Volume Title: Measuring Business Cycles

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Volume Publisher: NBER

Volume ISBN: 0-870-14085-X

Volume URL: http://www.nber.org/books/burn46-1

Publication Date: 1946

Chapter Title: Plan of Treating Secular, Seasonal and Random Movements

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Chapter URL: http://www.nber.org/chapters/c2982

Chapter pages in book: (p. 37 - 55)

CHAPTER 3

Plan of Treating Secular, Seasonal and Random Movements

I The 'Cycle of Experience' as the Unit of Analysis

In *Business Cycles: The Problem and Its Setting* various methods of 'eliminating' seasonal variations and secular trends from time series were described. Since the publication of that book in 1927, the technical devices available to the investigator of business cycles have multiplied abundantly. In the present studies we might utilize such of these tools, old and new, as approve themselves to our judgment, and begin the analysis of cyclical behavior by striving for the completest attainable isolation of cyclical fluctuations.

We do not follow that plan. In the first place, the isolation of cyclical fluctuations is a highly uncertain operation. Edwin Frickey once diligently assembled 23 trend lines fitted by various investigators to pig iron production in the United States, and found that some of the trend lines yield cycles averaging 3 or 4 years in duration while others yield cycles more than ten times as long.[1] This range of results illustrates vividly the uncertainty that attaches to separations of trends and cycles, though it perhaps exaggerates the difficulties. If an investigator fits a trend line in a mechanical manner, without specifying in advance his conception of the secular trend or of cyclical fluctuations, he may get 'cycles' of almost any duration. But an informed investigator who is seriously studying cycles of a given order of duration will use whatever guidance he can get from history and statistics; he will scrutinize the movements of the original data, seek to mark off in advance the cycles or traces of cycles that

[1] The Problem of Secular Trend, *Review of Economic Statistics*, Oct. 15, 1934; see also Ch. 7-8 below.

correspond to his basic conception, then choose a trend line that cuts through and exposes the cycles in which his interest centers. Yet this procedure also illustrates the difficulties of segregating trends and cycles. For it leaves room for choice of the trend line, the method of fit, and the method of trend elimination. Further, it makes the trend depend upon the cycles, and may not lead to the discovery of cycles that are obscured by the trend. To judge what features of the final result are merely technical and what features are a significant characteristic of the series is likely to require considerable testing and experimenting even on the part of a skilled technician.[2]

It is fairly common for statisticians to assume that the elimination of the secular trend from a time series indicates what the course of the series would have been in the absence of secular movements, and that the graduation of a time series, whether in original or trend-adjusted form, indicates what the course of the series would have been in the absence of random movements. There is no warrant for such simple interpretations. A 'least squares' trend line fitted, for example, to grocery chain store sales in the United States may move majestically on a chart, but the analytic significance of the trend line is obscure. At least some of the 'growth factors' impinging on this branch of business—the addition of meats and vegetables to the grocery line, the rise of supermarkets, special taxes on chain stores—have made their influence felt spasmodically. When a continuous 'trend factor' is eliminated from the data, it is therefore difficult to say what influences impinging on the activity have been removed and what influences have been left in the series. Cyclical graduations are no easier to interpret than trend adjustments. Systematic smoothing of a time series will, indeed, eliminate short-run oscillations produced by random factors; but can it eliminate the influence of powerful random factors—such as a protracted strike, or a succession of bad harvests, or a great war?

There is always danger that the statistical operations performed on the original data may lead an investigator to bury real problems and worry about false ones. When new commodities, new techniques of production, new methods of organizing business, new methods of financing are first introduced on a substantial scale, they affect the general business situation more profoundly than at a later time when they have fully penetrated the economic system and become a part of routine experience. For example, railroad investment in the United States shows long leads at cyclical revivals during the eighteen seventies and eighties; as the decades roll on the leads tend to become shorter, disappear, and finally are replaced by lags. A fact of this sort is of considerable importance

[2] See the interesting study of this problem by Edwin Frickey, *Economic Fluctuations in the United States* (Harvard University Press, 1942). Cf. Arthur F. Burns, Frickey on the Decomposition of Time Series, *Review of Economic Statistics*, Aug. 1944.

historically, for it suggests that railroad investment in this country gradually shifted from an 'active' to a 'passive' role in the process of recovery from business depressions.[3] It is also of theoretical significance, so far as it suggests a point that may be true generally of 'new' versus 'old' industries. But this point is likely to be lost or blurred when the secular trend is completely removed from the data, since that operation tends to standardize the features of successive cycles.[4]

If these observations are well founded, it follows that in attempting to explain business cycles, we should work with cyclical measures that take account of secular trends, and also of substantial random movements. The historical records of processes that represent volume of business, as distinct from prices, usually appear as a series of expansions followed by sharper and briefer but less considerable contractions. Also the few dwindling processes in business do not show a steady movement, but brief and mild expansions followed by larger contractions. Our aim in developing a technique for analyzing cyclical behavior has been to reproduce the *form of development* common to the industrial activities of nations whose economic life has been organized on a business basis. Hence we make no attempt to adjust for that portion of the secular trend which falls within the limits of a single cycle; but we do adjust the original data of time series for seasonal variations, and make allowance in the course of the analysis for the minor oscillations that diversify the cyclical expansions and contractions of economic processes.

