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AN ECONOMETRIC MODEL OF BUSINESS CYCLES

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INTRODUCTION

THIS is a progress report on an econometric model of business cycles. The main characteristics of business cycles have been summarized in a recent article by Burns.[1] This study has incorporated many, but by no means all, of the important elements in the Burns article. Moreover, it has included some relationships not explicitly covered there. Hence, this is by no means a perfect translation. In general, the material we present is a simplified, aggregative version of the earlier text.

Although economic activities of individual households, business firms, and industries do not uniformly follow the expansion and contraction of aggregate economic activities, much of the study of business fluctuations is concerned with the determination—through

NOTE: Arthur F. Burns participated in the early stages of formulation of this model, and we are deeply indebted to him for his constructive suggestions.

Builders of an econometric model always benefit from previous works. These include the Brookings Model, the FED-MIT Model, the models by T. C. Liu, the OBE Model, the Wharton Model, and their predecessors. The rationale for some of the demand equations can be found in G. C. Chow, "Multiplier, Accelerator, and Liquidity Preference in the Determination of National Income in the United States," *The Review of Economics and Statistics*, XLIX (February, 1967), pp. 1–15. Preliminary versions of this paper were presented before staff meetings of the National Bureau of Economic Research and of the Bureau of Labor Statistics, where valuable comments were received. Phillip Cagan, Jacob Mincer, and Thomas Juster participated in the initial formulation of the model, and have given frequent advice. The computations for Table 1 were done at MIT using the Troll system, with Mark Eisner offering considerable help. Gary Becker, Charlotte Boschan, Edwin Kuh, and Victor Zarnowitz have provided suggestions in various stages. In acknowledging our sincere thanks to the above-mentioned colleagues, we must confess that had their many suggestions been taken more seriously, this paper would have been improved.

[1] A. F. Burns, "Business Cycles: General," *International Encyclopedia of the Social Sciences* (New York, The Macmillan Company, 1968), pp. 226–45.

time, and through the stages of business cycles—of aggregate output and employment, total income and profits, the general price level, the average wage rate, and some index of interest rates. Aggregate output and the price level are determined by the forces of demand and supply. On the side of aggregate demand, it is useful to distinguish between the demand for consumption goods and the demand for investment goods. The former depends on the level of income, among other factors. The latter may be influenced by the rate of change in output or sales, but is also affected by profits and the rate of interest and, perhaps, their rates of change.

Consumption expenditures for durable goods may be treated differently from nondurable goods and services, since expenditures on durable goods, net of depreciation, are additions to the total stock of durable goods available for consumption, just as expenditures on producer durable goods, net of depreciation, are additions to the total stock of capital goods available for production. Insofar as the stock of durable goods is related to income, and the stock of capital goods is related to output, durable goods expenditures may be related to the change in income; and investment expenditures, to the change in output. Hence, it is reasonable to treat expenditures on nondurable goods and services as dependent on the level of income, while durable goods expenditures are dependent on the rate of change in income. In the case of investment expenditures, however, treating the demand for services from capital as a special case of the derived demand for inputs may be incomplete if the firm's decision to invest—as distinguished from its output decision for a given amount of capital—depends on the profitability of the existing enterprise relative to the possible return to capital available elsewhere. Moreover, the flow of profits may influence the timing of decisions to initiate investment projects through their influence upon the state of confidence, and through the ready availability of funds.

Since investment expenditures play a crucial role in business cycles, their determinants must be sufficiently explained. First, we should expect rates of interest to be inversely affected by the supply of money, but positively related to the supply of government debt, given the level of gross national product. Second, business profits will tend to increase with national income and the price level, but to decrease as wage rates and the level of employment increase. A major factor limiting the expansion of investment expenditures is the curtailment in the growth of profits. The latter occurs when the rate of growth of output is retarded, partly for reasons of supply to be discussed later, but also when increase in the wage rate catches up with the price increase, and when labor input per unit of output can hardly be reduced further in the later stages of a business expansion.

The above-mentioned behavior of wages and employment can be partly explained by a model of demand for labor and its supply. The demand for labor depends on output, the amount of capital, the price level, and the wage rate. Employment follows the demand for labor with time lags. The supply of labor—namely, the labor force—grows mainly with population, but is inversely influenced by the level of unemployment, through the discouraged-worker effect. Finally, the rate of change in the wage rate is governed by the gap between the demand for, and the supply of, labor, and by the rate of price change, with delays. Hence, with employment and the wage rate adjusting slowly to the expansion of output, profits will increase more rapidly in the early phase of expansion than in the later phase. Furthermore, in the later phase—with the supply of labor limited by population—the continued effort to increase employment forces the wage rate up; as the ratio of capital to output is diminished, labor requirement per unit of output can no longer be reduced.

In our model, the relationship between investment expenditures and their determinants is studied in two steps. In one, the determinants are related to orders or contracts, since these variables reflect more closely the decision stage. In the other, the orders and contracts are used to predict investment expenditures through a distributed-lag relationship. This relationship between investment expenditures and past orders may be affected by the state of the economy; the lags in completions of past orders may be longer, with fuller utilization of existing capital stock.

The forces of supply affect aggregate output and the price level in various ways. As we have just pointed out, given investment demand as expressed by orders and contracts, output of investment goods can vary according to the conditions of supply. The limitation of labor supply can raise the wage rate, thus adversely affecting profits and investment demand. The limitation of the stock of physical capital, relative to output, can raise labor requirements, also affecting profits adversely. The limitation of the supply of money and credit can raise interest rates and discourage investment.

The general price level can be explained by the interactions of demand and supply. An aggregate supply curve for the economy, relating aggregate supply positively to the price level (given other determinants) can be conceived as the sum of the supply curves of individual firms. The other determinants are the wage rate and the stock of physical capital. The price level is assumed to adjust toward the level determined by the aggregate supply function after equating aggregate demand (as measured by orders) with supply. Thus, when demand expands at full employment, and at a high rate of capacity utilization, the effect may be to increase the price level more than the physical output.

In the next section, we present a mathematical formulation of the model, following closely the above discussion of its structure.

MODEL FORMULATION

IN FORMULATING a dynamic model applicable to quarterly data, one has to be explicit about the lag structure of each equation, besides specifying the important variables involved. In this connection, economic theorizing and available empirical evidence provide only a partial guide, and much room is left for experimenting with data. In order to limit the amount of experimentation, we have decided to restrict the lag structure to a certain simple mathematical form. One of the simplest forms is geometrically declining weights applied to past values of the explanatory variables, which, after the familiar Koyck transformation, reduces to a linear combination of the current explanatory variables and the dependent variable, lagged one quarter. The form we have adopted results from replacing, in some cases, the current value of each explanatory variable in the above linear combination by an unweighted sum of its recent values.[2] This form is not

[2] This form was used in T. C. Liu, "A Monthly Recursive Econometric Model of United States: A Test of Feasibility," *The Review of Economics and Statistics,* XLXI (February, 1969), pp. 1–13.

unduly restrictive, in view of the fact that fairly different lag structures can fit the data almost equally well. It has the advantage of providing sufficient flexibility while economizing on the number of degrees of freedom in the face of multicollinearity.

For ease of exposition, the equations in this section will contain only the current value of an explanatory variable, whereas, in the empirical section to follow, that current value will sometimes be replaced by an unweighted sum of a small number of values in recent quarters. Similarly, when a lagged value of an explanatory variable is used in this section, it will sometimes be replaced by the corresponding sum of lagged values in the next section. Variables asserting a negative influence are indicated by a minus sign. All expenditures, incomes, and orders are in constant dollars.

Consumption expenditures on nondurable goods $C_{n,t}$ and on services $C_{s,t}$ are assumed to be distributed-lag functions of disposable personal income, a fraction g_t of personal income $Y_{p,t}$.

$$C_{n,t} = f_1(g_t Y_{p,t}, C_{n,t-1}) \tag{1}$$

$$C_{s,t} = f_2(g_t Y_{p,t}, C_{s,t-1}) \tag{2}$$

Consumption expenditures on durable goods $C_{d,t}$ depend positively on current disposable income and negatively on lagged disposable income. Since disposable income includes unemployment compensation and other Social Security payments, which may not induce the use of durable goods to the same extent as other components of disposable income, we include the rate of change in unemployment U in the expenditure equation for consumer durables.

$$C_{d,t} = f_3(g_t Y_{p,t}, -g_{t-1} Y_{p,t-1}, -\Delta U_t, C_{d,t-1}) \tag{3}$$

Investment expenditures on private residential construction I_p and on business plant and equipment I_b are related to the orders and contracts placed for them, J_p and J_b. Such a relationship may be affected by the stage of the business cycle. If the economy is close to full utilization of its capacity, i.e., if the capital stock K is small relative to output, one would expect current investment expenditures to be small, given the past contracts and orders.

$$I_{p,t} = f_4(J_{p,t}, K_t, I_{p,t-1}) \tag{4}$$

$$(5) \qquad I_{b,t} = f_5(J_{b,t}, K_t, I_{b,t-1})$$

Contracts for private residential construction are determined by the rate of change in disposable income and the rate of change in the long-term interest rate R_L, via a simple distributed-lag mechanism.

$$(6) \qquad J_{p,t} = f_6(g_t Y_{p,t} - g_{t-1} Y_{p,t-1}, R_{L,t} - R_{L,t-1}, J_{p,t-1})$$

Contracts and orders for business plant and equipment are determined by the rates of change in GNP and in the long-term rate of interest, and by corporate profits after corporate profit tax. Let Π denote corporate profits, and $h_t \Pi_t$ corporate profits after taxes.

$$(7) \qquad J_{b,t} = f_7(GNP_t - GNP_{t-1}, h_t \Pi_t, R_{L,t} - R_{L,t-1}, J_{b,t-1})$$

Inventory change $I_{i,t}$ is explained by recent orders J_t contributing to the inflow of goods, and by final sales of goods X_t contributing to outflow. The inflow is also affected by capital stock, as in the relations between investment expenditures and orders. Here J includes all orders: J_p, J_b, orders of goods for sale J_s, and government orders and contracts J_g.

$$(8) \qquad I_{i,t} = f_8(J_t, K_t, -X_t)$$

Order of goods for sale J_s, the third component of our order figures, is motivated by the desire to maintain an equilibrium stock of inventories while satisfying current sales. Insofar as the desired stock of inventories depends on sales X, the change in price level ΔP and the short-term rate of interest R_s, and insofar as new orders are related to the change in the desired stock, we have

$$(9) \qquad J_{s,t} = f_9(X_t - X_{t-1}, \Delta P_t - \Delta P_{t-1}, R_{s,t} - R_{s,t-1}, J_{s,t-1})$$

The short-term rate of interest R_s is determined by a demand for money equation involving the nominal stock of money M, GNP, P, R_s and M_{t-1}. Furthermore, an increase in government interest-bearing debt GD will tend to raise the interest rate.

$$(10) \qquad R_{s,t} = f_{10}(GNP_t, P_t, -\Delta M_t, \Delta GD_t, R_{s,t-1})$$

The long-term rate of interest R_L is determined by the short-term rate with a lag

$$(11) \qquad R_{L,t} = f_{11}(R_{s,t}, R_{L,t-1})$$

Corporate profits Π is a function of national income Y minus government expenditures on services G_s, money wage rate W, the price level P, and employment L.

(12) $$\Pi_t = f_{12}(Y_t - G_{s,t}, -W_t, P_t, -L_t)$$

Demand for labor L^D in the private sector, measured by private employment plus job vacancies, is dependent on output net of government expenditures on services $GNP - G_s$, money wage rate, the price level, and capital stock

(13) $$L_t^D = f_{13}(GNP_t - G_{s,t}, -W_t, P_t, -K_t, L_{t-1}^D)$$

Employment L in the private sector is assumed to adjust toward L_t^D with a lag

(14) $$L_t = f_{14}(L_t^D, L_{t-1})$$

Total labor supply L^s is dependent on population N of ages 16 and over, and on the level of unemployment U

(15) $$L_t^s = f_{15}(N_t, -U_t, L_{t-1}^s)$$

Money wage rate W is influenced by the difference between total labor demand (L^D plus government employment L_g) and labor supply, and by the price level

(16) $$W_t = f_{16}(L_t^D + L_{g,t} - L_t^s, P_t, W_{t-1})$$

The general price level P is determined by an aggregate supply function involving aggregate supply, P, W, and K, and by equating aggregate demand with aggregate supply. Aggregate demand is measured by total orders and contracts J plus consumption expenditures on services C_s minus imports IM.

(17) $$P_t = f_{17}([J + C_s - IM]_t, W_t, -K_t, P_{t-1})$$

Dividends D is a simple distributed lag function of after-tax corporate profits

(18) $$D_t = f_{18}(h_t \Pi_t, D_{t-1})$$

Imports IM is explained simply by consumption expenditures $C = C_n + C_s + C_d$, investment expenditures $I = I_p + I_b + I_i$, and by government expenditures on goods G_g with geometric lags.

$$(19) \qquad IM_t = f_{19}(C_t, I_t, G_{g,t}, IM_{t-1})$$

Capital consumption allowances CCA is a weighted sum of its own value in the preceding quarter and current investments in private residential construction and in business plant and equipment, provided there is no change in depreciation tax laws.

$$(20) \qquad CCA_t = f_{20}(I_{p,t}, I_{b,t}, CCA_{t-1})$$

When a major change in tax laws occurs, as in the first quarter of 1962, a dummy variable can be introduced which takes a positive value for that quarter.

These complete the specification of the 20 equations in our model. In addition, there are 5 identities:

$$(21) \qquad GNP = C_n + C_s + C_d + I_p + I_b + I_i + G_g + G_s + EX - IM$$

$$(22) \qquad Y = GNP - CCA - T_1$$

Here Y denotes national income plus business transfer payments plus statistical discrepancy, and T_1 is indirect business tax minus subsidies less current surplus of government enterprises.

$$(23) \qquad Y_p = Y + T_2 - \Pi + D$$

T_2 is government transfer payments plus interest paid by government and by consumers, minus contributions for social insurance.

$$(24) \qquad X = GNP - C_s - G_s - I_i$$

$$(25) \qquad U = L^S - L - L_g$$

The 25 dependent variables are the left-hand variables in the above equations. All other variables are treated as exogenous.

STATISTICAL RESULTS

THE parameters of the 20 structural equations in our model, all assumed to be linear, are estimated by taking four-quarter differences of postwar quarterly data for the United States. Sources of data are given in the Appendix. We have experimented with one-quarter differences and found results to be poor, probably because of errors in measurement. The use of four-quarter differences eliminates trends in the original data, reduces serial correlations in the residuals of most of the equations, concentrates our attention upon business cycle changes, and makes the multiple correlation coefficient as a measure of goodness of fit more discriminating. The sample period for the dependent variables runs from the third quarter of 1948 to the last quarter of 1967, giving a total of 78 observations, with a few exceptions to be noted below.

The method of estimation is two-stage least squares. In the first stage, all dependent variables (in four-quarter differences) are regressed on a selected set of predetermined variables (in four-quarter differences). These are selected from a more aggregative version of our model, including equations 1 for total consumption C; 5–7 for total investment I; 11 for long-term rate of interest; 12 for profits; 13, 15, and 16 for labor demand, supply, and wage; 17 for the price level; 18 for dividends; and 20 for capital consumption allowances, treating exports minus imports as exogenous. They are the four-quarter differences of 17 variables, namely, C_{t-1}, I_{t-1}, I_{t-2}, $R_{L,t-1}$, Π_{t-1}, W_{t-1}, P_{t-1}, P_{t-2}, D_{t-1}, CCA_{t-1}, G_t, G_{t-1}, M_t, M_{t-1}, N_t, $(EX - IM)_t$, $(T_2 - T_1)_t$. In presenting the results below, we use the same symbols to denote four-quarter differences of the variables defined in the previous section. For example, C_t, C_{t-1}, and $g_tY_{p,t}$ actually refer to $(C_t - C_{t-4})$, $(C_{t-1} - C_{t-5})$, and $(g_tY_{p,t} - g_{t-4}Y_{p,t-4})$, respectively.[3] The ratio, in absolute value, of each coefficient to its approximate standard error is given in parentheses. DW stands for the Durbin-Watson statistic, exhibited purely as a descriptive statistic for serial correlation of the residuals. SE is the standard error of the regression. As in most econometric studies, many of the equations presented below are not the ones we started with, but were modified when we found deficiencies in the fit. In view of this experimentation, the meaning of the usual statistics presented is obscured. We are also aware of the large (often the largest) contribution made by the lagged dependent variable in each equation where it appears.

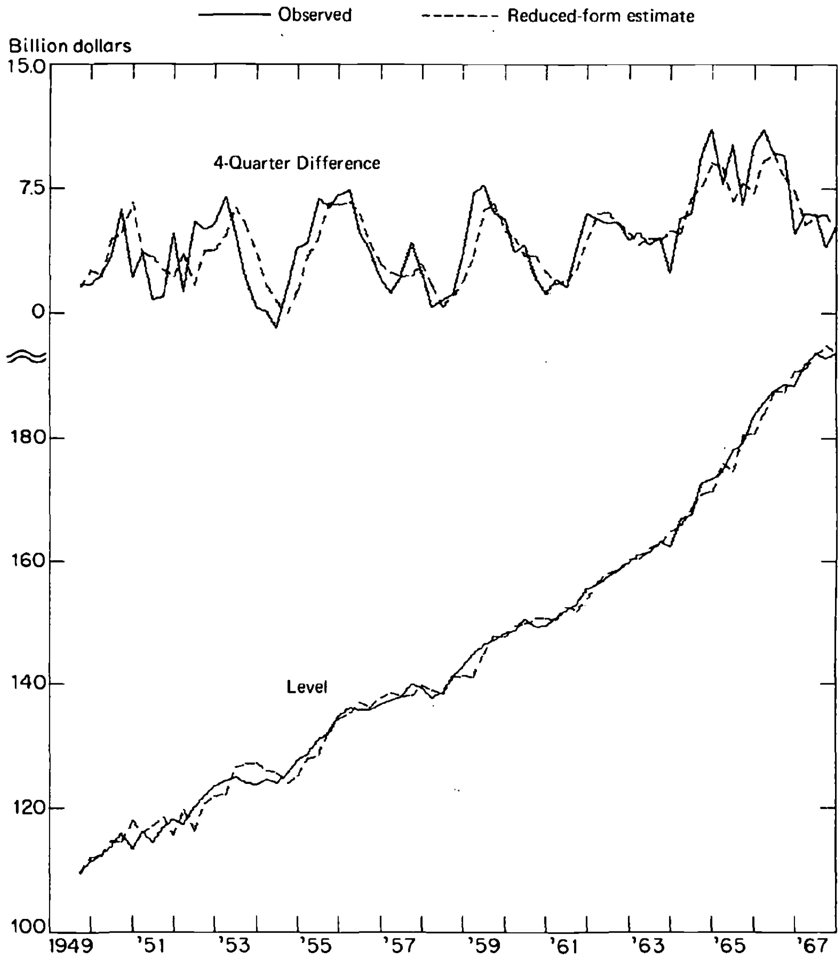
[3] This notation is consistent throughout, including equations 7 and 9. The term $GNP_t - GNP_{-4}$ in equation 7 actually means $(GNP_t - GNP_{t-4}) - (GNP_{t-4} - GNP_{t-8})$.

