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BUSINESS CYCLE ANALYSIS OF ECONOMETRIC MODEL SIMULATIONS

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1 INTRODUCTION

1.1 BACKGROUND AND PURPOSE

IN A pioneering study published ten years ago, Irma Adelman and Frank L. Adelman [2] calculated and analyzed the time paths of the main endogenous variables from the 1955 Klein-Goldberger (KG) econometric model of the United States [22]. They examined several forms of hypothetical long-term development of this system: (1) nonstochastic simulations based on smooth extrapolations of the exogenous variables; (2) stochastic simulations of "Type I," with random shocks superimposed upon the extrapolated values of the exogenous quantities; and (3) stochastic simulations of "Type II," with random shocks introduced into each of the fitted equations. Each of these different solutions was dynamic, in that it related current values of endogenous variables to their lagged values generated by the model from earlier data; each also involved some tentative assumptions about secular economic trends, in that it projected the exogenous variables far beyond the sample-period base of the KG estimates (1929–52) over one hundred years of the "future." The Adelmans were primarily interested in learning whether the KG Model can, internally, generate cyclical movements resembling cycles found historically in the United States economy. The nonstochastic simulations and those using the Type I shocks did not produce such movements, but the stochastic simulations with shocks of Type II did, as the Adelmans concluded from comparisons of the time paths computed for the KG Model with the NBER "reference cycle" measures for the series involved.

Since then – in the 1960's – increasingly ambitious efforts have been made to estimate economic relationships with more detailed and complex econometric models, and the simulation experiments performed upon these models have grown correspondingly in size and scope. Simulations of a quarterly model by Duesenberry, Eckstein, and Fromm [8] were designed to test the proneness to recession of the U.S. economy and the effectiveness of automatic stabilizers. Later, several quarterly models of the postwar U.S. economy were unveiled in quick succession, notably those by L. R. Klein [22], Klein and M. K. Evans [13], M. Liebenberg, A. A. Hirsch, and J. Popkin for the Office of Business Economics of the Department of Commerce (OBE) [23], the Brookings Model [9], and most recently, the FRB-MIT Model [3], [11], [27]. The last two systems represent efforts by sizable groups of economists, and each consists of a very large number of equations. At the NBER, an econometric model of business cycles was formulated in the last few years by G. C. Chow and G. H. Moore: its early sets of estimates are currently being evaluated [6].

The present Conference is concerned with these recent models, viewed as instruments for the analysis and prediction of general economic fluctuations. Our study, in particular, deals with experiments performed on some of these systems in a search for answers to the type of question which Irma and Frank Adelman asked with respect to the KG system. Do these models endogenously generate cyclical behavior, and, if so, to what extent, how, in which sectors, and over what predictive span? To what degree are the fluctuations produced by external impulses? How do such cycles as may originate in the nonstochastic and stochastic simulations compare with the relationships observed in the NBER business cycle studies? How do the models differ from each other in these respects?

The materials that can now be analyzed with a view to clarifying these issues are clearly much richer than those available in the late 1950's. It has long been recognized, for instance, that annual data are far less adequate in business cycle analysis than are quarterly or monthly data. The new quarterly models, therefore, should definitely be more appropriate for the purposes at hand than are the older annual models (such as the KG equation system). Furthermore, the present models draw on longer experience with, and better knowledge of, econometric estimation methods; and they cover larger data samples and a much greater number and variety of macroeconomic relations. These data were hardly tapped for studies of cyclical behavior before the present Conference. Such simulations as were made with these models were used primarily for general evaluation and for analyzing the effects of specific postulated policy changes [10], [17].

Although simulation is a powerful tool of economic analysis, its inherent limitations are substantial and should be recognized. As noted by Irma Adelman, "Any simulation experiment produces no more than a specific numerical case history of the system whose properties are to be investigated" [1, p. 272]. Hence, the inferences drawn from simulation results concerning the properties of the economic system are only as good as the model which is used as the analogue of that system. For example, the Adelmans' study has shown that the cyclical fluctuations in the KG Model are due to random shocks of a certain type; to what extent this study has verified the hypothesis that random perturbations were the major cause of business cycles of experience, depends essentially on the quality of the KG equation system as a representation of basic relationships in the U.S. economy.

While no simulation study can avoid being limited in this sense, evidence from studies based on different models, applications to different periods, and so forth, may to some extent cumulate and help reduce this weakness. This would be so if the different applications and models were complementary in their substantially valid parts, and if the evidence based on them were internally consistent. For example, should the simulations for a variety of differently structured quarterly models yield similar indications of the importance of exogenous erratic impulses, we would regard this as additional support for the random-shock hypothesis of cyclical behavior. In the light of possibilities of this sort, a plausible argument can be made in favor of comprehensive and diversified coverage of econometric model simulations in business cycle analysis.

1.2 PROGRAM AND DATA

According to the original plan for the Conference, the study was to cover five models: Brookings, Wharton-EFU, OBE, FRB-MIT-PENN, and Chow-NBER.[1] However, no simulation data were received for the Brookings Model before the time scheduled for delivery of the Conference papers, and the estimates for the Chow-NBER Model are still incomplete. The Wharton, OBE, and FMP model-builders have supplied us with the large amounts of required data, and have given us excellent cooperation. In its present version, therefore, our study covers the estimates produced by the current versions of these three quarterly models of the postwar U.S. economy.[2]

Twenty-two variables were selected for the cyclical simulations. The list includes *GNP* in constant (1958) dollars and five of its components: consumption, residential construction, nonresidential fixed investment, change in business inventories, and net exports. Also specified for the investigation were data on *GNP*, personal income, and corporate profit in current dollars, employment and the unemployment rate, average workweek, new and unfilled orders, construction contracts and housing starts, the implicit price deflator for *GNP*, labor compensation per man-hour and unit of output, money supply, and the short- and long-term interest rates. These variables were selected because of their importance for macroeconomic theory in general, and business cycle analysis in particular, and in view of their cyclical sensitivity and timing. With some exceptions and modifications, they appear in most of the recent econometric models of intermediate or large size.

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[1] See references in previous section. For brevity, the Wharton Econometric Forecasting Unit (EFU) Model and the FRB-MIT-PENN Model will henceforth be referred to as the Wharton and FMP Models, respectively.

[2] In the process of being developed and revised, each model has been undergoing changes of varying importance and frequency. Models with relatively long histories, such as the Wharton Model, have passed through several distinguishable versions, as described in the paper by Evans, Haitovsky, and Treyz [12]. The OBE Model, as used in this report and identified by the list of its equations in the paper by George Green and associates [18], differs from the earlier version introduced in 1966 [23]. The model variants on which our analysis is based are those developed by the spring and summer seasons of 1969, prior to the time when the simulation data were supplied to us by the model-builders. These models are explained in considerable detail in other reports prepared for this Conference [12], [14], [18]; we shall refer to this information as needed, without reproducing it at length.

Table 1.1 gives some descriptive detail and sources for the data actually used to represent the selected variables in each of the cooperating systems. It shows that, on the whole, the models agree rather well with each other in regard to coverage of the specified items. However, there are several differences among models, and some variables have been omitted. Thus, none of the three systems includes construction contracts. Only the OBE Model estimates housing starts, and labor costs per unit of real private *GNP*; only OBE and FMP have endogenous components of money supply. Also, the concepts and industrial coverage differ among the models for certain variables. For example, OBE uses both new and unfilled orders for durable-goods manufactures in real terms; Wharton, deflated unfilled orders for all manufacturing; and FMP, unfilled orders for machinery and equipment industries, in current dollars.

Most of the important differences in data definitions and coverage are brought out in Table 1.1, and some minor discrepancies in units of measurement are also annotated, but we do not claim to have identified all factors that impair comparisons across the models. Such factors are undoubtedly numerous and some are difficult to detect, notably the differences in the vintage of data used, which can be quite significant, as in the case of the frequently revised series for *GNP* and components.

For each of the models, three types of complete-model simulations were examined, namely: (*a*) nonstochastic simulations over selected six-quarter periods which include the dates of the turning points of recent fluctuations in aggregate economic activity; (*b*) nonstochastic simulations over the entire period covered by the models; (*c*) stochastic simulations projecting the models for a period of twenty-five years, starting at the end of the sample period.

Each set of simulations of a particular type consists of discontinuous sequences (for *a*), or continuous time-series (for *b* and *c*) of estimates for as many of the selected endogenous variables as are included in the given model. One set per model is sufficient to produce the simulations of type *a*; and one run, for those of type *b*; but for the stochastic simulations (*c*), as many as fifty runs per model were requested, with one hundred quarterly terms in each run. This was done to examine the variability of responses of a given system to different configurations of random shocks, and to avoid excessive reliance on

TABLE 1.1

List of Variables and Data Definitions for Business-Cycle Simulations of Three Econometric Models
(x *indicates simulated series available*)

Line	Symbol	Variable and Units	Wharton Model[a] (1)	OBE Model[a] (2)	FMP Model[a] (3)
1	GNP^a	Gross national product, annual rate, billions of dollars	x	x	x
2	$GNP58^a$	Gross national product, annual rate, billions of 1958 dollars	x	x	x
3	C^a	Personal consumption expenditures, annual rate, billions of 1958 dollars	x	x	x
4	IH^a	Investment in nonfarm residential structures, annual rate, billions of 1958 dollars	x	x	x
5	ISE^a	Investment in nonfarm nonresidential structures and producers' durable equipment, annual rate, billions of 1958 dollars	x	x	x
6	II^a	Change in nonfarm business inventories, annual rate, billions of 1958 dollars	x	x	x
7	NE^a	Net exports, annual rate, billions of 1958 dollars	x	x	x
8	YP^a	Personal income, annual rate, billions of dollars	x	x	x
9	P^b	Implicit price deflator for *GNP*, 1958 = 100	x	x	x
10	LE^b	Total civilian employment, millions of persons	x	x	x

11	UN^b	Unemployment rate, per cent of labor force	x	x	x
12	CPR^b	Corporate profits and inventory valuation adjustment, annual rate, billions of dollars	x	x	x
13	AWW^b	Average workweek, private employment, hours per week	x	x	
14	LH^a	Total hours per man in nonfarm private domestic sector, thousands of hours per year			x
15	OMD^b	New orders, durable manufacturers' goods, billions of 1958 dollars		x	
16	UMD^b	Unfilled orders, durable manufacturers' goods, at end of quarter, billions of 1958 dollars	x	x	
17	$OUME^b$	Unfilled orders, machinery and equipment industry, end of quarter, billions of dollars			x
18	HS^b	Private nonfarm housing starts, annual rate, thousands		x	
19	RS	Average yield, 4–6 month prime commercial paper, per cent per annum	x	x	x
20	RL	Average yield, corporate bonds, Moody's, per cent per annum	x	x	x
21	W	Private wage and salary compensation per man-hour, dollars	x	x	x
22	LC/O^b	Private employee compensation per unit of constant dollar private GNP, dollars		x	
23	M	Demand deposits, adjusted, and currency outside banks, daily average of quarter, billions of dollars		x	x

SOURCE NOTES FOR TABLE 1.1

Line

1. OBE definition. National Income Accounts (NIA) Table 1.1, line 1.
2. NIA Table 1.2, line 1.
3. *Ibid.*, line 2.
4. *Ibid.*, line 11. In FMP Model, hundred billion 1958 dollars.
5. *Ibid.*, line 8.
6. *Ibid.*, line 15.
7. *Ibid.*, line 17.
8. NIA Table 2.1, line 1.
9. NIA Table 8.1, line 1.
10. Based on monthly BLS figures. In Wharton Model, includes armed forces.
11. *Ibid.* In FMP Model, labor-force base includes armed forces.
12. Corporate profits before taxes, including inventory valuation adjustment.
13. Based on monthly BLS figures. OBE: 1957–59 = 1.00. Wharton: 40 hours = 1.00 (manufacturing and nonmanufacturing).
14. Unpublished BLS series.
15. Based on monthly Census figures. Deflated by Wholesale Price Index for durable-goods manufactures.
16. *Ibid.* In Wharton Model, equals unfilled orders for all manufacturing. Deflated by corresponding Wholesale Price Index series.
17. *Ibid.*
18. *Ibid.*
19. Based on monthly Federal Reserve System data.
20. Based on monthly Moody's Investors Service series. In FRB-MIT Model, the A bond yield.
21. Based on monthly BLS labor income and man-hours data. In Wharton Model, wage rate (quarterly earnings at annual rate), weighted average for manufacturing and nonmanufacturing. In FMP Model, rate of compensation in nonfarm private domestic sector.
22. Based on monthly OBE data.
23. Based on monthly Federal Reserve System data.

[a] Seasonally adjusted.

[b] Quarterly, seasonally adjusted.

the results of any particular distribution of the shocks that could well be highly idiosyncratic.

The sections that follow deal successively with these three types of simulations, thus proceeding from the shortest to the longest ones. The six-quarter simulations (*a*) can be viewed as conditional predictions over selected, relatively short periods. They are conditioned on the ex post values of the exogenous factors, and on the structure of the model estimated from the sample-period data, which include the turning point episodes covered here. Being reinitiated from actual base-period values in each new run, they predict six successive quarters. The sample-period simulations (*b*) are conditional, or ex post, in the same sense, but they have much longer predictive spans: up to (approximately) 11, 14, and 20 years, for the FMP, OBE, and Wharton Models, respectively. Finally, the stochastic simulations (*c*) start from initial conditions as of the end of the sample-period, and look into the future: a period which is for the most part unknown and—for a long time yet—unknowable. These simulations are based on nonstochastic projections (control solutions) of each of the models, which embody various assumptions—some, reasonably well founded; others, made for working purposes only. In a purely formal sense, these simulations are ex ante model forecasts over a long stretch of time, but they were not intended, or constructed, to serve any practical forecasting purposes. Instead, their function is to help us evaluate some important characteristics of the models and to compare the evolution charted in these experiments with the historical movements of the economy.

1.3 SOME PROBLEMS OF MEASUREMENT AND INTERPRETATION

Different types and aspects of simulations require different analytical methods and measures. For the six-quarter simulations around business cycle turning points (Part 2), the emphasis is on the reproduction of turning points, the timing of these turns, and the amplitudes of cyclical swings—all in comparison with the corresponding segments of the actual series. The measures applied to the sample-period simulations (Part 3) range widely, from the NBER reference-cycle patterns, cyclical timing and amplitude comparisons, through some results of time-series decomposition and correlation and regression analysis, to selected summary measures of absolute and relative accuracy of prediction. For the long stochastic simulations extending into the future (Part 4), broad comparisons with the sample-period actuals are made in terms of the average frequencies, durations, and relative size of movements. The relative timing of the various simulated variables is analyzed, and an attempt is made to find out whether the simulated series can be classified as leaders, coinciders, and laggers—in the same way in which the historical indicators were classified.

This diversification of the techniques and tools used (still understated in the above summary) reflects the difficulty of any attempt to establish the degree of verisimilitude of a model as an analogue of the economic system in motion. The task is necessarily intricate, for it involves study of relationships of various kinds, between different economic processes and over time. Incomplete knowledge of the past, and ignorance of the future, reduce the potential attainments of the analysis. Recurrent, diverse, cumulative, and widely diffused expansions and contractions in economic activities, which underlie aggregative cyclical fluctuations, have been a persistent feature of highly developed capitalistic economies of the modern era. To what extent they will continue in this role in the future, no one can predict with confidence: it depends on structural changes in the economy, the success of economic policies (and of the underlying forecasts), international developments, and so on. All we have as a measurable criterion for evaluating the model-results is the past evolution of the economy. This compels a particularly cautious interpretation of any findings for the long-term simulations.

The results for the different models are not directly comparable for at least two reasons. First, there are differences between the sample periods (e.g., the simulations start late in 1948 for the Wharton Model, in 1953 for OBE, and in 1956 for FMP). This can strongly affect the relative performance of the models. As a task for the future, it would be most desirable to recalculate the simulations with one common sample period for all included models. Secondly, models differ in coverage: in particular, what is endogenous in one model may be exogenous in another. This is a major problem for comparing models of different scope, with respect to their predictive performance [7, Sec. E-1], but it is not so serious for our study, which concentrates on a subset of selected variables that are basically common to, and endogenous in, all of the models covered. However, some points of difference ought to be noted. Comprehensive aggregates, such as *GNP*, include certain exogenous components in each case, but they are not always exactly the same across the models. Thus, in the Wharton Model, the parts of real *GNP* originating in the farm sector, and in the government sector, are exogenous; in the OBE Model, in addition to government purchases of goods and services, and investment in farm structures, housing services are treated exogenously; and in the FMP Model, only the federal part of government expenditures is exogenous, while the state and local government purchases are handled essentially as endogenous [13], [18], [25]. Furthermore, exports and imports are endogenous for Wharton, while exports are exogenous for OBE and FMP (as are military imports). Of variables other than real expenditure components, the money supply (*M*) deserves attention. In the OBE Model, *M* consists of currency outside banks (exogenous) and demand deposits (endogenous). The FMP Model, which is particularly concerned with monetary factors and financial markets, also adopts this differential treatment of the two components of *M*. The variable does not appear in the Wharton Model.

Because of these differences in sample-periods and scope, large parts of this report deal with each of the models separately. However, comparisons between the models will inevitably be made, and some of them may be justified if they are framed with caution. We shall have something to say on this subject in summarizing the results of the different types of simulation.

2 SIX-QUARTER SIMULATIONS AROUND REFERENCE TURNS

BUILDERS of cyclical models have stressed—correctly—that their models are short-term models, that cumulations of short-run errors would tend to distort the results of simulations which are run continuously for many quarters, and that, therefore, it would be inappropriate to test the efficacy of the models by long-run simulations only. Another rationale for this contention is the argument that dynamic relationships, such as consumption responses to cyclical swings of income, may be structurally different in the short run and in the long run. Thus, this type of model should be tested for its efficacy over relatively short time-spans. Since such tests may not be very interesting over stretches without any cyclical turns, we tested the models by a more stringent criterion; that is, by their performance during six-quarter periods which include cyclical turns in general business conditions. Specifically, simulations were carried out for six-quarter periods beginning, alternatively, three quarters, two quarters, and one quarter before each business cycle turn. In these simulations, the endogenous variables were derived by using actual values for the quarter preceding the simulation and letting the model determine subsequent values; exogenous variables were used throughout the simulation period at their historical levels. The resulting configurations of twenty specified variables were compared with the actual behavior of these variables during the corresponding periods. The following three behavioral characteristics of the simulated and actual series were investigated: (*a*) Did cyclical turns occur in simulated and actual behavior? (*b*) If so, what were the timing relations between simulated and actual turns? (*c*) What were the comparative amplitudes of simulated and actual cycle phases?

2.1 INCIDENCE OF TURNING POINTS

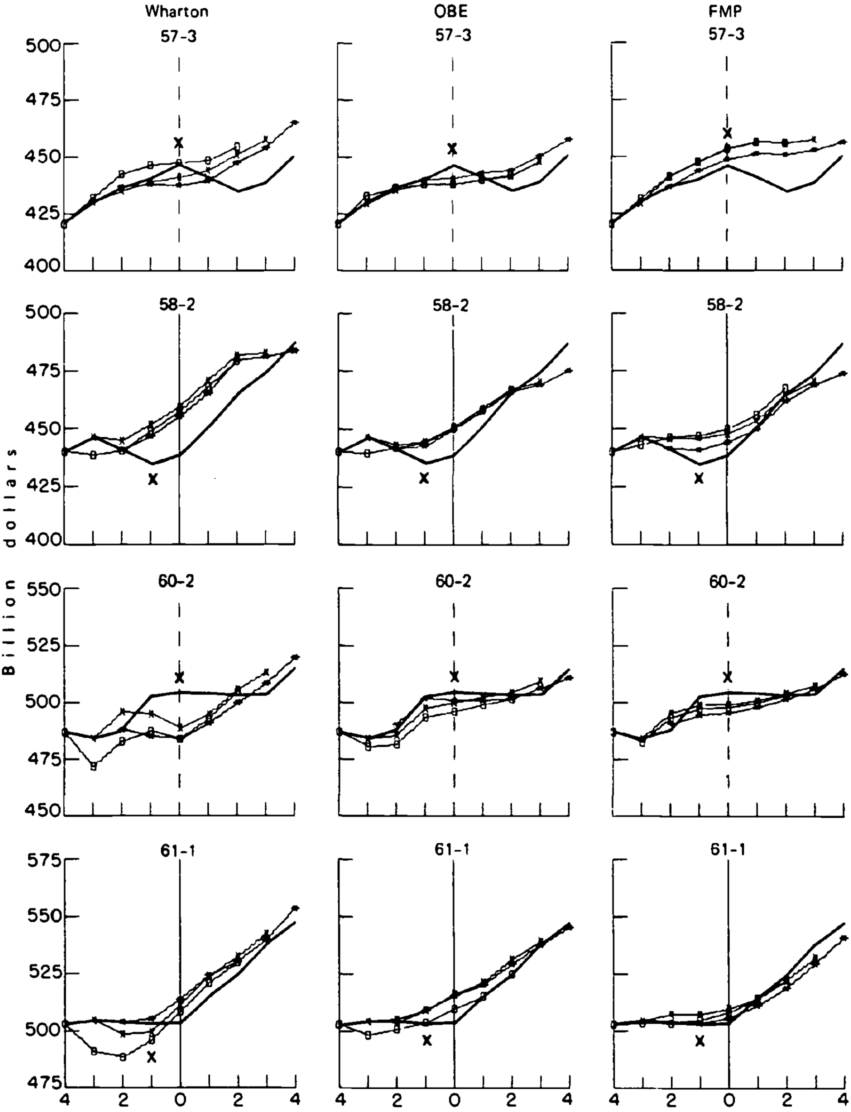
For the Wharton Model, the sample period starts in the third quarter of 1948 and ends in the fourth quarter of 1964, but the simulations are extended through the first quarter of 1968. Thus, they include four reference troughs (1949-IV, 1954-III, 1958-II, and 1961-I) and three reference peaks (1953-II, 1957-III, and 1960-II). The Office of Business Economics (OBE) sample period starts in 1953-II and ends in 1966-IV, including three troughs and two peaks. The Federal Reserve-MIT-PENN (FMP) Model has the shortest sample period, extending from 1956 to 1966 and covering two troughs and two peaks only.

For each variable and for each turning point covered by a given model, we compared the simulated behavior produced in the three simulation runs with the actual behavior of the particular variable. Chart 2.1 contains a selection of these comparisons. We reproduced charts only for those variables and turning points which were common to the three models. The charts are arranged in such a way that the Wharton Model is on the left, the OBE in the middle, and the FMP on the right. The top panel shows comparisons for the 1957 peak; the second panel, for the 1958 trough; and so on. In each diagram, the

CHART 2.1

Nonstochastic Six-Quarter Simulations

Gross National Product



(*continued*)

CHART 2.1 *(continued)*

Gross National Product, 1958 Dollars

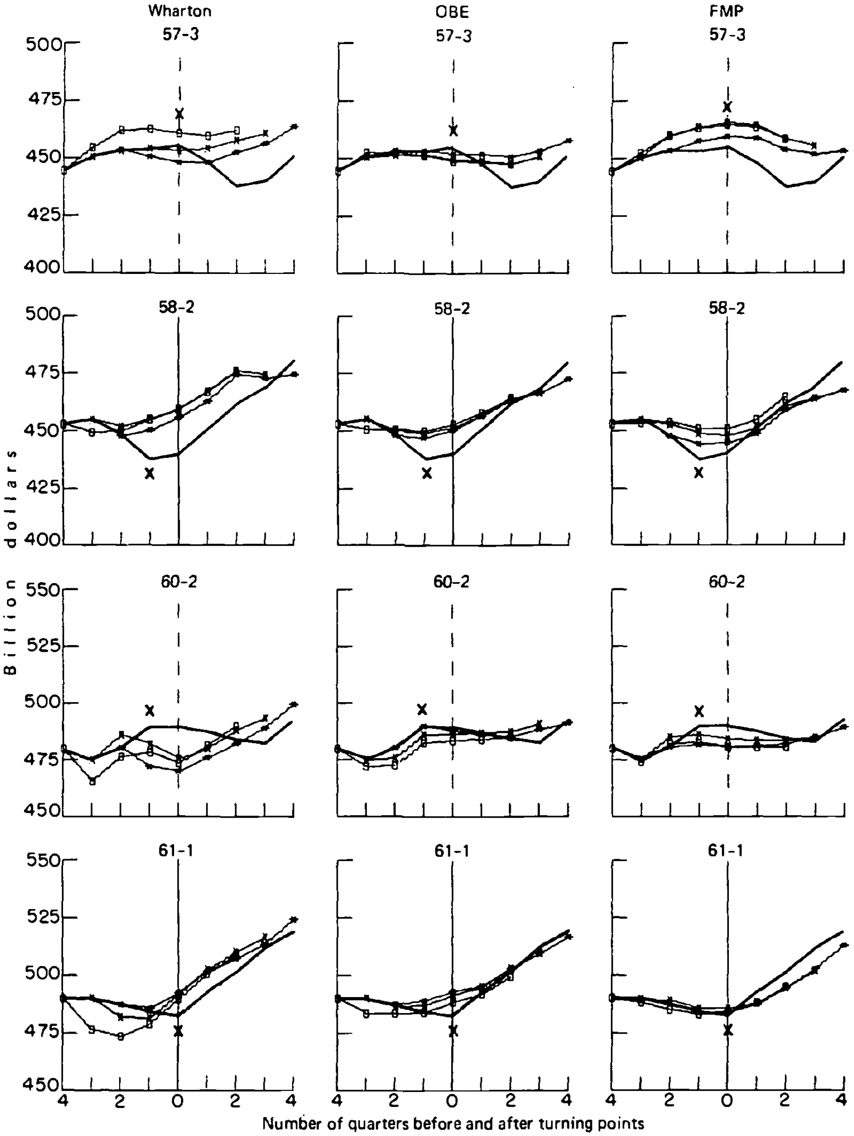
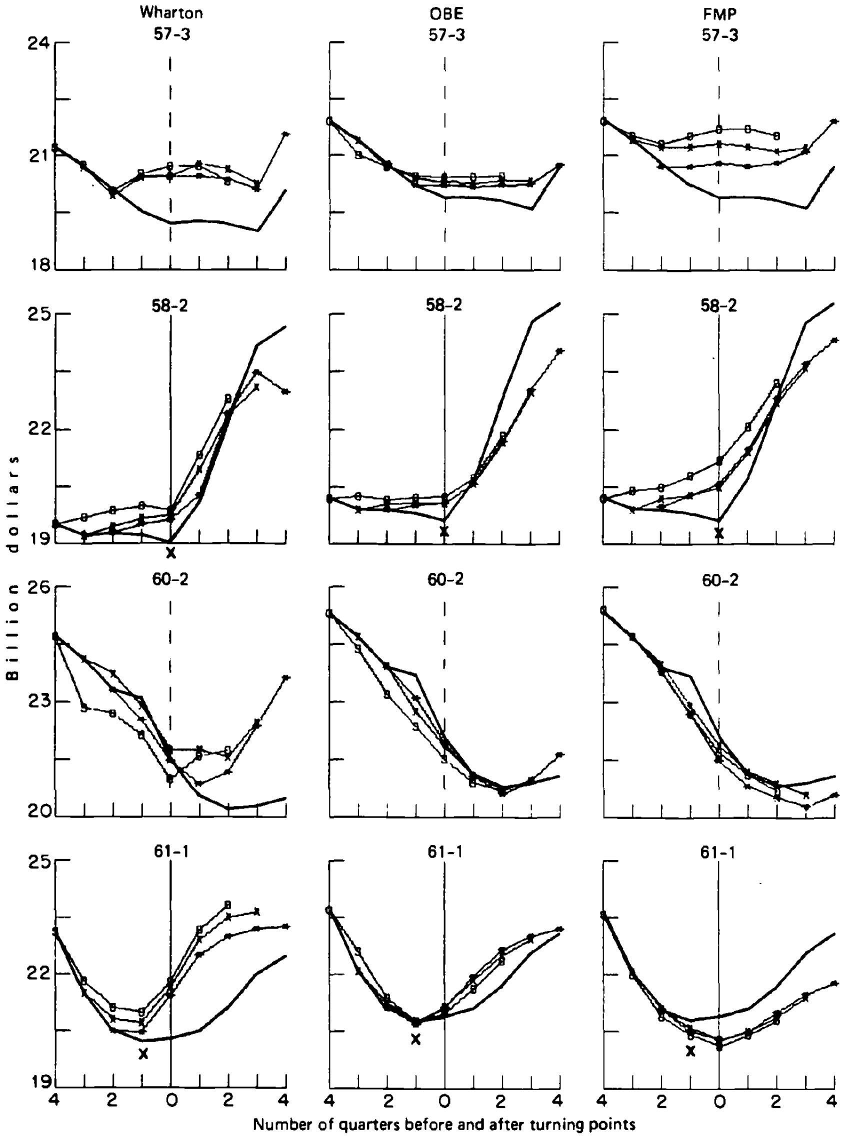


CHART 2.1 (*continued*)

Investment in Nonfarm Residential Structures



(*continued*)

CHART 2.1 *(continued)*

Investment in Plant and Equipment

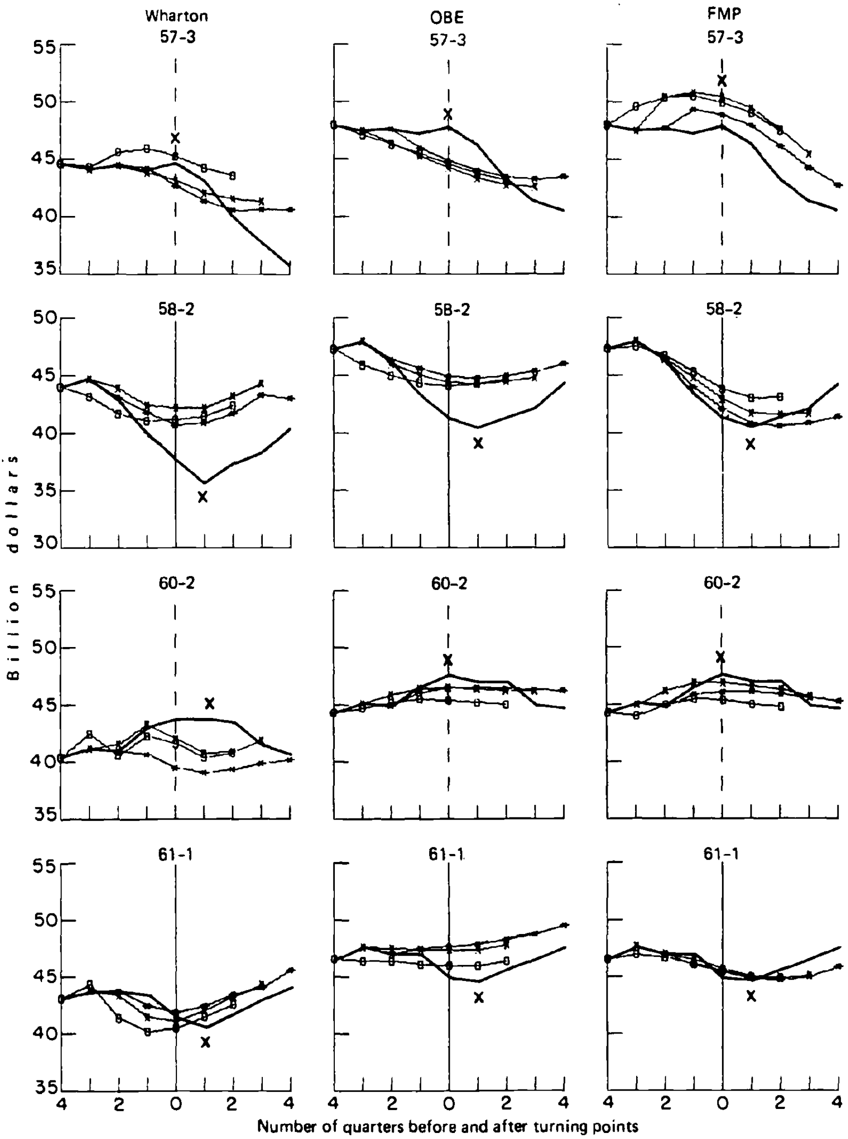
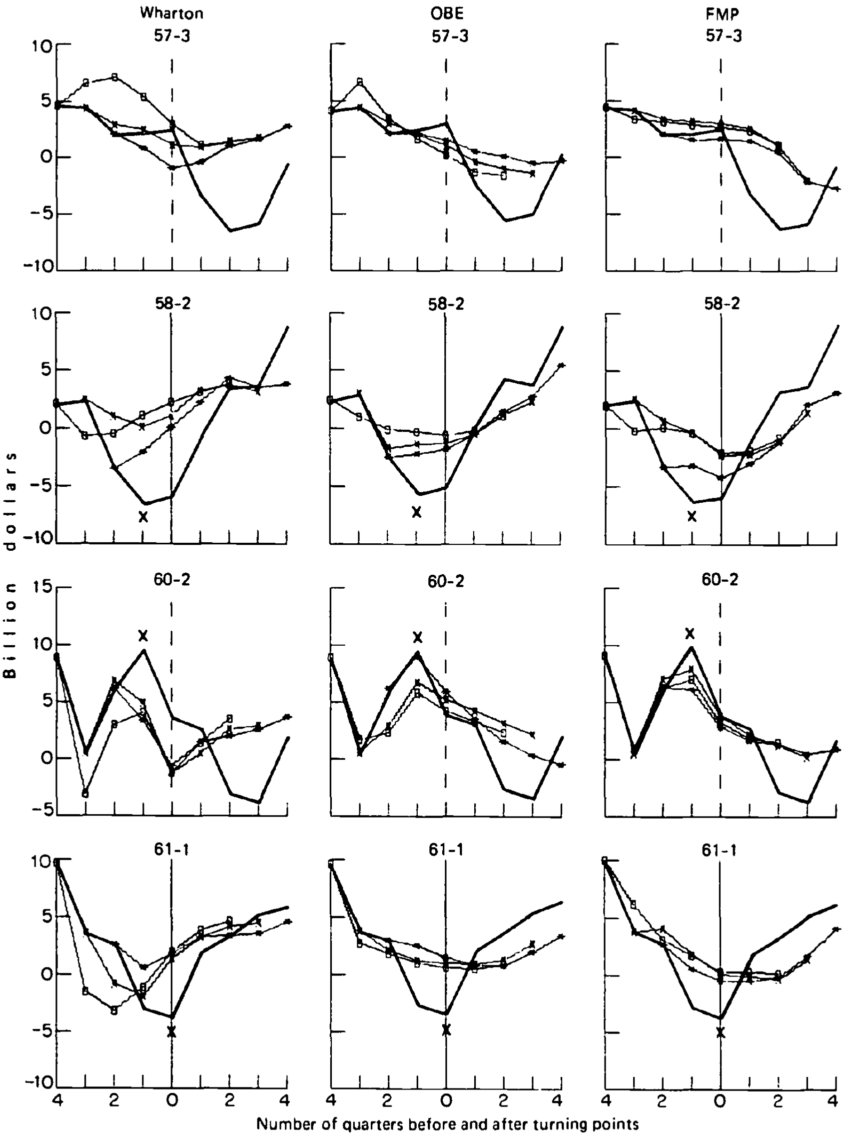


CHART 2.1 *(continued)*

Change in Business Inventories



(continued)

CHART 2.1 *(continued)*

Total Civilian Employment

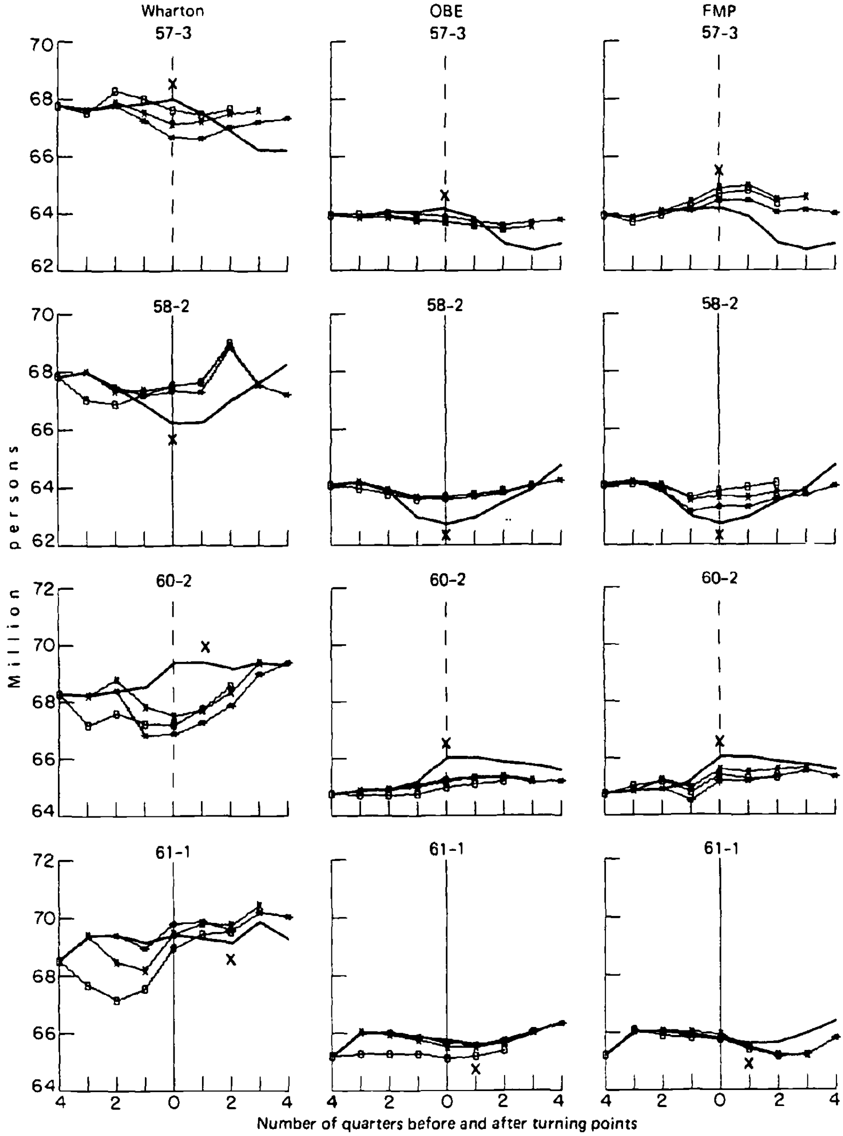
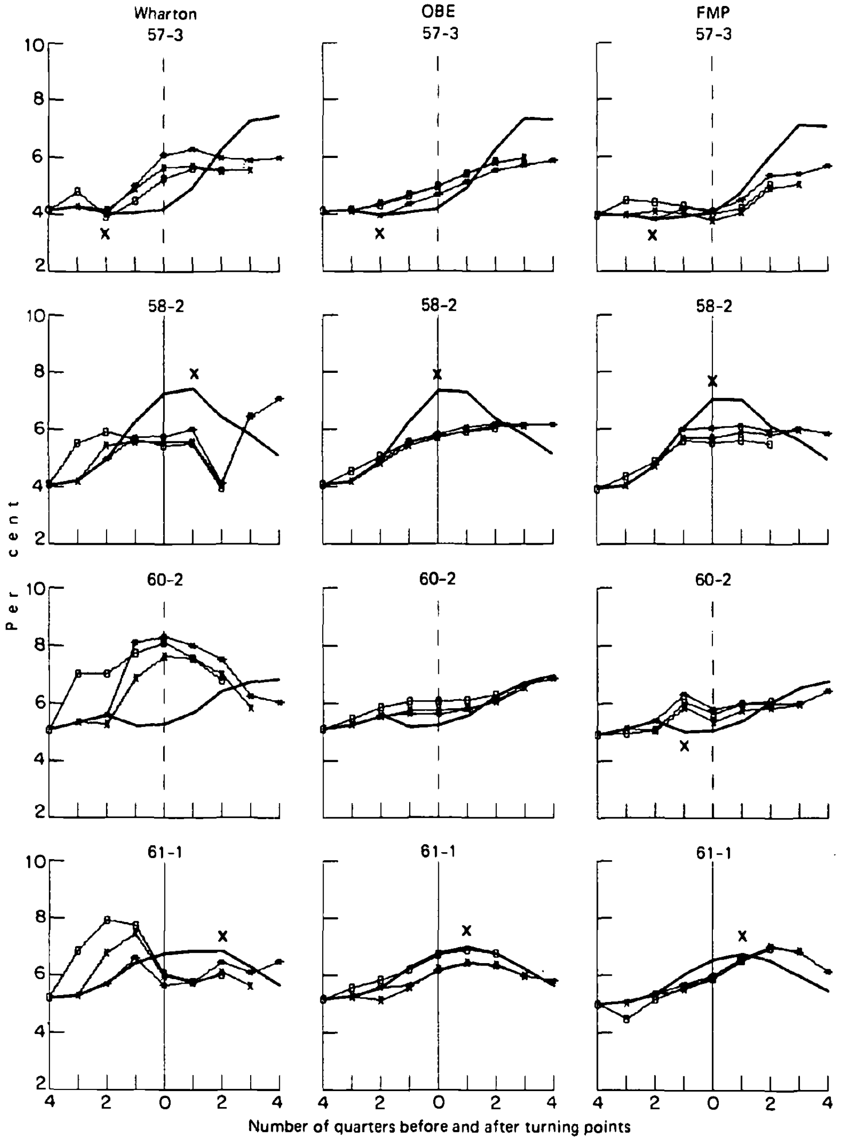


CHART 2.1 (*continued*)

Unemployment Rate



(*continued*)

CHART 2.1 *(continued)*

Corporate Profits and Inventory Valuation Adjustment

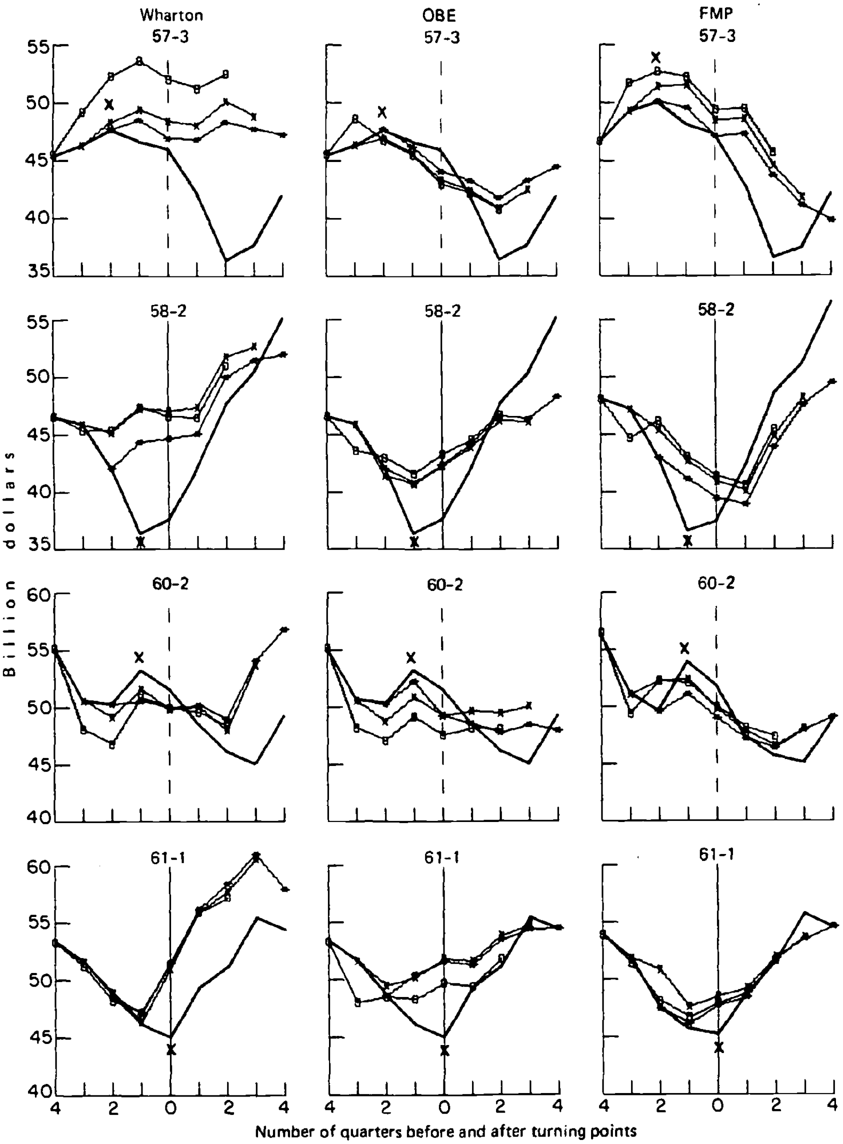
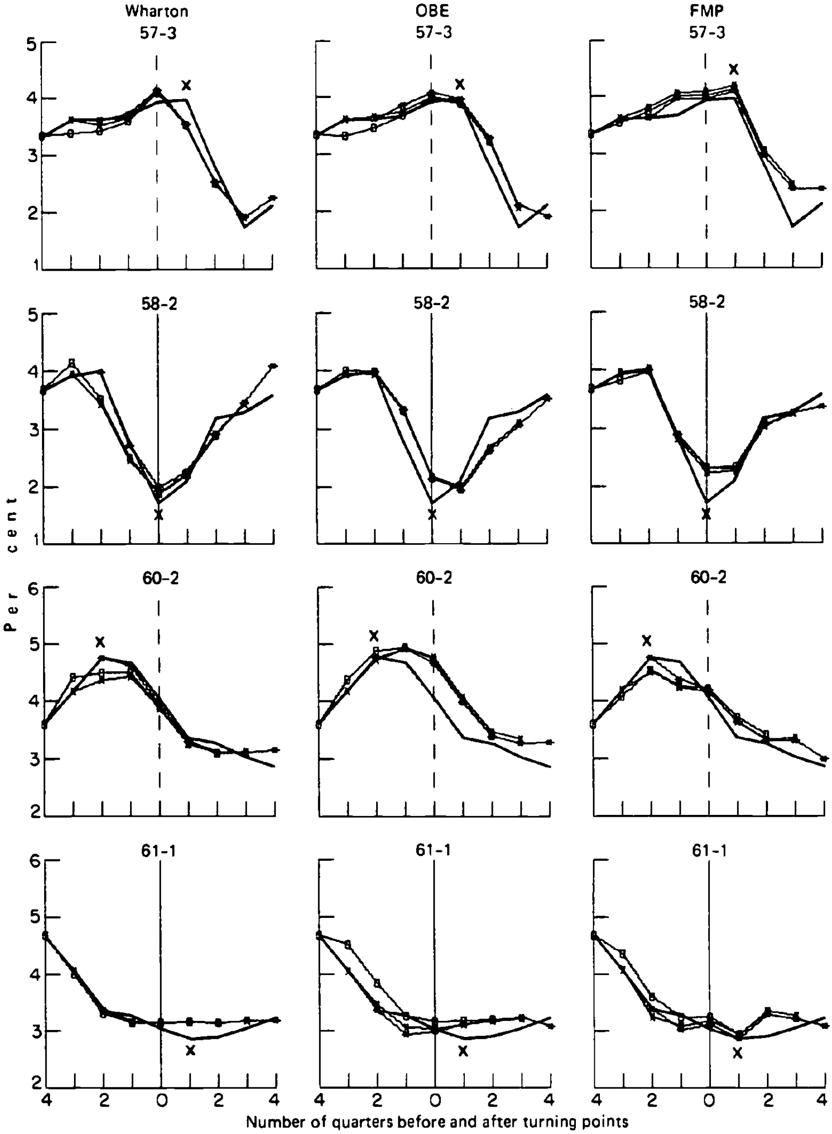


CHART 2.1 *(continued)*

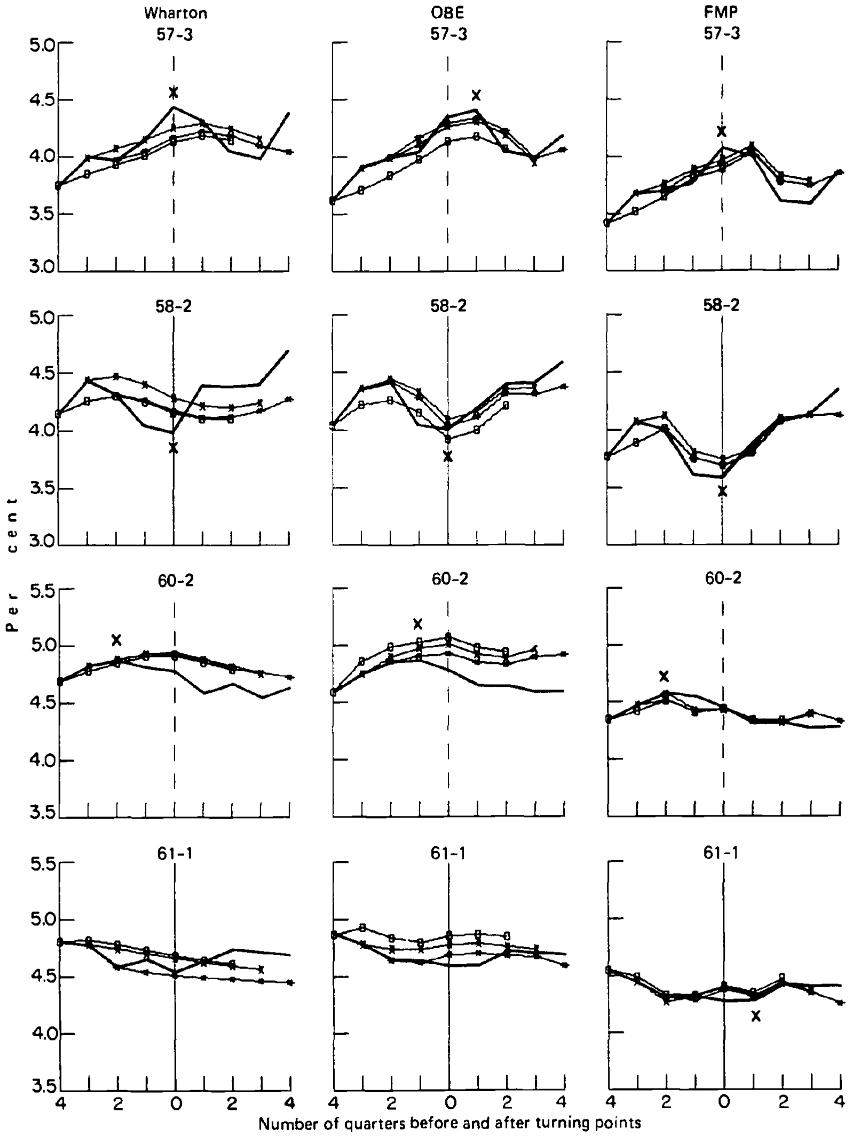
Average Yield, Short Term



(continued)

CHART 2.1 (*concluded*)

Average Yield, Long Term



actual is shown as a solid line; the simulation starting three quarters before the turn is delineated by boxes (☐); the simulation starting two quarters before the turn by x's; and the simulation starting one quarter before the turn by asterisks. In each case, the simulation results are connected to the last available actual observation. Turning points in the actuals are marked by X's.

