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Optimum Investment in Social Overhead Capital

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Introduction

In most industrialized countries, the "environment" has come to play a significant role in recent years both in the process of resource allocation and in the determination of real income distribution. This is primarily due to the fact that, in these countries, the environment has become scarce relative to those resources which may be privately appropriated and efficiently allocated through the market mechanism. There is no inherent mechanism in a decentralized market economy whereby the scarcity of social resources may be effectively restored. In decentralized economies, most research has been concerned with regulating the use of the environment; few researchers have analyzed the effects the accumulation of the environment may have upon the pattern of resource allocation and income distribution in general.

In order to analyze the role played by the environment in the processes of resource allocation and income distribution, it may be convenient to introduce a broader concept of "social overhead capital," of which the environment may be regarded as an important component. Social overhead capital is defined as those resources which are not privately appropriated, either for technological reasons or for social and institutional reasons, and which may be used by the members of the society free of charge or with nominal charges. Social overhead capital may be collectively produced by the society, as in the case of social capital such as roads, bridges, sewage systems, and ports, or it may be simply endowed in the society as in the case of natural capital such as air, water, etc.

Thus all scarce resources may be classified into two categories; private means of production and social overhead capital. In a decentralized market economy, private means of production are allocated through the market mechanism, resulting in an efficient allocation of given amounts of scarce resources, but the management of social overhead capital has to be delegated to a certain social institution.

Such social institutions in charge of social overhead capital are concerned with two functions. First, they have to devise regulatory measures in order to see to it that the given stock of social overhead capital may be efficiently allocated among the members of the society and effectively utilized by each member. Second, they are concerned with the construction of social overhead capital by using private and social means of production in such a way that the resulting pattern of resource allocation and income distribution is optimum from the social point of view. In this paper, I should first like to discuss the criteria by which social institutions in charge of social overhead capital allocate scarce resources in order to attain a dynamic pattern of resource allocation which is optimum from the social point of view. Next I should like to analyze the problem of what sort of criteria one has to impose upon the behavior of social institutions which are in charge of the management and construction of such social overhead capital. It will in particular be shown that, if the effect of social overhead capital is neutral in the sense precisely defined below, then the dynamically optimum pattern of accumulation of social overhead capital may result when the use of the services from such a social overhead capital is priced according to its marginal social cost and an interest subsidy is given to the extent to which the market (real) rate of interest differs from the social rate of discount.

Social Overhead Capital

Before I proceed with the main discussion, I should like to present a general framework in which some of the more crucial aspects of social overhead capital may be detailed. The services provided by social overhead capital are usually analyzed in terms of Samuelsonian public goods, as introduced in Samuelson's now classical papers [2, 3]. However, most of the familiar examples of social overhead capital such as highways, air, etc., do not satisfy the definition of pure public goods. There are two aspects of the services provided by social overhead capital with which I am particularly concerned. The first aspect is related to the choice made by each member of the society of the level at which he uses the services; the Samuelsonian concept of pure public goods excludes such a possibility. For most services provided by social overhead capital, the members of the society have, within a significant range, freedom in determining the amounts of the services they use.

The second aspect which has been neglected in the Samuelsonian analysis is related to the phenomenon of congestion. For most social overhead capital, the capacity of overhead capital is generally limited and the effectiveness of a certain amount of the services to be derived from the given stock of overhead capital is affected by the amounts of the same services other members of the society are using.

The concept of social overhead which has been introduced in Uzawa [6] may take care of these two aspects which are characteristic of most services provided by the government as public goods. The basic approach in Uzawa [6] may be briefly outlined as follows.

All the means of production are classified into two categories; private means of production and social overhead capital. Private means of production, simply referred to as private capital, may be privately appropriated and each member of the society may dispose of private capital which he possesses in such a way that his utility or profit is maximized. On the other hand, social overhead capital, simply referred to as social capital or overhead capital, may not be privately appropriated and the use of the services derived from overhead capital may be regulated either by the government or by a social institution to which the management of overhead capital is delegated.