On the broader subject of the differences between observations generated by a system of linear stochastic difference equations and the estimates formed by only the nonstochastic part, see G. C. Chow, "The Acceleration Principle and the Nature of Business Cycles," *Quarterly Journal of Economics*, LXXXII (August, 1968), pp. 403–18.

CHART 1

C_n

Consumer Expenditures, Nondurables



1. Consumption expenditures on nondurables.

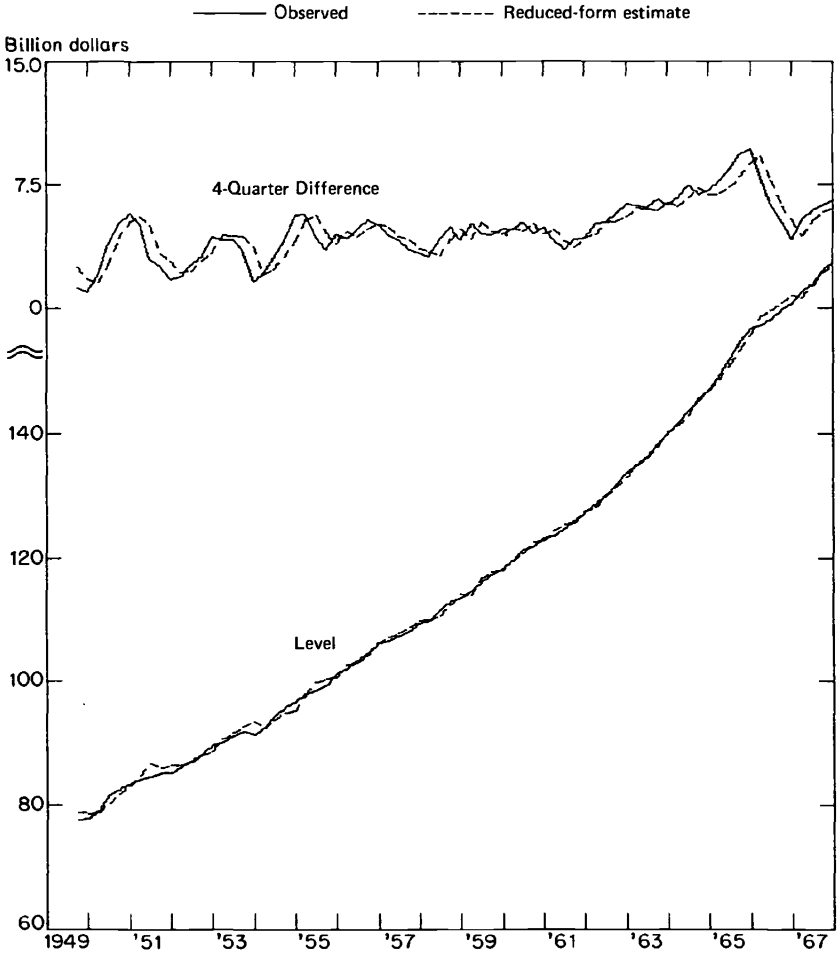
$$C_{n,t} = .5450 + .0906(g_t Y_{p,t} + g_{-1} Y_{p,-1}) + .3835 C_{n,-1}$$
$$\qquad (4.4) \qquad\qquad\qquad (3.6)$$

$$R^2 = .65 \qquad SE = 1.68 \qquad DW = 1.90$$

CHART 2

C_s

Consumer Expenditures, Services



2. *Consumption expenditures on services.*

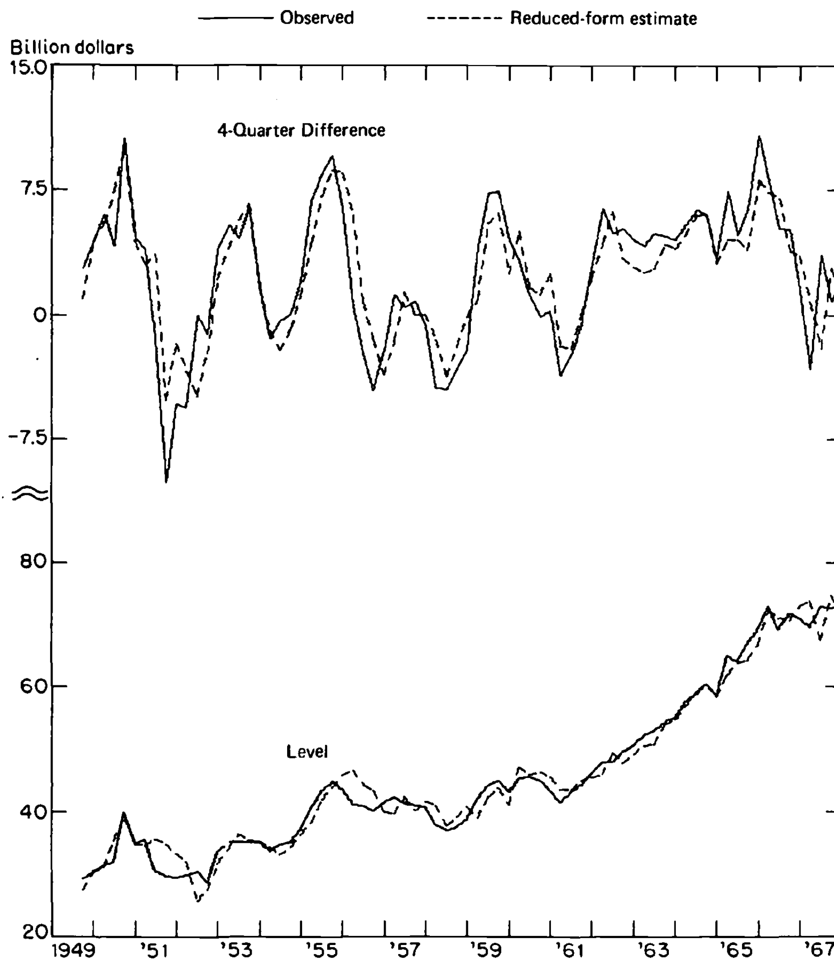
$$C_{s,t} = .6022 + \underset{(1.22)}{.0101}(g_t Y_{p,t} + g_{-1} Y_{p,-1}) + \underset{(12.2)}{.8301} C_{s,-1}$$

$$R^2 = .80 \qquad SE = .80 \qquad DW = 1.26$$

CHART 3

C_d

Consumer Expenditures, Durables



3. Consumption expenditures on durables.

$$C_{d,t} = .4422 + \underset{(2.9)}{.1061}(g_t Y_{p,t} + g_{-1} Y_{p,-1} + g_{-2} Y_{p,-2})$$
$$- \underset{(3.0)}{.0992}(g_{-1} Y_{p,-1} + g_{-2} Y_{p,-2} + g_{-3} Y_{p,-3}) - \underset{(2.0)}{1.8842}(U_t - U_{-1})$$
$$+ \underset{(3.6)}{4.2349}KD + \underset{(4.2)}{2.6479}SD + \underset{(7.9)}{.6967}C_{d,-1}$$

$R^2 = .74 \qquad SE = 2.12 \qquad DW = 1.76 \qquad$ (48.IV–67.IV)

where KD is a Korean War dummy variable, taking +1 for 50.III and −1 for 50.IV; SD is a strike dummy variable, taking −1 for 52.III, 59.IV, and 64.IV, and +1 for 52.IV, 60.I, and 65.I. Without the dummy variables, the equation would be

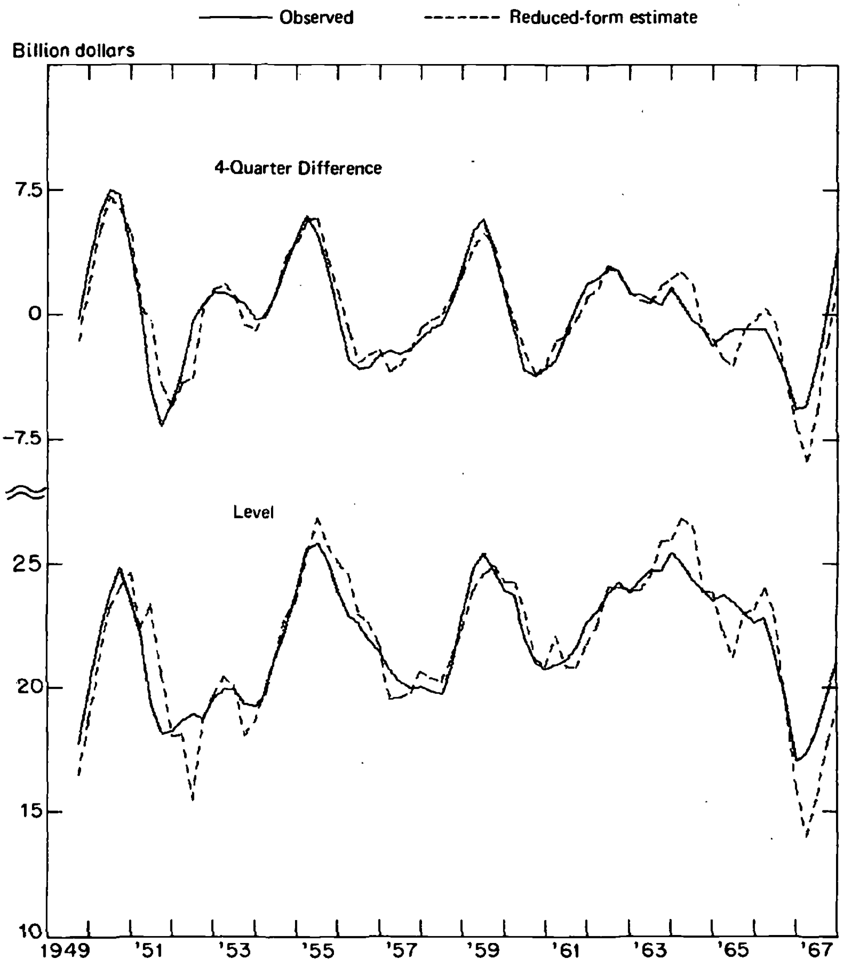
(*3a.*)
$$C_{d,t} = .2200 + \underset{(3.3)}{.1336}(g_t Y_{p,t} + g_{-1} Y_{p,-1} + g_{-2} Y_{p,-2})$$
$$- \underset{(3.0)}{.1109}(g_{-1} Y_{p,-1} + g_{-2} Y_{p,-2} + g_{-3} Y_{p,-3})$$
$$- \underset{(2.1)}{2.3251}(U_t - U_{-1}) + \underset{(5.6)}{.5521}C_{d,-1}$$

$R^2 = .63 \qquad SE = 2.52 \qquad DW = 2.09$

CHART 4

I_p

Investment, Residential



4. *Expenditures on private residential construction.*

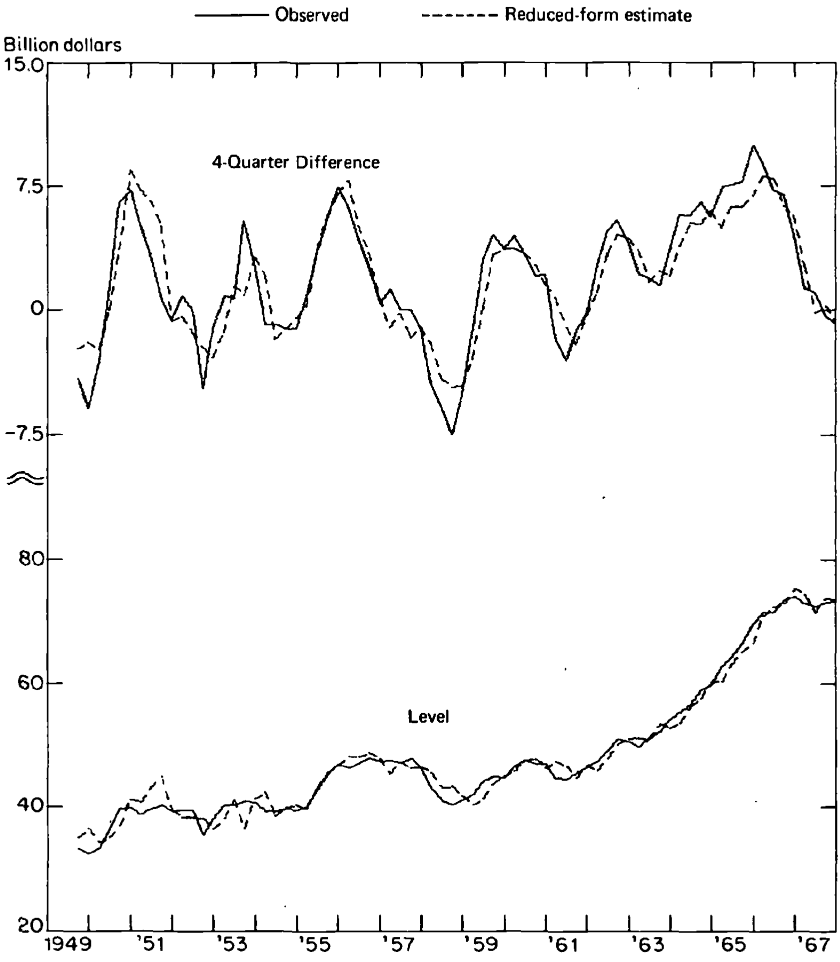
$$I_{p,t} = -.4181 + .4858(J_{p,t} + J_{p,-1}) + .0236CCA_{-1} + .3237I_{p,-1}$$
$$(12.9) \qquad\qquad (.20) \qquad\qquad (6.2)$$

$$R^2 = .91 \qquad SE = .93 \qquad DW = .90$$

CHART 5

I_b

Producers' Durable Equipment and Nonresidential Structures



5. Expenditures on nonresidential construction and business equipment.

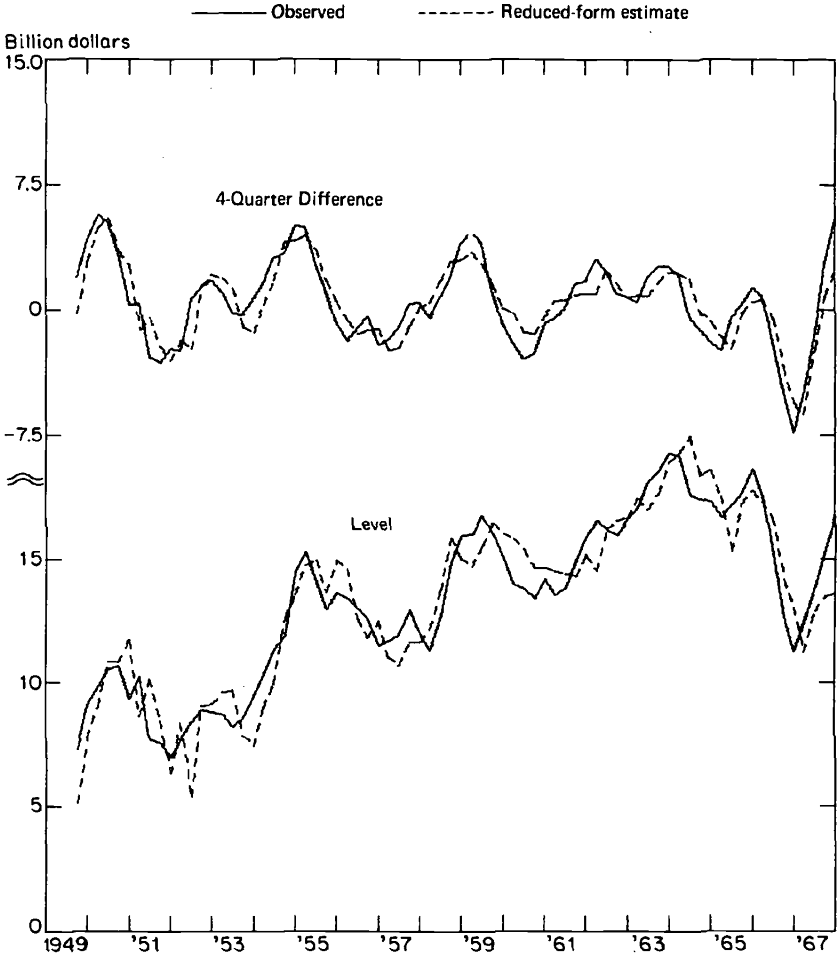
$$I_{b,t} = -.7702 + \underset{(5.8)}{.1029}(J_{b,t} + J_{b,-1}) + \underset{(2.4)}{.5720}CCA_{-1} + \underset{(10.2)}{.6405}I_{b,-1}$$

$R^2 = .80$ $\quad SE = 1.72$ $\quad DW = 1.41$ $\quad$ (49.II–67.IV)

