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Chapter Author: Karl Göran-Mäler

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Macroeconomic Aspects of Environmental Policy

Karl Göran-Mäler, University of Stockholm

Introduction

This paper is divided into two parts. The first part deals with long-term planning of the environment. The discussion focuses on a neoclassical growth model of the Ramsey type, in which environmental considerations have been introduced. The objective of the planning authority or the government is represented by an intertemporal welfare function which is equal to the present value of future utilities. The instantaneous utility function has as arguments consumption per capita and environmental quality. The environmental quality depends on the discharge of waste products generated in firms. The firms have two possibilities to reduce their waste discharges. They can decrease their rate of production or they can allocate more capital and labor to waste treatment. It is shown that if some natural assumptions are satisfied, there exists an optimal path, and that this optimal path will tend to a steady state in the long run.

The second part of the paper is devoted to a discussion of short-run macro problems when the economy switches from one steady state to another. These problems arise because the labor market is not perfect: the wage rate is fixed from outside in case there is an excess supply of labor. Only when there is an excess demand for labor will the wage rate rise. It is shown that if effluent charges are used as an instrument in environmental policy, these charges (or taxes) cannot be regarded as ordinary taxes, because an increase in the charge may have expansive effects on the economy in contrast to income taxes. In order to analyse these questions, it is necessary to have an explicit monetary mechanism by which the general price level is determined. In a pure barter economy, where the price level is arbitrary, the effluent charge will function as a numeraire, and a change in the charge will not affect the allocation of resources in the economy. Finally, a similar but very brief analysis is made for the situation in which effluent standards are used as main instruments in environmental policy.

Long-term Planning

The production structure

We will assume the existence of a single homogeneous good that can be used for private and public consumption, and for capital accumulation. This good is produced according to the production function

$$Q = H(K_1, L_1), \tag{1}$$

where Q is output (gross national product), K_1 is the capital stock allocated to production, and L_1 is the labor input in production. It is assumed that H is twice continuously differentiable and strictly concave, and that it has positive partial derivatives. Moreover, it is assumed that H satisfies the Inada conditions: $H_1(0, L_1) = H_2(K_1, 0) = +\infty$, $H_1(\infty, L_1) = H_2(K_1, \infty) = 0$. In the course of production wastes are generated. It is assumed that it is technically possible to reduce the amount of waste by treatment. Therefore, the need for ultimate waste disposal in the environment is given by the treatment function:

$$z = M(K_1, L_1, K_2, L_2), \tag{2}$$

where z is the amount of waste discharged into the environment. K_1 and L_1 appear as arguments in the M function because it is assumed that the amount of primary waste is determined not only by the amount of output, but also by the way this output is produced. K_2 and L_2 are the amounts of capital and labor input allocated to waste treatment.

M is assumed to be twice continuously differentiable and convex. Moreover, we assume that

$$\frac{\partial M}{\partial K_1} > 0, \quad \frac{\partial M}{\partial L_1} > 0, \quad \frac{\partial M}{\partial K_2} < 0, \quad \frac{\partial M}{\partial L_2} < 0.$$

The convexity implies that

$$\frac{\partial^2 M}{\partial K_1^2} \geq 0, \quad \frac{\partial^2 M}{\partial L_1^2} \geq 0, \quad \frac{\partial^2 M}{\partial K_2^2} \geq 0, \quad \frac{\partial^2 M}{\partial L_2^2} \geq 0,$$

which seems natural to assume. We also assume that

$$\frac{\partial^2 M}{\partial K_1 \partial K_2} > 0, \quad \frac{\partial^2 M}{\partial K_1 \partial L_2} > 0, \quad \frac{\partial^2 M}{\partial L_1 \partial K_2} > 0, \quad \frac{\partial^2 M}{\partial L_2 \partial K_2} > 0.$$

In order to simplify the notation, let us denote derivatives by subscripts:

$$\frac{\partial M}{\partial K_1} = M_1, \text{ etc.}$$

We assume that both factors are completely malleable. This seems to be a legitimate assumption in long-run modeling, and Arrow and Kurz [1] have shown that the removal of the malleability assumption does not change the asymptotic properties of optimal growth models. Only the transition from the initial situation to the ultimate equilibrium is changed. When we come to discuss short-run problems, in section 3, capital will no longer be assumed to be malleable.

We will assume that wastes are generated only in connection with production, and not in connection with consumption. This is obviously a far from realistic assumption. Moreover, we will not consider raw materials input, although the extraction of raw materials and their subsequent uses is very important in connection with environmental questions. For a model that takes both these factors into consideration see [9, Chap. 3].

It is now possible to aggregate the production function and the waste treatment function into a single production function by assuming a maximizing behavior. Consider the problem

$$\max H(K_1, L_1),$$

subject to

$$\begin{aligned} &K_1 + K_2 \leqq K \\ &L_1 + L_2 \leqq L \\ &M(K_1, L_1, K_2, L_2) \leqq z \\ &K_i \geqq 0, \quad L_i \geqq 0, \quad z \geqq 0; \quad i = 1, 2. \end{aligned}$$

It is easily seen that the constraints satisfy the constraint qualification; therefore, the Kuhn-Tucker theorem can be applied. This theorem yields nonnegative multipliers r, w, and q such that the following conditions are satisfied:

$$\begin{aligned} H_1 - qM_1 - r &\leqq 0 \\ H_2 - qM_2 - w &\leqq 0 \\ - qM_3 - r &\leqq 0 \\ - qM_4 - w &\leqq 0. \end{aligned} \tag{3}$$

As usual, the multipliers can be given an economic interpretation: r can be interpreted as the rental for capital, w, the wage rate, and q, the effluent charge. This maximization problem defines maximal output as a function of available factor inputs, K and L, and of the discharge of wastes, z:

$$Q = F(K, L, z). \tag{4}$$

It is proved in the theory of nonlinear programming [11] that F is a concave function of all its arguments. Since H is strictly concave, so is F. Moreover, it can be proved that $F_1 = r$, $F_2 = w$, and $F_3 = q$. It is however not possible to determine the sign of the second-order cross derivatives. The assumption that the Inada conditions are satisfied for H implies that they are also satisfied for F. For the rest of this section, the aggregate production function will be used.

Let C and G stand for private and public consumption respectively. Assume that due to wear and tear the capital stock depreciates at exponential rate μ. The allocation of output must then satisfy the conditions

$$\dot{K} + \mu K + C + G \leqq F(K, L, z), \tag{5}$$

where $\dot{K}$ is the rate of change of the capital stock or the net investment.

The environmental interaction function

Due to the production processes, wastes are generated in the amount z. These wastes must be disposed of in some way, and we assume that the only way is to discharge them into the environment (all other ways are thought to be included in the waste treatment function). This discharge will, however, affect the quality of the environment. We assume that it is possible to measure the quality of the environment in one variable Y.

This is a very restrictive assumption because it prevents a discussion of reversible and nonreversible changes in the environment simultaneously.

The scale we measure Y in is chosen so that $Y = 1$ when no discharge of wastes has ever been made. $Y = 1$ corresponds thus to a virgin environment. The discharge of wastes will cause Y to decrease, but the environment is assumed to have an assimilative capacity which starts a self-purification process. However, the strength of this purification process depends on the quality of the environment, and we assume that the closer Y is to 1, the slower the purification process works. We can summarize these assumptions in the following equation:

$$\dot{Y} = \lambda(1 - Y) - \gamma z, \tag{6}$$

in which λ and γ are constants. In the absence of any discharges of wastes, the solution to this equation will converge to 1. Formally this equation is identical to the celebrated Streeter-Phelps equation which governs the dissolved oxygen in a river into which organic wastes are discharged.

To each waste load z, constant over time, there is a stationary state to which the quality of the environment will approach. This stationary state is given by

$$Y = 1 - \frac{\gamma}{\lambda} z.$$

The objective function

We assume that the preferences of the government can be represented by a utility functional:

$$W \doteq \int_0^T U(c, g, Y)\, dt; \tag{7}$$

where c and g are per capita private and public consumption respectively, U an instantaneous utility function, and T the horizon. We assume that U is twice continuously differentiable and strictly concave, and has positive partial derivatives. Moreover, we assume that $U_1(0, g, Y) = +\infty$, $U_2(c, 0, Y) = +\infty$. The horizon T is predetermined in this model, but we assume that T is very large (one could replace the assumption of a finite horizon with an assumption of infinity without substantially changing the results). Furthermore, it is assumed that the capital stock at the horizon must not be smaller than a predetermined size K_T, i.e.,

$$K(T) \geqq K_T, \tag{8}$$

and that environmental quality at the horizon must not be smaller than a predetermined size Y_T, i.e.,

$$Y(T) \geqq Y_T. \tag{9}$$

Moreover the initial size of the capital stock and environmental quality is given by the past,

$$\begin{aligned} K(0) &= K_0 \\ Y(0) &= Y_0. \end{aligned} \tag{10}$$

We assume that K_T and Y_T are such that there exists at least one path from K_0, Y_0 to K_T, Y_T satisfying the constraints.

