Volume Title: Input-Output Analysis: An Appraisal

Volume Author/Editor:

Volume Publisher: Princeton University Press

Volume ISBN: 0-870-14173-2

Volume URL: http://www.nber.org/books/unkn55-2

Publication Date: 1955

Chapter Title: The Nature and Uses of Interindustry-Relations Data and Methods

Chapter Author: W. Duane Evans, Marvin Hoffenberg

Chapter URL: http://www.nber.org/chapters/c2865

Chapter pages in book: (p. 53 - 136)

The Nature and Uses of Interindustry-Relations Data and Methods

W. DUANE EVANS AND MARVIN HOFFENBERG

DIVISION OF INTERINDUSTRY ECONOMICS, BUREAU OF LABOR STATISTICS

A. Conceptual Framework

The national economy exists primarily to supply goods and services required for some individual or collective human purpose. In a complex modern economy, the task of assembling and processing goods destined for ultimate use has been divided among many industries. Some of these industries have little direct contact with the ultimate purchasers and users of finished products. Rather, they exist to perform specialized tasks for other processing industries. Nevertheless, many of the limitations on the size and the potentialities of an existing economic system are related directly to conditions in such industries. For many important national economic problems, it is essential to establish consistent connections between demand for finished products, on the one hand, and the implications of this demand for production, employment, capacity utilization, and resource use levels of industries that may be significantly though remotely involved, on the other. For example, a decision to produce additional munitions (which are end products) will affect not just the industries that assemble and deliver these items, but also in

The authors were initially requested to prepare two papers for the Conference. The first was to cover, in a general fashion, "The 1947 Input-Output Study" and to serve as a conspectus and background for other papers, which would examine specific problems in greater detail. However, the authors had recently published a paper with almost the same title ("The Interindustry Relations Study for 1947," *The Review of Economics and Statistics*, May 1952), and it was felt that this, together with other material being presented to the Conference, would meet the intended purpose. Accordingly, this earlier article was designated as a background paper for the Conference, taking the place of the projected first paper.

The article in *The Review of Economics and Statistics* covers more than a brief description of the conceptual and practical problems of the 1947 study. It includes a short historical note, comments on the input-output approach, discussions of computation problems, associated data requirements, areas of use, probable lines of development, etc. Some overlapping with the present paper is inevitable, but an effort has been made to keep it to a minimum. It is suggested that the present paper be read subsequent to the first.

References to other publications have in general been omitted from the present paper; however, a list of references to selected publications is appended.

some way virtually every other industry in the economy as well. The question as to whether any comprehensive addition to current schedules of demand will produce overloads, in terms of manpower, capacity, or resource limitations at critical points, cannot be answered unless some consistent connection can be found between an addition to the demand schedule and its impact throughout the economic system.

A direct attack on the problem is always available. Before a prospective increase in finished-goods deliveries, one may consult with the industries that perform the final processing steps to obtain estimates of the additional requirements that the increase will impose on other sectors of the economy. The immediate suppliers of these industries can then be visited for similar information. Going on to earlier stages, one can trace the connections systematically and cumulate the total impact.

This direct attack on the problem is obviously correct and proper, and, within the limits set by ability to define the problem adequately, to carry through the procedure, and to obtain accurate estimates, it will produce the answers desired. It is equally obvious that it will be time-consuming, expensive, and unwieldy. Few individual problems have such intrinsic importance as to justify the labor required; nevertheless, there are such problems and the approach has, in fact, been used. It has been implicit in much industrial mobilization analysis.

1. THE BASIC CONCEPT

The interindustry-relations approach is in concept very closely related to the direct attack described above, but it permits the capitalization of an expenditure for empirical research over a large number and variety of analytical applications. It is based on the observation that a sector's purchases of materials, components, or services from other parts of the economy are commonly related to that sector's output or production. Empirical considerations suggest that these functional relationships are not unworkably complex, and that institutional and technological influences impose on them a degree of temporal stability. In other words, and in a rather oversimplified form, a basic concept of the input-output approach is that in many cases the pattern of goods and services needed to carry on a given productive activity is identifiable through empirical research, exhibits strong elements of stability, and hence is useful for a variety of analytical purposes.

This basic concept is so simple in character and contains so much common sense that it has, in fact, been used by practical people from time immemorial. It is probably the principal tool used in all but a small fraction of planning for business production and purchasing. The automobile assembler who notes that a finished passenger car requires five wheels, five tires, and twenty-one wheel nuts, and balances his future production and purchasing plans accordingly, is applying a fundamental principle of the input-output approach.

The principle is relatively new only as a formal tool in quantitative national economic analysis. As an unformalized tool it has undoubtedly been used again and again. For example, it was used in the early thirties in an analysis of the relative employment-generating effects of direct work relief versus public-construction projects. The Works Projects Administration, in setting up its projects, required that the bulk of the funds granted be used directly to hire workers and closely limited expenditures for overhead and materials. The Public Works Administration, also with the intent of creating employment, engaged primarily in various forms of construction activity. These entailed substantial charges for materials, and hence the direct employment-generating effects per dollar of expenditure were less than those of work relief projects.

There was some controversy as to the relative employment-creating potentialities of these two forms of federal expenditure. The PWA pointed out that, while project employment per million dollars spent was less than for work relief projects, purchases from the depressed construction-materials industries created other employment opportunities, and that these spreading effects should be considered if any comparison were to be made.

The Construction Division of the Bureau of Labor Statistics was requested to establish the approximate indirect employment effects of federal public-construction expenditures. A sample of construction projects was chosen, and the man-years of on-site employment and the pattern of materials purchases for the projects were recorded. A sample of firms in the construction-materials industries was then selected. These firms were requested to supply estimates of the man-years of employment required in their plants, and the amounts and kinds of materials purchases from other firms per million dollars of construction materials delivered. The estimates were carried back in this way for a number of stages, and then cumulated to indicate the number of man-years of employment directly and indirectly created by a given type and amount of federal public-construction expenditure.

The procedure was adopted, not simply as a reasonable method, but as the only method available for attacking the specific problem. The approach lacked the conceptual completeness and many of the empirical cross-checks of current input-output analysis, but the basic idea was essentially the same.

As a conceptual scheme for considering interindustry balances within a national economy, the input-output approach might initially have been suggested by the simple observation that many physical interrelationships between industries are very nearly proportional for substantial periods of time. For example, the pounds of cotton fiber required for a square yard of broadcloth or the amount of iron ore needed to make a ton of pig iron would change very little from year to year. Hence, one might reasonably expect cotton purchases by the textile industry or iron ore usage by blast furnaces to vary proportionately with the production levels of these industries. As a purely conceptual exercise, one might assume that all purchases of materials and services among the industries of an economy obey rules of simple proportionality and then examine the properties and characteristics of a system where such relationships prevail.

Even though such an approach may have elements of oversimplicity, it also has points of strength as a framework for at least the coarser types of national economic analysis. And because of its simplicity, this formalization has very definite suggestive and heuristic value. However, even in its earlier stages, input-output analysis included additional elements of empirical sophistication.

2. EMPIRICAL DATA IN A CONCEPTUAL FRAMEWORK

Regrettably, some critics have seemed to view the input-output approach as if no broader conceptual foundation than that of general proportionality could be found for it, or as if there had been no conceptual development in the field since the approach was first suggested as a method for national economic analysis. It has even been suggested that this entire field represents only a sort of glorified exercise in accounting, quite ignoring the fact that much of the work has been purely conceptual and that the empirical work has been to give necessary content to a conceptual scheme. A science that purports to deal with the real world but that ignores its empirical and observational side is likely to appear a rather empty and unproductive discipline. A major virtue of the interindustry-relations approach is that it does not deal with empty boxes but permits the constant interplay between concept and measurement that is still conspicuously absent from many fields of economics, though a commonplace in the physical sciences.

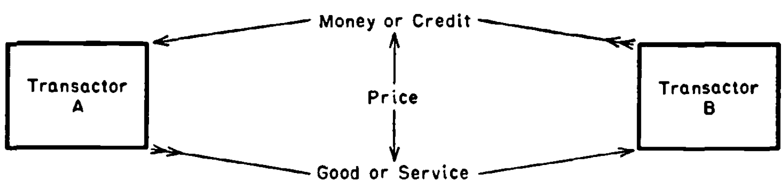
It is perhaps unfortunate that much of the discussion of the conceptual underpinnings of the approach has occurred primarily among the rather heavily burdened people working in the field and that it has not been readily accessible to those with a more general interest. Accordingly, it may be useful at this point to review briefly some of these more general ideas, at least as they are understood by the authors. It cannot be supposed that all people working in the field will view these matters in the same way.

Primarily, we are concerned with a framework for national economic analysis, in quantitative terms, that will be not only comprehensive but also sufficiently refined to distinguish a large number and variety of macroscopic details. To put this another way, there will clearly be concern with such aspects of the national economy as total employment, gross production, national income, and general price levels. It is also clear that for many problems there will be concern with such details as textile production and employment, electric power production, steel requirements, and construction activity. The first set of variables covers what might be called general business conditions. The second deals with characteristics for particular sectors or industries of the economy. These two sets of variables are clearly interdependent. General business conditions will affect the level of operation of the steel, textile, and power industries, and the influence will be reciprocal. Some method of analysis is required that will take account of the interrelationships among all these variables in a reasonable and consistent fashion. Since the approach is quantitative in intent, it must be linked to measurable and, more particularly, to customarily measured aspects of the economy.

Examination of a national system reveals that it embraces not only economic activity but many other types of activity as well. There is need, therefore, to establish a frame of reference. While economics may be defined as the study of the material means of satisfying human desires, this definition is rather abstract, and in the present context something more operational in content will be desirable.

The transaction provides us with a fundamental observable unit in the field of economic behavior. Economic behavior usually acquires observable content only when a transaction takes place. The notion that the transaction sets a limit between economic and non-economic behavior is embodied, at least in part, in our statistical system. For example, construction by a householder of a piece of furniture for home use does not in itself affect the national income or gross national product figures, nor is the householder imputed to have had an increase in income on which he must pay a tax. However, if the householder engages someone else to perform the task for him, or if he sells the product of his labor to someone else, the transaction, at least in concept, becomes part of the estimate of national production and income. It is even difficult to define the result of some economic activities in the absence of a transaction. For example, consider the artisan who invests his time in what he hopes will be accepted as a work of art. When the time comes to sell it, he may discover that he has been engaged, however involuntarily, in a labor of love, or he may be pleasantly surprised to discover that his enterprise has been profitable. The transaction would thus appear to be a good starting point for the study of economic behavior.

Examination of the typical transaction indicates that it normally has four major elements. In addition to the two transactors, there is usually a flow of a commodity or service from one transactor to the other, and in exchange a counterflow of money or credit. The relationship between the money- and commodity-flows constitutes price. It is worth noting that, unless both flows are measurably defined in different units, the price cannot be established. This description applies, of course, to an economy where transactions normally involve a pecuniary medium of exchange. Barter elements in such an economy are typically handled by imputing equal (and perhaps arbitrarily set) monetary values to the goods or services involved. The typical transaction is schematically shown below.



It is quite obvious that all the transactions occurring in an economy, even within a short period of time, cannot be examined separately. To bring the task of observation within reach, the transactions must somehow be grouped. There are two obvious immediate choices: one may group together those transactors who, on one basis or another, are expected to exhibit similarities in behavior, or one may attempt to group together transactions that involve similar things—that is, commodities or services that are similar in nature or affected by similar influences.

Regardless of the intrinsic merits of these two possible systems of classification for our purpose, the choice between them has in fact long since been made in connection with the establishment of our national statistical series. A preponderant part of the regularly available information regarding the productive activities of the economy is set up on an establishment basis. Hence, our economic studies will almost necessarily have to recognize groups of transactors. Within this primary scheme, of course, there will be an effort to preserve as much information regarding commodity- and service-flows as possible, and in a limited number of cases it will be found that the two systems nearly coincide.

Much of our interest revolves around production and the levels of productive activity. The major criterion used for grouping transactors (establishments) in current industrial statistics is that they be engaged in producing similar things, that is, that the goods and/or services they offer for sale constitute a relatively homogeneous group. It is, or it should be, a major objective to form groups (or industries) in such a fashion that an additive and preferably simple measure of output can be applied to the units making up the group. In some cases, this measure may be physical in nature; in others, the only unit available may be a monetary one. In the following discussion, it is assumed that, where transactions are grouped to form a sector or industry, and where the major purpose of the sector is to provide goods and services for sale to others within the economy, some measure of productive activity or output for the sector is available. It is not assumed that the unit of measurement is the same for all sectors, but in fact a monetary unit will frequently be employed.

For convenience, a group of transactors will be referred to as a sector of the economy. If the sectors usually distinguished in economic statistics are examined, it will be seen that they fall generally into two broad classes. Some of them engage primarily in extractive, fabricating, processing, assembly, or service activities, and constitute what are usually thought of as industries. These sectors do not operate independently of the rest of the system, but rather primarily in response to demands made on them. In general, if these demands slacken, their level of production or output declines. In a sense, these sectors may be considered as agents who perform activities commissioned by other purchasers. In the following discussion, these groups will be referred to collectively as the intermediate sectors of the economy, but they may also be called the endogenous, processing, or productive sectors.

The other broad class of sectors will generally acquire goods and services as end products. In general, no further processing activities will be performed on items bought, and the purpose of purchase will not be for resale. In the following discussion these purchasers will be collectively termed the autonomous sectors, but they may also be called the exogenous or final demand sectors. An example of such a sector might be consumers or households considered collectively; in general, such sectors will correspond with the major categories distinguished in national income and gross national product analysis.

In relation to different problems or different purposes, there may be sectors which could with logic be placed at one time in the intermediate, and at another time in the autonomous, category. While this is permissible, it will be assumed that for any stated purpose a dichotomy is possible, so that a given sector will appear as either intermediate or autonomous, but not as both.

To anticipate slightly, it may be noted that the purchasing activities of an intermediate sector will be restricted in various ways. In the first place, they will usually occur only in response to demands originating outside the sector. Second, since the sector will perform a limited range of activities, its pattern of purchases will be related to these activities and a given technology for performing them. The purchases of the autonomous sectors are less restricted. It is in these sectors that changing human purposes find their most immediate expression. With qualifications, the purchasing behavior of the autonomous transactors may be regarded as the driving force of the economy, with the activities of the intermediate sectors a driven mechanism responsive to this expression of human intentions and desires.

3. RECORDING OF TRANSACTIONS DATA

So far it has been stipulated that the economy is to be separated into two broad classes of meaningful transactor groups, and that for each of the intermediate (as opposed to autonomous) sectors a measure of output or production has been or can be established. It is an obvious step to record the transactions among the sectors during some stated period or periods of time.

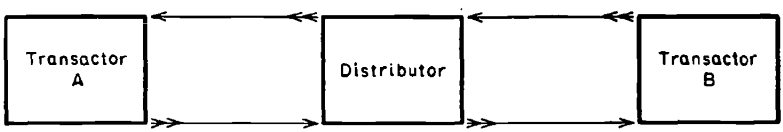
Let us consider the transactions record for a single sector. The sector may have transactions with any of the other sectors of the economy, and (since it is a composite) with itself as well. Two types of transaction can be distinguished; those in which money or credit is received, and those in which it is disbursed, corresponding in general with the instances when goods or services are sold to, or purchased from, other units. A systematic summary of the transactions of a given sector should, then, have both a receipts and a disbursements side. An item in the summary will record transactions occurring within a stated period between the given and another (or the same) sector. It may be presumed that transactions can always be recorded in monetary terms, with other details (such as physical volumes involved) set down as well where possible and relevant.

Since all transactions will be recorded at least in monetary terms, a monetary balance between the receipts and the disbursements sides of a sector's transaction record can be sought, and, with appropriate definitions, obtained. There are practical advantages to incorporating balances and controls in empirical measurement records wherever possible.

In its final form the monetary summary of transactions among sectors may resemble a set of double-entry books for the economy. However, the purpose of such a set of systematic observations is not simply one of accounting, but rather to provide a systematic and meaningful array of observations within which elements of stability or predictability for an economy may be found. In this sense, the measurements partake no more of bookkeeping than do the laboratory records of a research physicist.

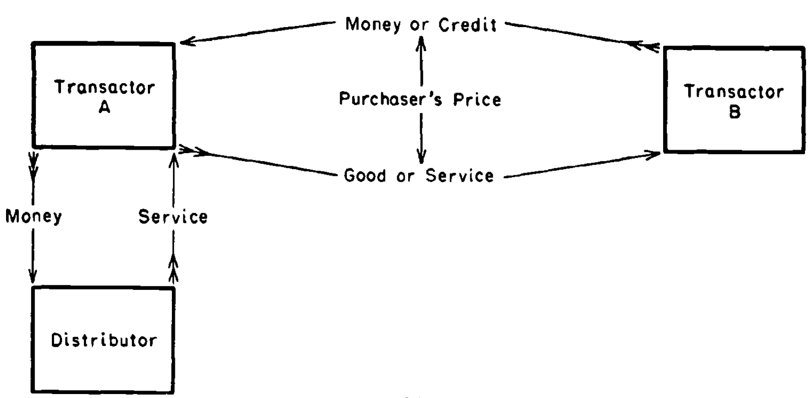
In keeping with this purpose, and guided by prior empirical experience, some features beyond those indicated will surely be incorporated within the observational framework. For one thing, the purchasing record of a firm or establishment in one of the processing sectors is likely to show some purchases related only indirectly to current production. In particular, certain items of expense will be charged to the current operating account and others to a capital account. While the former account may bear some fairly close relationship to the firm's current activity levels, the latter is likely to be much less intimately linked. For at least the processing sectors, then, it would seem expedient to separate purchases on current account from those on capital account. This might be done by setting up two complete accounting and measurement systems, and such an approach would have many attractive features. Unfortunately, current statistical resources are hardly adequate for such an approach. As an alternative, a hypothetical or fictitious autonomous sector may be set up, including, among other things, a consolidation of the capital accounts of all the intermediate sectors. The remaining measurements, relating only to transactions on current account, will then be independent of the perhaps erratic, and in any case less predictable, transactions on capital account.

Another modification will suggest itself quite readily. Many transactions between one sector and another occur with a jobber, wholesaler, or other distributor as an intermediary. There may be a technological necessity for the flow of goods from one sector to the other, but no special technological necessity for the flows to occur through a given distributive channel. The situation might be represented as illustrated below.

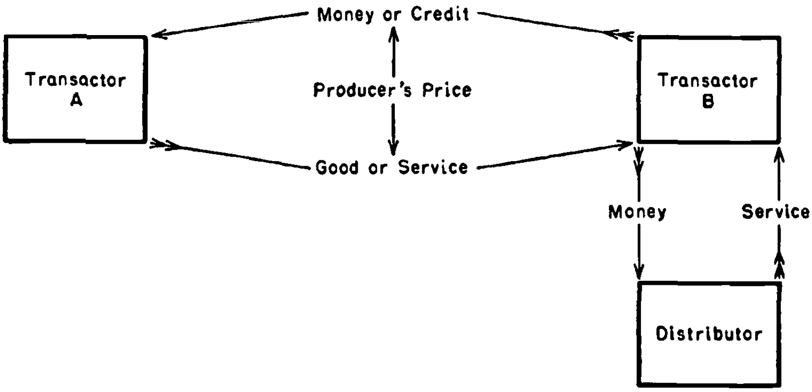


To bring producer and user together without the intervention of a heterogeneous trading structure, we might regard the transaction as having taken place directly between the two sectors, with the producing sector engaging the services of a distributive sector to effect and carry out the transfer. In this case, of course, the charge made on the second sector by the first will include the cost of distributive services rendered, and, in relation to the physical volume of goods or services transferred, the transaction may be said to be expressed in purchasers' prices. This situation may be represented as shown below.