CHART 6

J_p

Contracts, Residential Structures



6. *Contracts for private residential construction.*

$$J_{p,t} = .1654 + \underset{(3.0)}{.0673}(g_t Y_{p,t} - g_{-2} Y_{p,-2})$$

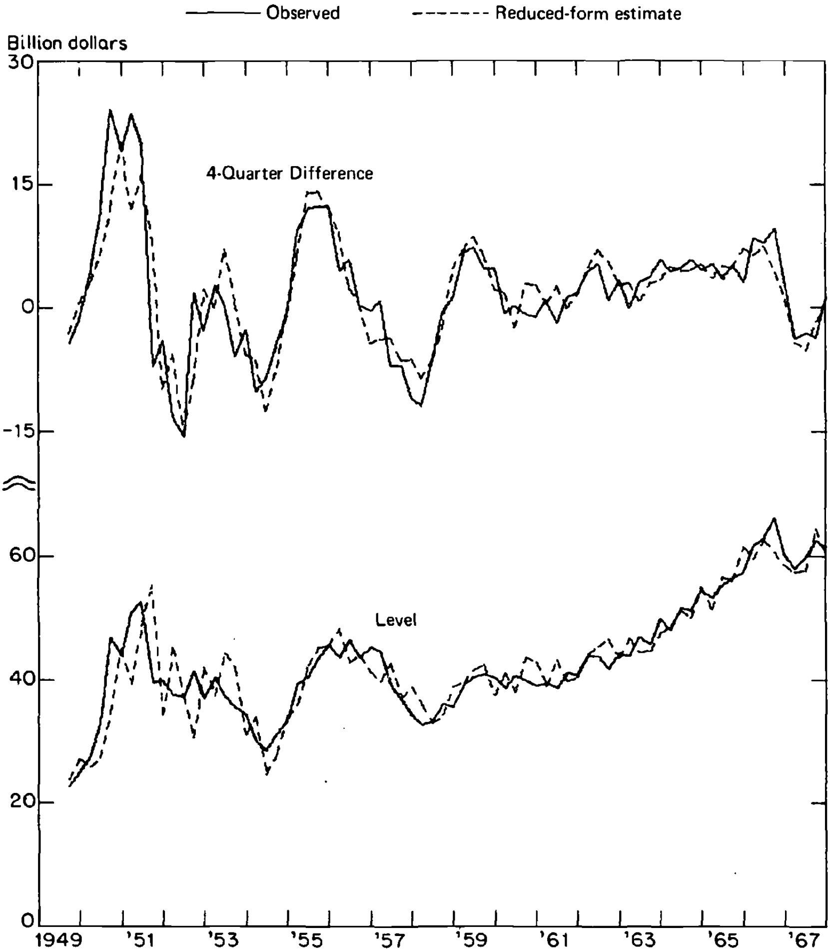
$$- \underset{(5.8)}{3.1394}(R_{L,-1} - R_{L,-3}) + \underset{(11.7)}{.7453} J_{p,-1}$$

$R^2 = .76 \qquad SE = 1.26 \qquad DW = 1.27 \qquad$ (48.IV–67.IV)

CHART 7

J_b

New Orders and Contracts for Plant and Equipment



7. *Contracts and orders for business investment.*

$$J_{b,t} = .4535 + \underset{(5.5)}{.1641}(GNP_t - GNP_{-4}) + \underset{(2.8)}{.1513}(h_{-2}\Pi_{-2}$$

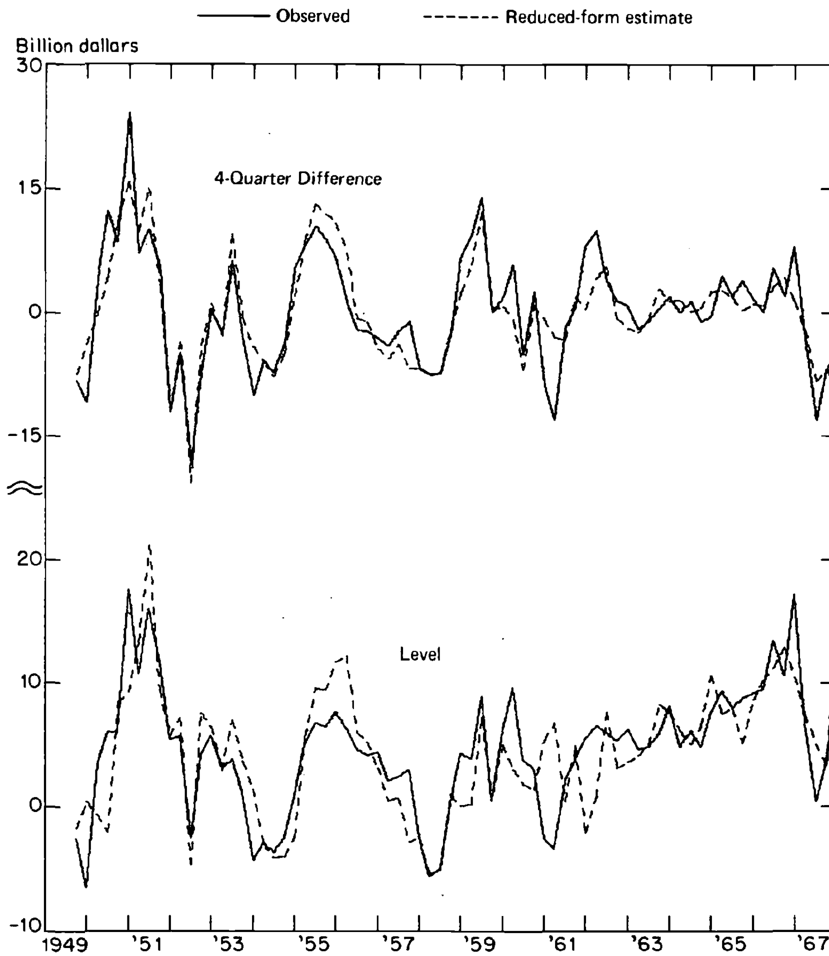
$$+ \cdots + h_{-5}\Pi_{-5}) - \underset{(3.5)}{4.8843}(R_{L,-1} - R_{L,-5}) + \underset{(6.3)}{.5370}J_{b,-1}$$

$R^2 = .70 \qquad SE = 4.20 \qquad DW = 2.27 \qquad$ (49.III–67.IV)

CHART 8

I_i

Inventory Investment



8. Inventory change.

$$I_{i,t} = -1.5470 + \underset{(4.1)}{.0793J_t} + \underset{(7.3)}{.1409J_{-1}} + \underset{(2.0)}{.9614CCA_{-1}}$$
$$+ \underset{(3.8)}{6.2211ID_t} - \underset{(4.5)}{.3018X_t}$$

$$R^2 = .76 \qquad SE = 3.58 \qquad DW = 1.68$$

where ID is a dummy variable for steel strikes, taking +.5 for 52.I and 59.II, and −1 for 52.II and 59.III.

9. Orders for goods for sale.

$$J_{s,t} = 1.0167 + \underset{(4.9)}{1.8914X_t} - \underset{(5.5)}{1.7740X_{-1}}$$
$$+ \underset{(5.0)}{9.9571(P_t - P_{-2})} - \underset{(5.4)}{9.2796(P_{-1} - P_{-3})}$$
$$- \underset{(2.4)}{3.5095(R_{s,-1} - R_{s,-5})} + \underset{(7.1)}{.7558J_{s,-1}}$$

$$R^2 = .76 \qquad SE = 13.67 \qquad DW = 1.87 \qquad (49.III\text{–}67.IV)$$

10. Short-term interest rate.

$$R_{s,t} = -.2340 + \underset{(4.8)}{.0169GNP_t} - \underset{(2.3)}{.0670(M_t - M_{-1})}$$
$$+ \underset{(2.2)}{.0332(GD_t - GD_{-2})} + \underset{(7.1)}{.6029R_{s,-1}}$$

$$R^2 = .73 \qquad SE = .40 \qquad DW = 1.01$$

CHART 9

J_s

Manufacturers' New Orders, Excluding Machinery and Defense Products

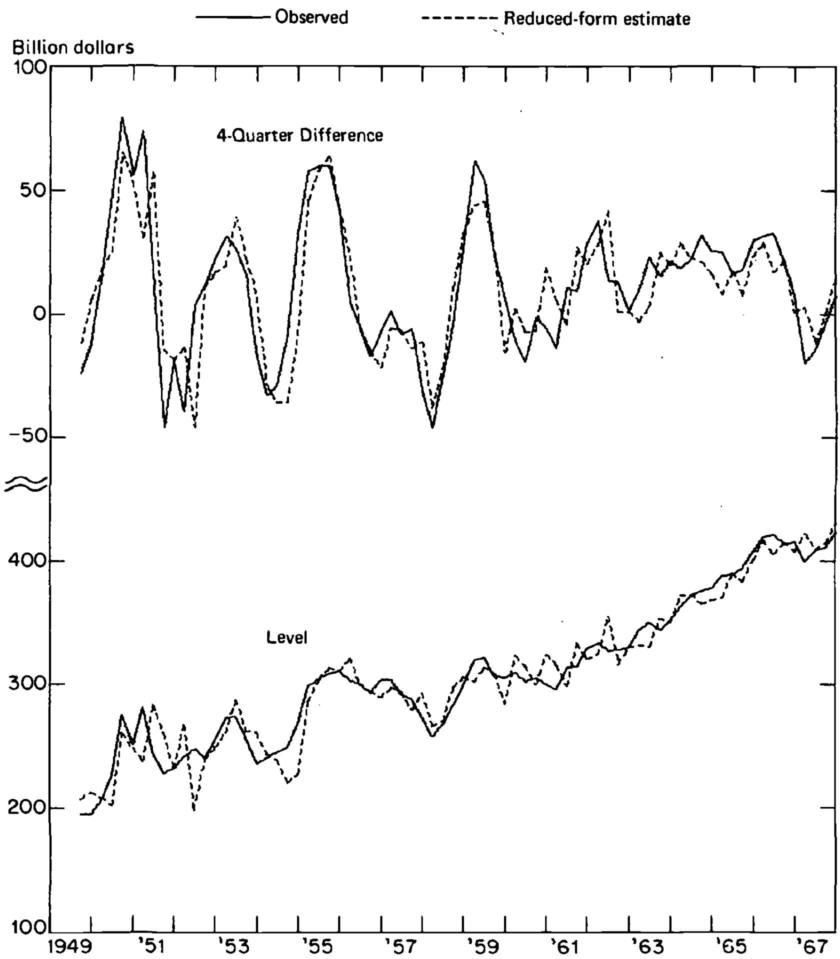


CHART 10

R_s
Treasury Bill Rate

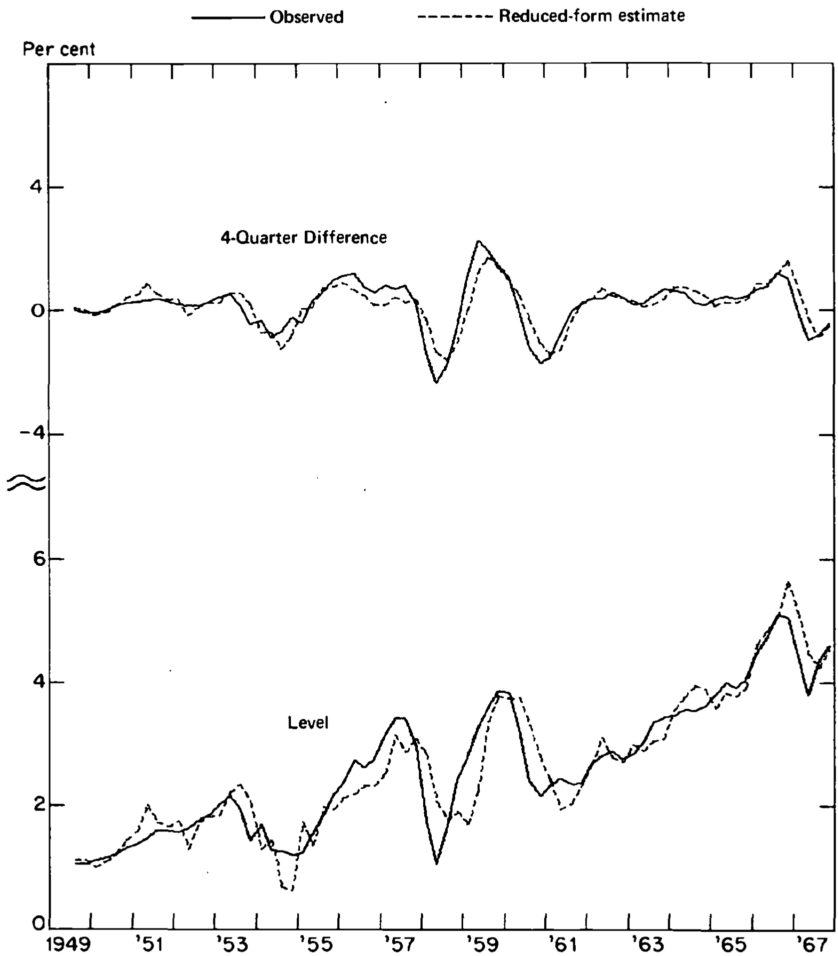
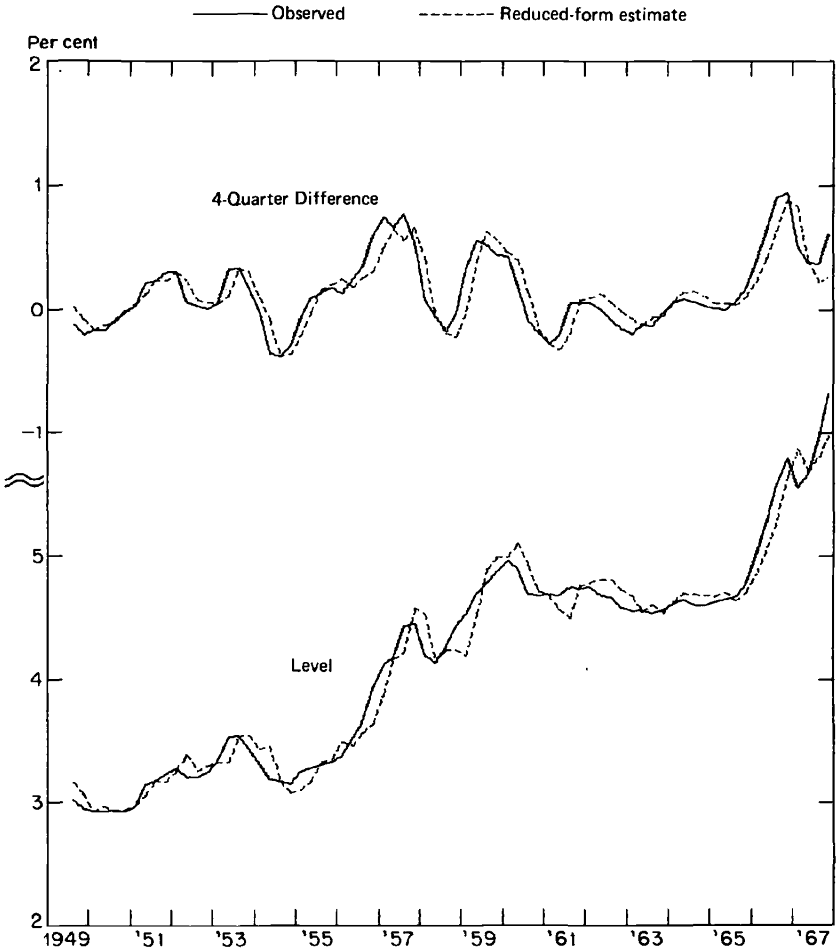


CHART 11

R_L

Corporate Bond Yield



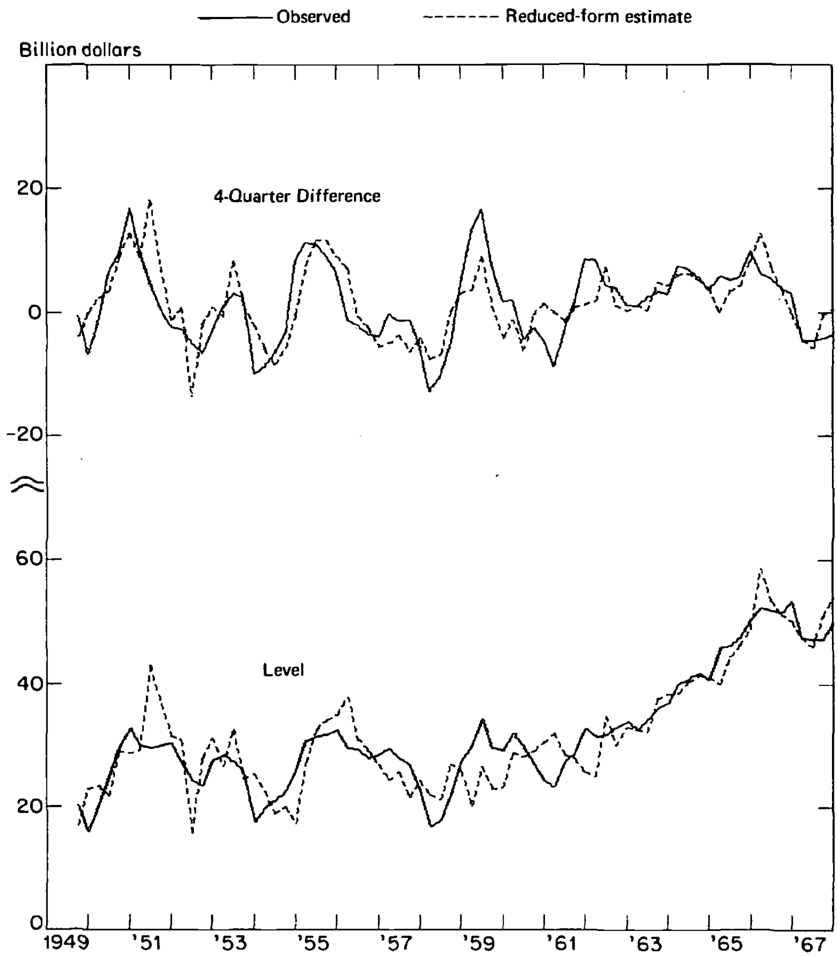
11. Long-term interest rate.

