The last two observations are reconciled by the greater severity of defaults when the trigger level is higher.

Since the true risk premium associated with assets is subject to considerable uncertainty, table 6.5 reports, for the base case for Fannie, the sensitivity of the reported actual bankruptcy probability to the assumption about

**Table 6.4** Effect of changing the bankruptcy trigger

| Trigger            | Equity value | Risk-neutral<br>bankruptcy<br>probability | Actual<br>bankruptcy<br>probability | Premium rate (bp) |
|--------------------|--------------|---|-------------------------------------|-------------------|
| <i>Fannie Mae</i>  |              |   |                                     |                   |
| 1.00               | 42.57        | 70.26                                     | 24.83                               | 14.17             |
| 1.03               | 48.85        | 58.12                                     | 17.84                               | 21.17             |
| 1.06               | 52.79        | 48.58                                     | 13.68                               | 24.74             |
| 1.09               | 54.87        | 41.42                                     | 10.98                               | 26.42             |
| 1.12               | 55.69        | 35.85                                     | 9.04                                | 27.01             |
| 1.15               | 55.60        | 31.27                                     | 7.48                                | 26.90             |
| <i>Freddie Mac</i> |              |   |                                     |                   |
| 1.00               | 38.24        | 67.38                                     | 17.11                               | 11.57             |
| 1.03               | 43.85        | 54.33                                     | 11.77                               | 18.09             |
| 1.06               | 47.01        | 44.57                                     | 8.67                                | 21.22             |
| 1.09               | 48.41        | 37.49                                     | 6.86                                | 22.65             |
| 1.12               | 48.71        | 31.75                                     | 5.51                                | 22.83             |
| 1.15               | 48.17        | 27.22                                     | 4.45                                | 22.46             |

*Note:* All parameters are those given in table 6.1.

**Table 6.5** Sensitivity of physical default probability to asset risk premium (Fannie Mae)

| Risk premium (bp) | Physical bankruptcy probability |
|-------------------|---------------------------------|
| 0                 | 34.22                           |
| 20                | 25.83                           |
| 40                | 18.83                           |
| 60                | 12.99                           |
| 80                | 8.40                            |
| 100               | 5.22                            |
| 120               | 3.11                            |
| 140               | 1.73                            |

*Note:* All parameters are those given in table 6.1 with a twenty-year horizon.

the risk premium on assets. When the risk premium is zero, the bankruptcy probability is the same as the risk-neutral default probability reported in table 6.3, and when the risk premium is 80 basis points, it is the same as the actual default probability reported in table 6.3. It is important to keep in mind that our cost estimates of the credit guarantees do not depend on the assumption about the risk premium.

Next, we consider the effect of discrete jumps down in asset value, where trend growth is adjusted up so that average asset growth is the same as the table 6.3 calculations. The probability of a jump is taken to be 3 percent per year, and the jump size is 5 percent. The results are reported in

**Table 6.6** 2005 Guarantee value estimates with jumps

|                                    | Fannie | Freddie | Fannie | Freddie |
|------------------------------------|--------|---------|--------|---------|
| Horizon                            | 10     | 10      | 20     | 20      |
| Guarantee cost (\$ billions)       | 16.54  | 19.30   | 38.25  | 32.32   |
| Premium rate (bp)                  | 23.62  | 13.30   | 29.23  | 25.23   |
| Implied equity value (\$ billions) | 51.80  | 47.37   | 58.96  | 51.90   |
| Default prob. (risk-neutral)       | .21    | .19     | .34    | .34     |
| Default prob. (actual)             | .068   | .053    | .112   | .090    |
| Trigger                            | 1.09   | 1.08    | 1.14   | 1.12    |

*Note:* 3 percent annual probability of 5 percent reduction in asset size.

table 6.6. The effect is to increase the probability of default and the value of the guarantee by \$10 to \$20 billion. Increasing the size of the jump to 10 percent increases the twenty-year cost for Fannie to \$43.8 billion, but also increase the equity value to \$65.8 billion, significantly higher than its observed value. It appears that plausible jump processes increase estimated cost, but not enough to reconcile options-based and spread-based cost estimates.