Optimal growth

For the discussion of optimal growth, only one final assumption is required, namely, that the labor supply is inelastic, constant over time, and is a constant fraction of the population. The problem of optimal growth can now be formulated:

Find K, Y, c, g, and z as functions of time, satisfying (5), (6), (8), (9), (10), (11) and the obvious nonnegativity constraints, so that the utility functional (7) is maximized. As L is constant, we can without loss of generality set $L = 1$, and $c = C$, $g = G$. This is a problem in calculus of variations. We state the necessary and sufficient conditions for an optimal growth path in terms of Pontryagin's maximum principle (in our application these conditions are the classical Euler-Lagrange equations; see [1] and [4]). According to this principle, if $K(t)$, $Y(t)$ is an optimal path, there exist two functions of time $p(t)$ and $\delta(t)$, such that the optimal values of the controls $C(t)$, $G(t)$ and $z(t)$ maximize the Hamiltonian

$$H = U(C, G, Y) + p[F(K, L, z) - \mu K - C - G] + \delta[\lambda(1 - Y) - \gamma z]. \tag{11}$$

Furthermore, the two auxiliary variables satisfy the following differential equations

$$\dot{p} = -\frac{\partial H}{\partial K} = -p(F_1 - \mu), \tag{12}$$

$$\dot{\delta} = -\frac{\partial H}{\partial Y} = -U_3 + \lambda\delta, \tag{13}$$

and satisfy the transversality conditions

$$p(T)[K(T) - K_T] = 0; \tag{14}$$

$$\delta(T)[Y(T) - Y_T] = 0. \tag{15}$$

Necessary conditions that the controls maximize the Hamiltonian are

$$\frac{\partial H}{\partial C} = U_1 - p \leqq 0; \tag{16}$$

$$\frac{\partial H}{\partial G} = U_2 - p \leqq 0; \tag{17}$$

$$\frac{\partial H}{\partial z} = pF_3 - \gamma\delta \leqq 0; \tag{18}$$

with equality when the corresponding variable is positive. That these conditions are necessary follows directly from Pontryagin's maximum principle. Our assumptions about concavity guarantee that they also are sufficient. This follows from a theorem proved by Mangasarian [6], but it is also easy to establish the sufficiency directly:

Let K^*, Y^*, C^*, G^*, z^* be a path that satisfies these necessary conditions, and let K, Y, C, G, z be another feasible path, i.e., a path that satisfies (5), (6), (8), (9), and the nonnegativity constraints. We can now make the following estimates. Since U is concave:

$$\int_0^T U(C, G, Y)\, dt - \int_0^T U(C^*, G^*, Y^*)\, dt \tag{i}$$

$$\leqq \int_0^T [U_1(C - C^*) + U_2(G - G^*) + U_3(Y - Y^*)]\, dt. \tag{ii}$$

By (16), (17), and (13)—if $U_1 - p < 0$, then $C^* = 0$, and $U_1(C - C^*) \leq pC$ because $C > 0$, etc.—

expression (ii)

$$\leqq \int_0^T [p(C - C^*) + p(G - G^*) + \lambda\delta(Y - Y^*) - \dot{\delta}(Y - Y^*)]\, dt. \tag{iii}$$

By equation (5) and partial integration of (iii),

expression (iii)

$$= -\delta(T)(Y)(T) - Y^*(T) + \delta(0)Y(0) - Y^*(0) \tag{iv}$$
$$+ \int_0^T \{p[F(K, L, z) - F(K^*, L^*, z^*) - (\dot{K} - \dot{K}^*) - \mu(K - K^*)]$$
$$+ \lambda\delta(Y - Y^*) + \delta(\dot{Y} - \dot{Y}^*)\} \, dt.$$

By equations (9), (11), and (15) and concavity of F and equation (6), expression (iv)

$$\leqq \int_0^T [p(F_1 - \mu)(\dot{K} - \dot{K}^*) - p(K - K^*) + pF_3(z - z^*)$$
$$+ \lambda\delta(Y - Y^*) - \lambda\delta(Y - Y^*) - \gamma\delta(z - z^*)] \, dt. \tag{v}$$

By equation (12) and partial integration of (v), expression (v)

$$= -p(T)[K(T) - K^*(T)] + p(0)[K(0) - K^*(0)]$$
$$+ \int_0^T [-\dot{p}(K - K^*) + \dot{p}(K - K^*) + (pF_3 - \gamma\delta)(z - z^*)] \, dt.$$

since (8), (10), (15), and (18) $\leqq 0$, therefore

$$\int_0^T U(C, G, Y) \, dt \leqq \int_0^T U(C^*, G^*, Y^*) \, dt,$$

which proves the sufficiency.

If we assume that K_T and Y_T are chosen so that there exists at least one choice of controls $C(t)$, $G(t)$, $z(t)$ such that the corresponding state variables K and Y satisfy $K(T) \geqq K_T$, $Y(T) \geqq Y_T$; then there exists also an optimal path. This follows from existence theorems in control theory (see [12]) or from existence theorems in the theory of ordinary differential equations (see [2]). It is also easy to see that the optimal path must be unique, because if K, Y, C, G, z differed from K^*, Y^*, C^*, G^*, z^* then we would have a strict inequality in the first estimate in the sufficiency proof above, because U is strictly concave. This shows that the optimal path must be unique.

It is obvious that p and δ can be interpreted as shadow prices: p is the imputed demand price on capital while δ is the imputed demand price for environmental quality. With this interpretation, (12) and (13) express the assumption of perfect certainty about future prices, and (14) and (15) express the usual demand and supply relation at the horizon, if capital is in excess supply, then the demand price is zero and similarly for environmental quality. It is possible to interpret $q = \gamma\delta$ as a demand price for waste disposal services, or as an effluent charge. The effluent charge is thus equal to the value of the deterioration of environmental quality that is caused by one more unit of waste discharge. From (18) it is seen that along an optimal path, firms will discharge wastes in such amounts that the marginal productivity of waste discharges will equal the effluent charge. We can also introduce the wage rate from the following condition:

$$pF_2 - w \leqq 0. \tag{19}$$

With these interpretations, the profit (π) of the firms can be written as

$$\pi = pF(K, L, z) - rpK - \mu pK - wL - qz, \tag{20}$$

where r is the interest rate. The interest rate is equal to

$$\frac{\dot{p}}{p} = -F_1 + \mu, \tag{21}$$

and the own rate of interest is

$$\frac{\dot{p}}{p} + F_1 - \mu = 0. \tag{22}$$

The own rate of interest is zero because there is no growth in labor supply or in technical knowledge in this model. If profits are maximized, it is seen that conditions (12), (18), and (19) are satisfied.

It is interesting to note that if we make the usual neoclassical assumption about the production function H, namely that H is linear homogeneous, and if we make the assumption that the waste treatment function M is homogeneous of degree zero (that is, doubling of production input and doubling of waste treatment input do not change the need for waste discharges), F will be linear homogeneous in K and L and profits will be negative. However, if M is assumed to be homogeneous of degree zero. we must drop the assumption that M is convex. Along an optimal growth path, the firms must therefore be subsidized. This is of course due to the increasing returns to scale that are embodied in this kind of technology. By increasing the use of labor and capital in the same proportion it is possible to increase output in the same proportion without increasing the use of the third factor, waste disposal services. If, however, effluent standards are used instead of effluent charges, the profits will be zero along an optimal path. In this case profit is equal to $pF - rpK - wL$, and profit is maximized subject to $z \leqq \bar{z}$. The first-order conditions are the same as before:

$$
\begin{aligned}
pF_1 - rp &= 0 \\
pF_2 - w &= 0 \\
pF_3 = q &= 0
\end{aligned}
$$

and profit becomes

$$pF_1K + pF_2L - rpK - wL = 0.$$

The explanation is of course that the firms need the scarcity rents that are created from environmental control to cover the deficits. In order to avoid subsidies, effluent standards may therefore be desirable. Effluent charges have other merits, however, which cannot be analysed in an aggregate model like this (see [7]). It is hardly realistic, however, to assume that M is homogeneous of degree zero, and we will not use that assumption.