Another alternative would be to have the cost of distributive



services borne by the purchaser. In this case the money or credit transferred to the producer would be less, and the transaction would be recorded in producers' prices, as illustrated in the following diagram.



Both forms of pricing are found in currently available economic statistics. Either method of handling distributive charges, systematically followed, will accomplish the major purpose of bringing producer and user into juxtaposition in a basic record of economic behavior. There are reasons, indicated elsewhere, for preferring the use of producers' rather than purchasers' prices in input-output analyses, but either may serve in a basic transactions record.

The above discussion has been concerned primarily with the establishment of a consistent and useful framework for recording in meaningful and suggestive form a large volume of observations of the economic system. Such a development is guided, of course, by prior empirical knowledge, a sense of measurement realities, and more than a hint of the conceptual scheme with which the measurements will be linked. Some of the interplay between the ultimate analytical framework and the form, content, and arrangement of the economic measurements has already been suggested, but of course this is a continuing process with endless modifications and improvements suggested by results.

4. DETERMINATION OF FUNCTIONAL RELATIONSHIPS

Let us turn now to another aspect of the initial problem. It has already been remarked that when goods or services are transferred from one to another intermediate sector the purpose is usually not arbitrary. Rather, the transfer is normally made to permit the second sector to carry on its appropriate activities. This conclusion is suggested and supported by direct observation of the processing system. It is hardly an hypothesis, but rather one of the most direct causal relationships that can be observed in the field of economic behavior. Without at the moment stipulating the exact form of the relationship, it is surely not inconsistent with our empirical knowledge regarding the economy to say that at least some of the shipments from the steel industry to the automobile industry are functionally determined by activity in the latter, that cotton fiber purchases by the textile industry are functionally related in some way to textile output, and so on. If such direct connections among processing activities of the economy, with their strong institutional and technological determinants, cannot be established, general economics is, and must remain, in a stage of painful indeterminacy.

That such functional relationships exist is a necessary assumption to provide real content for the more formal steps which follow. Let us consider the representation of an economy that includes n intermediate sectors. It has already been assumed that for any intermediate sector of the economy an additive measure of output or production is available. We may define the total output during a given period of some stated sector, say the ith, as X_i. Let us then define the part of the output of the ith sector that is determined by and varies with the level of output for the jth sector (where j may be the same as or different from i) as $F_{ij}X_j$. For convenience, we may stipulate that, if X_j equals zero, it will imply that $F_{ij}X_j$ also equals zero; any residual constant elements will be accounted for as described below. Continuing in this way, the parts of the output of the ith sector that are determined by, and vary with, each sector's output level can be set down. Any output of the ith sector that is destined for the intermediate sectors, but that is not functionally dependent on the values $X_1, X_2, \ldots, X_n$ (including any residual transfers when these production levels are zero), is placed in a separate category and denoted by the symbol B_i. The remaining output of the ith sector is necessarily taken by the autonomous sectors collectively and is denoted by Y_i. With these definitions, the distribution of the output of the processing sectors within the entire system can be accounted for formally as follows:

$$
\begin{aligned}
(1) \qquad & F_{11}X_1 + F_{12}X_2 + \ldots + F_{1n}X_n + B_1 + Y_1 = X_1 \\
& F_{21}X_1 + F_{22}X_2 + \ldots + F_{2n}X_n + B_2 + Y_2 = X_2 \\
& F_{n1}X_1 + F_{n2}X_2 + \ldots + F_{nn}X_n + B_n + Y_n = X_n\,.
\end{aligned}
$$

This may be more compactly represented in matrix notation as

$$[F_{ij}]\,X + B + Y = X \tag{2}$$

in which B, Y, and X are the indicated column vectors, $[F_{ij}]$ is the square matrix of interrelationship functions, and the symbol $F_{ij}X_j$ is understood to represent evaluation of the function F_{ij} at the point X_j. Since all entries in the system represent either money- or production-flows, they may be restricted to zero or positive values.

Let us now consider certain limitations on the forms of the functions that are suggested by, or not inconsistent with, observed characteristics of the real economy. In the first place, to avoid ambiguity, we may require that the interrelationship functions be single-valued for nonnegative indexes. The possibility that any of the functions can be constant for positive indexes has already been ruled out. Empirical considerations make it reasonable to assume that, at least in the positive quadrant, $F_{ij}X_j$ is everywhere a nondecreasing function of X_j. This implies that there is no point where an increase in the production level of the jth sector would result in lessened requirements for the products of the ith sector. Consideration of the real economy will indicate that any exceptions, if they exist, are likely to be trivial.

So much for the form of the interrelationship functions. It also seems reasonable to require that the system as a whole should have some purpose in terms of human requirements, that is, that the column vector Y must include some elements greater than zero. To avoid complete triviality in a mathematical sense, one might also require that at least some of the sectors have a purpose other than exclusively to supply autonomous demands, although this is not essential.

The conditions stated are sufficient to guarantee the following: If the functional relationships are identified, a practical computation method is available for determining the set of intermediate sector production levels consistent with any specified schedule of autonomous demands, provided that any such set of production levels exists.

This is a conclusion of some importance. It guarantees that the approach is conceptually practical under an extremely wide variety of circumstances. The restrictions imposed are of the lightest nature, and are consistent with observations of the functioning of the real economy. We may note also that, except for these light restrictions, no special appeal to empirical knowledge has been made in setting up the function system. The functions may be derived from input-output tabulations, from engineering studies, or from any source that appeals to the analyst using the approach. The functions embodied may in fact be quite hypothetical, referring to a situation that has never existed but is of interest to some analyst. In addition, there will be no contradiction if other variables are included in the functional system. For example, if the interrelationship functions are assumed to be dependent also on calendar time, the system can be evaluated once this is stipulated.

The conceptual system implied in equation (1) is, in fact, so very general that it outstrips available empirical knowledge regarding the functioning of the processing system of the economy. As a conceptual frame, it rather exceeds our practical requirements. With the limited resources available for empirical economic research, such precise identification, as is implied, of the way in which deliveries from one sector to a second depend on activity levels for the second will be possible in relatively few instances. Even in these cases, the exact knowledge may be of little moment, since the quality of results will be set more by the average quality of knowledge incorporated in an analysis than by a few exceptionally good or bad functions. This suggests that, even where an interrelationship function is established as nonlinear, a linear approximation to the function may suffice for all practical purposes. As has been pointed out, there are strong a priori reasons to expect not simply linearity but near-proportionality for many interrelationships among processing sectors. The uncomfortable limitations of research resources will undoubtedly make it necessary to use proportionality assumptions even in cases where they may not be strictly applicable. Under the circumstances, it is not unrealistic to suppose that a linear function system will suffice for practical use in an analytical framework. This is no real limitation, of course, since successive linear approximations could be brought as closely as desired to a point evaluation of a known continuous curvilinear situation.

If one is content to use a linear form for the representation of the interrelationship functions among intermediate sectors, the system represented previously may be put into a somewhat more familiar mold. If we represent by a_{ij} the coefficient attached to X_j in the linear functions (with any constant element in such a function becoming part of B_i), the system of equations shown previously may be represented as follows:

$$
\begin{aligned}
(3)\qquad & a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n + B_1 + Y_1 = X_1 \\
& a_{21}X_1 + a_{22}X_2 + \ldots + a_{2n}X_n + B_2 + Y_2 = X_2 \\
& a_{n1}X_1 + a_{n2}X_2 + \ldots + a_{nn}X_n + B_n + Y_n = X_n\,.
\end{aligned}
$$

In matrix notation, the system takes either of the following more compact forms:

$$
\begin{aligned}
(4)\qquad & AX + B + Y = X \\
& (I - A)X = B + Y\,.
\end{aligned}
$$

The conventional methods for solution of linear simultaneous equations are available for use with such linear function systems. These are supplemented by an ability to use certain iterative methods that depend on special properties exhibited by input-output systems.

The typical input-output problem to which the above equations refer might be stated as follows: Given a certain processing structure for an economy represented by the matrices A and B, what is the set of output levels for the intermediate sectors of the economy that will be consistent with the stipulated level of autonomous deliveries represented by the vector Y? The solution may be represented in matrix notation as follows:

$$
(5)\qquad X = (I - A)^{-1}(B + Y)\,.
$$

The matrix $(I - A)$ is readily shown to be nonsingular for all real situations.

Quite frequently, the problem takes the following form: A set of production levels is associated with a specific set of autonomous deliveries. If these autonomous deliveries were altered, what changes in production levels would be required to bring the system again into equilibrium? In this case, the column vector B is not relevant to the computations, as the following equations illustrate:

$$
\begin{aligned}
(6)\qquad & (I - A)X_1 = Y_1 + B \\
& (I - A)X_2 = Y_2 + B \\
& (I - A)(X_2 - X_1) = (Y_2 - Y_1)\,.
\end{aligned}
$$

It has already been mentioned that there will be a continuing interplay between efforts to record transactions within the system in a meaningful, suggestive, and useful framework on the one hand and attempts on the other to set down a reasonable and workable function system for representation of interrelationships within the processing structure of the economy. One aspect of such interplay is to decide whether or not a given sector shall be considered as part of the intermediate processing system. If its functional interrelationships with the remaining processing sectors cannot be identified, or if they cannot be established with a degree of accuracy satisfactory for the analytical purpose in mind, either the given sector must be ignored in the analysis or its operating levels and demands on the processing structure must be determined by some other means. Dropping the sector from consideration is equivalent to ruling it out of the frame of discourse. In a sense it becomes external to and not a part of the economic system being considered. This course is not one that will be frequently adopted. On the other hand, external determination of its operating levels and its requirements on the productive system is equivalent to placing it within the autonomous vector. Hence, the content of the autonomous vector will depend, not only on our ideas of what really constitutes the exogenous demands of the system, but also on the circumstances of a given problem and the state of knowledge at the time it is considered.

We may note, however, that except for such additions as may be made for purposes of a particular problem, the concept of autonomous or exogenous demands can be made to correspond as closely as one finds convenient with the concepts underlying the measurement of the gross national product on the product side. The convenience of this correspondence will, in turn, influence both measurement efforts and the setting up of the conceptual analytical framework.

In considering the mathematical representation for the processing structure, it was stipulated that for each processing sector some additive output measure be at hand. The units for each one might be quite different, but very frequently the quantities involved will be expressed in monetary terms. There is no necessary compulsion to do so when another measure is available. However, there are some sectors of the economy for which measures expressed in monetary units will ordinarily be the only ones available. For other sectors, where a choice between physical or monetary units is possible, it is usually a simple matter to convert from one basis to the other at any time. Hence, in these cases there will be no special advantage in using physical rather than monetary units.

In measurement systems, there are some advantages in a consistent use of monetary units, since they permit crosschecks (of receipts against disbursements) that would be meaningless or impossible if physical units were employed. There are also some advantages in expressing the parameters of function systems in monetary terms.

One of these is that monetary units may automatically give some notion of the relative importance of different items, and indicate, for example, the points where rounding of figures can be safely employed.

5. INTERRELATIONSHIPS WITH PRICE STRUCTURE

The preceding discussion has been exclusively in terms of production demands and flows, since it is an interest in problems of this kind that has been largely responsible for current developments in the field. As might be expected, though, the conceptual formulation also has close links with the analysis of price structure, and these may be briefly indicated. In equation (3), the quantity of output required from the first sector to support a given level of activity in the jth sector would be represented by $a_{1j}X_j$ plus b_{1j} (where the latter represents the constant term in the linear form subsumed as part of B_1 in the earlier equation). If this quantity is multiplied by p_1, representing the price for the first sector's output, the result will be the monetary value of the goods or services required from the first sector by the jth in order to support the latter's level of activity. The necessary disbursements to other intermediate sectors by the jth may be set down in the same way. In view of the dichotomy assumed earlier, all other disbursements are necessarily to the autonomous sectors, and the total of such payments for the jth sector may be represented by V_j. Finally, if definitions to insure equality between monetary disbursements and receipts have been employed, the total of the items mentioned above may be set equal to the output level for the jth sector multiplied by its price level. This will yield as an equation for the jth sector:

$$\begin{aligned}(7)\qquad & (a_{1j}X_j + b_{1j})p_1 + (a_{2j}X_j + b_{2j})p_2 + \dots \\ & \dots + (a_{nj}X_j + b_{nj})p_n + V_j = X_j p_j \,.\end{aligned}$$

The system is completed by similar equations for the other intermediate sectors.

The complete system of equations may be compactly represented in matrix notation as follows:

$$(8)\qquad \overline{X}A'p + B'p + V = \overline{X}p\,.$$

In this, $\overline{X}$ is the diagonal matrix of production levels; A' is the transpose of the matrix A in equation (4); B' is the transpose of the matrix of constant elements in the linear functions subsumed in (4); V is the column vector of payments to the autonomous sectors; and

p is the column vector of prices for the intermediate sectors. Solving for the price levels, one obtains

$$p = (I - A' - \overline{X}^{-1}B')^{-1}\overline{X}^{-1}V\,. \tag{9}$$

The column vector V, in general, will be comparable to the factor-payment side of the gross national product. The last two terms of the equation taken together are similar to a column vector of factor payments per unit of output in the intermediate sectors.

The system becomes substantially simpler if uniform proportionality of interrelationships among the intermediate sectors can be assumed. In this case, the matrix B vanishes, and (9) becomes

$$p = [(I - A)^{-1}]'\overline{X}^{-1}V\,. \tag{10}$$

In this case, the same inverse matrix that represents a general solution for production equilibrium problems in (5) becomes in its transpose form a general solution for problems of price equilibrium.

Equation (9) or (10), whichever may be considered applicable, shows how the unique set of prices consistent with a given set of production levels and factor payments, and with a known linear or proportional processing structure, may be determined. Similarly, the unique price changes consistent with a given set of changes in factor payments may be determined. However, for reasons that will be indicated later, problems of price equilibrium are given little emphasis in current work.

It may be noted that throughout the procedure described there has been no necessity to impute a functional dependence in current flows between sectors of the economy when such a functional dependence is lacking, or to ignore any functional dependence that can be shown to exist. Nor has there been any necessity, because of conceptual poverty, to use oversimplified functional forms. If simple forms are used, it will be for one of two reasons: first, lack of exact knowledge will dictate, according to the usual principle of economy, the use of functions with the minimum number of parameters required to be consistent with whatever observations are available; second, even where more exact information on functional form is available, it may be apparent that available time and resources can better be spent in other ways than in making a trivial or meaningless refinement. To put this another way, a timesaving linear approximation to a curvilinear relationship within a limited range may, and probably in most cases will, give results within realistically set accuracy limits.

In the preceding discussion, the framework for recording and classifying economic observations has been treated separately from the conceptual interrelationship system. The order was arbitrary—either topic might have been placed first—but the separation itself was a necessity. The first is not concerned with opinions or conclusions but with records and measurements; the second accommodates notions of purpose and hypotheses of cause and effect—the distinction between them is important. A similar separation is to be found in any science dealing with measured phenomena. The physicist or chemist does not expect exact correspondence between his measurements and his current concepts, since all physical measures, and probably concepts explaining them as well, are imperfect. But he attempts to make them correspond as closely as possible. As measuring instruments and measurements are improved, the concepts used are sharpened. We should expect nothing different in a quantitative study of an aspect of human behavior. It is felt that the conceptual system described is adequate, within the limits of present observational capacities, for what it purports to cover, but change and improvement should occur naturally as empirical knowledge increases.

6. INTRODUCTION OF DYNAMIC ELEMENTS INTO SYSTEM

Dynamic elements in the economy are in the main omitted from the functionally determined side of the present treatment. This is not intended to suggest that dynamic factors and their functional interconnections with the system are unimportant; exactly the contrary is true. Nor are these factors ignored; they are subsumed within the autonomous sectors, and hence supposed subject to external analysis and determination. Because of this method of treatment, the approach described will be most useful when dynamic factors are not of dominating importance in the analysis—and many real and important problems are of this type.

The conceptual scheme described can be extended to take explicit account of activities phased in time, necessary capital formation, various limiting factors on change, certain possibilities for choice or optimization, and so on. However, such generalizations are for the most part conceptually complicated, and they require consideration of additional measurement problems. They are regarded as beyond the intended scope of the present paper.

7. VALUE OF THE INTERINDUSTRY APPROACH

In general conclusion, it is the authors' belief that the interindustry-relations approach, broadly conceived, is a reasonable analogue for the macroscopic workings of an economy such as that of the United States, and hence that it provides a workable and useful framework for quantitative analysis of such an economy. The questions that trouble the authors are not those of concept, but relate rather to details of execution.

The conceptual framework of interindustry economics is believed to represent a sound scientific approach to the problems of quantitative analysis of national economic phenomena. The approach can accommodate in detail as much observational content and data as can be furnished, and substantially more than is available at present. It permits effective and direct use of current economic observations. It facilitates separate examination of the processing structure and of the demand structure of the economic system, and permits specific examination of the consequences of hypotheses regarding changes in either. It encourages constant crosschecking between assumptions regarding the system and actual measurements of it. It embodies the principle of economy in concept, and has a hard core of common sense. Like many fundamental and powerful tools, it may be approached from many sides, and it exerts a powerful integrative influence across many fields.

This same many-sidedness brings certain dangers. In its more elementary aspects, the approach can be couched in terms to appeal rather directly to the businessman, accountant, engineer, econometrician, industrial statistician, national income economist, market analyst, and others. But descriptions that are couched in terms and images familiar to and readily accepted by one group may have little meaning for another. We often fear that "practical" people may dismiss summarily a thoughtful but highly abstract presentation, but the converse happens as well. A simple presentation should not lead the student to suppose that some rather obvious difficulty has been naïvely overlooked or is insuperable. The approach has rich possibilities, and will reward the thoughtful observer with a deeper understanding of, and appreciation for, the intricacies and niceties of the most complicated mechanism yet created by man—the modern industrial national economy.

B. Analytical Framework

Virtually any nontrivial interindustry-relations analysis will involve one or another aspect of the complete national economy, with results that depend on the total set of interrelationships assumed for the processing structure. Hence, it is not unreasonable to refer to any such analysis as a model, that is, as a structural analogue of some aspect of a complete national economy. Throughout this text, the discussion will be limited, with only a few exceptions, to input-output models of the simplest type. These provide a rich field for exploration and exploitation quite aside from the possibilities offered by the more complex formulations.