Options-based estimates of guarantee value are quite sensitive to the assumed initial value of assets. In the months prior to Fannie and Freddie being put into receivership, their stock prices fell substantially and the underlying asset value for each firm clearly declined as well. It is interesting to see how the inferred guarantee value changes when the underlying assets suffer a loss. To illustrate the sensitivity to changes in initial leverage ratios, table 6.7 reports on guarantee values as a function of the initial ratio of market liabilities to market assets, holding other parameters the same as in the base case. In this table we simply reduce assets, holding asset volatility constant, and examine the effect on the value of equity and the insurance value.<sup>10</sup>

The optionality inherent in being an equity holder can be seen by considering the change in equity value as a function of the decline in asset value. A 5 percent decline in the value of assets for Fannie is about \$40 billion. The decline in imputed equity value is about half of that when assets decline by the first \$40 billion, and less than a third of that amount when assets decline by an additional \$40 billion. The increase in the guarantee cost for the two scenarios is substantial, at \$15 billion and \$37 billion. The physical default probability also increases at an increasing rate with a drop in asset values. Similar results, not reported here, are obtained for Freddie.

10. An alternative approach would be to use observed equity value and volatility during the summer of 2008. However, Fannie and Freddie were not typical defaulting companies, and there was great uncertainty about whether, when, and how the federal government would intervene. This uncertainty makes it problematic to interpret observed market volatility.

**Table 6.7** 2005 Guarantee value estimates, varying initial equity for Fannie

|                                    | Fannie | Fannie<br>-5% assets | Fannie<br>-10% assets |
|------------------------------------|--------|----------------------|-----------------------|
| Horizon                            | 20     | 20                   | 20                    |
| Guarantee cost (\$ billions)       | 35.49  | 50.84                | 72.50                 |
| Premium rate (bp)                  | 27.01  | 44.64                | 80.81                 |
| Implied equity value (\$ billions) | 55.78  | 36.02                | 23.84                 |
| Default prob. (risk-neutral)       | .34    | .43                  | .56                   |
| Default prob. (actual)             | .084   | .175                 | .394                  |

## 6.5 Conclusions

In this chapter we develop a valuation model for a firm that can continue to periodically issue insured debt that is a fixed percentage of the value of its operating (i.e., nonguarantee) assets as long as it remains solvent. We use the model to explore whether the presence of such a guarantee changes the relation between the equity value of the firm, and the value of operating assets. This is important because in derivative-based approaches to valuing debt guarantees, the unobservable value and volatility of assets is inferred from the observable value and volatility of equity. If the presence of the guarantee changes these relations (for instance, by affecting equity dynamics), the inferences could be biased.

The theoretical analysis reveals that in fact, the presence of the guarantee does not fundamentally change the relation between the volatility of levered equity and the underlying assets, leaving intact the standard equations underlying derivatives-based pricing. It does, however, create a wedge between the value of operating assets and the market value of debt and equity equal to the present value of the future stream of income generated by the guarantee. This affects the initial conditions for derivatives-based estimates. The analysis also reveals that the spread-based approach is upwardly biased when no correction is made for the lower predicted default rate for guaranteed firms that optimally default less often to preserve the value of future guarantees.

To provide estimates that take into account these adjustments and that also incorporate potentially important complications such as jumps in underlying asset value, time-varying volatility, and a more complicated default policy, we calibrate and simulate a computational version of the model. We find that an insurance premium of 20 to 30 bps on Fannie and Freddie debt would be fair compensation for the default risk assumed by the government in the benign economic environment of year-end 2005. However, when asset values decline by 10 percent, it causes the fair premium to more than double, all else equal. This highlights the sensitivity of guarantee

values to changes in equity value in highly levered financial institutions, and also demonstrates the usefulness of these types of models in setting risk-based insurance premiums.