Steady state

The steady state is defined as the singular solution to (5), (6), (12), and (13), or as the solution to

$$
\begin{aligned}
C + G + \mu K - F(K, L, z) &= 0 \\
\lambda(1 - Y) - \gamma z &= 0 \\
F_1 - \mu &= 0 \\
-U_3 + \lambda\delta &= 0 \\
U_1 - p &\leqq 0 \\
U_2 - p &\leqq 0 \\
pF_3 - \gamma\delta &\leqq 0
\end{aligned}
\tag{23}
$$

It is easy to show that there exists a unique solution to this system, as follows:

Consider the maximization problem

$$\max U(C, G, Y)$$

subject to

$$\begin{aligned}&C + G + \mu K - F(K, L, z) \leqq 0\\&\lambda(1 - Y) - \gamma z = 0\\&C \geqq 0, \quad G \geqq 0, \quad K \geqq 0, \quad z \geqq 0.\end{aligned}$$

The constraints are consistent; so there exist values of C, G, K, Y, and z which satisfy the constraints. Moreover, the constraints define a closed subset of R^5. Y is bounded above by 1, and $C + G$ is bounded above by $\max F - \mu K$. The continuous function U is thus bounded above on a closed set and therefore attains its maximum on this set. A maximum $(C^*, G^*, K^*, Y^*, z^*)$ thus exists. As U is strictly concave, this maximum is unique. It can be easily shown that the Kuhn-Tucker theorem may be applied to this problem. This step yields nonnegative multipliers p and δ, such that the Lagrangean function

$$L = U - p(C + G + \mu K - F) - \delta[-\lambda(1 - Y) + \gamma z]$$

has a saddlepoint at the maximum. A necessary and sufficient condition for L to have a saddlepoint at C^*, G^*, K^*, Y^*, z^* is that at this point the following conditions be satisfied:

$$\begin{array}{ll}U_1 - p \leqq 0 & C^*(U_1 - p) = 0\\U_2 - p \leqq 0 & G^*(U_2 - p) = 0\\-p(\mu - F_1) \leqq 0 & K^*p(\mu - F_1) = 0\\U_3 - \lambda\delta = 0 & \\pF_3 - \gamma\delta \leqq 0 & z^*(pF_3 - \gamma\delta) = 0.\end{array}$$

If $K^* = 0$, then $C^* = G^* = 0$, and the conditions are not satisfied. Thus $K^* > 0$, and $-p(\mu - F_1) = 0$. Obviously $p > 0$, and thus $\mu = F_1$. The system is now identical to system (23), which therefore must have a unique solution.

In this steady state or stationary equilibrium, the interest rate is zero, the demand price for environmental quality is equal to (except for a scale factor) the marginal utility of the environment.

Asymptotic behavior

It is well known that the Euler-Hamilton equations for autonomous problems in the calculus of variation have certain special properties that characterize the extremes. One such property is that the linear approximations to the systems have characteristic roots that appear pairwise symmetric around the origin. Let us derive this property.

In order to simplify the derivation, let us derive the property for an abstract system:

$$\dot{y} = \Delta_p H(y, p, x); \tag{24}$$

$$\dot{p} = -\Delta_y H(y, p, x); \tag{25}$$

$$0 = \Delta_x H(y, p, x); \tag{26}$$

where y is an n-vector corresponding to the vector (K, Y) in our application; p is an n-vector corresponding to (p, δ); and x is an m-vector corresponding to (C, G, z). The symbol Δ_p denotes the gradient of H regarded as a function of p only. The symbols Δ_y and Δ_x are used in an exactly analogous way.

Denote the singular solution by (y^*, p^*, x^*), i.e.,

$$\begin{aligned} \Delta_p H(y^*, x^*) &= 0 \\ \Delta_y H(y^*, p^*, x^*) &= 0 \\ \Delta_x H(y^*, p^*, x^*) &= 0. \end{aligned}$$

We assume that there exists a unique singular solution. Since differentiability is assumed, it is possible to expand (24), (25), and (26) in a Taylor series in the following way:

$$\begin{aligned} \dot{y} = H_{yp}(y - y^*) + H_{pp}(p - p^*) + H_{xp}(x - x^*) \\ + \zeta_y(y - y^*, p - p^*, x - x^*); \end{aligned} \tag{27}$$

$$\begin{aligned} \dot{p} = -H_{yy}(y - y^*) - H_{py}(p - p^*) - H_{xy}(x - x^*) \\ + \zeta_p(y - y^*, p - p^*, x - x^*); \end{aligned} \tag{28}$$

$$\begin{aligned} 0 = H_{yx}(y - y^*) + H_{px}(p - p^*) + H_{xx}(x - x^*) \\ + \zeta_x(y - y^*, p - p^*, x - x^*); \end{aligned} \tag{29}$$

where H_{yp} is a matrix of second-order derivatives. $\partial^2 H/\partial y_i \partial p_j$ and the other H's are other matrices of second-order derivatives. The ζ-functions are such that:

$$\lim_{(y,p,x)\to(y,p,x)} \frac{||\zeta(y - y^*, p - p^*, x - x^*)||}{||y - y^*|| + ||p - p^*|| + ||x - x^*||} = 0. \tag{30}$$

We assume that H_{xx} is nonsingular (which is the case in our application).

We can then solve for $x - x^*$, obtaining

$$x - x^* = -H_{xx}^{-1}H_{yx}(y - y^*) - H_{xx}^{-1}H_{px}(p - p^*) - H_{xx}^{-1}\zeta_x. \quad (31)$$

By substituting this expression for $x - x^*$ in (27) and (28) we obtain

$$\begin{aligned} \dot{y} = {} & (H_{yp} - H_{xp}H_{xx}^{-1}H_{yx})(y - y^*) \\ & + (H_{pp} - H_{xp}H_{xx}^{-1}H_{px})(p - p^*) + \epsilon_y; \end{aligned} \quad (32)$$

$$\begin{aligned} \dot{p} = {} & -(H_{yy} - H_{xy}H_{xx}^{-1}H_{yx})(y - y^*) \\ & + (H_{py} - H_{xy}H_{xx}^{-1}H_{px})(p - p^*) + \epsilon_p; \end{aligned} \quad (33)$$

where the ϵ functions satisfy a condition similar to (30).

Let us now put aside the ϵ-functions and consider only the linear system. It has the characteristic equation

$$\begin{bmatrix} H_{yp} - H_{xp}H_{xx}^{-1}H_{yx} - \tau & H_{pp} - H_{xp}H_{xx}^{-1}H_{px} \\ -(H_{yy} - H_{xy}H_{xx}^{-1}H_{yx}) & -(H_{py} - H_{xy}H_{xx}^{-1}H_{px}) - \tau \end{bmatrix} = 0. \quad (34)$$

This equation can also be written

$$\begin{bmatrix} H_{yy} - H_{xy}H_{xx}^{-1}H_{yx} & H_{py} - H_{xy}H_{xx}^{-1}H_{px} + \tau \\ H_{yp} - H_{xp}H_{xx}^{-1}H_{yx} - \tau & H_{pp} - H_{xp}H_{xx}^{-1}H_{px} \end{bmatrix} = 0. \quad (35)$$

But (35) implies that if τ_1 is a characteristic root, then so is $-\tau_1$, because apart from τ the determinant is symmetric. The characteristic roots are thus pairwise symmetric around the origin.

I now show that in this system, zero cannot be a characteristic root and that no root is purely imaginary. Note that the characteristic equation can also be written

$$\begin{bmatrix} H_{xx} & H_{xy} & H_{xp} \\ H_{yx} & H_{yy} & H_{yp} - \tau \\ H_{px} & H_{py} + \tau & H_{pp} \end{bmatrix} = 0. \quad (36)$$

Note also that H is linear in p, so that $H_{pp} = 0$, and that in our application H is strictly concave in x and y. This implies that the quadratic form

$$[dx, dy]\begin{bmatrix} H_{xx} & H_{xy} \\ H_{yx} & H_{yy} \end{bmatrix}\begin{bmatrix} dx \\ dy \end{bmatrix},$$

must be negative definite, in particular for dx, dy satisfying $H_{px}dx + H_{py}dy = 0$. (Here dx and dy are m- and n-dimensional vectors, respectively.) It is well known that this implies that the determinant

$$D = \begin{bmatrix} H_{xx} & H_{xy} & H_{xp} \\ H_{yx} & H_{yy} & H_{yp} \\ H_{px} & H_{py} & 0 \end{bmatrix},$$

has the same sign as $(-1)^{n+m}$. As H is nonconcave in x, the determinant

$$[H_{xx}],$$

has the same sign as $(-1)^m$. If we expand the characteristic determinant, we find that the constant term in the characteristic equation is equal to

$$\frac{1}{[H_{xx}]} D.$$

The constant term is accordingly positive, because in our application $n = 2$, $m = 3$. This shows that zero cannot be a characteristic root. In our application, the characteristic equation is fourth degree, and because of the symmetric distribution of the roots, the coefficients for τ^3 and τ are zero. A straightforward but complex computation shows that the coefficient for τ^2 is negative.