In the following discussion, it is assumed that both private capital and overhead capital are respectively composed of homogeneous quantities, and that the output produced by various production units is identical. The main propositions obtained in this paper may be extended, without much difficulty, to the general situation where there may exist several kinds of overhead capital as well as private capital. Is is also assumed that private capital is a variable factor of production in the sense that it costs less to shift its usage from one line to another. Again, the analysis may, with slight modifications, be extended to the general situation where some private capital goods are fixed factors of production, involving significant adjustment costs either in the process of accumulation or in the shift in their allocation.

The effects of social overhead capital upon the productive process are formulated in terms of the short-run production function which summarizes production processes of each producing unit. Namely, the output Q_β produced by a typical production unit β is assumed to depend upon the amount of the services of social overhead capital as well as that of private capital. Let K_β and X_β be respectively the amount of the services of private and social capital used by producing unit β, and let X and V

be respectively the total amount of the services of social capital being utilized and the stock of social capital existing at each moment of time. If α and β stand for the typical consuming unit and producing unit, respectively, then

$$X = \sum_{\alpha} X_\alpha + \sum_{\beta} X_\beta. \tag{1}$$

The production function for producing unit β may be written as:

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V). \tag{2}$$

The dependence of the production function upon X and V indicates that the phenomenon of congestion occurs with respect to the use of the services of social overhead capital. The effect of congestion may be brought out by the assumptions:

$$F_X{}^\beta < 0, \quad F_V{}^\beta > 0. \tag{3}$$

The Samuelsonian case of pure public goods may be regarded as the limiting case where the production function F^β is independent of X_β and X.

The standard properties concerning the production function are assumed for the present case. In particular, it will be assumed that the marginal rates of substitution between various factors are diminishing and that the production processes are subject to constant returns to scale. Namely, the production function F^β given by (2) is concave with respect to K_β, X_β, X, and V, and it is linear homogeneous with respect to all of its variables. Furthermore, it is assumed that private capital and social capital are complementary in the sense that the increase in the use of the services of social capital shifts the schedule of the marginal product of private capital upward; namely,

$$F_{K_\beta{}^\beta X_\beta} > 0. \tag{4}$$

Social overhead capital may be defined *neutral,* when the negative effect due to the increase in the aggregate level of the use of the services of overhead capital is precisely counterbalanced by the corresponding increase in the stock of social overhead capital. Such would be the case if, for each producing unit β, the production function is homogeneous of order zero with respect to the aggregate level of the use of social capital X and the stock of social capital V. In this case, the production F may be written in the following form:

$$Q_\beta = F^\beta(K_\beta, X_\beta, X/V). \tag{5}$$

(Using the convention that functional symbols are used to indicate the nature of dependency in general.)

If social overhead capital is homogeneous of order zero with respect to X and V, then the following condition is satisfied:

$$F_X{}^\beta X + F_V{}^\beta V = 0. \tag{6}$$

In this paper, social overhead capital will be defined neutral with respect to production, if the following condition is satisfied:

$$\sum_\beta F_X{}^\beta X + \sum_\beta F_V{}^\beta V = 0. \tag{7}$$

Thus, if, for each production unit β, the production function is homogeneous of order zero with respect to X and V, then social overhead capital in question is neutral with respect to production.

The neutrality condition may be stated in terms of various elasticities. Let the elasticities η_X and η_V be defined as follows:

$$\eta_X = \frac{-\sum_\beta F_X{}^\beta X}{\sum_\beta F^\beta}, \quad \eta_V = \frac{-\sum_\beta F_V{}^\beta V}{\sum_\beta F^\beta}. \tag{8}$$

Thus, social overhead capital is neutral with respect to the production if and only if the elasticity is equal to the elasticity η_X;

$$\eta_V = \eta_X. \tag{9}$$

The effects exerted by social overhead capital upon the processes of consumption may be similarly formulated. For consuming unit α, the level of utility U_α depends upon the amount of services of social overhead capital, X_α, as well as the amount of private consumption C_α. The phenomenon of congestion may occur with respect to the processes of consumption, so that the utility function for each consuming unit α may be in general written as

$$U_\alpha = U^\alpha(C_\alpha, X_\alpha, X, V). \tag{10}$$

The phenomenon of congestion may be again formulated by the following conditions:

$$U_X{}^\alpha < 0, \quad U_V{}^\alpha > 0. \tag{11}$$

It is assumed that the utility function of U_α exhibits the feature of diminishing marginal utility and that the variables appearing in (10) exhaust all the variables which are limitational in the processes of consumption; namely, it may be assumed that, for each consuming unit α, the utility function U^α is concave and linear homogeneous with respect to C_α, X_α, X, V.