As explained in the preceding chapter, most of our cyclical measures are based upon entries for three or more months instead of single months. This practice tends to moderate the influence of erratic movements on the results, and so too does the device of averaging the cyclical measures for all the cycles covered by a series. Our practice of first breaking the seasonally adjusted data of a series into specific cycles and then turning the data included within each of these segments into percentages of their average value during the segment, eliminates the portion of the secular trend that represents shifts in the level of the series from cycle to cycle, but retains the portion that lies within the limits of a specific cycle.[5] A similar result is obtained by breaking a series into reference cycles and then converting the data within each of these segments into reference-cycle relatives.[6] On the whole these statistical procedures yield fairly realistic pictures of the 'cyclical units' of economic experience in modern

[3] See Ch. 10, Sec. VIII.

[4] See Ch. 7, especially Sec. VI. Also, Joseph A. Schumpeter, *Business Cycles* (McGraw-Hill, 1939), Vol. I, Ch. V.

[5] This procedure implies that if the secular trend were represented instead by a continuous line, that line would be a flexible curve cutting through successive specific cycles. Cf. Arthur F. Burns, *Production Trends in the United States since 1870* (National Bureau of Economic Research, 1934), Ch. II.

[6] See Charts 14 and 18.

nations, and afford clues to the actual behavior of men from stage to stage of business cycles.

We treat these units of experience in seasonally adjusted form. Seasonal fluctuations vary endlessly from one business activity to another, but they are a comparatively regular factor within each activity. Since the seasonal pattern, by and large, is much the same in years of 'good' business and years of 'bad' business,[7] our analysis of cyclical movements can be facilitated by putting the seasonal fluctuations provisionally out of sight. The effects on business enterprises of an increase in activity that is expected to last at most a few months are very different from an increase that is expected to continue for years. For one thing, seasonal increases do not lead men to expand their investments as does rapid secular growth. To understand changes in contracting for the construction of new plants, ordering of industrial equipment, issuing of corporate securities, and the like, it is not necessary to differentiate activities that reach their seasonal peaks in January, April, and November; but it is necessary to differentiate activities that, seasonal variations aside, are growing, remaining constant, and shrinking. The business community 'allows for' seasonal fluctuations deliberately or tacitly, and we merely follow suit by erasing as well as may be their influence on time series.[8]

The larger the seasonal fluctuations the more essential is this step, both practically and theoretically. For example, the prices and inventories of many farm products are highly unsteady; but the wide fluctuations are rooted principally in seasonal factors, not in business cycles. If we ignored this knowledge and marked off their specific cycles on the basis of the original figures, we should conclude that their cyclical amplitudes are among the largest in economic records. That conclusion would rest on measures that reflect mainly the amplitude of the seasonal movements, and would be seriously misleading.

II Limitations of the Technique

Our technical procedures, designed as they are to aid in the explanation of the business cycles of experience, have certain disadvantages for the more modest task of describing the business-cycle behavior of individual time series. If secular trends were eliminated at the outset as fully as are

7 Since 1940, seasonal fluctuations have practically disappeared in numerous branches of industrial activity in this country. Such a result is to be expected when an industry is operating over a protracted period at 'full capacity'. But protracted plateaus of industrial output are not typical of cyclical fluctuations: as a rule, once output stops expanding, a decline sets in rather promptly.

8 Of course, no exact correspondence is implied. Our seasonal correction for a given year is usually based on the data of that year and of several preceding and following years. But when a businessman 'corrects' for the seasonal movement in recent months, he has only past experience to guide him. Again, the net result of seasonal adjustments applied to the sales of individual firms in an industry may differ materially from the seasonal adjustment applied to a single series representing aggregate sales of the industry.

seasonal variations, we would show that business cycles are a more pervasive and a more potent factor in economic life than our results indicate. For when the secular trend of a series rises rapidly it may offset the influence of cyclical contractions in general business, or make the detection of this influence difficult. In such instances our method may indicate lapses from conformity to contractions in general business, which would not appear if the secular trend were removed. Even when secular trends do not obscure the specific cycles, our cyclical patterns include an element of trend that confuses somewhat the 'cyclical component' proper. Another troublesome factor is the persistence of what seem to be random movements in the reference-cycle patterns of some series. If we could eliminate random movements from the original data we would probably find that our series on the whole conform more closely to business cycles than the present results indicate.

But we are not without safeguards against being misled by these shortcomings of method. Thus, when the secular trend rises with exceptional swiftness, we may put aside the early portions of a series, or recognize diminutive declines as contractions of specific cycles. Our index of business-cycle conformity is designed to show whether the rate of increase in a series is less rapid, or the rate of decrease more rapid, during a contraction in general business than during the next preceding and following expansions. Also we make measures that throw into relief the element of trend retained in the cyclical measurements. Those specific cycles which we know are dominated by random forces we exclude from the averages. We believe that these safeguards and certain others, combined with the checks and counterchecks that our many measures for each series and for related series make possible, overcome to a considerable extent the deficiencies of our technique for describing cyclical behavior.

III Need to Economize Effort

Doubtless the ideal procedure would be to make two sets of measures for each series: one set based on the original data adjusted only for seasonal variations, as is our present practice, the other based on the best attainable isolation of the 'cyclical component' of the data. But the resources at our disposal place grave obstacles to the realization of this ideal.

The isolation of cyclical components is a very costly operation. To assure sensible results, painstaking study of each series and considerable experimentation are likely to be necessary. If we attempted a full analysis, first, of the original data adjusted merely for seasonal variations, second, of data adjusted for secular trend and random movements as well as seasonal variations, we would be able to analyze comparatively few series, perhaps less than a tenth of the number we can analyze by our simple method. This restriction of the coverage would doom our efforts to lay a thorough factual foundation for the explanation of business cycles. As explained in Chapter 1, our working definition of business cycles makes necessary extensive observation on the cyclical behavior of economic activities. These activities embrace at least the production of commodities, construction work, transportation, pricing, carrying stocks of commodities, marketing, foreign commerce, getting and spending personal incomes, making business profits, saving, investing, borrowing, trading in securities, the circulation of money, and banking. Each of these broad processes must be divided into several or many parts, and for each subdivision enough time series are needed to show what cyclical behavior, or what varieties of cyclical behavior, are characteristic.