Assume now that all roots are purely imaginary. In that case the characteristic equation takes the form

$$(\tau - di)(\tau + di)(\tau - bi)(\tau + bi) = \tau^4 + (d^2 + b^2)\tau^2 + b^2d^2 = 0.$$

This is impossible, however, because the coefficient for τ^2 must be negative.

Assume now that two roots di and $-di$, are purely imaginary, and let the two other roots be $a + bi$, $a - bi$. Due to the symmetric distribution of the roots, $-a - bi$ and $-a + bi$ must also be roots, which is possible only if $b = 0$. The characteristic equation then takes the form

$$(\tau - di)(\tau + di)(\tau - a)(\tau + a) = \tau^4 + (d^2 - a^2)\tau^2 - a^2d^2 = 0,$$

which is impossible because the constant term is positive. We can therefore conclude that no root is purely imaginary.

We have now proved that the linear approximation of (5), (6), (12), (13), (16), (17), and (18) has four characteristic roots of which no one has a zero real part, and such that if τ_1 is a root, then so is $-\tau_1$. For a system of differential equations that possesses these properties, there exists an n-dimensional manifold, such that solutions with initial points on this manifold will converge to the singular solution, and all other solutions will ultimately leave any neighborhood of the singular solution. (See [2], Chap. 13, theorem 4.1.) This means in particular that for an infinite horizon, if the transversality conditions are satisfied (these conditions are no longer necessary with an infinite horizon), the prices must converge to a finite limit and the initial point must be on the stable manifold. (This presupposes that there exists an optimal path when the horizon is infinite; the existence theorems referred to earlier do not hold for an infinite horizon.)

For a finite horizon, the following theorem may be proved (see [8], Chap. 3): Given $m > 0$ and $t_1 > 0$, there exist positive numbers ϵ and N, such that if

$$|K_0 - K| + |Y_0 - Y| + |K_T - K| + |Y_T - Y| < \epsilon,$$

and if $T > N$, then it is true that

$$|K(t) - K^*| + |Y(t) - Y^*| < m\epsilon,$$

for $t_1 < t < T - t_1$.

The interpretation of this theorem is obvious. If the initial point (K_0, Y_0) and the terminal point (K_T, Y_T) are not too far away from the steady state, and if the horizon is long enough, then the optimal path will enter a small neighborhood of the steady state and stay there for most of the time. We have now characterized a very important property of optimal growth paths in this model. In particular, we can use this property as an argument to defend the procedure we will follow in the discussion of short-run macro problems. In this discussion we will assume that the economy is on a steady state. We know that this is approximately true if the economy has followed an optimal path for some time.

Some extensions

The model described above can be extended in several directions:

The Introduction of Time Preferences. We have so far assumed that the government does not discount future utilities, but regards them as equivalent to present utilities. We know however that future streams of goods and services are discounted to present values when actual decisions are made, and it seems therefore probable that the governments do have a positive time preference. Assuming that this time preference can be represented by a constant discount rate σ, the objective function should be written

$$\int_0^T U(C, g, Y)e^{-\sigma t}\, dt.$$

The necessary conditions now take the form

$$\begin{aligned} \dot{p} &= -p(F_1 - \mu - \sigma) \\ \dot{\delta} &= -U_3 + (\lambda + \sigma)\delta \end{aligned}$$

as well as (16), (17), and (18). The transversality conditions become

$$\begin{aligned} e^{-\sigma t}p(T)[K(T) - K_T] &= 0 \\ e^{-\sigma t}(T)[Y(T) - Y_T] &= 0. \end{aligned}$$

The sufficiency proof is carried out almost exactly as before, and the existence theorems referred to above are equally applicable in this case. The proof of the existence of a singular solution is not applicable here, but it is possible to give another proof of the existence.

The big difference is in the distribution of the characteristic roots. It is no longer true that the characteristic roots to the linear approximation are distributed symmetrically around the origin. It is, however, possible to prove that if τ_1 is a characteristic root, then so is $\sigma - \tau_1$. If the discount rate is not too large, then two of the roots will have positive real parts and two will have negative real parts. But the theorems referred to at the end of the last section require only that two roots have negative real parts and two have positive real parts, and they are therefore still applicable. If the discount rate is not too large the same turnpike property will therefore hold in this case, too.

The interest rate is now $r = (\dot{p}/p) + \sigma$; and the own rate of interest, σ. Even in the steady state, the interest rate will be different from zero.

Exogenous Growth of Labor Supply. If labor supply is growing exponentially at a rate n, a similar analysis may be performed only if the production function is linear homogeneous and the waste treatment function is homogeneous of degree zero. But if that is the case, we already know that the production units will be operated at a loss in a market economy (as long as the effluent charge is positive), and must be subsidized. Let $k = K/L$, i.e., the capital labor ratio. The constraints can now be written

$$\dot{k} = F(k, 1, z) - c - g - (n + \mu)k;$$
$$\dot{Y} = \lambda(1 - Y) - \gamma z.$$

The necessary conditions become

$$\dot{p} = -p(F_1 - n - \mu - \sigma)$$
$$\dot{\delta} = -U_3 + (\lambda + \sigma)\delta$$

as well as (16), (17), and (18). The transversality conditions require only the formal substitution of k for K.

The sufficiency and existence proofs are exactly as before. If $\sigma = 0$ the proof of existence of a singular solution may be repeated. Finally, the characteristic roots still have the property that if τ_1 is a root, then so is $-\tau_1$. The turnpike property is therefore still valid for the model with these changes. The interest rate now becomes $(\dot{p}/p) + \sigma + n$.

Autonomous Technical Change. It is possible to introduce Harrod-neutral technical change in the model without changing the basic qualitative properties of the optimal paths if the instantaneous utility function is such that the marginal utilities of private and public consumption are homogeneous of degree -1 as functions of c and g. We will not explore this further, however, because Harrod neutrality in the aggregate production function requires the same kind of technical progress in the function H as in the function M.

Short-run Stabilization Problems

Behavioral assumptions for firms

In the last section the optimal growth path was defined and some of its qualitative properties were derived. However, the possibilities of achieving this growth path by the use of some collection of fiscal and monetary policy instruments were not discussed. If the economy is completely centralized and if the government has perfect information on the production possibilities (which, in any case, is necessary for the definition of the optimal path), then it is of course possible to implement any feasible growth path, and in particular the optimal growth path. If, however, the economy is decentralized and if the government has only a limited collection of policy instruments (such as the income tax rate, the money supply, the effluent charge, etc.) then it is no longer certain that the optimal path can be implemented. Here, I do not go into this new and exciting field for economic analysis (but see [1] for one approach). My aim is much more modest. I start with the assumption that the economy is already at an optimal path and discuss the short-run adjustment problems (unemployment, inflation) that may appear due to various imperfections that are not accounted for in the model discussed above. Here, the discussion has as its starting point an employment model extended so as to include waste generation and environmental quality.

Let us recall the definition of profits from the second section. The present value of future profits is

$$\int_0^T (pF - p\dot{K} - \mu pK - wL - qz)e^{-rt}\,dt + p(T)K(T)e^{-rt}; \tag{1}$$

where r is the interest rate. Maximizing profits yields the following necessary conditions (which also are sufficient due to our concavity assumptions):

$$\frac{\dot{p}}{p} = -(F_1 - \mu - r); \tag{2}$$

$$w = pF_2; \tag{3}$$

$$q = pF_3. \tag{4}$$

At any given time, the demand for capital, labor, and waste disposal services are functions of the relative prices q/p, w/p, and the real interest rate $r - (\dot{p}/p)$. This implies that the demand for net investment is a function of the time derivatives of these real variables. The idea of a schedule of the marginal efficiency of investment, giving a functional relation between the rate of interest and the rate of investment, is thus not a meaningful concept in this kind of model. This point has been particularly stressed by Haavelmo [3] and by Arrow and Kurz [1]. We will however modify our assumptions somewhat in order to be able to derive an investment function which includes the real interest rate.

In the last section it was shown that an optimal path will for the most part stay in a neighborhood of the stationary equilibrium, provided that the time horizon is long enough and that the initial endowments of capital and environmental quality are not too different from the values in the stationary equilibrium. We can therefore assume that the economy is so close to the steady state that the actual economy can be sufficiently approximated by the steady state. From here on I will therefore characterize the economy by the stationary equilibrium:

$$
\begin{aligned}
&F(K, L, z) - \mu K - C - G = 0 \\
&Y = 1 - \frac{\gamma}{\lambda} z \\
&F_1 - \mu - r = 0 \quad pF_2 = w \\
&U_3 - \lambda\delta = 0 \qquad pF_3 = q \\
&U_1 = U_2 = p \qquad \gamma\delta = q
\end{aligned}
\tag{5}
$$

r is the interest rate and is equal to the utility discount rate. In this stationary equilibrium, net investment is by definition zero and gross investment equals μK.