Social overhead capital is defined *neutral with respect to consumption,* if the following conditions are satisfied:

$$\sum_\alpha \frac{U_{X^\alpha} X}{U_{C_\alpha}{}^\alpha} + \sum_\alpha \frac{U_{V^\alpha} V}{U_{C_\alpha}{}^\alpha} = 0. \tag{12}$$

Finally, it is assumed that the cost in real terms in providing the services of social overhead capital depends upon the stock of overhead capital as well as upon the amount of the services provided; namely, the current cost in real terms W is a function of X and V:

$$W = W(X, V). \tag{13}$$

It may be in general assumed that the larger the amount of the services of overhead capital provided, the higher is the current cost, but the larger the stock of overhead capital, the lower is the current cost. In symbols,

$$W_X > 0, \quad W_V < 0. \tag{14}$$

Furthermore, it will be assumed that the cost function $W(X, V)$ is linear homogeneous with respect to X and V, so that it may be written as:

$$W = w(x)V, \quad x = X/V. \tag{15}$$

It is now necessary to introduce two concepts which will play a central role in the analysis of social overhead capital. They are the marginal social cost associated with the use of social overhead capital and the marginal social product of overhead capital.

The marginal social cost associated with the use of overhead capital, θ, is defined as the aggregate of marginal losses incurred by all the economic units in the economy due to the marginal increase in the use of the services of social overhead capital. Thus the marginal social cost may be represented by the following formula:

$$\theta = \sum_\alpha \frac{-U_X{}^\alpha}{U_{C_\alpha}{}^\alpha} + \sum_\beta - F_X{}^\beta + W_X. \tag{16}$$

On the other hand, the *marginal social product of overhead capital, r,* is defined as the aggregate of the marginal increase due to the marginal increase in the stock of overhead capital. In symbols, the marginal social product r may be given by:

$$r = \sum_{\alpha} \frac{U_V^{\alpha}}{U_{C_\alpha}^{\alpha}} + \sum_{\beta} F_V^{\beta} + W_V. \tag{17}$$

Social overhead capital now may be defined as *neutral* if the following condition is satisfied:

$$rV = \theta X. \tag{18}$$

It may be easily seen that, if all the production functions and utility functions are homogeneous of order zero with respect to X and V, then social overhead capital is neutral in the sense defined here.

Social overhead capital may be classified as *socially biased* if rV exceeds θX. The implication of such a classification will be discussed later.

The marginal social cost θ and the marginal social product r both depend upon the relative magnitude of the endowments of private capital and social overhead capital, as well as upon the allocation of these resources between individual economic units. In general, the higher the ratio of the private capital over the stock of overhead capital, the higher is the marginal social cost and also the higher is the marginal social product of overhead capital. The marginal social cost may be regarded as an index to measure the scarcity of social overhead capital relative to the stock of private capital or relative to the level of economic activities.

Efficient Pricing for the Use of Social Overhead Capital

Let us first discuss the problem of how to make efficient use of the services derived from a given stock of social overhead capital. Suppose that the stock of overhead capital V and the stock of private capital K are given and remain constant. The allocation of private capital between producing units, the distribution of output between consuming units, and the distribution of the services of social overhead capital are termed efficient if they result in the situation where the level U of social utility, to be defined as the aggregate of individual utility levels, is maximized among all the feasible patterns of resource allocation. Mathematically, the problem is stated as follows. To find the pattern of resource allocation $(C_\alpha, X_\alpha, K_\beta, X_\beta)$ which maximizes the level of social utility

$$U = \sum_\alpha U_\alpha, \tag{19}$$

subject to the constraints:

$$\sum_\alpha C_\alpha + W = \sum_\beta Q_\beta, \tag{20}$$

$$\sum_\beta K_\beta = K, \tag{21}$$

$$\sum_\alpha X_\alpha + \sum_\beta X_\beta = X, \tag{22}$$

where

$$U_\alpha = U^\alpha(C_\alpha, X_\alpha, X, V), \tag{23}$$

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V). \tag{24}$$

As explained in Uzawa [6], such a problem may be easily solved, by applying the method of Lagrange multipliers. The solution then may be shown to be identical with the one which is obtained by the principle of the marginal social cost pricing. Namely, the efficient allocation may be obtained by the following mechanism.

Private capital is allocated through a perfectly competitive market, while the services of social overhead capital may be distributed among consuming and producing units in such a way that each individual unit is charged the marginal social cost for the use of the services rendered by social overhead capital. It may be noted that either the administrative costs associated with such a pricing scheme are assumed to be neglible or it is possible to find an alternative pricing scheme, without involving significant administrative costs, which results in the identical distribution of the services of social overhead capital.

As indicated in the previous section, when social overhead capital becomes scarce relative to the endowment of private capital, then the marginal social cost becomes higher and individual economic units are charged a higher price for the use of social overhead capital. However, it is generally the case that some of the scarce resources are classified as social overhead capital because of the impact they may have upon the equalization of the income distribution. Hence, if a high price were charged for the use of such social overhead capital, it would become difficult to justify the decision in classifying such resources as social overhead capital. The implications of such a phenomenon may be more explicitly brought out if the problem of accumulation of social overhead capital is discussed. In the next section, I should like to discuss the problem of optimum investment in social overhead capital.

Optimum Investment in Social Overhead Capital

In this section, I should like to extend the previous analysis to the situation where one is concerned with the process of capital accumulation for both private and social capital, and to examine the pattern of resource allocation over time which is optimum from a dynamic point of view. It will be shown that the principle of the marginal social costs may be extended to this dynamic case and the criteria for optimum allocation of investment between private and social capital will be obtained within the framework of the Ramsey theory of optimum growth.

In order to simplify the exposition, it will be assumed that the rate by which consumers discount their future levels of utility is constant and identical for all consumers in the society. Let δ be the rate of discount. The level of social utility U may now be expressed by

$$U = \int_0^\infty U(t)e^{-\delta t}\, dt, \tag{25}$$

where the utility level $U(t)$ at a point of time t may be given y

$$U(t) = \sum_\alpha U_\alpha(t), \tag{26}$$

with

$$U_\alpha(t) = U^\alpha[C_\alpha(t), X_\alpha(t), X(t), V(t)]. \tag{27}$$

Let V_0 be the stock of social overhead capital existing at the initial point of time 0. I am concerned with the problem of finding a path of private consumption for each consumer, of allocation of private and social capital between various economic units, and of capital accumulation for both private and social capital over time such that the resulting level of social utility (25) is maximized over all feasible paths. In order to discuss this optimum problem, I should like to pay particular attention to the difference between private and social capital with regard to the extent to which investment is used to increase the stock of capital (to be measured in the efficiency unit). In general, social overhead capital is difficult to reproduce in the sense that a significant amount of scarce resources have to be used in order to increase the stock of capital. For private capital however, investment may without much difficulty be converted into the accumulation of capital. It may be possible to formulate the relationships between the amount of investment and the resulting increase in the stock of capital in terms of a certain functional relationship. I have elsewhere discussed this problem for the case of private capital and a similar conceptual framework may be applied to the case involving social overhead capital (Penrose [1] and Uzawa [4, 5]).