We might have limited the sample to perhaps a hundred series by working with broad aggregates or index numbers. But experience early convinced us that this labor-saving shift would not do. Though comprehensive series reveal certain facts that might otherwise escape notice, and we analyze many series of this type, they hide differences among their constituents in respect of cyclical timing, duration, amplitude, pattern, and conformity—differences many of which seem highly significant for the understanding of business cycles. To get a clear view of the cyclical behavior of economic activity, it is necessary to go back of broad aggregates or index numbers to the series from which they are made.

In studying an economic process in detail, for example, the production of textiles, the construction of buildings, the prices of foodstuffs at retail, interest rates, we often find marked divergences in cyclical behavior and so must analyze a considerable number of series. If another group of activities behaves so similarly that any one series might be accepted as representative of the group, that important fact can be established best by making several sets of measurements and comparing them. Doubts concerning the trustworthiness of the original data add weight to these considerations. To dispel or confirm doubts we look for series compiled by two or more authorities, or series that record different aspects of the same activity. In so doing we are guided by the belief that comparison of the results yielded by different series relating to a given group of activities is the surest way of judging whether the measurements represent typical characteristics of cyclical behavior.[9] For example, we cannot be certain that an analysis of wheat harvests in the United States will give a representative picture of the cyclical behavior of agricultural production. To make sure, we examine also wheat crops in other countries, add studies of other crops, and investigate the output of animal husbandry. Our simple analysis of some forty series showing production by farmers, supplemented by numerous series on the acreage planted and the acre-yields of leading crops, also by the prices, sales, stocks, exports and imports

[9] Cf. Ch. 12, Sec. V.

of farm products and their processed derivatives, gives more trustworthy knowledge of the relation of agriculture to business cycles than could be achieved by expending equal effort on two sets of cyclical measures—one based on the 'original' data and the other on the best attainable isolation of the cyclical component—for perhaps only ten or twelve series.

Our desire to cover as many cycles as possible in each of several countries makes still more formidable the task of a double analysis. That desire has grown stronger as we have become increasingly familiar with the behavior of time series. "Strictly speaking, every business cycle is a unique historical episode, differing in significant ways from all its predecessors, and never to be repeated in the future."[10] The like is true of every specific cycle in every economic activity. It is part of our task to learn the respects in which and the degree to which business cycles and specific cycles vary: in particular, whether there are secular and perhaps cyclical changes in their duration and intensity, as well as the irregular variations that everyone recognizes.[11] Obviously, long series are necessary for such studies. And we can no more discover what are the uniform than what are the variable characteristics of our phenomena unless numerous instances are observed.

The bulk of materials required by our concept of business cycles not only rules out two analyses of each series; it also puts a premium upon simplicity in analysis. Fortunately the measures made from data adjusted only for seasonal variations promise to be more useful in explaining business cycles than would measures made from highly fabricated data. The force of this statement will become clearer after the reader has studied Chapters 7-8, where the influence of trend adjustments and smoothing operations upon our measures of cyclical behavior is analyzed;[12] but its full justification must await the theoretical analysis of the final volume.

IV Treatment of Seasonal Variations

While we consider it desirable to economize effort in handling secular trends and random movements, experience has taught us not to economize effort by working with annual data. As Chapter 6 shows in detail, annual data are exceedingly crude materials for comparing the cyclical behavior of different activities in the same period or of the same activity in different periods. They obscure timing relations, they make it impossible to trace cyclical patterns with confidence, often they obscure and sometimes they obliterate cyclical fluctuations. For these reasons our

10 See the section on The Distinctive Character of Each Business Cycle, in Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 354-7.

11 This problem is tentatively considered in Ch. 9-12.

12 Cf. also Ch. 10, Sec. VIII.

analysis, excepting crop harvests, is based chiefly on monthly and quarterly data; annual data are used only as a last resort.

Monthly and quarterly records are more difficult to compile and to analyze by our standard technique than annual data; they also impose the burden of removing seasonal fluctuations. A preponderant number of monthly and quarterly series show an unmistakable seasonal swing, that is, a repetitive intra-annual fluctuation. Although techniques of seasonal measurement have been greatly improved in recent years, the problem of adjusting time series for seasonal variations remains very troublesome in practice. Hard decisions must be made at every turn: Is the series characterized by a genuine seasonal movement? If definite evidence of seasonality does exist, how should it be measured? What period should the computation cover? Should a constant or a shifting seasonal index be constructed? If a shifting seasonal index seems preferable, may it remain constant over short segments of the data or should it change from year to year? Should the seasonal index be of the 'absolute' type, of the 'relative' type, or some cross between the two? If a relative seasonal index is used, by what method should it be constructed? Should the index be adjusted, and in what manner, for variations in seasonal amplitude? After these questions have been settled, it is still necessary to decide how to remove the seasonal variations and, most important of all, to judge the success of the seasonal adjustment.