Our investigation of the incidence of successfully simulated turns must begin with the determination of cyclical turning points. This task, never an easy one, is complicated by several circumstances. First, the data refer to short, nonconnected periods, making it impossible to use existing computer programs. Second, the shortness of the simulation period makes it difficult to decide whether observed changes in direction are cyclically significant or merely reflect random movements of short duration. This is particularly difficult whenever the suspected turns fall close to the beginning or the end of the six-quarter period, causing the observable part of one of the phases to become very short indeed. Third, it is difficult to determine whether the observable part of the amplitude of a particular movement is large enough for it to qualify as a cyclical phase. We resolved these problems largely by deciding in favor of recognizing turns if this seemed at all reasonable.

The determination of turning points for the six-quarter simulations may require consideration of events outside this period. This gives rise to a puzzling problem: if actual series experience turning points before or after the six-quarter period chosen for simulation, should a simulation be considered successful when it shows a turn because the actual series experiences a turn – albeit not within the six-quarter period? Or should it be considered successful when it does not show a turn, because during the simulation period the actual shows no turn either? We circumvented this problem by using two types of counts: the simplest way is to take only those instances of the actual series where cyclical turns occurred in the six-quarter period under observation and match them with turns in the simulations. Alternatively, we used an increased sample by utilizing our knowledge of the behavior of the actual series shortly before and after the simulation period. Thus, if the actual series showed a peak shortly before the beginning of the simulation period and the simulated series continued downward, the simulation was presumed to have produced a peak.[3] Unfortunately, this procedure has to be modified for series with long lags extending beyond the end of the simulation period. In such situations, only those simulations which still showed a turn in the sample period could be recognized as successful. No assumptions could be made about simulated turns that might occur beyond the sample period. The incidence of successfully simulated turns is summarized in Table 2.1. The upper panel refers to the larger sample, including the inferred prior turning points; the lower panel refers only to those turns which actually occurred in the six-quarter period.

On the whole, all three models were fairly successful in reproducing cyclical turns in the neighborhood of business cycle turns, particularly when the more liberal definition—which includes inferred turning points—was used. In almost all cases, the simulated series successfully showed "no turns" whenever there were no turns in the actual series. We, therefore, based the "attainment rate" for the simulation of cyclical reversals on the number of actual turning points.

We expected simulations starting one quarter before reference turns to reproduce more turning points than those starting two or more quarters before. We had this expectation because we thought that one quarter before a reference turn the cumulative strains before peaks, and the gradual restoration of profits before troughs, are included in the data used in the simulation. Conversely, data truncated as much as

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[3] Take, for example, the unemployment rate at the 1957 reference peak (Chart 2.1). The actual series has a trough in the first quarter of 1957, i.e., two quarters before the reference peak. For the Wharton Model (furthest left in the chart) the simulation starting in 1956-IV, i.e., three quarters before the turn, also shows a trough in 1957-I. This is unproblematic. For the simulation starting two quarters before the reference turn, a trough can be found in the first simulated observation if we connect the simulations with the last available actual before the start of the simulation. This trough is still located within the six-quarter simulation period and is counted as a turn in the larger, as well as in the more restrictive, sample. The last simulation, however, starts in 1957-II, i.e., one quarter after the trough in the actual. Although one can infer the existence of a trough from the upward direction of the simulated series, it did not occur during the simulation period. Hence, this simulation is counted as having successfully reproduced a turning point in the larger sample, but neither the actual nor the simulated turn is counted in the sample which is restricted to the six-quarter period.

The case is still more complicated for a configuration like that of the OBE simulations of this series at the same turning point (upper panel, middle row). Here all three of the simulations are higher than the last actual, so that in each case the inferred trough occurs in the quarter before the simulation starts. All are counted in the extended sample; none of the simulations and only the first two actuals are counted in the smaller sample.

TABLE 2.1

Nonstochastic Six-Quarter Simulations: Frequencies of Actual and Simulated Turning Points

		Wharton (1949–61)			OBE (1954–61)			FMP (1957–61)		
		Actual	Simulation		Actual	Simulation		Actual	Simulation	
	Start of Simulations	Number (1)	Number (2)	Per Cent of Actual (3)	Number (4)	Number (5)	Per Cent of Actual (6)	Number (7)	Number (8)	Per Cent of Actual (9)
	All turns[a]									
1	3 Q before reference turns	95	64	67	85	57	67	50	38	76
2	2 Q before reference turns	95	62	65	85	59	69	50	44	88
3	1 Q before reference turns	95	57	60	85	63	74	50	45	90
	Turns occurring in 6-quarter periods									
4	3 Q before reference turns	83	59	71	71	45	63	47	33	70
5	2 Q before reference turns	79	53	67	64	43	67	46	37	80
6	1 Q before reference turns	73	35	48	60	37	62	41	32	78

[a] Including the inferred prior turning points in simulations, corresponding to the known actual turns that occurred outside the simulation period. (*See text.*)

three quarters before changing business conditions should not be expected to reflect, to the same extent, the dynamic processes preceding cyclical turns. As it turns out, these expectations have not generally been met, either by the six-quarter periods (lower panel), or by the extended sample periods (upper panel). The reason may be as follows: as the beginning of the six-quarter period approaches the reference turn, fewer specific turning points occur, both in the actual series and in the simulated ones. This is because more specific turns occur two quarters before reference turns (the quarter omitted as we go from the second to the third simulation) than four quarters after reference turns (the quarter added). This is true for the actual series (see Table 2.1, second panel, columns 1, 4, and 7), as well as for the simulated series (columns 2, 5, and 8). Whenever the simulated series lead the actuals, as seems to be the prevailing tendency for the Wharton Model (see Section 2.2 below), more turns get lost in the simulations than in the actual. For the extended sample period, no turns are actually cut off in the beginning by shifting the sample period. All the same, it seems that for the Wharton Model, simulations starting one quarter before peaks showed fewer turns (actual or imputed) than those starting two quarters before. One series for which this happens rather consistently is the average workweek, which is known to be a leading series.

We also expected the simulations to be more successful in reproducing troughs than in reproducing peaks. The reasons for this expectation are basically that the contours of troughs are often more sharply defined, and that the turning points are more closely clustered around reference troughs than around reference peaks. This can be observed particularly during the postwar period and is largely due to long-term upward trends, government intervention to end recessions, the absence of "drag" factors (such as backlogs, contractual obligations, gestation periods—as they exist at peaks), and the rapid expectational changes, based on these elements. We thought that these characteristics might be reflected in the structure of the models and would certainly be imposed upon the simulations by actual reversals in the exogenous variables—much more decisively than in the neighborhood of reference peaks. We expected that all of these elements would increase the likelihood of reversals in the neighborhood of troughs falling within our six-quarter simulation period.

TABLE 2.2

Nonstochastic Six-Quarter Simulations: Incidence of Successfully Simulated Peaks and Troughs
(*frequencies as per cent of actual turns*)

		Wharton Model			OBE Model			FMP Model		
	Start of Simulations	Troughs	Peaks	All Turns	Troughs	Peaks	All Turns	Troughs	Peaks	All Turns
				All turns[a]						
1	3 Q before reference turns	65	71	67	69	65	67	65	88	76
2	2 Q before reference turns	65	66	65	69	71	69	92	83	88
3	1 Q before reference turns	70	46	60	75	74	74	100	79	90
				Turns occurring in 6-quarter periods						
4	3 Q before reference turns	64	83	71	69	50	63	62	81	70
5	2 Q before reference turns	61	77	67	79	57	67	85	75	80
6	1 Q before reference turns	50	44	48	55	81	62	92	53	78

[a] Including the inferred prior turning points in simulations corresponding to the known actual turns that occurred outside the simulation period. (*See text.*)

TABLE 2.3

Nonstochastic Six-Quarter Simulations: Relative Frequency of Inferred Prior Peaks and Troughs (as per cent of all turns)[a]

	Start of Simulations	Wharton Model (1)	OBE Model (2)	FMP Model (3)
	At reference troughs			
1	3 Q before troughs	3	3	6
2	2 Q before troughs	14	11	8
3	1 Q before troughs	37	37	8
	At reference peaks			
4	3 Q before peaks	14	50	19
5	2 Q before peaks	15	50	25
6	1 Q before peaks	42	48	58
	At all reference turns			
7	3 Q before turns	8	21	13
8	2 Q before turns	15	27	16
9	1 Q before turns	39	41	29

[a] The inferred prior turning points in simulations (*see text and Table 2.1, note a*) are expressed as percentages of all turning points in the simulations (as listed in Table 2.1, lines 1 to 3, columns 2, 5, and 8).

However, the evidence summarized in Table 2.2 does not show any systematic difference between successful duplication of peaks and of troughs.[4] Perhaps we should investigate to what extent this may reflect the failure of the models to distinguish the difference in cyclical dynamics in the neighborhood of peaks and troughs; and to what extent it portrays the relative weakness of the exogenous variables in imposing characteristic cyclical behavior on the simulation patterns.

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[4] There is a facet of this experiment where the wider spread of turning points in the neighborhood of peaks does cause the six-quarter periods before troughs to differ from the six-quarter periods before peaks: for the actual, as well as the simulated, series more turns had to be inferred before peaks than before troughs, relative to the turns actually occurring in the six-quarter period. (*See Table 2.3.*)

Although the three models cannot be directly compared—for the reasons stated in Section 1.3 above—it must be noted that in the matter of reproducing turning points, the FMP Model has higher attainment rates. The reason for this is not that fluctuations are more easily simulated for the period 1957–61, since the better performance of the FMP Model is retained even when comparisons for the three models are based on this shorter period. Nevertheless, it is possible that the better performance is caused, at least partly, by the better fits associated with a shorter sample period.

2.2 TIMING COMPARISONS

For Tables 2.4 and 2.5, timing comparisons between the turning points in the actual and the simulated time-series are based on observed cyclical turns falling into the six-quarter periods only. That is, we neglect the instances in which turns falling outside those periods were inferred.

The majority of turns in the simulated series occurred within one quarter of those in the actual series. Table 2.4 shows the percentages of turns of the simulated series which coincided with turns in the actual series, those one quarter away, and those yet further away from turns in the actual series.

The relative frequency of simulated turns outside the three-quarter span—centered around the turn in the actual—is lowest in the FMP simulations and highest in the Wharton Model, both for peaks and for troughs.

Given the smaller dispersion of turning points in the neighborhood of troughs, we would have expected simulated turns at troughs to be closer to the actual ones than simulated peaks are to actual peaks. The evidence does not show any systematic difference in performance, perhaps because the simulation period is so short that long leads and lags (which are more frequent at peaks) fall outside the span of observation. Alternatively, the unexpected similarity of behavior at peaks and troughs may be a consequence of the constancy of the lag structure used by the models.

The incidence of leads and of lags is different for the three models.

TABLE 2.4

Nonstochastic Six-Quarter Simulations, Timing Relationship of Cyclical Turns in Simulated Relative to Actual Series, Per Cent of Leads, Lags and Coincidences[a]

		Troughs			Peaks			All Turns		
	Start of Simulations	Coinci-dences (1)	Leads & Lags of 1 Q (2)	Longer Leads & Lags (3)	Coinci-dences (4)	Leads & Lags of 1 Q (5)	Longer Leads & Lags (6)	Coinci-dences (7)	Leads & Lags of 1 Q (8)	Longer Leads & Lags (9)
					Wharton Model, 1949–1961					
1	3 Q before turn	21	32	47	14	50	36	18	39	43
2	2 Q before turn	14	57	39	20	55	25	17	56	27
3	1 Q before turn	21	54	25	55	27	18	31	46	32
					OBE Model, 1954–1961					
4	3 Q before turn	34	49	17	18	46	36	30	48	22
5	2 Q before turn	41	47	12	31	46	23	38	47	15
6	1 Q before turn	59	30	11	34	60	6	50	40	10
					FMP Model, 1957–1961					
7	3 Q before turn	44	50	6	50	38	12	47	44	9
8	2 Q before turn	46	45	9	57	36	7	50	42	8
9	1 Q before turn	33	63	4	50	38	12	38	56	6

[a] In each line, figures add up to 100 in columns 1–3, 4–6, and 7–9 (subject to rounding errors).

The Wharton Model shows a clear preponderance of leads in the turns of simulated series compared to those in the actual ones; the FMP Model shows more lags than leads; and the OBE Model does not show any systematic preference for leads or lags (leads being more frequent in the simulations starting three quarters and two quarters before reference turns, with lags predominating in the simulations starting one quarter before). The numerical findings are given in Table 2.5.

One important question is whether the models generate simulated time-series which exhibit the same general timing characteristics as those shown by the actual variables. In other words, do turning points in the simulated leading series lead business-cycle turning points? Do coinciders coincide and laggers lag? The series included in each model were classified into leaders, coinciders, and laggers, according to their historical performance.[5] For each group, leads (including inferred leads), coincidences, and lags—expressed as a percentage of all turns that can be matched with business-cycle turns—are shown in Table 2.6. The evidence reveals a distinct bias toward early turns in the simulated series. For the Wharton Model, the majority of the simulated series in all three groups lead at reference turns. In the OBE Model, most of the simulated leading and coinciding series lead, while lagging series show a tendency to coincide. The FMP Model generally produces more leads than lags, but the bias is a little less strong. The percentage of leaders in the simulated leading series is actually smaller than that in the actual leading series.

The bias toward leads is substantially reduced if we exclude turns before the start of the six-quarter period (actual turns as well as inferred turns of the simulations). The evidence, presented in Table 2.7, shows that for the actual series only those classified as leaders are seriously affected by this exclusion. The proportion of leads for this group is reduced from about two-thirds (if all turns are considered) to about half (if only those in the six-quarter period are counted). Since few long leads—i.e., turns before the start of the six-quarter simulation

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[5] Wharton Model, leading series: *IH*. *II*, *CPR*, *AWW*, *UMD*: coinciding series: *GNP*, *GNP58*, *C*, *YP*, *LE*, *UN*; lagging series: *ISE*, *RS*, *RL*. OBE Model, leading series: same as Wharton, plus *OMD*, *HS*, and *M*; coinciding series: same as Wharton: lagging series: same as Wharton plus *LC/O*. FMP Model, leading series: *IH*, *II*, *CPR*, *LH* and *OUME*; coinciding series: *GNP*, *GNP58*, *C*, *Y*, *LE*, *UN*: lagging series: same as Wharton.

TABLE 2.5

Nonstochastic Six-Quarter Simulations: Leads, Coincidences, and Lags of Cyclical Turns in Simulated Series Relative to Those in Actual Series

	Start of Simulations	Total, All Comparable Turns[a] (1)	Leads		Coincidences		Lags	
			Number (2)	Per Cent of Total (3)	Number (4)	Per Cent of Total (5)	Number (6)	Per Cent of Total (7)
			Wharton Model, 1949–1961					
1	3 Q before turn	56	33	59	10	18	13	23
2	2 Q before turn	48	27	56	8	17	13	27
3	1 Q before turn	35	15	43	11	31	9	26
			OBE Model, 1954–1961					
4	3 Q before turn	46	21	46	14	30	11	24
5	2 Q before turn	45	18	40	17	38	10	22
6	1 Q before turn	42	9	21	21	50	12	29
			FMP Model, 1957–1961					
7	3 Q before turn	32	8	25	15	47	9	28
8	2 Q before turn	36	5	14	18	50	13	36
9	1 Q before turn	32	6	19	12	37	14	44

[a] Refers to number of turning points in the six-quarter periods which occurred in the actual series as well as in the simulated series. It may be less than the total number of simulated turns shown in Table 2.1, since some turns in the simulated series cannot be matched with actual turns, either because there was no turn in the actual series, or because the turn occurred outside the six-quarter period.

TABLE 2.6

Nonstochastic Six-Quarter Simulations: Timing Relative to Business-Cycle Turns in Leading, Coinciding, and Lagging Series: Actuals and Simulations (per cent of all turns)[a]

		Wharton Model			OBE Model			FMP Model		
		Leads	Coinci-dences	Lags	Leads	Coinci-dences	Lags	Leads	Coinci-dences	Lags
	Leading series									
	Actuals	67	27	6	74	21	5	59	23	18
	Simulations starting:									
1	3 Q before reference turn	88	4	8	80	10	10	60	27	13
2	2 Q before reference turn	83	4	13	90	3	7	50	22	28
3	1 Q before reference turn	87	0	13	81	3	16	59	12	29
	Coinciding series									
	Actuals	41	46	13	42	46	12	33	52	15
	Simulations starting:									
4	3 Q before reference turn	96	0	4	69	23	8	58	17	25
5	2 Q before reference turn	90	5	5	67	13	20	42	29	29
6	1 Q before reference turn	84	11	5	65	12	23	56	6	38
	Lagging series									
	Actuals	14	29	57	20	20	60	17	50	33
	Simulations starting:									
7	3 Q before reference turn	50	37	13	20	47	33	46	18	36
8	2 Q before reference turn	42	29	29	13	47	40	42	16	42
9	1 Q before reference turn	20	53	27	21	36	43	33	17	50

NOTE: For classification of series see text.

[a] Includes inferred prior turns in simulations (*see text*), and turns occurring outside the 6-quarter simulation period in actuals.

TABLE 2.7

Nonstochastic Six-Quarter Simulations: Timing Relative to Business-Cycle Turns in Leading, Coinciding and Lagging Series; for All Turns and for Turns Occurring in 6-Quarter Period Actuals and Simulations

	Wharton Model			OBE Model			FMP Model		
	Leads	Coinci-dences	Lags	Leads	Coinci-dences	Lags	Leads	Coinci-dences	Lags
				All turns[a]					
Leading series									
Actuals	67	27	6	74	21	5	59	23	18
Simulations	86	3	11	83	6	11	56	20	24
Coinciding series									
Actuals	41	46	13	42	46	12	33	52	15
Simulations	90	5	5	66	16	18	52	17	31
Lagging series									
Actuals	14	29	57	20	20	60	17	50	33
Simulations	37	40	23	18	43	39	40	17	43
				Turns in 6-quarter period					
Leading series									
Actuals	46	44	10	55	36	9	45	31	24
Simulations	79	4	17	70	10	20	33	30	37
Coinciding series									
Actuals	37	49	14	38	50	12	32	53	15
Simulations	88	6	6	55	21	24	47	18	34
Lagging series									
Actuals	12	31	57	15	21	64	12	53	35
Simulations	36	40	24	12	46	42	32	19	49

[a] Includes inferred prior turns in simulations and turns occurring outside the 6-quarter period in actuals.

period—occur in the actual coinciding and lagging series, the percentage distributions for those two groups are not much affected. For the simulated series, on the other hand, long leads were inferred for coincident series, as well as for some lagging ones. Thus, the bias toward leads is somewhat reduced by eliminating the inferred turns from the count. However, even for turns occurring in the six-quarter period only, leads are relatively more frequent in the simulations than in the actuals for all groups except the laggers in the OBE Model and the leaders in the FMP Model.

On the whole, then, the simulations discriminate only very weakly between the historically leading, coinciding, and lagging variables. In view of the importance of lead-lag relationships in economic dynamics, it should be worthwhile to investigate whether there exists any connection between the model formulations and the timing biases observed in the simulations.

2.3 AMPLITUDES

Amplitude measures—in the framework of the present investigation—can obviously describe only those segments of expansions and contractions which occur during the six-quarter simulation periods. In spite of this truncation, it is of interest to establish whether there are systematic differences between the observable portions of simulated and actual amplitudes: among different variables, cyclical phases, and variously timed simulations.

A glance at the charts shows one fact quite clearly: in most cases, the patterns of the simulated series are flatter—i.e., they have amplitudes which are more shallow than those of the actual series. A related finding is that the patterns of simulated phases are more similar to each other than to the actual ones. This is particularly striking for the three time-staggered simulations produced by the same model; but it is also often true across models. The similarity among the time-staggered simulations may be explained by the fact that cyclical conditions vary relatively slowly and, therefore, the initial conditions for the three simulations are fairly similar. Furthermore, forecasts for the first quarter are typically more successful than those for later quarters. The family resemblance of simulations produced by different models may be due to the fact that all models reflect only the systematic portion of cyclical interactions, which during any historical period, represent only a part of economic reality. It is, of course, also possible that the three models have some common biases and that this is the reason why they resemble each other more than they resemble reality.

In order to give some precision to these impressions, amplitudes were computed for the observable part of expansions and contractions in the actual and the simulated series for each simulation that contained a cyclical turning point.[6] As a first step in the analysis, we determined the frequencies with which simulated amplitudes were smaller than, similar to, or larger than, those of the actual series. Differences of one percentage point and less (for *UN*, *II*, *RS*, and *RL*, differences of 10 per cent and less in the absolute differences) were regarded as negligible, and the amplitudes were tabulated as similar. The frequencies are summarized in Table 2.8. For each model, for each type of time-staggered simulation, and for all expansions and all contractions, the frequencies are expressed as a percentage of all comparisons feasible in that class.

On the whole, simulated amplitudes underestimate actual amplitudes more often than they overstate or equal actual amplitudes. As the table shows, this is equally true for expansions and contractions, for each of the time-staggered simulations, and for each model.[7] The one exception occurred in the Wharton Model, for which this tendency is generally somewhat less pronounced. The incidence of underestimation amounts usually to more than half of all cases, except in the Wharton Model, where the incidence of underestimating expansions is only about 40 per cent.

Let us turn from the analysis of incidences to that of measured amplitudes. Table 2.9 presents average expansion and average contrac-

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[6] Since no direct comparisons between expansion and contraction amplitudes were intended, the percentage-base bias of amplitude measures could be neglected, and relative amplitudes could simply be measured as percentage changes from initial levels. In case of rates and differences (for *UN*, *II*, *RS*, and *RL*), absolute changes rather than percentage changes were computed.

[7] It is also true for each cycle, for phases before and after the turn, and for most activities. A minor exception, not shown in the summary table, is the expansion preceding the 1958 peak for the FMP Model, where the simulations overestimated amplitudes for almost all variables.

tion amplitudes of all variables included, irrespective of comparability. The average amplitude measures presented for each variable in each model cover all incidents for which amplitudes could be measured; thus, the composition of the measures is not strictly comparable among the different variables or models. Comparability exists only insofar as there is a corresponding expansion for each contraction, and a corresponding actual phase for each simulation phase. The number of simulations included in each average amplitude is indicated in the table. The comparisons show that for a large majority of variables, the averages of the simulated amplitudes are smaller than those of the actual amplitudes; for about 60 per cent of the possible comparisons, they are more than 20 per cent below the actuals. The Wharton Model simulations seem to underestimate less than the others, but comparisons are difficult to make, because of the heterogeneous composition of the amplitude averages.

In order to increase comparability, we present comparisons only of those cycle phases which could be measured for the same time period and the same well-defined economic process in all models. This attempt to increase comparability from model to model brought about a sharp reduction in sample size. Furthermore, for the sake of simplicity, and in view of the observed similarity among the time-staggered simulations, we used only the simulations starting two quarters before reference turns. The results appear in Table 2.10. Again we find that the incidence of underestimation of amplitudes by the simulations is pervasive, particularly for contractions. The magnitudes of underestimation vary widely. In spite of problems of summarization, due to the large variation in size among the amplitudes themselves, Table 2.11 provides averages for simulated and actual amplitudes, and for absolute and relative differences.[8] Again, the summary measures show the smaller amplitudes of the simulations. Intermodel comparison shows that for the simulations included, the Wharton Model comes very close to actual amplitudes during expansions. During contractions the FMP Model gives closer approximations than do the other two

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[8] Since the average of percentage differences gives a large weight to large percentage differences which may be based on very small amplitudes (see, for instance, the last expansion phase in Table 2.10), we also provide the percentage difference of the average amplitudes (which gives larger weight to large amplitudes, e.g., corporate profits).

models. However, comparisons among models remain very uncertain, even for this less heterogeneous selection, since the sample is small and the differences not very pronounced.

A major analytical interest concerns the reasons for the sweeping tendency toward underestimation of amplitudes shown by most simulations. It has been argued that most of the explanation can be found in

TABLE

Nonstochastic Six-Quarter Simulations: Amplitude
Incidence of Underestimation

		All Phases			
			Per Cent of All Phases Compared		
	Start of Simulations	Number of Phases Compared (1)	$S < A$ (2)	$S \approx A$ (3)	$S > A$ (4)
	Wharton Model, 1949-1961				
1	3 Q before turn	104	36	28	36
2	2 Q before turn	96	51	27	22
3	1 Q before turn	70	44	34	22
4	All simulations	270	44	29	27
	OBE Model				
5	3 Q before turn	90	64	29	7
6	2 Q before turn	84	67	27	6
7	1 Q before turn	76	59	30	11
8	All simulations	250	63	29	8
	FMP Model				
9	3 Q before turn	62	62	19	19
10	2 Q before turn	72	67	15	18
11	1 Q before turn	66	61	27	12
12	All simulations	200	63	21	16

NOTE: S and A denote amplitudes of simulated and actual series, respectively. For *UN*, *II*, *RS*, and *RL*, amplitudes were computed as absolute changes between two turning points; for all other series, as percentage changes from initial levels.

the systemic tendency of regression techniques to underestimate changes. Also, there is some bias inherent in the method used for selecting turning points. Randomly high observations tend to be selected as peaks; and randomly low ones, as troughs—leading to an overstatement of the cyclical component of actual amplitudes. The simulated series, on the other hand, are constructed without imposition of

2.8

Comparisons Between Actual and Simulated Series; and Overestimation

Expansions				Contractions			
Number of Phases Compared	Per Cent of All Compared Phases			Number of Phases Compared	Per Cent of All Compared Phases		
	$S < A$	$S \approx A$	$S > A$		$S < A$	$S \approx A$	$S > A$
(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
52	33	29	39	52	40	27	33
48	52	19	29	48	50	35	15
35	40	37	23	35	49	31	20
135	42	27	31	135	46	31	23
45	65	24	11	45	65	33	21
42	72	21	7	42	62	33	5
38	66	24	10	38	53	37	10
125	67	23	10	125	60	34	6
31	49	19	32	31	75	19	6
36	64	14	22	36	69	17	14
33	67	21	12	33	55	33	12
100	60	18	22	100	66	23	11

TABLE 2.9

Nonstochastic Six-Quarter Simulations: Simulated and Actual Amplitudes for All Simulations with Turning Points; Averages by Variable, Cycle Phase, and Model

I. Wharton Model

Variable	Number of Observations	Expansions[a]				Contractions[a]			
		S	*A*	*S/A*	*S-A*	*S*	*A*	*S/A*	*S-A*
GNP	8	7.2	7.2	1.00	0	−1.2	−1.1	1.09	−0.1
GNP58	12	5.6	6.0	0.93	−0.4	−2.1	−3.9	0.54	+1.8
Consumption	2	7.6	3.7	2.05	+3.9	−0.9	−0.6	1.50	−0.3
Investment in housing	4	9.9	10.4	0.95	−0.5	−5.0	−5.7	0.88	+0.7
Investment in plant and equipment	15	7.2	8.6	0.84	−1.4	−4.9	−7.1	0.69	+2.2
Change in inventory investment	9	6.8	9.6	0.71	−2.8	−5.2	−7.2	0.72	+2.0
Personal income	6	6.0	6.2	0.97	−0.2	−0.9	−0.5	1.80	−0.4
Employment	10	2.4	2.0	1.20	+0.4	−2.4	−1.9	1.26	−0.5
Unemployment (inverted)	11	3.2	2.3	1.39	+0.9	−1.4	−0.7	2.00	+.7
Corporate profits	9	18.0	22.0	0.82	−4.0	−9.1	−18.4	0.49	+9.3
Workweek	4	0.7	0.7	1.00	0	−0.9	−1.2	0.75	+.3
New orders	8	15.3	28.4	0.54	−13.1	−4.6	−9.2	0.50	+4.6
Unfilled orders	9	3.4	9.8	0.35	−6.4	−16.4	−20.6	0.80	+4.2
Interest rate, short	14	0.7	0.8	0.88	−0.1	−0.7	−1.6	0.47	+0.9
Interest rate, long	16	0.1	0.3	0.33	−0.2	−0.1	−.3	0.35	+0.2

II. OBE Model

Variable	Number of Observations	Expansions[a]				Contractions[a]			
		S	*A*	*S/A*	*S-A*	*S*	*A*	*S/A*	*S-A*
GNP	4	6.8	7.8	0.87	−1.0	−0.6	−0.5	1.20	−0.1
GNP58	12	3.5	5.3	0.66	−1.8	−1.0	−2.6	0.38	+1.6
Investment in housing	4	9.7	10.1	0.96	−0.4	−5.2	−5.6	0.93	+0.4
Investment in plant and equipment	11	2.9	6.1	0.48	−3.2	−2.8	−7.0	0.40	+4.2
Change in inventory investment	9	3.6	8.5	0.42	−4.9	−5.1	−9.7	0.53	+4.6
Employment	12	1.0	2.7	0.37	−1.7	−0.6	−1.3	0.46	+0.7
Unemployment (inverted)	6	0.3	1.2	0.25	−0.9	−1.4	−2.2	0.65	+0.8
Corporate profits	9	12.6	27.2	0.46	−14.6	−9.0	−14.9	0.60	+5.9
Workweek	4	0.1	0.6	0.17	−0.5	−0.9	−1.7	0.53	+0.8
New orders	6	1.9	4.5	0.42	−2.6	−18.1	−15.4	1.18	−2.7
Housing starts	4	18.4	23.8	0.77	−5.4	−5.9	−9.6	0.61	+3.7
Interest rates, short	15	0.6	0.7	0.86	−0.1	−1.5	−1.6	0.94	+0.1
Interest rates, long	12	2.7	3.2	0.84	−0.5	−2.5	−3.4	0.74	+0.9
Labor cost per unit of output	3	1.5	3.0	0.50	−1.5	−0.6	−0.3	2.00	−0.3
Money	3	2.9	3.3	0.88	−0.4	−0.2	−0.7	0.29	+0.5

(continued)

TABLE 2.9 (*concluded*)

III. FMP Model

Variable	Number of Observations	Expansions[a]				Contractions[a]			
		S	*A*	*S*/*A*	*S*-*A*	*S*	*A*	*S*/*A*	*S*-*A*
GNP	5	6.5	8.0	0.81	−1.5	−.6	−1.4	0.43	+0.8
GNP58	11	3.4	4.5	0.76	−1.1	−1.1	−2.6	0.42	+1.5
Consumption	8	4.0	4.0	1.00	0	−.4	−1.1	0.36	+0.7
Investment in housing	3	5.4	8.5	0.64	−3.1	−8.9	−6.8	1.31	−2.1
Investment in plant and equipment	11	2.9	4.7	0.62	−1.8	−7.4	−9.9	0.75	+2.5
Change in inventory investment	7	4.6	10.4	0.44	−5.8	−4.3	−8.7	0.49	+4.4
Employment	9	1.0	1.2	0.83	−0.2	−.9	−1.6	0.56	+0.7
Unemployment (inverted)	5	.4	1.3	0.31	−0.9	−2.8	−2.2	1.27	−0.6
Corporate profits	8	14.7	24.6	0.60	−9.9	−12.0	−18.6	0.65	+6.6
Workweek	6	.4	.8	0.50	−0.4	−.8	−1.0	0.80	+0.2
Unfilled orders	7	3.0	5.3	0.57	−2.3	−9.5	−12.2	0.78	2.7
Interest rates, short	11	.6	.8	0.75	−0.13	−1.4	−1.8	0.78	+0.4
Interest rates, long	9	.3	.4	0.75	−0.08	−.3	−.4	0.75	+0.1

[a] *S* and *A* denote amplitudes of simulated and actual series respectively. For *II*, *UN*, *RS* and *RL*, amplitudes were computed as absolute changes between two turning points; for all other series, as percentage changes from initial levels.

TABLE 2.10

Simulated and Actual Amplitudes, Comparable Phases Only

Variable	Turn	Wharton Model				OBE Model				FMP Model			
		S	*A*	*S*/*A*	*S*-*A*	*S*	*A*	*S*/*A*	*S*-*A*	*S*	*A*	*S*/*A*	*S*-*A*
						Expansions							
Investment in housing (*IH*)	T 61-II	14.5	8.9	1.63	+5.6	10.1	8.7	1.16	+1.4	5.4	9.2	0.59	−3.8
Investment in plant and equipment (*IP*)	T 58-II	5.0	7.3	0.68	−2.3	1.1	4.2	0.26	−3.1	0.2	4.2	0.05	−4.0
	P 60-III	5.1	6.8	0.75	−1.7	3.1	5.5	0.56	−2.4	4.2	6.2	0.68	−2.0
	T 61-II	7.8	6.2	1.26	+1.6	1.7	4.5	0.38	−2.8	0.5	4.7	0.11	−4.2
Change in inventory investment (*II*)	T 58-II	4.7	10.2	0.46	−5.5	4.0	9.9	0.40	−5.9	3.7	10.1	0.37	−6.4
	P 60-III	6.3	9.2	0.68	−2.9	6.2	9.1	0.68	−2.9	7.4	9.5	0.78	−2.1
	T 61-II	6.4	9.1	0.70	−2.7	1.8	8.9	0.20	−7.1	1.7	9.0	0.19	−7.3
Employment (*LE*)	T 58-II	2.4	2.1	1.14	+0.3	0.6	1.9	0.31	−1.3	0.4	1.9	0.21	−1.5
Corporate profits (*PCB*)	T 58-II	16.6	38.5	0.43	−21.9	13.5	38.5	0.35	−25.0	20.2	39.9	0.51	−19.7
	T 61-II	30.6	23.1	1.32	+7.5	10.7	23.1	0.46	−12.4	12.6	23.5	0.54	−10.9
Interest rates, short (*RS*)	P 57-III	0.59	0.36	1.64	+0.23	0.37	0.36	1.03	+0.01	0.56	0.36	1.56	+0.26
	T 58-II	1.57	1.58	0.99	−0.01	1.11	1.64	0.68	−0.53	1.03	1.59	0.65	−0.50
	P 60-II	0.26	0.58	0.46	−0.31	0.73	0.57	1.28	+0.16	0.34	0.57	0.60	−0.23
Interest rates, long (*RL*)	P 57-III	0.30	0.47	0.64	−0.17	0.41	0.52	0.79	−0.11	0.41	0.39	1.05	+0.02
	T 58-II	0.04	0.42	0.10	−0.38	0.27	0.41	0.66	−0.14	0.40	0.55	0.73	−0.15
	P 60-II	0.11	0.05	2.20	+0.06	0.26	0.12	2.17	+0.14	0.04	0.10	0.40	−0.16

(*continued*)

TABLE 2.10 (*concluded*)

Variable	Turn	Wharton Model				OBE Model				FMP Model			
		S	*A*	*S*/*A*	*S*-*A*	*S*	*A*	*S*/*A*	*S*-*A*	*S*	*A*	*S*/*A*	*S*-*A*
						Contractions							
Investment in housing (*IH*)	T 61-II	−3.7	−6.0	0.62	+2.3	−5.9	−5.9	1.00	0	−8.1	−6.3	1.29	−1.2
Investment in plant and equipment (*IP*)	T 58-II	−5.6	−20.1	0.28	+14.5	−7.7	−15.6	0.49	+7.9	−13.3	−15.6	0.85	+2.3
	T 60-III	−5.8	−5.5	1.05	−0.3	−0.6	−5.7	0.11	+5.1	−2.6	−5.9	0.44	+3.3
	T 61-II	−6.2	−7.3	0.85	+1.1	−0.4	−6.3	0.06	+5.9	−6.0	−6.5	0.92	+0.5
Change in inventory investment (*II*)	T 58-II	−3.4	−8.9	0.38	+5.5	−4.7	−8.6	0.55	+3.9	−4.8	−8.8	0.55	+4.0
	P 60-III	−8.0	−13.5	0.59	+5.5	−4.6	−13.0	0.35	+8.4	−7.7	−13.7	0.56	+6.0
	T 61-II	−5.5	−7.4	0.74	+1.9	−2.9	−7.2	0.40	+4.3	−4.4	−7.5	0.59	+3.1
Employment (*LE*)	T 58-II	−1.0	−2.6	0.38	+1.6	−0.8	−2.3	0.35	+0.5	−1.0	−2.3	0.43	+1.3
Corporate profits (*PCB*)	T 58-II	−1.5	−20.7	0.07	+19.2	−11.3	−20.7	0.55	+9.6	−15.0	−22.5	0.67	+7.5
	T 61-II	−10.1	−12.8	0.79	+2.7	−4.1	−12.8	0.32	+8.7	−8.1	−12.9	0.63	+4.8
Interest rate, short (*RS*)	P 57-III	−2.22	−2.27	0.98	+0.05	−1.93	−2.27	0.85	+0.34	−1.73	−2.27	0.76	+0.54
	T 58-II	−2.07	−2.27	0.91	+0.20	−1.96	−2.27	0.86	+0.31	−1.78	−2.28	0.78	+0.50
	P 60-II	−1.34	−1.74	0.77	+0.40	−1.58	−1.75	0.90	+0.17	−1.18	−1.75	0.67	+0.57
Interest rate, long (*RL*)	P 57-III	−0.14	−0.46	0.30	+0.32	−0.36	−0.43	0.86	+0.06	−0.31	−0.49	0.63	+0.18
	T 58-II	−0.27	−0.46	0.59	+0.19	−0.35	−0.42	0.83	+0.07	−0.39	−0.49	0.86	+0.10
	P 60-II	−0.18	−0.33	0.55	+0.15	−0.12	−0.28	0.43	+0.16	−0.19	−0.30	0.63	+0.11

NOTE: *S* and *A* denote amplitudes for simulated and actual series respectively. For *RS* and *RL*, amplitudes are computed as absolute changes between two turning points; for all other series, as percentage changes from initial levels.

TABLE 2.11

Nonstochastic Six-Quarter Simulations: Summary Amplitude Comparisons Between Simulated and Actual Series; 16 Comparable Phases Only

	Simulations	Actuals	$\frac{\text{Avg } S}{\text{Avg } A}$	$\text{Avg}\left(\frac{S}{A}\right)$
Expansions				
Wharton	6.39	7.80	0.81	0.99
OBE	3.50	7.37	0.47	0.71
FMP	3.69	7.61	0.48	0.56
Contractions				
Wharton	−3.56	−7.02	0.50	0.61
OBE	−3.08	−6.59	0.47	0.55
FMP	−4.78	−6.48	0.74	0.70

NOTE: For explanation of symbols, see footnote to Table 2.10.
SOURCE: Table 2.9.

random factors and thus their cyclical highs and lows are not "exaggerated" by such components.[9] It is true, however, that the described distortion of amplitudes by random elements is weaker for quarterly series than for monthly series, and is, perhaps, not likely to constitute a major part of the explanation of the observed underestimation of amplitudes.

In view of the weakness of the suggested explanation, the tendency of the models to underestimate amplitudes requires further investigation.

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[9] The effect of random factors on amplitude measures could be tested by imposition of random elements.

3 NONSTOCHASTIC SIMULATIONS FOR THE SAMPLE PERIODS

THERE are three main sections in this part of our report, one for each of the models covered. The same general format is used in each section, the material being organized around four tables that show respectively: (1) the average absolute and relative errors of the simulated series; (2) selected regression and correlation statistics summarizing the relations between the simulated and the actual changes; (3) comparisons of average cyclical amplitudes; and (4), comparisons of the cyclical timing of the simulated and actual series. A summary section concludes this analysis.

To compile the measures included in (3) and (4), dates of cyclical turning points had to be identified in all of the simulated series. For the sample-period data, this was done by the NBER computer program for the determination of cyclical turning points, and checked independently by at least two of the co-authors of this paper, who then jointly resolved any judgmental discrepancies involved. The process involved a careful examination of time-series charts. These charts, although very useful for the analysis that follows, are too numerous to be fully reproduced here; however, we do show them for a subset of selected variables at the beginning of each of the three main sections.

In addition, individual and average reference-cycle patterns are discussed for all of the actual (*A*) and simulated (*S*) series. Again, illustrations are provided in charts for the selected variables.

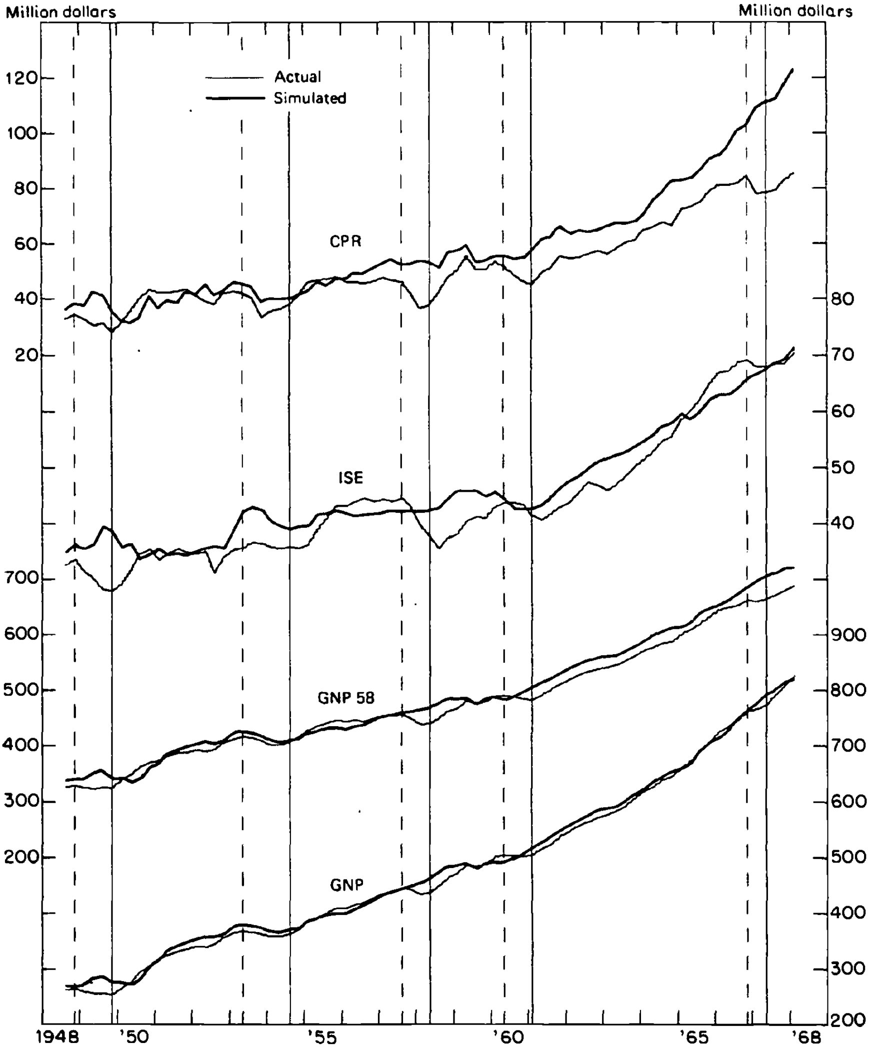
3.1 THE WHARTON MODEL

3.1.1 The extended sample-period simulations for this model embrace 79 quarters (from 1948-III through 1968-I) and include all four of the postwar contractions, as well as such milder retardations as those of 1962–63 and 1966–67.