The simplest input-output models are typically concerned with open full-equilibrium situations. A continued delivery of specified end products outside the processing system is assumed, and the activity levels for the intermediate sectors consistent with these deliveries and with an assumed set of structural interrelationships among the intermediate sectors are derived. Dynamic elements in the economy are treated externally to the functional system. For example, inventory changes and capital formation are considered as autonomous. No explicit resource limitations are incorporated in such models. Computations in these cases are usually limited to the solution of one or more sets of linear simultaneous equations of the type shown in equation (3) of the preceding section.

Certain dynamic elements that can be handled in more complicated models by explicit treatment within the model can sometimes be given effect in these simpler models through a form of iteration. For example, if the results of a model indicate that the capital formation originally specified is incompatible with the production-level results that are derived, one or more modified models may bring the results into balance. Similarly, postulated demand schedules leading to production levels that are unreasonable in the light of information on resource limitations may be adjusted iteratively.

The typical simple input-output analysis has four parts:

1. The demand schedule that is to be imposed on the processing system, and whose consequences are to be examined, must be stipulated. Essentially, this is equivalent to a detailed statement of the problem to be solved.

2. The system of interrelationships within the processing structure that will be used for computation must be established. It is typical of the input-output approach that the functional system to be employed must be determined by the analyst. It is not necessarily given by historical data or the accidents of the past, but is subject in every part to the analyst's judgment and decision.

3. The demand and processing structures must be conjoined in a computation, which is usually of substantial dimensions.

4. The results of the computations must be examined for their significance to the problem under consideration. For particular problems, a certain amount of translation of the results, using collateral data, may be required.

These four parts of the analysis will be discussed in sequence, but the general question of the degree and kind of aggregation to be used so pervades all other aspects that it will be treated as a separate topic.

1. AGGREGATION

An input-output analysis may embody a classification system for the intermediate sectors that ranges from fine detail to high aggregation. Whatever the degree of aggregation, there will be reciprocal effects between the classification system and all other parts of the analysis.

The reasons that will impel use of the most detailed classification system feasible in the compilation of basic input-output tabulations have been described elsewhere. In use, there will be varying amounts of reaggregation to a smaller number of sectors. The kind and amount of condensation will depend on the conditions of the problem to be attacked, and on the resources available for its solution.

It has already been pointed out that use of an establishment base in much of the national statistical reporting system sets this as the basic mode of classification that must be used in input-output systems. Finer details of measurement can generally be obtained only by pursuing additional commodity records within the established industry divisions. For most input-output purposes, an industry is perhaps best thought of as a collection of productive units engaged in supplying commodities or services that are similar in nature and for which some common output measure is available. The similarity in outputs is expected to enjoin a certain degree of similarity in production facilities and in input requirements for the producing units within a sector. To compensate for imperfections in this expectation, a general trend in basic data compilation undoubtedly will be to add commodity detail on both input and output sides within sectors. The greater the amount of industry and commodity detail that can be recorded, the greater will be the flexibility and adaptability in use of the tabulations.

Certain problems caused by changing product mix can be met through availability of information on commodity inputs and outputs within sectors. In particular instances, commodity input structures may be reweighted to give new sector input structures that will compensate for actual or anticipated changes in product composition. In other cases, commodity details may be retained within the analytical structure, effectively adding the commodities as additional intermediate sectors.

On the other hand, an increase in the number of sectors distinguished in an analysis adds to labor requirements at every stage. Considering that resources available for most research purposes are limited, there will be a strong incentive to reduce this burden. This is accomplished only by limiting the number of sectors to be handled, in other words, by aggregating to a smaller number of sectors.

Every aggregation sacrifices a part of the detail that delineates the demand and processing structures. Some sharpness will be lost, and the results to some extent will be blurred. Therefore, there will be a constant conflict between limited resources and the desire to retain in the analysis all possible relevant detail.

There are certain principles that can help to minimize the undesirable effects of a necessary sacrifice in detail. One of these is the principle of exclusive use. If the output of one sector is all (or nearly all) taken by a second sector, they may be combined without loss of information. For example, since the output of iron ore mining is used almost exclusively by blast furnaces, the combination of these two activities will have little effect on analytical results. Because of this, there is a general tendency in highly aggregated input-output tables to combine basic extractive industries with the initial processing sectors that make almost exclusive use of their output. This has led to uninformed comment that mining must have been overlooked or omitted in the smaller published tables. Actually, in small tables a considerable sacrifice in detail elsewhere would be required to preserve the customary divisions of agriculture, mining, manufacturing, etc. While it has been customary to leave agriculture as a separate sector in the small tables, it might be more reasonable to include agricultural fiber production with textiles and apparel, agricultural food production with food processing, and so on.

A second principle for aggregation is the combination of sectors with identical input structures. If two sectors make the same demands on the remainder of the processing structure per unit of output, nothing will be lost for analytical purposes in combining them. The same principle may be used where the input structures are similar but not identical, and in this case analytical results will be affected in degree as the basic assumption is violated.

A third principle is that of demand complementarity. If it can be accepted, for a problem under consideration, that the demand for the products of two or more sectors will rise or fall together, the sectors may be combined without significant loss of detail. It is difficult to establish complementarity, however, except when it appears as a special case of the exclusive use principle, with both sectors having the same dominant customer.

Perhaps the most important principle of all for the general analyst is that of irrelevance. If two or more sectors have little or no bearing on the particular problem with which he is concerned, these sectors may be combined with small loss for his purpose. For example, an analyst concerned with a metalworking-industry problem might very reasonably combine a number of consumer nondurable sectors, even though they were quite unlike in terms of any of the criteria mentioned above, with confidence that he had not changed significantly any conclusions he might reach, and that he had saved himself a substantial amount of work.

The basic BLS study of interindustry relations for 1947 distinguished some 450 intermediate sectors, with varying amounts of additional commodity detail. For current analytical uses within the government, these data have been aggregated to tables distinguishing 190 intermediate sectors. The principles listed above, not excluding the last one, were used in varying degree in making the aggregation. The 190-order classification system represents the pooled judgment of many people with a very specific problem in mind—an analysis of the impact of current or prospective munitions production schedules. Hence, it is not properly a general-purpose classification scheme. However, even these tables are quite detailed, so that for many purposes the general analyst may desire to aggregate them further.

The aggregation of a large input-output table to one of smaller dimensions is in itself no inconsiderable task. The basic 450-order tables include about 40,000 summary intermediate sector entries, and any handling of such a large number of figures is sure to be time-consuming. Even the 190-order tables include about 12,500 entries for the intermediate sectors. Additions must be made for both rows and columns, and constant checks on accuracy are necessary. Moreover, a series of adjustments for secondary products must be carried through. In the large tables, products that are secondary to a given sector are shown as transferred to the sector where they are primary. When sectors so connected are combined, the fictitious transactions involving secondary products must be eliminated.

A significant development will be the preparation and publication of both general- and special-purpose smaller matrices. This can most easily be done by agencies that have the basic data stored on punch cards or magnetic tapes, and have worked out methods for aggregation by mechanical means.

2. DEMAND SCHEDULES

The autonomous sector or demand schedule, in a typical input-output problem is in effect an explicit statement of the problem that the analyst wishes to solve. Within the framework of the equation system adopted, the analyst asks: if an economy with the specified technological interrelationship structure is required to produce goods for delivery to autonomous sectors in exactly the amounts specified, what output levels for every sector will bring the system to exact equilibrium in terms of demand and supply? Or the question may be of the following type: If this economy is required to deliver to autonomous sectors more (or less) goods in stated amounts than in fact it is delivering, what increase (or decrease) in the output levels for each sector will bring the system into equilibrium? These questions may have a slightly different mathematical formulation, as illustrated in equations (3) and (5) above.

The content of the autonomous sector is determined by the purposes of the analyst. In a simple case, it may contain only a single entry. For example, the problem might be to determine the increase in output level required for each sector if processed food deliveries are increased by $1 billion a year.

Let us consider a more complex case in which the demand schedule represents the total of all deliveries to ultimate users by all intermediate sectors within the economy under certain assumed conditions. The details of the content of the autonomous sector for such a problem will not be discussed here. Other papers presented to the Conference discuss the treatment of the final-demand sector in the basic 1947 tabulations, and the problems involved in stipulating end-product demand for a large-scale analysis. It will be pointed out here only that the content of the final-product or autonomous-demand schedule, as defined in basic input-output tabulations and as customarily treated in large-scale analyses, is controlled to a very considerable extent by the concepts underlying the gross national product accounts. In general, gross national product figures on the output side will provide the take-off point for estimating autonomous deliveries in any large-scale model.

However, there will be modifications. In a particular problem, sectors of the economy that normally would be considered to lie in the intermediate area may be moved into the autonomous category. This will occur when it is felt that the conditions of the problem make it dubious or impossible to arrive at a simple functional determination of the levels of demand for the output of the sector. For example, it might be felt that, under normal circumstances, advertising or insurance purchases by a given sector bear some identifiable relationship with the sector's volume of production. Under abnormal circumstances, one might feel that an independent estimate of the total volume of advertising and insurance purchases is preferable. In this case, the advertising and insurance sectors are eliminated from the intermediate structure, and the requirements of these sectors on the processing system are estimated independently and added to other autonomous demands. In general, any requirements on the system that cannot be represented within the context of a given problem as functionally determined by levels of activity for the intermediate sectors will be treated in this way.

Indiscriminate use of this device will create problems, however. In its simplest form, the input-output system is a very large interlocking mechanism. The amount of side computation required increases substantially as more elements are transferred from the intermediate to the autonomous area. The side computations, being relatively small, are not usually subject to mechanical procedures. Computations involving the intermediate sectors, on the other hand, will normally be programed for mechanical computation. Hence, it is advantageous to make transfers from the intermediate to the autonomous areas only when there is reason to believe that results for a particular problem will be significantly improved by the change.

A common problem in setting up an autonomous-demand schedule (or bill of goods, as it is frequently called) is that initial demand estimates that are furnished to the analyst may be stated in terms of purchasers' values, that is, the figures given will include payments to the distributive sectors linking producer and user. The functional matrix, on the other hand, will normally be expressed in producers' values; in order to bring the demand and processing systems into consistency, it is necessary to convert the autonomous entries to a producers' valuation base. This is done by decreasing each item in the autonomous deliveries by an estimate of the separate distributive margin charges involved. The latter estimates are then cumulated to arrive at the autonomous-demand entries for the distributive sectors.

Another problem that may require consideration in setting up a total autonomous-demand schedule involves administratively determined competitive imports. Imports as such are excluded from the intermediate processing structure, but they constitute an element in the cost structure of each sector. Hence, unless some contrary step is taken, there is an implicit assumption that with a rising level of demand for the products of a sector an equal proportional addition to the supply could be obtained from other countries. In particular instances, such an assumption may be considered untenable. To take a specific case, it might be assumed that the level of a specific import would be set by administrative fiat (or that the level of the import could best be set through independent analysis). One would then wish to establish the production level of the domestic industry at a figure that would satisfy domestic requirements beyond the stipulated level of imports. This can be accomplished by placing the stipulated import as a negative entry in the autonomous demands for the products of the domestic industry. To be more concrete, one might wish to stipulate in a given model the level of imports of natural rubber, with domestic synthetic rubber production furnishing the balance between demand and supply. The stipulated natural rubber imports (expressed in terms of the price of synthetic rather than natural rubber, if the computations are in monetary terms) would then be entered as a negative item among the autonomous demands for synthetic rubber. The same technique covers not only imports, but also stipulated net withdrawals from domestic inventories or stock piles.

Where the analysis is concerned not with equilibrium in production flows but with cost-price balances, the equivalent of the autonomous sector becomes an estimate of factor payments within the economy under the conditions considered. The take-off point for such estimates will normally be a consideration of the gross national product accounts on the income side. There may be special problems in treatment, but these will parallel somewhat the topics given above and will not be discussed here.

3. STRUCTURAL INTERRELATIONSHIPS

It is difficult to emphasize sufficiently that a base-year imput-output tabulation and the set of interrelation functions among sectors that may be used for any particular analytical purpose are separate and distinct things. The base-year tabulations are simply a record, within a consistent classification system, of the transactions among sectors during a stated period of time, as well as these can be established. This transactions record is intended to be a starting point for the analyst in his consideration of structure. However, the representation of the processing structure that is incorporated in any given model is exclusively the responsibility of the analyst. He may believe that strict proportionality and use of the base-year relationships will suffice for his problem. This is an assumption that may be forced on him by limited resources, but not by the existence of base-year data or the conceptual scheme.

The task of the analyst is, of course, easiest when he feels that he can use the proportionality assumption and establish the relationships empirically on the basis of base-year data alone. For many problems, especially those involving the examination of economic interconnections in the base year, the analyst may regard these as highly tenable assumptions. In any case, the assumption of proportionality may be expected to be least unsatisfactory when the autonomous sector to be investigated does not differ substantially from that incorporated in the base-year input-output table. In this case, the proportionality line and the correct function, whatever its form, will have passed through a common point in the neighborhood of the solution, and hence the former may represent a good, and even close, approximation to the latter.

For a more critical or more careful job, the analyst should examine the interrelationship function connecting every pair of sectors, using all information available to him from base-period tables, historical records, and other sources to determine the specific form that is most suitable for the intended purpose. Such a detailed analysis might appear time-consuming and laborious, but it may be kept from becoming excessively so by reference to the same principle of irrelevance that was mentioned in the preceding section. The analyst will normally limit his examination to those relationships that he has reason to feel will significantly affect the results he is seeking to achieve. The analyst concerned primarily with a textile problem will be satisfied with fairly crude functional representations for the metalworking areas, knowing that even a very approximate functional representation here will not alter any conclusions that he regards as significant. Even in such large-scale analyses as those referring to industrial mobilization, there will be numerous sectors that will clearly be of minor significance in the results. The greatest care in establishing the structural connections will be reserved for those sectors considered crucial through experience or foresight.

There are many reasons why an analyst might wish to alter base-year relationships. One of the most obvious of these is to account for a technological change that has occurred, or is expected to occur, between the base period and the reference period of the analysis. For example, there has been a steady decrease in the quantity of coal required to produce a kilowatt hour of electric energy. The assumption that this trend will continue into the future is based on knowledge of current and planned construction programs for additional electric generating facilities.

A change might be made for quite different reasons. In a mobilization analysis, for example, it might be felt that restrictions on the use of critical materials would force reduced use and substitutions in some areas. The analyst may, and should, take account of such changes as he can. A somewhat more difficult problem may be to account for possible price substitutions. However, if the prices appropriate to the analysis are known or can be estimated, the appropriate alteration in structural relationships can be made through use of whatever elasticity information is available. Unless information on both of these points is available or can be estimated, no adjustment can, of course, be made under the input-output or any other approach.

There have been some expressions of disappointment that changes in input structure, whether due to technological change, price substitution, or other reasons, were not somehow automatically determined within the input-output analytical structure, but rather required external consideration. What has sometimes been overlooked is that adjustment for such changes requires information beyond that presupposed to be available for simple models. If such additional information is supplied, account can be taken of it, and in general more easily and more explicitly with input-output than with other estimating systems. It is unfortunate, but no method has yet been found that will do the analyst's thinking for him; no model will yield results beyond the information and judgment put into it.

The range of data and information that can be used in establishing input relationships is definitely not limited to the range contained in input-output tables. The analyst may examine data on the use of materials by particular sectors covering many years (provided such data are available), and express the relationship between two sectors as a function of calendar time, if he desires. No contradiction with the input-output framework is involved, and, in general, no complication in the computations either. It is in the area of establishing functional interconnections between sectors that specific industry studies, including those of an engineering nature, can contribute a great deal. Perhaps the extreme of such an approach is represented by the establishment through engineering study of an input pattern for an industry that does not exist (as in certain mobilization problems) and its subsequent incorporation within a scheme of analysis. The possibilities in establishing a valid structural interrelation system are limited only by the analyst's purposes, resources, patience, and judgment.

There are a few special problems in setting up the functions representing the processing structure that may be mentioned here. One of these is the handling of sales where the volume may not be determined uniquely by the purchaser but rather by the activities of the producer. This problem perhaps can be best described in terms of an illustration. In its operations, the machine tool industry produces metal scrap, which has a market value. In order to accommodate such sales within a base-period input-output tabulation, a "dummy sector" to handle scrap metal may be set up. The sales of metal scrap by the machine tool industry are shown as flowing to this dummy sector. In turn, the dummy sector sells scrap metal to other industries, notably the iron and steel sector. If these sales are treated as equivalent to normal flows in setting up a structural system, they imply that, with an increase in demand for steel, its purchases from the metal scrap sector will increase. This in turn will lead to additional purchases of scrap from the machine tool industry and to the ridiculous effect of having greater machine tool output to produce scrap for consumption by the iron and steel industry.

The problem is met by assuming that production of scrap by the machine tool industry is functionally dependent only on the level of machine tool output. Flows of scrap are placed outside the intermediate processing system. The general effect of this is to set up a situation equivalent to the following: Production of scrap or waste materials by an industry is considered to be determined by its level of operation. The scrap and waste materials flow into a stock pile that is external to the processing system. Industries that use scrap obtain it by purchase from this external stock pile. No requirement is imposed by the system that additions to, and withdrawals from, the external stock pile be identical. If an implied change in scrap inventories contradicts known facts regarding the system, this must be determined by external calculation. An exact balance between flows to and from scrap inventory stock piles is sacrificed to obtain a greater degree of validity in the functional representation of interrelationships.

An identical line of reasoning may be applied to by-products. For example, domestic hide production, even though normally sold to the leather industry, will be determined primarily by the demand for meat products rather than for leather. Hence, in setting up a structural rather than a transactions system, it would be better to regard hide production as functionally set by the level of meat packing, with the hides flowing to an external stock pile. The leather industry would obtain its hide requirements from the stock pile. In this case, the lack of an imposed balance on flows to and from the stock pile of hides is probably desirable, since in actuality an excess demand for hides would very likely be met by increased imports.

In the 190-order tabulations currently available, a substantial number of specific by-products have been identified and treated in the manner described. In addition, two sectors that were set up in the 450-order basic tabulations to accommodate metal and nonmetal scrap have been eliminated, as suggested above. In the smaller, preliminary 44-sector tables previously published, no by-product or scrap adjustments have been made. For many kinds of problems, scrap or by-product adjustments will not alter results significantly, and hence the analyst may wish to ignore them. In certain applications, however, the specific identification and separate treatment of such items may be desirable.

Another problem that the analyst may find necessary to consider is the treatment of unallocated output. In basic transactions tables, some production within the system inevitably will not be identified by point of origin or by point of destination. These amounts are usually shown in a separate dummy undistributed or unallocated sector. For analytical purposes, undistributed output may be treated as a separate intermediate sector, with demands on it proportional to other activities, and demands by it on other processing sectors proportional to its total. This was done, for example, in computations based on the preliminary 44-sector tables that have been published.

However, unallocated production, if it is treated in this way, affects input-output computations exactly as if it had been distributed among the other sectors proportionately to its own row and column. If undistributed production is a reasonably small proportion of output throughout, this need not cause any particular concern. However, it may directly contradict well-founded judgments about the processing system. For example, suppose one industry had a substantial volume of unallocated output and a second industry a substantial volume of unallocated input. Treatment in the manner described would imply that a substantial volume of the unallocated production moved from the first to the second industry. However, it might be felt that movements from the first to the second sector had been well established both empirically and functionally, or even that any movement of goods in this way was improper. For example, it would be irrational to presuppose any substantial sales of textiles to the machine tool industry simply because the former had substantial unallocated output and the latter unallocated input.