Let I_V be the amount of real investment devoted to the accumulation of social overhead capital. If social overhead capital V is measured in a specified efficiency unit, the amount of real investment I_V may not necessarily result in the increase in the stock of capital by the same amount. Instead, there exists a certain relationship between the amount of real investment I_V and the corresponding increase $\dot{V}$ in the stock of social overhead capital on one hand, and the current stock of social overhead capital V on the other:

$$I_V = \phi_V(\dot{V}, V). \tag{28}$$

The relationship (28) may be interpreted as follows: in order to increase the stock of social overhead capital V by the amount $\dot{V}$, real investment I_V given by (28) has to be spent on the accumulative activities for social overhead capital. In what follows, it will be assumed that the function ϕ exhibits a feature of constant returns to scale with respect to $\dot{V}$ and V, thus one may write (28) as

$$I_V/V = \phi_V(\dot{V}/V). \tag{29}$$

Since it may be assumed that the marginal costs of investment are increasing as the level of investment is increased, the function ϕ_V satisfies the following conditions:

$$\phi_V'(.) > 0, \quad \phi_V''(.) > 0. \tag{30}$$

Similar relationships may be postulated for the accumulation of private capital for each producing unit; namely, for each producer β, the amount of real investment I_β required to increase the stock of capital K_β by the amount $\dot{K}_\beta$ may be determined by the following Penrose function:

$$I_\beta/K_\beta = \phi_\beta(\dot{K}_\beta/K_\beta), \tag{31}$$

where the Penrose function ϕ_β again satisfies the conditions:

$$\phi_\beta'(.) > 0, \quad \phi_\beta''(.) > 0. \tag{32}$$

For the current cost W, we have,

$$W = w\left(\frac{X}{V}\right)V, \quad w'(.) > 0, \quad w''(.) > 0. \tag{33}$$

The optimum problem may now be more precisely stated as follows: A path of resource allocation over time,

$$[C_\alpha(t),\ I_\beta(t),\ I_V(t),\ X_\alpha(t),\ X_\beta(t),\ K_\beta(t),\ V(t)], \tag{34}$$

is defined as a feasible path if it satisfies the following conditions:

$$Q(t) = \sum_\alpha C_\alpha(t) + \sum_\beta I_\beta(t) + I_V(t) + W(t), \tag{35}$$

$$Q(t) = \sum_\beta F^\beta[K_\beta(t), X_\beta(t), X(t), V(t)], \tag{36}$$

$$X(t) = \sum_\alpha X_\alpha(t) + \sum_\beta X_\beta(t), \tag{37}$$

$$\frac{I_\beta(t)}{K_\beta(t)} = \phi_\beta[z_\beta(t)], \quad \frac{\dot{K}_\beta(t)}{K_\beta(t)} = z_\beta(t), \tag{38}$$

$$\frac{I_V(t)}{V(t)} = \phi V[z_V(t)], \quad \frac{\dot{V}(t)}{V(t)} = z_V(t), \tag{39}$$

$$W(t) = w[X(t)]V(t), \quad x(t) = X(t)/V(t), \tag{40}$$

$$K_\beta(0) = K_\beta{}^0, \quad V(0) = V^0 \text{ given.} \tag{41}$$

I am then interested in finding a feasible path of resource allocation over time which maximizes the social utility (25). This optimum problem is difficult to solve, and I shall instead be concerned with finding a path of resource allocation which reasonably approximates the optimum path. Among such approximate paths, the one with the simplest structure will be obtained by examining the conditions which the imputed prices of private and social capital have to satisfy.

Let $p_\beta(t)$ and $p_V(t)$ be respectively the imputed prices, at time t, of private capital K_β and social overhead capital V, and let $p(t)$ and $\theta(t)$ be the imputed prices of output Q and the use of social overhead capital X.

These imputed prices correspond to the Lagrange multipliers associated with the constraints for the optimum problem. The Euler-Lagrange conditions which the optimum path has to satisfy may be rearranged to yield the following conditions:

$$U^{\alpha}{}_{C_\alpha} = p, \quad U^{\alpha}{}_{X_\alpha}/U^{\alpha}{}_{C_\alpha} = \theta, \tag{42}$$

$$F^{\beta}{}_{X_\beta} = \theta, \tag{43}$$

$$\theta = \sum_{\alpha} \frac{(-U^{\alpha}{}_{X})}{U_{C_\alpha}{}^{\alpha}} + \sum_{\beta}(-F^{\beta}{}_{X}) + W'\left(\frac{X}{V}\right), \tag{44}$$