As shown in Chart 3.1, the simulated *GNP* series runs more often above than below the actual series; but such differences are much less systematic here than they are for the series in constant dollars. The

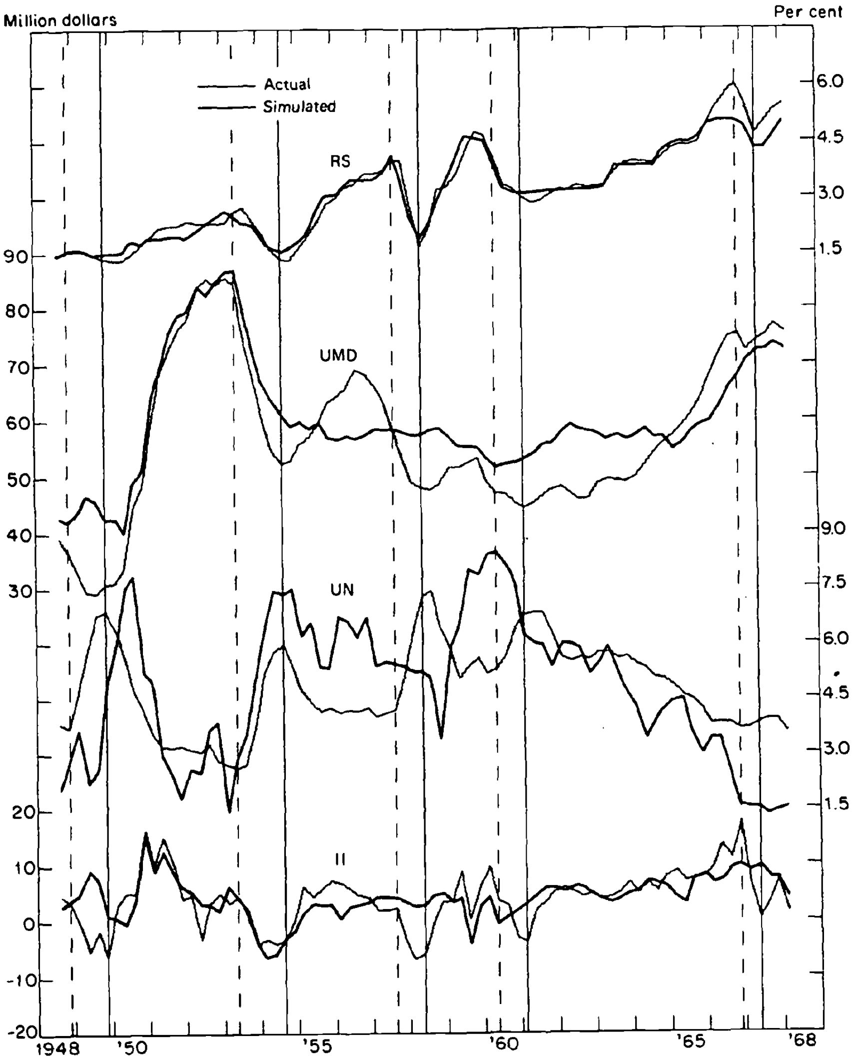
CHART 3.1

Nonstochastic Simulations for the Sample Period, Wharton Model: Simulated and Actual Series for Selected Variables (1948-III-1968-I)



(continued)

CHART 3.1 (*concluded*)



simulated *GNP58* figures exceed the corresponding actual levels, except for relatively brief stretches of time in 1950–51, 1955–56, and 1960. For real consumption (not shown in Chart 3.1), the levels of *S* exceed those of *A* throughout. Consistent with these results, the *S* series for the *GNP* price deflator (also not shown) runs first slightly above the *A* series in 1949–56, and then increasingly below it in 1957–68. Substantial and persistent level discrepancies of a similar sort can also be observed for such other "real" variables as plant and equipment investment (*ISE*), net exports (*NE*), and employment (*LE*).

For several variables with large cyclical and irregular fluctuations and relatively weaker trends, the outstanding feature of the charts is that the variations in *S* tend to be smaller than those in *A*, often by large amounts. The best examples of this are furnished by the investment series, particularly in housing and inventories, and by profits and unfilled orders (*IH*, *ISE*, *II*, *CPR*, *UMD*). On the other hand, opposite or mixed results are obtained in this respect for the unemployment rate and the hours worked in industry (*UN*, *AWW*). There is a very satisfactory, close comovement of *S* and *A* for the short-term interest rate (*RS*), except only in 1966–67 when *S* underestimated both the level and change of *A*. (*RS* is determined only by exogenous variables which describe the policy of the FRB.) For the long-term rate (*RL*), however, *S* looks very much like a heavily smoothed and lagged version of *A*, a frequent effect of relating a series to its own previous value (which is done here via the familiar Koyck transformation, with the aim of making *RL* a function of present and past values of *RS*).

On the average, of course, variations in *S* must be expected to be smaller than those in *A*, because the (nonstochastic) simulations do not include the component of random disturbances that is present in the actuals. Single-equation estimates for the sample period would be entirely consistent with this expectation; the complete-model simulations need not conform to it quite as well, because here the errors from different equations can interact, becoming magnified in various ways, both across the model and over time. For the same reason, the model simulations can show persistent drifts away from the course of the actual series, in contrast to the single-equation short-period "predictions," in which such bias is precluded by the estimation method itself. (This point applies only to the sample-period simulations.)

3.1.2 The prevalence of positive mean errors of levels (MEL) shown in Table 3.1, column 1, suggests that, on the average, overestimates outweigh underestimates in the Wharton simulations for the sample period 1948–68 (the errors are defined as differences $S_t - A_t$). The opposite errors of level underestimation prevail for the price, wage, and interest variables, as well as for personal income and unemployment (*P*, *W*, *RS*, *RL*, *YP*, and *UN*).

The MEC figures in column 2 of Table 3.1 suggest that increases in *GNP*, *P*, and *LE* (employment) were on the average underestimated, and that increases in *GNP58*, *C*, and *YP* were on the average overestimated. For variables with less pronounced trends and strong fluctuations, however, the signs of the MEC's cannot be interpreted so simply.[10] A very conspicuous and uniform characteristic of the MEC statistics is that they are much smaller absolutely than the corresponding MEL figures (compare columns 1 and 2). This reflects two facts: (1) the errors of change are typically much smaller than the errors of levels; (2) the errors of change vary in sign more than the errors of levels. Consequently, the cancellation of positive against negative figures has stronger effects on MEC than on MEL.

As usual, the mean absolute errors of levels and changes (MAEL and MAEC) are generally much larger (disregarding sign) than the corresponding mean errors, for here the positive and negative errors are not permitted to cancel each other (compare column 1 with column 3, and column 2 with column 4).[11] For all variables, the MAE figures are larger for levels than for changes; that is, MAEL > MAEC (see

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[10] Errors of absolute change (defined as $\Delta S_t - \Delta A_t$) are negative when: (1) increases are underestimated; (2) decreases are overestimated; or (3) the actual change is positive and the predicted one is negative. They are positive in the three converse cases (*cf.* [29, p. 51]). Thus the sign of the mean change error (MEC in column 2) does not in itself indicate whether changes have been under- or overestimated. For example, if both *A* and *S* were positive and rising, a negative MEC would denote understatement of the actual increases by the simulated series, while a positive MEC would indicate overstatement. Should both *A* and *S* be positive and declining, then the situation would be precisely the reverse. For series that fluctuate, the outcome will depend on the relative timing, durations, and amplitudes of rises and falls in *A* and *S*; for series that assume negative as well as positive values, it will also depend on the extent to which *S* and *A* agree in sign.

[11] Arithmetic means tell us something about the bias which occurs when a set of predictions typically understates or overstates the corresponding actual values. A set *S*, for example, is unbiased if the mean error ($\bar{E} = \bar{S} - \bar{A}$, where $\bar{S}$ and $\bar{A}$ are the simple averages of S_t and A_t, respectively, over the entire period covered) is not significantly different from zero (*cf.* [24, pp. 8–10]). If $\bar{E} \neq 0$, then there is a constant, common ele-

columns 3 and 4). Of the two factors that were identified above as accounting for an analogous relationship between the mean errors ($|\text{MEL}| > |\text{MEC}|$), only the first one applies here: the absolute errors of changes are typically much smaller than the absolute errors of levels.

The fact that the average errors are larger for levels than for changes is a familiar result from the analysis of forecasts, having a twofold explanation: (*a*) the error of each level forecast is the algebraic sum of the error in the base of the forecast (in the estimate of the preceding level of the series) and the error of the predicted change; (*b*) on the average, errors of the base have the same signs as the errors of the succeeding levels (e.g., if the prediction for period t is too low, that for period $(t + 1)$ is likely to be so, too). It is therefore not the common finding that $|\text{MEL}| > |\text{MEC}|$ and MAEL > MAEC that is particularly interesting but, rather, the fact that the differences involved are as large and pervasive as Table 3.1 shows them to be. The explanation is again simple but also important. When forecasts are made frequently over short spans, the errors of base values can be, and usually are, kept relatively small, so that the level errors are not very much larger than the change errors. In multiperiod forecasts made over long spans of time, however, errors for successive periods can cumulate and the predictions can be increasingly off base. Complete-model simulations over entire sample-periods, in which the errors of lagged dependent variables can cumulate, are comparable to very long multiperiod forecasts in this respect.

For most of the aggregative variables under study, quarterly changes are typically small relative to the levels of the series; hence, simulation errors may well be small when compared with the levels, but large when compared with the changes in the realizations. Typi-

ment in all errors for the given set. When $\bar{E}$ is subtracted from each observed error, the remainder reflects the variation among the errors measured from this average. The mean square error, which is the sum of the bias component ($\bar{E}^2$) and the variance of error (S_e^2), offers a comprehensive and mathematically convenient summary measure, which has been computed, and could be used here. (By taking the root of this MSE figure, an average is obtained which has the proper dimension, being expressed in the same units as the errors to be summarized.) However, for present purposes, it will be sufficient to use the simpler measure of the mean absolute error (MAE). The MAE figures are as a rule smaller than the root mean-square errors (RMSE) statistics for the same samples, since RMSE gives more than proportionate weight to large, as compared with small, errors.

TABLE 3.1

Nonstochastic Simulations for the Sample Period, Wharton Model: Average Errors and Their Ratios to Average Actual Values (1948-III–1968-I)

	Variable		Mean Error (ME)		Mean Absolute Error (MAE)			Ratio of MAE to Mean Absolute Actual Values (MAA)		
Line	Symbol[a]	Unit[b]	Level (MEL) (1)	Change (MEC) (2)	Level (MAEL) (3)	Change (MAEC) (4)	Relative Change[c] (MAERC) (5)	Level $\left(\frac{MAEL}{MAAL}\right)$ (6)	Change $\left(\frac{MAEC}{MAAC}\right)$ (7)	Relative Change[d] $\left(\frac{MAERC}{MAARC}\right)$ (8)
1	*GNP*	billion $	6.34	−0.15	9.298	4.901	1.171	.019	.604	.681
2	*GNP58*	billion 58$	11.75	0.26	15.093	5.069	1.125	.032	.818	.852
3	*C*	billion 58$	22.81	0.20	22.812	2.963	1.028	.079	.975	.985
4	*IH*	billion 58$	0.15	0.01	1.465	0.721	3.477	.070	.901	.898
5	*ISE*	billion 58$	1.99	−0.02	3.192	1.266	3.118	.072	1.013	1.036
6	*II*	billion 58$	0.11	0.06	3.054	2.752	[e]	.568	.894	[e]
7	*NE*	billion 58$	1.92	0.32	5.906	0.994	[e]	1.366	1.079	[e]
8	*YP*	billion $	−3.70	0.81	9.486	3.207	0.935	.025	.537	.600
9	*P*	1958 = 100	−0.01	−0.001	0.017	0.002	0.269	.018	.439	.453
10	*LE*	million	0.13	−0.01	1.101	0.500	0.745	.016	1.214	1.238
11	*UN*	per cent	−0.02	−0.004	1.411	0.755	17.795	.293	2.199	2.502
12	*CPR*	billion $	7.70	0.43	8.541	2.365	4.905	.166	1.058	1.037
13	*AWW*	40 hrs. = 1.000	0.01	0.0003	0.007	0.005	0.448	.007	1.484	1.436
14	*UMD*	billion 58$	2.97	−0.09	6.093	2.007	3.712	.106	.780	.802
15	*RS*	% per annum	−0.05	−0.01	0.209	0.163	5.808	.066	.677	.702
16	*RL*	% per annum	−0.04	0.01	0.191	0.096	2.339	.047	.924	.935
17	*W*	dollars	−0.10	0.02	0.207	0.063	1.107	.044	.714	.678

NOTES TO TABLE 3.1

[a] For meaning of symbols, see Table 1.1.

[b] The average errors, with and without regard to sign (ME and MAE), which are listed in columns 1 through 4, are expressed in these units, and so are the average actual values (MAA); hence the figures for MAE/MAA, in columns 6 and 7, are pure ratios.

[c] In percentage points. Mean of the differences: quarterly per cent change in the simulated series minus quarterly per cent change in the actual series, all taken without regard to sign.

[d] Ratio of the figure in column 5 to the corresponding mean of actual percentage changes.

[e] Not applicable: since net change in inventories and net foreign investment can assume negative values, these series can be analyzed only in absolute, not in relative, terms.

cally, then, the ratios of mean absolute errors of simulations to mean absolute actual values (MAE/MAA) are quite small fractions for the level measures and considerably larger for the changes. That is, $\frac{\text{MAEL}}{\text{MAAL}} < \frac{\text{MAEC}}{\text{MAAC}}$ (columns 6 and 7), although $\text{MAEL} > \text{MAEC}$ (implying that the MAAL figures exceed the corresponding values of MAAC by very large amounts). As elsewhere in the analysis of predictive accuracy, the comparisons with changes are on the whole much more meaningful than those with levels. Because the series have different units and levels, it is useful to express changes in the simulated series in relative terms, and to compute their errors, correspondingly, as deviations from the actual relative changes. The averages of the absolute errors of relative changes (in percentage points) are listed in column 5. They vary between 0.9 and 1.2 for the comprehensive income and consumption aggregates, and for the wage series (*GNP*, *GNP58*, *C*, *YP*, and *W*), but are considerably larger for most of the other variables (fixed investment, profits, and, particularly, unemployment) and smaller for only a few (*P*, *LE*, *AWW*).

Since the series also differ greatly in variability, the most meaningful comparisons are probably those that relate the average size of errors to the average size of actual changes. The ratios of the relative-change measures are generally quite similar to the ratios of the absolute-change measures for the same variables (columns 7 and 8). The lowest ratios (most favorable to the simulations) are those for current-dollar *GNP*, *YP*, *P*, *RS*, and *W*; they fall in the range of 0.4 to 0.7. The ratios for *GNP58*, *C*, *IH*, *II*, *UMD*, and *RL* exceed 0.75 but are smaller than one. Ratios in excess of unity signify that errors are on the average larger than the recorded changes, an adverse finding applying to the simulations of *ISE, NE, LE, UN, CPR*, and *AWW*. On the whole, the ratios tend to be relatively low for the more stable trend-dominated variables and high (unfavorable) for the more volatile and fluctuating ones.

3.1.3 The correlations between the actual and simulated *levels* of the series are, in general, quite high, as would be expected; they exceed .90 for thirteen, and .95 for eleven, of the seventeen variables. The lower r-coefficients, ranging from .331 to .659, are those for the volatile series *NE, UN, II*, and *IH*. However, the high correlations of levels reflect mainly common trends, rather than good agreement between the shorter movements in simulations and realizations. Correlations between the relative or absolute *changes* in *S* and *A* are drastically lower than those between levels. They vary from practically zero for *ISE* to .777 ($\bar{r}^2 = .598$) for *RS* (Table 3.2, columns 1 and 2). Nine of these r-coefficients are smaller than .4; five (*GNP, IH, II, NE*, and *YP*) are larger than .4 but smaller than .6; and only three (*P, UMD*, and *RS*) exceed .6.

Simple linear least-square regressions of actual on simulated changes yield the statistics on the constant intercept a and the slope coefficient b, listed in columns 3 and 4 of Table 3.2. Ideally, the true population parameters for the constant (α) and for the slope (β) should equal zero and one, respectively, in order for the simulation to be both unbiased and efficient. In terms of the limited-sample statistics available, this means that a and b should not be statistically different from zero and one, respectively. Tests of the corresponding null hypotheses (that $\alpha = 0$ and $\beta = 1$, jointly or separately) are summarized in columns 5 and 6.

The results are fair or good for some of the variables whose changes were relatively well simulated, judging from the correlation statistics: *IH, II, P, UMD, RS*, and *RL*. Here the constants are very small fractions, while the slope coefficients range from .72 to 1.06 and do not appear to be significantly different from one, using the conventional probability levels. Elsewhere, the intercepts are still generally small and—what is more meaningful—the differences between the means of simulated and actual changes are for the most part small. But

TABLE 3.2

Nonstochastic Simulations for the Sample Period, Wharton Model: Correlation, Regression, and Test Statistics (1948-III–1968-I)

		Correlation of Simulated with Actual Changes[b]		Regression of Actual on Simulated Changes[c]			
Line	Variable Symbol[a]	r (1)	$\bar{r}^2$ (2)	Constant (a) (3)	Slope (b) (4)	F-ratio for $(\alpha = 0, \beta = 1)$[d] (5)	t-test for $\beta = 1$[e] (6)
1	*GNP*	.409	.156	.010	.355	25.27	7.10
2	*GNP58*	.328	.096	.007	.278	31.07	7.88
3	*C*	.159	.012	.010	−.122	84.09	12.97
4	*IH*	.450	.192	.001	.808	0.56	1.04
5	*ISE*	.016	[g]	.010	.021	21.54	6.56
6	*II*[f]	.503	.243	−.052	.724	1.88	1.93
7	*NE*[f]	.407	.155	−.177	.458	13.86	4.60
8	*YP*	.538	.280	.009	.447	24.44	6.89
9	*P*	.666	.436	.001	.938	0.72	0.52
10	*LE*	.342	.105	.003	.208	72.73	12.06
11	*UN*[f]	.115	.0002	−.003	.060	125.07	15.81
12	*CPR*	.336	.102	.008	.356	16.02	5.64
13	*AWW*	.132	.004	−.001	.114	40.88	9.02
14	*UMD*	.637	.398	.004	.827	1.23	1.51
15	*RS*[f]	.777	.598	.011	.902	1.5	0.3
16	*RL*[f]	.343	.106	.010	1.065	0.5	1.0
17	*W*	.330	.097	−1.336	2.097	1.45	1.59

NOTES TO TABLE 3.2

[a] For meaning of symbols, see Table 1.1.

[b] Based on relative changes in the simulated series S_t, defined as $(\Delta S/S)_t = (S_t - S_{t-1})/S_{t-1}$, and on relative changes in the actual series A_t, defined as $(\Delta A/A)_t = (A_t - A_{t-1})/A_{t-1}$, except as noted in footnote *f*. Correlation coefficients r are listed in column 1, adjusted determination coefficients $\bar{r}^2$ in column 2.

[c] The regressions are of the form $(\Delta A/A)_t = a + b(\Delta S/S)_t + u_t$, except as noted in footnote *f*.

[d] See text. Some of the relevant percentage points of the F-distributions (for $n_1 = 2$, $n_2 = 77$) are: $F_{0.01} = 4.93$, $F_{0.05} = 3.12$, $F_{0.10} = 2.38$, and $F_{0.25} = 1.41$.

[e] See text. Some of the relevant percentage points of the t-distribution ($n = 77$, two-tailed test) are 2.65, 1.99, 1.67, and 1.16 (for 1, 5, 10 and 25 per cent significance levels, respectively).

[f] Based on absolute changes in S_t and A_t; that is, these regressions are of the form: $\Delta A_t = a + b(\Delta S_t) + u_t$.

[g] Correlation too low to give a meaningful adjusted coefficient $\bar{r}^2$ (unadjusted $r^2 = .00026$).

the regression coefficients *b* are much too low for comfort: they not only definitely differ from one (in the downward direction), but in several cases they are not significantly different from zero according to the standard *t*-test. (This is so for *ISE*, *UN*, *AWW*, and *C* where the slope coefficient is negative.) For these series, then, changes in *S* and *A* are apparently uncorrelated.

Among the probable reasons for these rather disappointing results is the effect of the "base errors" that were noted before, in connection with the large differences between the accuracy of the level and that of the change predictions, based on the *S* series. These base errors are likely to act as errors of observation in independent variables; that is, they would tend to lower the regression, as well as the correlation coefficients. Whatever their source, large deviations of the slope coefficients from unity indicate that the simulated series provide inefficient estimates of the actual changes.

3.1.4 Confirming and quantifying what has been broadly observed with the aid of our basic charts of the *A* and *S* series, Table 3.3 shows that the simulations understate the fluctuations of the actuals in most cases. The averages listed in columns 1 to 6 are based on amplitudes measured for both *A* and *S* over successive business-cycle expansions and contractions in the sample period. These amplitude meas-

TABLE 3.3

Nonstochastic Simulations for the Sample Period, Wharton Model: Average Amplitudes of Cyclical and Trend Movements (1948-III–1968-I)

Line	Variable Symbol[a]	Average Change Per Month in Reference-Cycle Relatives[b] During:						Mean Absolute Per Cent Change Trend-Cycle Component[c]	
		Expansions		Contractions		Full Cycle			
		Simulated (1)	Actual (2)	Simulated (3)	Actual (4)	Simulated (1) − (3) (5)	Actual (2) − (4) (6)	Simulated (7)	Actual (8)
1	*GNP*	.57	.67	.17	−.12	.40	.79	1.67	1.66
2	*GNP58*	.48	.40	−.18	.04	.66	.36	1.20	1.26
3	*C*	.34	.35	.22	.12	.12	.23	0.99	0.93
4	*IH*	−.01	−.09	.48	.48	−.49	−.57	1.78	3.59
5	*ISE*	.38	.67	−.10	−.87	.48	1.54	1.79	2.44
6	*II*	.08	.27	−.16	−.72	.24	.99	1.28	1.88
7	*NE*	.16	−.01	−.17	−.01	.33	0	0.76	0.76
8	*YP*	.52	.63	.19	−.03	.33	.66	1.42	1.56
9	*P*	.18	.20	.14	.06	.04	.14	0.46	.5
10	*LE*	.14	.18	−.01	−.16	.15	.34	0.65	0.53
11	*UN*	−.09	−.25	.03	.07	−.11	−.32	11.53	6.51
12	*CPR*	.70	.85	−.36	−1.16	1.06	2.01	2.10	3.59
13	*AWW*	−.02	−.01	−.06	−.11	.04	.10	0.22	0.23
14	*UMD*	.48	.93	−.91	−1.92	1.39	2.85	2.67	4.37
15	*RS*	.04	.05	−.11	−.09	.15	.14	6.39	7.33
16	*RL*	.014	.02	−.004	−.03	.018	.05	1.05	2.04
17	*W*	.35	.44	.34	.20	.01	.24	0.98	1.35

NOTES TO TABLE 3.3

The period covered by the measures in columns 1 through 6 extends from the 1948-IV peak to the assumed 1966-IV peak.

[a] For meaning of symbols, consult Table 1.1.

[b] Based on quarterly data but expressed as rate per month (quarterly rates would be three times as large). Figures for all series except *II, NE, UN, RS* and *RL* are expressed as reference-cycle relatives; that is, as a percentage of the average level of the series during each business ("reference") cycle. Figures for *II, NE, UN, RS* and *RL* are expressed in absolute units. (*See Table 1.1 for units.*)

[c] Average quarter-to-quarter percentage change, without regard to sign, in the trend-cyclical component: a smooth, flexible moving-average of the seasonally adjusted series.

ures refer to per month rates of change between standings at business-cycle peaks and at business-cycle troughs, both expressed in per cent of the average level of the respective series during the particular business cycle.[12] The differences between amplitudes of *A* and *S* depend not only on the relative size of the cyclical swings ("specific cycles") in the paired series, but also on the differences in their timing and conformity to business cycles. Thus, it is conceivable that, say, *A* showed systematically larger specific-cycle amplitudes than did *S*, while no such regular relationship applied to the reference-cycle amplitudes of the same two series. For this to be possible, *A* and *S* would have to have sufficiently different timing at the peaks and troughs of the business cycle, and *S* would have to conform more closely to the general business cycle than *A* does. Actually, substantial differences in timing and conformity are not uncommon for the compared *A* and *S* series.

The simulations for *GNP* and *YP* show small positive, instead of small negative, changes during contractions, while the reverse applies to *GNP58*. Also, for net exports, the expansion amplitude is negative in *A*, positive in *S*. In all other cases, the average amplitudes of *A* and *S* agree in sign (compare columns 1 and 2, 3 and 4). This includes eight series that show definite procyclical changes (positive in expansions and negative in contractions, except for the unemployment rate, where an inverted pattern is, of course, expected); three series that experienced only retardations of growth rather than declines during business contractions (*C*, *P*, and *W*); one series that moved downward

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[12] See below, Section 3.1.6.

throughout, though less so in expansions (*AWW*); and one whose movements were on the average countercyclical (*IH*). In most instances, the average changes per month are larger for *A* than for *S* in both expansion and contraction periods. In all but two cases—*GNP58* and *NE*—the full-cycle amplitudes (i.e., expansion amplitude minus contraction amplitude) are larger for *A* than for *S*. This is strong evidence for the tendency of these simulations to underestimate the observed cyclical movements.[13]

As another average amplitude measure, one that disregards the difference between rises and falls, and does not depend on the relative cyclical timing and conformity of the series, we use the mean absolute percentage change per quarter in the trend-cycle component of the series. This is based on changes in a weighted moving average, which contains practically all of the trend and cyclical movements and little, if any, of the seasonal and short irregular movements in the data.[14] Here the amplitude of *S* is smaller than that of *A* for twelve of the seventeen variables; and in most of the remaining cases, the differences between the paired measures are very small (columns 7 and 8).

The simulations, although usually underestimating the average changes in the series, rank the variables very well according to their typical amplitudes. The ranks based on the expansion averages (columns 1 and 2) show a positive correlation of .934; those based on the contraction averages (columns 3 and 4), a correlation of .919. The correlation between the ranks based on the mean absolute percentage changes in the trend-cycle components of *S* and *A* (columns 7 and 8) is .950. (These Spearman coefficients adjusted for tied ranks—r_s—are all high enough not to be attributable to chance.)

3.1.5 The Wharton series cover four business-cycle peaks (1948-IV, 1953-II, 1957-III, and 1960-II) and four troughs (1949-IV, 1954-III, 1958-II, and 1961-I) but in some cases their timing at the beginning of the sample period—that is, at the 1948 peak—could not

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[13] However, let us remember that these amplitude differences still reflect in some unknown degree the fact that the variance of *A* includes the random disturbance component, while the variance of *S* does not.

[14] This is a quarterly equivalent of "Spencer's graduation" (a weighted fifteen-term formula used as an estimate of the trend-cycle component of a series in the Census Bureau computer program of seasonal adjustment and time-series decomposition). See Julius Shiskin, "Electronic Computers and Business Indicators" [26, Chapter 17].

be determined. In addition, some of these series, notably those for inventory investment and short-term interest rates, show strong contractions in 1966–67 in both the actual and simulated values. These movements correspond to the business retardation that can be dated as having occurred in the period 1966-IV–1967-II, and they have been so treated here.

Table 3.4 summarizes the record of cyclical timing of the *A* and *S* series for fourteen variables. The price-level series show only one cyclical decline in *A* (in 1948–49) and none in *S*; the wage-rate series have no contractions at all, in either *A* or *S*. Net exports data conform poorly to business cycles, so that of the nine turning points in the *A* series for this variable, only four can be matched with reference dates; but the *S* series reproduced fairly well six of these episodes in the years 1948–58. (Afterward, its fit to *A* became quite inadequate, however.) Because so few comparisons of business revivals and recessions can be made for these data, the variables *P*, *W*, and *NE* are not included in Table 3.4.

The simulations for *GNP*, *GNP58*, *C*, *LE*, *AWW*, and *UMD* failed to match the contractions of 1957–58 and 1960–61; thus, each of these six series "skipped" four of the business-cycle turns that occurred in the Wharton sample period (Table 3.4, column 1). The corresponding *A* series, in contrast, did turn in conformity with these general economic reversals. The simulation for personal income matched only the 1953–54 contraction and missed six reference dates at other times, while the actual *YP* skipped only the two turning points marking the 1960–61 recession. For each of the other variables included in the table, the *S* series skipped two turning points (typically either in 1957–58 or in 1960–61), except that the *RS* simulation declined at each peak, and rose at each trough, of the business cycles covered. The corresponding actuals, on the other hand, matched these turns on practically all occasions.

Another manifestation of lapses from conformity is found in "extra turns," which occur when a series shows a specific-cycle peak or trough which cannot be matched with a business-cycle recession or revival. For a few variables—*GNP58*, *LE*, *UMD*—the simulations do show such extra turns, where the actuals have none; while in two cases—*C* and *ISE*—the opposite applies (Table 3.4, column 2). On the

TABLE 3.4

Nonstochastic Simulations for the Sample Period, Wharton Model: Timing at Business-Cycle Turns and Corresponding Measures for the Actual Values
(1948-III–1968-I)

Line	Variable Symbol[a]	Frequencies of Timing Observations for Series *S* and *A*						Dominant Type of Timing[b] (7)
		Business Cycle Turns Skipped[b] (1)	Extra Turns[b] (2)	Leads (3)	Exact Coinci- dences (4)	Lags (5)	Long Leads or Lags[c] (6)	
1	*GNP* *S*	4	0	1	1	2	1*	n.i.[i]
2	*A*	0	0	3	5	0	0	coincident
3	*GNP58* *S*	4	2	1	1	2	1*	n.i.
4	*A*	0	0	4	4	0	0	coincident
5	*C* *S*	4	0	2	0	2	1	n.i.
6	*A*	2	2	2	3	1	1	coincident
7	*IH* *S*	2	4	3	2	0	3	leading-irregular
8	*A*	2	4	4	2	0	3	leading-irregular
9	*ISE* *S*	2	0	2	1	3	3[d]	coincident-lagging
10	*A*	0	2	0	4	4	0	coincident-lagging
11	*II* *S*	2	2	4	1[e]	1	3	leading-irregular
12	*A*	0	2	6	4[f]	0	3	leading-irregular
13	*YP* *S*	6	0	1	0	1	0	n.i.
14	*A*	2	0	2	3	1	0	coincident
15	*LE* *S*	4	2	2	0	2	1*	n.i.
16	*A*	0	0	2	4	2	0	coincident
17	*UN*[g] *S*	2	0	3	0	2	4[h]	n.i.-irregular

(*continued*)

TABLE 3.4 (*concluded*)

		Frequencies of Timing Observations for Series *S* and *A*						
Line	Variable Symbol[a]	Business Cycle Turns Skipped[b] (1)	Extra Turns[b] (2)	Leads (3)	Exact Coinci-dences (4)	Lags (5)	Long Leads or Lags[c] (6)	Dominant Type of Timing[b] (7)
18	*A*	0	0	2	3	2	1	coincident
19	*CPR S*	2	0	3	0	2	1	leading-irregular
20	*A*	0	0	5	2	0	3	leading
21	*AWW S*	4	2	3	0	1	2	leading-irregular
22	*A*	0	1	4	2	1	3	leading
23	*UMD S*	4	4	0	1	2	1*	n.i.-irregular
24	*A*	0	0	4	2	1	1	leading-coincident
25	*RS S*	0	0	5[e]	3	2[c]	1	coincident-leading
26	*A*	0	0	1	3[f]	6	0	lagging
27	*RL S*	2	0	0	1	5	1*	lagging
28	*A*	0	0	1	4	3	1*	coincident-lagging

[a] For meaning of symbols, see Table 1.1. *S* refers to simulations, *A* to actuals.

[b] See explanation in text.

[c] Leads or lags of three or more quarters. Numbers marked by asterisks refer to lags, others refer to leads at business-cycle turns (*see also notes d and h below*).

[d] Includes two lags and one lead.

[e] Includes one observation relating to the 1966–67 retardation.

[f] Includes two observations relating to the 1966–67 retardation.

[g] Treated on the inverted plan: peaks in *UN* are matched with business-cycle troughs, troughs in *UN* with business-cycle peaks.

[h] Includes one lag and three leads.

[i] Not identified.

whole, however, extra turns are not an important source of discrepancies between *S* and *A*.

The frequencies of leads, coincidences, and lags, and also of those leads or lags that are longer than two quarters, are listed in columns 3 to 6 of Table 3.4 for each of the paired *A* and *S* series. It is often easier to identify the presence of a lead or lag than to measure its duration with adequate precision, especially at the beginning and end of a series; and the shortness of the simulated series makes this a fairly important consideration. Moreover, we are here interested mainly in whether the simulations agree with the actuals regarding the type of timing that is predominant for the given variable—not in comparisons of the length of leads and lags, which would be needlessly ambitious and could only be spuriously precise for the available data. For these reasons, the timing measures proper are not presented here; we show only the frequency distributions that are based on them. However, the entire evidence is used to determine the "dominant type of timing" of *A* and *S* in column 7.

For some variables, the paucity of the timing comparisons at turning points, or the varied composition of these observations, or both, make it impossible to identify the series as a leader, coincider, or lagger. This applies to the simulations of *GNP*, *GNP58*, *C*, *YP*, and *LE*: all cases in which the label "not identified" (n.i.) had to be used for the *S* series, whereas the actuals, as shown by the historical evidence, are clearly coincident. In addition, the simulations for the unemployment rate and unfilled orders are best described as "n.i.-irregular," while the actual data show *UN* as typically coincident and *UMD* as leading at peaks and coincident at troughs. For the other variables in Table 3.4, there is a good or fair correspondence between the timing of *S* and *A*, except that leads are more frequent in the simulated data for *RS* and lags more important in the actual data. For example, leads, often of long duration, prevail in both *A* and *S* for investment in housing (*IH*), and in inventories (*II*), while lags are more characteristic of *A* and *S* for business investment in plant and equipment (*ISE*).

3.1.6 One way of evaluating the models' capacity to simulate cyclical characteristics is the use of NBER reference-cycle patterns for simulated and actual series. These reference-cycle patterns represent the condensation of time-series data for each business cycle into nine cyclical stages, covering standings at business-cycle turning points, and average standings for thirds of expansions and contractions. In trough-peak-trough cycles, the initial, middle, and terminal turns are designated as Stages I, V, and IX respectively. Average standings during successive thirds of business expansions are Stages II, III, and IV; those of contractions, VI, VII, and VIII. Stage relatives are constructed by expressing these average stage standings as percentages of the average value of the series over the full cycle.

In the graphic presentation, the time scale can reflect chronological time or it can be standardized to represent all cycle phases (expansions and contractions) as equal time-spans before and after the central turn. A third possibility, the one used in this study, is to represent expansions and contractions in proportion to the average duration of these phases during the whole period covered by the series.[15]

The Wharton sample-period series cover three complete trough-peak-trough cycles: 1949–53–54, 1954–57–58, and 1958–60–61; but acceptance of the proxy dates for a peak in 1966-IV, and for a trough in 1967-II (actually, these dates identify a definite retardation in general economic activity), enables us to use the subsequent data in a fourth pattern for the period 1961–66–67. Similarly, with the aid of the same pair of extra reference dates, four successive peak-trough-peak patterns can be computed for each series: 1948–49–53, 1953–54–57, 1957–58–60, and 1960–61–66. Each set of computations inevitably fails to cover some of the data at the beginning and end of the sample period: in particular, the trough-to-trough patterns miss the 1948–49 contraction, and the peak-to-peak patterns miss the declines or retardation of 1966–67. In order to maximize the informational potential of this approach, we have computed, plotted, and inspected all the T-to-T and P-to-P patterns that the data allow, including a pair of average patterns for each series. For economy in presentation, the patterns have then been combined so as to show in one diagram the behavior of the series on both sides of each of three turns, T-P-T. Chart 3.2

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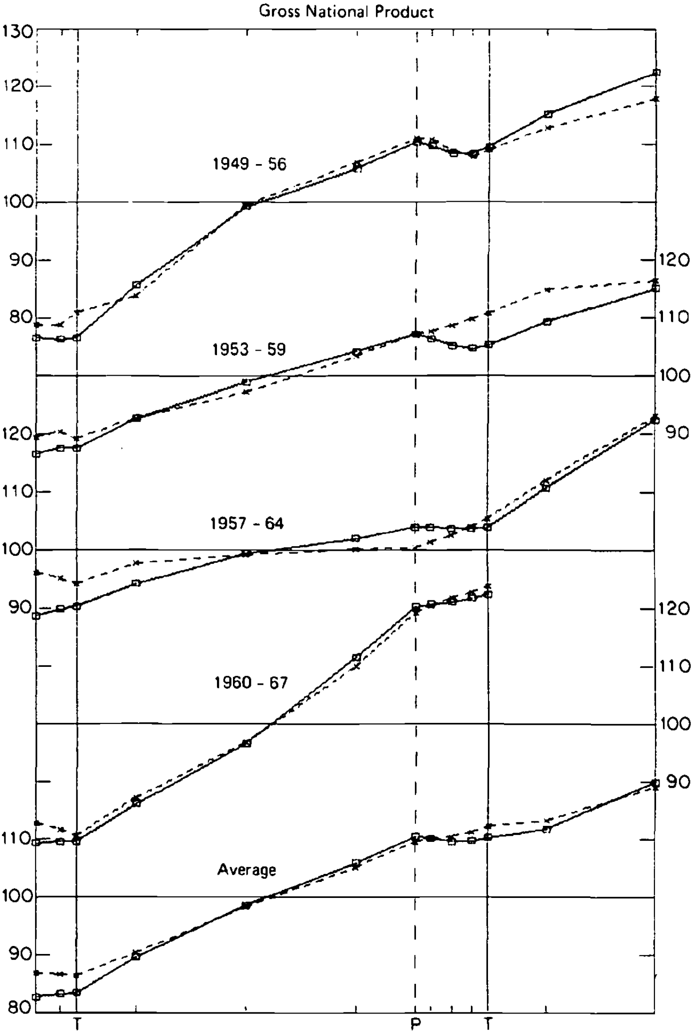
[15] Since this time period is different for each of the three models, the time-scales of the charts vary among models. In the present case of quarterly data, interpolations had to be used in dividing up the contraction phase when this phase was short.

For a full discussion of the reference-cycle patterns and their application to quarterly data, see [5, pp. 160–70, 200–202]. For a condensed discussion of the approach, description of related computer programs, and interpretation of output measures see [4, Part III].

CHART 3.2

Nonstochastic Simulations for the Sample Period, Wharton Model: Reference-Cycle Patterns for Simulated and Actual Series, Selected Variables (1949–1967)



NOTE: Scale in reference-cycle relatives or (for unemployment rate and short-term interest rate) in absolute deviations from cycle base. *P* and *T* stand for peaks and troughs, respectively.

(*continued*)

CHART 3.2 (*continued*)

Gross National Product, 1958 Dollars

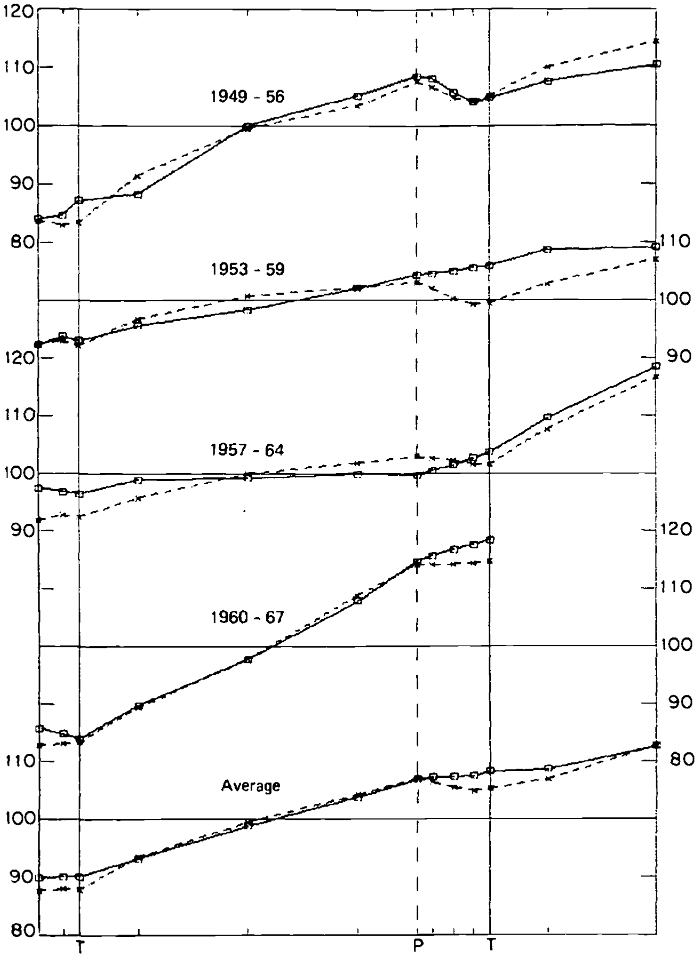
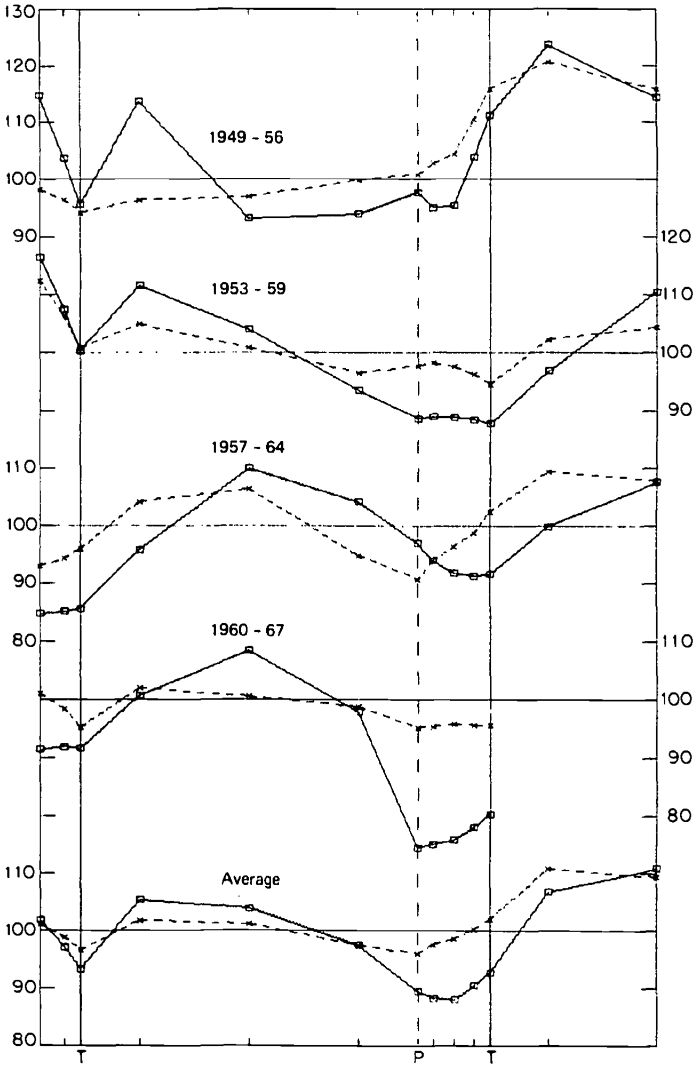


CHART 3.2 (*continued*)

Investment in Housing



(*continued*)

CHART 3.2 (*continued*)

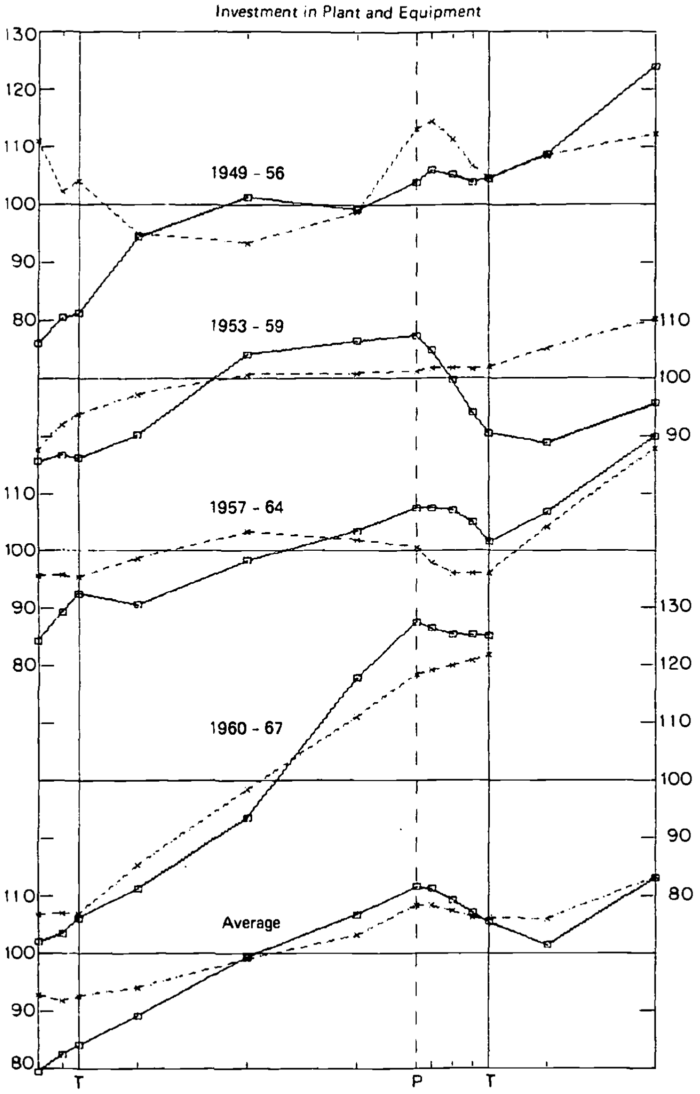
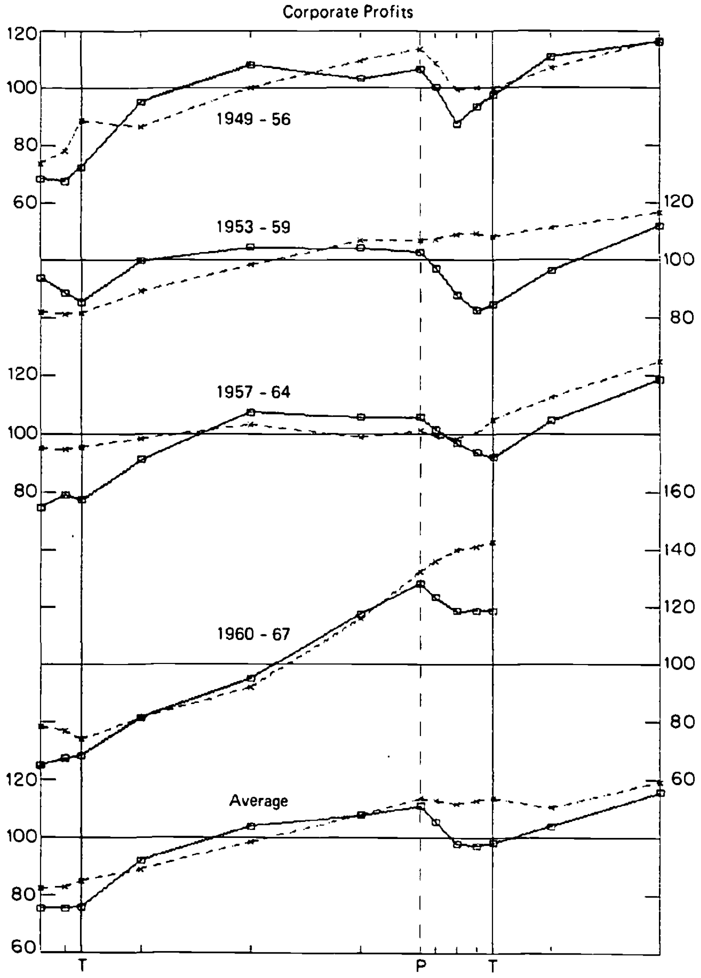


CHART 3.2 (*continued*)



(*continued*)

CHART 3.2 *(continued)*

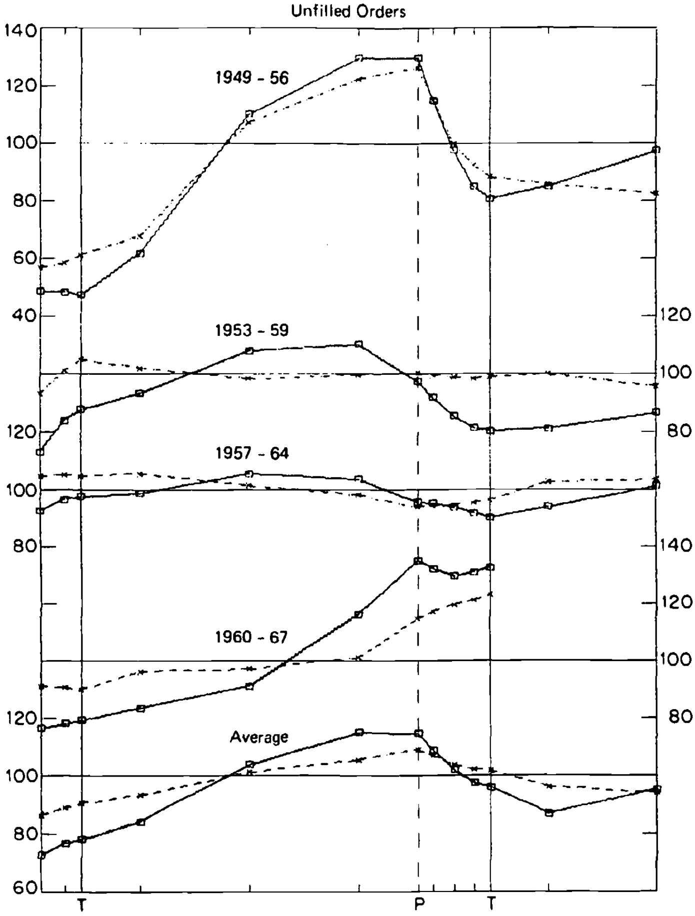
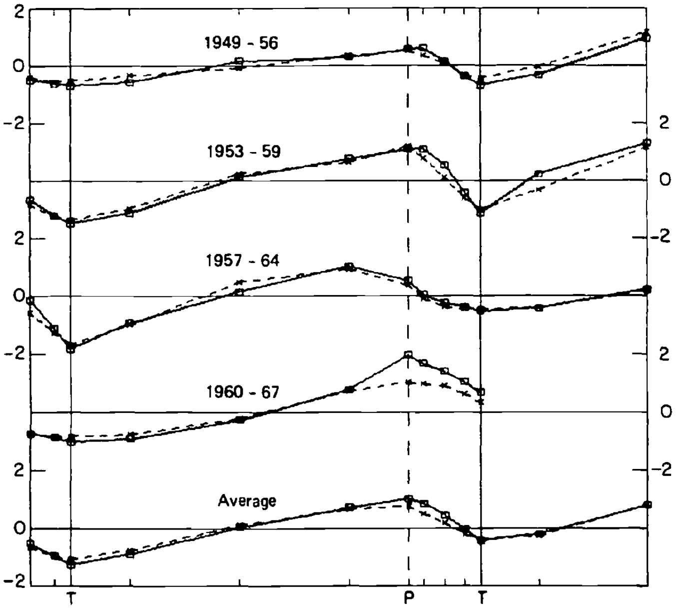


CHART 3.2 (*continued*)

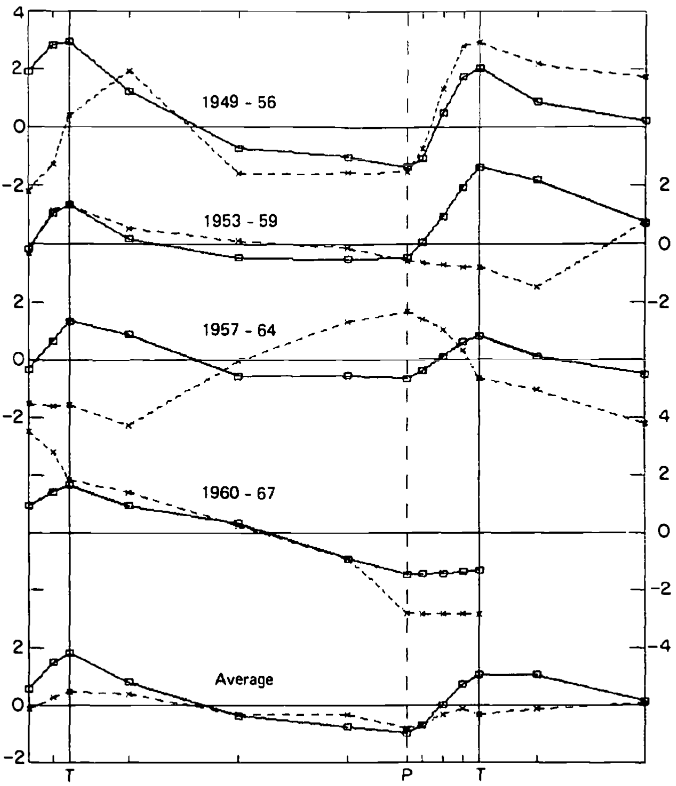
Short-Term Interest Rate



(*continued*)

CHART 3.2 (*concluded*)

Unemployment Rate



presents a sample of these diagrams for selected variables, while the discussion that follows is based on the entire material.