$$R_{L,t} = .0170 + .0991R_{s,t} + .7898R_{L,-1}$$
$$(3.9) \qquad (13.3)$$

$$R^2 = .77 \qquad SE = .14 \qquad DW = 1.19$$

CHART 12

II
Corporate Profits



12. Corporate profits.

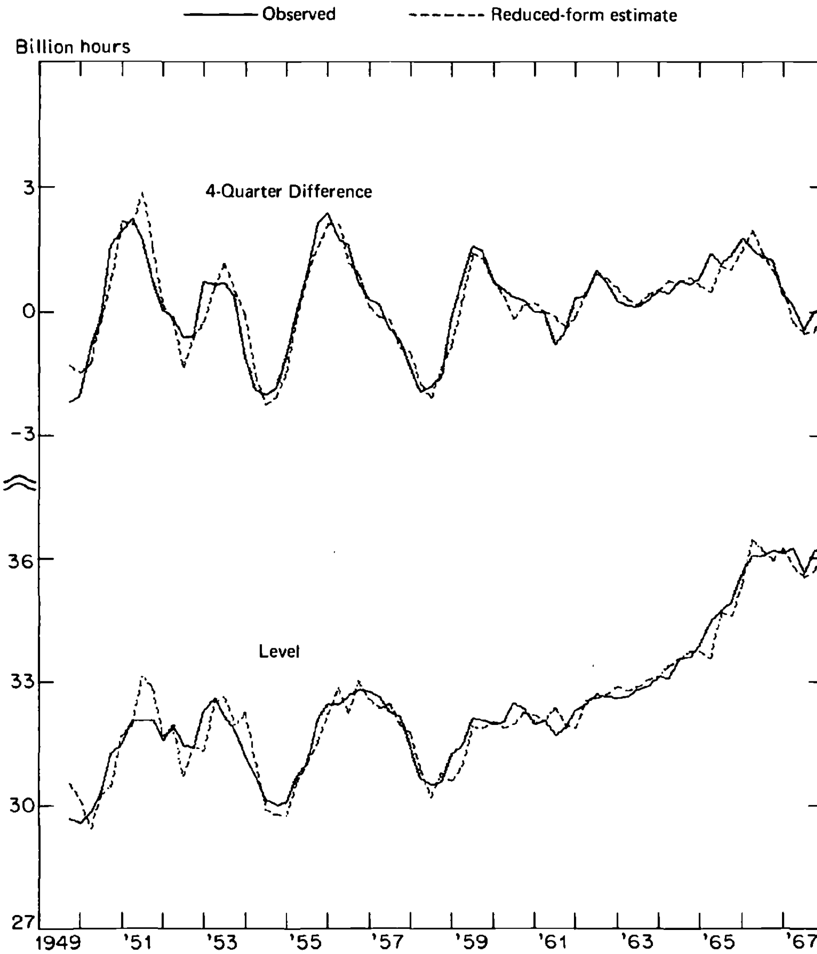
$$\Pi_t = -1.2986 + .5793(Y_t - G_{s,t}) - 1.1111W_t + .9962P_t - 2.8222L_t$$
$$\qquad\qquad (8.5) \qquad\qquad\qquad (3.8) \qquad\quad (2.7) \qquad\quad (2.3)$$

$R^2 = .68 \qquad SE = 3.52 \qquad DW = 1.58$

CHART 13

L^D

Labor Demand



13. Demand for labor.

$$L_t^D = .1027 + \underset{(8.6)}{.0408}(GNP_t - G_{s,t}) - \underset{(3.3)}{.0249}(W_t + \cdots + W_{-3})$$

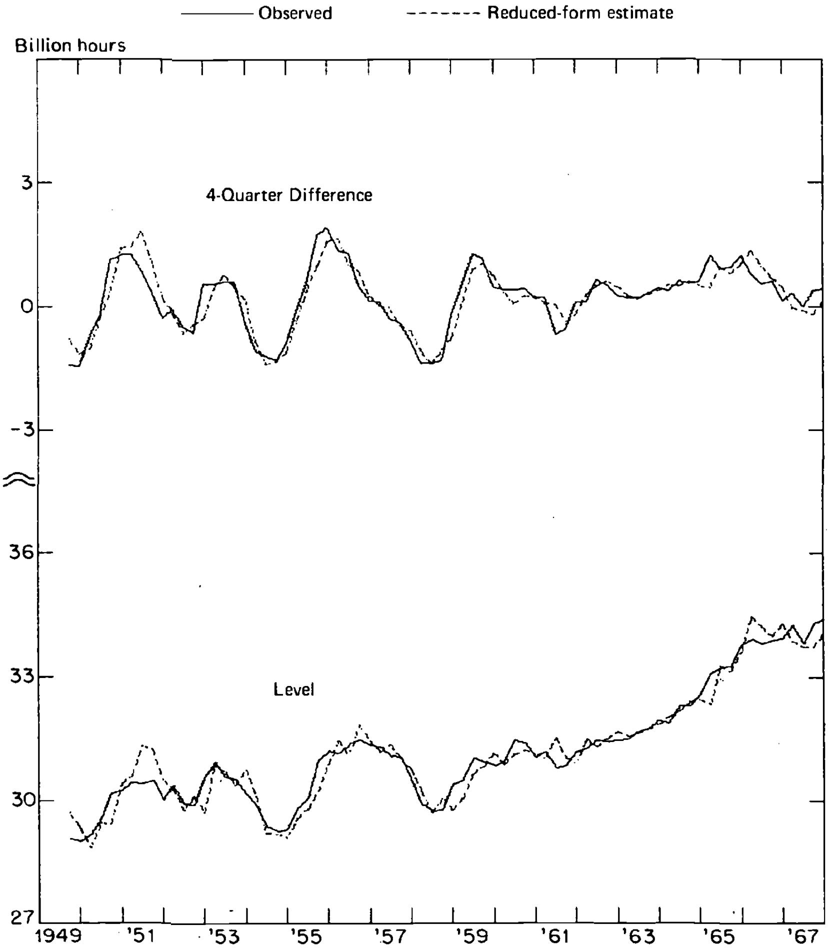
$$+ \underset{(1.2)}{.0553}P_t - \underset{(2.0)}{.1461}CCA_t + \underset{(8.6)}{.5632}L_{-1}^D$$

$R^2 = .87$ $SE = .40$ $DW = 1.55$ (48.IV–67.IV)

CHART 14

L

Employment



14. Employment adjustment.

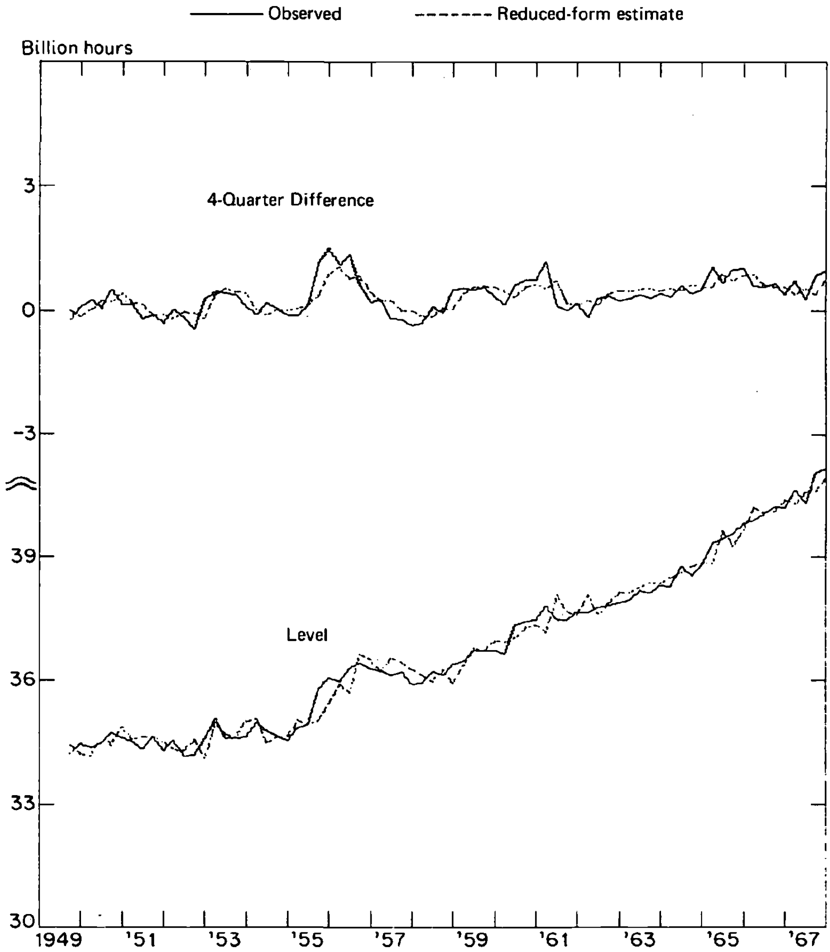
$$L_t = .0339 + \underset{(8.1)}{.4801L_t^D} + \underset{(4.1)}{.3190L_{-1}}$$

$R^2 = .81 \qquad SE = .34 \qquad DW = 1.43$

CHART 15

L^s

Labor Force



15. Supply of labor.

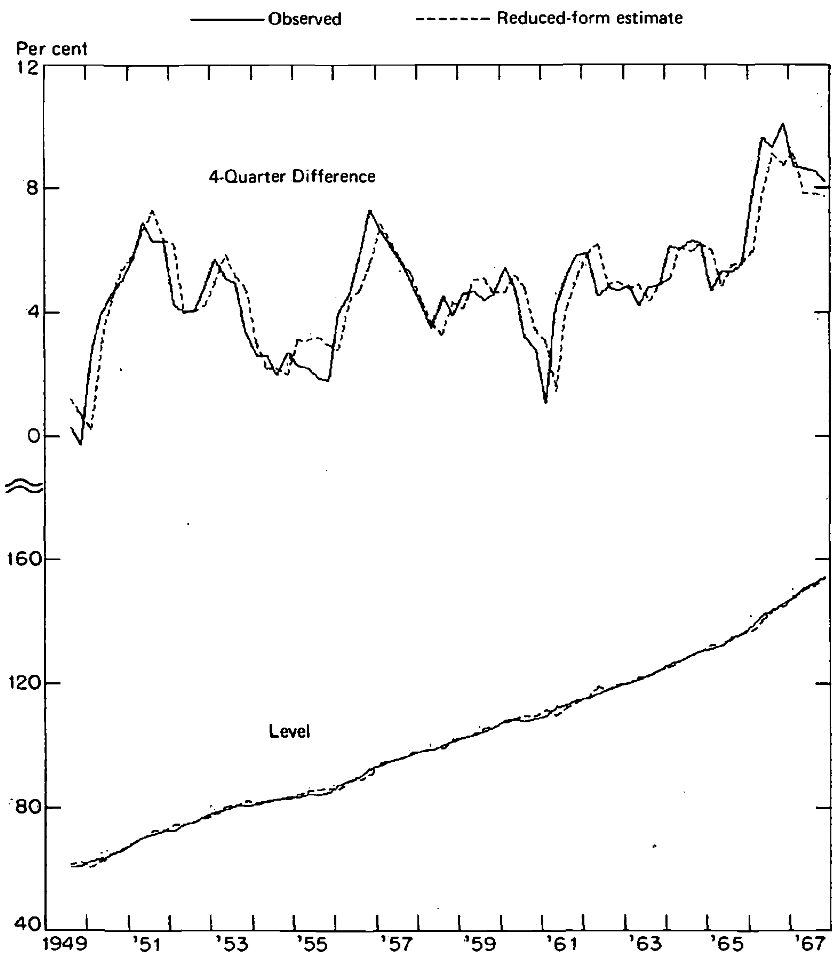
$$L_t^s = -.1236 + .2099N_t - .1636U_t + .5225L_{t-1}^s$$
$$(3.3) \qquad (2.5) \qquad (5.8)$$

$$R^2 = .52 \qquad SE = .29 \qquad DW = 2.01$$

CHART 16

W

Compensation Per Man-Hour



16. Wage adjustment.

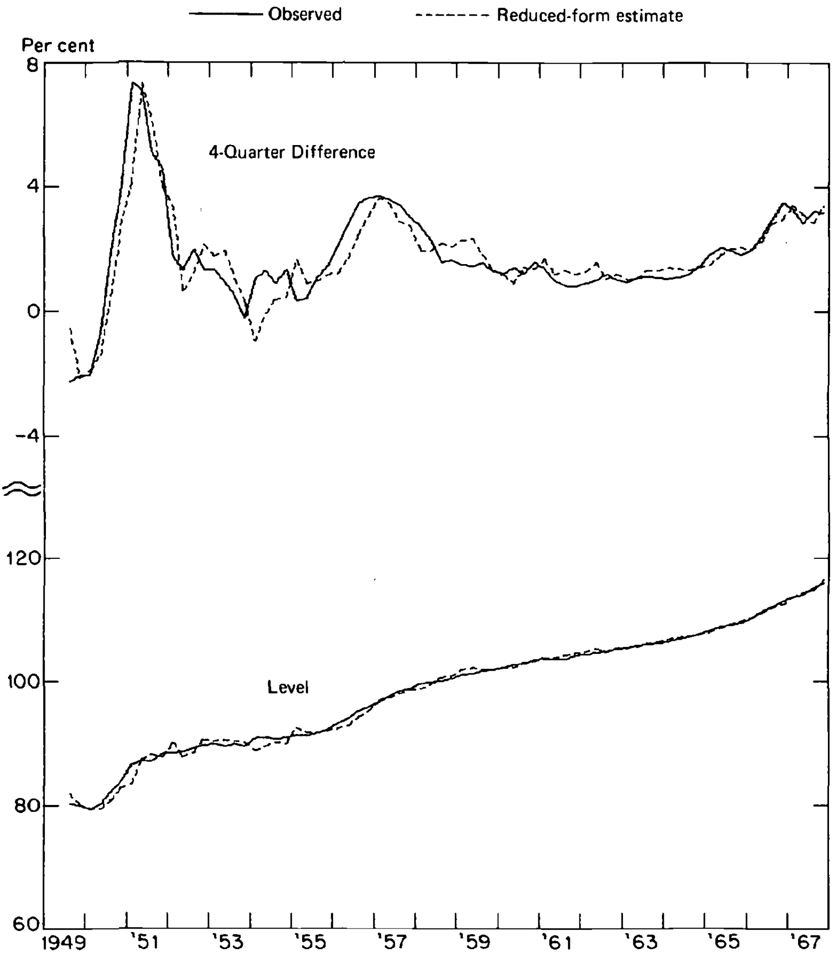
$$W_t = .8896 + \underset{(2.9)}{.3968}(L_t^D + L_{g,t} - L_t^s) + \underset{(13.9)}{.8240}W_{-1}$$

$$R^2 = .77 \qquad SE = .98 \qquad DW = 1.61$$

CHART 17

P

Implicit Price Deflator



17. Price adjustment.

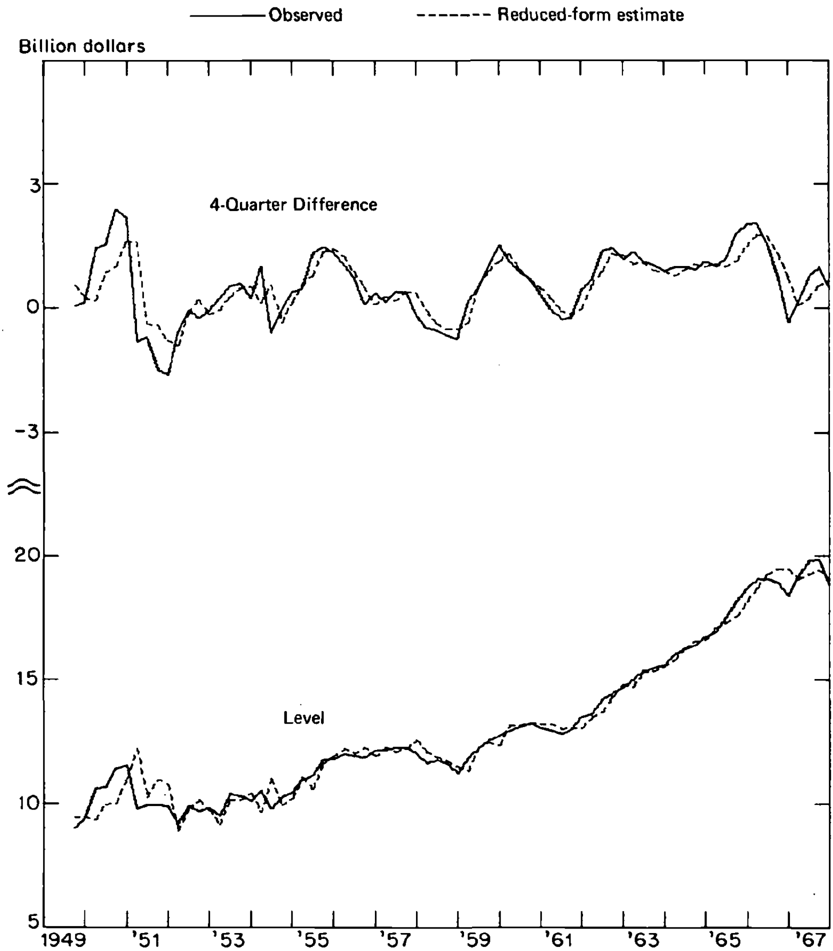
$$P_t = .2108 + \underset{(3.7)}{.0098}(J_{p,t} + J_{b,t} + J_{s,t} + J_{g,t} + C_{s,t} - IM_t)$$

$$+ \underset{(2.2)}{.1141}W_t - \underset{(2.3)}{.2796}CCA_t + \underset{(12.1)}{.7608}P_{-1}$$

$R^2 = .82 \qquad SE = .71 \qquad DW = 1.33$

CHART 18

D
Corporate Dividends



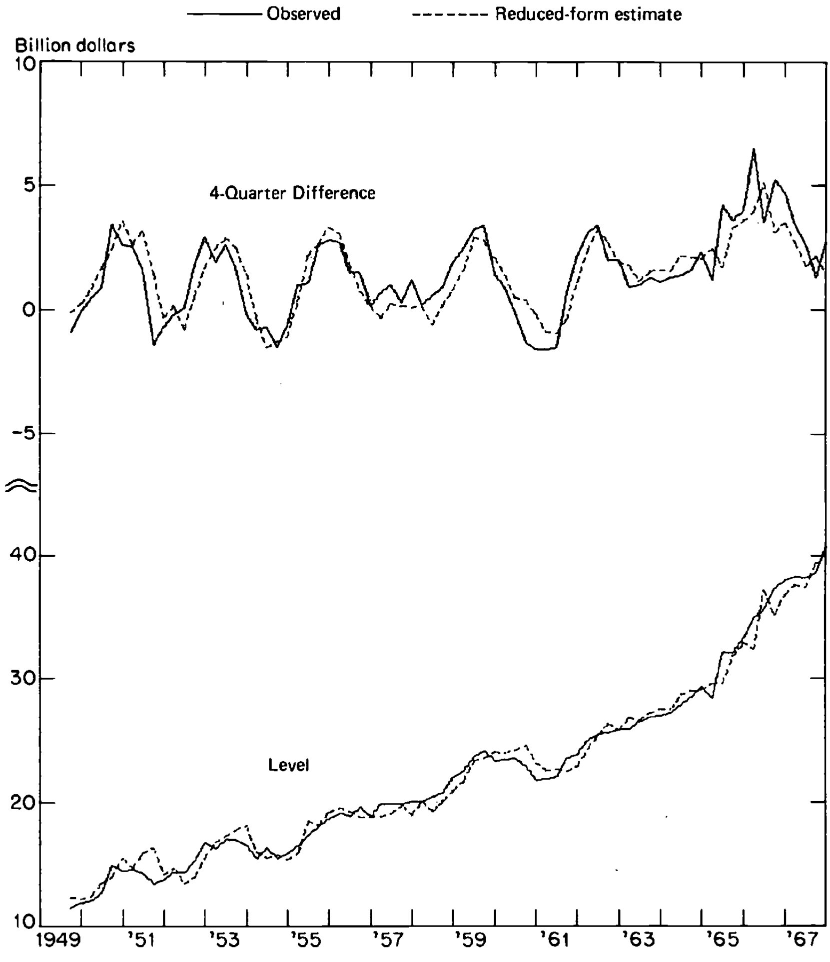
18. Dividends.