$$\frac{\dot{p}_\beta}{p_\beta} = \delta - z_\beta - \frac{r_\beta - \phi_\beta(z_\beta)}{\phi'_\beta{}'(z_\beta)}, \tag{45}$$

where

$$\phi_\beta{}'(z_\beta) = \frac{p_\beta}{p}, \quad r_\beta = F_{K_\beta}{}^{\beta}, \tag{46}$$

$$\frac{\dot{p}_V}{p_V} = \delta - z_v - \frac{r_V - \phi_V(z_V)}{\phi_V{}'(z_V)}, \tag{47}$$

where

$$\phi_V{}'(z_V) = \frac{p_V}{p}, \quad r_V = \sum_{\alpha} \frac{U_V{}^{\alpha}}{U_{C_\alpha}{}^{\alpha}} + \sum_{\beta} F_V{}^{\beta}. \tag{48}$$

I have omitted the time suffix t to avoid ambiguity.

The quantity on the right-hand side of equation (44) corresponds to the concept of the marginal social costs associated with the use of social overhead capital in the context of the dynamic optimization. It may be noted that the marginal costs associated with the depreciation of social overhead capital are evaluated in terms of its imputed price p_V/p measured in real terms. The quantity r_β defined in (46) is nothing but the marginal product of private capital, while the r_V defined in (48) is the marginal social product of social overhead capital measured in real terms. Namely, the r_V represents the marginal gain to the society measured in real terms due to the marginal increase in the stock of social overhead capital V.

The conditions (44–46) suggest that, in order to attain an optimum allocation of scarce resources in the short run, one has to impose charges equal to the marginal social costs for the use of social overhead capital, with the marginal social costs being defined in the modified sense (46). On the other hand, the pattern of accumulation of private and social capital may be described by the conditions (45–48) describing the rules by which the imputed prices change over time. In order to approximate the structure of the optimum path of capital accumulation, I should like to consider the case where the imputed prices are assumed constant at each point in time. Namely, the rates of accumulation of private and social capital are obtained by assuming that equations (45) and (47) are equated to zero. It can be shown that the path of capital accumulation obtained by such a procedure reasonably approximates the optimum path, although the sense in which reasonable approximation is used needs a more complicated formalization.

If the imputed prices are assumed constant over time, the rates of capital accumulation z_β and z_V may be obtained by solving the following conditions:

$$\frac{r_\beta - \phi_\beta(z_\beta)}{\delta - z_\beta} = \phi_\beta'(z_\beta), \tag{49}$$

$$\frac{r_V - \phi_V(z_V)}{\delta - z_V} = \phi_V'(z_V). \tag{50}$$

The determination of the rates of accumulation, z_β and z_V, is illustrated by the following diagrams.

It is easily seen, from the diagrams in Figures 1 and 2, that the rates of accumulation of private and social capital are uniquely determined, that the higher the marginal product of private capital, the higher is the

Figure 1

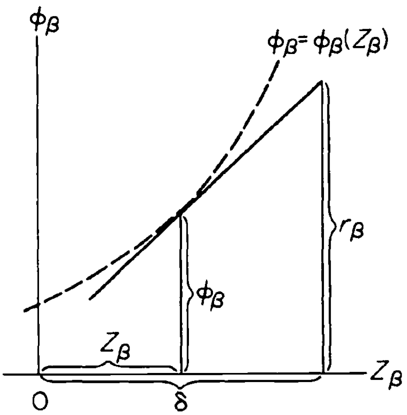
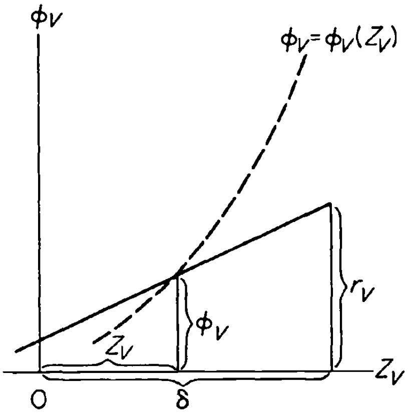


Figure 2



corresponding rate of accumulation for private capital, and that the higher the marginal social product of social capital, the higher is the rate of accumulation. On the other hand, an increase in the social rate of discount δ will lower the rate of accumulation both for private and social capital.