The method of measurement employed is such as to eliminate the intercycle trends and to smooth out much of the short "irregular" movements of the series within the stages of the business cycle. The apparent effect of this detrending and smoothing is, in many cases, to make S and A resemble each other much better than they do in terms of the original data (*cf.* Charts 3.1 and 3.2). The evidence of the patterns is also, in general, more favorable to the simulations than are some of the sets of measures discussed before, notably the average errors and correlations based on quarterly changes in S and A.

For the variables dominated by strong growth tendencies—*GNP*, *GNP58*, *C*, *YP*, *P*, *LE*, and *W*—the patterns show clearly that *S* decreased less often and by smaller relative amounts during business recessions than did *A*. Frequently, small cyclical declines in *A* are matched by reduced rates of increase in *S*; but then, in most such cases, the retardations of growth in *S* are so pronounced as to make the contrast between the patterns of behavior in expansions and contractions nearly as marked for *S* as for *A*. However, it is also true that the correspondence between the patterns for the paired series was, on the whole, considerably closer during expansions than during contractions. The agreement between *S* and *A* was, by and large, appreciably better during the 1953–54 recession than in the other three periods of contraction (including the 1966–67 retardation).

For the variables with large fluctuations and relatively weak trends, much greater discrepancies are observed between the patterns for the paired *A* and *S* series. Some of the largest discrepancies are found for the investment series (*IH*, *ISE*, *II*), the unemployment rate, unfilled orders, and net exports. The amplitudes of the *S* patterns tend to be smaller than those of the corresponding *A* patterns (only for *NE* is the opposite clearly indicated). The correspondence between the *A* and *S* patterns is particularly good for the short-term interest rate, *RS*.

3.2 THE OBE MODEL

3.2.1 The sample period for this model is 1953-II through 1966-IV (55 quarterly observations). It includes three business-cycle contractions and two minor retardations.

Chart 3.3 shows that the simulated *GNP* series declined only briefly in the first quarter of 1954. Because of the relatively large amplitude of the decline and the slowness of the subsequent recovery, this movement is considered to be cyclical, although one-quarter declines are usually thought to be too short for such consideration. Only one other interruption of the upward trend is observed in this series: it occurred in 1959-III, a reflection of the major steel strike, which similarly affected the actual *GNP* series. During the 1957–58 and 1960–61 recessions, the simulated *GNP* fails to show downward

CHART 3.3

Nonstochastic Simulations for the Sample Period, OBE Model: Simulated and Actual Series for Selected Variables (1953–1966)

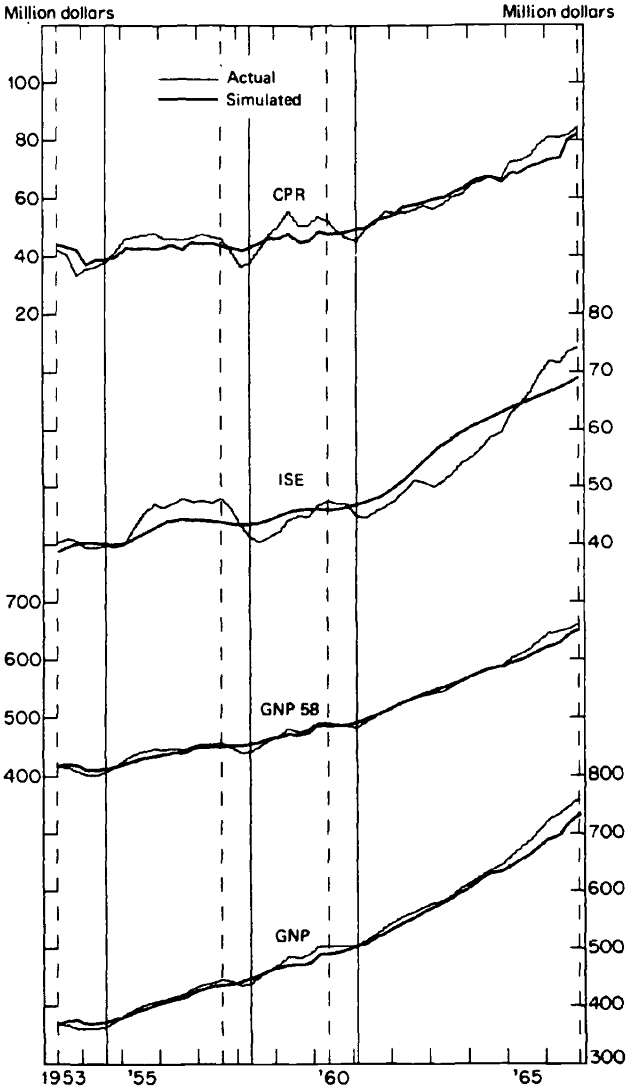
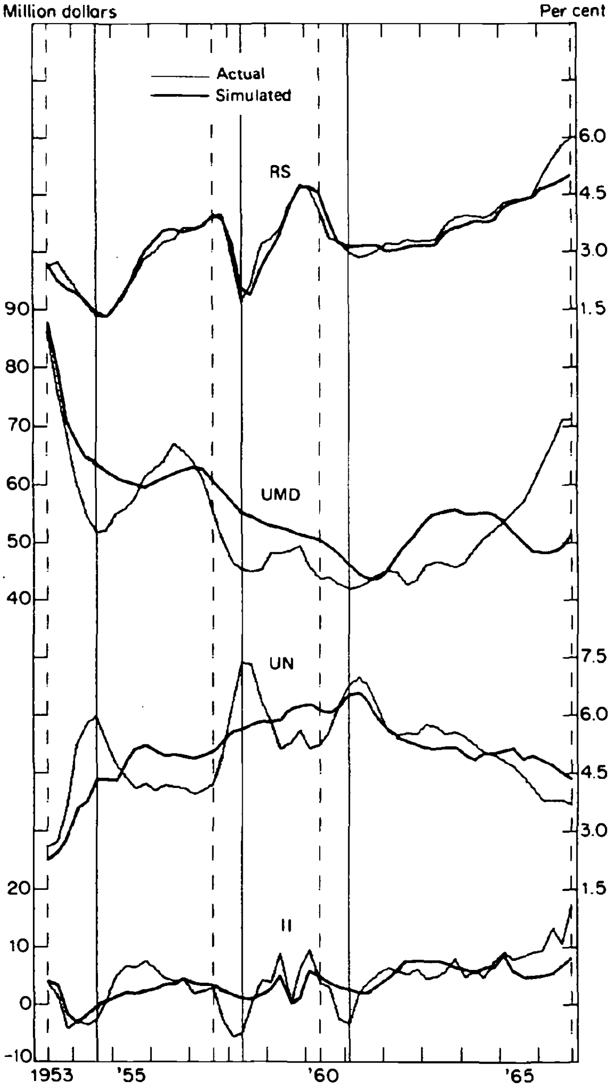


CHART 3.3 (*concluded*)



movements corresponding to those in actual *GNP*. The values of simulated *GNP* exceed the recorded values only during the recession years 1953–54 and 1958. At other times, i.e., in eleven out of the fourteen years covered, the level of *GNP* was persistently underestimated in the simulation. The amount of this underestimation is shown to have increased at the end of the sample period, in 1965–66.

Much the same can be said about the simulation of personal income in current dollars, except that, here, the 1953–54 decline lasted two quarters. Considerably less underestimation, however, is observed in the simulations of *GNP* in 1958 dollars, and of personal consumption expenditures in 1958 dollars; in particular, the fits here are very close for the years 1961–64.

The story is quite similar for other variables with strong upward trends and relatively small fluctuations, such as the *GNP* price deflator, private wage and salary compensation per man-hour, and private labor costs per unit of output; except for the first three or four years after the "initial shock" of starting the simulation, the levels of these series are consistently underestimated. On the other hand, for the more volatile variables with large fluctuations and much weaker, or less consistent, trends, the tendency for the simulated series to run below the levels of the actuals is not so apparent. What the charts do clearly indicate for these series is the tendency to underestimate *changes:* the curves for simulated values show fewer and smaller fluctuations than do their counterparts for the actuals.

3.2.2 Table 3.5 confirms the prevalence of underestimation errors in the OBE Model simulations for the sample period 1953–66. For all but four of the twenty-one variables used, the average level of the simulated series is lower than that of the actual series, $\bar{S} < \bar{A}$, as indicated by the negative signs of the mean level errors (MEL) in column 1. (The errors of simulation are defined as the difference, simulated minus actual value.) For all but five of the variables, the mean change errors (MEC) are negative (column 2). As noted above, in Section 3.1.2, the sign of MEC can mean different things under different circumstances; but where both A and S are positive and show predominantly rises rather than declines, negative MEC indicates that changes were, on the average, underestimated. This is the case for most of the variables covered by the OBE data. The variables with

TABLE 3.5

Nonstochastic Simulations for the Sample Period, OBE Model: Average Errors and Their Ratios to Average Actual Values (1953-II–1966-IV)

Line	Variable		Mean Error (ME)		Mean Absolute Error (MAE)			Ratio of MAE to Mean Absolute Actual Values (MAA)		
	Symbol[a]	Unit[b]	Level (MEL) (1)	Change (MEC) (2)	Level (MAEL) (3)	Change (MAEC) (4)	Relative Change[c] (MAERC) (5)	Level $\left(\frac{MAEL}{MAAL}\right)$ (6)	Change $\left(\frac{MAEC}{MAAC}\right)$ (7)	Relative Change[d] $\left(\frac{MAERC}{MAARC}\right)$ (8)
1	*GNP*	billion $	−7.51	−0.48	9.991	3.340	0.701	.020	.421	.459
2	*GNP58*	billion 58$	−1.95	−0.14	6.291	3.101	0.644	.013	.502	.518
3	*C*	billion 58$	−1.57	−0.02	3.153	1.926	0.612	.010	.578	.595
4	*IH*	billion 58$	−0.16	0.07	1.196	0.537	2.436	.053	.794	.794
5	*ISE*	billion 58$	0.16	−0.09	2.616	0.916	1.896	.053	.793	.812
6	*II*	billion 58$	−0.49	−0.16	2.540	1.928	[e]	.470	.846	[e]
7	*NE*	billion 58$	0.09	0.05	0.894	0.449	[e]	.208	.529	[e]
8	*YP*	billion $	−5.87	−0.40	8.106	1.774	0.460	.020	.308	.334
9	*P*	1958 = 100	−0.96	−0.05	1.353	0.234	0.239	.013	.469	.488
10	*LE*	million	−0.07	−0.01	0.570	0.220	0.348	.009	.418	.430
11	*UN*	per cent	0.02	0.01	0.657	0.297	6.000	.131	.909	.890
12	*CPR*	billion $	−1.16	−0.05	3.064	1.903	3.949	.057	.868	.881
13	*AWW*	hours/week	−0.01	0.002	0.170	0.148	0.359	.004	.938	.046

(continued)

TABLE 3.5 (*concluded*)

	Variable		Mean Error (ME)		Mean Absolute Error (MAE)			Ratio of MAE to Mean Absolute Actual Values (MAA)		
Line	Symbol[a]	Unit[b]	Level (MEL) (1)	Change (MEC) (2)	Level (MAEL) (3)	Change (MAEC) (4)	Relative Change[c] (MAERC) (5)	Level $\left(\frac{MAEL}{MAAL}\right)$ (6)	Change $\left(\frac{MAEC}{MAAC}\right)$ (7)	Relative Change[d] $\left(\frac{MAERC}{MAARC}\right)$ (8)
14	*OMD*	billion 58$	−2.05	−0.03	3.279	2.053	4.403	.066	1.035	1.033
15	*UMD*	billion 58$	2.04	−0.36	6.158	2.008	3.674	.113	.922	.976
16	*HS*	thousand/year	−16.48	7.01	109.642	64.837	4.723	.079	.925	.925
17	*RS*	% per annum	−0.07	−0.02	0.212	0.149	4.797	.062	.556	.512
18	*RL*	% per annum	−0.01	−0.002	0.145	0.074	1.828	.033	.778	.802
19	*W*	dollars	−0.05	−0.002	0.054	0.015	0.674	.022	.667	.579
20	*LC/O*	dollars	−0.01	−0.0005	0.013	0.003	0.552	.002	.784	.784
21	*M*	billion $	−1.02	−0.02	1.359	0.493	0.336	.001	.533	.532

[a] For explanation of symbols, see Table 1.1.

[b] The average errors, with and without regard to sign (ME and MAE), which are listed in columns 1 to 4 are expressed in these units, and so are the average actual values (MAA); hence the figures for MAE/MAA, in columns 6 and 7, are pure ratios.

[c] In percentage points. Mean of the differences: quarterly per cent change in the simulated series, minus quarterly per cent change in the actual series, all taken without regard to sign.

[d] Ratio of figure in column 5 to the corresponding mean of actual percentage changes.

[e] Not applicable: Since net change in inventories and net foreign investment can assume negative values, these series can be analyzed only in absolute, not in relative terms.

positive average errors (*ISE* and *UMD* for levels; *IH*, *AWW*, and *HS* for changes; and *NE* and *UN* for both levels and changes) all have large cyclical and irregular fluctuations in actual values. The differences between the average levels of *A* and *S* are relatively small in most of these cases and not very important. Furthermore, measures of average changes taken without regard to sign, or separately, for periods of expansion and contraction (Table 3.7), suggest that the fluctuations of *S* are usually smaller than those of *A*, even for these variables.

Table 3.5 shows that for all variables $|\text{MEL}| > |\text{MEC}|$ and $\text{MAEL} > \text{MAEC}$; also $\text{MAEL} > |\text{MEL}|$ and $\text{MAEC} > |\text{MEC}|$ (see columns 1 to 4). These relationships and the underlying causes are already familiar. (They were discussed for the Wharton Model in Section 3.1.2, above.) The fact that the level errors exceed the corresponding change errors by relatively large amounts must again be attributed to the cumulation of errors over time, which throws the *S* predictions increasingly off base.

The mean absolute errors of relative change in *S*, as compared with *A*, vary from 0.2 to 0.7 percentage points for the comprehensive income, consumption, and employment aggregates, and also for prices, wages, and money. (See the entries in column 5, lines 1 to 3, 8 to 10, 13, and 19 to 21.) The errors for the other variables—investment outlays and commitments, unemployment, corporate profits, and interest rates—are substantially larger in these terms, ranging from 1.8 to 6.0 percentage points.

Another relationship that is confirmed by the figures in Table 3.5 is that $\frac{\text{MAEL}}{\text{MAAL}} < \frac{\text{MAEC}}{\text{MAAC}}$ (columns 6 and 7). It is the ratios for the absolute and relative changes in the last two columns of the table (the two values are generally close to each other) that, here as elsewhere, deserve our particular attention. These figures are of the order of .3 to .6 for the most comprehensive aggregates, as well as for money supply and the price, wage, and short-term interest series. They are much higher (.8 to .9) for the more volatile investment series, such as *ISE*, *II*, and *HS* and for various—mainly leading—indicators, such as *UN*, *CPR*, *AWW*, *UMD*, *RL*, and *LC/O*. Only for one variable, new orders for durables, do the ratios exceed one; that is, the average errors slightly exceed the average actual changes.

3.2.3 The correlations between the actual and simulated levels of the series exceed .86 for sixteen, and .95 for twelve, of the twenty-one variables. For the volatile series *HS*, *UMD*, *UN*, *II*, and *IH*, the *r*-coefficients are significantly lower, varying from .597 to .721. Again, it is principally the common trends in *A* and *S* that explain the high correlations of levels, as there is much less agreement between the shorter movements in these series. Correlations between changes in *S* and *A* are much lower than those between levels, although they vary a great deal, from .185 to .980. (See Table 3.6, column 1.) The lowest correlations, between .1 and .4, are for the price, wage, profits, and a few other series, notably on investment (*ISE*, *HS*, and *AWW*). The highest correlations, exceeding .7, are for real *GNP*, net exports, short-term interest rate, and employment.

The regression results are favorable, in the sense that the intercepts are small (near zero) and the slope coefficients are not clearly different from one, for most of the variables. This is true even for some variables for which the correlations of simulated with actual changes are relatively low, notably *UN* and *AWW*. (See columns 3 and 4 and also the tests in columns 5 and 6.) As these cases illustrate, a simulation *S* (viewed as a set of predictions) can be unbiased and efficient (i.e., have errors that are unsystematic and uncorrelated with the values of the *S* series itself), although it is only weakly correlated with the realizations *A*. (That is, the residual variance in the regression of *A* on *S* is large, relative to the variance of *A*.) On the other hand, some of the estimates are clearly unsatisfactory if these criteria are accepted. For the series *P*, *OMD*, *W*, and *LC*/*O*, for example, the intercepts are too large and the slope coefficients, too small.

3.2.4 Table 3.7 shows the average amplitudes of rises and falls in the actual and simulated series, per month, in reference-cycle relatives (columns 1 to 4).[16] During the sample period, several actual series with particularly dominant trends grew on the average in both expansions and contractions, though at higher rates in the former than in the latter (*cf.* entries in columns 2 and 4 for *C*, *YP*, *P*, *W*, *LC*/*O*, and

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[16] There are only two complete trough-to-trough reference cycles in the period 1953–66 (1954–57–58 and 1958–60–61); but, if the proxy date for a "peak" in 1966-IV is accepted, three peak-to-peak cycles can be distinguished in the same period, namely 1953–54–57, 1957–58–60, and 1960–61–66. We chose the second rather than the first alternative in order to utilize as much of the available information as possible.

TABLE 3.6

Nonstochastic Simulations for the Sample Period, OBE Model: Correlation, Regression, and Test Statistics (1953-II–1966-IV)

Line	Variable Symbol[a]	Correlation of Simulated With Actual Changes[b] r (1)	$\bar{r}^2$ (2)	Regression of Actual on Simulated Changes[c] Constant (a) (3)	Slope (b) (4)	F-ratio for ($\alpha = 0, \beta = 1$)[d] (5)	t-test for $\beta = 1$[e] (6)
1	*GNP*	.617	.369	.266	.847	0.70	1.03
2	*GNP58*	.710	.494	−.032	1.068	0.14	0.47
3	*C*	.478	.214	.314	.674	1.83	1.91
4	*IH*	.500	.236	−.295	.747	1.24	1.43
5	*ISE*	.368	.120	.325	.828	0.28	0.60
6	*II*[f]	.581	.326	.146	1.175	0.41	0.78
7	*NE*[f]	.862	.738	−.036	.871	1.88	1.83
8	*YP*	.635	.392	.349	.786	1.75	1.63
9	*P*	.185	.016	.334	.342	4.00	2.64
10	*LE*	.980	.960	.014	1.089	4.43	2.97
11	*UN*[f]	.501	.237	−.013	1.022	0.03	0.09
12	*CPR*	.388	.135	.626	.630	1.64	1.80

(continued)

TABLE 3.6 (*concluded*)

Line	Variable Symbol[a]	Correlation of Simulated With Actual Changes[b]		Regression of Actual on Simulated Changes[c]			
		r (1)	$\bar{r}^2$ (2)	Constant (a) (3)	Slope (b) (4)	F-ratio for ($\alpha = 0, \beta = 1$)[d] (5)	t-test for $\beta = 1$[e] (6)
13	*AWW*	.294	.069	.005	1.097	0.02	0.20
14	*OMD*	.435	.174	.383	.531	4.84	3.11
15	*UMD*	.482	.217	.474	.792	1.21	1.05
16	*HS*	.375	.125	−.483	.930	0.18	0.22
17	*RS*[f]	.829	.681	.022	.928	0.54	0.84
18	*RL*[f]	.626	.380	.004	.941	0.08	0.37
19	*W*	.273	.057	.606	.493	2.56	2.12
20	*LC/O*	.412	.154	.245	.602	2.86	2.18
21	*M*	.665	.432	.040	.951	0.09	0.33

[a] For meaning of symbols, see Table 1.1.

[b,c,f] See the corresponding footnotes in Table 3.2.

[d] Some of the relevant percentage points of the F-distribution (for $n_1 = 2, n_2 = 53$) are: $F_{0.01} = 5.05$, $F_{0.05} = 3.19$, $F_{0.10} = 2.40$, and $F_{0.25} = 1.43$.

[e] Some of the relevant percentage points of the t-distribution ($n = 53$, two-tailed test) are: 2.68, 2.01, 1.68, and 1.16 (for *1, 5, 10, and 25 per cent significance* levels, respectively).

TABLE 3.7

Nonstochastic Simulations for the Sample Period, OBE Model: Average Amplitudes of Cyclical and Trend Movements
(1953-II–1966-IV)

		Average Change Per Month in Reference-Cycle Relatives[b] During:						Mean Absolute Per Cent Change Trend-Cycle Component[c]	
		Expansions		Contractions		Full Cycle			
Line	Variable Symbol[a]	Simulated (1)	Actual (2)	Simulated (3)	Actual (4)	Simulated *(1)* − *(3)* (5)	Actual *(2)* − *(4)* (6)	Simulated (7)	Actual (8)
1	*GNP*	.50	.60	.16	−.08	.34	.68	1.30	1.48
2	*GNP58*	.34	.42	.02	−.21	.32	.63	.91	1.14
3	*C*	.35	.39	.21	.03	.14	.36	.93	1.00
4	*IH*	−.02	−.14	−.13	.16	.11	−.30	1.58	2.82
5	*ISE*	.42	.66	.12	−.64	.30	1.30	1.14	2.12
6	*II*	.10	.27	−.27	−.68	.37	.92	.70	1.41
7	*NE*	.054	.007	−.061	.054	.115	−.44	.58	.65
8	*YP*	.48	.57	.23	.06	.25	.51	1.26	1.36
9	*P*	.15	.18	.14	.13	.01	.05	.43	.48
10	*LE*	.13	.18	−.02	−.15	.15	.33	.34	.52

(continued)

TABLE 3.7 (*concluded*)

		Average Change Per Month in Reference-Cycle Relatives[b] During:						Mean Absolute Per Cent Change Trend-Cycle Component[c]	
		Expansions		Contractions		Full Cycle			
Line	Variable Symbol[a]	Simulated (1)	Actual (2)	Simulated (3)	Actual (4)	Simulated (*1*) – (*3*) (5)	Actual (*2*) – (*4*) (6)	Simulated (7)	Actual (8)
11	*UN*	–.096	–.24	.008	.05	–.104	–.29	3.25	5.97
12	*CPR*	.55	.86	.23	–1.10	.32	1.96	2.03	3.47
13	*AWW*	–.02	0	–.07	–.13	.05	.13	.10	.24
14	*OMD*	.36	.53	–.44	–1.08	.80	1.61	2.22	3.79
15	*UMD*	–.01	.47	–1.74	–2.45	1.73	2.92	2.25	3.59
16	*HS*	–.15	–.38	.18	.32	–.33	–.70	1.88	4.08
17	*RS*	.05	.06	–.14	–.14	.19	.20	7.81	8.23
18	*RL*	.022	.024	–.019	–.028	.041	.52	1.56	2.06
19	*W*	.36	.42	.26	.15	.10	.27	1.00	1.06
20	*LC/O*	.13	.19	.11	.04	.02	.15	.39	.62
21	*M*	.17	.20	.18	.09	–.01	.11	.55	.60

NOTE: The period covered by the measures in columns 1 through 6 is from the peak in 1953-II to the assumed peak in 1966-IV (*see text*).

[a] For meaning of symbols, see Table 1.1.

[b,c] See the corresponding footnotes to Table 3.3.

M). The housing series, *IH* and *HS*, responded perversely; they show negative amplitude figures during expansions and positive ones during contractions (lines 4 and 16). Net exports increase more in contractions than in expansions (line 7). For all other variables—including the unemployment rate, which is treated on the inverted plan—a strong procyclical response is observed, with the average amplitudes in the actuals being positive in expansions and negative in contractions.

For business-cycle expansions, the average amplitudes of *S* and *A* agree in sign, with but two exceptions. (The figures for *AWW* and *UMD* are negative in the simulations.) For contractions, there are six cases of directional disagreement, relating to *GNP, GNP58, ISE, CPR, IH,* and *NE*. (All but the last two of these involve positive changes in *S*.)

During business-cycle expansions, the average amplitudes of the *S* series are smaller than those of the *A* series, except only for *NE* and *AWW*, where changes are very small (columns 1 and 2). The comparison of amplitudes, however, is less easily summarized for business-cycle contractions. In the eight cases where declines prevailed in the average contraction amplitudes of both *S* and *A* (including the inverted unemployment rate), average declines of *A* exceeded those of *S* in seven instances, the only exception being *RS*, where the declines are of the same magnitude. When both *S* and *A* showed retardations rather than actual declines (*C, YP, P, W, LC/O,* and *M*), *A* increased less than *S* throughout. In addition, as mentioned above, four series show average contraction amplitudes which are negative for *A* and positive for *S*. The two construction series (*IH* and *HS*), whose behavior seems to be countercyclical for *A*, are somewhat less countercyclical in *S*. The resulting full-cycle amplitudes (expansion amplitudes minus contraction amplitudes) are larger for *A* than for *S* in all cases except net exports and the two construction series. All of this constitutes strong evidence for the existence of a general tendency of simulations to underestimate fluctuations during historical business cycles.

When the amplitudes are measured independently of the timing of cyclical turns in the series, as the mean absolute percentage changes in trend-cycle components (columns 7 and 8), the tendency for *S* to underestimate the variability of *A* is again very strongly in evidence. Here the figure for *S* is in each case smaller than that for *A*.

The ranks of the variables based on the average reference-cycle amplitudes of *S* and *A* are positively correlated for both rises and falls, with Spearman coefficients (adjusted for tied ranks) of .937 and .714, respectively. The correlation between the ranks based on the mean absolute percentage changes in the trend-cycle components of *S* and *A* is .962.

3.2.5 The simulated series for nominal and real *GNP* fail to reproduce the contractions of the actual series in 1957–58 and 1960–61, which means that they skip four of the six business-cycle turns in the OBE sample period (Table 3.8, column 1). For consumption, *S* skips all six turns; and the simulations for eight other variables omit four or two turns each, while the actuals omit none. By this criterion, the *A* series conform to business cycles better than do the *S* series for twelve of the eighteen variables.

Turning points that are unconnected with general economic revivals and recessions constitute another class of indicators of nonconforming behavior. Only a few of the OBE simulations show such episodes where the actual series have none. (See the entries for *UN*, *OMD*, and *UMD* in column 2.)

A summary of the frequency distributions of leads, coincidences, and lags is followed here by an attempt to indicate the prevalent type of timing for each of the series (Table 3.8, columns 3 to 7). For eight variables, including *GNP* and several comprehensive, mostly coincident, indicators, the absence or paucity of turning points, or the heterogeneity of such timing observations as can be made, prohibit such a determination for the *S* series, and the labels used in these cases are "no turns" (n.t.) or "not identified" (n.i.). For the unemployment rate, the timing of *S*, instead of being coincident, is rather irregular but mostly lagging. However, for the nine remaining variables, the correspondence between the timing of *S* and *A* is, on the whole, good. And these comparisons cover a variety of timing patterns, including some with prevalent leads, as in housing starts and new orders; and others with prevalent lags, as in unit labor costs and interest rates.[17]

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[17] It must be recognized that the determination of the timing patterns is necessarily more uncertain for the *S* series than it is for the *A* series, because there is some additional evidence on *A* but not on *S*. Wherever possible, we have checked the timing of the *A* series in the sample period against the timing of longer series for the same variables (to cover, at least, the entire postwar period). In some cases, such comparisons could only

Table 3.8 does not include three variables: *P*, *W*, and *NE*. No cyclical turning points in either the actual or the simulated series can be identified for the price level and the wage rate. Net exports show five major turns in the period 1953–66 (or seven if one includes a short decline in 1955). Only two of these can be matched with the reference dates (relating to the 1957–58 recession), but all were reproduced in the *S* series, and rather well at that.

3.2.6 Chart 3.4 shows the reference-cycle patterns of *S* and *A* for selected variables. The patterns, again extended to show both sides of each turn, cover the two T-to-T cycles between 1954 and 1961, the last curve in each set representing the average of the two.

In general, the patterns show the simulations in better light than do the measures previously discussed, probably because they involve considerable smoothing and detrending of the data. They demonstrate that the *S* series often underwent marked retardations during business contractions, corresponding to mild declines in the *A* series: this is so for each of the comprehensive indicators of production, income, consumption, and employment—*GNP*, *GNP58*, *C*, *YP*, and *LE*. In this situation, amplitude figures have different signs for *S* and *A*, and the timing comparisons show skipped turns for *S*, so that a somewhat exaggerated impression of simulation errors may be created by these measures, which the patterns help to correct.

For most variables, the *S* patterns have smaller amplitudes than the *A* patterns, reflecting the underestimation of cyclical movements in the *S* series. The relatives for *S* are usually higher than those for *A* in contractions, and lower in expansions. However, apart from these differences (which, although apparently systematic, are not always pronounced), many of the *S* patterns resemble rather well the corresponding patterns for *A*, even where the latter show large fluctuations with diverse timing. Good illustrations of this statement are provided by the diagrams for corporate profits, *CPR* (except in 1960–61), and, particularly, for the interest series *RS* and *RL*. On the other hand, there are also some cases of drastic dissimilarity between the paired

be very tentative or approximate, being based on fairly short records, or on data for related, rather than the same, variables (for example, different interest-rate series or undeflated *GNP* components). However, allowing for such discrepancies as are likely to arise in some of these cases, it is possible to conclude that our identifications for *A* generally do agree with those historical classifications that are applicable.

TABLE 3.8

Nonstochastic Simulations for the Sample Period, OBE Model: Timing at Business-Cycle Turns and Corresponding Measures for the Actual Values (1953–1966)

		Frequencies of Timing Observations for Series *S* and *A*						
Line	Variable Symbol[a]	Business Cycle Turns Skipped[b] (1)	Extra Turns[b] (2)	Leads (3)	Exact Coinci-dences (4)	Lags (5)	Long Leads or Lags[c] (6)	Dominant Type of Timing[b] (7)
1	*GNP S*	4	0	1	0	1	0	n.i.
2	*A*	0	0	3	3	0	0	coincident
3	*GNP58 S*	4	0	1	0	1	0	n.i.
4	*A*	0	0	3	3	0	0	coincident
5	*C S*	6	0	0	0	0	0	n.i.
6	*A*	0	0	2	3	1	1	coincident
7	*IH S*	2	2	4	0	0	3	leading-irregular
8	*A*	2	2	3	1	0	2	leading-irregular
9	*ISE S*	2	0	1	1	2	2[d]	n.i.
10	*A*	0	2	0	3	3	0	coincident-lagging
11	*II S*	0	4	3	2	1	1	leading-irregular
12	*A*	0	4	4	2	0	2	leading-irregular
13	*YP S*	4	0	1	0	1	0	n.i.
14	*A*	2	0	2	1	1	0	coincident
15	*LE S*	4	0	1	1	0	0	n.i.

16		*A*	0	0	1	4	1	0	coincident
17	*UN*[e]	*S*	2	2	1	1	2	1*	lagging-irregular
18		*A*	0	0	2	3	1	0	coincident
19	*CPR*	*S*	2	0	3	1	0	1	leading-irregular
20		*A*	0	0	5	1	0	2	leading
21	*AWW*	*S*	4	2	0	1	1	0	n.i.
22		*A*	0	3	3	1	2	2	leading
23	*OMD*	*S*	0	2	5	1	0	1	leading
24		*A*	0	0	5	1	0	3	leading
25	*UMD*	*S*	2	2	2	0	2	2*	n.i.-irregular
26		*A*	0	0	3	2	1	1	leading-coincident
27	*HS*	*S*	2	2	4	0	0	3	leading
28		*A*	0	2	6	0	0	3	leading
29	*RS*	*S*	0	0	1	2	3	1*	lagging
30		*A*	0	0	1	1	4	0	lagging
31	*RL*	*S*	0	0	0	2	4	2*	lagging
32		*A*	0	0	1	1	4	1*	lagging
33	*LC/O*	*S*	4	0	0	0	2	1*	n.i.-lagging
34		*A*	0	0	0	0	6	2*	lagging
35	*M*	*S*	2	0	4	0	0	2	leading
36		*A*	2	0	3	1	0	2	leading

---

[a] For meaning of symbols, see Table 1.1.

[b] See explanation in text.

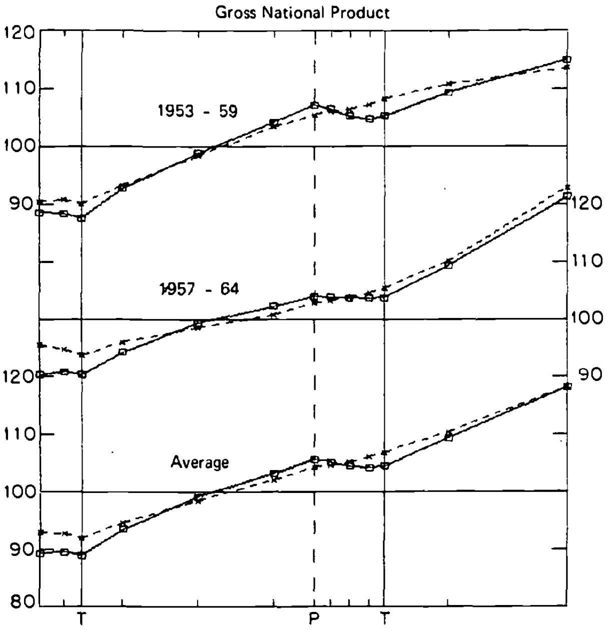
[c] Leads or lags of three or more quarters. Numbers marked by asterisk refer to lags, others refer to leads at business-cycle turns.

[d] Includes one lead and one lag.

[e] Treated on the inverted plan (*see Table 3.4*).

CHART 3.4

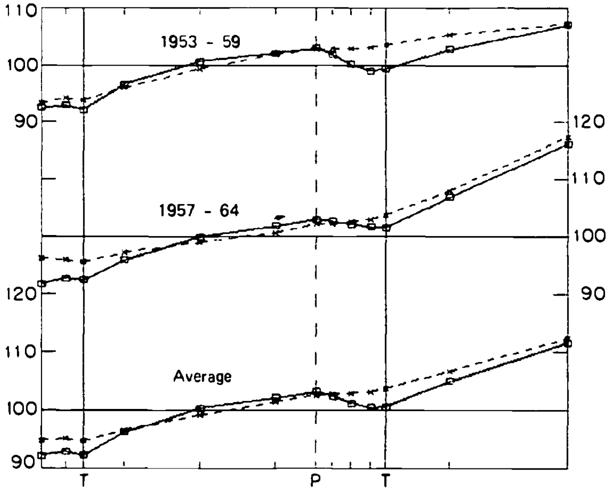
Nonstochastic Simulations for the Sample Period, OBE Model: Reference-Cycle Patterns for Simulated and Actual Series, Selected Variables (1953–1964)



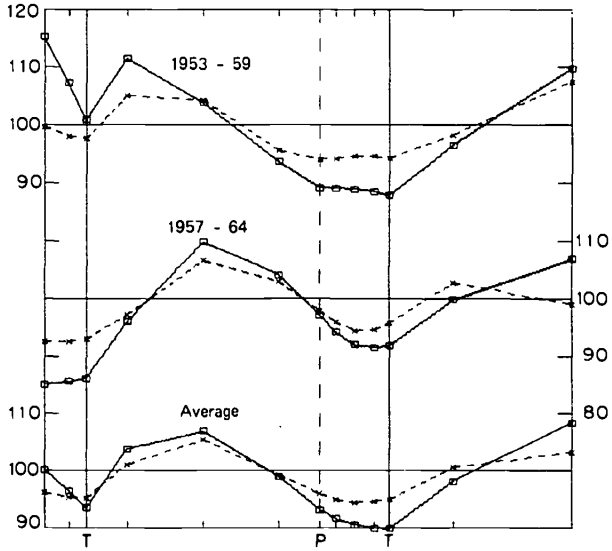
NOTE: Scale in reference-cycle relatives or (for unemployment rate and short-term interest rate) in absolute deviations from cycle base. *P* and *T* stand for peaks and troughs, respectively.

CHART 3.4 (*continued*)

Gross National Product, 1958 Dollars

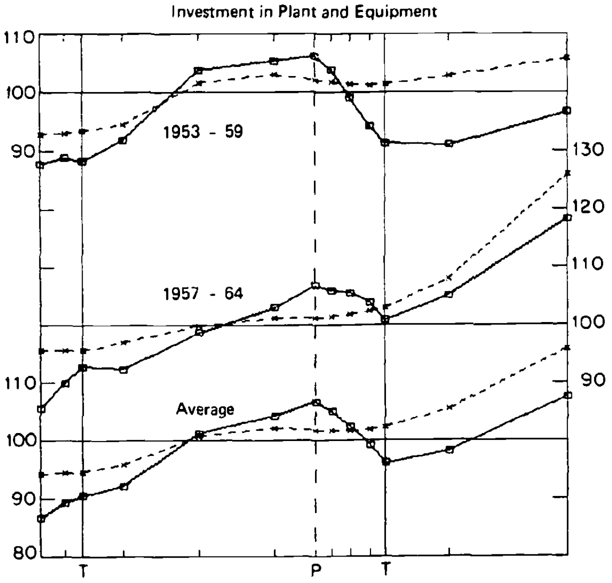


Investment in Housing



(*continued*)

CHART 3.4 (*continued*)



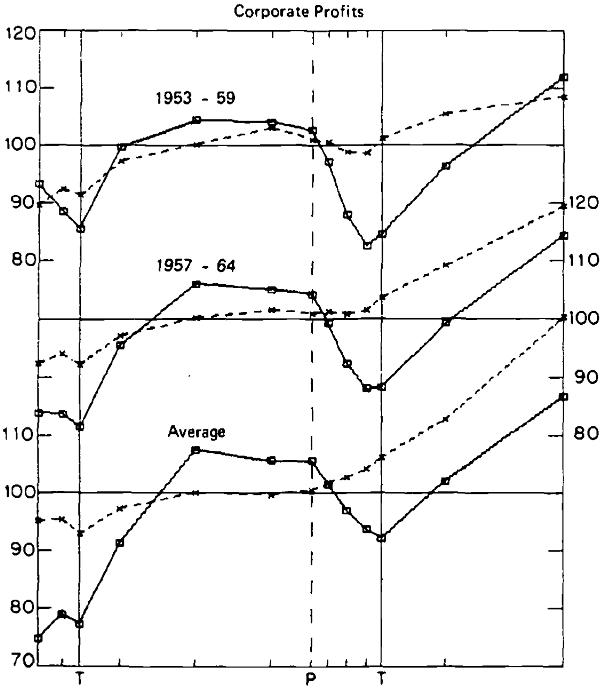
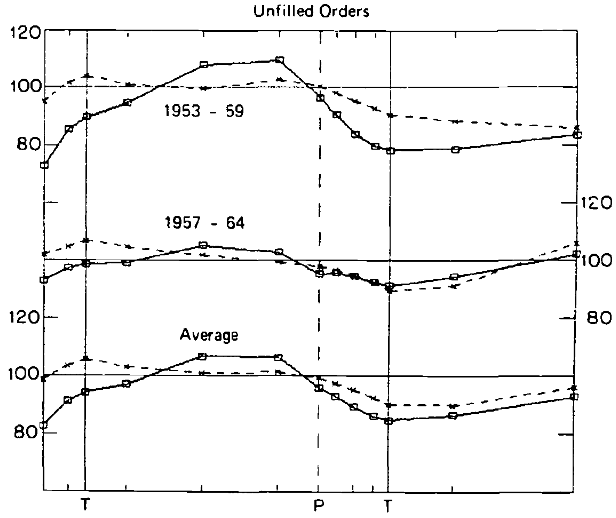
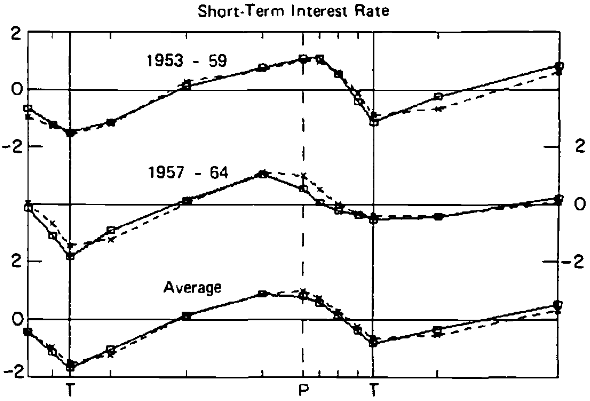


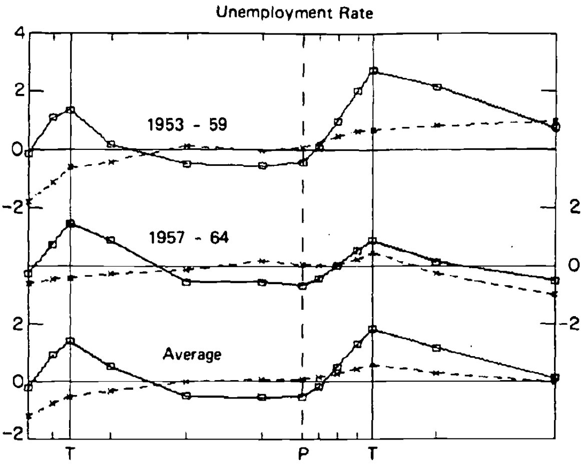
CHART 3.4 (*continued*)





(*continued*)

CHART 3.4 (*concluded*)



patterns, notably for the inventory change, unemployment, and unfilled orders.

3.3 THE FRB-MIT-PENN (FMP) MODEL

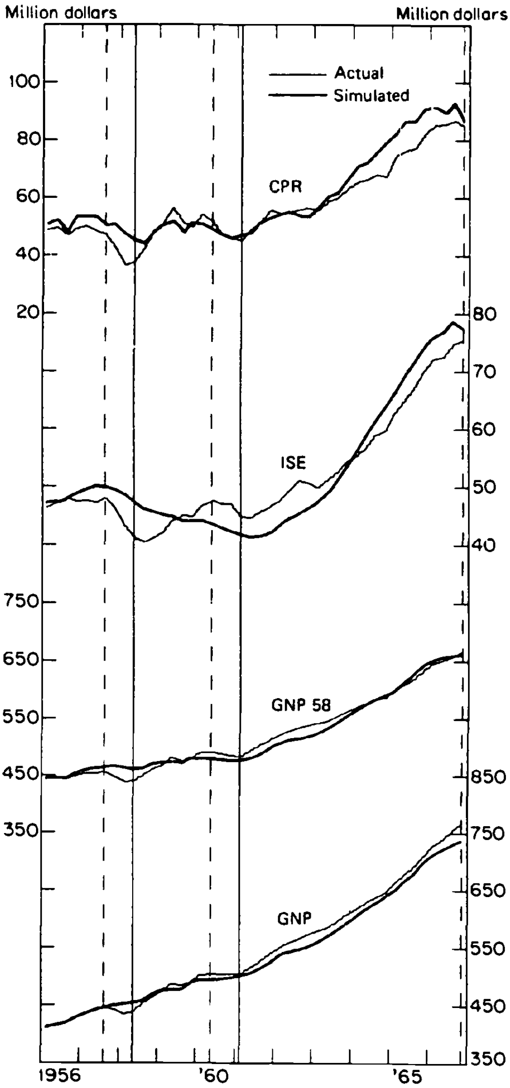
3.3.1 The sample period for this model is relatively short: it begins only in 1956 and covers the eleven years through 1966, or 44 quarterly observations. Thus, the contractions and retardations included are those that are also covered by the OBE sample-period simulations, except for the 1953–54 recession.

Chart 3.5 shows that the simulated *GNP* series declined only once for one quarter: in 1959-III, during the major steel strike. Recorded *GNP* also had two two-quarter declines in 1957–58 and 1960–61, but the S series continued to move upward during these recessions, although at lower rates. Through mid-1957, the levels of S and A are very close, and then, in 1957–58, A falls below S; but thereafter, in 1959–66—for nearly eight years—the levels of *GNP* are consistently underestimated; i.e., $S_t < A_t$.