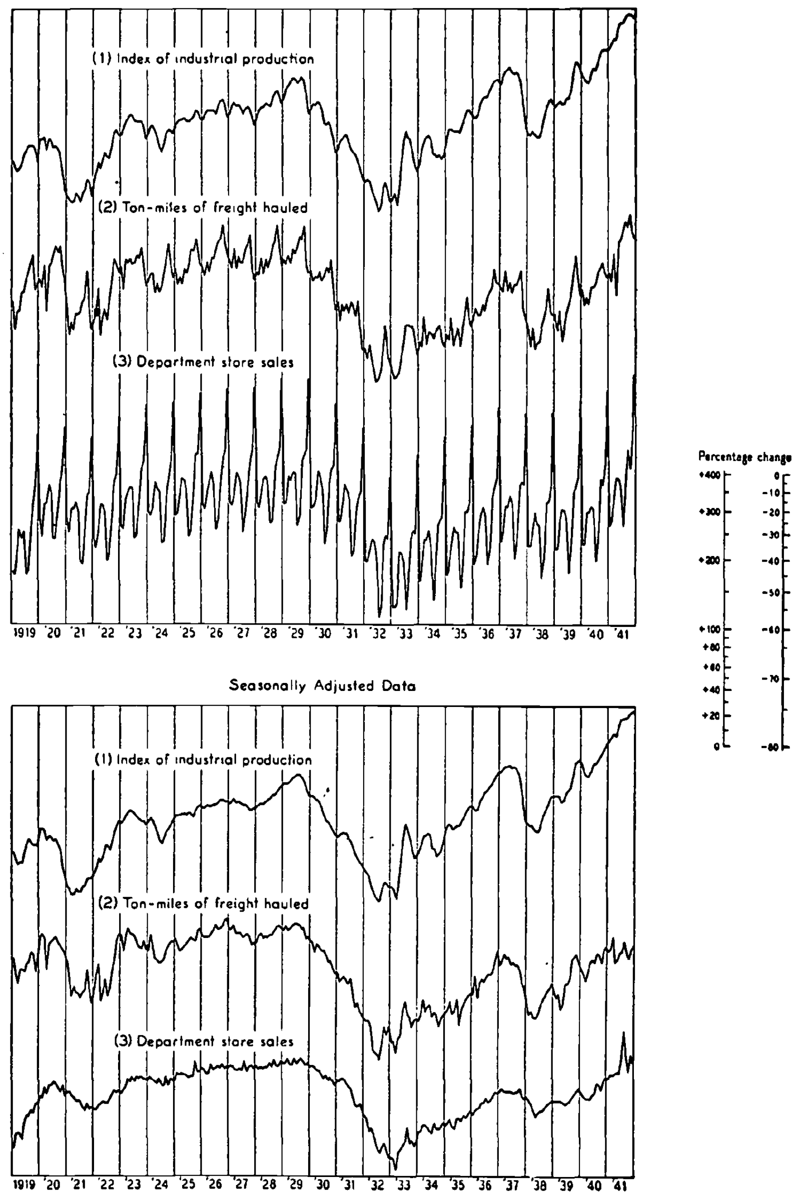
A statistician who has struggled with seasonal adjustments of numerous time series is not likely to underestimate the part played by 'hunch' and 'judgment' in his operations. If pressed hard, he may admit that what he does, and not too satisfactorily in many instances at that, is merely to erase the repetitive intra-annual tendency that he observes in the original figures. Seasonally adjusted figures no more show what the behavior of time series would have been in the absence of seasonal forces than indexes of physical production show what production would have been in the absence of changes in relative prices; or than figures on per capita income show what income would have been in the absence of income inequalities. But although it is difficult to give a satisfactory theoretical interpretation of seasonal adjustment, this operation like any other can be subjected to a pragmatic test. In the last resort, its significance must be judged by the results to which it leads. That the removal of seasonal variations can facilitate analysis of the interrelations of cyclical movements in different activities will be plain to any reader who takes the trouble to study Chart 3, which depicts the movement in recent years of industrial production, railroad freight traffic, and department store sales in the United States, before and after adjustment for seasonality. The seasonal variations follow an individual course in each series, and obscure the protracted fluctuations in freight traffic and department store sales. When the variations associated with the seasons of the year are removed, the underlying fluc-

CHART 3

Original and Seasonally Adjusted Data

Three American Series, 1919 – 1941

Original Data



For sources of data, see Appendix C.

tuations in economic fortune come clearly to the surface and can be readily traced in the several series.

The seasonal variations of some series are removed by their compilers, but in most instances we have had to carry through this operation ourselves. This step in the analysis precedes all others and often takes as much time as the subsequent operations put together. We have measured seasonal variations by a variety of methods, partly because different series require different types of adjustment, partly because our methods changed as we gained experience in using them. A brief description of the methods we currently use will have to suffice; they differ little from the techniques explained in treatises on statistics.[13]

In ascertaining the seasonal variations of monthly data we use two principal methods. Method (1) consists in taking averages of the original figures for successive Januaries, for successive Februaries, and for each of the other months, then adjusting these monthly averages for secular trend. Method (2) entails taking a twelve-month moving average of the original figures, placing each average in the seventh month of shifting twelve-month intervals, expressing the original figures as ratios to the 'centered' moving averages, and striking averages of these ratios, first for the Januaries, then the Februaries, and so on.[14] In applying each method we attempt to protect the twelve monthly averages from distortion by extreme items. Sometimes we compute two or three sets of positional means and select that set which includes the largest number of items without including any extreme item; at other times we drop extreme items or years at the start—that is, before striking averages. In either case the twelve monthly averages are adjusted so that their sum equals 12.[15] When the data come by quarters, the general procedure is the same, but the sum of the values of the seasonal index equals 4. When it is obvious that the amplitude of seasonal fluctuations is more nearly constant in absolute than in percentage units we vary the method; that is, the final seasonal index is expressed in units of the original data, the algebraic sum of the twelve monthly terms being zero.