Thus, the (approximate) optimum rates of accumulation for private and social capital will be determined once the marginal private or social product of these capital are known. However, the marginal products of both private and social capital depend upon the extent to which social overhead capital is used by the member of the society. The amount of the services of social overhead capital used is in turn related to the imputed price p_V/p of social overhead capital, as is seen from the definition of the marginal social costs.

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COMMENT

Robert Dorfman, Harvard University

The time constraint compels me to concentrate on a single aspect of Uzawa's interesting paper. In this paper, Uzawa makes powerful use of a conceptual device that a number of people have been experimenting with recently. I shall use my time to indicate some of the implications of this device, since I have found it to be illuminating in confronting some of the slippery conceptual problems that arise in the general area of public economics.

First, I would like to sketch the context in which the use of this device arises. In the traditional theory of consumption we imagine a typical consumer, Mr. α, who chooses the amounts he purchases of different commodities, x_i^α, so as to maximize the value of an indicator of his utility, $u^\alpha(x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha)$, subject of course, to a budget constraint. This formulation is adequate for studying Mr. α's personal consumption decisions, but when we turn attention to the economy-wide allocation of resources we have to bring some additional considerations into the picture. In particular we have to take public goods and externalities into account. In 1954 Samuelson demonstrated how to introduce public goods into the analysis. The essential idea is to include the level of provision of public goods in the individual's utility indicator by writing the typical indicator as $u^\alpha(x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha, Z_1, \ldots, Z_m)$. In this notation the Z_j indicate the levels of provision of public goods or collective goods. Notice, they do not have any superscripts.

Too much has already been written about the definition of these public goods and I shall not further burden the literature. The essential operational characteristics that distinguish them from private goods are that (1) each enters with the very same magnitude into the utility indicators of several or all consumers, and (2) the individual consumer does not select this magnitude, nor does it enter explicitly into his budget constraint. The levels of the public goods are regarded as being chosen by some economic entity, normally a governmental body or a nonprofit institution, that does not have a utility function of its own, but is supposed to be concerned with the effect of its choice on the utilities and productivities of the other decision units in the economy. The well-known upshot of the analysis in this form is a set of criteria for the optimal levels of these public goods.

But externalities are still absent from the formulation. Here is where the conceptual device that I have mentioned enters. Let us write the typical utility indicator as $u^\alpha(x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha, X_1, X_2, \ldots, X_n, Z_1, \ldots, Z_m)$. In this notation the X_i represent the levels of what I shall call externality-conveying goods. Like the public goods variables, they carry no superscripts and their magnitudes are common to the utility indicators of several or all consumers. They differ from the public goods in the locus of the decisions made about them. Indeed, no explicit decisions are made about them at all, but their levels are by-products of decisions made about private goods. We may formalize this notion by writing $X_i = f_i(x_i^\alpha, x_i^\beta, \ldots, x_i^\omega)$ so that the level of each externality-conveying good is some function of the consumers' choices concerning their private consumptions of that very same good. Uzawa concentrates on the special case where $X_i = \sum x_i^\alpha$ and so shall I for the most part, since it is an important special case and is sufficient to bring out most of the significant issues. In particular, this special case arises where the externality is some form of congestion. For example, if x_i^α represents the number of visits that the α family makes to a public park then u^α is likely to be positively affected by an increase in x_i^α, but negatively affected by an increase in $X_i = \sum x_i^\alpha$. In this formulation the number of acres in a public park would also be relevant, and would be indicated by one of the components Z_j since it is a public good. This simple formulation thus captures all three aspects of the enjoyment of a good used in common: the direct private decision, the interaction of private decisions, and the level of provision of the common facility.

In addition to congestion externalities, the simple summation formulation is appropriate for some types of environmental pollution. Street litter is a plausible instance, as is atmospheric pollution when effluents are considered evenly dispersed over a wide area. There are other forms of externality for which more complicated formulas are needed. The pollution of public waters is an important instance, but for this case a weighted sum is likely to be adequate in many applications.