As before, the profits of firms are

$$\int_0^T (pF - p\dot{K} - \mu pK - wL - qz)e^{-rt}\,dt + p(T)K(T)e^{-rt}.$$

In stationary equilibrium, the necessary conditions for maximum profits become

$$pF_1 = \mu + r; \tag{6}$$

$$pF_2 = w; \tag{7}$$

$$pF_3 = q. \tag{8}$$

We will study transitions between different stationary equilibria of firms. If any of the prices change, so will the optimal factor input. We will assume that firms can immediately adjust their input of labor and their waste generation to changing prices, but that adjustment of the capital stock takes time. Assume that the transition time between two different stationary equilibria is $\bar{t}$. The difference in the capital stock between the two times constitutes net investment demand during this time period. This net investment demand is determined from (6), (7), and (8), and is obviously a function of r, w, q, and p.

The demand for labor and for waste disposal services is not determined from this set of equations, however, because (6) and (7) describe perfect adjustment of all factors, while the capital stock is not adjusted immediately. Neither is the demand for labor and waste disposal capacity determined by (7) and (8) (keeping the capital stock constant), because the F-function is defined as the maximum output when capital and labor can be perfectly adjusted. In order to derive the short-run demand for labor and waste disposal services, we have to consider short-run profits:

$$\max pH(K_1, L_1) - w(L_1 + L_2) - qM(K_1, L_1, K_2, L_2); \tag{9}$$

where the maximum is taken over L_1 and L_2. This yields the following necessary conditions that determine the short-run demand for labor and short-run supply of goods and residuals:

$$pH_2 - qM_2 - w = 0; \tag{10}$$

$$- qM_4 - w = 0; \tag{11}$$

$$z - M = 0. \tag{12}$$

We assume that capital stock is constant in the transition period and then suddenly changes to the new stationary equilibrium. In consequence we also assume that short-run demand is constant in the transition period, and then at the end of this period suddenly changes to the new equilibrium demand. These assumptions are in contrast to the assumption, in the preceding section, of a gradually changing capital stock. This change in assumptions is not fundamental, however, because it is possible to reach the conclusions to be derived below by maintaining the idea of a continuous changing capital stock, but at the cost of some technical complications. In effect, what we are doing is switching from a model with continuous time to a model with discrete time.

We can without loss of generality assume that $\bar{t} = 1$. This implies that we can identify the total short-run demand during the transition period from the solution to (10), (11), and (12). Let us summarize the implications of these assumptions: The demand for capital is given by equations (6), (7), and (8); and the demand for labor and waste disposal services, by equations (10), (11), and (12). We thus obtain the following behavioral functions for firms:

$$I = I\left(\frac{w}{p}, \frac{q}{p}, r\right); \tag{13}$$

this gives the demand for new capital goods as a function of the real wage rate, the real effluent charge, and the interest rate;

$$L^D = L\left(\frac{w}{p}, \frac{q}{p}\right); \tag{14}$$

this gives the demand for labor during the transition period. Note that the interest rate does not appear as an argument in this demand function because the capital stock is kept constant.

$$z = z\left(\frac{w}{p}, \frac{q}{p}\right); \tag{15}$$

this gives the demand for waste disposal services:

$$S = S\left(\frac{w}{p}, \frac{q}{p}\right); \tag{16}$$

this gives the supply of goods produced and services. This function is defined by

$$S\left(\frac{w}{p}, \frac{q}{p}\right) = H\left[K_1, L_1^D\left(\frac{w}{p}, \frac{q}{p}\right)\right],$$

where L_1^D is the demand for labor input for production of goods.

Using the assumptions we already have made for the production function and the waste treatment function, it is possible to show that

$$\frac{\partial S}{\partial p} > 0, \quad \frac{\partial S}{\partial w} < 0, \quad \frac{\partial S}{\partial q} < 0$$

$$\frac{\partial I}{\partial p} > 0 \qquad \frac{\partial I}{\partial r} < 0.$$

$$\frac{\partial L^D}{\partial w} < 0$$

$$\frac{\partial z}{\partial p} > 0 \qquad \frac{\partial z}{\partial q} < 0.$$

If the following additional assumptions are made for the waste treatment function, that

$$M_{11} - M_{12} > 0 \tag{i}$$

$$M_{22} - M_{12} > 0 \tag{ii}$$

$$M_{11} - 2M_{12} + M_{22} > 0 \tag{iii}$$

$$M_2M_{11} - M_{12}(M_1 + M_2) + M_1M_{22} < 0, \tag{iv}$$

it is possible to show that

$$\frac{\partial L^D}{\partial q} = \frac{\partial z}{\partial w} > 0$$

$$\frac{\partial L^D}{\partial p} > 0.$$

We also assume that $(\partial I/\partial w) < 0$ and $(\partial I/\partial q) > 0$.

This last assumption can be justified in several ways. The argument used here is the following. An increase in q means that the cost for waste disposal increases. If, as seems to be the case, waste treatment is a capital-intensive activity, more capital will be allocated to waste treatment than the corresponding fall in the optimal capital stock in production. The net result is thus an increase in the demand for new capital goods.

These assumptions are to a very large extent quite arbitrary, but the point is that they are not completely unreasonable. The conclusions drawn will therefore have some validity, and they will show the necessity for great care in discussing the macroeconomic short-run effects of changes in the effluent charge.

We have now completely characterized the behavioral functions for firms. We have however not introduced any financial factors. The analysis has been carried out in real terms. I will shortly show that it is necessary to introduce some monetary theory into the model in order to study the effects of changes in the effluent charge.

The necessity of a monetary theory

Before discussing the role of the government and the behavior of consumers in any detail, let us note the following important implications of a general equilibrium model for a barter economy in which an effluent charge is imposed on the waste dischargers. It is well known that price homogeneity implies that only relative prices are determined. In the absence of credits and money the government's budget must be balanced, and Walras's law implies that it is sufficient to study only one market, say the labor market (recall that in our model there are only two markets, one for labor and one for produced goods; the effluent charge is a tax). We have already derived the demand for labor as a function of the real wage rate and the real effluent charge:

$$L^D\left(\frac{w}{p}, \frac{q}{p}\right).$$

If the supply of labor is completely inelastic, which we assume it to be, and equal to L, we have as the condition for equilibrium in the economy

$$L^D\left(\frac{w}{p}, \frac{q}{p}\right) = L,$$

where q is determined by the government. But this means that given q, all relative prices are determined. But as long as the price p is not determined in the model, a change in q will not have any effect on the real variables, because the price level will change in the same proportion. It is therefore not possible to achieve a decrease in the flow of wastes by increasing the effluent charge, because the generation of wastes is itself a function of relative prices only.

The simple explanation for this result is that q is not an equilibrium price determined on a market. If we had added a supply function for waste disposal services to the model, this result would not have been obtained. But—and this is the important point—a supply function implies a kind of effluent standard, which is not implied by an exogenously determined effluent charge. If there is a market for waste disposal services, then the waste dischargers must not discharge more residuals than the supply of such services and they have to pay the equilibrium price on this market. On the other hand, effluent charges imply that the discharge of residuals is in no way limited by the supply, and the charge cannot therefore be interpreted as an equilibrium price.[1]

It seems necessary, therefore, to introduce some mechanism by which the price level is determined, in order to analyze in a fruitful way the effects of effluent charges. In a later section, I will specify a Patinkin-like monetary model [10], in which the price level is determined endogenously, but for the moment assume that the price level is in some way fixed and that $p = 1$. The equilibrium condition can now be written as $L^D(w, q) = L$, and it is possible to investigate the effects of a change in q. We know that

$$\frac{dw}{dq} = -\frac{L_2{}^D}{L_1{}^D},$$

which can be positive or negative depending on the sign of $L_2{}^D$. If the demand for labor increases with an increase in the effluent charge, the wage rate will increase. This gives us a hint that for short-run stabilization policies, an increase in the effluent charge may imply an increase in the activity in the economy. This possibility will now be studied more closely by using an extension of the simple model set out above. In doing this, I will follow [5] closely.

1. If instead of using a charge as a tool in environmental policy, the authorities construct property rights which are negotiable on a market, this difficulty would not have shown up.

Government and Consumers

We have already derived behavioral functions for the firm, giving the demand for capital goods, labor, and waste disposal services and the supply of goods as functions of the real wage rate, the real effluent charge, and the interest rate. These functions were derived, however, for the case of a barter economy. We now assume that the firms do not want to hold money balances and that investments are financed by borrowed funds. This means that we can mantain the behavioral functions already derived. We have to add one more behavioral function, however, giving the net demand for credit instruments.