Since even negative judgment of the type mentioned is better than no judgment at all, there is some reason for eliminating the unallocated sector by distribution of its content to other sectors on any reasonable basis available. This has been done for the 190-order tables. The statistical foundation for this, almost by definition, is extremely scanty, but it is felt that the result improves the tables as a basis for establishing a function system. However, the amount of undistributed production in these tables was, in the first instance, relatively small, so that its distribution on even a wholly arbitrary basis could not have much effect.

In general, base-period transactions records will be published with undistributed production specifically identified. The analyst, in using these tables to set up a function system, must decide for himself how undistributed production should be treated.

4. COMPUTATION

Computation problems will not be discussed here, except to point out the obvious fact that the volume of work required increases rapidly with the number of sectors considered. For most problems, the computation load varies approximately with the square of the number of sectors. Satisfactory computation methods for the solution of input-output problems exist, and the size of the linear simultaneous systems of this type that can be handled is limited only by resources and the speed of the computation machinery available.

Error control and rounding problems, which may be troublesome in some types of large simultaneous systems, are mainly absent from input-output computations. The larger-scale computations, in particular, are best left to specialists with access to modern high-speed computing equipment.

5. INTERPRETATION AND ANALYSIS OF RESULTS

The immediate end result of input-output computations of the type considered here is a set of production figures representing the levels of activity implied for the different sectors of the economy under the assumed conditions. The computations usually are carried out in monetary units, and the results expressed in base-year price levels. As indexes, the production figures may be considered as quantities weighted by base-year prices. These indexes are not the same as the implicitly value-added weighted indexes of the Board of Governors of the Federal Reserve System, and this should be kept in mind in making any comparisons. The production indexes implicit in input-output calculations are of such a type that, when used with a proper simple aggregative production-weighted price index, the movements of total value of output are defined.

In some cases, the production index results, considered in their proper conceptual context, will provide the essential answer to the problem initially posed. For other problems, however, the production level results may represent only an intermediate stage. Real interest, for example, may lie in the employment implications of the conditions initially assumed. Here, additional estimates of productivity and of changes in working hours may be used with the production figures to yield industry-by-industry employment estimates. A great many other translations may be required in given cases, each making use of varying amounts of associated data. The problems involved will typically be those of the special interest fields, and outside the scope of the present discussion.

A question that is frequently raised may be mentioned briefly at this point. It is sometimes feared that a large number of relatively minor or inconsequential errors may cumulate within the large equation systems used and serve to produce substantially larger and serious errors in calculated production level results. A paper by one of the authors, published elsewhere, deals with this question; it is beyond the scope of the present discussion.[1] However, it may be

[1] W. Duane Evans, "The Effect of Structural Matrix Errors on Input-Output Estimates," *Econometrica*, October 1954, pp. 461-479.

said here that, while errors of commission or omission anywhere within the estimating structure will affect the results derived from it, one of the powerful features of the input-output approach is that it tends to make the inevitable errors of detailed empirical measurement compensating rather than cumulative. These properties are so strong that for some types of problem one might almost consider the approach as a device for getting answers of a better quality than the analyst has any right to expect, considering the quality of the raw materials.

One general consideration may be pointed out again. The amount of work that the analyst will normally consider necessary in carrying through an input-output analysis will depend most particularly on the problem he is trying to solve. For example, in setting up a complete industrial mobilization analysis, almost any part of the demand and processing structures assumed may turn out to be significant. Hence, a relatively detailed classification system will be used, and the demand structure will be established only after much critical examination. New and detailed studies of input requirements for some sectors may be required before it is felt that an adequate functional representation of the system can be put together. Because of the large size of the system, the computation load will be heavy, and analysis of the results obtained will explore many potential resource limitation areas and involve use of a large amount of collateral data. A careful analysis of this general type can hardly be undertaken except with a large staff, and even then it may require a substantial period of time for completion.

Consider, in contrast, the problem of a market analyst for some basic metals industry; he is concerned with the probable effect on the demand for the products of his industry of certain shifts in ultimate demand that he thinks may occur. In the first place, as pointed out previously, he may aggregate to a rather small number of sectors, retaining detail only where he thinks it is significant and making extensive consolidations in other areas. When setting up his assumptions regarding the demand structure, he will not produce pinpoint estimates for areas that will not affect his results substantially. He might well regard estimates for motor vehicles, other major consumer durable goods, construction, industrial machinery, and a few other items as the critical elements for his problem and concentrate the bulk of his attention on estimates of demand in these areas.

His consideration of structure will also be simplified. Presumably, he will be acquainted with, and have ready access to, information on technological changes in metals-use that affect the more immediate markets of his sector, and will exercise considerable care to insure that whatever knowledge he has of such changes is reflected in his analysis. On the other hand, he may have access to, but may ignore, information on shifts in relative uses of synthetic and natural fibers, feeling that a change in his estimating structure to take account of his knowledge will not improve his conclusions enough to repay the effort it costs him. The small system used will reduce the volume of computations. Finally, because he is interested only in the implied change in production levels for a single industry, the interpretation and evaluation of his results will likewise be simple.

There is no typical volume of work implied by an input-output problem; it depends on many things, but mainly on the question that the analyst seeks to answer.

C. Work Materials

The interindustry-relations or input-output approach is specifically intended to permit the use of large amounts of empirical data, and to a substantial extent these data must be specially compiled. It was felt that it might be useful in this paper to catalogue, briefly but with as complete coverage as possible, the special collections of current information and data that are now, or in the near future may be, available for general use.

The first materials resulting from the BLS study of interindustry transactions in 1947 were published in 1951. This release included a transactions table, an input coefficient table, an inverse matrix computed on the basis of a proportionality assumption, and accompanying descriptive material. The transactions table distinguished 45 intermediate and 5 autonomous-demand sectors. In the computation of the inverse matrix, new and maintenance construction was moved from the intermediate to the autonomous area, resulting in a 44-sector inverse matrix. Undistributed production was treated as a processing sector, and no special adjustments for scrap and waste materials or by-products were made. These tables have been widely reproduced and are now very generally available.

A transactions matrix for 1947 distinguishing 190 intermediate and 10 autonomous-demand sectors has also been prepared. A coefficient matrix and a corresponding inverse matrix incorporating the proportionality assumption have been computed, and all three are available for general distribution. As indicated previously, certain adjustments not contained in the earlier small tables have been carried through in these computations. Adjustments for scrap and identified by-products have been made, and all originally unallocated production has been distributed within the tables.

A detailed description of the classification system embodied in the 200-order tables in terms of both the Standard Industrial Classification and the 500-order system used in the preparation of the basic data is also available. The document includes an identification of the main sources for the basic information, the definition of the sectors in terms of gross output, a descriptive statement on the content of each sector, and a reconciliation of the accounts with the *Census of Manufactures: 1947.*

The published tables refer to the year 1947 exclusively. Partial revisions of the processing structure to reflect more current operating conditions have been made for a number of sectors in connection with the interagency program. The most important revisions refer to uses of steel, copper, aluminum, fuels, and synthetic fibers. Security restrictions prevent general publication of these revisions at the moment, but it is hoped that they can be released in the future. An inverse of the revised matrix is available within the government.

Back of all the summary tables are the basic industry reports for the sectors distinguished in the 1947 study. Many of these include, in addition to summary statements of the transactions among industries, varying amounts of information on the sale or use of specific commodities. These basic industry reports are being put into form for general publication as rapidly as possible. However, those that have nearly reached this stage refer, in the main, to relatively minor industries. Reports for the larger industries are typically more complex and take longer to prepare. The gradual release of the detailed industry studies will be a slow process, because only a small part of available staff time can be allocated to this general information purpose.

Substantial amounts of auxiliary data have been reorganized within the 200-order classification system. Special employment estimates to correspond with the sector definitions used have been compiled for 1947 and 1951, and special-purpose indexes to bring them up to date are being prepared. Production indexes corresponding with the 200-order classification are in preparation. Special studies of productivity movements for the processing sectors of the 200-order matrix are also being made. These are intended not only to furnish estimates of changes after 1947 but also to provide a basis for estimating probable changes in productivity under various assumed conditions.

Some special-purpose price indexes to correspond with the 200-order classification system have been prepared. These are being superseded by a more careful analysis now under way. All current wholesale price quotations collected by the Division of Prices and Cost of Living, Bureau of Labor Statistics, are being coded to correspond with both the 190- and the 450-order classification systems. Special weights for the purpose of combining the price quotations to correspond with movements for input-output industries are being prepared. A related project, which is not yet under way, is the coding of results from the recent large-scale study of consumer expenditures to correspond with input-output classifications.

The Bureau of Mines has assembled a substantial amount of data on the use of critical materials and on requirements for fuel and power, all organized within input-output classifications. These data, while intended primarily for use in industrial mobilization feasibility tests, will be of value for many other purposes as well.

A substantial amount of additional data has also been collected within consistent categories for use in the dynamic extensions of the input-output approach, particularly for industrial mobilization programing and analysis work. Some of this information refers to construction and equipment requirements for the expansion of industrial capacity in specific areas. There have also been studies of inventory holdings within the productive system, and of time lags between processing stages (as represented by the different sectors), which are characteristic of United States industrial operations. It is expected that all of this information will gradually be organized for general use. There are substantial interlocking data compilations relating specifically to the production of munitions items, but virtually all of these are classified as containing security information, and there is no prospect of their release for general analytical purposes in the near future.

In addition to the above, there are many smaller data compilations and a large number of research memoranda and minor unpublished studies that have been prepared in the agencies working on the inter-industry-relations program. It is impossible to catalogue these, but some will undoubtedly be published in a variety of periodicals.

D. Areas of Use

1. NONANALYTICAL VALUE OF TABULATIONS

It has been pointed out elsewhere that input-output tables are valuable and useful in their own right, independently of any particular analytical or conceptual scheme in which they may be used. The tables represent a vast extension and synthesis of information from the *Census of Manufactures*, the gross national product accounts, the statistics of income, the foreign trade statistics of the United States, and a great variety of other sources, all organized and reconciled within a consistent framework. The value of the basic tables is greatly enhanced by collateral information on production, prices, employment, productivity, capacity, capital structure, and so on that has been or is being organized around them. There is no better way to acquire a personal appreciation of the immensity, diversity, and complexity of the American economy than to examine a set of detailed input-output tabulations.

The value of these tables in spotlighting gaps, discrepancies, or redundancies in the available body of national economic measurements has also been mentioned elsewhere. Such tables exert a powerful influence toward rationalizing and improving the national statistical reporting structure. However, since it is presumed that the immediate interest is more in the potential analytical uses for interindustry-relations data and methods, these nonanalytical applications will not be discussed further here.

2. ANALYTICAL USES NOT FULLY DEVELOPED

The following description of some of the analytical purposes to which interindustry-relations methods may contribute is not intended as a catalogue, but rather as suggestive of the possibilities for obtaining through such methods a greater insight into important economic problems. Any discussion of the potential areas for use of input-output data and techniques will almost surely be incomplete in any case. This field of research is just emerging from an initial research and development stage. Only recently has a volume of empirical data sufficient to permit general analytical use become available. Efforts at practical application have been largely limited to a few very specific problems. Other potentially important areas to which input-output data and techniques will contribute may easily not have occurred to the present authors.

3. RELATIONS OF MAJOR STRUCTURAL ELEMENTS

There are three major elements or areas that may be distinguished and considered separately within the interindustry-relations framework:

1. The processing structure of the economy (considered as a set of functional interconnections among the intermediate sectors and determined in large measure by technological and institutional factors);

2. The demand structure (which will usually be closely related to concepts and measures of the gross national product from the product side);

3. The factor payment structure (which will be closely related to the gross national product from the income side).

The analyst, as he requires, may consider the first, the first plus one other, or all three of these major structural elements as established for a reference period for which an analysis is to be made. The reference period may or may not be the base period for a set of input-output tabulations.

The analyst may limit his attention exclusively to the implications of the interrelationships among the variables that he has distinguished in the three major structural areas. For example, he may be concerned with estimating the part of iron and steel production associated with consumer demand for processed foods in a reference period. Alternatively, the analyst may change some part (or all parts) of any of the three major structural areas to observe the effect of the change on any other variables embraced within the system. For example, he may change the processing structure to observe the consequences of an expected material substitution or other technological change, or he may even add currently nonexistent sectors to the intermediate processing structure. He may increase a part of the demand structure to determine the additional requirements that would be laid on all sectors throughout the system. He may alter factor payments to observe the consequences in terms of price levels. Some problems may be concerned with the joint effect of changes in all three of the major structural areas.

Finally, the analyst may bring in other variables that are external to the immediate input-output analysis. These may be used in setting the frame of the problem. For example, consumer demands might be set in relation to expected changes in expenditure patterns and income distributions, or factor payments specified in relation to expected wage or tax rate changes. Perhaps more frequently, external variables will be brought in to assist in interpreting or evaluating the results of an analysis. For example, it may be desired to examine estimated output levels in terms of their implications for employment or industrial capacity utilization.

Simple input-output models might perhaps be classified on the basis of the type and kind of assumptions made regarding the processing, demand, and factor payment structures, whether the analyst was concerned with totals or differences, the kind and amount of collateral data brought into the analysis, and so on. The number of possibilities, however, is so great that it is doubtful whether such a classification scheme would be very useful. No consistent classification of possible models is followed in the ensuing discussion. At the beginning, a few of the more general model possibilities are listed, and the discussion then turns to some subject-matter areas in which the input-output approach may be used.

4. PRODUCTION-REQUIREMENTS ANALYSIS

The desire to determine the specific production requirements that may be laid on the complete processing system by a specified amount and kind of end-product or finished-goods delivery accounts for the main current interest in interindustry-relations research. The reasons for an interest in this particular type of problem have been touched on briefly in the first part of this paper, and others will appear later.

In its simplest form, a production-requirements problem may involve only the processing structure of the economy. One may ask what additional output would be required from each intermediate sector to support exactly an increase of a single unit in deliveries to the autonomous sector by a specified intermediate sector. The mathematical statement of the problem will be as in equation (6) above with all entries on the right-hand side reduced to zero, except for the one sector under examination. The computation is equivalent to finding a single column in the inverse of the matrix $(I - A)$.

The result of a computation of the type described might be verbalized as follows: To permit the continued delivery outside the intermediate sectors of an additional unit of output from a given sector, additional production will be required not only in this sector but in all other sectors of the economy called on directly or indirectly to supply needed materials or services. The computation establishes quantitatively the increases that are needed. For example, one might ask what additional production requirements will be laid on the economy by the delivery to a purchaser in the autonomous sector of one additional unit of motor vehicle production. The motor vehicle industry will require for this purpose additional electric power, steel, and other materials and services. The industries supplying these requirements may also require additional electric power, steel, and so on. The total increased power requirements will be the sum of all the direct and indirect increments throughout the processing system. Clearly, there will be cases where indirect requirements will exceed by many times the direct purchases of an item by a given sector. For example, on the basis of the 1947 study, total requirements for electric power by the motor vehicle industry cumulate to more than five times the direct purchases.

For a wide variety of problems, the very broad and general importance of figures showing the specific impacts, throughout the economic system, that can be anticipated because of a particular and specific change in demands is readily apparent. It is an obvious extension to consider not simply unit changes for single sectors but a complete schedule of changes in autonomous deliveries by all sectors. Such problems may arise in many ways, some of which will be illustrated later.

5. FACTOR-PAYMENT EXTENSIONS OF PRODUCTION-REQUIREMENTS ANALYSIS

For many purposes, the results of production-requirements computations will be of interest as they stand, but in many instances there will be a desire to appraise the results in terms of associated variables. Among these are the factor payments that may be generated by a given amount and type of autonomous demand.

In delivering goods or services in response to autonomous demands, a sector will normally make payments for factors of production (that is, to the autonomous sectors on the income side) as well as purchase goods and services from other intermediate sectors. These sectors, in turn, will make factor payments and transmit a diminishing series of demands for goods and services to other intermediate sectors. Eventually, value equivalent to the total of the initial autonomous demands will have been disbursed outside the processing structure in terms of factor payments.

Given a fixed processing structure, the factor payments associated with any stipulated demand schedule may be estimated both by type and by sector location. For example, one may compute the wage and salary payments in each intermediate sector corresponding to any given schedule of autonomous demands. The computation may be done directly or through use of an inverse matrix.

The effect of the computation is to associate a given amount and composition of factor payments with a unit volume of output for each sector. This type of analysis may be considered as a direct extension of the production-requirements analysis. For example, if the delivery of one unit of output from sector *A* to the autonomous area required directly and indirectly one-tenth unit of output from sector *B*, and if each unit of output in sector *B* was associated in the period of analysis with thirty cents of wage and salary payments, it would be implied that delivery of one unit of output by sector *A* to the autonomous area would be accompanied by three cents of wage and salary payments in sector *B*.

An analysis of the type described presents many interesting possibilities. For example, an autonomous demand for $1 million worth of agricultural products will correspond with a total of $1 million of factor payments, but these will be distributed in a definite pattern. Comparing, say, agricultural products with petroleum refining, one might expect induced factor payments to be weighted percentagewise in the first case more to wage and salary payments and in the second case to capital consumption allowances. Tables showing the percentage distribution of induced factor payments for each processing sector in a recent period would shed light on the implications for various forms of income payment of changes in demand schedules. For example, a particular shift in demand might have the tendency throughout the economy to move production toward higher-wage industries. The same total of end-product demand with a different distribution might make relatively heavier demands on capital-intensive industries. The tables would also permit judgment of the effect on noncompetitive imports of a shift in demand structure.

There is the possibility of looking at the figures in a slightly different way. The delivery to end-product consumers of agricultural products, for example, will induce factor payments not only in the agricultural sectors of the processing system but in extractive, manufacturing, distributive, service, and utilities sectors as well. Since the factor payments do not contain duplicated elements, they are additive. Thus, we may determine the percentage distribution of factor payments among agricultural, manufacturing, distributive, and other broad sectors induced by the delivery of agricultural products to end-product consumers. In a general way, this would indicate which sectors in the processing system have added the value delivered to ultimate buyers. It is an extension of the familiar value-added-by-manufacture notion, but takes into account indirect as well as direct contributions. Thus, it is possible to speak, for example, of the value added by distribution in the production of agricultural products as compared with textiles or durable goods.

6. RELATION OF PRODUCTION REQUIREMENTS TO EMPLOYMENT

The argument so far has run as follows: Given the processing system of an economy as represented by an equation system, the requirements on each of the intermediate sectors imposed by unit autonomous deliveries from ony one can be computed. If these computations are joined with any given demand schedule, the production levels required in each sector to support the deliveries may be estimated. These production levels may be associated with given amounts and kinds of factor payments. So far, all quantities have been those that are normally directly distinguished and recorded within basic input-output tabulations. The implications of the analysis may, however, be readily extended to other elements in the economic system, if these can be associated functionally with any variables that are a part of the basic analysis. A natural extension is to the area of employment analysis.