Our two principal methods of constructing seasonal indexes rest on the same logic: to ascertain the seasonal variations of a time series, all nonseasonal movements must be eliminated. By averaging the arrays for successive Januaries, Februaries, and so on, we allow the random move-

13 See also Mitchell, *Business Cycles: The Problem and Its Setting*, pp. 233-49.

14 In practice we use ratios to twelve-month moving totals instead of to moving averages. The two yield the same final results, but the former is a more economical method of calculation. A thirteen-month moving average centered on the seventh month, the first and last months receiving half weight, is preferable in principle to a simple twelve-month moving average; but experiments by one of our colleagues, Julius Shiskin, have demonstrated that the gain is negligible and definitely is not worth the extra cost. Even a simple four-quarter moving average centered on the third quarter is not appreciably inferior to a five-quarter moving average in which the two end quarters receive half weight each.

15 See Note 1 of the Appendix to this chapter.

ments of a series to cancel one another. The larger the number of observations the better are the chances that the random movements will cancel out in fact; hence extreme values are dropped only when there is fairly clear indication that they distort the averages. Each method rests on the assumption that the process of averaging will tend also to make the cyclical components of a series sum to zero. This desideratum is likely to be met better by method (2) than by method (1); for in (2) we average values each of which contains, as a rule, merely a small portion of the cyclical component, while in (1) we average values that contain the full cyclical component. In applying each method we analyze periods covering whole years instead of periods covering whole cycles. The former practice reduces the chances of eliminating the cyclical component when method (1) is used, but it helps to prevent distortion of the seasonal index by the secular component of the series. In method (1) the secular component is eliminated by adjusting the month-by-month averages of the original data for the average monthly increment of a linear trend; while in method (2) the adjustment for trend is automatically accomplished by expressing the original data as ratios to twelve-month moving averages. The elimination of the secular movement is likely to be more successful in method (2), which takes account of a trend of any degree of flexibility.

Method (1) is obviously less laborious, but method (2) is superior on most other counts. Hence we confine method (1) to series having seasonal variations that seem both clear-cut and fairly constant over many years. When seasonal variations are obscured by other types of fluctuation, or when they seem to change materially over time, we begin the seasonal analysis by expressing the original figures as ratios to twelve-month moving averages. The ratios are then studied with a view to finding periods, if any, during which the seasonal variations were fairly uniform. The periods selected rarely cover less than eight or more than fifteen years. The operations called for by method (2) are then performed separately upon each segment. The final seasonal measurements sometimes seem excellent, more often they are merely tolerable. For some series the best seasonal indexes we can make after much labor are poor approximations. Especially troublesome are the cases in which the timing of what seem to be seasonal peaks and troughs varies irregularly from year to year by a month or two.

In some series the pattern of seasonal variations shifts from one year to the next in regular fashion; in some others the pattern of what seems to be the seasonal movement remains constant but the amplitude varies sharply from year to year. When series of these types are encountered, we do not construct 'constant' seasonal indexes. To take care of the first group we compute 'moving' seasonal indexes; to take care of the second we compute seasonal indexes that are constant in pattern but vary in amplitude from year to year.

Our method of constructing 'moving' seasonal indexes consists of the following steps: (1) a twelve-month moving average of the original data is taken and placed in the seventh month; (2) the original monthly figures are expressed as ratios to the centered moving averages; (3) the ratios for successive Januaries are arranged in chronological order, so also are the ratios for the Februaries and the other months; (4) these figures are plotted on twelve diagrams, one for each month; (5) a moving average, usually covering five points, is taken of the ratios for successive Januaries, and likewise for each of the other months; (6) the moving averages for the Januaries are plotted on the diagram showing the ratios for successive Januaries, and the moving averages for other months are plotted on the other monthly diagrams; (7) the lines of the moving averages are smoothed freehand and are extended to cover the initial and the terminal years for which there are no moving averages; (8) the values for the months of each year are read from the smoothed curves; (9) if the sum of the monthly values for each calendar year is not equal to 12, the smoothed curves are experimentally adjusted to produce this result.

The series in which the amplitude of the seasonal swings varies sharply and irregularly from year to year are treated on a different plan. The first step is to make a 'constant' seasonal index by one of the methods described above. The seasonal observations [16] for a given year are then adjusted so that their sum is 12. Next, the constants of a 'least squares' straight line are computed for the year, the seasonal index being treated as the independent variable and the adjusted seasonal observations for the year as the dependent variable. Finally, monthly values for the year that correspond to the monthly values of the seasonal index are computed from the equation of the straight line; this step yields a seasonal index having the same pattern as the constant seasonal index but a different amplitude. These operations are repeated for each year covered by the series, or each year of that portion of the series subjected to a seasonal correction of shifting amplitude.

This refinement upon current practice has been employed extensively by Simon Kuznets,[17] who refers to the slope of the straight line just described as the 'amplitude ratio'. As Kuznets points out, the amplitude ratio rests on the hypothesis that the nonseasonal movements within each year are uncorrelated with the seasonal movements. The amplitude ratio cannot be used properly except when the seasonal observations for a given year are positively correlated with the constant seasonal index; we do not employ it when the coefficient of correlation between the two is less

[16] That is, the values averaged to make the seasonal index: the original data in method (1), ratios to moving averages in method (2).