$$D_t = .1400 + \underset{(2.7)}{.0270}(h_{-1}\Pi_{-1} + h_{-2}\Pi_{-2}) + \underset{(8.1)}{.6509}D_{-1}$$

$R^2 = .56 \qquad SE = .54 \qquad DW = 1.76$

CHART 19

IM
Imports



19. Imports.

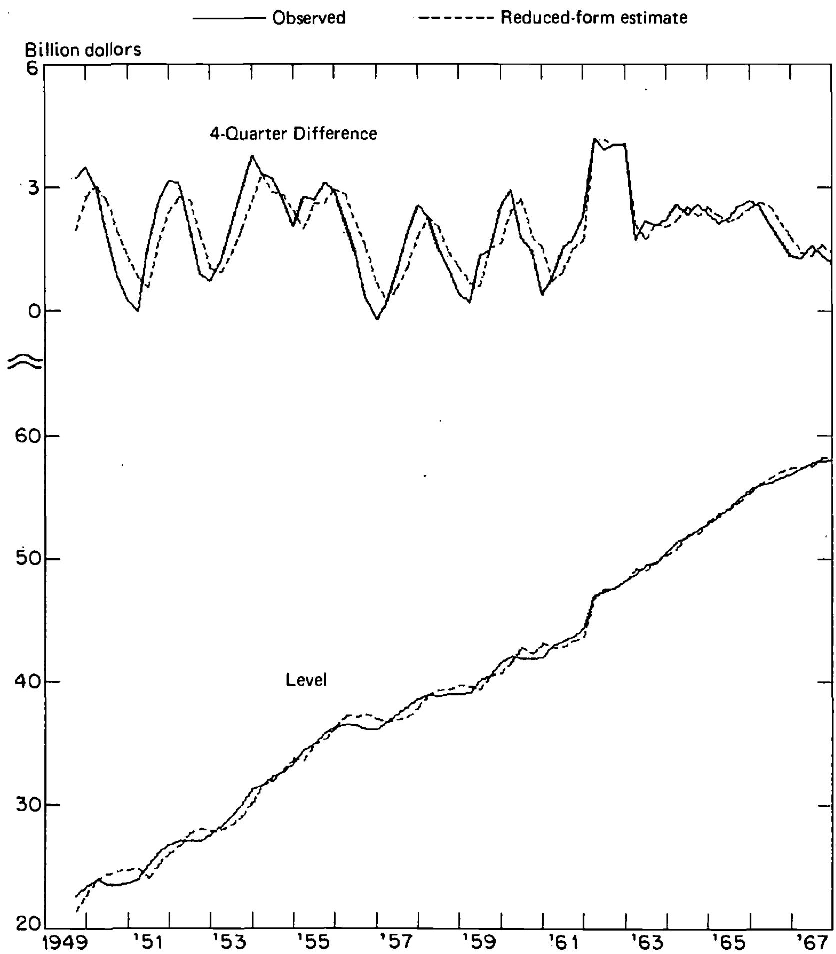
$$IM_t = -.1268 + \underset{(3.9)}{.1019}(C_{n,t} + C_{d,t}) + \underset{(2.6)}{.0424}(I_{p,t} + I_{b,t} + I_{i,t})$$

$$+ \underset{(2.7)}{.0573}G_{g,t} + \underset{(5.9)}{.4541}IM_{-1}$$

$$R^2 = .73 \qquad SE = .87 \qquad DW = 1.91$$

CHART 20

CCA
Capital Consumption Allowances



20. Capital consumption allowances.

$$CCA_t = .4137 + .0057I_{p,t} + .0269I_{b,t} + .7527CCA_{-1} \quad (10.1)$$

$R^2 = .57 \qquad SE = .59 \qquad DW = 1.00$

As we have pointed out, some of these estimated equations are the results of experimentation in fitting, rather than the original equations that we started with. Nevertheless, it is important to note that, in terms of the signs and the relative magnitudes of the coefficients, each of the above equations contains a reasonable explanation of the dependent variable in question. Especially noteworthy are the successes that we have had in using acceleration relations in the explanation of consumer-durable expenditures (equation 3), contracts for private residential construction (equation 6), contracts and orders for business investment (equation 7), and orders of goods for sale (equation 9), where the major explanatory variables are assumed to be rates of changes rather than levels. As a preliminary check, then, our explanations are consistent with the data. A more severe test in terms of the errors in long-term simulations will be discussed later.

A few additional comments on the estimated equations are required. First, capital consumption allowance *CCA* (deflated) has been used to measure the capital stock *K* throughout. We fully recognize the possible deficiencies of this measure based on original costs. If increases in the prices of capital goods have been gradual and fairly smooth, and if the age distribution of capital goods has not been subject to violent changes, this measure will be adequate. Its main advantage, as compared with the capital stock figures based on accumulating depreciated expenditure-figures in constant dollars, is that it is a continuous census. We have experimented with capital stock series estimated by the latter method, and found the *CCA* series to perform better.

In every instance when *CCA* is used, it provides the correct qualitative effect. As a factor enhancing the completion of business investment, given past orders (equations 5 and 8), it shows a positive effect. Because completion of private residential construction has been estimated mechanically from past contracts, it shows no effect (equation 4). It gives the correct substitution effect between capital and labor in the demand for labor (equation 13); and it does shift the aggregate supply schedule to the right, thus lowering the price level (equation 17).

The reader should note that in the capital consumption allowance equation (20), we have introduced a dummy variable which takes the value of 1 for 62.I in order to allow for the major change in tax laws. This variable has a significant coefficient of 1.9709. We have subtracted 1.9709 from the original *CCA* series (in four-quarter differences) for the four quarters of 1962. It is this adjusted *CCA* series which has been used in the above estimated equations, including equation 20.

In equation 7, for orders of business plant and equipment, we have found both the rate of change in *GNP* and the level of profits to have significant effects. This result can, perhaps, be rationalized by regarding the first effect as representing the effort to adjust capacity approximately to the level of output, while the second – profitability – modifies the *timing* of such adjustments.

Concerning equation 10, for the short-term interest rate, it may be argued that if we had started with a demand equation for money in real terms, the relevant money variable should also be in real terms – or the price level should be introduced together with the nominal stock of money. We have tried using M/P instead of M, but could not improve the result – the ratio of the coefficient of $(M/P)_t - (M/P)_{t-1}$ to its standard error is reduced slightly from 2.3 to 2.0. We have introduced P linearly but have obtained a negative and highly insignificant coefficient. The same has happened to $P_t - P_{t-1}$. Thus, we have not found sufficient evidence to insist on the distinction between the change in the real stock of money and the change in nominal stock in their effects on interest rates in the short run. A related issue occurs in wage-adjustment equation 16. It may be argued that real wage W/P should adjust to the difference between labor demand and supply. We have used money wage, finding that introducing the price level does not produce a significant effect.

ERRORS OF THE MODEL

A MAIN purpose of this study has been to ascertain whether a set of aggregative econometric relationships formulated according to the selected elements of business cycle theory can explain past observations. Undoubtedly, a stronger test of our model will have to be based on its predictive performance in the future. Nevertheless, examining its behavior during the sample period does help locate some of its weak points and provide some guidance for future research. At this time, however, we have not fully completed our study of the model's performance during the sample period. Hence our conclusions are tentative.

In this section we will report on some aspects of the errors in our model. By an error, we mean the difference between an observed value and the corresponding value estimated from the model, bearing in mind that the parameters of the model are themselves estimated from the same set of data. Parts of the errors are inherent in the mathematical form of our model, while the remainder may be due to mis-specification of particular equation or equations. After forming some notion of the former errors, one can identify the latter and pinpoint certain equations that are causing the discrepancies.

Beside the standard errors of the individual structural equations reported in the previous section, we will discuss the errors of the reduced-form equations and of the (74 quarter) nonstochastic simulations for the entire sample period, given the initial conditions as of 1949.III, and given the true values of all exogenous variables. The latter errors will be expressed in terms of both four-quarter differences and the levels of the variables. We will show how these errors are related to one another simply by the mathematical form of our model. Any errors seemingly too large to be so explained will suggest possible defects in particular equations.

Comparing the standard errors of the structural equations previously reported with the root mean-squared errors of the reduced form presented in Table 1, column 2, one finds that they are almost identical. Since, in the second stage of the method of two-stage least squares used to estimate the structural equations, each dependent variable is explained by the set of predetermined variables—as in the case of each reduced-form equation—this result is not unexpected. Hence, we can begin with the errors of the reduced-form equations.

Consider a univariate system explaining the four-quarter difference y_t of a certain economic variable by the following stochastic difference equation:

$$(a) \qquad y_t = ay_{t-1} + bx_t + e_t$$

where x_t is an exogenous variable. This one-variable model brings out the relationships between different errors, and its multivariate generalization is straightforward. Equation (a) corresponds to our reduced-form equation, where the coefficients a and b are estimated from the data, and e_t is the error of the reduced-form estimate; its root mean square corresponds to column 2 of Table 1.

TABLE 1

Errors of Reduced-Form and Sample-Period Simulation

		Reduced-Form Estimate			Nonstochastic Simulation for Sample Period			
		4-Quarter Difference		Level	4-Quarter Difference		Level	
Variable	Level 1967.IV	RMS Error	R^2	R^2	Mean Error	RMS Error	Mean Error	RMS Error
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
C_n	193.6	1.67	.64	.995	0.23	2.53	−2.35	4.47
C_s	167.4	0.80	.79	.999	0.06	1.55	−4.29	5.23
C_d	73.0	2.16	.73	.972	−0.04	4.48	−4.00	6.13
I_p	21.0	1.20	.86	.834	−0.19	3.16	3.03	4.33
I_b	73.4	1.69	.81	.977	−0.85	4.68	−17.55	19.14
J_p	16.7	1.25	.77	.872	−0.09	2.63	2.85	3.70
J_b	61.4	4.20	.69	.811	−1.47	8.29	−19.54	22.11
I_i	8.7	3.54	.76	.570	−0.07	6.21	−1.00	4.93
J_s	423.2	16.51	.65	.924	0.50	29.79	1.62	25.77
R_s	4.61	0.41	.74	.873	0.00	0.75	−0.19	0.73
R_L	6.32	0.14	.77	.972	0.00	0.27	−0.40	0.51
Π	69.7	4.46	.53	.794	−0.47	7.58	−6.92	9.79
L^D	36.2	0.40	.87	.945	0.03	1.06	−0.39	1.08
L	34.4	0.35	.80	.936	0.01	.80	−0.50	0.90
L^S	41.2	0.28	.56	.980	−0.01	0.36	−0.18	0.38
W	154.3	0.93	.79	.999	0.13	1.65	−3.46	4.41
P	116.0	0.76	.78	.994	0.25	1.36	3.14	3.65
D	18.9	0.53	.59	.970	−0.10	0.95	−3.04	3.40
IM	40.7	0.98	.66	.983	−0.04	1.73	−3.82	4.26
CCA[a]	58.0	0.59	.68	.997	−0.08	1.00	−0.70	1.35
GNP	679.6	6.43	.82	.995	−0.81	15.63	−22.38	28.50

[a] This *CCA* series refers to the original series, while the *CCA* variable in equation 20 is adjusted by the coefficient of the dummy variable.

When a nonstochastic simulation for the sample period is performed, given y_0 and all x_t, the estimate of y_t is

$$(b) \qquad \hat{y}_t = a\hat{y}_{t-1} + bx_t$$
$$= a^t y_0 + bx_t + abx_{t-1} + a^2bx_{t-2} + \cdots + a^{t-1}bx_1$$

whereas equation (*a*), by repeated substitutions, can be written as

$$(c) \qquad y_t = a^t y_0 + bx_t + abx_{t-1} + a^2bx_{t-2} + \cdots + a^{t-1}bx_1$$
$$+ e_t + ae_{t-1} + \cdots + a^{t-1}e_1$$

The error (denoted by u_t) of nonstochastic simulation, measured by columns 5 and 6 of Table 1, is therefore

$$(d) \qquad u_t = y_t - \hat{y}_t = e_t + ae_{t-1} + a^2e_{t-2} + \cdots + a^{t-1}e_1$$

It is a weighted sum of past errors of the reduced-form estimates.

When the nonstochastic simulations in four-quarter differences are converted into levels, the estimate of the level, denoted by Y_t, is

$$(e) \qquad \hat{Y}_t = \hat{y}_t + \hat{y}_{t-4} + \hat{y}_{t-8} + \cdots$$

Since the actual level is

$$(f) \qquad Y_t = y_t + y_{t-4} + y_{t-8} + \cdots$$

the error of nonstochastic simulation in terms of level is

$$(g) \qquad v_t = Y_t - \hat{Y}_t = u_t + u_{t-4} + u_{t-8} + \cdots$$

It is a sum of past nonstochastic simulation errors in four-quarter differences, and is represented by columns 7 and 8 of Table 1. Thus, the errors of the reduced-form estimates are magnified twice, through equations (*d*) and (*g*), to form errors in the sample-period nonstochastic simulations in terms of the levels of the variables, as seen by comparing columns 2, 6, and 8 of Table 1.

Looking through the magnifying glass—namely, columns 7 and 8—one finds relatively large errors for I_b, J_b, J_s, and *GNP*. J_s, orders for goods for sale, has a poor equation to begin with, as evidenced by a standard error of 13.67 billion in the structural equation and of 16.51 billion in the reduced-form estimate. The errors in J_b, orders for business investment, are not so large to begin with, but are magnified tremendously from columns 6 to 8. One also witnesses a drift of the mean error from −1.47 in column 5 to −19.54 in column 7. By equation (*g*), this can happen if a few *u*'s in an early period are large, since their influence persists for all later *v*'s. The errors in J_b account for most of the errors (both in mean and in root mean square) in I_b, which is essentially a weighted average of the former, and thus in *GNP*. Hence, the main errors in the model have come from the equations for orders J_b and J_s. We have found that the errors e_t in the reduced-form equations for these two variables are abnormally high during the early quarters of the Korean War, as a result of anticipations of coming shortages not explicitly treated in our equations 7 and 9. These errors alone can account for the large errors v_t in our sample-period simulations of the levels of J_b and J_s.

Turning back to the errors in the reduced-form equations, we have plotted the estimates, together with the actual series, in the accompanying charts—with the top showing four-quarter differences and the bottom showing the levels of the variables. If one tries to compare the turning points of the estimates with the actual series, and their lead-lag relationships in general, he will find that for many, but not all, series the estimates often appear to lag. The appearance of the lag can be attributable to the mathematical form of our model. If a series of observations y_t were generated by equation (*a*), after specifying the coefficients *a* and *b* and using independent unit normal drawings for the error e_t, comparison of the estimate $ay_{t-1} + bx_t$ with the actual y_t so generated, would reveal a tendency for the estimate to lag. This argument cannot explain away all possible deficiencies of our model in estimating turning points. It does point out, however, that, even if the model were correct, the estimate would still tend to lag behind the actual, as long as the model is in the form of stochastic difference equations relying heavily on y_{t-1} to explain y_t.[4] The degree of reliance may, however, be excessive, since it could arise from our inability to specify the structural equations correctly.

An examination of the errors of the reduced-form estimates also

[4] The estimated coefficients for $I_{p,t}$ and $I_{b,t}$ are obtained indirectly. The quarterly rate of depreciation for business capital stocks, .0269, was provided by the Office of Business Economics, U.S. Department of Commerce. The rate for private residential buildings, .0057, was derived from Goldsmith's tables (Tables B-5, B-7, B-10) concerning private nonfarm housekeeping units in his *The National Wealth of the United States in the Postwar Period,* Princeton, Princeton University Press, 1962.

reveals other equations that might be improved. They include the equations for inventory change and for profits, having standard errors of 3.6 billions and 3.5 billions respectively (or 3.5 billions and 4.5 billions respectively in the reduced form). These variables are known to show large fluctuations and are important in business cycles. For the profit equation, productivity might be introduced as a separate variable, while our present equation 12 includes an output (in fact, income) variable and an employment variable, separately, in a linear fashion.

Some observations should be made concerning the model's ability to represent the business cycle movements of the sample period. In terms of real *GNP* (Chart 21), each of the major swings in the rate of change (four-quarter spans) that correspond with business recessions identified by the National Bureau is reflected in the estimates. So, too, are the minor swings (in 1952–53, 1956–57, 1962–63, and 1966–67). The timing of these swings is also closely represented by the estimates, though with a tendency to lag by one quarter. In twelve observations of peaks or troughs in the rate of change, there is one instance of a lead of one quarter, five exact coincidences, five lags of one quarter, and one lag of two quarters.