The simulation of *GNP* in 1958 dollars looks better, in that here S declined along with A in 1957–58 and 1960 (as well as briefly in 1959). The level comparisons give similar results to those for *GNP*

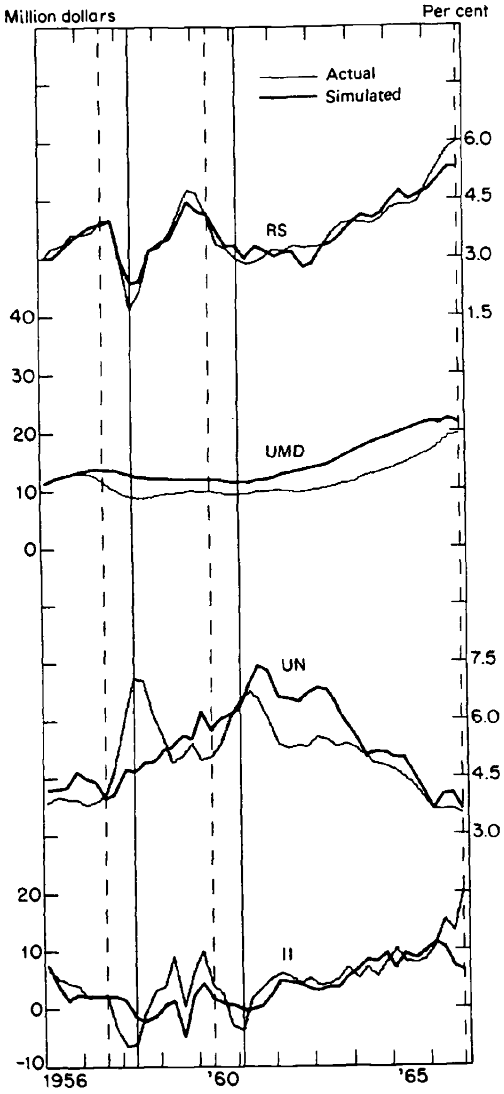
CHART 3.5

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model: Simulated and Actual Series for Selected Variables (1956–1966)



(*continued*)

CHART 3.5 (*concluded*)



in current dollars in the period 1956–63; but in 1964–66, the *S* series rises above *A*, so that there is less underestimation for real, than for nominal, *GNP*. The *A* and *S* series for consumption in real terms are similarly related, except that here *S* shows only a retardation, not a decline, in 1960.

There are some analogous features in the graphs for the other trend-dominated, relatively smooth and stable variables. The one mild decline of personal income, in 1957–58, is not reproduced in the simulated series; also, $S_t < A_t$ persistently in 1959–66 for this aggregate. Simulated employment, too, fails to match the recorded contraction in 1957–58 and underestimates actual employment, except between mid-1957 and mid-1959; but here *S* does move down along with *A* in 1960–61. No turning points occurred in either *A* or *S* for the price index and the wage rate. The price level was underestimated in 1956–59, and again, more strongly, in 1963–66. The wage level was underestimated throughout and, again, more so in the last three or four years of the sample period.

For the more cyclical variables, trends are generally weaker and less consistent, and so are any differences between trends in *S* and *A* and the resulting discrepancies in levels. Once more, the striking feature of these graphs is that some fluctuations in *A* are entirely skipped by *S* (for example, the 1957–59 cycle in unemployment; the 1958–61 cycle in man-hours) and others are heavily muted (e.g., the 1958–61 movements in inventory investment). For one variable in this group—unfilled orders for machinery and equipment—a persistent difference in levels of the two series is observed, with *S* exceeding *A* throughout the period 1957–66. Rather close fits to *A* are provided by the *S* series for the short- and long-term interest rates.

3.3.2 The MEL figures are negative for most variables, indicating that the actual levels tend to be underestimated in the sample-period simulations (Table 3.9, column 1).[18] There is more variation in sign among the MEC statistics, but they, too, are negative for most of the series with strong upward trends, reflecting the tendency of *S* to have smaller rates of growth than *A* (column 2). As elsewhere, the

[18] The values of MEL are positive for *ISE*, *NE*, *UN*, *CPR*, and *OUME*—all variables that had relatively weak trends and marked fluctuations in the period covered by these simulations.

TABLE 3.9

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model: Average Errors and Their Ratios to Average Actual Values (1956–1966)

	Variable		Mean Error (ME)		Mean Absolute Error (MAE)			Ratio of MAE to Mean Absolute Actual Values (MAA)		
Line	Sym-bol[a]	Unit[b]	Level (MEL) (1)	Change (MEC) (2)	Level (MAEL) (3)	Change (MAEC) (4)	Relative Change[c] (MAERC) (5)	Level $\left(\frac{MAEL}{MAAL}\right)$ (6)	Change $\left(\frac{MAEC}{MAAC}\right)$ (7)	Relative Change[d] $\left(\frac{MAERC}{MAARC}\right)$ (8)
1	*GNP*	billion $	−7.31	−0.70	10.034	3.159	0.609	.018	.357	.377
2	*GNP58*	billion 58$	−2.02	−0.13	8.807	3.235	0.652	.017	.497	.524
3	*C*	billion 58$	−1.18	0.10	4.839	1.618	0.493	.14	.463	.480
4	*IH*	100 billion 58$	−0.003	0.002	0.010	0.004	2.000	.044	.698	.733
5	*ISE*	billion 58$	0.30	0.03	3.057	0.876	1.792	.059	.704	.746
6	*II*	billion 58$	−1.17	−0.34	2.794	2.129	[e]	.469	.874	[e]
7	*NE*	billion 58$	0.23	−0.04	0.947	0.352	[e]	.196	1.017	[e]
8	*YP*	billion $	−7.57	−0.47	8.769	1.425	0.355	.020	.223	.245
9	*P*	1958 = 100	−0.84	−0.08	1.203	0.228	0.218	.012	.443	.429
10	*LE*	million	−0.33	−0.01	0.755	0.222	0.341	.011	.694	.723
11	*UN*	per cent	0.29	−0.004	0.670	0.338	6.256	.133	1.194	1.155
12	*CPR*	billion $	4.27	−0.02	5.490	5.513	12.174	.095	0.141	0.928
13	*LH*	thousand hours/year	−0.0004	0.0002	0.007	0.005	0.251	.004	.962	.962
14	*OUME*	billion $	3.01	0.04	3.021	0.334	2.661	.258	.768	.741
15	*RS*	per cent	−0.03	−0.02	0.219	0.196	6.002	.059	.741	.761
16	*RL*	per cent	−0.04	−0.004	0.092	0.078	1.913	.022	.764	.767
17	*W*	dollar	−6.38	−0.36	6.378	0.795	0.309	.025	.294	.280
18	*M*	billion $	−1.14	0.0006	1.398	0.562	0.386	0.001	.555	.570

[a] For meaning of symbols, see Table 1.1.

[b,c,d,e] See corresponding footnotes to Table 3.1.

change errors are on the average much smaller than the level errors; i.e., |MEL| > |MEC| and MAEL > MAEC, which again is largely attributable to the cumulation of errors over time (columns 1 through 4). The absolute means of the relative change errors, MAERC, range from 0.2 percentage points for *P* to 12.2 percentage points for *CPR*, and are somewhat larger than 0.6 for *GNP* in current and constant dollars (column 5).

The relative accuracy analysis once more shows the level ratios, MAEL/MAAL, to be quite low, and much smaller than the corresponding change ratios, MAEC/MAAC and MAERC/MAARC, which tend to be similar (columns 6 through 8). The ratios of the average change errors fall in the range between .2 and .4 for *GNP*, *YP*, and *W*, and in the range .4 to .6 for *GNP58*, *C*, *P*, and *M*; and they exceed .6, but are less than one, for the other variables, except *UN* and *NE* where they are alone in exceeding unity (columns 7 and 8).

3.3.3 The correlations between the simulated and actual levels are very high for this model, exceeding .95 for twelve variables and .7 for fifteen variables; the lowest of these coefficients is that for the unemployment rate (.67). Correlations between the changes, ΔS_t and ΔA_t, are substantially lower, though still generally respectable; they vary from .27 to .95, but all except three exceed .5, while the correlations between the relative changes, $(\Delta S/S)_t$, and $(\Delta A/A)_t$, shown in Table 3.10, are in most instances lower still, but not by much. The highest of the change correlations, ranging from .6 to .8, are recorded for the two *GNP* series, *C*, *IH*, *YP*, *P*, *NE*, *M*, and the two interest rates. The lowest are those for *UN* and hours per man, *LH*, which are close to .3 (Table 3.10, columns 1 and 2).

The regressions of actual on simulated changes give encouraging results. For most variables, the hypothesis that $\alpha = 0$ and $\beta = 1$ cannot be rejected on any of the considered significance levels (columns 3 to 6). However, there are good grounds for rejection in the cases of *P*, *LE*, *UN*, and *M*, and for at least some doubts concerning *ISE* and *CPR*. Quite generally, there is little indication of bias here, and such problems as are suggested by the tests relate principally to inefficiency; i.e., deviations of the slope coefficients from unity (as a rule in the downward direction).

3.3.4 Peak-to-peak reference cycles are used for the measure-

TABLE 3.10

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model: Correlation, Regression, and Test Statistics (1956–1966)

Line	Variable Symbol[a]	Correlation of Simulated With Actual Changes[b]		Regression of Actual on Simulated Changes[c]			
		r (1)	$\bar{r}^2$ (2)	Constant (a) (3)	Slope (b) (4)	F-ratio for ($\alpha = 0$, $\beta = 1$)[d] (5)	t-test for $\beta = 1$[e] (6)
1	*GNP*	.633	.387	.003	.848	0.78	0.94
2	*GNP58*	.652	.411	.002	.772	1.34	1.63
3	*C*	.674	.440	.002	.802	1.06	1.44
4	*IH*	.740	.536	.0002	1.196	0.67	1.15
5	*ISE*	.573	.312	.004	.655	2.78	2.36
6	*II*[f]	.488	.219	.337	.742	1.07	1.24
7	*NE*[f]	.905	.813	.043	.979	0.18	0.25
8	*YP*	.690	.464	.004	.805	1.66	1.48
9	*P*	.601	.346	.003	.564	9.44	3.72
10	*LE*	.535	.269	.002	.586	4.14	2.87

TABLE 3.10 (*concluded*)

Line	Variable Symbol[a]	Correlation of Simulated With Actual Changes[b]		Regression of Actual on Simulated Changes[c]			
		r (1)	$\bar{r}^2$ (2)	Constant (a) (3)	Slope (b) (4)	F-ratio for ($\alpha = 0$, $\beta = 1$)[d] (5)	t-test for $\beta = 1$[e] (6)
11	*UN*[f]	.275	.053	−.004	.341	6.26	3.54
12	*CPR*	.556	.292	.006	.675	2.13	2.06
13	*LH*	.328	.086	−.0003	.577	1.34	1.63
14	*OUME*	.578	.318	−.002	.954	0.14	0.22
15	*RS*[f]	.799	.630	.013	.975	0.08	0.21
16	*RL*[f]	.676	.444	.009	.901	0.25	0.65
17	*W*	.565	.302	.002	.876	1.76	0.62
18	*M*	.729	.520	.204	.613	9.24	4.30

[a] For meaning of symbols, see Table 1.1.

[b,c,f] See the corresponding footnotes in Table 3.2.

[d] Some of the relevant percentage points of the F-distribution (for $n_1 = 2$, $n_2 = 42$) are: $F_{0.01} = 5.16$, $F_{0.05} = 3.21$, $F_{0.10} = 2.44$, and $F_{0.25} = 1.44$.

[e] Some of the relevant percentage points of the t-distribution ($n = 42$, two-tailed test) are: 2.70, 2.02, 1.68, and 1.17 (for 1, 5, 10, and 25 per cent significance levels, respectively).

ment of the average amplitudes in Table 3.11 because, assuming a peak in 1966, there are two such complete cycles in the sample period of the FMP Model (1957–58–60 and 1960–61–66), whereas there is only one complete trough-to-trough cycle (1958–60–61).

During expansions, *S* rose on the average less than *A* for fourteen variables, and declined less than *A* for the unemployment rate, whose movement is typically countercyclical. In the single case of *RL*, *S* increased more than *A*. Both *A* and *S* had negative signs for investment in housing and net exports, and their signs differed for hours per man in the private nonfarm sector, *LH*. However, the average changes per month were exceedingly small in each of these cases.

During contractions, *S* tended to decline less than *A* for seven variables and to increase less than *A* for the unemployment rate. The net exports, personal income, price level, wage-rate, and money-supply series continued to rise, and the corresponding simulations had still larger average increases. For *GNP58*, consumption, and employment the *A* series show small declines and the *S* series show small rises. Finally, both *A* and *S* fell by about equal average amounts in the case of *IH*, while for *RL* the decline in *S* was somewhat larger than that in *A*.

When amplitudes for the full cycle (expansion minus contraction) are compared for simulated and actual series, actual amplitudes exceed simulated ones in all but four cases: net exports, long term interest rates, and the two perversely behaving series, prices and money supply. These measures confirm the fact that cyclical fluctuations in the actual series tend to be underestimated by those in the simulations.

The strongest expression of the tendency for the *S* series to vary over time less than the actuals is provided by the mean absolute percentage changes in the trend-cycle components (columns 7 and 8). According to these measures, *S* had a smaller average amplitude than *A* for each of the included variables.

The differences in average variability across the series are reproduced very well in the simulations. The ranks based on the expansion measures in columns 1 and 2 show a correlation of .967; those based on the contraction measures in columns 3 and 4 have a correlation of .991; and those based on the trend-cycle component measures in columns 7 and 8 have a correlation of .991.

TABLE 3:11

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model:
Average Amplitudes of Cyclical and Trend Movements
(1956–1966)

		Average Change Per Month in Reference-Cycle Relatives[b] During:						Mean Absolute Per Cent Change Trend-Cycle Component[c]	
		Expansions		Contractions		Full Cycle			
Line	Variable Symbol[a]	Simulated (1)	Actual (2)	Simulated (3)	Actual (4)	Simulated (5)	Actual (6)	Simulated (7)	Actual (8)
1	*GNP*	.52	.62	.14	.11	.38	.51	1.36	1.55
2	*GNP58*	.40	.47	–.07	–.26	.47	.73	1.02	1.16
3	*C*	.40	.42	.03	–.06	.37	.48	.97	1.01
4	*IH*	–.07	–.00	–.40	–.39	.33	.39	1.94	2.47
5	*ISE*	.60	.74	–.50	–1.12	1.10	1.86	2.07	2.23
6	*II*	.10	.36	–.31	–.88	.41	1.24	.91	1.47
7	*NE*	–.05	–.07	.28	.13	–.33	–.20	.56	.64
8	*YP*	.50	.57	.25	.08	.25	.49	1.34	1.42
9	*P*	.11	.14	.23	.16	–.12	–.02	.42	.49
10	*LE*	.13	.18	.01	–.15	.12	.33	.37	.46
11	*UN*	–.07	–.25	.02	.05	.09	.30	3.65	4.71

(continued)

TABLE 3.11 (*concluded*)

		Average Change Per Month in Reference-Cycle Relatives[b] During:						Mean Absolute Per Cent Change Trend-Cycle Component	
		Expansions		Contractions		Full cycle			
Line	Variable symbol[a]	Simulated (1)	Actual (2)	Simulated (3)	Actual (4)	Simulated (5)	Actual (6)	Simulated (7)	Actual (8)
12	*CPR*	.72	1.00	–.79	–1.72	1.51	2.72	2.94	3.61
13	*LH*	–.00	.01	–.07	–.11	.07	.12	.13	.21
14	*OUME*	.61	.99	–.70	–1.74	1.31	2.73	2.40	3.49
15	*RS*	.04	.06	–.13	–.18	.17	.24	5.98	6.80
16	*RL*	.021	.018	–.037	–.010	.058	.028	1.78	2.36
17	*W*	.32	.38	.29	.18	.03	.20	.95	1.05
18	*M*	.19	.21	.29	.13	–.10	.08	.66	.66

NOTE: The period covered by the measures in columns 1 through 6 from the peak in 1957-III to the assumed peak in 1966-IV (*see text*).

[a] For meaning of symbols, see Table 1.1.

[b] Based on quarterly data but expressed as rate per month (quarterly rates would be three times as large). Figures for all series except *II*, *NE*, *UN*, *RS*, and *RL* are expressed as reference-cycle relatives; that is, as a percentage of the average level of the series during each business (reference) cycle. Figures for *II*, *NE*, *UN*, *RS*, and *RL* are expressed in absolute units. (See Table 1.1 for units.)

[c] Average quarter-to-quarter percentage change, without regard to sign, in the trend-cyclical component—a smooth, flexible moving average of the seasonally adjusted series.

3.3.5 Table 3.12 represents an attempt to identify the timing characteristics of twelve *S* series produced by the FMP Model, and to compare them with the record of the corresponding *A* series.

The simulated series in current dollars for *GNP* and *YP* show no declines corresponding to those in actual *GNP* during the 1957–58 and 1960–61 recessions. For the price level and the wage rate, neither *A* nor *S* shows any cyclical contractions during the period covered. Hence these variables are omitted from the timing comparisons of Table 3.12. Also excluded is net exports, for which the series show pronounced and well correlated fluctuations but poor conformity to business cycles.

Because of the shortness of the sample period and scarcity of observations at turning points, it has been particularly difficult to infer the timing properties of the *S* series for this model. For six of the twelve variables included in Table 3.12—*C, ISE, LE, UN, LH,* and *OUME*—our verdict in column 7 had to be "not identified." However, this is due primarily to the relatively weak conformity of these simulated series, which is shown by the frequency with which they skipped the business-cycle turns (column 1). It does not necessarily follow that the timing of *S* was very different from that of *A* for the variables concerned; in fact, where comparisons can be made, similarities definitely prevail. Thus, of the 38 comparisons (there were 38 turns in *S* and 50 in *A* for the data covered in Table 3.12, including the "extra" turns), 21 indicate complete agreement; i.e., coincident timing of matched turning points for the paired series, 10 consist of leads or lags of one quarter, and only 7 involve larger timing discrepancies. Also, the average leads or lags of *A* and *S* at the reference turns are not greatly different, being not more than 1.5 quarters apart in any case, and less than one quarter apart for all but three variables.

3.3.6 This section summarizes what can be learned from the reference-cycle patterns for the FMP sample-period simulations. Chart 3.6 presents a selection of such diagrams for the two peak-trough-peak cycles covered (1957–58–60 and 1960–61–66) and the corresponding average patterns; as before, each graph matches the pattern for *S* against that for *A*.

On the whole, the patterns for the *S* series resemble those for the *A* series rather well, but the differences between them tend, again, to be systematic, in that the *S* patterns are "flatter"; i.e., have the smaller

TABLE 3.12

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model: Timing at Business-Cycle Turns and Corresponding Measures for the Actual Values (1956–1966)

		Frequencies of Timing Observations for Series *S* and *A*						
Line	Variable Symbol[a]	Business-Cycle Turns Skipped[b] (1)	Extra Turns[b] (2)	Leads (3)	Exact Coincidences (4)	Lags (5)	Long Leads or Lags[c] (6)	Dominant Type of Timing (7)
1	*GNP58 S*	0	0	2	1	1	0	coincident?
2	*A*	0	0	2	2	0	0	coincident
3	*C S*	2	0	1	0	1	0	n.i.
4	*A*	0	0	1	2	1	0	coincident
5	*IH S*	0	1	2	1	0	2	leading-irregular
6	*A*	0	1	2	1	0	1	leading-irregular
7	*ISE S*	2	0	1	0	1	0	n.i.
8	*A*	0	2	0	2	2	0	coincident-lagging
9	*II S*	0	2	1	1	1	0	coincident?
10	*A*	0	2	2	1	0	0	leading-coincident
11	*LE S*	2	0	0	1	1	0	n.i.
12	*A*	0	0	0	3	1	0	coincident
13	*UN*[d] *S*	2	0	0	1	1	0	n.i.

TABLE 3.12 (*concluded*)

Line	Variable Symbol[a]	Frequencies of Timing Observations for Series *S* and *A*						Dominant Type of Timing (7)
		Business-Cycle Turns Skipped[b] (1)	Extra Turns[b] (2)	Leads (3)	Exact Coinci-dences (4)	Lags (5)	Long Leads or Lags[c] (6)	
14	*A*	0	0	2	1	1	0	coincident-leading
15	*CPR S*	0	0	3	1	0	0	leading
16	*A*	0	0	3	0	1	0	leading-irregular
17	*LH S*	2	1	1	0	1	1	n.i.
18	*A*	0	1	2	1	1	2	leading-coincident
19	*OUME S*	2	0	1	0	1	0	n.i.
20	*A*	0	0	2	0	2	1	leading-lagging
21	*RS S*	0	0	1	1	2	1	lagging-coincident
22	*A*	0	0	1	1	2	0	lagging-coincident
23	*RL S*	0	0	1	1	2	1	lagging-coincident
24	*A*	0	0	1	2	1	1	coincident-lagging
25	*M S*	0	0	4	0	0	2	leading
26	*A*	0	0	4	0	0	2	leading

[a] For meaning of symbols, see Table 1.1. *S* refers to simulations, *A* to actuals.

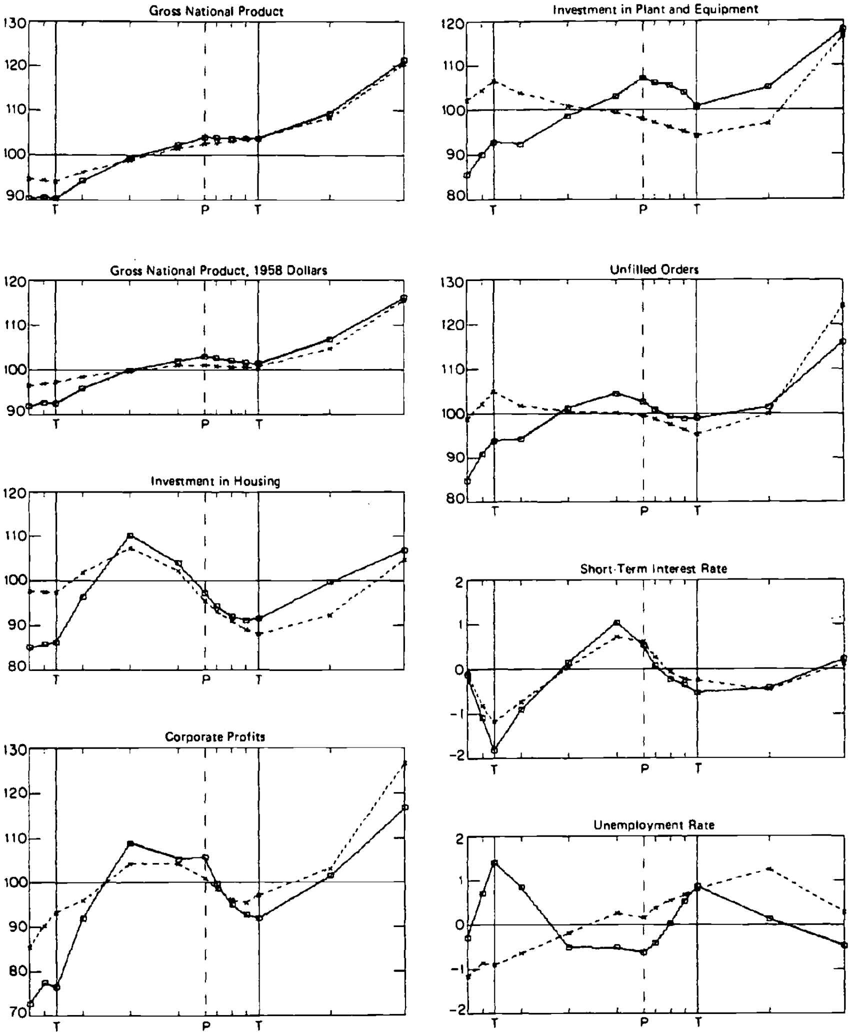
[b] See explanation in text.

[c] Leads or lags of three or more quarters. All figures here refer to leads.

[d] Treated on the inverted plan; see Table 3.4.

CHART 3.6

Nonstochastic Simulations for the Sample Period, FRB-MIT-PENN Model: Reference-Cycle Patterns for Simulated and Actual Series, Selected Variables
(1957–1964)



NOTE: Scale in reference-cycle relatives or (for unemployment rate and short-term interest rate) in absolute deviations from cycle base. *P* and *T* stand for peaks and troughs, respectively.

amplitudes. For series that declined very mildly during the contractions of 1957–58 and 1960–61, the *S* patterns show still smaller declines (as for *GNP58*), or virtually no change (*C*), or very small rises (*GNP*, *YP*). Yet these discrepancies are generally small, and the expansion-contraction contrasts brought out in the *A* patterns reappear nearly as strongly in the corresponding *S* patterns. The diagrams for employment, *LE*, present a similar picture, except for poorer correspondence between *S* and *A* in the 1957–60 cycle. Prices and wages show less retardation of growth during the two recessions covered, and this is broadly reflected in the *S* patterns for these variables, too. The patterns for total hours per man, *LH*, show very little change in terms of reference-cycle relatives during any of the episodes covered, for *S* as well as for *A*.

Some impressive similarities between the cyclical movements of *S* and *A* can be observed in the patterns for several variables that are subject to large fluctuations with diverse timing. This can certainly be said about the difficult to estimate investment in housing *IH*, particularly with reference to the expansion periods 1958–60 and 1961–66. (Interestingly, the fit in the early 1957–58 phase is appreciably less satisfactory.) For plant and equipment *ISE*, the patterns of *A* and *S* also bear good family resemblance, except for the 1958–60 expansion, during which *A* rose but *S* fell. Even for inventory investment, the agreement between the patterns is good as regards the direction of change and the timing of the turning points, although the diagram for the 1957–60 cycle shows large discrepancies between the reference-cycle relatives for *A* and *S* in all except the trough stage. A remarkably close agreement is disclosed by the patterns for net exports. There is much less conformity among the patterns for the unemployment rate and the unfilled orders for machinery and equipment industries, but, interestingly, it is again the early-1957–60 cycle (and particularly its expansion phase, 1958–60) that accounts for the largest divergencies between the simulated and the actual patterns in these variables (*UN* and *OUME*). The general shape of the cyclical movements in corporate profits is rather well reproduced in the *S* patterns, but the deviations of the relatives for simulated *CPR* from their counterparts for the recorded values are large during the 1957–60 cycle. Finally, very good agreement exists between the *S* and *A* patterns for both the short-term and the long-term interest rate.

3.4 SUMMARY INTERPRETATIONS AND COMPARISONS

3.4.1 The cyclical aspects of the nonstochastic sample-period simulations reviewed in this part of our study can be summarized briefly by concentrating on the behavior of the simulations for real *GNP* during each of the general business recessions covered. (See Charts 3.1, 3.3, and 3.5.) These comparisons indicate that each model reflects cyclical behavior substantially better in the early part of its simulation period than in the later part. Each of the models shows the economy (measured by real *GNP*) as declining during the first recession period covered (1948–49 for Wharton, 1953–54 for OBE, and 1957–58 for FMP), or at least during part of this period. The three models also have *GNP58* contracting, or at least flattening out, during the contractions in 1953–54, 1957–58, and 1960–61, respectively. The Wharton Model does not produce a fall in *GNP58* during the recession of 1957–58, and neither the Wharton nor the OBE Model produces one in the 1960–61 recession. Although the FMP Model does produce such declines in these two periods, it would be wrong to conclude that it is better, inasmuch as the initial conditions for this model. being as of 1956-I are much closer to these episodes than are the initial conditions for Wharton and OBE. Had the FMP Model been started in 1948 or 1953, its results for the 1957–58 and 1960–61 declines might have been similar to those obtained for the other models. Or, to put it the other way around, if the initial conditions were selected as of a late date (say 1956) for Wharton and OBE, then it is likely that these models would perform better in the last two recessions, perhaps in a way similar to the FMP predictions.[19]

The important conclusion is that there appears to be a progressive *dampening* of the fluctuations the further away a model's simulation proceeds from its initial-conditions period. This type of movement would be characteristic of a hypothetical economy representing a stable macro-dynamic system insulated from external disturbances. The diminishing oscillations in such a model originate in the divergencies

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[19] According to A. L. Nagar, "Stochastic Simulation of the Brookings Econometric Model" [10, Chapter 12; see in particular, pp. 443–44]. "Better predictions of 1957–58 and 1960–61 would have been observed [in the Brookings simulations] if initial conditions closer to those dates had been selected." As evidence on this point, Nagar cites results obtained in [10, Chapter 11] and [17].

from equilibrium that are likely to exist in any initial state of the system; they tend to disappear as the system approaches its equilibrium rate of growth.

The hypothesis is then naturally completed by the notion that external disturbances, or "erratic shocks," do, in fact, impinge upon the economy continually. The response of the system to these irregular but persistent shocks is such that the damped fluctuations are converted into a maintained movement of the type historically observed as the recurrent business cycles. Following an important paper by Ragnar Frisch [16], this hypothesis gained a strong foothold in business-cycle theory, and became particularly influential in regard to aggregate econometric model-building.[20]

It is possible for a simple macroeconomic model to produce a heavily damped time-path of aggregate real income (output) when solved deterministically (i.e., without random disturbances), but to produce a maintained quasi-cyclical movement when solved stochastically (i.e., with addition of random disturbances).[21] It might appear, therefore, that the failure of nonstochastic sample-period simulations to re-create the continuous cyclical developments that did occur at the time need not, under the above hypothesis, constitute any adverse evidence about the structure of the underlying model. Instead, such results could be due to the suppression of the disturbance terms.

However, it must be noted that the simulations here reviewed use ex post values of exogenous variables and incorporate the effects of any changes in these variables. The latter include a large subset of "autonomous" shocks, such as changes in government defense and nondefense expenditures on goods and services, transfer payments, tax rates, monetary base, reserve requirements, population, exports, and so on.[22] This class of disturbances covers the major impact of monetary and fiscal policy changes, and is presumably very important,

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[20] Frisch, writing in 1933, credits a 1907 Swedish address by Knut Wicksell with the first formulation of this hypothesis. Another important antecedent here is the 1927 paper by Slutsky [28].

[21] For hypothetical examples, see the 1940 paper by Haavelmo [19] and the 1952 article by G. H. Fisher [15]. The 1959 study by Irma and Frank L. Adelman [2] provides an empirical illustration.

[22] The models differ with respect to the identity and contents of exogenous variables, as noted in Section 1.3, but it seems safe to view the autonomous shocks as generally important.

particularly since these (partly nonstochastic) shocks may often cause relatively prolonged repercussions within the economic system. What the sample-period simulations suppress, then, is not these exogenous factors but rather the stochastic components of the endogenous variables. The nondefinitional structural relationships among these variables involve disturbance terms that reflect the impact of a variety of "unique" events, as well as errors of sampling, aggregation, and other aspects of measurement and specification.[23]

We cannot be certain that it is the disregarding of these sources of irregularity that is predominantly responsible for the errors (deviations from *A*) of the nonstochastic sample-period simulations. It can hardly be doubted that there are mis-specification errors in the models, which could be just as important. The autocorrelations of the disturbance terms in some of the original structural equations are high enough to be disturbing. The failure of the simulations to track major cyclical movements can often be traced to certain specific relations that seem weak; e.g., those for inventory investment or the price levels. Under an alternative hypothesis that business cycles are generated endogenously by a deterministic economic system, the absence of confluent specific cycles in the *S* series would have to be judged as indicative of serious specification errors in the given model. (Hypotheses in this class may well be, to a large extent, implausible or unsubstantiated, but to dismiss all of them a priori would be illegitimate, just begging the issue.)

The point of the argument is simply that the evidence of the nonstochastic sample-period simulations alone is inadequate as a basis for discriminating between the different hypotheses. If the performance of these simulations is deemed unsatisfactory, the next logical step is to construct and examine stochastic simulations—which could prove considerably more realistic, thereby lending support to the random-shock hypothesis. On the other hand, it is possible to give more emphasis to the similarities between the nonstochastic *S* series and the actuals; that is, to the capacity of the model to reproduce the economy's short-term movements even when the random error terms in the

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[23] It is the existence of the random error terms in the behavioral equations of the KG Model that explains the introduction of shocks of Type II by the Adelmans. See [2, Section 8].

determination of the endogenous variables are omitted. But then one must remember that these similarities are rather short-lived and must ask next whether nonstochastic simulations beyond the sample period reflect the expected type of fluctuations in any substantial measure. And, since no one can seriously deny that models of the economy must be treated as stochastic because of the importance of random elements of behavior, gaps in knowledge, and inevitability of aggregation, it would still be necessary to study stochastic simulations in an effort to learn what difference the disturbances make or how much they matter.

Another pertinent consideration is that we are dealing here with long-run simulations, whereas the models on which these calculations are based were designed to serve primarily as short-term predictive and analytical devices. Simulations of this kind, therefore, may impose "a severe strain on the underlying assumptions and rationale used to justify the model structure," according to another paper prepared for this Conference.[24] Errors in lagged dependent variables may well cumulate, causing increasing errors at later points of time in the sample period. This argument, of course, relates to stochastic as well as to nonstochastic simulations; but, in the latter, the disregard of disturbance terms is an additional source of errors, which are subject to interaction and cumulation over time through the effects of the lagged variables. This factor must be recognized as a potentially severe handicap for the nonstochastic simulations, which is likely to counteract the favorable factors that tend to cause overstatement of the closeness of fit for simulations which cover, or largely overlap, the sample period. (Such simulations, of course, benefit from the fact that the coefficients of the model have been estimated from data for the same period, as well as from being based on ex post values of the exogenous variables.)

3.4.2 How do the models compare with one another in terms of the relative accuracy of their simulations? For reasons already noted (Section 1.3 above), this question cannot be answered with confidence on the basis of the available materials. Table 3.13 collects some measures of the kind that would be helpful in this context, but it is not conclusive because of the differences in coverage among the models.

The table lists first the mean absolute errors of relative change, in

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[24] See [18, p. 67]; also [14, p. 147].

TABLE 3.13

Nonstochastic Sample-Period Simulations for Three Models: Average Errors of Relative Change and Their Ratios to Average Values of Actual Relative Change

	Model and Period	Simulations for the Variables[a] *GNP* (1)	*GNP58* (2)	*P* (3)	*ISE* (4)	*UN* (5)
	Mean Absolute Error of Relative Change (MAERC), in percentage points[b]					
1	Wharton, 1948-III-1968-I	1.17	1.12	0.27	3.12	17.80
2	OBE, 1953-II-1966-IV	0.70	0.64	0.24	1.90	6.00
3	FMP, 1956-I-1966-IV	0.61	0.65	0.22	1.79	6.26
	Ratio of MAERC to Mean Absolute Relative Change in Actuals $\left(\frac{MAERC}{MAARC}\right)^c$					
4	Wharton, 1948-III-1968-I	0.681	0.852	0.453	1.036	2.502
5	OBE, 1953-II-1966-IV	0.459	0.518	0.488	0.812	0.890
6	FMP, 1956-I-1966-IV	0.377	0.524	0.429	0.746	1.155

[a] For meaning of symbols, see text or Table 1.1.

[b] Source: Tables 3.1, 3.5, and 3.9 (*see column 5 in each table*).

[c] Source: Tables 3.1, 3.5, and 3.9 (*see column 8 in each table*).

percentage points, for five selected variables: *GNP* in current and constant dollars, the price level, business expenditures on plant and equipment, and the unemployment rate (*GNP, GNP58, P, ISE, UN*). According to these figures, the errors of the Wharton simulations are on the average considerably larger than those of either the OBE or the FMP simulations, except for *P*, where the differences are small (compare lines 1, 2, and 3). The MAERC measures for the OBE Model are not very different from those for the FMP Model. However, the Wharton simulations cover a much longer period than the others, including the unsettled and difficult to fit developments of the late 1940's and the Korean War, which could account for the larger deviations between *S* and *A* for this model.

Dividing MAERC by the mean absolute values of actual relative change (MAARC) is a standardizing procedure which probably tends to correct for the differences in the sample periods but does not guarantee an unbiased comparison. The ratios MAERC/MAARC (lines 4 to 6) show smaller differences between the models than do the MAERC figures, but the models would be ranked very similarly according to the two measures. The FMP simulations show the smallest ratios, except for *GNP58* and *UN*, where the ratios for the OBE Model are lower. The differences between the simulations for OBE and FMP are still small according to the MAERC/MAARC figures; and for the price-level simulations, the differences remain small among all three models.

3.4.3 Finally, it is also instructive to compare the models with respect to their ability to simulate the diverse timing characteristics of selected endogenous variables. Table 3.14 assembles the relevant evidence, which indicates that the simulations do discriminate broadly between the groups of series that are typically leading or lagging at business-cycle turns, but that they do not carry this differentiation nearly as far as the actual timing distributions do. For example, among the five leading variables in the Wharton Model, leads outnumber lags by 23 to 2, according to the actuals; and by 13 to 6, according to the simulations (in percentage terms, 62 to 5 and 56 to 26, respectively). See Table 3.14, lines 1 and 2. Similarly, among the three lagging variables, actual lags outnumber leads 13 to 2; whereas in the simulations, there are 10 lags and 7 leads (the corresponding proportions here being 50 to 8 per cent for the actual, and 46 to 32 per cent for the simulated, scores). See lines 5 and 6. The worst results are obtained for the six roughly coincident indicators, where exact coincidences make up 51 per cent of the timing observations for the actual series, but only 9 per cent of those for the simulated series. (The leads and lags are nearly balanced for the latter, as shown in lines 3 and 4.)

The results for the other two models point to the same general conclusion. The OBE Model differentiates between the leaders and laggers better than does the Wharton Model but still not as well as the historical series (lines 9 and 10, 13 and 14). For the coinciders, the performance of the OBE Model is poor in that coincidences constitute a small minority of the timing comparisons for the simulated series, while the proportions of leads and lags are both large (lines 11 and 12). As for the FMP Model, it gives good results for the leaders, while overstating somewhat the proportion of lags among the laggers (lines 17 and 18, 21 and 22). But the FMP simulations, too, are disappointing on the group of roughly coincident series, where they show lags to

TABLE 3.14

Nonstochastic Sample-Period Simulations for Three Models; Absolute and Relative Frequency Distributions of Leads and Lags at Business-Cycle Turns

			Timing Observations at Business-Cycle Turns							
			Actuals				Simulations			
Line	Group of Variables[a] (number and symbols in parentheses)	Number or Percentage (1)	Total (2)	Leads (3)	Exact Coincidences (4)	Lags (5)	Total (6)	Leads (7)	Exact Coincidences (8)	Lags (9)
			Wharton Model (1948–68)[b]							
1	*Leading* (5: *IH*, *II*, *CPR*, *AWW*, *UMD*)	number	37	23	12	2	23	13	4	6
2		per cent	100.0	62.2	32.4	5.4	100.0	56.5	17.4	26.1
3	*Coincident* (6: *GNP*, *GNP58*, *C*, *YP*, *LE*, *UN*)	number	43	15	22	6	23	10	2	11
4		per cent	100.0	34.9	51.2	14.0	100.0	43.5	8.7	47.8
5	*Lagging* (3: *ISE*, *RS*, *RL*)	number	26	2	11	13	22	7	5	10
6		per cent	100.0	7.7	42.3	50.0	100.0	31.8	22.7	45.5
7	*All* (14)	number	106	40	45	21	68	30	11	27
8		per cent	100.0	37.7	42.5	19.8	100.0	44.1	16.2	34.7

					OBE Model (1953–66)[c]					
9	*Leading* (8: *IH*, *II*, *M*, *CPR*, *AWW*, *OMD*, *UMD*, *HS*)	number	44	32	9	3	34	25	5	4
10		per cent	100.0	72.7	20.5	6.8	100.0	73.5	14.7	11.8
11	*Coincident* (6: *GNP*, *GNP58*, *C*, *YP*, *LE*, *UN*)	number	34	13	17	4	12	5	2	5
12		per cent	100.0	38.2	50.0	11.8	100.0	41.7	16.7	41.7
13	*Lagging* (4: *ISE*, *RS*, *RL*, *LC/O*)	number	24	2	5	17	18	2	5	11
14		per cent	100.0	8.3	20.8	70.8	100.0	11.1	27.8	61.1
15	*All* (18)	number	102	47	31	24	64	32	12	20
16		per cent	100.0	46.1	30.4	23.5	100.0	50.0	18.8	31.2
					FMP Model (1956–66)[d]					
17	*Leading* (6: *IH*, *II*, *CPR*, *OUME*, *LH*)	number	22	15	3	4	18	12	3	3
18		per cent	100.0	68.2	13.6	18.2	100.0	66.7	16.7	16.7
19	*Coincident* (4: *GNP58*, *C*, *LE*, *UN*)	number	16	5	8	3	10	3	3	4
20		per cent	100.0	31.2	50.0	18.8	100.0	30.0	30.0	40.0
21	*Lagging* (3: *ISE*, *RS*, *RL*)	number	12	2	5	5	10	3	2	5
22		per cent	100.0	16.7	41.7	41.7	100.0	30.0	20.0	50.0
23	*All* (13)	number	50	22	16	12	38	18	8	12
24		per cent	100.0	44.0	32.0	24.0	100.0	47.4	21.1	31.6

[a] Classified according to the timing of the historical data.

[b] Based on Table 3.4.

[c] Based on Table 3.8.

[d] Based on Table 3.12.

be more frequent than coincidences, and leads to be as frequent (lines 19 and 20).

For each model and in each timing category, the simulated series offer fewer observations than do the actual series, as can be seen by comparing columns 2 and 6 in Table 3.14. This reflects our finding that the *S* series "skipped" business-cycle turns more frequently than did the *A* series, particularly for the roughly coincident indicators.[25]

When all the variables included in the comparisons for a given model are combined, without regard to their historical timing, the resulting summary distributions show that the proportion of coincidences was heavily underestimated in the simulations; the proportion of leads somewhat overestimated; and that of lags, strongly overestimated. This is observed for each of the three models (lines 7 and 8, 15 and 16, and 23 and 24).

These findings suggest that the models are wanting in ability to identify the leaders and laggers, and to separate them from the coinciders. It is true that the procedure favors the actuals somewhat, in that they were used in classifying the variables according to timing, but the importance of this factor should not be exaggerated. The classification was, in fact, based to a large extent on historical information other than that contained in the sample-period actuals (e.g., *GNP*, *C*, or *YP* would always be treated as coinciders, although leads were more, or at least not less, frequent than coincidences in some of the periods covered).

Neither does it appear that the results are attributable to the exclusion of the stochastic elements from the simulated series. Suppose that the true timing of a variable is coincident but that this is obscured by erratic movements which cause some turning points to be misdated in the direction of extremes—leads or lags. Given small-sample data—evidence limited to short time-series—misclassification could result. Had this happened often enough, however, we should have found the proportion of coincidences to be greater in the nonstochastic simulations than in the actuals; yet, in point of fact, the opposite is found to apply. Actually, the distinctions between the leaders and laggers are in

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[25] It should be noted that Table 3.14 covers only the series listed in the underlying Tables 3.4, 3.8, and 3.12. These tables omitted a few variables for which too few—or no—timing observations could be made for either *S* alone or for both *S* and *A*. *GNP* and *YP* (both coincident) were excluded from the comparisons for the FMP Model because of the lack of turning points in the *S* series.

large part based on sound a priori or theoretical considerations, and on substantial empirical evidence of business-cycle history—such as recurrent, and presumably typical, timing sequences. To this extent, then, the timing classifications represent systematic differences not random phenomena.

The greater frequency of laggers among the *S* series could be due to some induced smoothing effects; in particular, the use of distributed lag equations. In future work, it may prove interesting to check out this possibility.[26]

4 HUNDRED-QUARTER EX ANTE STOCHASTIC SIMULATIONS

THIS part of our report presents an analysis of replicated simulations in which random shocks are applied on a continued basis to estimated equations of selected systems. Such simulations were received for two models only, Wharton and OBE. As proposed in the plans for this Conference, each of these simulations covers twenty-five years beyond the model's sample period. For the Wharton Model, fifty simulations use serially uncorrelated random shocks, and fifty use serially correlated shocks. For the OBE Model, there are twenty-five runs with non-autocorrelated shocks and twenty-five runs with autocorrelated shocks.

The random shocks used in the stochastic simulations for both models were generated according to a procedure developed by Michael McCarthy.[27] The method is such that the expected value of the var-

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[26] Note that ordinary smoothing of a time-series by means of moving averages can shift the timing of the turns in the series in either direction, and in random or systematic ways: the outcome depends on the statistical structure of the series and the smoothing formula applied. (See A. F. Burns and W. C. Mitchell, *Measuring Business Cycles*, pp. 316–326.) Lags will often be produced at terminal turns of brief but large cyclical movements, especially at troughs, while leads of smoothed data may be more frequent at peaks. However, it is important to recognize that smoothing does not eliminate the irregular component movements of a series; it merely redistributes them over time in successive values of that series. In contrast, stochastic elements are presumably excluded from the *S* series here considered.

[27] See M. D. McCarthy, "Some Notes on the Generation of Pseudo Structural Errors for Use in Stochastic Simulation Studies" [14, Appendix].

iance-covariance matrix of the shocks over the simulation period is equal to the variance-covariance matrix of the observed residuals over the sample period. In those runs where the shocks are serially correlated, such lag correlations are also, for a sufficiently large number of observations, equal to the corresponding sample values obtained for the residual matrix.

It will be noted that these procedures differ in several respects from the approach adopted in the Adelmans' study, where the simulations are annual, unreplicated (from a single run), and based only on serially uncorrelated shocks, on the assumption of zero covariance of errors. On the whole, the innovations enrich the potential of the simulations and their analysis. But doubt has been expressed about another deviation from the Adelmans' method [18, p. 77]. They used the ratio of the standard deviation of the residuals to the average value of the normalized dependent variable in the sample period as the basis for scaling their shocks in the simulation period, whereas, here, the basis is the standard deviation of the sample-period residuals itself. The latter standard could result in unduly small shocks if the variances of the true normalized equation errors were heteroscedastic—increasing over time with the levels of the simulated series.

For each model, the initial topic of discussion will be the major properties of the nonstochastic simulations beyond the sample period; that is, of the "control solution." This is necessary in order to introduce the main body of our analysis, which is concerned with the stochastic simulations. We shall present measures relating to the frequency, duration, and relative size of (*a*) rises and declines and (*b*) cyclical expansions and contractions in the stochastically simulated series. The difference between the two sets is that in (*a*) any upward movement, however short or small, is treated as a rise, with downward movements of any magnitude being treated as a decline, while in (*b*) movements must be sufficiently long and pronounced to qualify as "specific cycle" expansions or contractions, under the rules of NBER cyclical analysis. Thus in (*a*) any directional change in a series separates a rise from a decline, which permits an entirely objective identification of these movements; whereas in (*b*), the selection of the turning points between expansions and contractions is in principle a matter for trained judgment, although computer procedures for a mechanical approximation to this task have recently been designed and tested with generally good results [4].

The distributions of both the (*a*) and (*b*) measures should be compared with those of their counterparts for the sample-period actuals. But in some of the simulated series, there are very few declines and no cyclical contractions. This leads us to apply the measures not only to the shocked series proper, but also to the relative deviations of these series from the corresponding control series. The analysis is carried out for simulations with serially uncorrelated shocks, as well as for those with serially correlated shocks, so that the two sets can be compared at each point with respect to their relative performance.

Finally, the relative timing of the simulated series is analyzed with the aid of the cyclical turning points, as determined in (*b*), to see whether the typical sequence of leading, coinciding, and lagging indicators tends to be reproduced in these measures. This phase of the analysis is also applied to both types of simulation and, as required by the nature of the data, to either the levels or the deviations from trend or both.

It would have been excessively costly to execute this full program for all the simulation runs of each model, but it would also be undesirable to discard much of the potentially useful information. As a compromise, therefore, all runs were used in the analysis of the periodicities in the *GNP* and *GNP58* series; but elsewhere, measures were compiled and interpreted for random samples of a few simulation runs of a given type.