To accomplish this, employment and production must be related within a given sector. For example, a hypothetical problem might imply that production in a sector would fall 10 per cent from base-period levels. A simple but not necessary assumption would be that employment would fall by 10 per cent as well. The conditions of the problem might lead one to assume that productivity would rise or fall, or that working hours would be increased or lessened. Given assumptions about productivity and working hours, however, it will be possible to convert a statement about changes in production levels into a statement of changes in employment requirements.

Perhaps the simplest manpower analysis would be a set of estimates of the amount of increased employment expected in each sector because of a stipulated increase of one unit in autonomous deliveries by a single sector, assuming no changes in productivity or working hours. A complete table of this kind (which has been called an employment inverse) would show quite directly the amount and location of employment change that would be expected because of any shift in end-product demand schedules.

Such tables would contribute to the appraisal of proposed alternative actions designed to alleviate unemployment. The federal government is committed by the Employment Act of 1946 to take corrective action in the event of recession or serious unemployment. To the extent that any proposed course of action or policy can be formulated in terms of the specific additions to autonomous demands that it would imply, the amount and industrial location of employment increase to be expected could be estimated. Since some industries are labor intensive relative to others, two courses of action that were identical in total fiscal effect might yet have different employment-generating possibilities. Perhaps of greater importance, all sectors of the economy are not equally affected by recession conditions. For example, the dip of 1948 was mainly a decline in soft goods, but the depression following 1929 most seriously affected the heavy industries. The detail afforded by an input-output analysis offers the possibility of appraising different proposals for corrective action in terms of their adaptation to the specific problems of the moment.

Such tables can also be used to determine the amount and location of employment associated with current or expected patterns of exports from the domestic economy. Several such analyses have been made in the past. It may be of significance to those in an industry to know that, even though no direct exports of their products are made, they are nevertheless used by other sectors in carrying on activities that produce exported commodities. Previous analyses, for example, have shown that dependence of the steel industry on export demands is typically about double the direct export of steel products. The relationship of domestic employment to foreign trade can also be extended in the other direction. Estimates can be prepared of the amount of domestic employment that would be required to balance reduced competitive imports. It is possible to appraise the extent to which current patterns of exports and imports are relatively labor intensive.

Employment may be associated with other parts of the demand structure. For example, production and hence employment in one industry may be shown to depend primarily on ultimate consumer demands; in another, on capital formation, or perhaps on government purchases. Shifts in the relative importance or composition of these major categories of demand may be followed by changes in both the amount and the industrial location of employment.

Labor as a potential limiting resource may be considered when a full schedule of all autonomous demands is joined with a processing structure to give production and employment estimates. In such an economy-wide model, the first question investigated will often be whether estimated total employment will exceed or fall short of an estimate of the labor supply. Collateral information may permit examination of the sector employment estimates in terms of occupational composition, with perhaps some conclusions on feasibility in relation to available skills or requirements for future training. Some inferences about possible shifts in the geographical location of employment may also be made.

A further angle is that of productivity. Final-demand deliveries from a given sector will be associated at some (historical) time with employment in the sector, and also with employment in other sectors that supply required materials and services. An input-output calculation permits one to estimate the cumulative man-years or man-hours throughout the economy associated with, say, $1 million of delivered output from a given sector. Such a figure is interesting in itself, and its reciprocal becomes a statement of productivity for the industry that takes into account not only labor employed by the industry but also all labor embodied in the materials and services it requires from the economy as a whole in connection with its current activities. Such productivity figures have some interesting properties. To the extent that trends in productivity can be measured through indexes of this general type, some measurement problems that have afflicted the more conventional forms of productivity index can be met. In particular, such indexes will account for economies in the use of materials and the labor-saving potentialities of new materials.

7. RELATION OF PRODUCTION REQUIREMENTS TO OTHER RESOURCE IMPACTS

A fairly typical input-output problem might be the following: A processing structure that is felt to be an adequate representation of a period a few years in the future is established. This is joined with a possible demand schedule whose implications it is desired to investigate. The set of sector production levels consistent with the assumed demand and processing structures is computed. A series of investigations may then be made to determine whether, in the light of existing current information, these production schedules seem feasible or practical. An investigation of manpower feasibility has already been described. A second check might be in terms of industrial capacity.

Adequate measures of industrial capacity exist for some, but by no means all, sectors of the economy. Where a suitable measure is available, a comparison with an expected production level can be made. If the comparison indicates no imbalance, there is reason to feel that at least in this area lack of industrial facilities will not make the proposed production schedule impractical. If it appears that current capacity will be exceeded, one might investigate whether the implied addition to capacity would be physically practical and perhaps form conclusions as to whether it would be likely to occur in the normal course of economic development, or whether it could only be brought about by extraordinary efforts. One might conclude that changes in the processing structure (for example, materials substitutions) could eliminate the discrepancy. Finally, it might appear that the assumed demand schedule would not be industrially feasible. Of course, the analysis may not show capacity overloads, but rather the possibility of unused facilities.

Estimates of the impacts on physical natural resources may also result from an analysis of the type described. Physical resources typically enter the processing system at a limited number of identifiable points. If the use of given natural resources or materials can be functionally related to the operating levels of one or more sectors of the economy, the implied disappearance of natural resources can then in turn be related to a given schedule of autonomous demands or finished-goods deliveries. Any general treatment of the physical resource problem will require extensive use of collateral data and analyses. A substantial volume of work in this area currently under way at the Bureau of Mines is intended to be used not only in connection with current critical materials problems, but also to give insight on longer-range resource impact problems.

8. PRICE EQUILIBRIUM PROBLEMS

It was pointed out in section A that, given a processing structure and a set of output levels, the additional stipulation of total factor payments in each sector will permit computation of the unique set of prices consistent with the assumptions. Similarly, the set of price changes corresponding with a change in part or all of the factor payments structure can be estimated. Given the necessary data and assumptions, computations of this type are perfectly straightforward. The interpretation of results, however, may be anything but routine.

A great deal of the confidence that people in the field have in the use of input-output methods for investigating production-requirements problems is due to the structural similarity between the method of analysis and comparable operations in the real economy. A demand for additional motor vehicles does, in fact, induce the automobile maker to order more steel, more tires, and other materials. A continuing greater demand for automobile upholstery fabric requires the textile manufacturer either to reduce production of other textiles or to purchase more textile fibers. The analogy between the form of input-output computations and the way in which increases or reductions in orders for materials and services are passed through the economy is a very close one.

The same analogy could exist (but it probably does not) in the transmission of changes in factor costs through the economy. The analyst may be given a problem of the following type: Assume that the current processing and demand structures remain the same, and that all elements in factor payments remain unchanged, except that wage levels in a designated sector are increased a given amount; what changes in prices may be expected to result?

The use of an input-output model for such a problem presupposes an orderly passing forward through the economy of a change in costs, with each sector making the exact adjustment necessary to maintain its former profits position, and with an eventual arrival at an equilibrium position before other dynamic elements have intervened. The actual mechanism of price change in the real economy bears no close resemblance to this placid process. There is the initial question as to whether the assumption that other factor payments within a sector will remain constant with a change in wage rates is at all realistic. Beyond this, there may be anticipatory price or factor cost changes in other sectors that are in no sense a necessary consequence of changes in the first. These, in turn, may set in operation a chain of price effects. Many human, and hence fallible, judgments on the extent and timing of desirable, proper, or even possible price adjustments will be involved. The model can show the *necessary* changes in price required for equilibrium under the assumed new conditions, but the *actual* responses of the economy in such a situation may be quite different.

These cautions are directed mainly toward attempts to determine the short-run price effects of relatively minor changes. They apply with less force to an analysis of the expected consequences of broader changes in factor costs over longer periods, or to efforts to identify factors that may prominently affect or determine price changes for a given sector. They apply hardly at all to a *de facto* analysis of causal interconnections in historical price movements.

There will probably be circumstances under which the determination of price levels consistent with factor payment changes (or, carrying the analysis a little further, the interrelations among factor payments at the same or different price levels) may be useful, important, realistic, and hence well justified. However, care and good judgment must be exercised by the analyst at all points to insure that such analyses do not lead to unrealistic or misleading conclusions.

9. MARKETING ANALYSIS

The situations discussed so far in this section have been rather general in character in that, while they might be complete in themselves, they might also be incorporated as parts in models constructed for widely different purposes. The remainder of the discussion will be directed toward specific areas of interest.

It has been pointed out elsewhere that any single row in a base-period transactions table is equivalent to the usual market analysis showing the distribution of the output of an industry among its various immediate purchasers. A similar distribution can be obtained for any period for which complete demand and processing structures are stipulated. A production-requirements analysis will show the total demand (and, if desired, its implied distribution among immediate purchasers) for the output of each sector.

The production-requirements analyses so far described have been backward-looking, in the sense that one started with a statement of autonomous demands and worked back through the processing system to determine the requirements upon production. It is equally possible, given the demand and processing structures, to trace the output of a given sector forward to determine its ultimate dependence on specific expressions of demand by autonomous sectors. It is considered that the reason for an intermediate sector's activity is either to deliver goods and services directly to specific autonomous purchasers or to provide goods and services to other intermediate sectors. These sectors in turn will produce and deliver goods with the same objectives. Eventually, all parts of a sector's output level may be considered as caused by specific autonomous demands. The dependence may be quantitatively established through a routine input-output computation. This possibility of establishing a measurable connection between the amount and type of end-product demands and the production level of a sector whose direct customers may be mainly industrial is of the greatest potential importance for general market analysis in many industries.

A useful extension of input-output data in the marketing field that does not depend on use of a general equation system is possible. Suppose it is assumed that the materials required to produce a sector's output is independent of the geographical location of plants classified in that sector—for example, that a paint factory will require the same inputs whether it is located in Massachusetts or Tennessee. The industries supplying paint manufacturers can be ascertained from the input-output reports or more aggregative tabulations. Combining existing data on the geographic location of paint establishments, these suppliers may then estimate the proportion of their sector's immediate market that is located in various regions of the country. There will be numerous exceptions to the assumptions made, but there will be enough validity in many cases to make the results valuable for considering such problems as the desirable distribution of regional sales efforts.

10. REGIONAL AND INTERNATIONAL ANALYSES

The establishment of any part of a national input-output analysis, whether in the processing, demand, or factor payments structures, quite naturally has implications with respect to the regions within the country. Changes in national demands that may lead to changes in employment, production, or incomes will imply regional changes as well, and the regional impacts may be unequal. An expected reduction in aggregate consumer demand for new motor vehicles, for example, might be expected to have immediate repercussions in the Detroit area. Unless this reduction was balanced by increases in demand for other steel- or rubber-using products, some of the effects would be transmitted to Pittsburgh and Akron. Any of the many areas in the country that supply goods and services needed directly and indirectly in the manufacture of motor vehicles could be expected to feel some effects. This approach to regional impacts is, of course, enormously oversimplified, but it suggests that some rough regional inferences might be drawn from strictly national input-output computations.

A more adequate consideration of regional economic problems by input-output methods will typically require the availability of regional input-output data. For rough analysis, national patterns of materials and service requirements by industry may simply be applied to regional production figures (as in the tables recently published for the Eighth Federal Reserve District). This will give some effect to regional differences in the composition of production, and will permit the carrying through on a regional basis of some of the analyses previously described. In particular, many questions relating to the regional balance of trade with the remainder of the country and the world may be examined. The approach suggested is, of course, at best a first approximation. More careful studies of processing and demand structures within a given region will be a prerequisite for refined analyses.

When data on demand and processing structures are available for two or more regions and the details of trade among them can be specifically identified, the input-output framework offers many possibilities for the study of problems of mutual economic balance among them and with the rest of the world. Any hypotheses regarding changes in the demand or processing structure for any region, or changes in the structure of trade among regions, can be investigated in a logical manner. Analyses of this general type will usually refer to nations, rather than regions within a nation, because of data limitations. Information on foreign trade by country of origin and destination is commonly available, but such figures for regions within a country are usually lacking.

Many nations have very difficult problems of internal production balance in relation to their foreign trade because exports may require imports in their manufacture, and an attempt to manufacture at home goods that can be imported may affect export possibilities. A large number of indirect production-requirements balances are clearly involved. The input-output approach, by its nature, should contribute to the solution of such problems. In an advanced form, the approach offers the possibility of analyzing the conditions under which a domestic standard of living may be enhanced, keeping in mind resource limitations and the necessity for maintaining a balance between imports and exports. Such a model would fall in the linear-programing field.

11. INDUSTRIAL MOBILIZATION ANALYSIS

It is because of the necessity for doing a better job in industrial mobilization analysis, as applied to both partial and full mobilization periods, that most current developments in the field of interindustry economics are under way. Many of the potential applications of input-output analysis to industrial mobilization problems have been suggested in the earlier text and elsewhere. There is no need to describe the applications in detail, since any large-scale industrial mobilization analysis will almost surely be carried on within the government itself. However, some of the uses may be indicated briefly.

First, the detailed production requirements laid on the processing system by any specific item of munitions procurement may be estimated. Production of $1 million worth of tanks implies the preempting of a part of the output of a great many industries. The amounts can be established with reasonable confidence through input-output analysis. The pattern of requirements for a heavy bomber will be quite different. A catalogue of implied requirements per unit delivered can be compiled for various items of munitions procurement; with information on the critical or tight areas of the economy, the compilation will help to indicate quickly and directly where changes in procurement may run into difficulties.

An obvious extension is to apply to such patterns any proposed total schedule of munitions deliveries. With some knowledge of the resources available for military production within the economy, probable problem areas can be located. The problem can be set up in converse form. End-product delivery allowances for the nonmilitary side of the economy under conditions of varying stringency may be estimated. The implications of these for manpower and industrial-capacity use in the processing system may be computed, and an estimate obtained of the resources that can be devoted to military production under varying conditions.

End-product deliveries for both military and nonmilitary purposes may be stipulated and joined with a processing structure to determine the implications of these schedules in terms of limiting economic factors, such as manpower, industrial facilities, and natural resources, as previously indicated. Given data in the proper form, many alternative possibilities can be considered quite rapidly to permit quick judgment of possibilities that should receive further attention. The possibility of rapid consideration of many alternatives might be quite important, for example, in connection with questions of adjustment to bomb damage.

A somewhat more elaborate formulation permits stipulation of desired or postulated military and nonmilitary autonomous deliveries in a number of succeeding time periods, and incorporation in the estimating system of estimates of necessary lags between processing stages. More advanced models may incorporate additional dynamic elements.

12. FORECAST MODELS

The future is not always seen as through a glass, darkly, but sometimes as through a brick wall. Economic forecasters, in particular, have had their vicissitudes. Poor results from forecasting, however, have not shaken interest in it. The reason, of course, is that so many contemporary actions must be conditioned by, and based on, expectations for the future.

Curiosity about the future, however, seems to go far beyond any necessity for providing a basis for current decisions. Speculations about the future seem always to have a ready market and, if cryptic enough, even a steady market. The veiled prophecies of Nostradamus, four centuries old, still have their adherents.

Under the circumstances, it is inevitable that any new tool of economic analysis should be examined in terms of whether or not it helps in forecasting. Input-output methods have received a due (and in the authors' opinion, an undue) amount of attention in this respect.

Before continuing, a digression to clarify the content of the word "forecast" seems desirable. Some lack of agreement on the role of input-output methods in forecasting seems to have arisen because this term is sometimes used without distinction in two rather different senses. Take, for example, the statement, "If it rains tomorrow afternoon, the baseball game will be canceled." In some limited sense this may be called a forecast, but to call it so is to debase the meaning of the word. One can imagine people with widely different expectations about the weather agreeing on the statement itself. The truth of the statement does not depend on what the weather will be, but rather on well-established and well-known practices in connection with baseball games, and any forecast element involved in the statement refers only to this latter element. To permit such statements to be called forecasts is to permit any forecaster to fatten his reliability average as much as he likes.

Consider the alternative statement, "It will rain tomorrow afternoon and the ball game will be canceled." This includes the preceding statement and an important additional element—a belief that the conditional statement of the preceding proposition will be fulfilled. Any confidence that we may have in the truth of the statement will depend primarily on our feelings about the weather; for example, on how frequently in our remembered experience days similar to today have been followed by rain the next day. Statements of this kind constitute forecasts in the full sense of the word. The distinction is useful, and it is commonly accepted. The Weather Bureau, for example, does not permit its forecasters to improve their accuracy ratios by making statements like, "If it rains tomorrow, the sky will be cloudy." In the following, the word forecast will be used in its complete sense.

These remarks have seemed necessary, since there has been some disposition to treat all input-output analyses as if they were forecasts. The majority are not forecasts in the full sense of the word, but rather conditional statements that are established as well as possible under the circumstances. The input-output approach is primarily a tool to help the analyst make conditional statements about the economy. It is a tool that may be used by forecasters as well, but it is not primarily an instrument of prophecy.

Before continuing on this subject, we may consider the features of an input-output forecast model. In setting up such a model, the analyst will be concerned with a specific time in the future, say five years from now. He will first examine current and historical records that may help him establish what he regards as a reasonable functional representation of the processing system for the year he is considering. It is clear that, if he has the research resources, he may draw on an enormous range of data in this task, and that it will give full scope for his ingenuity and judgment. The analyst may then set down the complete schedule of autonomous demands that he feels will characterize the selected year. Again his possibilities are almost unlimited. He may examine changes in patterns of consumer taste. He may consider the changing role of government in society, whether other countries will require more or less goods from the United States, whether cold war tensions will be important, whether capital formation will be more or less important, and so on. Having established quantitatively his expectations for the processing and the demand structures, he may join them to determine the sector production levels that they imply. Quite typically, he will extend his consideration to other questions. For example, taking into account expected changes in population, labor force participation rates, productivity, and working hours, will the demand schedules that he assumed provide adequate levels of employment? What changes in the capacity of industry may be implied by new and higher production levels, and are these consistent with current trends? What uses of natural resources are implied, and are these compatible with today's plans for resource development?

The analyst may in addition set down his expectations on payments to the factors of production if he desires, giving his model a price structure as well. A model in which both the demand and the income structures are specified, with some logical interconnection between them, is usually called a closed model, indicating that a balance between factor payments and demands has been sought. Input-output forecast models may be elaborated in various ways.

It has been suggested that the utility of input-output models is limited by the "ability to forecast," referring specially to forecasts of the probable outlines of demand structure in a future period. This limitation obviously applies only to forecast models, that is, to those in which the analyst has prepared a processing structure, a demand structure, and/or an income payment structure that he believes will characterize some future period. In such models, the analyst's results will clearly depend on his ability to forecast every element included. The limitation does not apply, however, to interconnection analyses of the expected consequences of current or historical changes in parts of the demand, income, or processing structures, in short, to a majority of the uses that have been described. It does not even apply to many analyses that in fact extend into the future, including very particularly many industrial mobilization analyses. Typically, there is in these no commitment on the part of the analyst, or the policy maker for whom he may be acting, that any specific schedule of demands incorporated in such models is one that is to be implemented, or that anyone expects will be carried into effect in its entirety. Most such input-output analyses are of the "if-then" type; if such and such takes place, then, within the limits of current empirical knowledge and analytical ability, the following consequences are implied. Input-output models that are true forecasts, in the sense that the "if" statements are expected to be fulfilled, are the exception rather than the rule in current work.