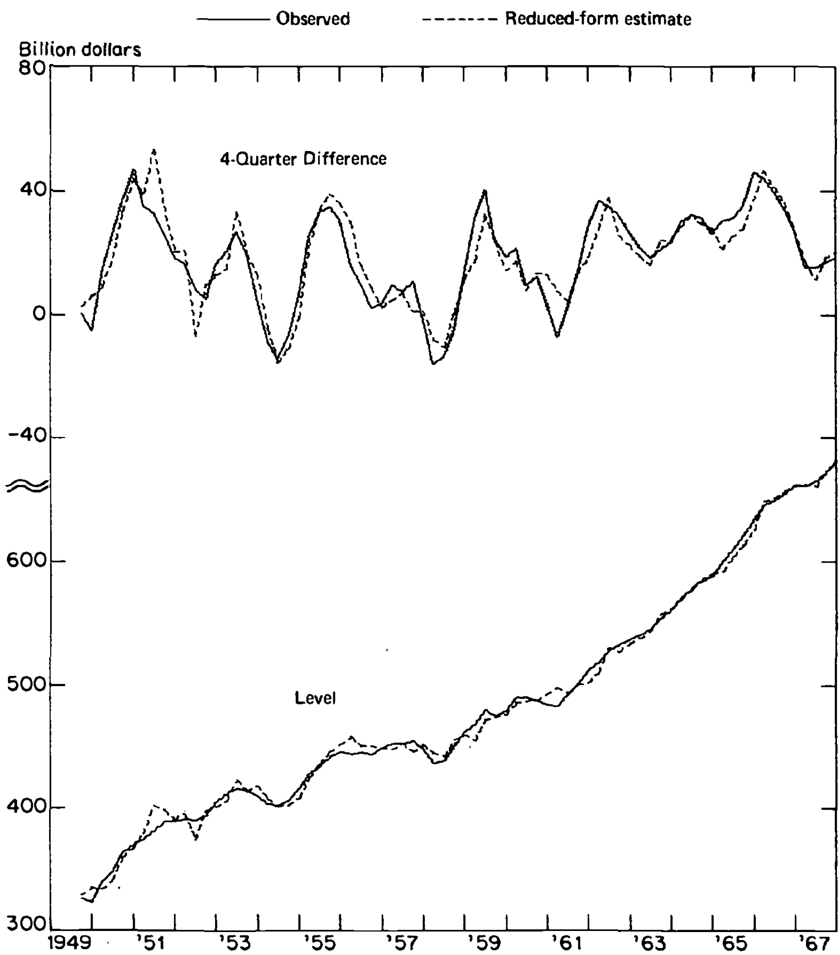
In assessing these results, it should be recalled that the reduced-form estimates essentially represent forecasts for one quarter ahead. For each successive quarter, the true values of the predetermined variables up to and including the preceding quarter are used. Hence, at turning points, a lag of more than one quarter is not very likely; by the same token, a lag of one quarter when the forecast is for only one quarter ahead must be regarded as a failure to predict the turning point. On the other hand, such a lag may mean that the model does have some ability to *recognize* a turning point one quarter after the event (i.e., as soon as data for the quarter subsequent to the turning point quarter are available). Before even this is vouchsafed, however, one would need to examine forecasts over two, three, or more, quarters (since the results for later quarters may contradict those for the initial quarter); forecasts constructed without the aid of the true values of the exogenous variables; and forecasts developed outside of the sample period.

When the rate of change estimates are converted to levels, the following observations can be made:

(1) The 1951–52 slowdown in *GNP* growth is exaggerated.
(2) The 1953–54 recession is traced quite well.
(3) The estimates level off too early and too much in 1956–57, and understate the magnitude of the 1957–58 recession.
(4) The estimates fail to reflect the 1960–61 recession at all.
(5) The slowdown in 1966–67 is well represented.

CHART 21

GNP
Gross National Product



Before concluding, it should be emphasized that this is a progress report. Beside the limitations already mentioned, the present model has not yet incorporated many possibly important elements in business cycles. The influence of expectations, especially on investment decisions, is not adequately treated. Construction costs may also be important for investment, while the present model has included the wage rate as the only factor price. The influence of money and credit is allowed to assert itself only through the rate of interest. In general, the monetary sector can be strengthened. Linear approximations may not be adequate for some relationships. We have not even tried to tackle the problem presented by the fact that aggregative variables do not explicitly reveal the phenomenon of diffusion: neither businessmen nor consumers act in unison or react to the same variables in the same way. The list of possible improvements is long. However, we believe that the present report is useful as a first step in bringing together some of the important elements of business cycles into a manageable system; in evaluating how well it can explain past fluctuations; and in detecting its possible shortcomings, thereby providing significant directions for further research.

APPENDIX: LIST OF VARIABLES AND SOURCES OF DATA

VARIABLES are listed alphabetically below. Unless otherwise noted, the flow variables are seasonally adjusted, quarterly at annual rates, and in billions of 1958 dollars (whenever data in constant dollars are not available, the current dollar data are deflated by either *GNP* deflator or another appropriate deflator). Variables with asterisks are exogenous in the model.

CCA Capital consumption allowances. Data for 1947–63 from *The National Income and Product Accounts of the United States, 1929–65, A Supplement to the Survey of Current Business,* August, 1966; for 1964–66, the July, 1967, issue of the *Survey;* and for 1967, the June, 1968, issue of the *Survey.*

C_d Personal consumption expenditures on durable goods. For data, see *CCA*.

C_n Personal consumption expenditures on nondurable goods. For data, see *CCA*.

C_s Personal consumption expenditures on services. For data, see *CCA*.

D Corporate dividends. For data, see *CCA*.

EX^* Exports of goods and services. For data, see *CCA*.

GD^* Federal Government interest-bearing debt in billions of current dollars. For data, see each August issue of *Treasury Bulletin* by the United States Treasury Department.

G_g^* Government purchases of durable goods, nondurable goods, and structures. Unpublished data from the Department of Commerce. For data on $G_g + G_s$, see *CCA*.

GNP Gross national product. For data, see *CCA*.

G_s^* Government purchases of services, including compensations. Unpublished data from the Department of Commerce.

I_b Gross private domestic investment in producers' durable equipment and nonresidential structures. For data, see *CCA*.

ID^* Steel strikes, taking +.5 for 52.I and 59.II, and −1 for 52.II and 59.III.

I_i Gross private domestic investment in change in business inventories. For data, see *CCA*.

IM Imports of goods and services. For data, see *CCA*.

I_p Gross private domestic investment in residential structures. For data, see *CCA*.

J_b New orders and contracts for plant and equipment. Data on contracts are from F. W. Dodge Co. and data on orders from *Manufacturers' Shipments, Inventories, and Orders, Series M3-1,* United States Bureau of the Census, Department of Commerce.

J_g^* New orders, defense products industries, plus contracts for government construction. For data on contracts, see J_p. For data on orders, see J_b, noting that the series before 1953 has been extrapolated by using an estimated relationship between orders and Federal Government expenditures on national defense.

J_p Contracts for private residential buildings. Data are from F. W. Dodge Co. The series before 1956 has been adjusted upward because of its narrower coverage.

J_s Manufacturers' new orders, excluding machinery and equipment and defense products. For data, see J_b.

KD^* Korean War (taking +1 for 50.III and −1 for 50.IV).

L Employment in billions of man-hours per quarter, total private economy (total employment minus government employment). For data on number employed, see *Employment and Earnings* by Bureau of Labor Statistics. For data on man-hours worked, see *Current Population Reports* by Bureau of the Census, and *Employment and Earnings*.

L^D Demand for labor in man-hours (employment plus non-agricultural job-openings unfilled), total private economy. Data on job openings unfilled are from Bureau of Employment Security, Department of Labor. For other data, see L.

L_g^* Government employment in billions of man-hours per quarter. Data on number of wage and salary workers in government from unpublished tabulation, and *Employment and Earnings* by Bureau of Labor Statistics; those on man-hours worked interpolated from unpublished data from J. W. Kendrick.

L^s Total labor force in man-hours (i.e., $L + L_g + U$). For data, see L and U.

M^* Money supply, including time deposits in commercial banks, billions of current dollars. Data from *Federal Reserve Bulletin,* Board of Governors of the Federal Reserve System.

N^* Population sixteen and over, excluding armed forces. Derived from data in *Current Population Reports,* Series P-25.

P Implicit deflator of gross private output, i.e., $(GNP - G_s)$ deflator.

Π Corporate profits before tax ($h\Pi$ is corporate profits after tax). For data, see CCA.

R_L Yields on corporate bonds, average of Moody's *Aaa* and *Bbb* in per cent per annum. For data, see M.

R_s Market yields on 3-month treasury bills in per cent per annum. For data, see M.

SD^*	Shortage of supply due to strikes (taking −1 for 52.III, 59.IV, and 64.IV; and +1 for 52.IV, 60.I, and 65.I).
T_1^*	Indirect business tax minus subsidies less current surplus of government enterprises. See *CCA* for source of data.
T_2^*	Government transfer payments plus interest paid by government and by consumers, minus contributions for social insurance. See *CCA* for source of data.
U	Unemployment in billions of man-hours per quarter. Derived from data on labor-force time lost, *Employment and Earnings;* and those on man-hours of employed labor-force. For latter, see L.
W	Index (1957–59 = 100) of labor compensation per man-hour in money terms, total private economy. Data from *Productivity, Wages, and Prices* by Bureau of Labor Statistics.
X	Final sales of goods.
Y	National income plus business transfer payments plus statistical discrepancy.
Y_p	Personal income plus statistical discrepancy (gY_p is personal disposable income).

DISCUSSION

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1.

How might we express in terms of an econometric model the National Bureau of Economic Research's approach to an explanation of business cycles? This is a question that has frequently been asked and, I take it, the paper under review represents an attempt to answer it. As such, the report is much to be welcomed.

A further question immediately arises. What shall we take as the National Bureau's approach to a theory of business cycles? So far as I know, the National Bureau, as an institution, has never officially adopted any particular theory or model of the business cycle. Nonetheless, over a period of nearly half a century, the work of Wesley Mitchell, Arthur Burns, and Geoffrey Moore—not to mention the empirical work of others for the Bureau—has suggested the main elements that might go into a business cycle model. The main outlines were laid out originally by Mitchell. I agree with the authors of the present paper when they imply that what might be called the "National Bureau approach" is fairly summarized in Arthur Burns' article on business cycles in the new *International Encyclopedia of the Social Sciences.*

In their opening sentence, the authors state that this is a progress report "on an econometric model of business cycles." This statement immediately raises the question, In what sense is this model any more specifically a model of business cycles than any of the other dynamic econometric models with which we are familiar? With its use of differences and lags, the model is certainly dynamic. But I cannot see that the present model presumes the existence of business cycles any more than does, for example, the Wharton or Brookings Model. The model under discussion is not formulated in such a way as to emphasize possible differences in behavior in different phases of the business cycle, nor does it put particular stress on what happens in the neighborhood of turning points. Indeed, less is done in this respect than in some other recent econometric work.

I am sorry that the authors have been content to work with such an aggregative model. It has always seemed to me that the National Bureau's approach to the study of business cycles emphasizes what goes on behind the aggregates. Certainly, this was true of Mitchell's work. It is also true of Dr. Burns' *Encyclopedia* article. Thus, the Burns article puts considerable emphasis on the diffusion process—the spread of expansive and contractive tendencies among different industries or sectors.[1] In this connection, it is surprising that the present model makes no use at all of diffusion indices, a subject on which Geoffrey Moore has done pioneering work. Nor does it attempt to disaggregate important aggregative variables and look for significant timing relationships among the components.

I shall mention some of the variables with respect to which, in my opinion, failure to carry out a modest degree of disaggregation is

[1] *International Encyclopedia of the Social Sciences,* Vol. 2, p. 233.

particularly unfortunate. Business fixed investment could have been divided; at least, between manufacturing and other investment. There is no attempt to subdivide inventory investment—as between manufacturing and trade, or according to durability, or according to stage of processing (i.e., finished goods, work-in-process, and raw materials). No distinction is made between direct and overhead labor, although these two types of employment behave differently over the cycle. (There has also been a structural change in the relative importance of these two types of employment.) Moreover, there is only one wage index and one price level.

Burns' work has not been adequately considered. Only limited use is made of variables which he lists in his *Encyclopedia* article as leading at the turning points. Building contracts and new orders are included, but—to mention a few others—no attention is paid to additions to private debt or new equity issues (both presumably leading business investment), or to profit margins, stock prices, investment in material inventories, raw material prices, or length of the work week. I recognize that it would probably not be profitable to include a number of these series, particularly as endogenous variables, but I mention this as an example of failure to tailor the model at all closely to the suggestions in the Burns article mentioned above.[2]

Burns also puts considerable emphasis on the widely different amplitudes among different economic series, with the result that the "turmoil that goes on within aggregate economic activity" is "in no small part systematic" (page 234). Again, the present model makes no attempt to build on this suggestion.

I shall just indicate briefly some of the additional variables stressed by Burns which fail to show up in the present model. The current model does not take explicit account of the behavior of unit labor costs, profit margins, and labor productivity[3]—all important elements in

[2] I am not arguing here for indiscriminate disaggregation for its own sake. But I am suggesting systematic experiments in disaggregation, following—but not necessarily confined to—the suggestions in the Burns article, in order to (1), look for additional variables that might have explanatory value; and (2), uncover additional aspects of the dynamic process that presumably generates business cycles.

[3] The fact that these variables show up implicitly as ratios between other variables in the model does not answer the point that I am making here. In the Mitchell-Burns framework, these ratios—like unit labor costs and profit margins—should themselves have been tested for their explanatory value, and for the light they throw on the cyclical process.

the Mitchell-Burns explanation of cyclical turning points—although there are equations relating employment to output, prices, and wages; and relating the price level to a measure of aggregate demand and to wages. (These equations, like the others in the model, also include the lagged value of the dependent variable as an argument.) Both the demand-for-labor and the price equation attempt, in a peculiar way, to bring in the influence of the degree of capacity utilization by use of a proxy for the capital stock—a matter to which I shall return later.

The price of capital goods does not enter into the investment equations, although this variable is included by Burns, along with interest rates, as variables strongly influencing investment. To cite a few other examples, the model fails to include unfilled orders, monetary variables such as loans and investments, and free reserves—or the proportion of firms experiencing rising profits. The last is a diffusion-index type of variable which plays an important role in Burns' explanation of the upper turning point particularly. To give one final illustration: as the authors themselves admit, the "influence of expectations . . . is not adequately treated" in the model.

2.

I turn now to a few other general criticisms of the model. It seems to me that there is a much too undiscriminating reliance on the use of the lagged dependent variable—the justification cited being the particular lag structure, with geometrically declining weights, involved in the Koyck transformation. A variety of other lag structures could have been tried, of course. I am especially bothered by the assumption that precisely the same lag structure holds for nearly every equation, whether we are trying to explain the behavior of fixed investment, the short-term interest rate, or the wage rate. The lagged value of the dependent variable fails to appear in only two of the twenty behavioral equations—inventory investment and profits—and we are not told why it is missing in these two cases. I should like to submit that it is highly unlikely that precisely the same lag structure holds for every one of the eighteen equations to which the Koyck transformation has apparently been applied.

One not unexpected result of this assumed lag structure is that the lagged value of the dependent variable comes to play a very large role in explaining the behavior of the current value of the dependent variable. An extreme example of this is the model's equation for consumption expenditures on services. Here the coefficient on the sum of the current and last quarter's disposable income (expressed as four-quarter differences) is very small, not significantly different from zero; and expenditures on services are explained almost entirely by the constant term and the lagged dependent variable. In about half the equations, the *t*-ratio for the lagged dependent variable is much larger than those for the other explanatory variables, and in many of the equations a good part of the "explanation" is apparently accounted for by the lagged dependent variable.

Another complaint I feel obliged to express concerns the fact that all equations have been fitted for the entire period, chiefly from the third quarter of 1948 to the fourth quarter of 1967—i.e., from the end of the period of pent-up postwar demand, through the Korean War years, through the rest of the 1950's, and through the years of virtually uninterrupted expansion from 1961 on. The author of the Chow test makes no effort to discover whether or not, in some of his equations, there might have been statistically significant changes in parameters. Dummy variables—for the Korean War and strikes—enter into just two of the equations. Allowance is apparently made for changes in personal and corporate income taxes, and the capital consumption allowance is adjusted for the effects of the 1962 tax legislation. In the latter case, in effect, a dummy variable is introduced for the first quarter of 1962 (see page 770), although this does not show up in equation 20. Nonetheless, beyond these few obvious examples, there is no recognition of the possibility of structural changes in parameters. I might add here that some experiments on the Brookings Model indicated that it was not safe in all cases to include the pre-Korean years along with the remainder of the postwar period. I believe that in the latest version of the Brookings Model, most of the equations have been fitted for the period since the latter half of 1953.

The authors develop the logic of their model in terms of the absolute levels of the variables which they include. In fitting the actual regressions, however, all variables are apparently converted into four-quarter differences. Several advantages are cited for this procedure, including elimination of trend,[4] reduction in serial correlation in the residuals, and making R^2 more meaningful as a measure of goodness of fit. At the same time there are some disadvantages. One is a rather marked lag that seems to be built into a number of the equations, with the calculated values for the four-quarter differences lagging behind the actual differences. As the authors point out also, the errors of estimate in the difference equations are magnified when we make the transformation into absolute levels.

3.

I should like, finally, to offer a few comments on some of the specific equations in the model:

Consumption expenditures on durables. It would have been desirable to have a separate equation for automobiles, which the model combines with all other durables. More important, the level of expenditures on durables is assumed to be related to the change in disposable income through a capital-stock adjustment process based on the simple accelerator. The stock of durable goods does not enter into the equation. In neither the rationale on p. 740, nor the statement on p. 743 in connection with the generalized form of the equation, is replacement mentioned. Unemployment, which is not infrequently included in automobile equations, here is assumed to affect expenditures on all durables. The *level* of expenditures is assumed to be influenced by the *change* in unemployment. This is because unemployment is used as a proxy for transfer payments, for which the authors want to adjust disposable income in determining the desired stock of durables. Since expenditures depend on the *change* in income, allowing for transfer payments, it is the *change* in unemployment that shows up in the generalized equation on page 743. I might also point out that neither liquid assets nor relative prices enter into the equation.

In the actually fitted regression, four-quarter differences are used, as in the other equations. If we convert equation 3 on page 751 back to absolute levels, and cancel and combine terms as needed, we wind

[4] Actually, a constant term (usually small) does appear in each of the equations, and this term does reflect some trend.

up with an equation somewhat different from that with which the authors started on page 743. Chart 3 suggests that the fit leaves something to be desired in the case of both four-quarter differences and levels.

Consumption expenditures on services. I have already mentioned the curious result that current income has virtually no effect on expenditures on services, changes in which are "explained" by the constant term and the lagged dependent variable.

Residential construction. Here, as in the case of business fixed-investment, two equations are used—one representing the decision to invest (contracts and orders) and one representing actual expenditures. In the equation for building contracts, a capital-stock adjustment process is implied, in which the equilibrium stock of housing depends on disposable income and the long-term interest rate. The level of contracts depends on changes in these two explanatory variables. When the equation is converted to four-quarter differences, these two explanatory variables enter as second-order differences. Contracts, rather than housing starts, is the dependent variable here; and no account is taken of a number of variables which presumably affect residential building, for example, vacancies and the relation of rents to building costs. In the equation for actual expenditures on residential building, an overly simple lag between expenditures and contracts is assumed. In this equation, we get our first example of the authors' peculiar use of the variable for capital consumption allowance. Here, as elsewhere in the model, this variable is used as a proxy for the total capital stock—not for just the stock of housing. This variable is included here as a measure of capacity utilization. In the authors' words, "if the capital stock K is small relative to output, one would expect current investment expenditures to be small, given the past contracts and orders" (page 743). As noted, capital consumption allowance is used instead of a direct measure of the capital stock. No effort is made to develop a specific measure of capacity that affects residential building. And, in this case, the coefficient on this variable, lagged one quarter, turns out not to be significantly different from zero.