4.1 THE WHARTON MODEL

4.1.1 These simulations start in 1968-III, which is already beyond the space of sample experience, and run for one-hundred quarters into the future, to end in 1993-II. Initial values of predetermined variables were set at levels assumed to be realistic, and the further course of the exogenous factors during the entire simulation period was determined so as to keep the unemployment rate within the narrow range of 3.7 to 4.7 per cent, and the short-term and long-term interest rates within the narrow ranges of 4.4 to 4.6 per cent, and 5.3 to 5.9 per cent, respectively, beginning in 1971. In other words, the exogenous variables are assumed to take on values that would keep the model economy moving along a steady long-run growth path, at least as far as the overall aggregates of national income and output are concerned. The exogenous variables reflect primarily U.S. fiscal and monetary policies.

In the first few years of the simulation period, some of the generated series show substantial disturbances, due mainly to the repercussions of the anticipated settlement of the war in Vietnam. In 1970 and the first half of 1971, reductions of military personnel by 350,000 men and of spending by $11.1 billion in 1958 prices are assumed—to follow a cease-fire.[28] The tax surcharge is discontinued and civilian expenditures are gradually increased, so that total government spending in current prices does not decline (though in real terms it does decline slightly for two quarters). The discount rate is reduced by 1/2 of 1 percentage point and net free reserves are maintained at $200 million.

The main consequence of the postulated changes is that the unemployment rate increases sharply from 4.3 to 5.5 per cent in 1969 and early 1970, only to fall again to nearly 4.1 per cent in mid-1972. The short-term interest rate declines from 5.8 to 4.6 per cent; and the long-term rate, from 6.5 to 5.7 per cent. Corporate profits wobble briefly in 1968–70, as do unfilled orders for durable manufactures and investment expenditures on plant and equipment; also, investment in housing pauses somewhat later, in 1971–73. But no general recession develops as personal income, consumption, and *GNP* in both current and constant dollars all rise steadily throughout the simulation period.

In fact, apart from the mild effects of the initial shock and transition, none of the nonstochastic simulation series that represent the "control solution" of the model display any significant fluctuations. There are some minor oscillations in variables such as profits and net exports, which are in the nature of residuals; and in the average workweek, unemployment, and interest rates. (These last-named series—simulated *AWW*, *UN*, *RS*, and *RL*—differ from all others in showing downward rather than upward drifts.) But the dominant feature of any and all of these series is simply persistent trends representing the simulated long-term growth of the economy. *GNP* grows from about $850

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[28] See the section on "Long Run Simulations" in [14] for more detail on the assumptions discussed at this point and in the rest of the paragraph.

billion to $3,160 billion, or approximately 3.7 times; *GNP58*, from $700 billion to $1,660 billion, or nearly 2.4 times. These figures suggest that the projected rates of growth are, on the average, about 5.5 per cent per annum for *GNP* and 3.5 per cent per annum for *GNP58*.

It is important to recall that for the 1948–68 period, the nonstochastic simulations of the Wharton Model did show a considerable degree of cyclical response in several variables, including the most comprehensive aggregates, such as *GNP*, which had, at least, substantial retardations at the time of recessions in general economic activity. This is in marked contrast to the long post-sample-period nonstochastic simulations now considered, which are virtually cycle free, particularly for the over-all aggregates of national output, employment, and so on. Now, the main difference between the two sets of simulations lies in the treatment of the exogenous variables. In the 1948–68 calculations, these variables take on their "true" (i.e., ex post) recorded values, which include some large and long fluctuations. In the 1968–93 control solution, exogenous variables are constrained to assume pure trend values consistent with a long-run growth path in real *GNP* that keeps the unemployment rate at close to four per cent. It is, therefore, tempting to speculate that stronger cyclical elements might have been obtained had the exogenous variables been subjected to shocks or somehow made to fluctuate. It should be very interesting to test this hypothesis by means of experiments with shocked or auto-regressively fluctuating exogenous variables.[29] To be sure, there are other feasible explanations of the obtained results. It is possible, for example, that specification errors in the model account largely for the differences between the sample-period, and the post-sample-period, nonstochastic simulations.

In any event, since the latter simulations are based on very tentative projections of exogenous variables, they should be regarded merely as a "base-line solution," to be used for subsequent experiments with stochastic shocks, not as preferred long-period model predictions. This is stressed by the authors of both the Wharton Model and the OBE Model [14, p. 150], [18, p. 68]. But it is also necessary to empha-

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[29] That is, we advocate (here, as well as for the simulations of other models) the addition of "shocks of Type I" to the "shocks of Type II," to use the Adelmans' terminology [2].

size two other facts: (1) it appears to be quite difficult, for either model, to produce reasonable behavior over long stretches of time in the chosen time-series included in the control solution; (2) at least, in the solutions here adopted, what seemed to be a satisfactory over-all course for the most comprehensive indicators of economic activity, such as *GNP*, was "purchased" at the expense of rather implausible behavior patterns for some other variables, notably unemployment and the interest rates. (This second point, too, applies to both the Wharton Model and the OBE Model. See Section 4.2.1 below.)

4.1.2 Chart 4.1 shows two randomly selected pairs of stochastic simulations for *GNP* and *GNP58*: one drawn from the fifty runs with non-autocorrelated shocks, and the other from the fifty runs with autocorrelated shocks. These curves are clearly dominated by growth trends. Inspection of similar charts for all runs discloses no important differences among the individual simulations in this respect.

The trends in the simulated series simply reflect the assumptions about the smooth growth in the exogenous variables that underlie the nonstochastic control solution of the model. They represent the common component of the series, whereas the effects of the random shocks show up in the oscillations of the series around the trends. As illustrated in Chart 4.1, there is considerable variation in the rates of change in the *GNP* and *GNP58* simulations from quarter to quarter. In the series with serially uncorrelated random shocks, growth is frequently interrupted by declines. The declines are generally short and relatively small, but they appear to be larger and more frequent in the constant-dollar *GNP* series than in the current-dollar *GNP* series.

In the *GNP* simulations with autocorrelated shocks, there are few declines and virtually none of more than one-quarter duration; many of these series show no downward movements at all. Fluctuations are again more frequent, and not quite so small, in the *GNP58* series, but here, too, the use of serially correlated shocks results in a reduction of both the number and the size of the declines.

The impressions conveyed by the charts are confirmed and quantified in Table 4.1, which summarizes several distributional measures. In each of the fifty runs with serially uncorrelated random shocks (S_u), there are one-quarter declines in the *GNP* series; in eighteen of the runs, one or two declines of two quarters each are also observed, but

CHART 4.1

A Random Sample of Stochastic 100-Quarter Simulations for GNP in Current and Constant Dollars, Wharton Model (1968-III–1993-II)

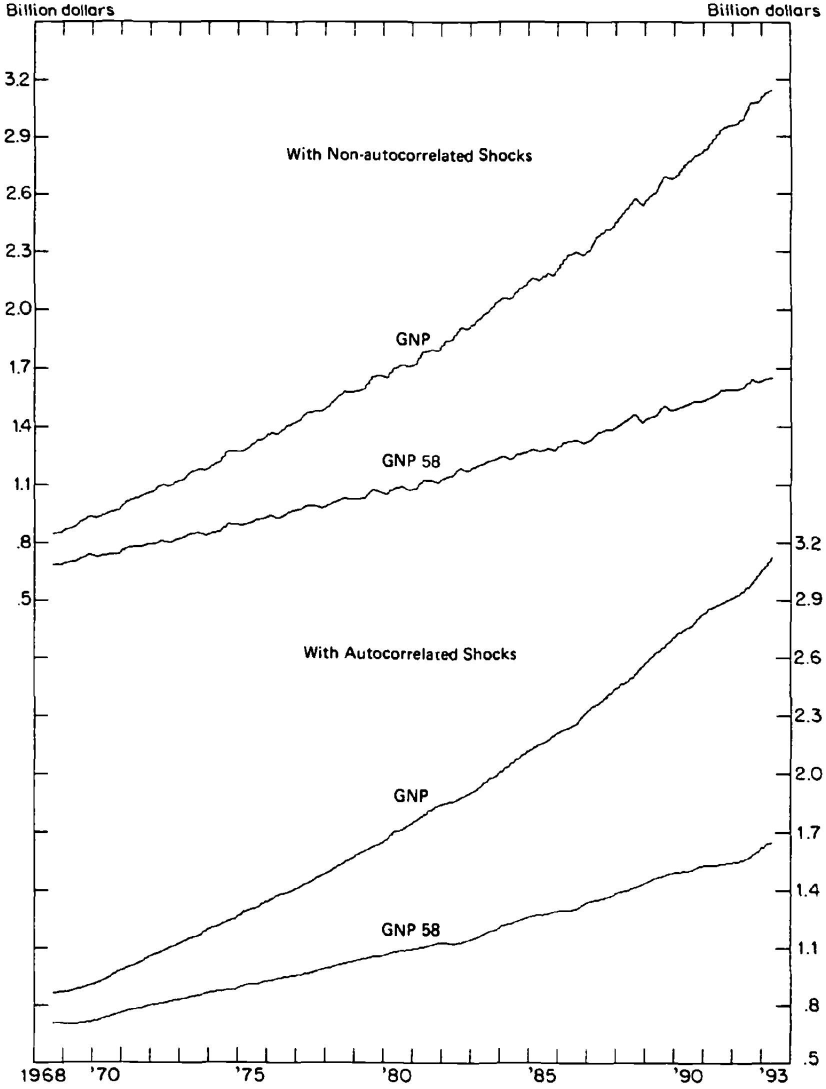

TABLE 4.1

Stochastic 100-Quarter Simulations, Wharton Model, and the Corresponding Sample-Period Actuals: Summary Statistics on Frequency, Duration, and Relative Size of Rises and Declines in Series for GNP *and* GNP58

		Frequency (number)		Duration (quarters)			Amplitude (per cent)	
Line	Type of Series and Movement	Mean or Total[a] (1)	Standard Deviation (S.D.)[b] (2)	Mean (per run)[c] (3)	S.D. (between runs)[b] (4)	S.D. (within runs)[d] (5)	Mean (per run)[e] (6)	S.D. (between runs)[b] (7)
				GNP *in Current Dollars*				
	Simulations with non-autocorrelated shocks: S_u							
1	Rises	10.71	2.87	9.02	3.05	7.53	1.88	0.20
2	Declines	9.94	2.82	1.05	0.07	0.12	0.55	0.16
	Simulations with autocorrelated shocks: S_e							
3	Rises	1.61	0.76	73.75	28.66	13.61	2.28	0.48
4	Declines	0.69	0.82	0.51	0.54	0.00	0.14	0.23
	Sample-period actuals: *A*							
5	Rises	8		8.12		9.69	1.85	
6	Declines	7		1.86		0.31	0.71	

TABLE 4.1 (*concluded*)

Line	Type of Series and Movement	Frequency (number): Mean or Total[a] (1)	Frequency (number): Standard Deviation (S.D.)[b] (2)	Duration (quarters): Mean (per run)[c] (3)	Duration (quarters): S.D. (between runs)[b] (4)	Duration (quarters): S.D. (within runs)[d] (5)	Amplitude (per cent): Mean (per run)[e] (6)	Amplitude (per cent): S.D. (between runs)[b] (7)
	GNP *in Constant Dollars* (GNP58)							
	S_u simulations							
7	Rises	21.35	2.83	3.60	0.65	2.43	1.64	0.16
8	Declines	21.02	2.68	1.14	0.08	0.33	0.75	0.15
	S_r simulations							
9	Rises	7.04	2.25	15.00	7.33	12.11	1.06	0.10
10	Declines	6.41	2.33	1.17	0.17	0.30	0.31	0.10
	A: actuals							
11	Rises	13		4.46		2.31	1.33	
12	Declines	11		1.73		1.19	0.68	

NOTE: All simulations refer to the 25-year period, 1968-III–1993-II; each average covers 50 runs. The actuals refer to the sample period, 1948-III–1968-I.

[a] For simulations: mean number per run; for actuals: total number (of rises or declines).

[b] Standard deviation of means per run (standard deviation between runs).

[c] For simulations: mean duration per run; for actuals: mean duration (of rises or declines).

[d] For simulations: mean standard deviation of the durations of rises or declines (standard deviation within runs); for actuals: standard deviation of the observed durations.

[e] For simulations: mean amplitude per run; for actuals: mean amplitude (of rises or declines). In per cent, at quarterly rate.

there are no longer contractions. Almost the reverse applies to the rises, among which one- or two-quarter movements are relatively few, movements of five or more quarters representing a majority. The mean durations are approximately one and nine quarters for falls and rises, respectively. However, there is no such contrast between the frequencies of occurrence per run, which average 10.7 for rises and 9.9 for declines (and range from 5 to 16 or 17 in either subset). Finally, the average amplitude per quarter is 1.9 per cent for the upward movements and 0.6 per cent for the downward ones (Table 4.1, lines 1 and 2).

In comparing such statistics for the simulations with figures on the corresponding attributes of historical series, it seems appropriate to stress the measures of frequency and duration, rather than those of amplitude. The random-shock hypothesis here considered asserts, in the formulation by Frisch [16, p. 171], that "the majority of the economic oscillations . . . seem to be explained most plausibly as free oscillations. . . . The most important feature of the free oscillations is that the length of the cycles and the tendency towards dampening are determined by the intrinsic structure of the swinging system, while the intensity (the amplitude) of the fluctuations is determined primarily by the exterior impulse." This suggests that the amplitudes of movements in the stochastic *S* series would depend mainly on the simulator's decision as to the magnitude of the shocks applied; they may be quite different from the amplitudes of the actuals, not because of any failure of the model to reproduce the basic structure of the economy, but because the impulses or shocks have not been properly scaled.

We have only one "run" that history has performed to produce the recorded "actuals"; we can compare its outcome with the over-all average from many experimental runs relating to the hypothetical future, allowing for the dispersion of the components of that average (the means of the individual runs). For example, the number of rises in *GNP* during the sample period is confronted with the mean frequency per run of rises in the simulated *GNP* series; i.e., of the corresponding averages for the individual runs. The declines are treated similarly. Accordingly, in Table 4.1, the entries for the *S* series in columns 1 and 2 are to be compared with those for *A* in column 1, and analogously for the duration and amplitude measures (columns 3 and 4, 6 and 7, respectively).

In addition, Table 4.1 shows the standard deviations of the durations of rises and declines in the sample-period actuals. These figures should be related to the mean standard deviations of the durations of rises and declines in the *S* series (that is, to the averages of the corresponding S.D. measures for the individual runs; see column 5).

During the period of nearly twenty years (1948-III–1968-I) used for the Wharton Model calculations in Part 3 of this report, seven declines occurred in the recorded quarterly *GNP* series. This does not appear inconsistent with the mean frequency of declines per run of 9.9 in the twenty-five year S_u simulations, with a standard deviation of 2.8 (Table 4.1, columns 1 and 2). However, the downward movements lasted on the average 1.86 quarters in the sample-period data and only 1.05 quarters (with a very small S.D.) in the S_u series for *GNP* (columns 3 and 4). The "within run" dispersion of the durations of movement in S_u is smaller than the dispersion of the actual duration figures (column 5). One-quarter declines account for over 95 per cent of all declines in these simulations; in contrast, *GNP* in 1948–68 underwent three contractions of two quarters each and one of four quarters, in addition to three one-quarter declines. As the simulated falls are shorter, so the simulated rises are longer than the actual ones (9.0 vs. 8.1 quarters; see column 3).

The mean percentage amplitudes of the declines are 0.71 for actuals and 0.55 for the S_u series. For the rises, the corresponding amplitudes are virtually identical— 1.85 and 1.88 per cent for *A* and S_u, respectively (column 6).

To conclude, the simulations with serially uncorrelated random shocks produce declines that are somewhat shorter and smaller than the declines observed in the postwar *GNP* series. But the differences are not really large. The declines are about as frequent in S_u as in *A* (the average length of rise-plus-decline is approximately the same in both cases, 10 quarters). The amplitude differences could, perhaps, be reduced to negligible size by the use of somewhat stronger shocks.

On the other hand, there can be no doubt that the *GNP* simulations with autocorrelated shocks (S_c) differ drastically from the actual data in that they show no recurrent fluctuations in levels. Half of these projections show no downturns at all, only continuous rises, so that the S_c series have very long expansions and just a few very short declines

(Table 4.1, lines 3 and 4, columns 1–5). The upward movements, also, are considerably larger, and the downward movements smaller, in S_c than in S_u (columns 6 and 7). The use of autocorrelated shocks has a powerful smoothing effect, eliminating many declines and reducing others. The behavior patterns represented by the S_c simulations seem implausible in the light of historical experience.

Turning next to the simulations for *GNP58*, we observe that they are subject to much more frequent directional changes than are the simulations for *GNP*: the numbers per run of both rises and falls are greater here, and the expansions are much shorter and smaller. Differences of the same kind also exist between the actuals for *GNP* and *GNP58*. (All this can be seen by comparing the corresponding measures in Table 4.1, lines 1–6 and 7–12.) However, in the simulated series these differences are exaggerated. The simulations for *GNP58* deviate from the sample-period actuals in several respects.

First, the mean frequencies per run of rises and falls are too large for the S_u series (with non-autocorrelated shocks) and too small for the S_c series (with autocorrelated shocks), as compared with the numbers for the recorded *GNP58* (columns 1 and 2). Second, the movements in S_u are shorter than those in A: the mean duration of rises and declines are 3.6 and 1.1 for these simulations, 4.5 and 1.7 quarters for the actuals. In the S_c runs, the declines are similarly short, but the expansions are much longer, averaging 15 quarters (columns 3–5). Finally, the relative amplitudes in S_u exceed, and those in S_c fall short of, their counterparts in the real *GNP* series for the sample period. But the differences are not large, except that the declines in the S_c series are apparently less than half the size of the declines in S_u or A (columns 6 and 7).

Thus, the pattern of movement in *GNP58* is not reproduced closely in simulations of either type. The S_u series are rather too erratic and the S_c series too smooth; i.e., the fluctuations are too frequent and short in the former, and too infrequent and long—because of long rises—in the latter. However, the simulations are not very far off the mark on the average, according to some of the criteria applied. In general, the S_u series come out better in these comparisons than the S_c series. It is true (as noted in [14, p. 159]) that the average length of the rise-and-decline sequence in the S_c series—about 16 quarters—is approximately equal to the average length of business cycles in the United States (50 months in 1854–1958, or 52 months in 1945–58, for example; see [26, p. 671]). But we are dealing here with rises and declines of any duration rather than with the expansions and contractions of the NBER chronology (where one-quarter declines, in particular, would generally fail to qualify as cyclical contractions). In terms of the present measures, the average duration of movements is much shorter in the actuals and better approximated by the S_u than by the S_c simulations (Table 4.1, lines 7–12, columns 3 and 4).

4.1.3 Stochastic simulations of the components of *GNP* in constant dollars and other indicators, when based on the equations of the Wharton Model with non-autocorrelated shocks (S_u), tend to show frequent directional changes, from rather short rises to still shorter declines, and so on. Many of these series are highly erratic, with very large up and down movements of short duration; others show relatively smaller short oscillations superimposed upon longer waves; in still others, trends are more important. There are large differences between series for different variables. When autocorrelated shocks are used, the resulting series (S_c) are generally much smoother, though no less differentiated. Chart 4.2 shows some randomly drawn examples of these S_u and S_c simulations.

Table 4.2 lists the frequencies and average durations (AD) of rises and declines for one set of the S_u series and for two sets of the S_c series. There are no apparent reasons to suspect that the selection of these particular runs tends to bias our results, but it may be desirable to check up on this point with measures based on larger numbers of different runs. The table also contains comparable data on the number and AD of rises and declines in the recorded series for the same variables, using the sample period 1948-III–1968-I.

Given that the actual data cover less than twenty years and the simulated series, twenty-five years, Table 4.2 suggests that rises and declines alternate much more frequently in S_u than in *A* (columns 1 and 3). Consequently, both rises and declines are virtually all shorter in the S_u series than in the corresponding actuals (columns 2 and 4).[30]

The S_c series have smaller frequencies of both rises and declines

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[30] The only exceptions are for rises in *GNP*, *P*, and declines in *RL* (lines 1, 17, and 32).

CHART 4.2

A Random Sample of Stochastic 100-Quarter Simulations for Selected Variables, Wharton Model (1968-III–1993-II)

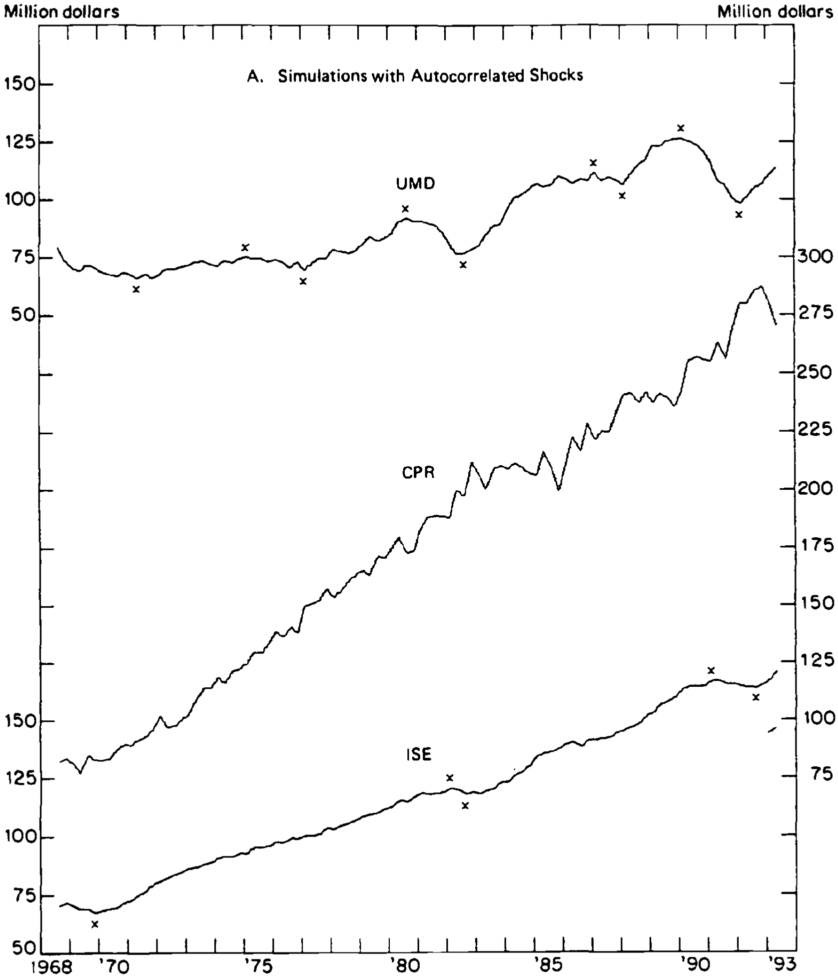
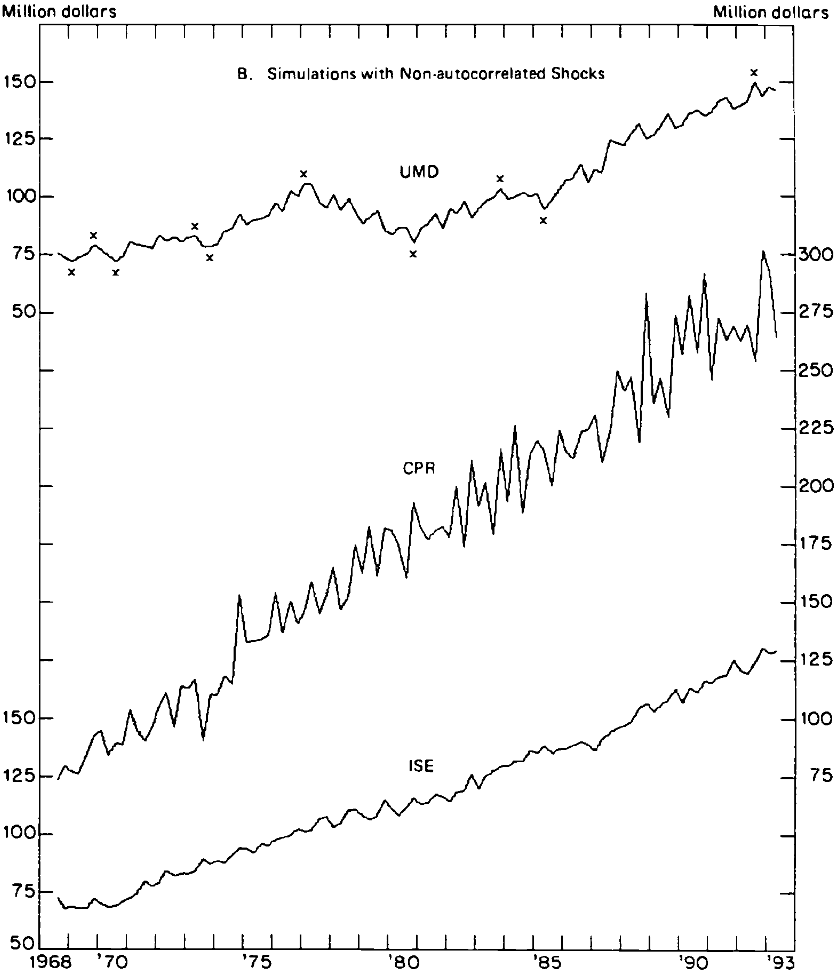


CHART 4.2 (*continued*)



(*continued*)

CHART 4.2 (*continued*)

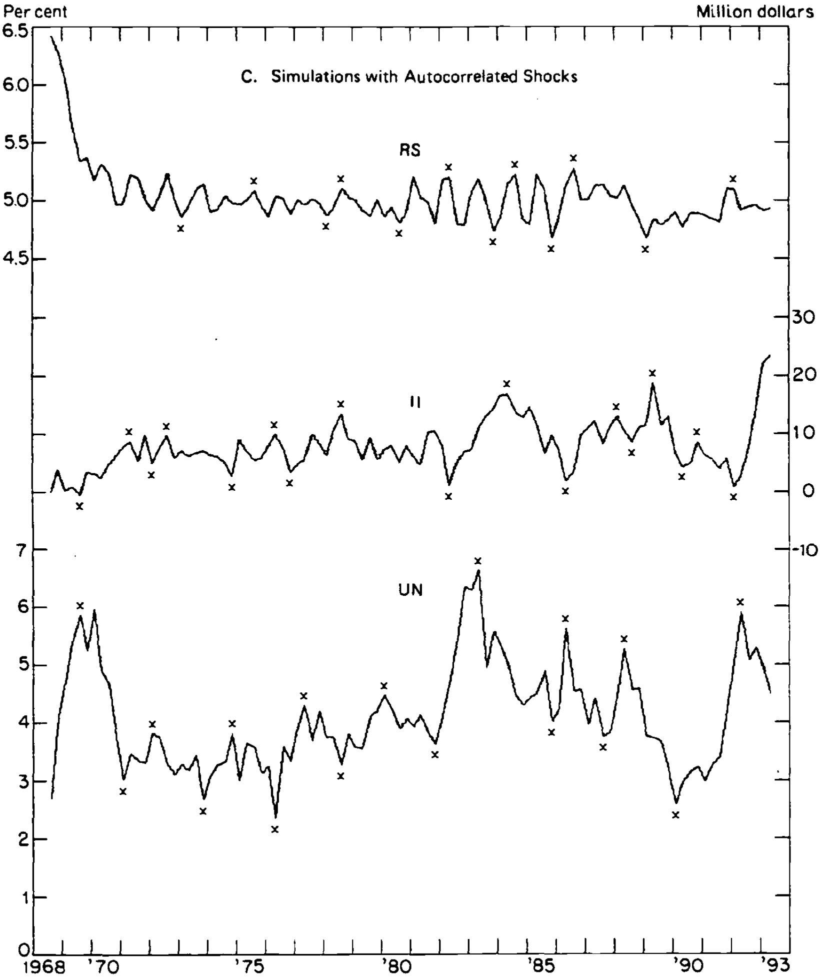


CHART 4.2 *(concluded)*

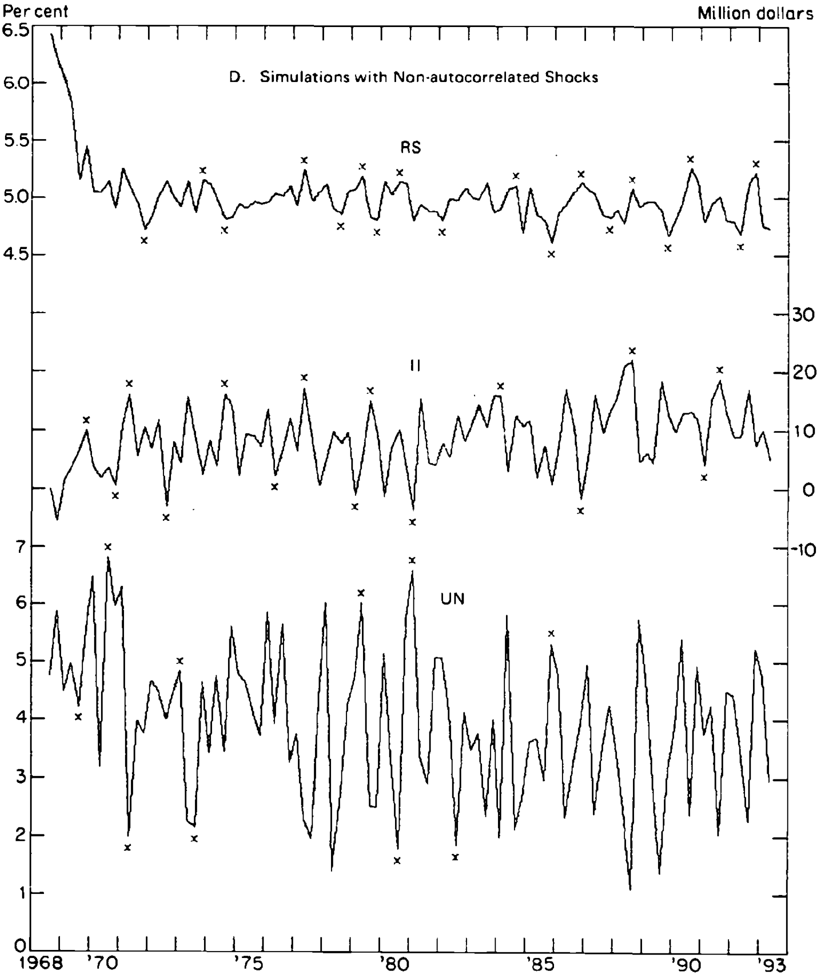


TABLE 4.2

Stochastic 100-Quarter Simulations, Wharton Model: Frequency and Average Duration of Rises and Declines in Seventeen Variables, Comparison of Three Simulation Runs and Actuals

					Stochastic Simulations (Three Runs)					
			Actuals for the Sample Period 1948-III–1968-I		With Uncorrelated Shocks (Run 31)		With Serially Correlated Shocks (Run 14)		With Serially Correlated Shocks (Run 26)	
Line	Variable Symbol[a]	Rise (R) or Decline (D)	Number[b] (1)	AD[c] (quarters) (2)	Number[b] (3)	AD[c] (quarters) (4)	Number[b] (5)	AD[c] (quarters) (6)	Number[b] (7)	AD[c] (quarters) (8)
1	*GNP*	R	8	8.1	4	23.8	1	99.0	1	99.0
2		D	7	1.9	3	1.3	0	0	0	0
3	*GNP58*	R	12	4.9	20	3.7	4	23.8	7	12.9
4		D	11	1.7	20	1.3	3	1.3	6	1.5
5	*C*	R	7	9.9	22	3.5	5	18.8	11	8.0
6		D	6	1.1	21	1.0	5	1.0	10	1.1
7	*IH*	R	11	3.7	29	1.9	22	2.9	20	3.1
8		D	11	3.4	29	1.6	22	1.6	19	1.9
9	*ISE*	R	14	3.7	26	2.6	12	6.8	16	5.0
10		D	13	2.0	26	1.2	11	1.6	16	1.2
11	*II*	R	21	2.0	34	1.5	26	2.2	23	2.4
12		D	22	1.6	35	1.4	25	1.7	22	2.0
13	*NE*	R	14	2.6	31	1.7	30	1.9	24	2.0
14		D	15	2.7	30	1.5	30	1.4	23	2.1

15	*YP*	R	5	13.5	23	3.3	3	32.2	5	18.8
16		D	4	2.2	22	1.0	2	1.0	4	1.2
17	*P*	R	4	17.8	4	24.0	2	49.0	1	99.0
18		D	4	1.8	3	1.0	1	1.0	0	0
19	*LE*	R	13	4.3	32	2.0	7	13.3	7	13.3
20		D	12	1.8	31	1.2	6	1.0	6	1.0
21	*UN*	R	15	2.3	33	1.4	29	1.8	25	2.2
22		D	16	2.6	33	1.5	29	1.6	25	1.7
23	*CPR*	R	15	3.4	38	1.4	30	2.2	28	2.2
24		D	14	1.9	38	1.2	29	1.1	28	1.3
25	*AWW*	R	15	3.0	28	1.9	25	1.5	23	1.7
26		D	16	2.0	29	1.6	25	2.4	22	2.7
27	*UMD*	R	8	6.0	28	2.1	20	2.7	18	3.1
28		D	9	3.3	29	1.4	20	2.2	17	2.6
29	*RS*	R	10	5.2	28	1.6	27	1.6	24	1.7
30		D	9	2.9	29	1.9	27	1.9	24	2.4
31	*RL*	R	17	2.5	26	1.7	25	1.6	26	1.7
32		D	17	2.0	25	2.2	26	2.3	27	2.0
33	*W*	R	4	18.5	25	3.0	9	10.0	12	7.2
34		D	3	1.3	24	1.0	8	1.1	12	1.0

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[a] For meaning of symbols, see Table 1.1.

[b] Number of rises (R) or declines (D).

[c] Average duration, in quarters, of rises (R) or declines (D).

(fewer directional reversals) than the corresponding S_u series (compare columns 3, 5, and 7). There are only three exceptions to this among the sixty-eight comparisons that can be made.[31] Accordingly, movements in either direction, but particularly rises, tend to be longer in the S_c than in the S_u simulations. For rises, AD figures are larger for S_u than for S_c in only three of the thirty-four comparisons; for declines, they are larger in seven instances.[32]

There are nineteen instances in which the average length of rises is larger for the actuals than for the S_c series, and fifteen instances in which the reverse applies. For the most part, the upward movements in S_c, being longer than those in S_u, approximate better the duration of such movements in A. However, large deviations in either direction are apparently not uncommon in these comparisons (see the R lines for *YP*, *P*, *LE*, and *W*, for example). As for the AD of declines, the S_c series still underestimate this dimension in most cases (26 out of 34), according to the yardstick of the sample-period actuals, but the differences here are often small. (See the D lines, columns 2, 6, and 8.)

Table 4.3 compares the average percentage amplitudes (APA) of quarterly rises and declines in the actuals and in the selected S_u and S_c series. It shows a distinct tendency for the simulations with uncorrelated shocks to have larger APA than the sample-period realizations. (There are only a few exceptions here, notably for *GNP*, *ISE* and the interest-rate series; see columns 1 and 2.) In contrast, the relative changes in the simulations with serially correlated shocks tend, just as strongly, to be smaller than the APA for the actuals. (The only exceptions to this rule are found for *GNP*, *UN*, *AWW*, and the declines in *P*. See columns 1 and 3 and 4.)[33]

To sum up, the evidence for the selected variables seems on the whole more favorable to the S_c than to the S_u simulations, mainly because the latter are too erratic and have much smaller AD of rises than those observed in the historical series. The corresponding figures for S_c often differ by large margins from those of A, but apparently not in any strongly systematic fashion. The changes in S_u are too large, and the changes in S_c are too small, when compared with A. This criterion,

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[31] All exceptions occur in the long-term interest rate *RL*. See lines 27 and 28.

[32] However, there are eight cases in which the AD of declines are equal for S_u and S_c. For rises, there are only two such cases.

[33] Table 4.3 omits the variables *II* and *NE*, which can, and do, assume negative values.

TABLE 4.3

Stochastic 100-Quarter Simulations, Wharton Model: Average Percentage Amplitudes, Per Quarter, of Rises and Declines in Fifteen Variables, Comparison of Three Simulation Runs and Actuals

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Average Amplitude of Movement Per Quarter[b]			
			Actuals for the Sample Period 1948-III–1968-I (1)	Stochastic Simulations (Three Runs)		
				Uncorrelated Shocks (Run 31) (2)	Serially Correlated Shocks	
					(Run 14) (3)	(Run 26) (4)
1	*GNP*	R	1.85	0.80	2.63	2.71
2		D	0.71	0.28	[c]	[c]
3	*GNP58*	R	1.33	1.68	.80	1.10
4		D	0.68	1.18	.28	0.28
5	*C*	R	1.32	1.83	1.14	1.09
6		D	0.87	1.34	0.25	0.21
7	*IH*	R	3.50	6.06	1.67	1.65
8		D	3.27	5.26	1.46	1.48
9	*ISE*	R	2.83	2.51	1.28	1.13
10		D	2.74	2.02	0.66	0.60
11	*YP*	R	1.96	2.78	1.63	1.56
12		D	1.11	2.02	0.10	0.20
13	*P*	R	0.55	0.84	0.35	0.58
14		D	0.39	0.60	0.53	[c]
15	*LE*	R	0.59	1.35	0.45	0.48
16		D	0.36	0.77	0.08	0.19
17	*UN*	R	6.25	78.19	12.22	14.35
18		D	4.97	32.18	11.15	12.11
19	*CPR*	R	4.85	11.55	3.34	3.52
20		D	4.48	8.05	2.59	1.80
21	*AWW*	R	0.31	0.99	0.59	0.63
22		D	0.33	1.33	0.58	0.56
23	*UMD*	R	4.45	4.55	2.67	2.52
24		D	3.33	3.78	1.97	1.90
25	*RS*	R	9.10	3.93	3.17	4.08
26		D	7.23	3.38	2.99	3.03
27	*RL*	R	3.27	1.82	1.85	1.39
28		D	1.57	1.76	1.51	1.57
29	*W*	R	1.31	1.82	1.12	1.05
30		D	0.64	2.17	0.41	0.38

[a] For meaning of symbols, see Table 1.1.

[b] All figures are at quarterly rates.

[c] No declines.

the APA comparisons, is given less weight in our judgment than the criterion of the AD comparisons. If the projections for exogenous variables were shocked, these additional disturbances could well increase the APA figures for the simulations. This would then tend to reduce the amplitude discrepancies between S_c and A, but it would tend to increase such discrepancies between S_u and A.

4.1.4 Arguments have been made in favor of analyzing the long ex ante simulations in the form of deviations from trend as represented by ratios of the shocked series to the control series [18, pp. 78–80]. Examples of such ratio series for *GNP* and *GNP58* are shown in Chart 4.3. The series with non-autocorrelated shocks, like those in Part A of the chart, are highly erratic; the series with autocorrelated shocks, in Part B, are much smoother. Movements with the attributes of "specific cycles" of the NBER analysis can be identified in both groups of the ratio series, though they are much more distinct in the runs with autocorrelated disturbances. The turning points in these movements were determined by the computer method of dating and are identified on the chart.

The series shown have been picked randomly from the fifty runs in either category. Inspection of charts for all runs discloses numerous and large differences of detail, but no systematic deviations from the general characteristics noted in the previous paragraph. Any of the ratio series is likely to show fluctuations that vary greatly in size and duration, but these variations appear to be randomly distributed over the simulation period, with no tendency either to decrease or increase.

Table 4.4 presents the summary measures of frequency, duration, and relative amplitude of movements in these series, using all of the experimental runs for *GNP* and *GNP58*. This table, which has the same format as Table 4.1, shows that the rises in the ratio series are on the average very close to the declines—in terms of frequency and duration, as well as in relative size. This applies to the simulations with non-autocorrelated and with autocorrelated shocks; to *GNP* and *GNP58*; and to the averages and the standard deviations (compare, column by column, the paired entries in lines 1 and 2, 3 and 4, and so forth). This aspect of strong symmetry suggests that the control solution provides workable estimates of the trends in *GNP* and *GNP58* for both the S_u set and that of the S_c simulations.

CHART 4.3

A Random Sample of Stochastic 100-Quarter Simulations for GNP in Current and Constant Dollars, Ratios to Control Solutions, Wharton Model (1968-III–1993-II)

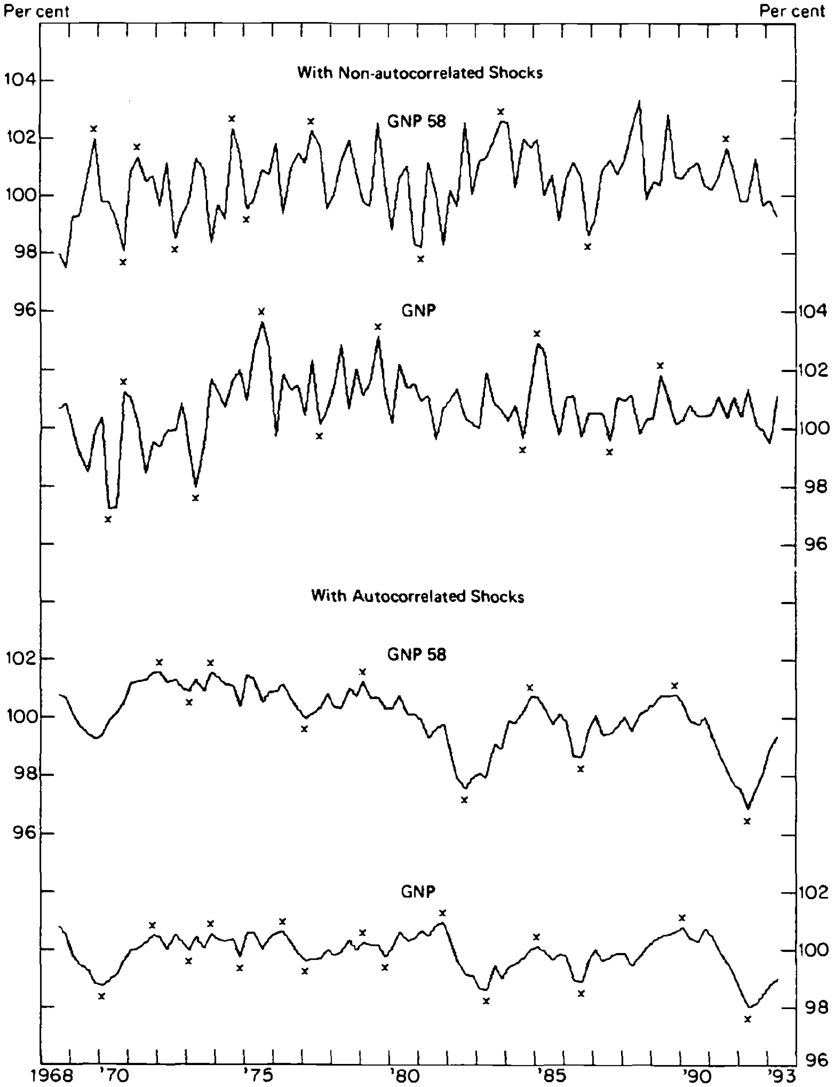

TABLE 4.4

Stochastic 100-Quarter Simulations, Wharton Model, and the Corresponding Sample-Period Actuals: Summary Statistics on Frequency, Duration, and Relative Size of Rises and Declines in Relative Deviations from Trend in GNP and GNP58

		Frequency (number)		Duration (quarters)			Amplitude (per cent)	
Line	Type of Series and Movement	Mean or Total[a] (1)	Standard Deviation (S.D.)[b] (2)	Mean (per run)[c] (3)	S.D. (between runs)[b] (4)	S.D. (within runs)[d] (5)	Mean (per run)[e] (6)	S.D. (between runs) (7)
		GNP: Ratio to Control Solution (S) or Trend (A)[f]						
	Simulations with non-autocorrelated shocks: S_u							
1	Rises	29.50	1.82	1.67	0.14	0.79	0.93	0.14
2	Declines	29.54	1.85	1.68	0.15	0.79	0.90	0.11
	Simulations with autocorrelated shocks: S_c							
3	Rises	21.86	2.51	2.25	0.28	1.41	0.36	0.05
4	Declines	21.82	2.27	2.29	0.29	1.49	0.36	0.05
	Sample-period actuals: A							
5	Rises	13		3.38		2.07	0.92	
6	Declines	13		2.62		1.64	0.95	

TABLE 4.4 (*concluded*)

		Frequency (number)		Duration (quarters)			Amplitude (per cent)	
Line	Type of Series and Movement	Mean or Total[a] (1)	Standard Deviation (S.D.)[b] (2)	Mean (per run)[c] (3)	S.D. (between runs)[b] (4)	S.D. (within runs)[d] (5)	Mean (per run)[c] (6)	S.D. (between runs) (7)
		GNP58: Ratio to Control Solution (S) or Trend (A)[f]						
	S_u simulations							
7	Rises	30.02	1.80	1.64	0.15	0.80	1.11	0.15
8	Declines	29.98	1.80	1.66	0.14	0.80	1.07	0.13
	S_c simulations							
9	Rises	22.80	2.37	2.18	0.27	1.33	0.46	0.06
10	Declines	22.80	2.39	2.18	0.29	1.32	0.46	0.08
	A: actuals							
11	Rises	12		3.83		1.93	0.73	
12	Declines	12		2.67		1.43	1.02	

NOTE: See note to Table 4.1.

[a,b,c,d,e] See corresponding footnotes to Table 4.1.

[f] The nonstochastic control solution series for *GNP* and *GNP58* are taken to represent the trend components of the corresponding stochastic series, S_u and S_c. Exponential trends have been fitted to the *A* series for *GNP* and *GNP58*.

Ratios of the quarterly *GNP* and *GNP58* values to their exponential trends in the sample period of the Wharton Model (1948–68) were computed to provide measures for actuals that correspond to those for the *S*-ratios. As would be expected, the results show the declines in the recorded deviations from trend to be as frequent as—but on the average, shorter and larger than—the rises (Table 4.4, lines 5 and 6, and 11 and 12). Compared with the fluctuations in these reference series, the rises and falls in the S_u ratios are much more frequent and shorter, but of similar relative size (*cf.* lines 1 and 2, and 5 and 6; also, lines 7 and 8, and 11 and 12). The S_c ratio series still overestimate the frequency—and underestimate the average duration—of movements in the *A* series, but by much smaller margins; on the other hand, the amplitudes of these ratios are much smaller than the corresponding measures for S_u and *A* (*cf.* lines 3 and 4, and 5 and 6; also, 9 and 10, and 11 and 12).

4.1.5 Chart 4.4 illustrates the behavior of the simulated ratio series for selected variables. The series that incorporate serially uncorrelated random shocks, S_u, are generally very erratic; those that incorporate autocorrelated shocks, S_c, are considerably less so. The large irregular up-and-down variations in S_u often obscure any longer movements that may exist in these series. In S_c, the longer movements of specific-cycle duration are more readily discernible.

As shown in Table 4.5, both rises and declines in the S_u ratios are short, varying from 1.1 to 2.1 quarters, but concentrated heavily in the range of 1.3 to 1.7 quarters (column 2). In a stationary random series, the expected value of the "average duration of run" would be 1.5 unit periods, for rises and falls alike.[34] In the S_u ratios, the rises are often longer than the declines, but the differences between these AD statistics are, in general, very small.