An element of forecasting relating to the future does occur in industrial mobilization models, but not in the demand structure. It is the processing structure incorporated in such a model that may be considered almost in its entirety as a forecast. The analyst feels that he can make such a forecast because he believes that technological or other changes will not be sufficiently widespread to alter current materials and service requirement patterns beyond use, or that he knows enough about such changes to make adequate allowances for them. In this area of forecasting, there is some reason to feel that the analyst is on firmer ground. Most people would feel more confident in stipulating that two years hence automobiles will still have four wheels and a gasoline engine and will use steel as a major structural material than in trying to guess just how much consumers will spend for new passenger cars. The analyst, like the baseball fan in the example given earlier, feels that in the main the rules will not change, and when they do he will get advance warning.

To summarize, input-output analyses may fall in one of three general classes. There are those that relate exclusively to the present or past; such analyses are usually designed to increase our understanding of the complexities and interdependence of the economy, and they involve no forecast element. A second class, which includes most mobilization analyses, conjoins a particular demand structure (as a hypothesis) with a forecast of the processing structure appropriate to the situation. The purpose of such models normally is to facilitate a rational choice among possible alternatives by examining the logical consequences of each. The *validity* of such models depends on the analyst's ability to forecast the main features of the processing structure; their *utility* depends on the judgment and common sense exercised in portraying the hypothetical demand structure. Finally, there are models in which both the processing and demand structures are forecasts. The validity and usefulness of such pure forecast models will depend on the degree to which the future structures are actually approximated. The problems of making a realistic forecast of demand structure are likely to be the more difficult, and the value of the model will chiefly depend on success in this area. It may be noted in passing that the problem of forecasting the structure of demand is not peculiar to input-output forecast models; it is encountered in connection with nearly any kind of forecast model.

So far, only one reasonably complete input-output analysis covering a future period has been released; this was made in 1946 and published under the title "Full Employment Patterns, 1950" in the *Monthly Labor Review* for February and March 1947. Although it has been treated as such, this was not a forecast model. In fact, two sharp alternative situations were covered in the study, primarily to insure that it could not be considered as a forecast. On the other hand, it must be said that, if the authors had felt able to make reasonable forecasts of specific variables for 1950, they would have been incorporated in the model. Some of the figures that went into the model, such as estimates of labor force, population growth, and so on, were essentially forecasts. Just as definitely, others were not.

There is no point at this late date in attempting to discover the extent to which this early analysis could properly be considered to contain forecast elements; there are other things that should be said about it. The study was a part-time venture for a few people. It was a pioneer effort using a new method. Many of the techniques were improvised for this first use, and many of the data assembled from unpromising beginnings. (The latest *Census of Manufactures* data, for example, referred to 1939.) The study used as a starting point a matrix for 1939; it was not very detailed, distinguished only a limited number of processing sectors, left a substantial portion of the economy's output in the undistributed sector, and had been subjected to few of the checks now considered essential. There was little opportunity for analysis of the validity of the matrix, or for adequate revision of its structural contours to reflect postwar conditions. The entire analysis was extrapolated from a year of heavy unemployment at the end of a depression period (1939) to a year following a major war and a postwar readjustment. Under the circumstances, some slight deficiencies might be pardoned.

Hindsight being clearer than foresight, it is quite easy today to point out the many deficiencies in this study. This is useful to the extent that it helps avoid similar deficiencies in current work. The point in bringing up the study here, however, is to record the authors' feelings that too much attention has been given to this initial effort as a basis for appraising the potentialities of current input-output work. This early analysis has little relevance to the potentialities and validity of current input-output data and techniques. This comment is not made to disarm any criticism that may be made of the early study; its shortcomings were many, but the authors feel that it compares favorably with its contemporaries.

Forecasters have used, and will continue to use, many methods with varying proportions of objective content. Forecasting being what it is, one should in no sense deprecate the importance of the subjective element. However, there has probably been a gradual tendency to adopt methods in which the objective element is somewhat strengthened through use of increasing amounts of empirical data and a greater degree of mathematical complication. Perhaps the most important general development of the last decade or two has been an increase in the use of regression methods to supplement the intuitive elements in forecasting. The principal alternative to input-output methods for forecasting purposes today would probably be one or another form of regression analysis. Hence, some special comments on the comparative characteristics of these methods may be desirable.

In the typical regression analysis, a single dependent variable is to be estimated. A number of independent variables that are thought to be related to the dependent variable, and for which it is felt that a forecast can be made, are selected. The dependent variable might, for example, be steel production. The independent variables used might be the gross national product, the relation of some of its major components to the total, more specific figures, such as estimated construction activity or motor vehicle purchases, or others. A few independent variables might be selected rather casually, or a quite intensive and time-consuming analysis carried through to determine those best suited for the purpose. In any case, the parameters of an estimating equation that relates the dependent with the independent variables are finally established, on the basis of recorded historical values for the variables, usually by least-squares methods, and frequently including calendar time as one of the independent variables.

The regression approach is attractive primarily because it may be economical, especially if estimates are required for only a few independent variables. In general, a separate estimating equation is set up for each dependent variable and adapted to this single use. The approach has some limitations that, in particular instances, may be overlooked in favor of the speed and economy with which an estimate can be prepared.

The clearest limitation to the regression approach is that relationships among the variables used must not change, or must change in some functionally simple way during the historical period used to estimate the parameters, and this situation must continue through the future period for which an estimate is to be made. The assumption may be justified in a particular case, but it is difficult to establish that it necessarily will be true, and it interdicts the use of the method for cases where the conditions of a problem contradict the assumption. This would clearly be the case in many types of mobilization analysis.

Another limitation is not so well understood or so easily explained. It is unsafe to use a regression equation for estimating purposes when the combination of values of the independent variables in the forecast is not included among the combinations found in the historical period used in determining the parameters. (That is, the combination of independent variables in the forecast, considered as a vector, is outside the range of any scalar multiples of the same vectors in the historical period.) Some indication of the force of this limitation may be found in regression equations that attach negative coefficients to one or more of the independent variables. This clearly cannot be interpreted as implying that, if these variables were large and the other independent variables small, the dependent variable (say, steel production) would be a negative quantity. However, the limitation applies even where a regression equation contains only positive coefficients. It may be useful to illustrate this by using hypothetical figures taken from a memorandum on this point by Ellison Burton of the Bureau of Mines. The figures are shown in Table 1, which purportedly represents records over a period of

TABLE 1

	SECTOR 1	SECTOR 2			SECTOR 3		
	Total Output (X_1)	*Total Output* (X_2)	*Use of First Sector's Output*	*Input Ratio*	*Total Output* (X_3)	*Use of First Sector's Output*	*Input Ratio*
YEAR	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1941	30	45	23	.511	20	7	.350
1942	29	42	21	.500	33	8	.242
1943	51	73	36	.493	46	15	.326
1944	46	65	33	.508	43	13	.302
1945	45	64	33	.516	40	12	.300
1946	39	60	29	.483	35	10	.286
1947	35	54	26	.481	35	9	.257
1948	32	49	24	.490	31	8	.258
1949	29	42	22	.524	23	7	.304
1950	41	62	30	.484	30	11	.367

ten years showing the output level of a first sector whose product is entirely consumed by a second and a third, the output levels of these sectors, and their actual consumption of the output of the first. Input ratios (that is, the consumption of the products of the first sector per unit of output of the others) are also given and vary from year to year without noticeable trend. It is assumed that at any given time inputs and outputs vary proportionally.

A homogeneous regression equation between the output of the first sector (column 1) and the outputs of the other two sectors (columns 2 and 5) is fitted by least-square methods to give

$$X'_1 = .610X_2 + .115X_3 .$$

The standard error of estimation is 1.21 and the corrected coefficient of multiple correlation is .983.

Comparing the regression equation with Table 1, one makes the salient observation that, despite the low standard error of estimate and the high value of the correlation coefficient, the regression coefficients bear little resemblance to the input ratios found in any of the years. The first coefficient exceeds the maximum for any year, and the second figure is less than half the lowest.

There are surely real cases where it is possible to specify the minimum relative amounts of a material that are technologically possible for a given purpose. For the present example, let it be supposed that the minimums indicated in Table 1 can be used, that is, without a technological change (unforeshadowed in the data) sector 2 will always require per unit of output at least .481 units from sector 1, and sector 3 at least .242 units.

Considering these figures simultaneously with the regression equation, one finds that, if the latter is applied to any situation in which the production level in the third sector exceeds that for the second, an estimate for requirements on the first sector that is below minimum technological possibilities will be obtained. For example, a combination of the 1942 production level for sector 2 with the 1943 production level for sector 3, taken with the regression equation, will yield an estimate for sector 1 below the assumed technological minimum.

For many peacetime problems, the restriction illustrated may not be an important one. The general contours of demand assumed for a future period may be quite similar to those recorded in the present and immediate past. But exactly the contrary to this must be assumed for many mobilization analyses, and hence regression methods will be severely restricted in this area.

Returning to the example, suppose that some averaged or middle structural equation were used. For example, one might take by inspection

$$X_1 = .5X_2 + .3X_3 .$$

This equation on the average would not give results as good as the regression equation for the ten reported years. However, it would yield rather reasonable estimates for each year. More to the point, it will continue to give reasonable estimates so long as the technological situation remains unchanged, regardless of the balance between production levels in the second and third sectors.

The above argument depends, of course, on there being some reasonably stable, defined linear structure underlying the physical situation being considered. But this is exactly what is required if a linear regression equation is to yield anything other than a statement of coincidences. It may also be noted that the same development and conclusions would follow if the regression equation and underlying input relationships had been assumed to be linear rather than homogeneous.

The difficulties with the regression equation illustrated may be traced to the correlation of the production levels of the second and third sectors. If the data are rearranged so that the underlying structural situation is preserved but the correlation between the production levels is eliminated, a regression equation that represents an averaging of the input ratios shown will be obtained. While this is interesting, it is not necessarily useful. Economic time series, and especially those that may be used as independent variables for forecasting purposes, are likely to be quite highly intercorrelated.

It should not be inferred from the example given that a sufficiently perspicacious regression analyst would necessarily be trapped by the data. The intercorrelation in production levels, if investigated and properly evaluated, should give the analyst a sufficient warning against a false conclusion; it would also prevent him, if limited to regression methods, from reaching a helpful one.

To summarize, a reasonable structural relationship that accounts directly and positively for demand should give sensible results regardless of the values of the independent variables in the estimating equation. A regression relationship based on historical data, on the contrary, may in some instances yield estimates that contradict physical possibilities. The degree to which past variation is "explained" by the equation, as judged by the coefficient of correlation, is not evidence in determining whether a representation of the underlying structural situation has been obtained.

To leave the example, one may point out a few other restrictions applicable to regression analysis. The regression approach usually uses a rather small number of independent variables, in contrast to input-output analyses, where the number may be very large. It is not always realized that this, while a convenience when resources are limited, is also a flat necessity; it is imposed on the analyst, whatever his desires in the matter. The addition of more independent variables to improve a regression relationship is something to be undertaken with care. Whether the addition of another independent variable will bring about a real, or only a spurious, improvement in regression estimates is not easily settled without rather extensive and perhaps time-consuming analysis. In any case, the point of diminishing returns in terms of new variables is soon reached. For example, if observations over a period of ten years are to be used, a perfect explanation of any dependent variable (with no predictive value whatever) can be obtained from any ten random series.

Another point may be mentioned. If one *must* forecast, there is the advantage with input-output methods of being able, after the fact, to make point-by-point comparisons to determine just what estimates of structure or demand were in error, by how much, and perhaps even why. Thus, one can hope to learn from experience. With regression methods, it is possible after the fact to do little but shrug and hope for better luck next time. This last observation points up quite sharply a very fundamental difference between input-output and regression methods. The input-output approach is based throughout on an attempt to establish specific causal sequences. Errors in estimates are not considered to be the result of a stochastic process but rather the result of failure to identify accurately the parameters of the demand and the processing systems. Regression methods, in contrast, while they imply the existence of a structure, do not imply any direct structural connections between dependent and independent variables. The rationale of regression estimates may be presented in various ways, but it is significant that, regardless of rationale, the full machinery of stochastic inference is brought into play.

The above implies no criticism of the use of regression methods. It is perhaps a plea for a more realistic appraisal of the uses and limitations of regression analysis, for these methods do have many and important uses. They are now, and will be in the future, used as integral parts of input-output economy-wide models. There are many parts of the economy where the functional determinants of economic behavior, in a structural sense, are inaccessible. To deal with such areas at all, reliance must usually be placed on an averaging historical process, that is to say, on the regression approach. Both regression and structural methods will be found to have their special virtues, and, when limited to their proper fields, will tend to complement rather than compete.

We may now turn to a more general appraisal of the relative advantages of input-output and regression methods in forecast models. To start with the simplest situation, let us suppose that the analyst is considering a year not far in the future, and he feels that from the present to the reference year there will be little or no change in the processing structure of the economy or in the relative importance of the different demand elements. In general, he assumes that the economy remains approximately the same in structure throughout but that it may be changed in size. Ezra Glaser has pointed out that such an assumption, which has been called naïve, might in a particular instance be considered quite shrewd. There certainly have been periods when such an assumption would have been realistic. Under the conditions specified, the use of either input-output or regression methods could properly be called wasteful. The cheapest and best course would probably be to assume an equivalent growth in all quantities to be estimated.

Suppose now that, instead of no change, it were assumed that there were some changes in the demand and/or processing structure, but that these were not great and were within the range of available historical data. In this case, one might use either input-output or regression methods. If only a few estimates were required, economy in time and effort would surely recommend use of regression methods. Depending on the number of different estimates needed, on the requirements for consistency among them, and on the extent of changes that it was felt might have occurred, a point might be reached where an input-output approach would be preferred.

Finally, if it were supposed that a major alteration in processing structure would occur, or that the pattern of demands in the future period would be radically different than the patterns found in the available historical data, the basic assumptions on which regression methods rest would be violated, and they could be used, if at all, only with risk. Under these circumstances, an input-output approach to the problem would become almost mandatory if any solution were to be attempted at all.

The weakness of the input-output approach in forecast models, such as it is, lies in its size; it will presuppose in most cases a substantial amount of work. When the question to be answered is a simple one to which other methods are applicable, input-output methods will not be used. It has its strengths as well. One of these, and an important one, is that the processing structure for the economy is separately available for the analyst's consideration and open to any changes in details that his judgment may dictate. For example, it is perfectly feasible to examine the demands for steel in an economy where the analyst may feel that tantalum has substituted for some alloys, or that reinforced plastics have replaced steel for auto bodies. The analyst may, if he feels it necessary, add entirely new industries to his processing structure to represent the availability of new materials or new uses for old ones.

The demand structure of the economy is likewise open to change without restriction. One may increase, or one may limit, the production of consumer durable goods. One may presuppose a large or a small demand for munitions. This flexibility of treatment of both the demand and the processing structures is hardly available by any other method.

The input-output approach has a third feature, which may be an advantage in some problems. Separate results obtained from an input-output analysis will normally be consistent with one another, in the sense that they will represent possible outcomes for an economy that has technologically existed, or that the analyst has a reason for feeling could exist. This consistency is not usually a feature of regression estimates for a number of variables. For example, an economy-wide model (which was a contemporary of "Full Employment Patterns, 1950," mentioned above) included the reasonable estimate that construction volume in 1950 might be 90 per cent higher than in 1940. However, the study also included an estimate that lumber production might fall during the same decade. Since the demand for lumber is mainly determined by construction activity, some inconsistency would seem likely. Estimates for other major construction materials were included in the study, but none increased as much as the estimate for construction in the period covered. Forecast models using input-output methods may for any of a number of reasons yield estimates that will turn out to be wrong, but in general the estimates will at least be technologically consistent.

In conclusion, input-output data and methods may be of some assistance to forecasters, especially when it is believed that some fairly radical alteration in processing or demand structures will occur. At the same time, the authors wish to stress that these methods, while they may be helpful, are not intended primarily for such purposes.

E. General Remarks

Considering the scope of the interindustry-relations approach, the magnitude of input-output tabulations, and the variety of different problems to which both may be relevant, it is perfectly natural that a great many serious questions are raised about both the conceptual and the empirical content of the field. The preceding discussion has in part been prompted by such questions. This last section of the paper touches briefly on questions that did not fit into the earlier exposition.

By its nature, the interindustry-relations approach is a large-scale system of analysis, presupposing a substantial investment in empirical research before it can be used. One may well inquire, then, whether or not it will pay its way in terms of results. This question implies others: How important are the problems that one seeks to attack through input-output methods? Will these methods give satisfactory answers? Are cheaper or more efficient alternative approaches available?

It is easiest to establish the importance of the problems that are being attacked. The current large-scale developments in the interindustry-relations area are almost exclusively directed toward improvement in methods used in industrial mobilization and feasibility analysis.

These problems were simpler for earlier and more primitive economies, in which the organization of industry was more direct. There were clearly marked channels from slaughterhouse to tannery to leather worker; from farmer or fisherman to consumer; or from ironmaster to ironmonger. The specialized activities and intricate networks of supply that characterize the modern industrial economy were largely absent.

The problems of military supply were simpler too. Troops might be expected to feed and quarter themselves on the country through which they moved, to requisition their transport, and to create their own defenses. Armies were cohesive units, and military operations were usually limited to their neighborhoods. Munitions, while costly to manufacture, were simple in design.

It is far different today, when every resource of a complex industrial structure may be called into play by war, when any important factory or concentration of facilities throughout a country may be a target, and when equipment may be so costly as to overshadow the lifetime earnings of the men who use it, and so complex as to tax the understanding of experts.

There can hardly be any question of the importance of mobilization problems today when the fear of possible war can cause a nation to devote a sixth of its total production to defense, and an actual war would probably raise the proportion to more than half. Industrial mobilization analysis clearly must be made comprehensive, reasonably accurate, flexible, and efficient; every resource of common sense should be used to improve it, so far as possible.

It is not so easy to say whether the interindustry-relations method constitutes a necessary addition to the techniques of mobilization analysis for which there is no substitute or alternative. In the form in which it is used, it represents a rather direct analogue to the way in which production requirements are actually laid on processing units because of demands for finished goods. So far as one can now tell, the approach will yield the results expected and wanted from it. It is difficult to imagine another analytical method that would be conceptually more appropriate for an attack on the problems of production requirements. At the same time, the method is still in a developmental stage, and many real problems in its proper use remain to be solved. It is the belief of those who are now concerned with these problems that no difficulties so far encountered are insuperable and that the interindustry-relations approach in total will provide a very major advance in the techniques of mobilization analysis, but a wholly adequate appraisal cannot now be made. It cannot be ruled out that alternative and superior techniques may be developed. At the moment, however, none is in sight.

There have been suggestions from time to time that alternative methods that might perform as well and be cheaper in money or time could perhaps be found. The methods, however, are left unnamed and undescribed, suggesting that this is a hope rather than a conviction based on consideration of specific alternatives.