Business investment. No attempt has been made to disaggregate business investment; the change in GNP has been used as an explanatory variable, instead of privately produced output; no consideration is given to capital-goods prices, although this variable is specifically mentioned in the Burns article. Although both the orders and the expenditure equations seek to explain the behavior of gross investment, no specific reference is made to replacement. It is true that the capital consumption allowance variable is included in the equation relating investment expenditures to orders and contracts; but here, as in the case of residential construction, this variable is included as a proxy for the degree of capacity utilization. If I understand the logic of this, the relevant measure of capacity utilization would be one applying to the suppliers of capital goods. Actually, the capital-consumption proxy for the capital stock that is used is the figure in the national income accounts for the entire economy. Limited space prevents further discussion of equations 5 and 7 for business investment, including such matters as the inclusion of profits as well as change in output, and the particular set of lags that is used. The authors admit that they get a poor fit in the case of both these equations.

Inventory investment. It is not clear to me what precise stock-adjustment process is implied by the equation for inventory investment. The basic hypothesis seems to be that total orders, of all kinds, add to inventories, and that sales reduce inventories. The lagged stock of inventories is ignored. On page 744, there is passing reference to the hypothesis that the desired stock of inventories depends on sales, the change in price level, and the short-term rate of interest (presumably among other things), but change in neither the price level nor the interest rate is included in the fitted regression for inventory investment. The regression does include, however, that peculiar variable—capital consumption allowance—which, here, as in the fixed investment equations, acts as a proxy for the inverse of the degree of capacity utilization. As noted earlier, this is one of the two equations in which the lagged dependent variable does not appear. While the regression catches the turning points fairly well, the over-all fit is rather poor.

Corporate profits. This is made to depend on the national income, minus government payment for services, and on hourly earnings, the price level, and private employment. I have two chief complaints.

First, while the dependent variable is corporate profits, all of the explanatory variables refer to more than the corporate sector. Thus, corporate gross product would have been better than national income. Second, I would have preferred a different way of showing the effect of changing wages, prices, and employment on profits—for example, by taking the ratio of wages to prices and, perhaps, including a measure for labor productivity. This is a case in which more explicit attention might have been paid to the Mitchell-Burns emphasis on changing profit margins. It is implied, of course, in equation 12, by the negative coefficients on wages and employment, and by the positive coefficient on prices. Finally, Chart 12 indicates that the fit leaves much to be desired. The 1958–61 cycle in the level of profits is completely missed; and in other cycles, the peak in profits is regularly exaggerated.

Wages and prices. As has already been noted, one regrets that the model contains only one wage index and one price index. In the case of wages, the authors, quite logically, first construct equations for labor demand and labor supply. The wage rate (actually an index of hourly earnings in the private sector) is then made to depend on the difference between the demand for, and supply of, labor; and on wages lagged one period. (One must remember that all of these variables are expressed as absolute four-quarter differences. Unlike most equations for changes in wages, this model uses absolute, rather than relative, changes in the wage rate.)

While it seems quite logical to make wage changes depend on changes in the relation between labor demand and supply, in this case the procedure leads to a rather odd result. If in equation 16 for the change in wages, we substitute for labor demand and supply the variables on which the latter depend (from equations 13 and 15), we wind up with an equation in which wages depend on privately produced *GNP*, government employment, the price level, capital consumption allowance (the model's proxy for capital stock), the civilian population sixteen and over, the unemployment rate, the difference between labor demand and supply lagged one quarter, and the wage rate lagged one, two, and three quarters. (Remember that all of these variables are expressed as four-quarter changes.) Of the lagged wage variables at issue, only that with a one-quarter lag is of importance, and it explains much the largest part of the current change in wages.

An odd feature of this cumbersome collection of variables is that unemployment (expressed not as a rate but in billions of man-hours) shows up with a *positive* coefficient. Other things given, the larger the absolute increase in unemployment, the greater the absolute increase in wages! This peculiar result arises because unemployment occurs in the labor-supply equation with a negative coefficient (the discouraged-worker effect); labor supply, too, appears in the wage equation with a negative coefficient. So far as I can judge from a quick calculation, taking into account the units in which the variables are measured, unemployment changes do not have much of an effect on wage changes.

I shall conclude these overly long comments with a brief consideration of the price-adjustment equation. The authors state that the equation reflects the interaction of aggregate demand and aggregate supply. It seems to me that this is very crudely done. For one thing, the price index and the measure of aggregate demand do not relate to precisely the same bundle of goods and the same markets. Thus, the price index is the implicit deflator for *GNP* minus government purchase of services, i.e., gross private output. The measure of aggregate demand is the sum of new orders and contracts relating to residential building and to business fixed-investment; manufacturers' new orders, excluding machinery, equipment, and defense products; government defense orders plus government building contracts; and consumer expenditures on services minus imports. Some of these items may roughly measure the corresponding components of final demand, such as business fixed-investment, residential building, and government construction and defense purchases. But manufacturers' new orders—excluding machinery, equipment, and defense products—are hardly a satisfactory measure of the sum of consumers' expenditures on goods, inventory investment, and exports, for which these new orders are presumably a proxy. And this item—these residual new orders—make up much the largest component of aggregate demand, as our authors measure it. Further, in the case of consumer goods, the prices associated with new orders are wholesale prices, not the prices at the point of final sale.

In addition to this crude measure of aggregate demand, the price equation includes the wage index; our old friend, the capital consumption allowance (again apparently standing as a proxy for the rate of capacity utilization); and the lagged value of the dependent variable.

To me, this seems a highly unsatisfactory treatment of price determination in a model supposedly geared to Burns' article on business cycles and, by implication, to Mitchell's earlier work. In this context, I feel that it is highly important to study the dynamics of price relationships. Important econometric work has been done in recent years on the determination of wholesale, and implicit value-added, prices; and increasing attention is being paid to the relation between these prices and retail prices, or the implicit deflators for the various components of final demand. I hope that in future work on this model, considerable attention will be paid to substantial elaboration of the price section—and to better specification of the price equations used.

The need for brevity prevents consideration of the other equations in the model, although I might quickly express my agreement with the authors when they say that "the monetary sector can [and I would add 'should'] be strengthened."

We are indebted to the authors for this first attempt to formulate the National Bureau—or, perhaps, I should say the Burns-Mitchell—approach to business cycles in terms of an econometric model. I regret having to conclude that this first attempt has not been notably successful. The model needs considerable elaboration; the hypotheses underlying various of the equations need reexamination; more effort should be made to reflect essential elements of the Burns article; and the decision to work with the variables in the form of absolute four-quarter differences should, I feel, be reevaluated.

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I was quite disappointed by this paper. Its authors are a leading econometrician and an eminent authority on business cycles. At an earlier stage in its gestation, Arthur Burns was a participant in the research. With a parentage such as this, a great deal more might have been expected.

To make the best use of limited space, let me go directly to the aspects of the paper that disturb me. First, I shall discuss the general specification and estimation of the model. Then, I shall consider some of the individual equations. Finally, I shall present and comment on the dynamic multipliers that I obtained by solving the system.

1.

My primary objections to the over-all structure of the model are: (1) it is completely linear; (2) it is estimated from four-quarter first differences, with a constant term in every equation, implying that the basic structural equations in level form all have time trends; and (3) the dynamic specification is inadequately developed.

(1) Restricting all of the equations to linearity has a variety of costs. First, as the authors note, the equations often relate to absolute changes when proportional changes would be more appropriate. Closely related to this is the point that in several functional relationships, a priori considerations suggest the *ratio* of two explanatory variables rather than their separate absolute values. For example, in an economy with a growing labor force, the rate of wage increase should depend on the ratio of labor demand and supply, and not on their difference; a linear formulation implies that a one per cent excess demand for labor would cause a greater wage increase now than in earlier years, simply because the labor force is larger.

One of the important tasks of an econometric model of business cycles is to describe the behavior of the economy in the critical regions near the cyclical peaks and troughs. It is at just these points that a linear model is least adequate. A 1 per cent fall in unemployment from 3 per cent to 2 per cent has very different implications than does a fall from 6 per cent to 5 per cent. This has been allowed for in other models by using the reciprocal of the unemployment rate.

More generally, economic theory sometimes suggests not only the variables that should enter into an equation but also something about the form of the relation. I am thinking especially about much of the recent work on investment behavior. I realize that this is still a controversial subject and emphasize only that it seems unwise to preclude such nonlinear relations by insisting that the equations be linear.

(2) All of the equations have been estimated after taking four-

quarter first differences. Because each of these estimated equations has a constant term, the authors have implicitly introduced a linear trend into every basic structural relation. If the level equation is

$$Y_t = a + bT + cX_t + dY_{t-1}$$

where T is a time trend, the authors estimate

$$Y_t - Y_{t-4} = 4b + c(X_t - X_{t-4}) + d(Y_{t-1} - Y_{t-5})$$

For convenience in notation, they suppress the four-quarter lagged values in their writing. However, they never note the important implication of the constant term as a measure of an annual autonomous trend in the dependent variable.

Although a trend would seem theoretically reasonable in a number of equations—particularly those in which technical progress is relevant—there seems no reason to impose a trend on all equations. Why, for example, should the passage of a year have the immediate direct impact of raising consumer expenditure on nondurables by $500 million, plus further direct and indirect impacts through personal income and the lagged dependent variable? Unfortunately, no standard errors (or t statistics) are given for the constant term, and the significance of the trend is never tested. I suspect that the authors did not actually intend to introduce a trend but did so inadvertently by not suppressing the constant—or testing its significance—after taking four-quarter first differences.

(3) Unfortunately, the presence of the trend terms has a very important effect on the behavior of the model. In particular, it is probably responsible for the implausible adjustment dynamics in many equations. Consider, for example, the employment adjustment equations (14). Employment (L) adjusts to the demand for labor (L^d) according to the relation

$$L_t = 0.0339 + 0.4801L_t^d + 0.3190L_{t-1}$$

There is no economic reason for a time trend. The ultimate determinant of employment is the demand for labor; the passage of time, as such, is irrelevant. But if the time trend is ignored, the equation implies that an increase of one billion man-hours in the demand for labor leads eventually to only 705 million more man-hours of employment. Perhaps if the constant term were omitted, the coefficients of L_t^d and L_{t-1} would sum to one without any further constraint. If not, such a constraint should be imposed or, preferably, a more general dynamic adjustment of L to L^d should be developed.

Similar peculiar adjustments can be observed in the residential construction and business investment equations (4 and 5). The long-run impact of a one-dollar increase in a constant level of contracts (J_p) is to raise the flow of expenditure on residential housing by \$1.44; this is balanced by a strong negative trend which keeps actual and fitted values from departing through time. In other equations, there is no clear standard for assessing the reasonableness of the response speeds, but I would assume that they are affected in the same way by the imposed trend.

It may be objected that adding a term (the constant) whose true value is zero does not affect the expected value of the estimates of the other parameters. This is, of course, true if strong conditions, such as the Gauss-Markov assumptions, are imposed. In the current context, it is unlikely to be true for two reasons. First, the lagged dependent-variable adjustment model is a simple approximation of what may be a much more complex dynamic adjustment process. Second, the lagged dependent-variables are treated as exogenous for estimation. If the disturbances are serially correlated, the coefficients would not be consistently estimated, even if the constant were suppressed. Inconsistency of the estimated coefficient of the lagged dependent-variable will, in general, imply inconsistency of the constant term.[1]

Much work in econometrics in the last decade has been devoted to developing methods of estimating a variety of dynamic specifications. Many of the important policy issues hinge on questions of timing. It is disappointing, therefore, to read that "fairly different lag structures can fit the data almost equally well" (p. 743). If it is true that we cannot hope to know much about the time structure of responses, this is a very important fact, which should be demonstrated by showing, for example, the likelihood function associated with different time patterns.

[1] This implies, moreover, that ordinary t-tests of the constant terms will be biased unless a more sophisticated estimation method is used.

2.

In discussing the issue of dynamic specification, I have already touched on one of the problems of estimation: the probable inconsistency of the estimates, due to the presence of lagged dependent-variables in the equations. Closely related to this is the use of lagged endogenous variables in the "first stage" of the two-stage least squares process. Of the seventeen variables used in the first stage, the only ones that are not lagged endogenous variables are: population; the money supply, including time deposits; exports minus imports; transfers minus taxes; and the ratio of disposable income to personal income. All of these, except population, are generally treated as endogenous in more elaborate models. The result of using lagged endogenous variables (and current endogenous variables) in the first stage is to make two-stage least-squares estimates no longer consistent. Unfortunately, this is still a common procedure. Its persistence can be explained, in part, by a lack of sufficient exogenous variables in the model being estimated; the current model is particularly deficient in this respect. Perhaps it can be partially remedied by the use of more detailed models. I hope that this problem will receive explicit attention.[2]

I am confused by the authors' discussion of their reason for using four-quarter differences. Given any specification of an equation in level form, consistent estimates of the parameters could in principle be obtained in level form or after taking one-quarter or four-quarter first differences. The use of one- or four-quarter first differences may increase the efficiency of estimates by more closely approximating a generalized least squares estimator,[3] but it need not change the economic specification. It is difficult, therefore, to understand what seems to be the authors' primary reason for using four-quarter first differences rather than one-quarter differences (or, presumably, levels). They have

[2] There is a further technical problem with the Chow-Moore quasi-2SLS procedure, as described on page 747. Estimating the equations of a simultaneous model by ordinary least squares after replacing each of the endogenous variables by a fitted value based on a first-stage regression on a *subset* of truly exogenous variables, does *not* yield consistent parameter estimates. If an equation contains an exogenous variable which was not used in the first-stage regression, the second-stage estimates are not, in general, consistent.

[3] Although this is a much more complex problem in the context of simultaneous-equation estimation than when all the explanatory variables are exogenous.

argued previously that the one-quarter differences gave "poor results" because their model "attempts to explain somewhat longer-run 'cyclical' relationships rather than the evanescent factors such as strikes, unusual weather, etc., that importantly affect quarter-to-quarter changes." Although a one-quarter difference model might have lower R^2's, the parameter estimates need not be worse. The fact that the four-quarter difference also "reduces serial correlations in the residuals of most equations" (p. 747) is of little significance when the use of lagged dependent variables is taken into account.

3.

In considering the individual equations, I felt repeatedly that one or more crucial variables had been omitted. Tax variables and investment allowances do not affect the demand for capital. Relative prices do not affect the mix of consumer spending or the amount of imports. Neither inflation nor the size of the money supply (or the monetary base) affects the nominal interest rate. In discussing the individual equations, however, I shall generally not comment further on such omissions.

The three consumer-expenditure equations (1–3) relate current expenditure to a recent average of disposable income and the lagged dependent variable. In the durables equation, a lagged average income and a cyclical variable (the change in the unemployment rate) is added. The sum of the long-run marginal propensities to consume out of disposable income is only 0.48, reflecting, no doubt, the positive trend terms in all of the consumption equations.

Orders for business investment (equation 7) depend on the annual change in *GNP*, on corporate profits after tax, and on the change in interest rates. Investment has always had a primary role in business-cycle theory. I am, therefore, surprised that even in a preliminary version of their model, the authors have not developed a more sophisticated equation. There is certainly no lack of previous work on which to build.

Inventory change depends on orders, sales, and capital-consumption allowances (equation 8). Although the separate components of orders are generated in the model, the authors have used only the aggregate variable. The effect of orders for goods for sale (J_s) should certainly have a different effect on inventories than that of contracts for private residential construction (J_p). Use of the *CCA* variable as a proxy for capital or capacity utilization troubles me in this equation, as it does elsewhere in the model. Since the primary motivation for including it in many of the equations is to represent pressure on capacity, why not incorporate a capacity-utilization variable into the model?

I am pleased that the short-term interest variable is not made to depend on the Federal Reserve discount rate. It seems strange, however, that the *level* of the money stock or government debt is not included. As currently specified, the equation implies that a change in the money supply would have no permanent direct effect on the interest rate. Chow and Moore have also followed all of the other econometric models of the past few years in ignoring the impact on interest rates of sustained periods of inflation.

The long-term interest equation implies that the long-run effect of a one per cent increase in the short rate is a one-half per cent increase in the long rate. Although in the short run, the short-term rate would generally move much more than the long rate, in the long run they should move much more closely together. The estimate, no doubt, reflects the upward drift of rates during the observation period, the inclusion of a constant term, and the simple lagged adjustment model.

There were two equations in which I expected that technical progress would introduce a negative trend (i.e., constant term): the demand for labor (13), and the level of prices (17). In both, the constant term was positive. I find it hard to explain either of these equations.

4.

I want to conclude by commenting briefly on the historical fit of the model and its dynamic multipliers. I have heard it said that an application of the principles of aeronautical engineering would show that the common bee could not possibly fly. And yet it does. I have been quite critical of the components of this model, and yet the twenty-one charts seem to show that it is capable of fitting the historical period rather well. However, these simulations use the true values of the

TABLE 1

Change in Quarterly Levels of Endogenous Variables in Response to a Unit Increase in Government Spending on Goods and Services in Quarter Zero

	Quarter							
Variable	0	1	2	3	4	5	6	7
GNP	.9565	.5080	.1636	.0176	−.0947	−.1857	−.2034	−.1787
U	−.0165	−.0219	−.0174	−.0100	−.0026	.0038	.0080	.0095
P	.0252	.0242	.0197	.0136	.0055	−.0012	−.0055	−.0074
IB	.0161	.0432	.0592	.0652	.0477	.0138	−.0124	−.0298
IM	.0652	.0635	.0452	.0241	.0055	−.0097	−.0187	−.0219
RS	.0162	.0183	.0138	.0086	.0036	−.0010	−.0040	−.0054
RL	.0016	.0031	.0038	.0038	.0034	.0026	.0016	.0008

lagged endogenous variables up to and including the preceding quarter. As the authors acknowledge, they are essentially a series of one-quarter forecasts. It is therefore a quite disappointing performance when the forecasts miss (i.e., lag behind) seven out of twelve turning points in the rate of change of *GNP*.