[17] See his Seasonal Pattern and Seasonal Amplitude: Measurement of Their Short-time Variations, *Journal of the American Statistical Association*, March 1932; and *Seasonal Variations in Industry and Trade* (National Bureau of Economic Research, 1933), pp. 322-42.

than +.70. The amplitude ratio may break down completely, as when the calculated seasonal index for some month is negative.[18] Even over the range for which the amplitude ratio is serviceable, it makes merely approximate adjustments possible. We believe, nevertheless, that if used judiciously, this device improves the results.[19]

Having constructed by one or another method measures of seasonal behavior, we proceed to free the series from their seasonal variations. As a rule we follow the standard procedure; that is, the original figure for each month is divided by the seasonal index for the month.[20] This operation implicitly assumes that the seasonal correction, regarded as an additive term, depends not only on the trend and cyclical components but also on the random component—an assumption not easy to justify and one that sometimes leads to very poor results. To meet this difficulty, we occasionally substitute for the standard method of eliminating seasonal variations another method based on the assumption that the seasonal and random components are independent. This method involves (1) multiplying a centered twelve-month moving average of a series by its seasonal index, month by month, (2) subtracting these products from the original figures, (3) adding the resulting differences to the twelve-month averages. The first step estimates the nonrandom component of the series, the second estimates the random component, the third estimates the nonseasonal component by combining the trend-cycle and random components.[21] The results yielded by this method will not diverge appreciably from those obtained by using the standard method of eliminating seasonal variations, except when the seasonal and random movements are both very large.

After the laborious process of eliminating the seasonal variations from a time series has been carried out, two tests are applied before we are content to leave the matter of seasonality. First, the mean annual 'level' of the seasonally adjusted data is compared with that of the original data. If a discrepancy of more than 10 per cent turns up in any twelve-month interval, we consider the adjustment unsatisfactory and resort to new

18 This difficulty, however, may be avoided by making the computation in terms of logarithms.

19 The case for applying an amplitude ratio is clearest when the activity represented by a series has a 'natural' business year, with a definite beginning and end, as in movements of products from farms. The amplitude ratio should then apply to the natural year. In the absence of a natural year, there is no basis other than convention for selecting the boundaries of the year; the amplitude ratio and the final seasonal adjustment will then vary with the boundaries selected.

In some series, what look like enormous differences in seasonal amplitude are confined to the values in the immediate vicinity of the seasonal troughs or the seasonal peaks and are not characteristic of the month-by-month differences. If the troughs vary considerably in intensity from year to year, but not the peaks, the simple procedure described in Note 2 of the Appendix to this chapter (method B) is likely to yield better seasonal adjustments than do 'amplitude ratios'.

20 This is the standard procedure for a 'relative' seasonal index. In the case of an 'absolute' seasonal index (one expressed in units of the original data, the algebraic sum of the twelve monthly terms being zero), the index for each month is subtracted from the original value for the month.

21 For a fuller description, see Note 2 of the Appendix to this chapter.

devices, such as computing the seasonal index on the basis of a slightly different period, replacing a constant seasonal index by a changing seasonal index, substituting our modified method of removing seasonal variations for the 'standard' method, or discarding a seasonal index expressed in percentage units in favor of a seasonal index expressed in units of original data. Second, the adjusted figures are scrutinized to see whether they still show similar monthly movements in successive years, or movements correlated with the seasonal index.[22] If not, we deem the operation successful and pass on to the measurement of cyclical behavior. If clear traces of seasonal variations do remain, we must find what is wrong or deficient in the calculation and try again. Perhaps the correction for secular trend was too mechanical; perhaps method (1) was applied to data that call for the more elaborate treatment of method (2); perhaps the need of working with shorter periods was overlooked, or the need of moving seasonal indexes or seasonal indexes of constant pattern but varying amplitude. A second or third trial usually yields results we are willing to accept. But there remain instances in which we feel far from satisfied with the seasonal indexes as we have left them.

To free time series from seasonal variations is not the sole use to which we put indexes of seasonal variations. At a later stage we plan to compare seasonal movements with cyclical behavior. Meanwhile our worksheets show the average amplitude of the seasonal movements during the period covered by a series. The average amplitude is measured in two ways: by the range [23] and by the average deviation of the seasonal index. When more than one seasonal index is computed for a series, we take a weighted mean of the average amplitudes of whatever seasonal indexes are used, the weights being proportionate to the number of years to which each seasonal index is applied. In the few instances where the seasonal index is expressed in units of the original data, it must be converted into percentage units before calculating the average amplitude.

[22] See Note 3 of the Appendix to this chapter.

[23] Strictly speaking, we take the rise of the seasonal index from trough to peak, plus the fall from peak to trough. This equals twice the range. Of course, the erratic component that remains in the seasonal index tends to increase the range.

APPENDIX

Notes on the Elimination of Seasonal Variations

Note 1

The reasons for adjusting a monthly seasonal index so that the sum of its twelve terms equal 12 should be made explicit. This note attempts to show why the sum is 12, or 1200 in percentage terms, rather than some other number.

Let o_{ij} and n_{ij} stand respectively for the original and seasonally adjusted figures in the i^{th} month of the j^{th} year, and let s_i stand for the seasonal index in the i^{th} month.