The preceding remarks have been directed to the use of interindustry-relations methods in mobilization analysis. The value of the method must clearly be considered in this context, since this is the main purpose behind the current work. If the present work program can be justified on these grounds, then other questions of value are irrelevant so far as continuance of the work is concerned. However, it is the belief of those who work in the program that there are many other values, and that the data and methods being developed represent a substantial contribution for many other fields of work as well.

It has been remarked that the consistent data collections represented by input-output tabulations and associated studies can be directly useful in many ways. Economists concerned with an empirical base for their study of the economy cannot fail to find these data collections both useful and suggestive. These general uses presuppose no particular acceptance or use of the conceptual notions underlying input-output analysis. But these as well, limited in whatever ways the analyst thinks suitable, should contribute to our understanding of fundamental economic problems.

The benefits from input-output data and methods are not limited to economists. The health, vitality, and efficiency of the American economy rest on the wisdom of decisions made daily by those who direct its productive system. The authors do not believe that it is coincidence that, on the one hand, business decisions in the United States are typically made with the use of a wider range of more accurate data and information than anywhere else in the world and, on the other, industry is typically more progressive, adaptable, and efficient in the United States than elsewhere. A wide range of information about the economy as a whole, consistently established and of satisfactory quality, is just as vital for rational business decisions as for government uses. As before, it is felt that not simply the data, but the conceptual and analytical frameworks of input-output analysis as well, may be of direct value for business purposes.

The interindustry-relations approach does not supersede any of the current tools used in economic analysis. It is of special interest to this Conference, of course, that it does not replace, but rather extends, the gross national product accounts, and embraces them firmly within the analytical approach. As with most tools, the input-output technique may be highly useful in one context and of little relevance in another. In general, it will make its greatest contribution to problems where macroscopic details are important, where consistency in estimates over broad areas of the economy is required, and where major changes in demand or processing structure are involved. As these conditions are relaxed, the approach adds less to the analysis. There will remain many important areas of quantitative national economic analysis where the use of other tools will be not simply competitive but preferable. Perhaps the major feature of the approach is that the demand and processing structures of the economy are laid before the analyst in full detail, subject to whatever modification he feels may be required for his problem and without limitation to the accidents of historical processes.

There has been a fair amount of talk about how input-output, as a method of analysis, can be "tested." One suggestion has been to produce a "forecast" model, then check its results against actuality after the fact. This approach, unfortunately, would seem to check the forecaster's ability more than the method. Another, more sophisticated, approach is to take the processing structure for one year, the demand structure for another, place both in comparable units, join them to obtain the production levels implied by the demand structure, and then check the results against actual production data for the year to which the demand structure relates. While better, this approach has some flaws. It requires one to leave out of account knowledge of impending changes in the processing structure that an analyst might very well have had in his possession at the time the original structure was set up. (The alternative, to permit such changes, would surely compromise the test.) Of greater importance, the test depends on our abilities to solve the problems of measuring changes through time in production, prices, and demands—indeed, on our ability to measure some economic variables accurately at all. Errors in simple measurement, which have no relevance to a test of the method as such, will nevertheless result in discrepancies between estimated and actual figures, and economic time series in general are far from being error free. The process is very much like trying to test the gun, the ammunition, the marksman, and the firing range simultaneously.

It appears that adequate tests, if they are made, will refer more particularly to the basic assumptions on which the approach rests. As with all such higher-order abstractions, a host of minor presuppositions are involved, ranging down to confidence in the multiplication table. Some of the more important assumptions will be implicit in nearly any form of detailed national economic analysis. For example, it is assumed that the processing side of the economy can be divided into distinct sectors for which quantitative measures of some economic attributes (in particular, a meaningful additive measure of output or activity) can be established. Something like this would seem to be involved in almost any attack on production-requirements problems. The unique assumption of the input-output approach is that a reasonably uncomplicated functional form, which will describe within required accuracy limits the changes in the movement of commodities from a first to a second sector in response to changes in the production level of the second, can be found and used. This assumption, as represented in the functional forms used in any analysis, is subject to direct study. Such studies are to be encouraged not simply as tests, but as positive contributions to the method. The interindustry-relations approach is essentially a direct analogue system; hence, any addition to information on functional interrelations almost automatically provides the basis for an immediate improvement in the system.

The creation of a major new analytical tool is likely to be accompanied by fears, some real, some illusory, that it may be misused. The possibilities for real misuse can be guarded against. There is, of course, a strong temptation when a new and powerful but untried tool is available to give it a trial on any handy material around. It may be fortunate that the sheer size of the input-output tabulations is likely to militate against any casual attempts of this kind. As indicated in the previous text, the authors feel that the greatest possibilities for mischief lie in the price-wage-tax-profits-balance area.

An illusory fear is that the approach constitutes a potentially undesirable planning device. The word "planning" has acquired a rather unsavory semantic content, especially when linked with the word "government." It has come to imply some kind of belief that productive operations should be directed by a central authority; in other words, a belief in some form of socialism. This has been extended to imply that any device that might make planning more practical is somehow undesirable. When clearly stated, this is an obvious *non sequitur.*

A good deal of misunderstanding about what the interindustry-relations approach can do, or is intended to do, undoubtedly comes about through the vague meaning of the word planning. What is obviously needed is a good short phrase to connote "the rational analysis of the probable economic consequences of alternative proposed actions, an implicit alternative always being the decision to take no action at all." This is a description of what it is hoped to accomplish with input-output methods in the field of mobilization analysis. It is also a description of a course of action that we would condemn an individual for not following, that we would expect as a matter of course from any responsible business firm, and that we should require of our government in its operating functions.

Any continuing organization, whether it is concerned with the security of the United States, with making steel, or with selling peanuts, has at least two major functions. One is to carry out as efficiently as possible its current operations, which are the result of past commitments. The second is to assure as well as it can its viability in a constantly changing environment by making a wise choice among the alternative courses of action open to it. In doing this, the use of the best available means for appraising the probable future consequences of current actions is not just important—in a period of stress it may be a condition of survival.

The suggestion that interindustry relations, as a technical device, might help to make socialism more "practical" is arguable but irrelevant. Fanatic exponents of economic or social change have never failed to claim that their proposals were fully practical. The availability of a new technique thus will hardly alter the claims of socialists or the reception of their claims by others. The real issue is not whether socialism can in some fashion be made to work, but whether we are to have socialism at all, and this will be decided not on technical, but on political grounds. Economic collectivism has never had much appeal for Americans, and the surest way to maintain this attitude is to keep the free enterprise system fruitful, efficient, and secure. If the interindustry-relations approach can contribute to this objective, it becomes a weapon against collectivism, not a device that might favor it.

Another fear of misuse related to planning is that input-output methods may somehow be used in connection with the imposition of production controls and materials allocations. Nothing of the sort is implicit in current developments. The current interindustry-relations program is not concerned with current operating problems, but with analysis of possible future developments. Production and materials controls fall in the operating area, which is quite independent of the research area. Perhaps recourse may again be had to analogy. Most large firms will include a relatively small staff group that will be charged with the responsibility for the future prosperity of the corporation. The group will be planning, in the sense that they will be attempting to determine the probable consequences to the firm of taking or failing to take various actions open to it. Most large businesses consider this function both necessary and important, but the firm would not require and would not permit this group to handle or to participate directly in the daily affairs of the corporation. Mobilization analysis occupies a somewhat similar position in the business of government, and it should be permitted to enjoy a similar separation from current operating programs.

The reader may judge from the above that people who work in the interindustry-relations field take little joy in being considered planners. They take less in having their activities described as "push-button" planning, with perhaps an added implication that any thinking involved is done by "giant brains." The push-button description conveys an impression of ease and leisure that fills them with envy and a wish that it could all be true.

The materials distributed to this Conference should dispel any notion that interindustry-relations data are acquired by easy methods. One comes by such figures only through hard work, with perhaps more than the usual complement of false starts and disappointments. It is easy to deal in large round figures, just as it is easy to speak in generalities; details come hard. Attempts to use these large collections of data in such comprehensive problems as those of mobilization analysis are just as difficult. The questions to be considered must be formulated in more detail and with greater care than ever before. Many of the individual figures incorporated in the analyses require specific attention and the careful judgment of specialists. Many of the stages of data handling are not adapted to mechanical processing. The use of a greater volume of empirical information than heretofore in similar problems entails a greater volume of computation.

The giant brains, or electronic digital computers, are used simply as very fast, large-capacity calculating and tabulating machines. They are not capable of original thinking, and the researchers involved must still supply whatever judgment is used. The large-scale computers are used to achieve greater efficiency, to lower computation costs, and to reduce the elapsed time necessary for a long sequence of computations. All the calculations implicit in a mobilization analysis could be done on an adding machine, but too slowly and too inefficiently to be practical.

Finally, it should be stated that the people now using interindustry-relations methods for mobilization analysis purposes are not in any special way proponents of this particular approach, although they are easily put in a position to seem so. It is difficult to put forward the advantages of a method, or even to respond to normal queries and doubts about it, without some appearance of advocacy. It is almost inevitable that those working outside the field should be conscious of, and ask questions about, potential limitations or inadequacies. It is almost equally inevitable that the responses will point out advantages and virtues. An appearance of disagreement or even controversy is entirely too easy to create.

It should be said that the people working in the field are primarily concerned with solutions to certain specific problems, and not initially with methods. They are interested in the interindustry-relations approach, mainly because it seems to afford a means for attack on previously intractable problems. The principal objective, however, is to obtain answers in the most efficient way possible. Any part of the interindustry-relations approach, or all of it, will be discarded if a more acceptable, more efficient, or cheaper alternative can be found.

These remarks can be taken as an apology by the authors, if any parts of the preceding discussion have seemed argumentative. Such was not their intention. In the main, the authors feel that the spirit that is now guiding the development of work in the interindustry-relations field follows the lines laid down by Francis Bacon, "Whether knowledge is possible or not must be settled not by argument but by trying."

SELECTED REFERENCES

(The authors wish to acknowledge the help they had in preparing this list from an extensive bibliography compiled by Robert N. Grosse, Office of Statistical Standards, Bureau of the Budget.)

Aukrust, Odd, *National Accounts, 1930-1939 and 1946-1951,* Central Bureau of Statistics of Norway, Oslo, 1952. (Contains Norwegian input-output tables for 1948.)

Barna, T., "The Interdependence of the British Economy," *Journal of the Royal Statistical Society, Series A (General)*, 1952, pp. 29-77. (Contains input-output table for the United Kingdom based on pre-World War II data.)

Bronfenbrenner, Jean, *Lead Times in Interindustry Models: Concepts and Computations*, Office of Business Economics, Dept. of Commerce, August 1952, 48 pp.

Burtle, James, "Input-Output Analysis as an Aid to Manpower Policy," *International Labor Review*, May 1952, pp. 600-625.

Capital Requirements Arising from the Drilling of Wells for Oil, Gas, and Condensate, Department of Economics, Petroleum Research Project, Rice Institute, July 1, 1952.

Chabert, Albert, "Le système d'input-output de W. Leontief et l'analyse économique quantitative," *Economie appliquée*, 1950, pp. 173-205.

Charnes, A., W. W. Cooper, and Alexander Henderson, *An Introduction to Linear Programming*, Wiley, 1953.

Chenery, Hollis B., "Engineering Production Functions," *The Quarterly Journal of Economics*, November 1949, pp. 507-531.

——— and Paul Clark, "Applications of Input-Output Analysis in Italy" (abstract), *Econometrica*, January 1953, pp. 197-199.

Cornfield, Jerome, W. Duane Evans, and Marvin Hoffenberg, "Full Employment Patterns, 1950," *Monthly Labor Review*, February and March 1947.

Dorfman, Robert, *Application of Linear Programming to the Theory of the Firm*, University of California Press, 1951.

Edmonston, J. Harvey, "A Treatment of Multiple Product Industries," *The Quarterly Journal of Economics*, November 1952, pp. 557-571.

Evans, W. Duane, "Marketing Uses of Input-Output Data," *Journal of Marketing*, July 1952, pp. 11-21.

Evans, W. Duane, *The Effect of Structural Matrix Errors on Input-Output Estimates*, Bureau of Labor Statistics, October 1952.

———, Jerome Cornfield, and Marvin Hoffenberg, "American Economy under Full Employment," *Dun's Statistical Review*, July 1947, pp. 17ff., and "Requirements of Productive Capacity under Full Employment," *Dun's Statistical Review*, September 1947, pp. 17ff.

——— and Marvin Hoffenberg, "The Interindustry Relations Study for 1947," *The Review of Economics and Statistics*, May 1952, pp. 97-142.

Fox, Karl A., and Harry C. Norcross, "Some Relationships between Agriculture and the General Economy," *Agricultural Economics Research*, January 1952, pp. 13-21.

Freutel, Guy, "The Eighth District Balance of Trade," *Monthly Review*, Federal Reserve Bank of St. Louis, June 1952, pp. 60-78, and "Regional Interdependence and District Development," *Monthly Review*, August 1951, pp. 105-113.

Georgescu-Roegen, Nicholas, "Leontief's System in the Light of Recent Results," *The Review of Economics and Statistics*, August 1950, pp. 214-222.

Glaser, Ezra, "Interindustry Economics Research," *The American Statistician*, April-May 1951.

Goodwin, Richard M., "The Multiplier as Matrix," *The Economic Journal*, December 1949, pp. 537-555.

Hamilton, J. E., "Method of Synthesizing an Autonomous Shipbuilding Industry for an Interindustry Model" (abstract), *Econometrica*, January 1953, p. 182.

Hoffenberg, Marvin, "Employment Resulting from U.S. Exports," *Monthly Labor Review*, December 1947, pp. 675-678.

Holley, J. L., "Note on the Inversion of the Leontief Matrix," *Econometrica*, July 1951, pp. 317-320.

———, "A Dynamic Model: 1. Principles of Model Structure," *Econometrica*, October 1952, pp. 616-642.

"Input-Output Relations," *Proceedings of a Conference on Inter-Industrial Relations Held at Driebergen, Holland*, Netherlands Economic Institute, 1953. (Papers by Wassily Leontief, W. Duane Evans, Richard M. Goodwin, Tjalling C. Koopmans, Odd Aukrust, T. Barna, G. F. Loeb, J. Sandee, D. B. J. Schouten, J. R. N. Stone, J. E. G. Utting, and Ezra Glaser.)

Interindustry Bibliography, Office of Statistical Standards, Bureau of the Budget, Feb. 17, 1953, and *Supplement No. 1 to Interindustry Bibliography*, Office of Statistical Standards, Bureau of the Budget, Apr. 24, 1953.

Isard, Walter, "Interregional and Regional Input-Output Analysis: A Model of a Space-Economy," *The Review of Economics and Statistics*, November 1951, pp. 318-328.

Kaplan, Norman, *et al.*, "A Tentative Input-Output Table for the USSR, 1941," Project Rand Research Memorandum (RM-924), Sept. 2, 1952.

Koopmans, Tjalling C., editor, *Activity Analysis of Production and Allocation*, Wiley, 1951. (Papers by Kenneth J. Arrow, George W. Brown, Yale Brozen, Ansley Coale, George B. Dantzig, Robert Dorfman, David Gale, Murray A. Geisler, Nicholas Georgescu-Roegen, Murray Gerstenhaber, Clifford Hildreth, Tjalling C. Koopmans, Harold W. Kuhn, Oskar Morgenstern, Stanley Reiter, Paul A. Samuelson, Herbert A. Simon, Harlan Smith, Albert W. Tucker, and Marshall K. Wood.)

Leontief, Wassily, "Quantitative Input and Output Relations in the Economic System of the U.S.," *The Review of Economics and Statistics*, August 1936, pp. 105-125.

———, "Structural Matrices of National Economies," *Proceedings of the International Statistical Conferences, 1947*, Vol. V, pp. 273-280.

———, *The Structure of American Economy, 1919-1939*, 2nd ed., Oxford, 1951.

———, "Some Basic Problems of Structural Analysis," *The Review of Economics and Statistics*, February 1952, pp. 1-9.

———, *Domestic Production and Foreign Trade: The American Capital Position Re-Examined*, Harvard Economic Research Project, Mar. 26, 1953.

———, *et al.*, *Studies in the Structure of the American Economy*, Oxford, 1953. (Papers by Wassily Leontief, Hollis B. Chenery, P. G. Clark, J. S. Duesenberry, A. R. Ferguson, A. P. Grosse, Robert N. Grosse, M. Holzman, Walter Isard, and H. Kistin.)

L'Industria, Numero Speciale Dedicato alla Matrice Italiana, Milan, Numero 4, 1952. (Contains input-output table for Italy, 1948, and papers by Hollis B. Chenery, Wassily Leontief, P. G. Clark, V. Cao-Pinna, C. Righi, F. Pilloton, S. Guidotti, and B. Barberi. English summaries, pp. 622-625.)

McPherson, James L., S. N. Alexander, H. B. Horton, and Ezra Glaser, "Information Processing in Social and Industrial Research," *Scientific Monthly*, February 1953, pp. 100-108.

Metzler, Lloyd A., "Taxes and Subsidies in Leontief's Input-Output Model," *Quarterly Journal of Economics*, August 1951, pp. 433-438.

Morgenstern, Oskar, editor, *Economic Activity Analysis*, Wiley, 1954.

———, and Max A. Woodbury, "The Stability of Inverses of Input-Output Matrices" (abstract), *Econometrica*, April 1950, pp. 190-192.

Moses, Leon, "Regional Input-Output. A Method of Analyzing Regional Interdependence," Ph.D. dissertation, Harvard University, June 1952.

Shephard, Ronald W., *An Econometric Model of Interindustry Material Flows*, Rand Corporation, 1951.

———, *A Survey of Input-Output Research*, Rand Corporation, 1952.

Solow, Robert, "Structure of Linear Models," *Econometrica*, January 1952, pp. 29-46.

200-Order Transaction, Coefficient, and Inverse Tables for 1947, Division of Interindustry Economics, Bureau of Labor Statistics, 16 sections, October 1952, and *Industry Classification Manual for the 1947 Interindustry Economics Study*, Division of Interindustry Economics, Bureau of Labor Statistics, March 1953.

von Neumann, John, "A Model of General Economic Equilibrium," *The Review of Economic Studies*, 1945-1946, pp. 1-9.

Waugh, Frederick V., "Inversion of the Leontief Matrix by Power Series," *Econometrica*, April 1950, pp. 142-154.

Wood, Marshall K., "Scientific Techniques of Program Planning," *Air University Quarterly Review*, Winter 1949, pp. 49-65.

COMMENT

CLARK WARBURTON, *Federal Deposit Insurance Corporation*

Various passages in the papers of Wassily Leontief and of W. Duane Evans and Marvin Hoffenberg, presented to this Conference or published elsewhere, indicate a belief that input-output analysis is a suitable technique for approaching many problems of economic change in addition to those of mobilization, for which it has principally been used.[1] There is a recognition that for wider use some

[1] Wassily Leontief, *The Structure of American Economy, 1919-1939*, 2nd

refinements of the technique are necessary, particularly along the lines of what is called a dynamic model.[2] My remarks here relate to a few of the refinements we need to deal with one of the problems of economic dynamics, that of business cycles.