## References

- Ambrose, B., and A. Warga. 2002. Measuring potential GSE funding advantages. *Journal of Real Estate Finance and Economics* 25 (2/3): 129–50.
- Carnell, R. S. 2005. Handling the failure of Fannie Mae and Freddie Mac. Conference on Receivership Powers, American Enterprise Institute, Discussion Draft.
- Collin-Dufresne, P., and R. S. Goldstein. 2001. Do credit spreads reflect stationary leverage ratios? *Journal of Finance* 56 (5): 1929–57.
- Congressional Budget Office (CBO). 2001. Federal subsidies and the housing GSEs. Available at: <http://www.cbo.gov/doc.cfm?index=2841>.
- . 2004. Updated estimates of the subsidies to the housing GSEs. Available at: <http://www.cbo.gov/doc.cfm?index=5368>.
- . 2008. Cost estimate for H.R. 3221, Housing and Economic Recovery Act of 2008, July 2008. Available at: <http://www.cbo.gov/ftpdocs/95xx/doc9597/hr3221.pdf>.
- Crosbie, P., and J. Bohn. 2003. Modeling default risk. Moody's KMV White Paper. Available at: <http://www.moodykmv.com/research/whitepaper/ModelingDefaultRisk.pdf>.
- Demsetz, R., M. Saidenberg, and P. Strahan. 1996. Banks with something to lose: The disciplinary role of franchise value. *Economic Policy Review* 2 (2): 1–14.
- Feldman, R. 1999. Estimating and managing the federal subsidy of Fannie Mae and Freddie Mac: Is either task possible? *Journal of Public Budgeting, Accounting, and Financial Management* 11 (Spring): 81–116.
- Frame, W. S., and L. J. White. 2005. Fussing and fuming over Fannie and Freddie: How much smoke, how much fire? *Journal of Economic Perspectives* 19 (2): 159–84.
- Hubbard, R. G. 2004. The relative risk of Freddie and Fannie. *Fannie Mae Papers* 3 (3)
- Jaffee, D. 2003. The interest rate risk of Fannie Mae and Freddie Mac. *Journal of Financial Services Research* 24 (1): 5–29.
- Keeley, M. 1990. Deposit insurance, risk, and market power in banking. *American Economic Review* 80 (5): 1183–200.
- Leland, H. E. 1994. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance* 49 (4): 1213–52.
- Lucas, D., and R. McDonald. 2006. An options-based approach to evaluating the risk of Fannie Mae and Freddie Mac. *Journal of Monetary Economics* 53 (1): 155–76.
- Marcus, A., and I. Shaked. 1984. The valuation of FDIC deposit insurance using option-pricing estimates. *Journal of Money, Credit and Banking* 16 (4): 446–60.
- Merton, R. C. 1974. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29 (2): 449–70.
- . 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3 (1): 125–44.
- . 1977. An analytic derivation of the cost of loan guarantees and deposit



- insurance: An application of modern option pricing theory. *Journal of Banking and Finance* 1 (1): 3–11.
- Naranjo, A., and A. Toevs. 2002. The effects of purchases of mortgages and securitization by government sponsored enterprises on mortgage yield spreads and volatility. *Journal of Real Estate Finance and Economics* 25 (September-December): 173–95.
- Nothaft, F. E., J. E. Pearce, and S. Stevanovic. 2002. Debt spreads between GSEs and other corporations. *Journal of Real Estate Finance and Economics* 25 (September-December): 151–72.
- Passmore, W. 2005. The GSE implicit subsidy and the value of government ambiguity. *Real Estate Economics* 33 (3): 465–86.
- Seiler, R. S. 2003. Market discipline of Fannie Mae and Freddie Mac: How do share prices and debt yield spreads respond to new information? Office of Federal Housing Enterprise Oversight Working Paper no. 03-4.
- Sharpe, W. F. 1976. Corporate pension funding policy. *Journal of Financial Economics* 3 (3): 183–93.
- Stiglitz, J. E., J. M. Orszag, and P. Orszag. 2002. Implications of the New Fannie Mae and Freddie Mac risk-based capital standards. *Fannie Mae Papers* 1 (2): 1–10.

## **Comment**      Alan J. Marcus

This chapter is a valuable contribution to the literature on the too-often unacknowledged growth in explicit and implicit government guarantee programs. While the Federal Government did not originally provide explicit “full faith and credit” backing of Fannie Mae and Freddie Mac debt, its implicit support was, as Lucas and McDonald (henceforth, LM) point out, widely acknowledged and visibly apparent in the yields at which the two firms were able to issue their bonds. More recently, of course, that guarantee became explicit.

Lucas and McDonald estimate the ex-ante present value in 2005 of the combined guarantee to the two firms at around \$65 billion over twenty years. This is a considerable amount; moreover, as a mean, it is actually a conservative estimate of the government’s potential exposure. A value-at-risk estimate focusing on bad- or worst-case scenarios obviously would be multiples of this value. When comparing this implicit guarantee with some of the others discussed in this volume, it is good to remember that the risks of some programs have been assessed by worst-case scenarios and others by midpoint estimates. By either standard, Freddie-Fannie guarantees must be ranked among the more important contingent government obligations, and a careful demonstration of this point is by itself an important contribution of the chapter.