We assume that bonds are the only type of credit instrument in the economy. The bonds are perpetuities and yield \$1 per period. The price of a bond is therefore equal to $1/r$. Let B^f be the number of bonds the firms want to hold. The real value of these bonds is B^f/pr, and the net demand for holding bonds is given by

$$\frac{B^f}{pr} = -I\left(\frac{w}{p}, \frac{q}{p}, r\right) - K. \tag{17}$$

The aggregate profit becomes (we will ignore capital depreciation for simplicity):

$$\pi = pS - wL^D - qz + B_0{}^f; \tag{18}$$

where $B_0{}^f$ is the interest to be paid on the initial holding of bonds. This definition of aggregate profit is consistent with our earlier discussion of the firm.

Let us now turn to the government. The government demands goods and services in the quantity G. It finances its expenditure by the proceeds from the effluent charge (qz), by raising the income tax T, and from interest on government bonds B^g. Any deficit is financed either by an expansion in the money supply or by selling bonds. Let us assume that initially the government has no debt. The government budget can then be written:

$$pG - T - qz = \Delta M + \frac{B^g}{r}. \tag{19}$$

B^g will in general be negative, ΔM is the expansion (or contraction if negative) in the money supply. G, T, q, and B^g/pr are assumed to be determined by the government. z is determined from (15), and ΔM is determined as a residual.

Let us now turn to the consumers. Let M_0 be the cash balance held by households at the beginning of the transition period, and let $B_0{}^h$ be the number of bonds held by households at the same point of time. The gross income or wealth of consumers is then

$$E = wL^s + B_0{}^h + \pi + M_0 + \frac{B_0{}^h}{r}; \tag{20}$$

that is, the sum of wage income, interest on bondholdings, profits, initial cash balance, and value of initial bondholding. The income is spent on the purchase of consumer goods C, on demand for real cash balances, on demand for real bondholding, and on income taxes. The budget constraint can therefore be written:

$$E - T = pC + M + \frac{B^h}{r}.$$

The demand functions can now be written (we maintain the assumption that the supply of labor is completely inelastic and equal to L):

$$C = C\left(\frac{E - T}{p}\right) \tag{21}$$

$$\frac{M}{p} = m\left(\frac{E - T}{p}\right) \tag{22}$$

$$\frac{B^h}{pr} = b^h\left(\frac{E - T}{p}\right). \tag{23}$$

The Complete Model

We have now specified the behavioral functions and are now in a position to give a complete presentation of the short-run macro model.

Equilibrium in the market for commodities:

$$S\left(\frac{w}{p}, \frac{q}{p}\right) = G + C\left(\frac{E - T}{p}\right) + I\left(\frac{w}{p}, \frac{q}{p}, r\right). \tag{24}$$

Equilibrium in the labor market:

$$L^D\left(\frac{w}{p}, \frac{q}{p}\right) = L. \tag{25}$$

Equilibrium in the bond market:

$$-I\left(\frac{w}{p}, \frac{q}{p}, r\right) - K + b^g\left(\frac{E - T}{p}\right) + B^g = 0. \tag{26}$$

Equilibrium in the money market:

$$m^h\left(\frac{E - T}{p}\right) = \frac{M_0}{p} + \Delta M. \tag{27}$$

If we substitute the expression for total profit given in (18) into (20) we get the definition of income:

$$E = pS\left(\frac{w}{p}, \frac{q}{p}\right) - qz\left(\frac{w}{p}, \frac{q}{p}\right) - prK + B_0{}^h + \frac{B_0{}^h}{r} + M_0. \tag{28}$$

Finally we have the change in money supply:

$$\Delta M = T + qz\left(\frac{w}{p}, \frac{q}{p}\right) - pG - \frac{B^g}{r}. \tag{29}$$

We have thus six equations in the five unknowns, w, p, r, E, and ΔM. According to Walras's law one equation is redundant, however, and we can drop (27). At the same time we can also drop equation (29), and we are left with four equations in the unknowns w, p, r, and E. The variables K, $B_0{}^h$, M_0 are historically given; and G, T, b^g, and q are determined by the government. We assume that the system has a unique, economically meaningful solution.

It is now possible to examine the effects of changes in the governmentally controlled variables. The system is, however, too complicated to admit a straightforward determination of these effects. We will introduce one imperfection in the model, namely, that the wage level is fixed from outside at a higher level than the equilibrium level. This means that there will be unemployment. If, through an expansion in aggregate demand the demand for labor increases the full employment level, we assume that the wage rate will adjust upward. Our assumption then is simply that the wage rate will not fall when there is excess supply of labor, but will increase to a new equilibrium level if there is excess demand for labor.

We will also assume that a goal of the government is to maintain the interest rate at the level associated with the new steady state. The interest rate will therefore be considered as a constant (and equal to $\bar{r}$). The government's demand for goods and services is determined by the long-term goal discussed in the previous section. The effluent charge is determined by environmental considerations.

These assumptions imply that we can drop equation (27) and substitute $\bar{r}$ for r in the rest of the equations, and that we can drop equation (25) and substitute $\bar{w}$ (the exogenously given wage rate) for w. The system is now reduced to two equations, (24) and (28), in the two unknowns p and E. If we differentiate these two equations totally and solve for dp/dq, we obtain

$$\frac{dp}{dq} = \frac{p\,\dfrac{\partial S}{\partial q} - p\,\dfrac{\partial I}{\partial q} - C'\left[S + p\,\dfrac{\partial S}{\partial p} - z(1 + s_q{}^z)\right]}{-p\,\dfrac{\partial S}{\partial q} + p\,\dfrac{\partial I}{\partial q} + C'\left(S + p\,\dfrac{\partial S}{\partial p} - q\,\dfrac{z}{p}\right) - C'\,\dfrac{E - T}{p}}; \tag{30}$$

where $\epsilon_q{}^z$ is the elasticity of z with respect to q.

If we assume that the market for goods and services is stable, an application of Samuelson's correspondence principle shows that the denominator is negative. If the waste generation function z is such that an increase in the effluent charge increases the revenue from the charge (i.e., if the function is inelastic as a function of q), the expression inside the brackets in the numerator is negative. It is, however, impossible to say anything about the signs of the first two terms in the numerator. The effect on the price level of a change in the effluent charge is therefore ambiguous. The first two terms represent the direct effects on supply and demand for goods and services, while the third term represents the effect on demand for goods and services from a change in disposable income. If this last term can be neglected, then dp/dq is positive. Disregard of the third term can be justified in the following way: If we had divided the consumers into wage earners and capitalists, then the effect on disposable income would presumably have fallen mainly on the capitalists, and as their marginal propensity to consume presumably is lower, the third term will be small. An increase in the effluent charge can therefore be assumed to cause an increase in the price level.

Let us now investigate the effect on the demand for labor. Differentiating (14), we have:

$$\frac{dL^D}{dq} = \frac{\partial L^D}{\partial_q} + \frac{\partial L^D}{\partial p}\frac{dp}{dq}.$$

All terms are positive, and an increase in the effluent charge will therefore increase the demand for labor. This shows that the effluent charge, considered as a tax, has quite different effects than, for example, the income tax. While an increase in the income tax will have contracting effects on the economy, an increase in the effluent charge may have expanding effects. In particular, this implies that if the government wants to increase its demand for goods and services, it cannot finance this increase by an increase in the effluent charge if the economy is close to full employment, because both these measures will presumably increase the total demand to such an extent that inflation will result.

This analysis is based on several almost arbitrary assumptions, and the only way to "prove" the results we have obtained is of course by careful empirical research. The analysis is, however, not void due to this, because it points out the possibility that the effects on total demand of effluent charges may be quite different from the effects of other kinds of taxes. It may therefore not be true that an effluent charge can be regarded as an ideal tax, at least not in the short run with imperfections in the labor market.

Effluent Standards

We can very easily adapt the previous model to the situation in which effluent standards instead of effluent charges are used as instruments in environmental policy. The demand and supply functions of firms will now be functions of the real wage rate, the effluent standard $\bar{z}$, and the interest rate.

$$S = S\left(\frac{w}{p}, \bar{z}\right);$$

$$L^D = L^D\left(\frac{w}{p}, \bar{z}\right);$$

$$I = I\left(\frac{w}{p}, \bar{z}, r\right).$$

The government's budget will now look like

$$M - \frac{b^g}{r} = G - T;$$

and the disposable income of households is

$$E - T = pS - rpK + B_0^h + \frac{B_0^{hh}}{r} + M_0.$$

If we make the same assumptions concerning imperfections in labor market and goals of government, we can compute the effect on the price level of a change in the effluent standard:

$$\frac{dp}{dz} = p\frac{-\dfrac{\partial S}{\partial \bar{z}} + \dfrac{\partial I}{\partial \bar{z}} + C'\dfrac{\partial S}{\partial \bar{z}}}{p\dfrac{\partial S}{\partial p} - p\dfrac{\partial I}{\partial p} + C'\left(\dfrac{E-T}{p} - S - p\dfrac{\partial S}{\partial p} + \bar{r}K\right)}.$$

Now the sign of the derivative can be determined unambiguously, and $dp/d\bar{z}$ is negative. Stricter standards will therefore mean an increase in the price level, a fall in the supply, and may imply an increase in the demand for labor.