In the standard method of removing seasonal fluctuations, the original figure for a month is divided by the seasonal index for the month; that is, $n_{ij}=\frac{o_{ij}}{s_i}$, or $o_{ij}=n_{ij}s_i$. The mean value of the original figures during any twelve-month interval is $\frac{\Sigma o}{12}$ or $\frac{\Sigma ns}{12}$. The mean value of the seasonally adjusted figures during the year is $\frac{\Sigma n}{12}$. The relation between the two means is as follows:

$$\frac{\Sigma ns}{12}\cdot\frac{12}{\Sigma s}=\frac{\Sigma n}{12}+\frac{r_{ns}\sigma_n\sigma_s}{\bar{s}} \tag{1}$$

Here σ_n is the standard deviation of n during the year, σ_s the standard deviation of the seasonal index, r_{ns} the coefficient of correlation between n and s during the year, and $\bar{s}$ the mean value of the seasonal index.[24]

As may be seen from the above expression, $\frac{\Sigma ns}{12}=\frac{\Sigma n}{12}$ if two conditions are fulfilled: (i) $r_{ns}=0$, (ii) $\Sigma s=12$. Now, if seasonal variations have been eliminated properly, we should expect n and s to be uncorrelated. In any given year their numerical values may be correlated positively or inversely. But it seems clear, *a priori,* that in any given series the average value of r_{ns} (over a moving twelve-month interval) will closely approximate zero. The average value of r_{ns} for many series will approximate zero more closely still. It follows that the sum of the seasonal index must equal 12 in order that the *expected value* of the sum of the seasonally adjusted figures for a year equal the sum of the original figures for the year.[25]

[24] G. U. Yule, *An Introduction to the Theory of Statistics* (Charles Griffin, London, 1927), p. 221.

[25] If an 'absolute' seasonal index is used, $n=o-s$. For any year, $\frac{\Sigma n}{12}=\frac{\Sigma o}{12}-\frac{\Sigma s}{12}$. Hence if Σs is made 0, $\frac{\Sigma n}{12}$ will equal $\frac{\Sigma o}{12}$.

Note 2

Call the standard procedure of eliminating seasonal variations 'method A', and the alternative procedure sketched in the text 'method B'. Let the seasonally adjusted figures be represented by n_{ij} when method A is used and by n'_{ij} when method B is used.

Method A assumes that $o_{ij} = n_{ij}s_i$.

Method B assumes that $o_{ij} = m_{ij}s_i + R'_{ij}$, where m represents a composite of the trend and cyclical components of the series, and R' the random component. A rough estimate of m can be obtained by taking a centered twelve-month moving average of the original figures and that is the method we use in practice; but the argument that follows does not depend on this or any other specific method of estimating m. We assume R' to be independent of s. Whether or not R' should also be treated as independent of m is critical for the problem of obtaining an optimum estimate of s. But we are not concerned with that problem at present; we take s as given, and merely consider the consequences of alternative methods of removing s.

From the above definitions it is evident that $n_{ij} = \frac{o_{ij}}{s_i}$, and that

$$n'_{ij} = m_{ij} + R'_{ij}.$$

There is an advantage in writing these expressions in another form:

$$n_{ij} = \left[\frac{o_{ij}}{s_i}(1 - s_i)\right] + o_{ij} \tag{2}$$

$$n'_{ij} = \left[m_{ij}(1 - s_i)\right] + o_{ij} \tag{3}$$

The bracketed expressions in (2) and (3) show the seasonal *correction*, treated as an additive term, in methods A and B respectively. The implicit seasonal *component* is obtained by reversing the sign of the bracketed expressions. Clearly, the seasonal correction in method B is proportional to the trend-cycle component alone, while in method A it is proportional to the seasonally adjusted figure which includes the random component besides the trend-cycle component.

These relations can be put another way. The random component is the seasonally adjusted figure minus the trend-cycle component. Hence in method B the random component $(R'_{ij}) = n'_{ij} - m_{ij} = o_{ij} - m_{ij}s_i$. In method A the (implicit) random component $(R_{ij}) = n_{ij} - m_{ij} = \frac{o_{ij} - m_{ij}s_i}{s_i}$. Algebraically, R_{ij} is the seasonally adjusted equivalent of R'_{ij}; but it should be noted that the seasonal (s) applies to plus and minus values.

We may, finally, write n and n' as follows:

$$n_{ij} = R_{ij} + m_{ij} = \left(\frac{o_{ij} - m_{ij}s_i}{s_i}\right) + m_{ij} \tag{4}$$

$$n'_{ij} = R'_{ij} + m_{ij} = (o_{ij} - m_{ij}s_i) + m_{ij} \tag{5}$$

From these expressions it is plain that the results yielded by methods A and B cannot differ much when the seasonal variations of a series are relatively small. Even when the seasonal variations are large, the results will tend to be similar so long as the random variations are small. But when the seasonal and random variations are both large, n and n' may diverge sharply. If $s_i < 1$, the variance of R_i will exceed the variance of R'_i, and R_{ij} will be greater or less than R'_{ij} according as R'_{ij} is plus or minus; the larger the gap between s and 1, the larger will be these differences. On the other hand, if $s_i > 1$, the variance of R_i must be smaller than the variance of R'_i.

The basic assumption underlying method B is that the random component is independent of s. If this condition is fulfilled, the random component will tend to be a larger part, in percentage terms, of the original values at months of seasonal trough (and immediately adjacent months) than of the original values at months of seasonal peak (and immediately adjacent months). The original values at months of seasonal trough will therefore tend to vary more, logarithmically, than the original values at months of seasonal peak. Hence, if the seasonal fluctuations of a particular series are large, and the logarithms of its original figures at months of seasonal trough vary much and irregularly from year to year while the logarithms of the figures at peaks are relatively stable, method B is preferable to method A despite the added burden of arithmetic calculations. In such a series the application of method A is bound to introduce a seasonal factor into the seasonally adjusted figures, the imprint of s on n being positive in some years and inverted in others. We have found that in series of this type the seasonal adjustment obtained through method B is ordinarily a considerable improvement upon the results got by method A; for the application of method B yields, in effect, an amplitude correction.[26]