To assess the dynamic properties of the model over more than one-quarter intervals, I have solved for the eight-quarter dynamic multipliers.[4] Table 1 presents the effects on the levels of several endogenous variables of a billion-dollar increase in government purchases of goods and structures (G_g) in the first quarter only. The impact multipliers are not implausible, but after four quarters the effect on *GNP* becomes negative and rather substantial. Correspondingly, after the first year, unemployment actually rises; prices and investment begin to fall; and so on. Such a short-run reversal seems quite implausible, despite the theoretical possibility of destabilizing fiscal policy.

Table 2 presents the dynamic effects on *GNP* of a unit change in several other exogenous variables. Government expenditure on services (G_s) has the same rapid reversal-pattern as expenditure on goods. The variable T_2 is primarily net government transfers including interest; even if one ignores the sign change after one year, the effect is

[4] Because the two tax variables g_t and h_t are exogenous and enter multiplicatively, these are actually approximate multipliers for the period 1966:I to 1967:IV.

too small. Similarly, the effect of an increase in indirect business taxes (T_1) is less than would be expected. The most surprising result is the estimated effect of an increase in the money supply. The maximum impact is achieved rapidly (in the third quarter) and the effect is negative by the sixth quarter. The positive and negative effects cancel almost identically after eight quarters, returning *GNP* to its old level. Presumably, the longer-run effect of a temporary increase in *M* would be an actual decrease in *GNP*.

5.

The somewhat technical problems of estimation discussed in Sections I and II are not matters of detail. The use of a constant term in a first-difference form, with the treatment of lagged dependent-variables as if they were exogenous, is almost certain, on a priori grounds, to bias the coefficients and the estimated time-structure substantially. The labor adjustment and investment equations are examples of the importance of such biases. The imposed time trends act as a stabilizing influence and may explain why the system fails so often to find turning points. The downward bias of the consumption propensities helps to explain the low impact of dynamic multipliers.

The authors emphasize that their paper is a progress report, a "first step in bringing together some of the important elements of

TABLE 2

Change in Quarterly Levels of GNP in Response to a Unit Increase of Several Exogenous Variables in Quarter Zero

	Quarter							
Variable	0	1	2	3	4	5	6	7
G_g	.9565	.5080	.1636	.0176	−.0947	−.1857	−.2034	−.1787
G_s	1.2391	.4263	.3374	.0733	−.1073	−.2328	−.2562	−.2265
T_1	−.0871	−.1493	−.1340	−.0636	−.0386	−.0352	−.0283	−.0105
T_2	.2069	.3510	.2856	.0577	−.0591	−.1147	−.1398	−.1349
M	.0000	.0411	.1097	.0923	.0441	−.0342	−.1188	−.1285

business cycles into a manageable system . . ." (p. 778). Of course, first steps and much more substantial strides have previously been taken by several builders of econometric models since the early work by Tinbergen. It is not clear what the current work adds to what others have already done. Indeed, this first step by Chow and Moore may be a misdirected jump to a seemingly complete model. I hope that it is not too late to turn their work toward developing a very different model —one that will incorporate both recent econometric studies and the insights about business cycles contained in earlier writing by Burns and Moore.

REPLY

CHOW AND MOORE

We are grateful to R. A. Gordon and M. S. Feldstein for their thoughtful comments on "An Econometric Model of Business Cycles." Many of the issues that they raise could have been discussed more thoroughly in our paper. In view of their comments, we believe that our presentation will be better understood if these issues are more fully discussed.

Let us first reply to Gordon's four general comments. First, in what sense is this model any more specifically a model of business cycles than any other econometric model? Gordon does not think that "the present model presumes the existence of business cycles" any more than do other models. In the sense of being a system of stochastic difference equations to explain economic fluctuations, this model is just like any other econometric model. We do not claim that it is *more* specifically a model of business cycles than *any* other model. Econometric models have been constructed for many purposes. In using the term "business cycles" in the title, our intention was to convey our specific interest in its cyclical properties, not to the exclusion of the interests of others. What distinguishes one model from another, and for that matter, what distinguishes one business-cycle theory from another, is often not the unique inclusion of certain theoretical ingredients never utilized elsewhere, but rather the emphasis placed on certain elements as compared with others, and the way in which the different elements are put together in a theory or model.

In formulating an econometric model, one cannot "presume the existence" of all of its cyclical characteristics. If one could, no mathematics or computer simulations would be required. In the model-formulation stage, one can only infer, from his experience, what some of the cyclical characteristics might be, while most of the characteristics—including leads and lags at turning points and precise differences in amplitudes—have to be ascertained after the model is estimated and analyzed.[1] Insofar as we could, we did try to formulate the model with certain cyclical characteristics in mind. The emphasis on the price-wage-profit mechanism; the use of capital capacity to limit output, given demand; the role of orders and contracts in leading output; the stock-adjustment or acceleration formulations of the demand functions for consumer durables, residential construction, business investment, and for inventory change; the increase in interest cost through the limited supply of money; and the delays in adjustment of employment and the wage rate, as part of the cyclical mechanism, are some of the theoretical ingredients included in our model. A summary picture was provided in the Introduction section of our paper. We will comment on the relative amplitudes of different variables implicit in our model later on.

The second point is that the model is too aggregative. Our research strategy has been to construct an aggregative model as a first step, in order to find out what sectors are weak and will require further theoretical work. It seemed both worthwhile and feasible to determine what characteristics of business cycles could be captured by such a model. Further disaggregation may well prove to be necessary to improve the model's capability in reproducing the business cycles experienced.[2]

[1] For the analysis of cyclical properties, including leads and lags, the reader may refer to G. C. Chow, "The Acceleration Principle and the Nature of Business Cycles," *Quarterly Journal of Economics*, August, 1968, and G. C. Chow and R. E. Levitan, "Nature of Business Cycles Implicit in a Linear Economic Model," *Quarterly Journal of Economics*, August, 1969.

[2] One of us feels more strongly about the usefulness of aggregative models, having been affected by his experience in connection with G. C. Chow, "Multiplier, Accelerator, and Liquidity Preference in the Determination of National Income in the United States," *Review of Economics and Statistics*, February, 1967, and does not consider disaggregation, including the specific instances suggested by Gordon, necessarily desirable.

Third, Mr. Gordon remarks that the present model excludes a number of variables mentioned in Burns' article. The exclusion of some of these variables, including certain leading indicators such as "additions to private debt" and "new equity issues," depends on a judgment as to how essential they are to the structure of the model. Other variables have been included in our model indirectly. For example, labor productivity, or the ratio of output to labor input, is explained once its numerator and denominator are explained by the model. Still others probably should be included as a part of our structure, but we have not yet found a suitable mathematical and statistical formulation for them. Stock prices and the state of expectations are examples. The model does incorporate some elements of expectations, since consumption and investment equations involving distributed lags can be interpreted as describing responses to expected variables, but we have not gone beyond that. Many important notions in economics, such as "a feeling of confidence about the economic future" stressed by Burns, are not yet formulated and tested in econometric form. Since Gordon refers to "the approach of the National Bureau of Economic Research to an explanation of business cycles" as our implicit intent, we would like to point out that Burns' survey article itself covers more than the work of the National Bureau, and that we are drawing from all the good works that we can find, including the important contributions of the Bureau.

Fourth, according to Mr. Gordon, the present model fails to emphasize the widely different amplitudes among different economic series. This criticism is invalid. Larger fluctuations in production than in sales are accounted for by large fluctuations in inventory change, which are, in turn, explained by the principle of acceleration; durables fluctuate more than nondurables because of the stock-flow adjustment mechanism; prices can fluctuate more than the wage rate because of the slow adjustment of the latter; profits can fluctuate more than the wage bill, because the former depends partly on fluctuating output, while both the wage rate and employment adjust slowly; corporate profits fluctuate more than dividends according to our dividends equations; personal income fluctuates more than consumption according to our distributed-lag consumption equations; residential construction contracts fluctuate more than construction executed because the latter is partly a weighted average of the former, and the same applies to the relation between contracts and orders for business plant and equipment and their realization; long-term interest rate fluctuates less than the short-term rate according to our equation for the long rate.

Let us next turn to Gordon's other general comments. First, there is a misunderstanding concerning the use of a certain mathematic form for distributed lags. It is not true, as Gordon states, that precisely the same lag structure holds for nearly every equation. The mathematical form which we have adopted, while not being the most general for reasons of estimation stated in the paper, does allow for very different lag structures—the inverted V, and others. In this connection, a legitimate question can be raised—as we have done in our paper—as to whether too much reliance is placed on the lagged endogenous variable. Whatever its pros and cons, one should not be disturbed by the mere fact that its coefficient is large relative to its approximate standard errors (not strictly a *t* ratio, as it is sometimes mistaken to be); the question is whether the other explanatory variables are important, also. After all, most economic time series do not change sizably in a quarter. One should utilize recent information to explain or predict its current change, provided one allows other variables to modify what the recent information suggests. In the case of consumption expenditures on services, current change is explained mainly by the recent change, and since we know that this is a trendlike variable, there is nothing wrong in recording it as such.

Second, Gordon suggests that the entire postwar period is too long for one set of equations, that it should be broken up into subperiods, and that more dummy variables should be introduced. Our viewpoint is just the opposite; the longer the sample period, the better; and the fewer dummy variables, the better, other things being equal. A good model should explain a long period and employ no dummy variables or very few. We might not have succeeded in building a good model, but at least we tried. There has been one minor misunderstanding regarding the use of a dummy variable for *CCA* due to the change in depreciation tax laws: we did use one, and have stated that the empirical equation 20 explains the "adjusted *CCA*" series after the effect of the dummy variable was subtracted.

We now come to Gordon's specific comments on individual equations. In regard to consumer expenditures on durables, we do not see why expenditures on automobiles should necessarily be separated, in spite of the early interest of one of us on the subject.[3] This is again the question of aggregation versus disaggregation, a question difficult to settle on a priori grounds alone. At least, later works on consumer durables show that essentially the same mathematical form applies to different components of consumer durables.[4] Our own empirical result (equations 3 and 3a) confirms beautifully the working of the acceleration principle for aggregate expenditures on durables—witness the negative sign and similar absolute value of the coefficient of lagged income (or a sum of lagged incomes), as compared with the coefficient of current income (or a sum of recent incomes).

A more important point has to do with a misunderstanding of the basic formulation of our equations for consumer durables, residential construction, business investment, and inventory change. We take some responsibility for this misunderstanding, since we did not spell out the basic formulation in our paper, only noting, in footnote 1, that "the rationale for some of the demand equations can be found" in a paper in *The Review of Economics and Statistics,* February, 1967. We do employ a stock-adjustment model for these equations, in which the existing stocks of the durable goods in question, as well as depreciation, play an important role. According to the stock-adjustment model, current stock is explained by the levels of certain economic variables (income, output, the rate of interest, unemployment, for example) and by the stock in the preceding period. When we take the first-difference of the stock-adjustment equation, we are explaining net investment by the first-difference of those economic variables, and by net investment of the preceding period (which takes the place of the stock of the preceding period). This also explains why the rate of interest and unemployment enter in first-differences. If we take quasi-differences, in the sense of $S_t - (1 - \delta)S_{t-1}$ with S denoting stock and δ denoting the percentage rate of depreciation, we get an equation

[3] G. C. Chow, *Demand for Automobiles in the United States: A Study in Consumer Durables* (Amsterdam, North-Holland Publishing Company, 1957).

[4] A. C. Harberger, ed., *Demand for Durable Goods* (Chicago, The University of Chicago Press, 1960).

explaining gross investment by the quasi-differences of the economic variables and by lagged gross investment. This is how depreciation is incorporated, as explained in the reference cited. There is no difference between our theoretical equation 3 and empirical equation 3 if one notes, again, that an income variable can be replaced by a sum of its recent values.

The last paragraph should answer most of Gordon's comments about residential construction, business investment, and inventory change, as well. The *CCA* variable is not intended as a proxy for the lagged stock in the stock-adjustment model, the latter having been converted to lagged investment. It is a measure of the capital capacity of the entire economy, which will affect the actual deliveries of these goods, given their demands as explained by the stock-adjustment model (plus profits and other important variables). In the case of residential construction, we did not expect the *CCA* variable to exercise an important influence in the relation between realization and contracts, since the data for the former are constructed mechanically from the latter, and since the use of capital goods in housing construction is not very important, but we did use that variable in equation 4 for the purpose of checking. The formulation of our inventory equation is analogous to the formulation of the other investment equations insofar as inflows depend on past orders and the conditions of supply (measured by *CCA*), except that outflows have also to be allowed for, in the form of sales. The variables, such as "sales, the change in price level, and the short-term rate of interest," which affect the desired stock of inventory are used in the order stage in equation 9, in the same way that the economic determinants of other investments enter in the contract or order stage. By the way, one should note the excellent result of equation 9, in terms of the alternating positive and negative coefficients for sales and the *change* in price level.

Our profits equation is a linear approximation to the identity $\pi = Y - \frac{W}{P} L -$ other costs, after omitting the other costs. It is possible that the ratio $\frac{W}{P}$ will do better in some mathematical approximation to the profits equations than W and P entering linearly. We did try using a profits variable including proprietors' income and did not find it to be a better variable in explaining investment. True, the Y variable includes the noncorporate sector, as we realized, but the questions are whether the profits and income variables for these two sectors have similar cyclical variations, and whether it is worthwhile to disaggregate, thus requiring the introduction of more equations.

Gordon's main point about the wage equation appears to be a failure to distinguish between the total effect and the partial effect (given other variables) of unemployment on wage change. In our equation, wage change depends on labor demand minus labor supply. Gordon then substitutes the variables for labor demand and for labor supply, and finds that, other things given, unemployment has a positive effect on wage change, since it has a negative influence on labor supply (the discouraged-worker effect). There is nothing peculiar about this result, since it is only a partial effect. The total effect can be more readily seen by considering the difference between labor demand and supply, which equals job vacancies minus unemployment. Unemployment thus has a negative effect on wage change. If there were anything peculiar in the partial effect cited, one should be able to trace it to our formulations of the labor demand and labor supply equations. Good economics manipulated by mathematics can never produce bad economics; bad economics, if deduced by correct mathematics, must be due to bad economics to begin with.

Finally, on the price equation, the use of an aggregate price-index does not call for apologies. It is true, as Gordon points out, that the measure of aggregate demand refers to commodities which are not completely identical with those whose prices are included in the price index, but as long as the demand for the latter commodities is closely related to our measure of demand (for those components which do not coincide), or as long as the majority of the commodity components are shared by both measures, our equation will be a good approximation. Economists have long been searching for a good explanation of the general price level after discovering that the crude quantity theory of money does not work very well for short-run cyclical variations. Keynes, in Chapter 21 of *The General Theory*, suggested the idea that aggregate supply, together with aggregate demand, determines the price level. It is natural to include the wage rate and the stock of capital as factors affecting aggregate supply. We have formulated and tested a price equation consistent with some of Keynes' ideas, and regard the price equation as among the possible useful contributions of our work.

Feldstein's comments raise a number of technical points which are worthy of some discussion, but most of which cannot be settled in a clear-cut fashion. We happen to disagree with his points of view. First comes his objection to linearity. We do not believe that there is much evidence in favor of using the ratio, rather than the difference, between labor demand and supply in determining the wage rate, or of using the reciprocal of the unemployment rate, in spite of the fact that some other models have used it. Nonlinearities have been introduced in some business-cycle theories partly because the theories are nonstochastic. Once we allow a stochastic model, a linear one can capture many aspects of business fluctuations.[5] Experience with large, nonlinear econometric models has indicated that the nonlinearities introduced often do not produce results much different from the linear versions. These remarks should not be interpreted to mean that we will adhere to a linear model at all costs in our future research; in our paper, we have explicitly stated otherwise. At the present stage, when the main objective is to find out how well our first approximation works, a linear model does serve its purpose.

Our second area for discussion is the use of four-quarter differences. We simply do not know enough to make a definitive choice among levels, one-quarter differences, four-quarter differences, or other possibilities. Our tentative decision in this regard is based partly on the notion that the covariations between two economic time series observed in a long period can be very different from their covariations in short periods. For example, they can both increase in long periods because both have positive trends, thus producing a positive correlation in the levels, and yet they can move in opposite directions through the cycles, thus producing a negative correlation in four-quarter differences. The latter comovements appear to be more relevant for our purpose, and the use of four-quarter differences is an attempt to capture them. When four-quarter differences are used, there is less need to adjust the parameters of the model every time a quarterly

[5] See, for example, Chow and Levitan, *op. cit.*

forecast is made, as some forecasters are doing with their models. Trendlike variations will be, at least partly, eliminated and absorbed in the intercepts. Since we are not concerned about these trends at the present stage, we do not bother to study them. In imposing a trend on all equations, we are following exactly what sound statistical method would recommend. If one is mainly interested in measuring the effect of A, but fears that the effect of B may contaminate his measurement, he introduces B explicitly in the model, not to study the effect of B but to safeguard his measurement of A.

Another debatable issue is that of the method of estimation. With so many facets involved, the econometrics profession has not come out with a clear-cut recommendation, and we certainly are not able to do so here.

As for the comments on our individual equations, we prefer to let the reader form his own judgment, except for a few obvious points. Corporate-profits tax does affect investment through after-tax profits. Do we recognize that there is a discipline called macroeconomics, and that aggregates, such as total consumption expenditures, are worth studying irrespective of composition? The supply of money affects the rate of interest through a stock-adjustment demand equation for money, on which our equation 10 is partly based, explaining the opposite effect of M_{t-1} to that of M_t. We have tried to measure the effect of price change on interest rate, but have not succeeded.

Feldstein's exercise on dynamic multipliers is worth doing, but we have not had time to check his calculations and, therefore, cannot comment on them.

In evaluating and conducting econometric work, one should not put his major emphasis on technical details, although he should be aware of the technical issues involved; nor should he attribute his failure, if failure does come, to linearity or to the omission of certain second-order variables as first guesses. Most likely, there is something basically wrong with the specification that nonlinearity or the adding of minor variables cannot cure. Most of all, he should not lose sight of the forest while examining the trees. At the present time, further progress in economy-wide models will probably come mainly from viewing the forest as a whole. The existence of many econometric models has not provided conclusive answers to such basic questions as the relative importance of fiscal and monetary policies, or the essential mechanism generating business cycles. The production of more statistically fitted equations for more variables does not necessarily represent progress. Progress comes from sound theory and careful observation, and both may be jeopardized if economists are preoccupied with more components of *GNP* (and variables appearing in the equations for them), with statistical fitting, and with computer simulations of more elaborate sets of "empirical" equations.

PART THREE

EVALUATION OF FORECASTS