References

1. Arrow, K., and M. Kurz. *Public Investment, the Rate of Return, and Optimal Fiscal Policy*. Baltimore: Johns Hopkins Press, 1970.
2. Coddington, E., and N. Levinson. *Theory of Ordinary Differential Equations*. New York: Wiley, 1955.
3. Haavelmo, T. *A Study in the Theory of Investment*. Chicago: University of Chicago Press, 1961.
4. Hestenes, M. *Calculus of Variations and Optimal Control Theory*. New York: Wiley, 1966.
5. Lindbeck, A. *Monetary-Fiscal Analysis and General Equilibrium*. Helsinki: 1967.
6. Mangasarian, O. "Sufficient conditions for the optimal control of nonlinear systems," *SIAM Journal of Control* 4:139:52.
7. Mäler, K. G. "Effluent Charges versus Effluent Standards." Paper prepared for Conference on Urbanization and the Environment of the International Economic Association. Copenhagen, 1972. Mimeographed.
8. ———. "Studier i Intertemporal Allokering," Stockholm School of Economics, 1969. Mimeographed.

9. ———. *A Study in Environmental Economics.* Forthcoming.
10. Patinkin, D. *Money, Interest, and Prices.* London: Harper and Row, 1962.
11. Rockefeller, T. *Convex Analysis.* Princeton: Princeton University Press, 1970.
12. Strauss, A. *An Introduction to Optimal Control Theory.* Heidelberg: Springer-Verlag, 1969.

COMMENT

Colin Wright, Claremont Graduate School

Two problems that seem to plague individuals who accept invitations to discuss conference papers are those of complete agreement or complete disagreement. When under the spell of the former, the reviewer might experience difficulty in writing a respectable quota of nonsycophantic prose while with the latter the difficulty lies in exercising judicious constraint in not submitting an article in the guise of review comments. Though I do not suffer from either of these maladies, I am afflicted with a problem that might prove more embarrassing than the role of submitting unctuous praise or unremitting criticism—I am not sure that I fully understood the contents of this paper. So in the sense that the paper under review was a "preliminary paper" so shall this review be a "preliminary critique."

This is a most difficult paper to review because the author attempts too much. Though I would not impose the requirement that papers at conferences such as this be polished pieces of scholarship, neither would I wish that an author subject his readers to *several* papers under the guise of one paper. Nor would I refer him at crucial points in the development of the paper to as yet unpublished or generally unavailable papers. A further difficulty that I experienced and, I would think, many others would experience, is related to our specialization of research interest and research techniques. My bag of tricks includes rudimentary aspects of Pontryagin's maximum principle, but falls short of manipulating n-dimensional stationary manifolds with fourth degree characteristic equations having symmetric roots! Because of these gaps in my understanding I accepted unquestioningly the author's pronouncements of certain steady state conditions and have allowed him to chauffeur me along the turnpike.

Outline of the model

Mäler has introduced pollution into an aggregate growth model and a short run macroeconomic model. Pollution (z) is generated by the production of a single good (Q) produced in an idealized economy. Thus we have a utility or felicity function that depends not only upon consumption (divided into private and public in this model and noted as c and g respectively) but also depends negatively upon an index of environmental quality (y). The index of environmental quality is related to the output of waste by the differential equation

$$\dot{y} = \lambda(1 - y) - \gamma z. \tag{1}$$

It should be noted for future reference that the current waste disposal rate (i.e., emission rate) does enter the utility functional though indirectly through the environmental quality index (y). At a stationary equilibrium we have

$$y = 1 - \frac{\gamma}{\lambda} z. \tag{2}$$

A production function for the single good is noted as

$$Q = H(K_1, L_1) \tag{3}$$

and what might be called a pollution production function is written as

$$z = M(K_1, L_1, K_2, L_2) \tag{4}$$

with the presence of K_1 and L_1 to signify that not only the amount of Q but the way in which Q is produced may affect the production of waste.

The production function for Q and the pollution production function are aggregated to obtain

$$Q = H(K, L, z). \tag{5}$$

This is accomplished by engaging in "technical optimization." It is then noted that F is a concave function of all of its arguments. (It is also claimed that if H is strictly concave, so is F. With a "little help from my friends" I have been informed that this is not technically correct, and that in the theory of technological optimization the "strictly concave' argument holds only if the constraints and conditions in the optimization problem hold with equality. With inequality conditions only concavity can be proved. This is only a minor point, however. (See Daniel C. Vandermeulen. *Linear Economic Theory*. Englewood Cliffs, N.J.: Prentice-Hall, 1971, Chapter 8.)

After all of these preliminaries, the author invokes Pontryagin's maximum principle to determine the characteristics of the optimum path for the controls c, g, and z. Sufficient conditions are derived and certain characteristics of the steady state are outlined—specifically the characteristic roots of a linear approximation to the system are shown to appear pairwise and symmetric around the origin. A few extensions to the basic model are considered briefly—time preference is introduced, exogenous growth in the labor supply is introduced, and autonomous technical change is considered.

This dynamic model is then set aside for a consideration of short run stabilization problems. The way in which the short run problems in this section are handled is rationalized by the discovery that the optimal path will stay most of the time in a neighborhood of the stationary equilibrium provided that the time horizon is long enough and that the initial endowment of capital and environmental quality are not too different from the values in the stationary equilibrium. The last assumption is, in my view, an assumption of heroic proportions if some semblance of relevance and or reality lurks behind his model and does, I think, take the author beyond the province of those necessary assumptions involved to make the mathematics "manageable."

Taxes and standards

The development of Mäler's dynamic model appears to be technically flawless and some of his conclusions appear to be intuitively obvious. What I should like to consider is some of his pronouncements relative to his steady state conditions. I will do so, however, on my own terms which, I feel, are more enlightening and yet do not fall into the category of responding unduly to the second of the reviewer's curses.

To make these comments I first introduce the Lagrangean

$$\begin{aligned} u^I[c, g, y(z)] + \sum_{i=1}^{I-1} w^i[u^i(\cdot) - u^{-i}] + \phi_1[H(K_1, L_1) - c - g] \\ + \phi_2[z - M(K_1, L_1, K_2, L_2)] + \phi_3[K - K_1 - K_2] + \phi_4[L - L_1 - L_2]. \end{aligned} \tag{6}$$

Note that I use the same utility function as Mäler, but adjusted only by defining (y) as a function of (z), as in (2), and thereby eliminating the need for a constraint in the form of (z). I do not engage in technical optimization. The w^i are Lagrangean multipliers, which may be interpreted as welfare weights, all normalized on w^I.[1] Differentiating this system with respect to c, g, z, K_1, K_2, L_1, and L_2, rearranging, and noting

$$-w^i u_y{}^i \frac{\partial y}{\partial z} \text{ as } \lambda,$$

which we shall note as λ, we obtain

$$\begin{array}{lll}
\text{a. } w^i u_c{}^i - \phi_1 = 0 & i = 1, \ldots, I & \\
\text{b. } w^i u_g{}^i - \phi_1 = 0 & i = 1, \ldots, I. & \\
\text{c. } \phi_1 H_1 + \lambda M_1 - \phi_3 = 0 & & \\
\text{d. } \qquad + \lambda M_3 - \phi_3 = 0 & & (7) \\
\text{e. } \phi_1 H_2 + \lambda M_2 - \phi_4 = 0 & & \\
\text{f. } \qquad + \lambda M_4 - \phi_4 = 0 & &
\end{array}$$

These equations define Pareto optimum conditions—the choice of the w^i singling out a specific optimum. What is required is a mechanism by which utility maximizing individuals and profit maximizing firms effect such an optimum.

Utility maximizing individuals confronted with given prices will equate their marginal rates of substitution to the price ratio. Interpreting ϕ_2 as the shadow price of c and g (which are the same in a model such as this and such as Mäler's) assures us that conditions (7:a, b) are satisfied. The w^i, which are the reciprocals of the individual's marginal utility of income, are manipulated by the choice of head taxes h^i. It is in this manner that a specific Pareto optimum is determined.