Whether method A or B is applied to a given series, the 'level' of the seasonally adjusted figures will normally differ from the 'level' of the original figures; that is, the mean of seasonally adjusted figures for any twelve-month period will normally differ from the mean of the original figures. But method B has a stronger tendency to satisfy the 'equality condition' than method A. If method A is used, $\frac{\Sigma o}{12} - \frac{\Sigma n}{12} = r_{ns}\sigma_n\sigma_s$. This follows from equation (1), since $\bar{s} = 1$. If method B is used, $\frac{\Sigma o}{12} - \frac{\Sigma n'}{12} = \frac{\Sigma ms}{12} - \frac{\Sigma m}{12}$. But since $\bar{s} = 1$,

$$\frac{\Sigma ms}{12} = \frac{\Sigma m}{12} + r_{ms}\sigma_m\sigma_s.$$

where σ_m is the standard deviation of m for a twelve-month interval and r_{ms} the coefficient of correlation between m and s during this interval. Hence $\frac{\Sigma o}{12} - \frac{\Sigma n'}{12} = r_{ms}\sigma_m\sigma_s$. But $|r_{ns}\sigma_n\sigma_s|$ will tend to exceed $|r_{ms}\sigma_m\sigma_s|$ for two reasons: (i) σ_n is likely to be larger than σ_m since n includes the random component while m is a smoothed variable; (ii) inasmuch as m is smooth, low values of $|r_{ms}|$ are likely to rule, while the erratic values of n will now and then produce high values of $|r_{ns}|$.

[26] One technical disadvantage of method B (shared by an absolute seasonal) is that it *may* yield negative figures.

An empirical test demonstrates that method B approximates the 'equality condition' far better than method A. We applied the test to eight series, five of which were chosen because very large inequalities were known to result occasionally from the application of method A. In each series a (moving) twelve-month interval was taken as the unit of comparison. Method A yielded inequalities of 10 per cent or more in 159 instances, of 25 per cent or more in 52 instances, of 50 per cent or more in 19 instances; the largest inequality that turned up was 67 per cent. Method B yielded only eight inequalities of 10 per cent or more, the maximum discrepancy being 11.8 per cent.

A word may be added about the relevance of the 'equality condition'. The problem has been largely ignored by statisticians, and as yet has not entered the stage of open debate. But one way or another, the equality condition is involved in much of the work done on seasonal measurements. As explained in Note 1, this condition underlies the practice of making $\Sigma s = 12$. Again, the equality condition is the only logical defense for not recognizing a seasonal problem in work on annual figures. The statistician who disavows this condition has a seasonal problem on his hands even when he is concerned exclusively with the behavior of annual figures. For from his point of view, it may be desirable to remove seasonal variations from monthly or quarterly figures, and operate with annual sums or averages of the seasonally adjusted figures instead of annual sums or averages of the original figures.

If the 'equality condition' were made a crucial test of the goodness of seasonal adjustment, the standard method of removing seasonal variations (method A) would be ruled out from the start. That would be embarrassing in practice, since 'relative' seasonals usually give satisfactory results when judged by other criteria. The method we have followed represents a compromise, which seems wise in the present rough state of our knowledge of seasonal variations. Other things equal, we regard a seasonal adjustment as satisfactory in the degree to which the equality condition is fulfilled. But there is a point beyond which we are not willing to tolerate differences in the 'level' of a series produced by seasonal adjustments. Our limit of 10 per cent for a twelve-month interval is one of those practical rules for which no more can be said than that it is convenient, leads to recomputation infrequently, and yet yields results that usually look sensible.

Note 3

A common method of judging the goodness of a seasonal adjustment is to see whether the adjusted figures show similar movements in successive years. If a positive (or inverse) correlation exists between n and s, year by year, over the entire period covered, or if the correlation is positive at one end of the period and inverse at the other, it is plain that the seasonal adjustment is defective.[27] But it is more difficult to judge the seasonal adjustment if n and s do not behave in this fashion, yet are correlated in individual years. Some correlation, positive or inverse, is practically bound to exist; that is why the 'equality condition' is not met by method A (see the preceding note). The correlation

[27] Here n stands for seasonally adjusted data, irrespective of the method used in their derivation.

may result from a poor seasonal adjustment but that need not be the case: there is no good reason for assuming that n and s are invariably uncorrelated in single years.

It may be well, therefore, to put the trend-cycle component out of sight, and to judge the goodness of a seasonal adjustment by comparing s with an estimate of the random component of the series. In any single year the patterns of s and of the random component may be correlated; but that is not very likely to happen. Of course, the 'isolation' of the 'random component' is beset with grave difficulties. But if several methods of seasonal adjustment are tried, there will be as many (explicit or implicit) estimates of the random variations. The behavior of these estimates may then be compared with s, year by year, and any correlation between the two noted. The emergence of such correlations must count against the method that yields them.

Other things equal, that method of seasonal adjustment which involves the 'best' estimate of the random component may be regarded as yielding also the 'best' estimate of the nonseasonal movement of the series. For example, methods A and B, described in the preceding note, involve different estimates of the random component. Now, if R were truly a random series, we should expect the variance of R_i to equal the variance of R_{i+1}, of R_{i+2}, etc. The like applies to R'. (The variances should be measured from zero.) In any actual case the twelve variances of R will almost certainly not be the same, nor will the twelve variances of R'. But a preference may be expressed for R or R' according as the one or the other shows substantially smaller variability, when judged, say, by the coefficient of variation of the twelve variances.

This note is intended merely to be suggestive. The problem raised requires careful exploration.