The general formula suggested above does appear to cover these and a great many other kinds of externalities, but can be made more general. One obvious generalization is to permit a particular externality-conveying good, X_i, to be a function of the private consumptions of a number of different private goods. Actually this generalization is not very useful since the effects of different forms of consumption on a particular externality can be regarded as additive to a linear approximation. A more interesting generalization is to recognize that the functional relationship of an externality to private levels of consumption may be different for different consumers. This is the case for some kinds of congestion, and also for concentrations of atmospheric pollutants which are not uniform over a wide area.

In spite of these possibilities for elaboration, the simple formulations are perfectly adequate for conveying the main insights, which are likely to be obscured by attempting to achieve generality. Besides, in practical applications the data requirements increase rapidly with the complexity of the expression. It seems wisest to rest with simple expressions of the relationship between externalities and the private decisions that give rise to them.

To this point I have mentioned only consumers, but producers are also affected by public goods and externalities, and precisely the same considerations can be used to introduce the levels of public goods and externality-conveying goods into their production functions or technology sets. From this point on I shall presume that this has been done.

I have now sketched a tripartite vision of economic goods: private goods, externality-conveying goods, and public goods. The essential distinction among them is the locus of decisions concerning them and the considerations taken into account in making these decisions. More familiar formulations rest on such difficult concepts as "appropriability," "excludibility," "rivalness," and so forth, and I shall not recount the complications that those concepts lead into when we try to apply them. All those interrelationships are comprehended in the simple formulation $u^\alpha(x_1^\alpha, x_2^\alpha, \ldots, x_n^\alpha, X_1, X_2, \ldots, X_n, Z_1, \ldots, Z_m)$.

The virtue of this formulation is that it enables us to think about private goods, externalities, and public goods within a single unified framework. The conditions for Pareto optimality can be deduced from it, and there are no surprises. In the case of a pure private good, one for which X_i = constant, both the conventional marginal inequalities for consumer choice and the standard requirement that each consumer's marginal rate of substitution between any two such goods must be equal to producers' marginal rate of transformation between them emerges. With respect to pure public goods, this formulation leads to the familiar Bowen-Samuelson requirement that the marginal rate of transformation between a pure private good and a pure public good should be equal to the sum of consumers' marginal rates of substitution between that private good and that public good.

The condition for the optimal provision of externality-producing goods is a bit more complicated, but still much as expected. I do not see any way to state these conditions precisely without doing the underlying algebra, from which I forbear. The essential idea is that a wedge has to be driven between the private consumer's perception of the value of such private goods (expressed by his marginal rates of substitution) and the social resource cost of providing the goods (expressed by marginal rates of transformation). This wedge has to be equal to the sum of the externalities conveyed by the use of the good, to all consumers and producers. (These externalities are also marginal rates of substitution, exemplified by the number of units of some private good that a typical consumer would be willing to relinquish for a one-unit diminution in some X_i.) The formulas are complicated by the presence of externalities that bridge the consumption and production spheres: externalities that consumers impose on producers, producers on consumers, consumers on each other and producers on each other. One interesting consequence is that the social production function is not well defined when external effects are present, because consumption decisions change the range of choice for production decisions.

This point of view provides, as I said before, some significant clarifications in thinking about public economics. It emphasizes the importance of institutional arrangements in characterizing commodities and their social consequences, and subordinates technological characteristics. It explains why a public park is a public good while a commercial ski lift or resort is not, in spite of the virtual identity of their technological properties. It explains why a highway is a public good although the consumption of its services may be fiercely competitive. It circumvents the truly baffling problems of defining such concepts as excludability and appropriability, which may be important legally or technically, but are irrelevant to the analysis of resource allocation. It sorts out the private, external, and public aspects of such intangibles as education and law and order.

For all these reasons, I commend it to you strongly as the most satisfactory conceptual framework for dealing with public goods, spillover effects, and related phenomena. I predict that Professor Uzawa's paper is only the first in a great number that will invoke this point of view in the analysis of public goods and externalities.