We now introduce the following profit function

$$\pi = pH(K_1, L_1) - \rho(K_1 + K_2) - \omega(L_1 + L_2) - T[M(K_1, L_1, K_2, L_2)]. \quad (8)$$

This differs slightly in form from Mäler's but not in content. The change I have introduced is that of a tax function. Such a function is directly related to the amount of pollution (z). However, since z is a function of the distribution of capital and labor, we can invoke the function of a function rule and express the tax function as $T[M(K_1, L_1, K_2, L_2)]$. Differentiating this function with respect to those variables under the control of the firm, the K_i and the L_i, we obtain

1. See Robert Strotz, "Urban Transportation Parables," *The Public Economy of Urban Communities*, J. Margolis, editor (Baltimore: Johns Hopkins Press, 1965).

$$\begin{aligned} &\text{a. } pH_1 - \rho - T'M_1 = 0 \\ &\text{b. } \quad\ \ - \rho - T'M_3 = 0 \\ &\text{c. } pH_2 - \omega - T'M_2 = 0 \\ &\text{d. } \quad\ \ - \omega - T'M_4 = 0. \end{aligned} \tag{9}$$

Noting that ϕ_3 and ϕ_4 are the shadow prices of capital and labor respectively we assert their equivalence to ρ and ω. Defining T' as the change in the tax bill due to a change in (z) and $T'M_i$ as the marginal tax rate (or subsidy since M_3 and M_4 are negative) due to the incremental use of the *i*th factor as it enters the pollution production function, we note that $\Sigma w^i u_y{}^i(\partial y/\partial z)$, the welfare weighted sum of marginal disutilities due to the deterioration of environmental quality, is equal to T'. Thus the tax imposed upon a firm is related to the use of the various inputs and the effects such inputs have upon the production or reduction of pollution and the effect such changes have upon the level of utility. Thus the firm will be taxed in its use of K_1 and L_1 and subsidized in its use of K_2 and L_2.

Mäler asserts that linear homogeneity in H and zero homogeneity in M require that a firm be subsidized in the face of a pollution charge, i.e., the tax in (8). We should first note that the assumption of linear homogeneity in models having many firms implies that firm size and the number of firms is indeterminate and marginal products, and hence factor prices, are determined as though there existed a single producer who equates price and marginal cost. This is essentially what Mäler is or should be doing with his single firm model.

We next note that from (9:a) the price of the product (p) in the absence of an appropriately chosen corrective tax-subsidy function is $p = \rho/H_1 = \omega/H_2$, which can be interpreted as price equal to (private) marginal cost. In the light of an optimum correction to the existence of pollution we have $p = \rho + T'M_1/H_1 = \omega + T'M_2/H_2$. Since L_1 and K_1 contribute to pollution and since as z increases the tax would increase, we have $T'M_1$ and $T'M_2$ positive. Thus the cost to the firm of its inputs has increased though not necessarily by the same proportion. As costs increase and price remains momentarily constant, losses will be experienced; but by a change in the industry supply function caused by a decrease in the size and or number of firms brought about by the change in the firms' cost functions, price will increase to the level noted above and output will diminish. Simultaneous with the tax on K_1 and L_1 the firm is encouraged to use L_2 and K_2 through their subsidization (i.e., $T'M_3$ and $T'M_4$ being negative). Surely profits for the remaining firms will be normal, not negative; total profit will decrease though not necessarily its rate.

Either Mäler's confusion, or my confusion of Mäler, comes from his assertion that the characteristics of the H and M functions result in technology exhibiting increasing returns. As he says "by increasing the use of labor and capital (presumably L_1 and K_1 and L_2 and K_2) in the same proportion it is possible to increase output without increasing the use of the third factor, waste disposal." But such a system does not require firm subsidization in the sense I believe Mäler intends it—analogous to the standard case of increasing returns to scale firms operating at price equal to marginal cost. In his model the use of factors in production must be taxed and the use of factors in waste treatment subsidized—the size of the tax and subsidy being determined by the equilibrium value of the emission rate z.

Whether the tax bill is smaller or larger than the subsidy receipts is irrelevant for the analysis at this level since what is required is that factors be paid the value of the (social) marginal product. It can be shown that if the tax/subsidy function $T[M]$ is homogeneous of zero degree, then tax payments would equal subsidy receipts though such equality will not be interpreted as a zero tax by the firm. This follows, of course, because of the effect of the $T'M_i$ upon the relative prices of the factors in different uses.

Should a net subsidy be the case, as Mäler suggests, the question arises as to where funds to pay the subsidy are obtained. In the model presented in this review the receipts from the effluent tax may be used parametrically to pay the subsidy, the balance being paid out of the forced surplus of the head taxes and subsidies that were chosen to achieve a particular Pareto optimum. The "forced surplus" thus narrowing the range of Pareto optima that can be chosen. Mäler's model provides no direct answer to this query.

To investigate certain aspects of effluent standards, which I take to imply that some direct control is imposed upon the rate at which z can be emitted, I wish to return to equations (7). Solving these equations would result in an emission rate of say z^*. If z^m denotes the emission rate in the absence of the corrective tax, then the quantity $(z^m - z^*)$ denotes the level of abatement that the firm has engaged in. This abatement imposes costs upon the firm, but such costs are smaller than the tax saving $[T(z^m) - T(z^*)]$. Emitting pollution at the rate z^* does, in the model using effluent fees, require that the firm not only expend money to hire factors of production for abatement activities but also that it pay a tax of $T(z^*)$. These tax receipts in models such as this could be paid to those suffering the pollution in the form of a lump sum transfer or returned to the polluting firm in the form of a lump sum payment, regarded as being parametric to the firm. Such maneuvers produce income effects only and do not alter the formal marginal conditions.

If effluent standards are imposed and are imposed in such a manner that $\bar{z} = z^*$, the difference between the tax solution and the effluent standards solution is the presence in the former of a tax payment of $T(z^*)$. Such a tax, if viewed as parametric, has only income effects, and does not alter the marginal conditions. The tax formulation and the direct control methods differ in producing the appropriate marginal conditions when $z^* \neq \bar{z}$.

Firms do not operate at zero (normal) profits, as Mäler states, for adjustments in price and output counteract the use of unproductive resources (i.e., K_2 and L_2) by the firm.

Macroeconomic implications

In the second major part of his paper Mäler again causes me some puzzlement. His definition of the demand for waste disposal services as a function of the real wage and the real effluent charge seems at variance with previous sections. Previously such a function was noted as $z = M(L_1, K_1, L_2, K_2)$ which would seem to require that the interest rate be an argument in the new version of this function in the macro-model. In addition, we see that whereas the characteristics of the effluent charge (i.e., the $T'M_i$) were determined by the M function now the new waste disposal function has the effluents as a variable. These confusions arise, I think, because Mäler seems to vacillate between calling and using the M function as a *treatment* function and a *disposal* function. These are conceivably two different genres. If z^m is the amount of pollution emitted in the absence of control and z the amount after control, then $(z^m - z)$ is the amount treated and z the amount disposed of. Such a discrepancy is not sloughed off by letting z^m be constant—especially in a dynamic model—though I have not sorted out all of the ramifications of this difference.

In his discussion of the effect of an increase of the effluent charge on the emission rate Mäler makes the following points: (1) given the effluent charge relative prices are determined, (2) as long as the price level is not determined in the model an increase in the effluent charge will not decrease the emission rate because (3) all prices will change in the same proportion, therefore (4) "it seems necessary to introduce some mechanism by which the price level is determined" and (5) point (3) occurs because a supply function for waste disposal services was not given.

Are not (3) and (4) contradictory? The former implies a certain monetary mechanism—albeit a faulty one. Also, if the effluent charge changes relative prices (as is implied by (1) and which certainly is a characteristic of the model in this review) will not this be sufficient to change the emission rate? That is to imply that an increase in the effluent charge should change the relative prices of labor and capital used in production relative to abatement with a resulting increase in the relative price of output and a resulting decrease in its consumption. Perhaps, however, a one commodity model gets the theorist into trouble on such matters!

What does it mean to have a supply of waste disposal services? It would seem to mean a schedule of alternative quantities supplied at alternative prices. Is not this what the effluent charge *function* does? This function is not exogenously determined as Mäler claims, but it is determined by the marginal products of the treatment function and the (presumed) increasing marginal disutility of degradations to environmental quality. This was reflected in

$$T'M_i = \sum_i w^i u_y{}^i \frac{\partial y}{\partial z} M_i.$$

I personally find the attempt to determine the effect an increase in the effluent has upon the price level uninteresting and again puzzling. On the one hand Mäler assumes an inelastic supply of labor and on the other he assumes that the wage rate may only adjust upwards. The latter allows him to play around with a dichotomous economy of capitalists and workers by which he increases the probability of finding a positive relationship between the effluent charge and the price level. Yet such an assumption raises in my mind the question of how the firm responds to the effluent charge, particularly when it is noted that in the earlier part of the paper such a charge should logically fall upon all factors of production and treatment. Perhaps this is the way by which he is able to assert that the firm needs to be subsidized along its optimal path.

In summary let me add that I found this to be a stimulating paper with certain rough edges and certain problems in moving from Pontryagin's world to Patinkin's world. I am not convinced that the move was made—at least not along my turnpike.