Some of the speakers at this Conference have pointed out that input-output analysis is not, in fact, a direct application of the Walrasian system of equilibrium, as would be assumed from some of the statements in the writings of its sponsors. Input-output analysis does not allow for changes in relative prices, or for the "price mix," to use Mr. Shephard's term, or for interrelations of the price mix with the "product mix," and the "process mix," or the "materials mix," to-which several references have been made (without using that term). These "mixes" are the four concealed jokers to which John DeWitt Norton refers.[3]

To these inadequacies I want to add two more—both recognized, as are the preceding four, in traditional economic theory. The first relates to changes in prices—not in reference to each other but in their general level. Changes in the general level of prices (or monetary parameter, to use the late Professor Schumpeter's term) and the interrelation of changes in the price level to the price mix, the product mix, the process mix, and the materials mix are highly important in the understanding of business fluctuations. Somehow, the theory of the price level—which means the theory of the value of money—must be put into the conceptual scheme and its statistical implementation. We might call this the problem of the "money mix." Incidentally, insertion of the money mix into the conceptual scheme is much easier than insertion of the others, because there are only a few kinds of money but many kinds of prices, products, processes, and materials.

The other element from traditional theory that needs consideration is also not in the conceptual scheme of input-output analysis but is in the statistical data, and operates as a built-in device for the concealment of disequilibrium. In fact, it makes disequilibrium look like equilibrium. The beautiful balances arrived at everywhere in our newly acquired mammoth tables are not merely

ed., Oxford, 1951, p. 139; W. Duane Evans and Marvin Hoffenberg, "The Interindustry Relations Study of 1947," *The Review of Economics and Statistics*, May 1952, p. 97; and section D of their paper herein.

[2] See Leontief's paper in this volume, "Some Basic Problems of Empirical Input-Output Analysis."

[3] See Norton's paper in this volume, "Research Required for the Application of Interindustry Economics."

something different from the equilibrium of the equations of Walras, as Rutledge Vining and Carl Christ have pointed out[4]—they are the embodiment of all the departures from equilibrium. This potent element—the prime joker—which appears to have deluded the sponsors of input-output analysis into the error of thinking they have equilibrium when they really have disequilibrium—is profits, using the word in its Schumpeterian sense, or what some economists call "pure profit." We have, then, a problem of incorporating the theory of profits in the conceptual scheme to match the presence of profits in the statistical results. We might call this the problem of the "profits mix."

These problems of the money mix and the profits mix are of much greater importance when input-output analysis is oriented toward problems of business fluctuations than when it is oriented toward problems of mobilization. There is a good reason for this. The problems of mobilization are problems arising from centralized management of the economy. In the words of Evans and Hoffenberg: "If an economy with the specified technological interrelationship structure is required to produce goods for delivery to autonomous sectors in exactly the amounts specified, what output levels for every sector will bring the system to exact equilibrium in terms of demand and supply? Or the question may be of the following type: If this economy is required to deliver to autonomous sectors more (or less) goods in stated amounts than in fact it is delivering, what increase (or decrease) in the output levels for each sector will bring the system into equilibrium?"

Let us make no mistake about it. These are the kinds of questions that are appropriate for economic analysis in an authoritarian, or totalitarian, state. The problems of business fluctuations in an economy of free consumer choice and free enterprise are vastly different, and so are the questions to be posed to and by the economic analysts of a democratic state. For the authoritarian economy, problems of the price mix, the money mix, and the profits mix can be subordinated, while attention is concentrated on problems of the product mix and perhaps those of the process mix. In a free economy, problems of the price mix, the money mix, and the profits mix must occupy the center of the stage, where economists and their statistician collaborators perform.

[4] See Vining's comment on Leontief's paper in this volume, and Christ's paper "A Review of Input-Output Analysis."

OSKAR MORGENSTERN, *Economics Research Project, Princeton University, and* THOMSON M. WHITIN, *School of Industrial Management, Massachusetts Institute of Technology*

Papers presented at this Conference have mentioned various problems that touch upon topics considered by members of the Economic Research Project at Princeton University. For example, Carl Christ has concerned himself with the problems involving errors; W. Duane Evans has mentioned some aspects of the aggregation problem; and Tjalling C. Koopmans has referred to one aggregation experiment we are conducting. Therefore, we take this opportunity to describe a few areas of the research at Princeton, which we hope are of general interest to the participants in this Conference.[1] A more extensive description of this work, other than that already available in various publications, is contained in the volume *Economic Activity Analysis*, Wiley, 1954.

The problem of the accuracy of economic observations has received much attention, with respect to both economics in general and computation problems in input-output studies in particular, where the effects of changes in Leontief coefficients (due, for example, to corrections of observational errors) on the inverse are discussed.[2] As these results have been published elsewhere, we shall not examine them here, except to say that we still feel that the problem of determining the sensitivity of any economic model to the necessarily encountered, but generally unknown, errors in the data is of vital, indeed overriding, importance. The possibility of using electronic computers in economics focuses attention on these

[1] This discussion of aggregation and errors is based on research done under Office of Naval Research Contract N6onr-27009 at Princeton University.

[2] The general problem is discussed in Oskar Morgenstern, *On the Accuracy of Economic Observations*, Princeton University Press, 1950; 2nd ed., 1955. There, particular attention is paid to the basic work of John von Neumann and H. H. Goldstine on the origins and propagation of errors in matrix inversions. Carl Christ, in his contribution to this volume, has taken up the same range of questions already examined in this literature.

The specific problems of changes in Leontief matrices are discussed in Oskar Morgenstern and Max A. Woodbury, "The Stability of Inverses of Input-Output Matrices," *Econometrica*, April 1950. Stronger results are obtained by Y. K. Wong, "Quasi-Inverses Associated with Minkowski-Leontief Matrices," *Logistics Papers*, George Washington University, 1951, and the paper under the same title in *Econometrica*, July 1954.

Finally, Morgenstern discussed this matter in commenting on Leontief's paper, "Input-Output Relations," in *Proceedings of a Conference on Inter-Industrial Relations Held at Driebergen, Holland*, Netherlands Economic Institute, 1953. Cf., in particular, pp. 27-32, 96-98, 110.

questions. A thorough understanding of them is required before one can have a degree of confidence in the results of large-scale computations sufficient for the intended applications.

Whether one investigates the data and "noise" problems associated with input-output tables or proceeds to the investigation of the mathematical properties of the associated matrices, one is inevitably confronted with the need for rather advanced mathematics. However, some current studies of mathematical properties of input-output matrices make use of advanced mathematics to obtain merely weak results. For example, some recent proofs that each off-diagonal element in the row of the inverse is less than the diagonal element make use of the properties of characteristic roots and even of Brouwer's fixed-point theorem. Y. K. Wong, of this project, has been able to obtain stronger inequalities through the use of elementary algebra. He has also discovered many other inequalities that are applicable to various problems in input-output analysis.

Many of the results of mathematical studies of matrices of input-output type are restricted to indecomposable matrices. A matrix is decomposable if, by interchange of rows and columns, it can be subdivided into a matrix of the form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

where either C or B consists of only zero elements and A and D are square matrices. The process of interchanging rows and columns to test for decomposability is extremely tedious from a computational point of view. For example, a well-known result is that an input-output matrix is nonsingular if it is indecomposable and at least one column sum is positive. Wong has provided a method of testing for nonsingularity that does not involve indecomposability conditions.[3]

In connection with computation problems, the number of zeros contained in the matrix plays an important role. The number of zeros occurring in input-output tables increases rapidly with increases in the size of the matrix. An inspection of the 192-by-192 matrices distributed at this Conference is sufficient to show the

[3] Y. K. Wong, "Some Inequalities of Determinants of Minkowski Type," *Duke Mathematical Journal*, June 1952, pp. 231-241, in particular p. 237, Corollary 1. More general results are given in "Inequalities for Minkowski-Leontief Matrices," *Economic Activity Analysis*, Oskar Morgenstern, editor, Wiley, 1954, pp. 242-245. Also "On Non-Negative-Valued Matrices," *Proceedings of the National Academy of Sciences*, February 1954, pp. 121-124.

large proportion of the entries that are marked zero. In the event that these entries in the input-output value and coefficient matrices were small numbers instead of zeros, there would be a tremendous increase in the amount of noise involved in the computation of the inverse matrix.

A second problem with which we have been concerned is that of aggregation.[4] Most of our work has been directed toward aggregation problems that arise in connection with input-output models, although the results may have wider application to other econometric models. Here again, work has progressed along both computational and theoretical lines. In the absence of much good theory about aggregation, we believe that experimentation with the statistical material is indicated. Thus, we have aggregated large matrices into small matrices by various methods, inverted the latter, and compared the results of the inverses of the large and the small matrices. Particular coefficients may be compared if one or more "industries" are defined in the same manner in the aggregated system as in the original—that is, if they are kept distinct in the aggregated system. Also, for the distinct industries, the column sums of the two inverses may be compared, each sum representing the number of dollars of increased output in the economy required by a $1 increase in consumption in the particular industry under consideration. (See footnote 3.)

Tjalling C. Koopmans mentioned in the present discussion one experiment we have carried out that involves a high degree of aggregation for some sectors of the economy, and a very low degree of aggregation for others. These experiments are not yet completed, but on the basis of computations performed, we might arrive at a tentative belief that good approximations to the inverses of large matrices may be obtained from highly aggregated ones. The economic implication of this is that the particular effects lost by aggregation are small. This is, in many ways, a most peculiar result.

Difficulties of computing solutions to very large systems of simultaneous equations limit the amount of use that can be made of

[4] Wassily Leontief explicitly recognized the importance of the aggregation problem to input-output models when he stated before the American Economic Association, "There are many alternative ways of aggregating the ninety-eight [basic classifications] original industries under some forty-eight broader headings. Each classification will lead to a different system of fifty simultaneous equations and most likely also to a different solution." "Recent Developments in the Study of Interindustrial Relationships," *Papers and Proceedings of the American Economic Association*, May 1949, p. 217.

highly detailed data. Our computations indicate that it may be possible to use these data far more than is currently realized. The advantages of detail would then be combined with the advantage of easier computational procedures.

Suppose one is interested in some particular aspect of the economy, such as the effect of an increase in consumption of the products of industry i on the output of a second industry j, where the subscripts i and j represent industries in a large system of the usual input-output equations. These two industries can be kept distinct, and the remainder of the economy can be aggregated together into a large heterogeneous "mass industry" designated by subscript Γ. It is of interest to have some approximations for a particular element in the inverse of a Leontief matrix without computing the whole inverse. (See "Inequalities for Minkowski-Leontief Matrices," as cited, pp. 266-268. For the sum of the columns of the inverse, see "On Non-Negative-Valued Matrices," as cited, and Y. K. Wong, "Quasi-Inverses Associated with Minkowski-Leontief Matrices," *Econometrica*, July 1954, pp. 358-359.)

The equations of our aggregated Leontief system may then be written as

(1)

$$\begin{aligned} X_i - x_{ii} - x_{ij} - x_{i\Gamma} &= Y_i \\ - x_{ji} + X_j - x_{jj} - x_{j\Gamma} &= Y_j \\ - x_{\Gamma i} - x_{\Gamma j} + X_\Gamma - x_{\Gamma\Gamma} &= Y_\Gamma \end{aligned}$$

The inverse of the coefficient matrix corresponding to (1) may be written as

(2)

$$\begin{aligned} A_{ii}Y_i + A_{ij}Y_j + A_{i\Gamma}Y_\Gamma &= X_i \\ A_{ji}Y_i + A_{jj}Y_j + A_{j\Gamma}Y_\Gamma &= X_j \\ A_{\Gamma i}Y_i + A_{\Gamma j}Y_j + A_{\Gamma\Gamma}Y_\Gamma &= X_\Gamma \end{aligned}$$

Computations indicate that in most cases the coefficients A_{ii}, A_{jj}, A_{ij}, and A_{ji} in (2) do not differ "greatly" (i.e. not greatly from the point of view of economic application) from the corresponding coefficients of the inverse of the larger system of equations. Approximations to any or all inverse coefficients of matrices of high order may be obtained in this manner. In the event that large coefficients occur in the row corresponding to industry i or industry j in the disaggregated coefficient matrix, it is likely that the approximation will not be so good as would otherwise be the case, for the aggregated method excludes indirect effects through industries lumped into our "industry" Γ. These excluded effects involve the product of at least three coefficients, and hence will be small in most cases. Where it is evident that these excluded effects are large, additional industries can be kept distinct and a matrix of order larger than three inverted. Matrices of order other than three may therefore be used; we have chosen these to minimize computational difficulty.

The figures in Table 1 indicate the closeness of approximation of coefficients of our 3-by-3 inverse (column 1) to corresponding (rounded) coefficients of the BLS 44-by-44 inverse (column 2) distributed in 1951. Fourteen 3-by-3 matrices were inverted, each inversion making possible the comparison of four coefficients.

The results show that Leontief was correct in his supposition that different aggregations give rise to different solutions. The entries in our two columns are directly comparable, line by line, being given the same economic interpretation. To the extent that those figures differ from one another, an error or noise in some sense has occurred, as at most only one of the coefficients can possibly correspond with reality. However, to ascertain which figure is in error is an extremely difficult conceptual matter, for one does not know the "true" figures from which errors must be measured. The point is, of course, that even the 44-by-44 matrix is a highly aggregated version of something much bigger, for example of a 200-by-200 or a 500-by-500 matrix, not to mention the millions upon millions of interrelations from which any one of these has been obtained. Thus, there is no true matrix that could ever be inverted. We are always dealing with aggregates. The relevant question is whether one can speak of errors intuitively, in any direct sense, and then whether the errors are of a magnitude to cast doubt on the validity of the results. Percentagewise, the differences between some of the coefficients are fairly large, sometimes as high as 50 per cent. However, as the large percentage errors occur only in the smaller entries of the table, the error in terms of cents is, in most cases, quite small. Whether or not these errors are large enough to constitute an important bias in the results depends on the particular purpose at hand.

There are several advantages to our highly aggregated approach. Not least among these is the fact that particular effects may be

TABLE 1

Comparison of Coefficients

Inverse Coefficients Obtained from 3-by-3 Aggregation (1)	*Inverse Coefficients Obtained from 44-by-44 Table* (2)	*Inverse Coefficients Obtained from 3-by-3 Aggregation* (1)	*Inverse Coefficients Obtained from 44-by-44 Table* (2)
1.3646	1.4145	1.4559	1.4575
.0437	.0465	.0547	.0562
.2027	.2092	.0085	.0127
1.2583	1.2720	1.0196	1.0207
1.0258	1.0251	1.4985	1.5076
.0166	.0150	.07657	.0798
.0211	.0200	.0306	.0487
1.0234	1.0209	1.6957	1.7229
1.3627	1.4144	1.0194	1.0207
.0117	.0061	.0369	.0340
.3640	.3795	.0036	.0035
1.1637	1.1614	1.4992	1.5076
1.4991	1.5076	1.2313	1.2307
.0025	.0030	.0012	.0020
.1379	.1707	.1745	.1735
1.0335	1.0364	1.0175	1.0045
1.1689	1.1700	1.4993	1.5076
.0257	.0273	.0050	.0026
.0631	.0598	.3791	.4055
1.0511	1.0490	1.0223	1.0209
1.4568	1.4575	1.1684	1.1700
.0298	.0181	.0190	.0182
.1161	.1195	.0495	.0542
1.0550	1.0577	1.4983	1.5076
1.4991	1.5076	1.4930	1.5076
.0017	.0007	.0059	.0121
.1809	.2005	.1847	.2351[a]
1.0172	1.0199	1.4530	1.4575

[a] This difference was large, both in terms of percentage error and in terms of difference. A 5-by-5 matrix, keeping two more industries distinct, increased the .1847 coefficient to .2200, a much better approximation to .2351.

calculated without inverting the whole nonaggregated matrix.[5] Only those figures necessary for the purpose at hand must be calculated, rather than the whole inverse matrix. Also, full advantage may be taken of whatever detailed data on the economy can be gathered. The process of approximating an inverse may be carried on simultaneously by many individual computers, as the computation of each coefficient is independent of the computation of every other. In this respect it resembles the Monte Carlo method. Finally, the method is more flexible than those currently in use, allowing changes in the value table, and also technological changes to be incorporated more easily into the inverse.

The approximation to the inverse is probably sufficiently accurate so that there is no great advantage in obtaining the true inverse, for the underlying economic data are not precise. It is clear, of course, that these observations relating to 3-by-3 aggregations do not hold for arbitrary matrices. Neither is it certain, as yet, that anything along the lines of this empirically observed behavior of the aggregations of these special Leontief matrices can be established theoretically by rigorous proof. But the matter is of sufficient interest to be reported here. It may stimulate other investigators to consider these questions.

On the theoretical side of the aggregation problem, members of the Princeton group have established various criteria of aggregation, along with mathematical conditions required for aggregation, without introducing "errors." For example, necessary conditions have been described for the assertion that no difference obtains between the bill of goods of an aggregated system and that of the original system. These conditions are described in our volume *Economic Activity Analysis*, Wiley, 1954.

Another unusual approach to the aggregation problem with which we are experimenting is that provided by oriented "trees." Making use of some of Sylvester's results of the 1870's in circuit theory, R. Bott has proved that the existence of a tree implies the existence of a nonvanishing determinant for input-output matrices. An economic interpretation of the trees in terms of dollar flow has been given that indicates that the existence of a tree for an input-output matrix of order N implies the existence of a tree for principal minors

[5] In fact, the number of multiplications involved in obtaining the whole inverse by our procedure of large matrices is much smaller than for usual techniques of inversion. The additions necessary for the 3-by-3 inversions are trivial, because the basic tables always have control totals for the industries.

of this matrix.[6] The tree approach provides a striking geometrical method of viewing aggregation problems. A tree exists, and therefore a "static equilibrium" as currently defined for Leontief's system,[7] if every good receives money from the consumer, either directly or indirectly through other industries.

The aggregation problem is unquestionably one of the most profound facing the economist. In a sense, therefore, it is puzzling that our experiments, involving extremely heterogeneous and highly aggregated industries, have not led to results that emphasize the need for special aggregation procedures. On the other hand, it is conceivable that some especially desirable but at present unknown property of the model is responsible for our paradoxical result.

It may be possible to use other models to interpret input-output data. One interesting aspect of the study of alternative models would be the role assumed by the aggregation problem in each.[8] These models would have to take into explicit consideration such phenomena as monopoly and limitation of free competition, which are of nonadditive nature. It would lead too far, however, to discuss these matters at present.

[6] Thomson M. Whitin, "A Note on the Existence of a Static Equilibrium in the Leontief System," *Weltwirtschaftliches Archiv*, December 1952, pp. 188-198.

[7] Nicholas Georgescu-Roegen, "Leontief's System in the Light of Recent Results," *The Review of Economics and Statistics*, August 1950, pp. 214ff.

[8] A very different approach, requiring a direct establishment of production functions, is suggested by Ronald W. Shephard in *Cost and Production Functions*, Princeton University Press, 1953. The application of this approach would naturally also involve the aggregation problem. It appears there in the need of forming index numbers, a field that in many ways is better explored than that of input-output relations as described by input-output tables.