Upward and downward movements in the S_c series are predominantly longer than their counterparts in the S_u series. The AD figures for S_c exceed those for S_u in over 80 per cent of cases (as seen by comparing the entries in column 2 with those in columns 3 and 4). Nevertheless, both rises and declines are still, for the most part, shorter in the S_c simulations than in the corresponding actuals. In fact, the op-

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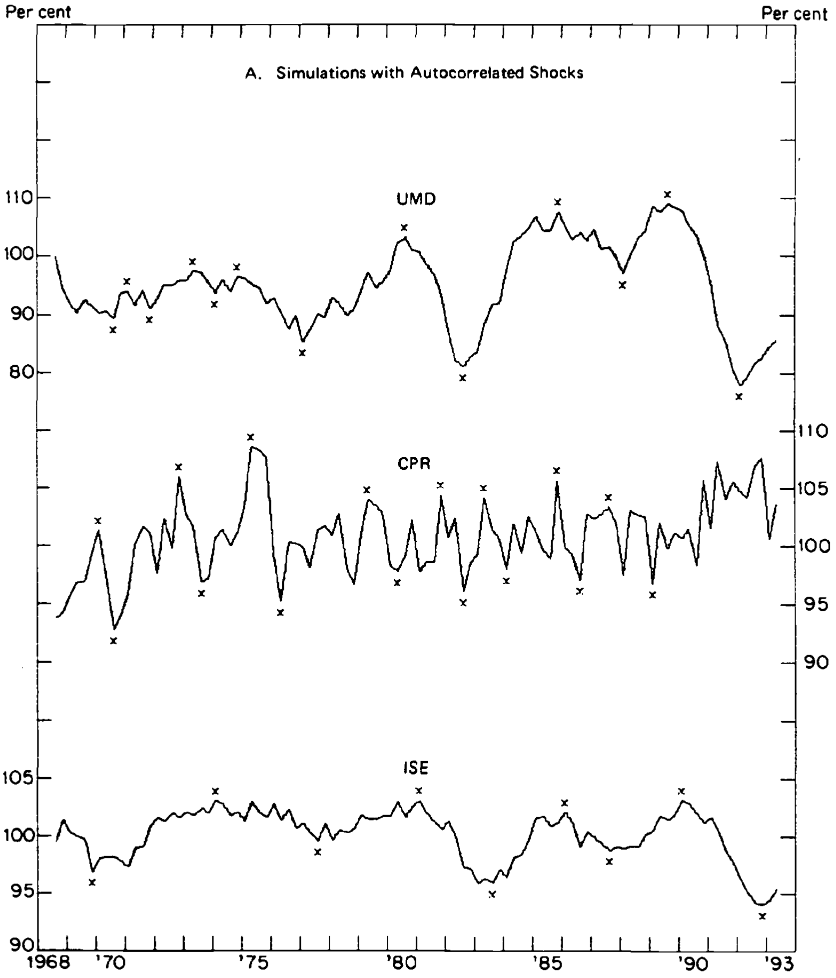
[34] A run in this context denotes an uninterrupted movement in one direction (rise or decline).

CHART 4.4

A Random Sample of Stochastic 100-Quarter Simulations for Selected Variables, Ratios to Control Solutions, Wharton Model (1968-III–1993-II)



(*continued*)

CHART 4.4 (*continued*)

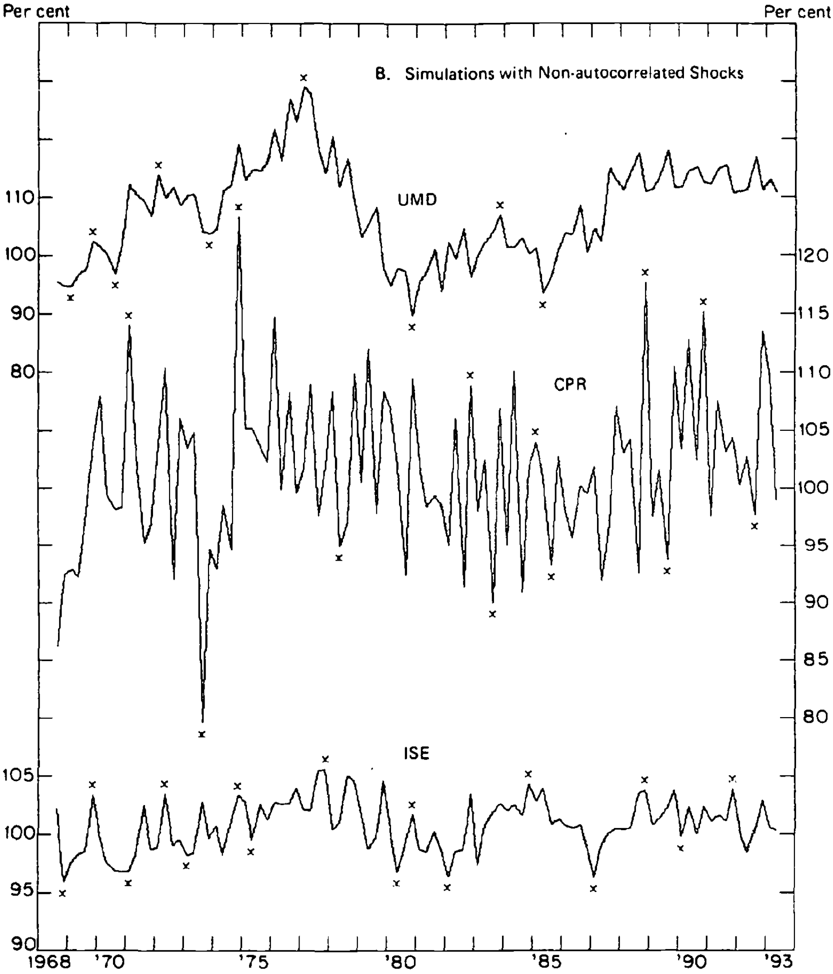
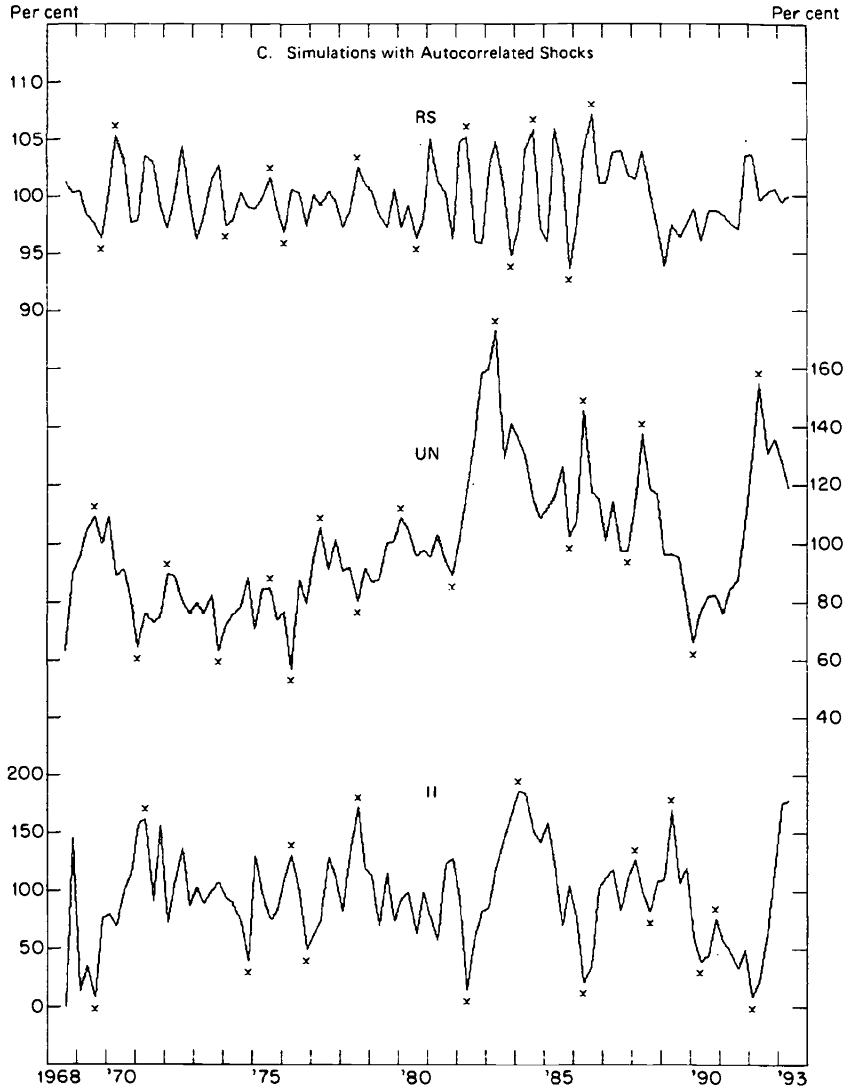


CHART 4.4 (*continued*)



(*continued*)

CHART 4.4 *(concluded)*

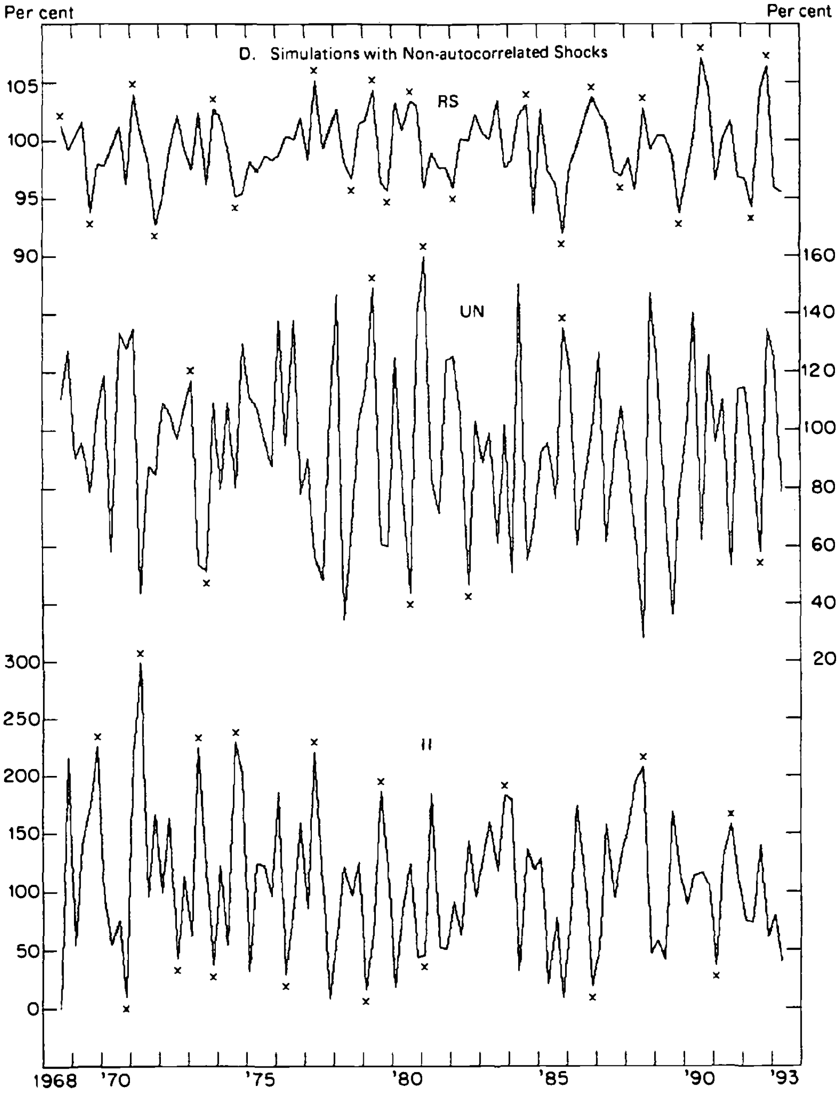


TABLE 4.5

Stochastic 100-Quarter Simulations, Wharton Model: Average Duration of Rises and Declines in Relative Deviations from Trend in Quarters, Seventeen Variables, Actuals and Three Simulation Runs

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Actuals: Ratio to Exponential Trend 1948-III–1968-I (1)	Stochastic Simulations: Ratio to Control Solution		
				Uncorrelated Shocks (Run 31) (2)	Serially Correlated Shocks (Run 14) (3)	Serially Correlated Shocks (Run 26) (4)
1	*GNP*	R	3.4	1.7	2.4	2.2
2		D	2.6	1.5	2.2	2.3
3	*GNP58*	R	3.8	1.7	2.3	2.1
4		D	2.7	1.5	2.4	2.0
5	*C*	R	2.2	1.7	1.5	1.5
6		D	1.9	1.3	1.5	1.7
7	*IH*	R	3.2	1.7	2.2	2.1
8		D	3.0	1.6	1.8	2.1
9	*ISE*	R	2.9	1.7	1.6	1.7
10		D	2.5	1.5	1.9	2.0
11	*II*	R	3.5	1.5	2.2	2.3
12		D	3.2	1.4	1.7	2.1
13	*NE*	R	2.5	1.7	2.1	2.2
14		D	4.8	1.5	1.4	1.9
15	*YP*	R	3.2	1.6	2.5	1.7
16		D	2.4	1.4	2.0	1.8
17	*P*	R	2.8	1.9	3.0	2.4
18		D	3.8	1.4	1.5	1.3
19	*LE*	R	3.5	1.5	1.9	2.6
20		D	3.0	1.5	1.8	2.4
21	*UN*	R	2.3	1.5	2.0	2.2
22		D	2.8	1.5	1.6	1.8
23	*CPR*	R	2.5	1.3	1.8	1.5
24		D	2.2	1.3	1.9	1.6
25	*AWW*	R	1.9	1.8	1.9	2.3
26		D	1.7	1.5	1.5	2.3
27	*UMD*	R	5.2	1.8	2.2	3.0
28		D	3.1	1.5	2.3	3.6
29	*RS*	R	3.1	1.6	1.8	1.7
30		D	3.2	1.6	1.9	2.1
31	*RL*	R	2.4	1.8	1.7	2.0
32		D	3.6	2.1	2.0	1.8
33	*W*	R	2.4	1.8	1.7	1.9
34		D	1.6	1.1	1.4	1.6

[a] For meaning of symbols, see Table 1.1.

posite applies in only four instances in Table 4.5 (where sixty-eight comparisons can be made between columns 1, and 3 and 4). However, the deviations between the AD measures for S_c and the actuals are, in about one-third of the comparisons, fairly small.[35]

The average percentage amplitudes (APA) of quarterly rises and declines are systematically larger for the S_u than for the S_c simulations, as would be expected; this is shown in Table 4.6, columns 2 to 4. There is only one contrary case among all the comparisons here, and the amplitude differences between the two sets of series are often large.

The relative size of movements in S_u is sometimes considerably smaller than the relative size of movements in the sample-period deviations from trend (A), as illustrated by the figures for C, ISE, RS, and RL; but frequently the opposite applies as well, e.g., in the cases of $GNP58$, IH, P, LE, UN, CPR, and AWW (compare columns 1 and 2). Elsewhere the differences between the APA figures for S_u and A are mostly small and of either sign (as for GNP, YP, UMD, and W).

The average percentage changes in the S_c series are, with few exceptions, smaller than those in A, mostly by relatively large margins (columns 1, and 3 and 4). The exceptions are concentrated in the measures for AWW, P (declines only), and W (rises only).

On the whole, the evidence for ratio-to-trend series conforms and complements the evidence for the level series. The major conclusion to be reached is that the stochastic simulations of the Wharton Model generally understate both the average durations and the average relative amplitudes of the sample-period actuals. There are considerable differences among the results for the different variables, which for the most part cannot be readily explained. On the other hand, the differences between the S_u and the S_c simulations are, for the most part, systematic as well as pronounced, and have clear technical reasons.

4.1.6 We proceed with the analysis of specific-cycle movements in the three sets of simulated series (Run 31 for S_u, and Runs 14 and 26 for S_c; see Sections 4.1.3 and 4.1.5 above for charts of some of these series in level and ratio form, with identification of their cyclical turning

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[35] There are ten cases in which the differences between the corresponding entries in columns 1 and 3 equal one-half of one quarter or less, and the same statement can be made about the differences between columns 1 and 4. On the other hand, only four deviations so small are found in comparing columns 1 and 2 (for S_u).

TABLE 4.6

Stochastic 100-Quarter Simulations. Wharton Model: Average Percentage Amplitudes Per Quarter of Rises and Declines in Relative Deviations from Trend, Fifteen Variables, Actuals and Three Simulation Runs

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Actuals: Ratio to Exponential Trend 1948-III–1968-I (1)	Stochastic Simulations: Ratio to Control Solution: Uncorrelated Shocks (Run 31) (2)	Serially Correlated Shocks (Run 14) (3)	Serially Correlated Shocks (Run 26) (4)
1	*GNP*	R	0.92	1.01	0.32	0.38
2		D	0.95	0.95	0.33	0.35
3	*GNP58*	R	0.73	1.25	0.47	0.45
4		D	1.02	1.21	0.36	0.49
5	*C*	R	4.57	1.16	0.46	0.85
6		D	2.95	1.42	0.47	0.81
7	*IH*	R	3.31	5.66	1.32	1.50
8		D	2.96	5.38	1.44	1.55
9	*ISE*	R	2.35	1.81	0.82	0.84
10		D	3.10	2.24	0.70	0.72
11	*YP*	R	0.90	0.77	0.35	0.55
12		D	0.59	0.59	0.44	0.51
13	*P*	R	0.34	0.45	0.20	0.23
14		D	0.28	0.48	0.45	0.40
15	*LE*	R	0.54	1.08	0.22	0.23
16		D	0.40	0.98	0.23	0.26
17	*UN*	R	6.25	76.57	11.72	14.69
18		D	4.66	32.07	11.49	11.42
19	*CPR*	R	4.00	9.76	3.37	2.83
20		D	4.19	8.47	2.59	2.42
21	*AWW*	R	0.32	1.03	0.54	0.63
22		D	0.34	1.21	0.57	0.58
23	*UMD*	R	3.94	3.93	2.17	2.84
24		D	3.23	3.87	2.09	2.10
25	*RS*	R	7.80	3.66	2.96	3.90
26		D	5.39	3.21	2.73	2.90
27	*RL*	R	3.56	1.94	1.94	1.52
28		D	2.34	1.61	1.36	1.46
29	*W*	R	0.44	1.31	0.49	0.55
30		D	2.23	2.06	0.58	0.64

[a] For meaning of symbols, see Table 1.1.

points). Of particular interest here is the relative timing of these series for variables that have historically typified the sequence of cyclical leaders, coinciders, and laggers.

Measuring this timing efficiently requires that a "reference chronology" be established for each simulation run. Some analysts of current business conditions would use a single, comprehensive aggregate such as *GNP58* as a basis for dating the business cycle (requiring, perhaps, a two-quarter decline, or lack of growth, in that series as a minimum condition for identifying a business recession). However, even in historical situations this criterion is not always available or reliable.[36] In dealing with simulations, the problem is aggravated by the paucity of cyclical turns in the levels of the *S* series for *GNP* in current and constant dollars. It is, therefore, desirable to use more information in determining the reference dates, namely the evidence on the bunching in time of the specific-cycle peaks and troughs in the *S* series for different variables.[37]

The so-called "historical" diffusion indexes provide a suitable method for organizing this evidence. After the cyclical turning points have been identified in each of the *S* series in a given set, the percentage of the series undergoing specific-cycle expansion is calculated for each successive quarter.[38] The deviations of these percentage figures from 50 are cumulated, to give a relatively smooth series—the so-called cumulated diffusion index (CDI)—whose peaks (troughs) would be centered on the periods with the greatest concentration of specific cycle peaks (troughs) in the component *S* series.[39]

Two CDI series have been computed for each of the three randomly chosen sets of Wharton Model simulations: namely, an index

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[36] For a critique of using *GNP* data alone in the dating of business cycles, see [30] and the references therein.

[37] It should be noted that this traditional NBER approach was also used in [1, Sec. 9].

[38] This implies that all movements contrary in direction to the cyclical phases of the series are ignored. A positively conforming series is treated as expanding in each quarter that falls between a specific trough and a specific peak in the data; it is treated as contracting in each quarter situated between a peak and a trough.

[39] At the culmination of business expansion, peaks tend to be most frequent and troughs least frequent. As the contraction begins, the proportion of series expanding falls below 50 per cent; the deviations from 50 that are cumulated in our index shift from + to − and the index passes through a peak. Analogous statements apply *mutatis mutandis* to the situation at the trough. For more information on measurements of cyclical diffusion, see [26, Chapters 8, 9, and 20].

based on the S series proper, and an index based on the ratios of these shocked series to the corresponding nonstochastic (control) series. However, only the indexes of the second type include all seventeen variables covered by the Wharton simulations. Unlike the simulated ratio series (all of which are divisible into specific cycles), the levels of the S series for some variables show only prolonged growth trends, sporadically interrupted by a very few short—predominantly one-quarter—declines (see Table 4.2). Series of this kind have no specific cycles and therefore cannot be included in diffusion indexes based on specific cycles. Thus, only ten of the seventeen variables are represented in the cumulated diffusion index (CDI) for the simulated series of Run 14. The series included are the more volatile ones, relating to types of investment, corporate profits, average workweek, unemployment, unfilled orders, and interest rates; excluded are the comprehensive aggregates for national output, income, and consumption, and the indicators of the general price and wage levels. The diffusion indexes for the level series of Runs 26 and 31 have the same composition.

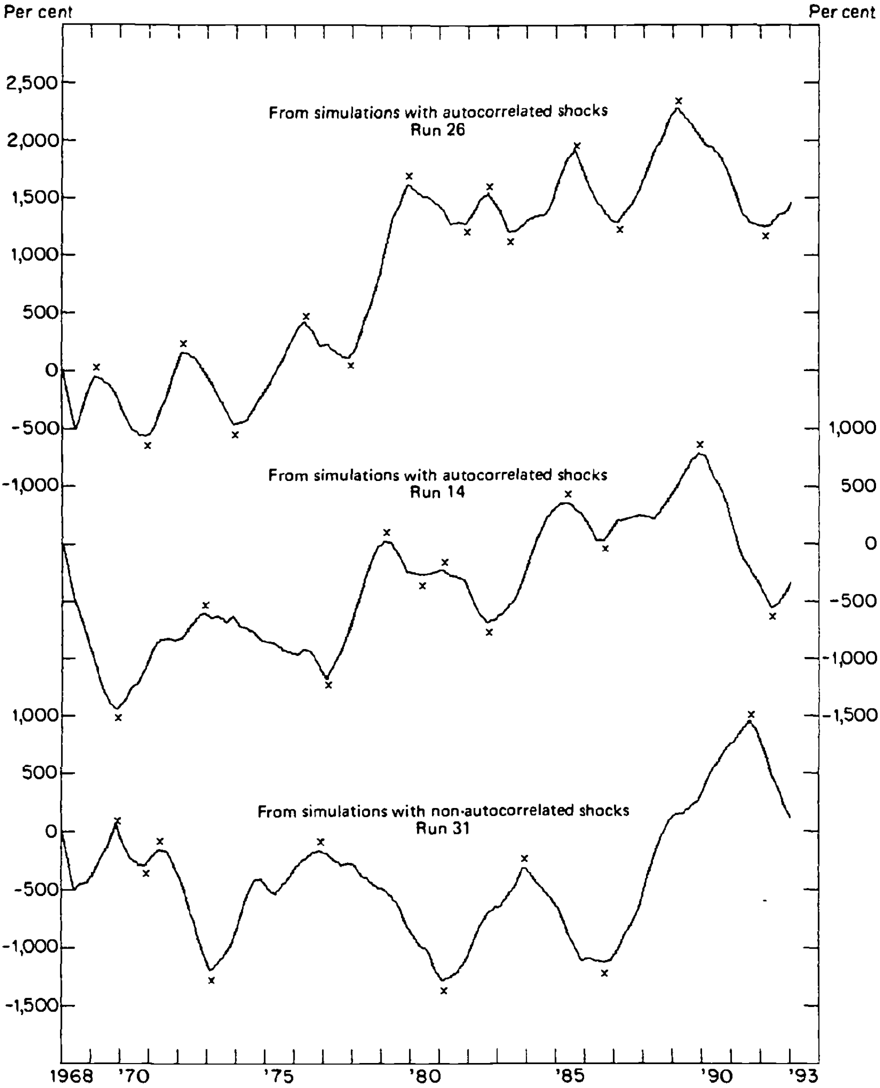
According to these summary diffusion measures for the simulations with autocorrelated shocks (Runs 14 and 26), the average duration of expansions would be about 9 to 11 quarters; that of contractions, more than 5 quarters. In the postwar period, the mean actual duration was approximately 12 to 16 quarters for expansions (the latter figure includes the long expansion of the 1960's up to the end of the Wharton sample-period), and 3 to 4 quarters for contractions. The figures for the S_c series are, at least, of a similar order of magnitude.[40] However, because the simulations of the comprehensive income and product aggregates cannot be included in this analysis, no further use will be made of the cyclical diffusion and other measures based on the S series proper in this paper; instead, we shall concentrate on the corresponding measures for the ratios of shocked to control series.

Chart 4.5 shows that the indexes of cumulated percentage expanding (CDI) for the ratio series have well-defined cyclical movements, with contractions that are relatively long—often as long as the expansions. There are some upward drifts in these indexes, but they

[40] The average duration of expansions in the index for the S_u series (Run 31) is considerably larger: 20.2 quarters. The average duration of contractions in this index is again similar, 4.5 quarters.

CHART 4.5

Cumulated Historical Diffusion Indexes for Selected Sets of Simulated Series, Ratios to Control Solutions, Wharton Model (1968-III–1993-II)



are small and not persistent (except in the first half of the simulation period for Run 26). In contrast, the indexes for the *S* series proper have relatively short and weak contractions and strong upward trends (cumulation due to the prevalence of expansions over contractions). This contrast reflects differences of the same kind between the movements in the levels of the simulated series on the one hand; and the movements in the ratio series, on the other.

The diffusion indexes vary greatly with respect to the timing and amplitudes of fluctuations, but this merely reflects fortuitous differences among the runs in the incidence of large and small shocks. What matters, again, is the *presence* in these composite series of reasonably smooth movements of cyclical dimensions. The main criterion is duration, and the averages in the accompanying tabulation show that the fluctuations in the cumulated diffusion indexes (CDI) for the ratio series are not very different in this respect from cycles in trend-adjusted *GNP* data for the postwar period.[41]

	Ratios to Control Solution, Wharton Model: Indexes of Cumulated Per Cent, Expanding			Ratios to Exponential Trend, Actual Data, 1948–68	
	Run 14	Run 26	Run 31	*GNP*	*GNP58*
	(1)	(2)	(3)	(4)	(5)
	Average Duration of Movement, in Quarters				
Expansions	9.4	7.2	12.0	12.5	11.0
Contractions	8.6	7.0	9.8	7.0	6.5
Full cycle	18.0	14.2	21.8	19.5	17.5

4.1.7 This section summarizes the results obtained by comparing the timing of the CDI for the Wharton Runs 31, 14, and 26, with the timing of each of the component series in the same run. In the process, measures of conformity are also computed, in the form of the fre-

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[41] The figures for the historical data are sensitive to the choice of the sample period and the figures for simulations depend on the idiosyncrasies of the given run. The deviations between the tabulated measures fall comfortably within the range of such sampling variations. Other related duration measures, based on average historical "growth cycles" in sets of selected indicators, confirm the above conclusion.

quencies of those turns in CDI and the simulated ratio-series that cannot be matched (Table 4.7, columns 1 to 4). The distributions of leads and lags of the simulated series at the reference (CDI) peaks and troughs are given in Table 4.7, columns 5 to 12. The means and medians of these distributions are listed in Table 4.8.

The frequencies of specific-cycle movements and turning points that have no counterparts in the over-all reference index are much greater for the series in Run 31 than for those in either of the other sets (Table 4.7, columns 3 and 4).[42] This might have been expected, since series with autocorrelated shocks (S_c) are generally much smoother than the series with serially noncorrelated shocks (S_u). On the other hand, there is no reason why S_u should score systematically worse (or better) than S_c in matching the reference turns, and there is no evidence that they do.[43] The over-all conformity record tends to be better for the S_c than for the S_u series, because the former show fewer extra turns and no greater rate of failures to match the reference turns.

Comprehensive aggregates of national product and income should naturally be among the best conformers, with approximately coincident timing, and the historical record fully confirms this presumption.[44] Thus, it is good to find that the simulations of these variables (*GNP*, *GNP58*, and *YP*) are among those with the lowest proportions of unmatched turns in columns 2 and 4 of Table 4.7. Also in this group are the series for two large real-expenditure components, *C* and *ISE*, and unfilled orders, *UMD* (a well-behaved, cyclical "stock" aggregate). Business investment in plant and equipment, a lagging series, has had an excellent historical record of moving with the general business cycle. Personal income and consumption (both coinciders) have conformed less well in that they underwent merely retardations of growth

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[42] All but one of the component series of Run 31 have some extra turns, while there are four series in Run 14 with no extra turns and four again in Run 26. The average percentages of such turns are 36.9, 19.4, and 13.6 for Runs 31, 14, and 26, respectively.

[43] There are eight series that fail to match all reference turns in Run 31, eleven such series in Run 14, and six in Run 26 (which, here as elsewhere, happens to yield particularly favorable results). The average percentages of unmatched reference turns are 10.4, 16.5, and 4.5 for the three runs listed in the same order (Table 4.7, columns 1 and 2).

[44] See [26, Chapters 3 and 7]. For the latest review of the performance of these and other indicators, see Geoffrey H. Moore and Julius Shiskin, *Indicators of Business Expansions and Contractions*. Occasional Paper 103. New York, Columbia University Press for the National Bureau of Economic Research, 1967.

TABLE 4.7

Cyclical Conformity and Timing of Simulated Ratio-Series, with Reference Chronologies Based on Cumulated Diffusion Indexes, Wharton Model, Three Runs

		Reference Turns Not Matched[b]		Extra Specific Turns[c]		Number of Timing Observations							
						At Reference Peaks				At Reference Troughs			
Line	Variable Symbol[a]	Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Coincidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Coincidences (10)	Lags (11)	Long Leads and Lags[d] (12)
						Run 31: with non-autocorrelated shocks[e]							
1	*GNP*	2	22.2	3	30.0	2	0	2	3	1	0	2	2
2	*GNP58*	0	0	2	18.2	0	4	1	0	1	2	1	0
3	*C*	0	0	3	25.0	0	2	3	1	1	1	2	1
4	*IH*	2	22.2	6	46.2	4	1	2	2	1	0	2	3
5	*ISE*	0	0	7	43.8	0	1	4	3	1	1	2	1
6	*II*	0	0	8	47.1	0	4	1	0	1	2	1	0
7	*YP*	0	0	1	10.0	0	2	3	1	2	1	1	0
8	*P*	1	11.1	3	27.3	0	0	4	3	1	1	2	3
9	*LE*	0	0	10	52.6	0	3	2	0	2	1	1	1
10	*UN*[h]	2	22.2	4	36.4	1	1	2	3	1	2	0	1
11	*CPR*	2	22.2	7	50.0	3	0	1	3	1	0	2	2
12	*AWW*	2	22.2	8	53.3	1	2	1	1	0	1	2	1
13	*UMD*	0	0	0	0	0	3	2	1	3	0	1	2
14	*RS*	1	11.1	12	60.0	2	0	2	2	3	0	1	4
15	*RL*	2	22.2	8	53.3	1	1	2	2	0	2	1	0

(*continued*)

TABLE 4.7 (*continued*)

Line	Variable Symbol[a]	Reference Turns Not Matched[b]		Extra Specific Turns[c]		Number of Timing Observations							
						At Reference Peaks				At Reference Troughs			
		Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Coincidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Coincidences (10)	Lags (11)	Long Leads and Lags[d] (12)
Run 14: with autocorrelated shocks[f]													
16	*GNP*	0	0	4	26.7	2	1	2	2	1	3	2	1
17	*GNP58*	2	18.2	0	0	2	2	0	1	0	5	0	0
18	*C*	2	18.2	2	18.2	2	1	1	1	1	3	1	0
19	*IH*	2	18.2	4	30.8	4	0	0	3	3	1	1	3
20	*ISE*	2	18.2	0	0	0	1	3	2	0	1	4	2
21	*II*	2	18.2	2	18.2	4	0	0	2	5	0	0	0
22	*YP*	0	0	4	26.7	1	1	3	1	2	2	2	0
23	*P*	4	36.4	2	22.2	1	0	3	4	1	0	2	3
24	*LE*	2	18.2	3	25.0	1	1	2	1	1	1	3	2
25	*UN*[h]	0	0	6	35.3	1	0	4	2	3	1	2	1
26	*CPR*	1	9.1	4	28.6	1	1	3	2	2	3	0	2
27	*AWW*	0	0	2	15.4	2	2	1	0	4	1	1	0
28	*UMD*	2	18.2	0	0	0	0	2	0	1	2	2	2
29	*RS*	0	0	3	23.1	0	0	1	4	2	1	3	3
30	*RL*	5	45.4	4	40.0	1	1	2	1	1	0	2	2
31	*W*	5	45.4	0	0	0	0	2	2	1	0	2	3

TABLE 4.7 (*concluded*)

		Reference Turns Not Matched[b]		Extra Specific Turns[c]		Number of Timing Observations							
						At Reference Peaks				At Reference Troughs			
Line	Variable Symbol[a]	Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Coincidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Coincidences (10)	Lags (11)	Long Leads and Lags[d] (12)
						Run 26: with autocorrelated shocks[g]							
32	*GNP*	0	0	0	0	1	4	2	0	2	3	2	2
33	*GNP58*	0	0	0	0	1	6	0	0	4	3	0	2
34	*C*	0	0	2	12.5	2	4	1	0	2	3	2	2
35	*IH*	0	0	3	17.6	3	2	2	1	4	1	2	2
36	*ISE*	0	0	2	12.5	0	1	6	3	0	7	0	3
37	*II*	0	0	5	26.3	4	3	0	1	5	1	1	1
38	*YP*	0	0	1	6.7	0	5	2	0	1	4	2	0
39	*P*	2	14.3	0	0	0	0	6	4	0	0	6	5
40	*LE*	2	14.3	3	20.0	0	1	5	3	1	3	2	2
41	*UN*[h]	0	0	2	12.5	0	6	1	0	3	0	4	2
42	*CPR*	1	7.1	2	13.3	2	1	3	2	5	0	2	5
43	*AWW*	0	0	8	36.4	3	3	1	2	2	2	3	0
44	*UMD*	0	0	0	0	4	3	0	1	2	3	2	2
45	*RS*	1	7.1	6	31.6	4	1	2	2	2	1	3	2
46	*RL*	3	21.4	2	15.4	2	0	4	4	2	0	3	5
47	*W*	1	7.1	2	13.3	2	0	4	4	5	0	2	2

NOTES TO TABLE 4.7.

[a] For meaning of symbols, see Table 1.1.

[b] Turns in the cumulated diffusion index (CDI) not matched by turns in the simulated series. For numbers of the reference turns (in CDI), see notes *e*, *f*, and *g*.

[c] Turns in the simulated series that have no matching turns in CDI.

[d] Leads and lags of three or more quarters.

[e] Nine reference turns (5 peaks and 4 troughs).

[f] Eleven reference turns (5 peaks and 6 troughs).

[g] Fourteen reference turns (7 peaks and 7 troughs).

[h] Inverted (peaks matched with reference troughs; troughs, with reference peaks).

rather than absolute declines in some of the recent mild recessions. Total employment (*LE*) should have a very good conformity record but does not (it has relatively large combined percentages in columns 2 and 4, which we have ranked for each run in making these comparisons). On the other hand, the unemployment simulations score fairly well here, particularly for the S_c runs, 14 and 26. The series for profits, interest rates, and wages rank lowest for conformity: i.e., have the largest total proportions of unmatched and extra turns. Actually, price and wage levels have been poor conformers in recent business fluctuations, although considerably better results would be obtained for these variables in trend-adjusted or first-difference form. Profits and, even more, interest rates (average corporate bond yields and rates on 4- to 6-month commercial paper) have been very sensitive, not only to major cyclical movements but to minor retardations and speedups as well.

4.1.8 The evidence on timing of the simulated series is substantially more difficult to summarize than that on their conformity. In an attempt to determine what, if any, are the typical timing characteristics of these experimental data, one must take into account the relative frequencies of leads, coincidences, and lags generated by the comparison of turns in the individual *S* series with turns in CDI (Table 4.7, columns 5 to 12), as well as the length of the resulting average leads or lags (Table 4.8). The timing at peaks and at troughs must be compared, and the consensus (or lack of it) between the corresponding measures for the different runs must be noted. Finally, one ought to consider any additional uncertainties due to the paucity of timing comparisons per run and the frequency of lapses from conformity.

Where leads and lags are in balance or (better) where coincidences

TABLE 4.8

Average Timing of Simulated Series (Ratios to Control Solutions) at Reference Dates Based on Cumulated Diffusion Indexes, Wharton Model, Three Runs (Mean (M) and Median (Md) Lead (−) or Lag (+) in Months)

Line	Variable Symbol[a]	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns
		M (1)	Md (2)	M (3)	Md (4)	M (5)	M (6)	Md (7)	M (8)	Md (9)	M (10)	M (11)	Md (12)	M (13)	Md (14)	M (15)
		Run 31[b]					*Run 14*[c]					*Run 26*[c]				
1	*GNP*	+0.8	+1.5	+3.0	+3	+1.7	−0.6	0	+1.0	0	+0.2	+0.9	0	+0.4	0	+0.6
2	*GNP58*	+1.2	0	−0.8	0	+0.3	−3.7	−3	0	0	−1.7	−0.4	0	−4.2	−3	−2.3
3	*C*	+3.0	+3	+2.2	+1.5	+2.7	+0.7	−1.5	−0.7	0	0	−0.4	0	+0.4	0	0
4	*IH*	−9.7	−7.5	+4.0	+9	−3.8	−10.5	−12	−3.6	−6	−6.7	−1.7	0	−1.7	−3	−1.7
5	*ISE*	+7.8	+12	0	+1.5	+4.3	+6.7	+6	+7.2	+6	+7.0	+6.9	+3	+6.5	+3	+6.6
6	*II*	+1.2	0	−0.7	0	+0.3	−11.2	−10.5	−3.0	−3	−6.7	−3.4	−3	−3.9	−3	−3.6
7	*YP*	+5.4	+3	−0.7	−1.5	+2.7	+3.0	+3	−0.5	0	+1.1	+1.3	0	+1.0	0	+1.1
8	*P*	+17.2	+15	+7.5	+7.5	+12.4	+7.8	+15.5	+12.0	+24	+9.6	+13.0	+12	+12.0	+9	+12.5
9	*LE*	+1.8	0	−4.5	−3	−1.0	+3.0	+1.5	+5.4	+3	+4.8	+7.5	+7.5	+3.0	0	+5.2
10	*UN*[d]	+1.5	+4.5	−3.0	0	−0.4	+4.8	+6	+0.5	−1.5	+2.4	+0.4	0	0	+3	+0.2
11	*CPR*	−5.2	−6	+2.0	+6	−2.1	−1.8	+3	−6.0	0	−3.9	+0.5	+1.5	−10.3	−12	−5.4
12	*AWW*	−0.7	0	+5.0	+6	+1.7	−1.2	0	−5.5	−3	−3.5	−1.3	0	−0.4	0	−0.9
13	*UMD*	+2.4	0	−3.0	−3	0	+0.8	+1.5	+4.8	0	+3.0	−3.0	−3	−3.4	0	−3.2
14	*RS*	0	+1.5	−6.8	−12	−3.4	−9.0	−6	−4.0	+1.5	−6.3	−0.9	−3	+1.5	+1.5	+0.2
15	*RL*	+4.5	+1.5	+1.0	0	+3.0	+5.0	+3	+6.0	+6	+5.5	+8.5	+12	−0.6	+9	+4.4
16	*W*[e]						+11.0	+18	+9.0	+9	+10.0	+4.0	+6	−2.6	−3	+0.5

NOTES TO TABLE 4.8.

The average leads and lags listed in this table cover the timing observations that are included in the frequency distributions of columns 5 to 12 of Table 4.7.

[a] For meaning of symbols, see Table 1.1.

[b] With non-autocorrelated shocks.

[c] With autocorrelated shocks.

[d] Inverted. (*See note h in Table 4.7.*)

[e] Not computed for Run 31 because the series is highly erratic, and its specific cycles (if any) could not be reliably identified.

prevail, the timing may be classified as *roughly coincident* (*RC*). The timing averages in this class should fall in the range from −3 to +3 months (leads or lags of one quarter or less, and exact coincidences). The medians are often more reliable than the means of the timing observations, because the latter are sensitive to extremely long individual leads or lags, which are sometimes particularly uncertain.[45] Where leads prevail and the averages exceed 3 months with minus signs, the timing is classified as *leading* (*L*). Where lags prevail and the averages are similarly large, but with plus signs, the series is called *lagging* (*Lg*). The determinations were made separately for peaks and troughs, and for each of the runs, and they are not always the same for a given variable.

The following can be classified with a relatively high degree of assurance:

(1) *GNP*, *GNP58*, *C*, *YP*, and *UMD*. These five variables all belong to the *RC* (roughly coincident) group. In the means for *GNP* and *YP*, lags prevail slightly, while in those for *GNP58*, leads prevail, but the figures are small. The distributions in Table 4.7 and the medians in Table 4.8 clearly indicate the *RC* classification at both peaks and troughs. The same applies to *C*, where some short lags appear in Run 31, which, however, is entirely compatible with the *RC* classification). As for *UMD*, lags are somewhat more frequent in one of the runs (14), and leads in another (26), but they are on the whole short and the entire evidence, including the averages, argues for inclusion of this variable in the *RC* category.

(2) *ISE*, *P*, and *RL*. These must be included in group *Lg* (laggers).

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[45] It is for this reason that we include the counts of "long leads and lags" (of three or more quarters) in Table 4.7, columns 8 and 12.

Lags dominate the distributions for business fixed-investment in every case, and they are intermediate or long, except for troughs in Run 31. Long lags prevail throughout in the timing of the price-level simulations. For the long-term interest rates, the lags are more variable and, on the average, much shorter.

For the remaining variables, the evidence is rather mixed, but it indicates these additional groupings:

(3) *IH*, *II*, *CPR*. Predominantly *L*, *leaders*. For housing investment, there are long, or intermediate, leads at peaks in Run 31; and at both peaks and troughs in Run 14. However, the average leads are small in Run 26 and *IH* can be included in the *RC* group there; and at troughs in Run 31, there are lags. Inventory investment is definitely a leader in Run 14, a short leader in Run 26, but a rough coincider in Run 31. Corporate profits have over-all mean leads in every case, but closer inspection shows them to be definitely leading only at peaks in Run 31 and at troughs in Run 26; elsewhere, *CPR* is better described as a rough coincider, with some short lags in the averages.

(4) *UN*, *AWW*. For the most part, *RC* (roughly coincident). Unemployment simulations show some tendency to lag, particularly at peaks in Runs 31 and 14. The series for the average workweek frequently lead, but mostly by short intervals (and *AWW* lags at troughs in Run 31).

(5) *LE*. Would be classified as *RC* in Run 31 but as *Lg* in Runs 14 and 26. These lags of the employment series, however, are on the whole not long, except at peaks in Run 26.

(6) *RS* and *W*. The simulated ratio-series for these variables are particularly erratic, and it is difficult to identify, let alone date, their specific-cycle movements. Hence, our results here are quite uncertain; moreover, they vary considerably for the different episodes and sets of data. For the short-term interest rates, coincident timing prevails at peaks in Run 31, and at either turn in Run 26, but leads are dominant elsewhere. For the wage rate, there are long lags in one run; offsetting lags and leads at peaks and troughs, respectively, in another.

There is much evidence on the historical timing-patterns of the important economic variables under study, but it generally refers to the series proper (usually after seasonal adjustment), rather than to their deviations from trend or other similar transformations. Adjusting for a rising trend would often tend to shift peaks in the series backward —and troughs, forward—in time; adjusting for a declining trend would have opposite effects. But such shifts seldom appear to be large [5, Chapter 7]; and in particular, very seldom large enough to alter the typical timing sequence of the indicator series. Also, diffusion indexes, which are highly correlated with the rates of change in the corresponding aggregates, have timing sequences that tend to parallel those between the aggregates for the same variables (see, e.g., [26, Chapter 9]). We shall proceed on the assumption that series of relative deviations from trends (or control solutions) should have the same relative timing properties as the corresponding series without any trend adjustments. This should be, at least, a justifiable first approximation, but it will deserve some checking in further research.

Historically, then, most of our variables are readily classifiable as either "rough coinciders," such as *GNP*, *GNP58*, *YP*, *LE*, and *UN*; or "leaders," such as *II*, *CPR*, and *AWW*; or "laggers," such as *ISE*. The above nine series are, indeed, all so designated in the basic list of the NBER business-cycle indicators. (See reference in footnote 18.) Real consumption expenditures, *C*, represent, as far as one can tell, another series in the *RC* group (as does the related indicator of retail sales). Unfilled orders for durables tended to lead at peaks and roughly coincide, often with short lags, at troughs [26, Chapter 14]. However, in less vigorous expansions, the lead of *UMD* at peaks may not be long, and the over-all timing of this series has been denoted as roughly coincident in the recently compiled comprehensive list of the NBER indicators.

The timing of the simulated series reviewed earlier in this section conforms to the historical over-all patterns for nine of these eleven variables (counting *UMD* simply as in the *RC* group). According to the measures in Tables 4.7 and 4.8, *LE* would be classified as lagging (instead of *RC*), and *AWW* as roughly coincident (instead of *L*).

Residential construction, *IH*, has led at each of the four postwar business-cycle peaks, at the last two of them by very long intervals; it has led by one or two quarters, or coincided, at troughs. Its conformity record was not very good. The prevalent leading patterns of the *IH* simulations, including the long leads at peaks in Runs 31 and 14, are not in conflict with this experience.

The implicit price deflator, *P*, has failed to decline in any of the three recessions since 1953, but did show retardations during each of these episodes. Historically, the wholesale price-index (except farm products and foods) has been a roughly coincident series. While many prices react sluggishly, the very long lags of the *P* simulations appear rather dubious when judged by past price-level behavior.

Compensation per man-hour, *W*, is even more dominated by upward trend and resistant to cyclical declines than *P*. In trend-adjusted form, this series could show a coincident-lagging timing broadly similar to that recorded for Run 26.

The Treasury-bill rate has been classified as roughly coincident in the comprehensive list of NBER indicators (1967), and the commercial-paper rate (*RS*) has a very similar timing. However, according to recent studies by Phillip Cagan, both *RS* and the high-grade bond yield have tended to lag.[46] The leading tendencies of the *RS* simulations are at variance with the historical pattern. The simulations show the average corporate bond yield (*RL*) as lagging, which agrees with the evidence of the long series studied by Cagan (but the NBER 1967 list classifies corporate bond yields as *RC* on the basis of data gathered since 1948).

To sum up, the timing of the simulations for *LE*, *AWW*, *P*, and *RS* seems definitely at odds with the historical patterns; but for the twelve other variables included in Tables 4.7 and 4.8, this is not the case—according to the above comparisons. We are inclined to regard this as a rather good total score, and would find it encouraging if confirmed by further testing. The latter is necessary, however, because there are many pitfalls in this kind of analysis. Perhaps the major one is that some of the simulated ratio-series are so erratic that it is difficult to identify their specific cycles, and the selection of a particular turning point date may involve considerable error. Averaging presum-

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[46] See P. Cagan, "The Influence of Interest Rates on the Duration of Business Cycles," *Essays on Interest Rates*, Volume I (J. M. Guttentag and P. Cagan, editors). New York, Columbia University Press for the National Bureau of Economic Research, 1969, p. 7 (with a reference to Cagan's earlier work).

ably helps here, but we cannot count on all individual errors to offset each other neatly. High proportions of unmatched turns and extra turns, and of very long leads or lags, provide danger signals. Inspection of charts confirms that the series for *CPR*, *AWW*, *W*, and the interest rates (particularly *RS*) are especially volatile; hence, all generalizations based on their behavior are most uncertain. The series with non-autocorrelated shocks (Run 31) are generally more irregular than the others, and include some additional items for which the results are similarly dubious. Moreover, for one variable—net exports (*NE*)—it was impossible to make any meaningful cyclical measurements at all.[47]

4.2 THE OBE MODEL

4.2.1 The post-sample-period simulations for this model begin in 1966-I and end in 1990-IV. Several modifications in the model structure were made for the purpose of these simulations.[48] As for the treatment of exogenous variables, all tax rates, the discount rate, and the time deposit rate were held at constant levels, while most of the other factors were set to grow at the average rates of change observed for them during the sample period. However, the growth rates of several series, including government purchases and government employment, were adjusted to produce results deemed to be plausible. Census population projections were used in determining the time-paths of some series. In the control solution, free reserves were kept at zero throughout. The resulting series of unborrowed reserves of banks was used

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[47] This series, it should be noted, conformed poorly in the past, which is not surprising; however, the Wharton simulated ratio-series for *NE* have a different and rather arbitrary appearance.

[48] Capital consumption allowances were made dependent on the value of the net stock of plant and equipment, instead of being treated as exogenous. Constant trend-increments were added to the equations for housing starts and merchandise imports, while negative trend expressions were eliminated from the equations for labor force participation and hours worked. The price level of government purchases from the private sector was made endogenous, to grow at the same percentage rate as the price level of private *GNP*, excluding housing services. The empirical tax and transfer relationships used during the sample period were replaced by equations linking taxable income to personal per capita income, and tax payments to liabilities; for state and local payments, some arbitrary assumptions about rising marginal rates and time trends had to be made. Improved equations for manufacturers' shipments and unfilled orders were adopted, as well as certain relatively minor changes affecting investment in plant and equipment, and some interest rates. For more detail, see [18, Sec. 4.1].

in the runs with stochastic shocks, where free reserves were not restricted. This implies that the money stock increases smoothly throughout in the OBE control solution: monetary policy is apparently growth-oriented rather than cyclical [18, pp. 72–75].

The control solution of the thus modified OBE Model shows *GNP* growing at annual rates varying from 7.5 to 5.7 per cent, and *GNP58* growing at rates varying from 5.3 to 3.4 per cent. The rates tend to fall off from year to year. Had they been held constant, given the projected growth patterns for the population and labor force, the unemployment rate would have exhibited a sharp decline. Actually, it is unemployment that is held within a narrow range of variation (it decreases from 4.2 to 3.9 per cent).

Unlike the Wharton Model control-solution, apparently no special assumptions were made here about the transition period involving the Vietnam War. (The starting point of these OBE simulations is 1966-I —seven quarters earlier than the beginning of the Wharton run—and its selection is said to have "minimized difficulties in the transition from actual data to the model solutions" [18, pp. 69–70].) There are, indeed, still fewer movements other than trends in the OBE control-solution series than in those for the Wharton Model. *GNP* grows persistently from $736 billion in 1966 to $3,413 billion in 1990, or more than 4.6 times. *GNP58* increases likewise from $649 billion to $1,675 billion, or nearly 2.6 times. The implied rates of growth are significantly higher here than in the solution of the Wharton Model (see Sec. 4.1.1 above), so that the *GNP* series reach higher levels sooner.

Of the twenty-one variables covered, all but seven have continuous upward trends in the OBE simulations. The others are: *II* and *OMD* (which show slight initial declines and a few minor, sporadic irregularities superimposed on their basic growth-trends); *NE* (which declines smoothly in the first eight years, then rises smoothly for the rest of the simulation period); *UN* (which trends downward, with a few discontinuities); *AWW* (which shows a short dip in 1966–68, then a rise in the 1970's, and a smoother decline in the 1980's); and finally, the interest rates *RS* and *RL* (which, like *AWW*, but still more smoothly, increase in the first half and decrease in the second half of the simulation period). The behavior patterns of these seven series are for the most part quite different for the OBE Model than for the Wharton

Model, but they all seem rather arbitrary when compared with the historical movements and, perhaps, may best be viewed as concomitants of the search for broadly satisfactory end results in terms of the control solution for the over-all aggregates.

In short, the conclusion reached in Sec. 4.1.1 for the Wharton Model applies here at least as strongly: the nonstochastic simulations for the sample period (1953–66 for the OBE Model) contain substantial cyclical elements, whereas the corresponding simulations for the subsequent (largely future) period of twenty-five years contain practically no such elements. One likely reason for this is that the exogenous variables are not permitted to fluctuate but are assumed, in many cases, to grow strongly throughout the simulation period. Notably, the federal defense purchases and total government nondefense expenditures on goods and services are set to increase persistently at average annual rates of 5.4 and 10.9 per cent, respectively [18, Sec. 4.2].

4.2.2 Against this background of growth, the *GNP* series that are derived by stochastic simulations of the OBE Model show, primarily, strong upward trends and very few declines (as illustrated by some randomly chosen runs in Chart 4.6). Indeed, only three of the twenty-five runs with non-autocorrelated random shocks, S_u, produce any downward movements at all in the *GNP* series (Table 4.9, column 1). Declines are much more frequent in the *GNP58* series computed from these runs (column 2), but they are very rare in both *GNP* and *GNP58* for the simulations with autocorrelated disturbances, S_c (columns 3 and 4). In all of the one-hundred series of 100 quarters each that are covered in Table 4.9, we count only 31 declines, none longer than one quarter (lines 2 to 4). Moreover, the few downturns that do show up are quite small, averaging from 0.15 to 0.24 per cent per quarter for the four sets of simulations (line 7). Being seldom interrupted, the expansions in the S_u series for *GNP* and in the S_c series for both *GNP* and *GNP58* account for nearly the whole length of the simulation period, averaging from 91 to 99 quarters; and in the S_u series for *GNP58* their mean duration is still not less than 59 quarters (line 5).[49]

These results contrast sharply with the observations for actual

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[49] In a series with no declines, the expansion lasts 99 quarters; in one with a single one-quarter decline and two expansions, the average length of the latter is 49 quarters. These are the most frequent outcomes for the runs considered here.

CHART 4.6

A Random Sample of Stochastic 100-Quarter Simulations for GNP in Current and Constant Dollars, OBE Model (1966-I–1990-IV)

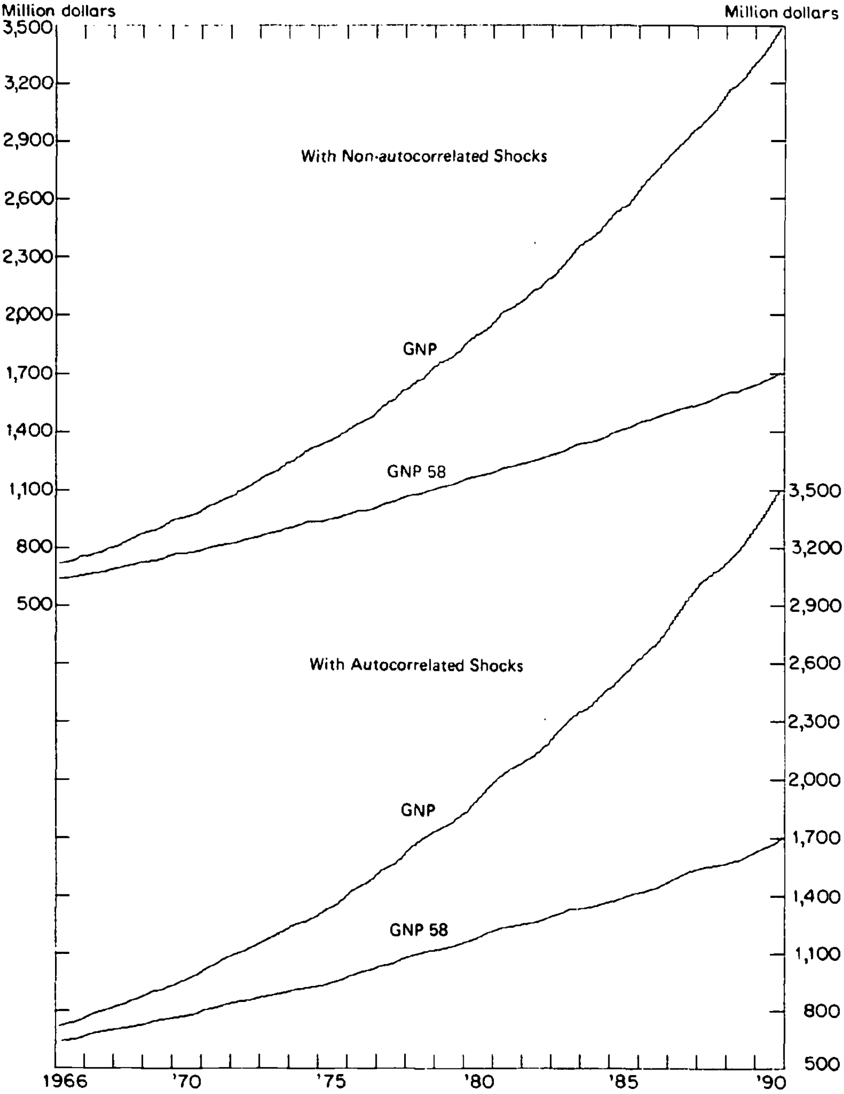

TABLE 4.9

Stochastic Simulations for 100 Quarters Beyond the Sample Period, OBE Model: Frequency, Duration, and Relative Size of Movements in GNP in Current and Constant Dollars (1966-I-1990-IV)

		Simulations With Serially Uncorrelated Shocks (S_u)		Simulations With Serially Correlated Shocks (S_c)	
	Number of Series Showing—	*GNP* (1)	*GNP58* (2)	*GNP* (3)	*GNP58* (4)
		Distribution of series by frequency of declines[a]			
1	No declines	22	6	24	20
2	One decline of one quarter each	3	17	1	5
3	Two declines of one quarter each	0	1	0	0
4	Three declines of one quarter each	0	1	0	0
		Average duration and amplitude[b]			
5	Mean duration of rises[c]	93.0	59.3	99.0	91.0
6	Mean size of rises[d]	3.8	1.5	3.9	1.6
7	Mean size of declines[d]	0.15	0.18	0.24	0.17

[a] Twenty-five stochastic simulation runs of each type were made: hence the entries in lines 1 to 4 of each column add up to 25.

[b] Each entry represents a mean of the averages for the given set of simulations. The averages in lines 5 and 6 include 25 figures each (means for all twenty-five runs in every case). The averages in line 7 exclude all runs with no declines; if all runs were included, the entries in this line (from left to right) would read: 0.02, 0.14, 0.01, and 0.03.

[c] In quarters.

[d] Per cent per quarter at quarterly rates.

GNP and *GNP58* in the postwar period (see Table 4.1, lines 5 and 6, 11 and 12). The differences are smaller, but are still significant, when the basis of the comparison is the sample period for the OBE Model (1953-III–1966-IV). During these 54 quarterly intervals, recorded *GNP* had two one-quarter declines with average amplitude of 1.33 per cent per quarter. *GNP58* had six declines (one of them of three quarters), averaging 1.3 quarters in duration and 0.32 per cent in amplitude. The mean duration of rises was 17.3 quarters for *GNP* and 6.6 quarters for *GNP58*.

It is clear that the simulated time-series for gross national income and output that result from shocked solutions of the OBE Model do not contain movements of the kind represented by the historically observed cyclical fluctuations in nominal and real *GNP*. However, the following two questions are pertinent: (1) Are cyclical elements also absent in simulations of various components of *GNP* and other sectoral indicators or—if they are present—do they tend to offset each other so as to disappear in the most comprehensive aggregates? (2) To what extent do our results for *GNP* reflect a state in which cyclical forces are latent but are overwhelmed by the strength of the assumed growth trends? To shed some light on (1), we present an analysis of simulations for variables other than *GNP* and *GNP58*. To attempt an answer to (2), we shall then turn to an evaluation of ratios of the shocked series to the control series for all variables covered by the OBE simulations.

4.2.3 Inspection of charts indicates that frequent and relatively large fluctuations are common in the stochastic simulations of the OBE Model for most of the variables covered. However, there are several variables to which this statement definitely does not apply, notably consumption, personal income, the price level, the annual wage rate per private employee, and the money supply (currency and demand deposits). The simulated series for these indicators (*C*, *YP*, *P*, *W*, and *M*), like those for *GNP* and *GNP58*, either show no declines at all or very few small and short declines. In addition, the projections for employment and labor costs per unit of output (*LE* and *LC/O*) also contain relatively few downward movements. It will be noted that these are all variables for which the recorded series have been particularly smooth and dominated by strong trends. In contrast, the simulated series with large and frequent fluctuations all refer to indicators that have varied greatly in past business cycles, such as investment in plant and equipment, housing, and inventories; unemployment, average workweek, manufacturers' orders for durable goods, net exports, and interest rates.

Simulations from three runs have been included in the complete analysis of the results for all variables. These randomly chosen runs are: 205 (with serially uncorrelated shocks) and 107 and 110 (both with serially correlated shocks).[50] Chart 4.7 illustrates the behavior of some of these series (including, for future reference, the identification of such specific-cycle turning points as can be identified).

Since the simulation period is almost twice as long as the sample period for the OBE Model, the frequencies of rises and declines would have to be about twice as large in the former as in the latter in order for the average durations (AD) of rises and declines to be approximately equal in the corresponding *S* and *A* series. Several cases of this sort are found in Table 4.10, relating to net exports, corporate profits, new orders, inventory investment (Run 107), employment (Run 205), the unemployment rate (Run 110), and the wage rate (Run 205). However, simulations for series that are typically rather volatile generally show much more numerous alterations of rises and declines than do the actual data in the sample period—often three and more times as many. Accordingly, the average durations of upward and downward movements are, as a rule, smaller in these simulations than in the actual series for the same variables—relating to investment, unemployment, the average workweek, unfilled orders, and interest rates. On the other hand, the simulations for consumption, personal income, employment, the price and wage levels, unit labor costs, and money generally have much longer expansions than those observed in the corresponding historical series. Indeed, nearly half of these simulations show monotonic growth, i.e., no declines at all. This includes all examined *S* series for *GNP*, *GNP58*, *YP*, and *M*. However, except in these extreme cases, the average durations of declines are not particularly underestimated for the variables in this set, where declines are infrequent and short in the actual series.

Thus the simulations can be divided into two groups: (1) the series with persistent expansions and few (or no) declines; and (2) the series in which both rises and falls are relatively frequent and short. Of the variables listed in Table 4.10, *GNP*, *GNP58*, *C*, *YP*, *P*, *LE*, *W*, *LC/O*, and *M* belong to Group 1, the others belong to Group 2. For the for-

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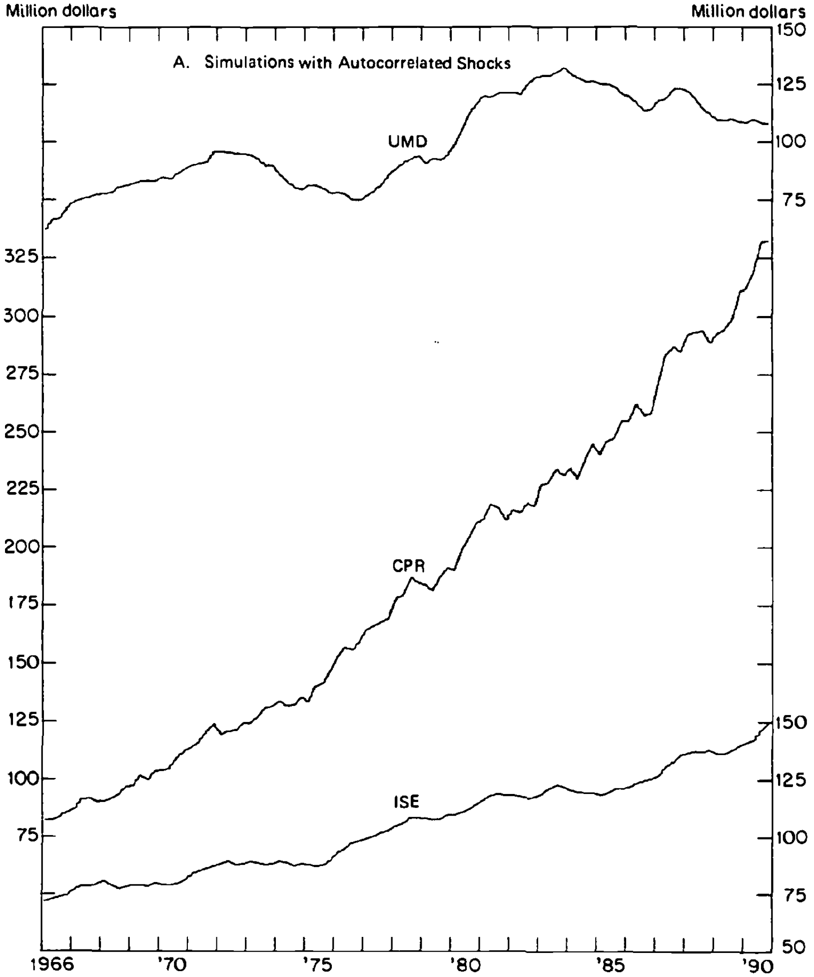
[50] If and when more time and resources become available, a larger sample of simulated series of both types should be analyzed to check on the results reported in the sections that follow. Data collected and calculations made for this study will make such replications possible.

CHART 4.7

A Random Sample of Stochastic 100-Quarter Simulations for Selected Variables, OBE Model (1966-I–1900-IV)



(*continued*)

CHART 4.7 (*continued*)

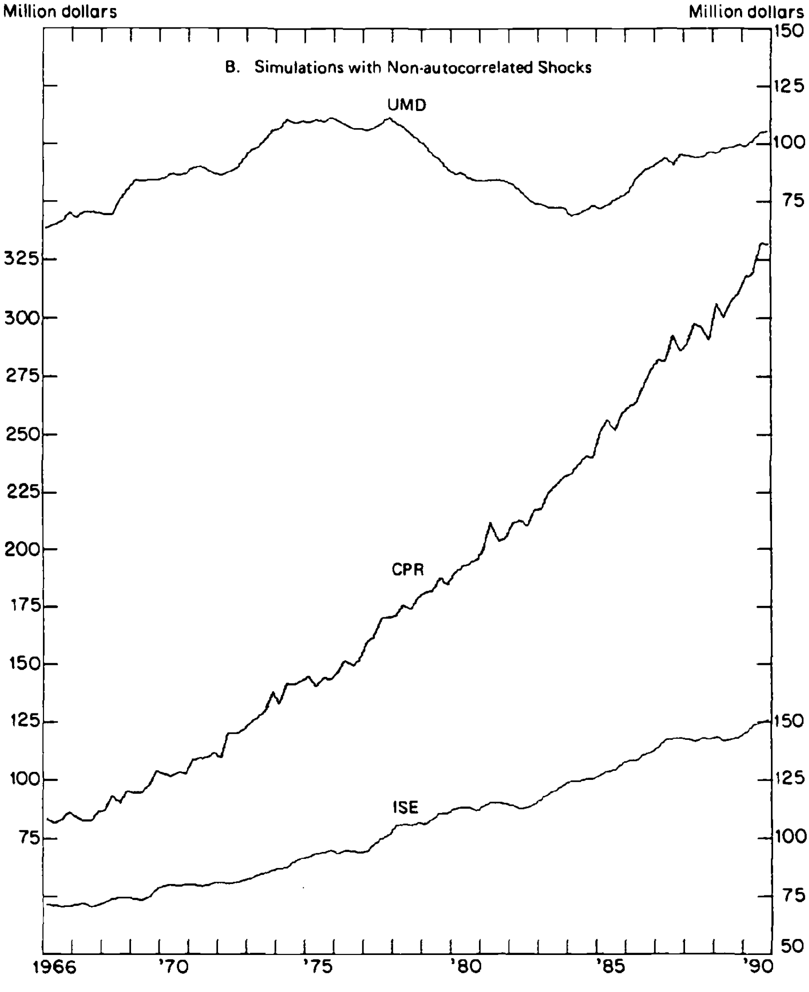
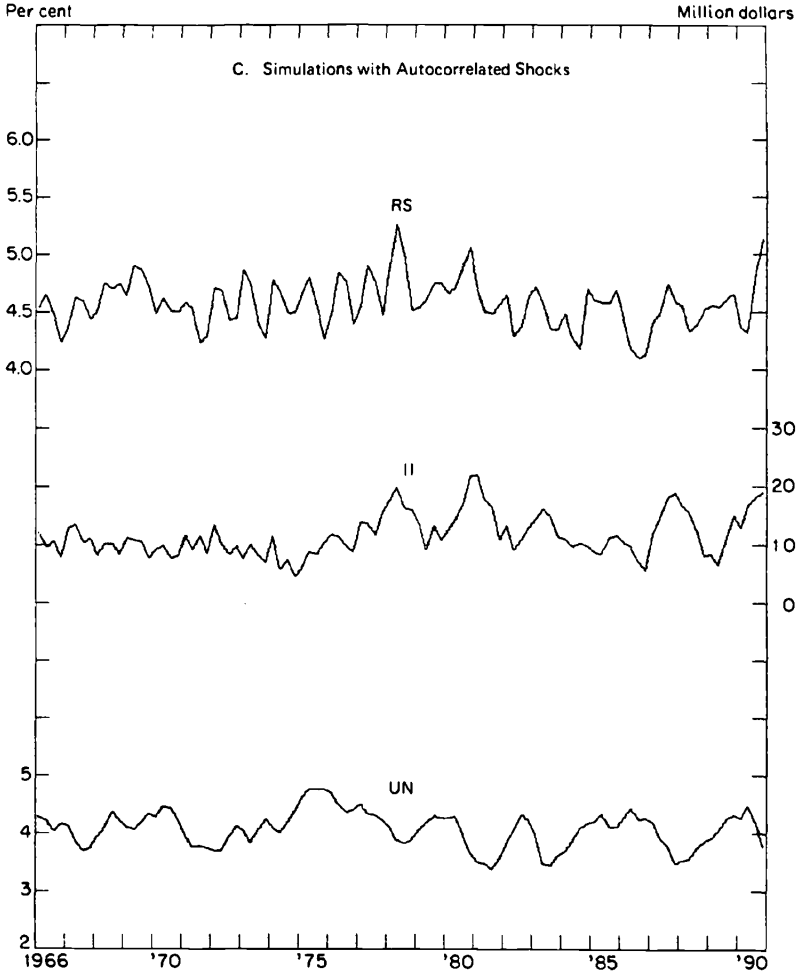


CHART 4.7 (*continued*)



(*continued*)

CHART 4.7 (*concluded*)

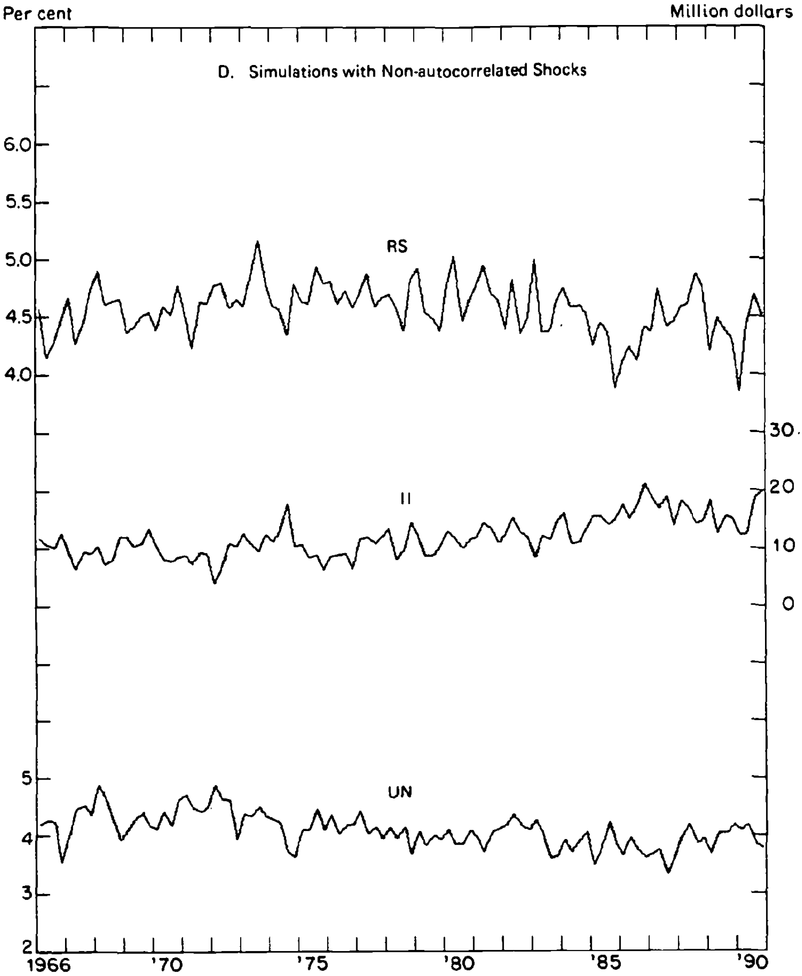


TABLE 4.10

Stochastic 100-Quarter Simulations, OBE Model: Frequency and Average Duration of Rises and Declines in Quarters for Twenty-One Variables, Comparison of Three Simulation Runs and Actuals

					Stochastic Simulations (Three Runs)					
			Actuals for the Sample Period 1953-II–1966-IV		With Uncorrelated Shocks		With Serially Correlated Shocks			
					(Run 205)		(Run 107)		(Run 110)	
Line	Variable Symbol[a]	Rise (R) or Decline (D)	Number[b] (1)	AD[c] (2)	Number[b] (3)	AD[c] (4)	Number[b] (5)	AD[c] (6)	Number[b] (7)	AD[c] (8)
1	*GNP*	R	3	17.3	1	99.0	1	99.0	1	99.0
2		D	2	1.0	0	0	0	0	0	0
3	*GNP58*	R	7	6.7	2	49.0	1	99.0	1	99.0
4		D	6	1.3	1.0	1.0	0	0	0	0
5	*C*	R	3	17.3	2	49.0	1	99.0	2	49.0
6		D	2	1.0	1	1.0	0	0	1	1.0
7	*IH*	R	5	4.0	22	2.8	18	3.7	17	4.0
8		D	6	5.7	21	1.8	17	1.8	16	1.9
9	*ISE*	R	4	11.0	16	4.8	18	4.2	12	6.1
10		D	3	3.3	16	1.4	17	1.4	11	2.4

(continued)

TABLE 4.10 (*concluded*)

					Stochastic Simulations (Three Runs)					
			Actuals for the Sample Period 1953-II–1966-IV		With Uncorrelated Shocks		With Serially Correlated Shocks			
					(Run 205)		(Run 107)		(Run 110)	
Line	Variable Symbol[a]	Rise (R) or Decline (D)	Number[b] (1)	AD[c] (2)	Number[b] (3)	AD[c] (4)	Number[b] (5)	AD[c] (6)	Number[b] (7)	AD[c] (8)
11	*II*	R	10	2.9	30	1.8	23	2.2	27	1.9
12		D	10	2.5	30	1.5	24	2.0	27	1.8
13	*NE*	R	11	2.5	23	2.7	23	2.6	21	2.9
14		D	12	2.2	22	1.6	23	1.7	21	1.8
15	*YP*	R	2	26.0	1	99.0	1	99.0	1	99.0
16		D	1	2.0	0	0	0	0	0	0
17	*P*	R	1	54.0	2	49.0	1	99.0	1	99.0
18		D	0	0	1	1.0	0	0	0	0
19	*LE*	R	5	9.2	8	11.5	5	19.0	4	24.0
20		D	5	1.6	7	1.0	4	1.0	3	1.0
21	*UN*	R	10	3.0	32	1.6	22	2.1	17	2.9
22		D	10	2.4	32	1.5	23	2.3	18	2.8

23	*CPR*	R	12	3.0	23	3.1	23	3.1	20	3.8
24		D	12	1.5	24	1.2	22	1.2	19	1.2
25	*AWW*	R	7	2.0	28	1.8	34	1.6	32	1.4
26		D	7	5.7	27	1.9	33	1.4	31	1.7
27	*OMD*	R	15	1.9	25	2.2	24	2.3	24	2.2
28		D	15	1.7	26	1.7	23	1.9	23	2.0
29	*UMD*	R	4	4.8	20	2.9	20	3.0	15	3.6
30		D	4	8.8	19	2.2	19	2.0	14	3.1
31	*HS*	R	6	3.8	32	1.8	29	1.8	28	2.0
32		D	5	6.2	32	1.3	29	1.6	28	1.5
33	*RS*	R	7	5.1	30	1.7	24	2.0	25	2.0
34		D	7	2.6	31	1.5	25	2.1	24	2.1
35	*RL*	R	6	6.5	25	1.9	21	2.5	21	2.4
36		D	6	2.5	26	2.0	21	2.2	21	2.3
37	*W*	R	3	17.3	6	15.7	3	31.7	2	49.0
38		D	2	1.0	5	1.0	3	1.3	1	1.0
39	*LC/O*	R	11	3.7	12	7.2	5	18.6	1	99.0
40		D	10	1.3	11	1.0	4	1.5	0	0
41	*M*	R	4	11.2	1	99.0	1	99.0	1	99.0
42		D	3	3.0	0	0	0	0	0	0

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[a] For meaning of symbols, see Table 1.1.

[b] Number of rises (R) or declines (D).

[c] Average duration, in quarters, of rises (R) or declines (D).

mer, the average durations of rises are heavily overestimated when compared with sample-period actuals; for the latter, they are for the most part underestimated. The average durations of declines tend to be underestimated in both groups of the simulated series, but particularly in Group 2. The frequencies are as follows:

	Group 1		Group 2	
Number of cases where—	Rises	Declines	Rises	Declines
AD is larger for S than for A	25	3	9	3
AD is equal for S and A	0	6	1	1
AD is smaller for S than for A	2	18	26	32
Total	27	27	36	36

For the most part, the simulations with serially uncorrelated shocks (S_u) have more frequent turning points and shorter rises and falls than the simulations with serially correlated shocks (S_c); the use of autocorrelated shocks, then, has the usual effects of smoothing (compare columns 3 and 4 with columns 5 and 6 and 7 and 8 in Table 4.10). The tabulation below shows the average durations of rises and declines in S_u and S_c for the two groups of series identified in the previous paragraph.

	Group 1		Group 2	
Number of cases where—	Rises	Declines	Rises	Declines
AD is smaller for S_u than for S_c	11	3	17	17
AD is equal for S_u and S_c	7	10	3	4
AD is larger for S_u than for S_c	0	5	4	3
Total	18	18	24	24

The S_u series in Group 1, having less persistent upward movements than the S_c series, overestimate the average durations of rises less in the actuals. However, they still show much longer expansions than those observed in these variables during the sample period, except for *P*, *LE*, and *W* (lines 17, 19, and 37; compare columns 2 and 4). With regard to the duration of declines, there is little difference between the S_u and S_c series of this group. In Group 2, where the length of both rises and declines in *A* tends to be underestimated, the relative advantage is often on the side of the series with autocorrelated shocks. That is, the movements in S_c—being on the average longer than those in S_u—differ less in duration from the movements in the actuals.

Table 4.11 shows the average per cent change per quarter of rises and declines in actuals and in the selected *S* series. It covers the same data as Table 4.10, except that *II* and *NE*, which can assume negative values, are omitted. For the relatively smooth, trend-dominated variables of Group 1, the average percentage amplitudes (APA) of rises are all larger in *S* than in *A*. In contrast, the APA of declines are here smaller in *S* than in *A*, with only two exceptions. The results are quite different for the more volatile variables in Group 2, where the average percentage changes are smaller in *S* than in *A* for both rises and falls in about 60 per cent of the cases, as shown by the accompanying figures.

	Group 1		Group 2	
Number of cases where—	Rises	Declines	Rises	Declines
APA is larger for *S* than for *A*	27	8	12	13
APA is smaller for *S* than for *A*	0	19	18	17
Total	27	27	30	30

In more than half of the comparisons for Group 1, the simulations with non-autocorrelated shocks, S_u, show smaller rises but larger declines than their counterparts with autocorrelated shocks, S_c. On the other hand, for the variables in Group 2, both upward and downward movements tend to be larger in the S_u than in the S_c series, as would be expected of a procedure with smoothing effects (see the tabulation on the following page).

The S_u series, having on the average larger percentage amplitudes than the S_c series, often underestimate less the relative size of movements in the actuals. By the same token, in those cases where the sim-

	Group 1		Group 2	
Number of cases where—	Rises	Declines	Rises	Declines
APA is smaller for S_u than for S_c	12	2	4	3
APA is equal for S_u and S_c	1	6	0	0
APA is larger for S_u than for S_c	5	10	16	17
Total	18	18	20	20

ulations overestimate the relative amplitudes of the actuals, the S_c series often differ less from A than do the S_u series. Although our sample permits twice as many comparisons for S_c as for S_u, the outcomes favor S_u about as often as S_c. Hence, the S_u series appear to have an edge over the S_c series in this respect, but this is so far merely a tentative inference from limited and rather mixed evidence.

A few general conclusions can, however, be reached with considerable confidence. The "errors" in the *GNP* and *GNP58* simulations reflect, to a large extent, similar differences vis-à-vis the actuals that are observed for the simulations of real consumption expenditures. Simulations of other comprehensive aggregates and indexes—personal income, employment, the general price and wage levels, the money stock—also consist mainly of upward trends. Rises are predominantly longer and larger for S than for A, and declines are shorter and smaller. Indeed, the behavior of the S series here (Group 1) contains very few cyclical elements of the type recorded in the past. In contrast, frequent fluctuations are characteristic of the simulations for the investment variables, net exports, profits, orders, and interest rates (Group 2). These fluctuations tend to be shorter than their sample-period counterparts but are otherwise very diversified. The differences in relative size between the S and A series in Group 2 vary greatly, but not in any clearly systematic fashion: the average percentage changes in S fall short of those in A in 35 cases and exceed them in 25 cases.

4.2.4 We now turn to the analysis of ratios of the shocked to the control series for *GNP* in current and constant dollars. Chart 4.8 illustrates the behavior of these ratio-series. It shows that they contain frequent fluctuations, which tend to be shorter and more irregular for

TABLE 4.11

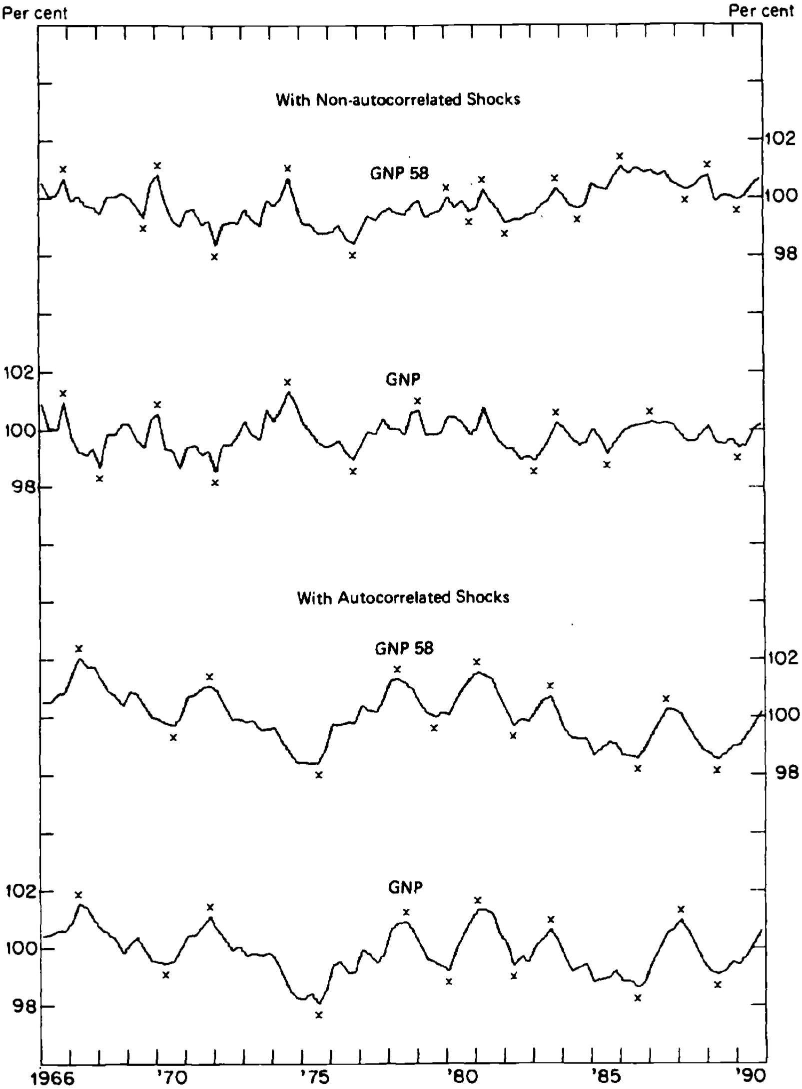
Stochastic 100-Quarter Simulations, OBE Model: Average Per Cent Amplitudes, Per Quarter, of Rises and Declines in Nineteen Variables, Comparison of Three Simulation Runs and Actuals

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Actuals for the Sample Period 1953-II–1966-IV	Stochastic Simulations (Three Runs): Uncorrelated Shocks (Run 205)	Stochastic Simulations (Three Runs): Serially Correlated Shocks (Run 107)	Stochastic Simulations (Three Runs): Serially Correlated Shocks (Run 110)
			(1)	(2)	(3)	(4)
1	*GNP*	R	1.36	3.86	3.98	3.90
2		D	1.33	[c]	[c]	[c]
3	*GNP58*	R	0.88	1.32	1.73	1.66
4		D	0.32	0.01	[c]	[c]
5	*C*	R	1.17	1.39	1.77	1.39
6		D	0.46	0.02	[c]	0.12
7	*IH*	R	2.03	1.74	1.10	1.19
8		D	1.30	1.15	0.69	0.81
9	*ISE*	R	1.41	1.14	1.04	1.28
10		D	0.24	0.71	0.51	0.87
11	*YP*	R	1.71	3.89	3.95	3.94
12		D	0.46	[c]	[c]	[c]
13	*P*	R	0.50	0.65	0.82	0.85
14		D	[c]	0.07	[c]	[c]
15	*LE*	R	0.28	0.50	0.44	0.43
16		D	0.17	0.20	0.06	0.04
17	*UN*	R	4.09	5.84	2.91	3.34
18		D	2.32	5.45	3.02	3.22
19	*CPR*	R	3.72	3.32	2.51	2.22
20		D	2.10	1.52	1.09	1.22
21	*AWW*	R	0.04	0.35	0.37	0.43
22		D	0.07	0.36	0.42	0.40
23	*OMD*	R	4.56	3.21	2.53	3.07
24		D	2.70	3.05	2.34	1.98
25	*UMD*	R	1.67	1.81	1.72	1.87
26		D	2.12	1.50	1.17	1.27
27	*HS*	R	1.80	5.01	3.58	2.95
28		D	1.58	4.26	2.22	2.28
29	*RS*	R	9.29	4.98	4.63	4.44
30		D	5.52	4.73	3.69	3.35
31	*RL*	R	1.44	2.11	1.43	1.56
32		D	1.98	1.64	1.24	1.13
33	*W*	R	1.22	1.50	1.69	1.95
34		D	0.04	0.39	0.12	0.31
35	*LC/O*	R	0.59	0.81	0.68	0.89
36		D	0.27	0.12	0.16	[c]
37	*M*	R	0.52	5.12	5.15	5.11
38		D	0.15	[c]	[c]	[c]

[a] For meaning of symbols, see Table 1.1. [b] All figures are at quarterly rate. [c] No declines.

CHART 4.8

A Random Sample of Stochastic 100-Quarter Simulations for GNP in Current and Constant Dollars, Ratios to Control Solutions, OBE Model (1966-I-1990-IV)



the simulations with serially uncorrelated random shocks than for the simulations with serially correlated shocks. The less erratic time-paths produced by the autocorrelated runs have been assessed by the OBE Model builders themselves as being "more in line with our expectations." This would indicate a preference for these simulations over the ones with non-autocorrelated disturbances.[51]

Since the runs used were chosen arbitrarily, there is no reason to suspect that the general observations do not apply to all runs. The ratio-series contain fluctuations that are broadly comparable to specific-cycle movements of the NBER analysis. They highlight the irregularities in the growth rates of the corresponding S series proper. The persistent upward trends that clearly dominate these simulations (see Chart 4.6) apparently conceal a great deal of variability in the deviations of the S series from the growth paths of the hypothetical shock-free solution.

The rises in the ratio series for *GNP* and *GNP58* are about as long and as large as those in the corresponding declines. As shown in Table 4.12, this near-equality of upward and downward movements in the ratios applies to the simulations with non-autocorrelated shocks, S_u, as well as to those with autocorrelated shocks, S_c (compare columns 1 and 2 and 3 and 4). The symmetry extends not only to the averages, but also to the dispersion of the means for the different runs in each of the four sets.

The rises and declines in the S_u series are, on the average, more frequent, shorter, and larger than the corresponding movements in the S_c series. This, too, is a firm finding to which there are no exceptions in Table 4.12 (compare columns 1 and 3, and 5). Using serially correlated

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[51] See [18, p. 80]. Messrs. George R. Green *et al.* also stress the contrast between the presence of "cyclical movements" in the deviations of shocked from control series for *GNP58* and the absence of such movements in the shocked series themselves. They note that "if the criterion for the presence of cycles is that protracted downturns must occur, then the present results do not depict cyclical behavior adequately." This, however, is associated with the fact that "these simulations incorporate very strong growth elements in the exogenous variables, and such elements have to be overcome by the effects of stochastic shocks for actual downturns to occur."

It should be noted that absolute deviations (differences) are used in [18], whereas we analyze relative deviations, i.e., ratios of shocked to control series, in per cent. Our approach has some advantages in terms of standardization of measurement units and with respect to heteroscedasticity problems.

TABLE 4.12

Stochastic 100-Quarter Simulations, OBE Model, and the Corresponding Sample-Period Actuals: Summary Statistics on Frequency, Duration, and Relative Size of Rises and Declines in Relative Deviations from Trend of GNP and GNP58 (Duration in Quarters, Amplitude in Per Cent)

Line	Type of Series and Movement	Frequency: Mean or Total[a] (1)	Frequency: Standard Deviation (S.D.)[b] (2)	Duration: Mean (per run)[c] (3)	Duration: S.D. (between runs)[d] (4)	Duration: S.D. (within runs)[a] (5)	Amplitude: Mean (per run)[c] (6)	Amplitude: S.D. (between runs)[b] (7)
	GNP: Ratio to Control Solution (S) or Trend (A)[f]							
	Simulations with non-autocorrelated shocks: S_u							
1	Rises	26.72	2.07	1.86	0.21	1.09	0.39	0.05
2	Declines	26.72	2.05	1.87	0.17	1.09	0.38	0.05
	Simulations with autocorrelated shocks: S_c							
3	Rises	19.16	2.37	2.70	0.43	1.93	0.30	0.04
4	Declines	19.04	2.57	2.56	0.25	1.82	0.31	0.05
	Sample period actuals: A							
5	Rises	10		3.10		3.44	0.61	
6	Declines	10		2.30		1.85	0.61	

TABLE 4.12 (*concluded*)

Line	Type of Series and Movement	Frequency: Mean or Total[a] (1)	Frequency: Standard Deviation (S.D.)[b] (2)	Duration: Mean (per run)[c] (3)	Duration: S.D. (between runs)[d] (4)	Duration: S.D. (within runs)[e] (5)	Amplitude: Mean (per run)[c] (6)	Amplitude: S.D. (between runs)[b] (7)
	GNP58: Ratio to Control Solution (S) or Trend (A)[f]							
	S_a simulations							
7	Rises	28.40	2.36	1.74	0.16	1.00	0.34	0.05
8	Declines	28.28	2.37	1.77	0.23	1.03	0.34	0.04
	S_c simulations							
9	Rises	17.42	1.93	2.92	0.38	2.00	0.28	0.14
10	Declines	17.42	2.02	2.83	0.35	1.99	0.26	0.04
	A: actuals							
11	Rises	12		2.33		3.27	0.60	
12	Declines	12		2.17		1.83	0.50	

NOTE: All simulations refer to the 25-year period, 1966-I–1990-IV; each average covers 25 runs. The actuals refer to the sample period, 1953-III–1966-IV.

[a,b,c,d,e] See the corresponding footnotes in Table 4.1.

[f] See footnote *f* in Table 4.4.

shocks has simply, here as elsewhere, the effects associated with smoothing.

In an attempt to construct comparable measures for the actuals, ratios of the recorded values to their exponential trends were computed from the quarterly *GNP* and *GNP58* data for the sample period of the OBE Model (1953–66). The average frequencies, durations, and amplitudes of movements in the resulting series are listed in the last two lines of the table.

Allowing for the difference in length between the sample period and the simulation period, we observe that rises and declines in *GNP* are, on the average, about as frequent in the S_c series as they actually were in 1953–66 (lines 3 to 6, columns 1 and 2). The mean duration of rises is somewhat smaller for these simulated series than for the actuals, and the mean duration of declines is slightly larger, but the differences are small. On the other hand, both the upward and downward movements in the simulated ratios with uncorrelated shocks, S_u (lines 1 and 2), are much shorter than the corresponding movements in the actual ratios (columns 3 to 4). In terms of the size of quarterly percentage changes, rises and declines in the *A* ratios are underestimated a little less in the S_u ratios than in the S_c ratios (columns 5 and 6).

The comparisons for *GNP58* yield results of the same general nature, with one exception. Unlike the case of current dollar *GNP*, S_c is here not clearly superior to S_u with regard to approximating the frequency and average duration of rises and declines in the *A* ratios. When judged by this criterion, the movements in S_c are too long in about the same measure as those in S_u are too short (columns 1 and 3, lines 7 to 12).

Because of the shortness of the OBE sample-period, one might also wish to consider the actual ratios for the Wharton sample-period, which starts in 1948 and ends early in 1968; the measures for these ratios are given in Table 4.4, lines 5, 6, 11, and 12. The trend estimates are better for the longer Wharton period, but otherwise the measures for the OBE period are more appropriate in the present context. The average duration and amplitude figures for 1948–68 are larger than those for 1953–66, and the measures for the OBE simulated ratios generally underestimate the former. Comparison with the longer sample-period results, for the most part, in larger discrepancies between the averages for the S and A ratios.

4.2.5 For most of the variables covered, the stochastic simulations for the OBE Model, in their original form, do show fluctuations; that is, declines as well as rises. (See Section 4.2.3 above.) For these variables, therefore, it is less important to use transformations of the simulated series. Nevertheless, the analysis of relative deviations from trend has been extended to variables other than *GNP* and *GNP58*, for the same reasons that suggested this approach earlier on. The data came from the three stochastic simulation runs which supplied the basic measures of Tables 4.10 and 4.11. The method is the same as that used in the preceding section for the *GNP* data. Cost-benefit considerations argued against application of more refined and diversified techniques.[52]

Chart 4.9 shows the behavior of the simulated ratio-series for selected variables. The runs using uncorrelated random shocks, S_u, produce more ragged series than those using serially correlated shocks, S_c, as elsewhere. There is a good deal of variation even in the ratios derived from those simulations that show almost no declines in levels (as, e.g., in the ratio series for *C*, *YP*, or *P*). Again, the specific-cycle turns marked on these graphs were selected by the computer method.

According to the information summarized in Table 4.13, movements in the ratios of shocked to control series (S) tend to be shorter than movements in the actual ratio-to-trend series (A). This applies to rises and declines alike. The average durations (AD) are smaller for S than for A in over four-fifths of the comparisons.

Both rises and declines tend to be longer in the ratio-series involving autocorrelated shocks (S_c) than in the ratio-series involving uncorrelated shocks (S_u). The AD figures are larger for S_c than S_u in about 82 per cent of the cases. Accordingly, the S_c series underestimate the length of the movements in the A ratios less than the S_u series, and even overestimate it in a few cases (notably for *GNP*, *GNP58*, *CPR* and *LC/O*).

Table 4.14 includes 68 instances in which the average percentage

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[52] Different types of fitted trends are likely to be appropriate for variables with diverse characteristics, but the benefits of such selections are uncertain, and the costs would exceed the available time and resources.

CHART 4.9

A Random Sample of Stochastic 100-Quarter Simulations for Selected Variables, Ratios to Control Solutions, OBE Model (1966-I–1990-IV)

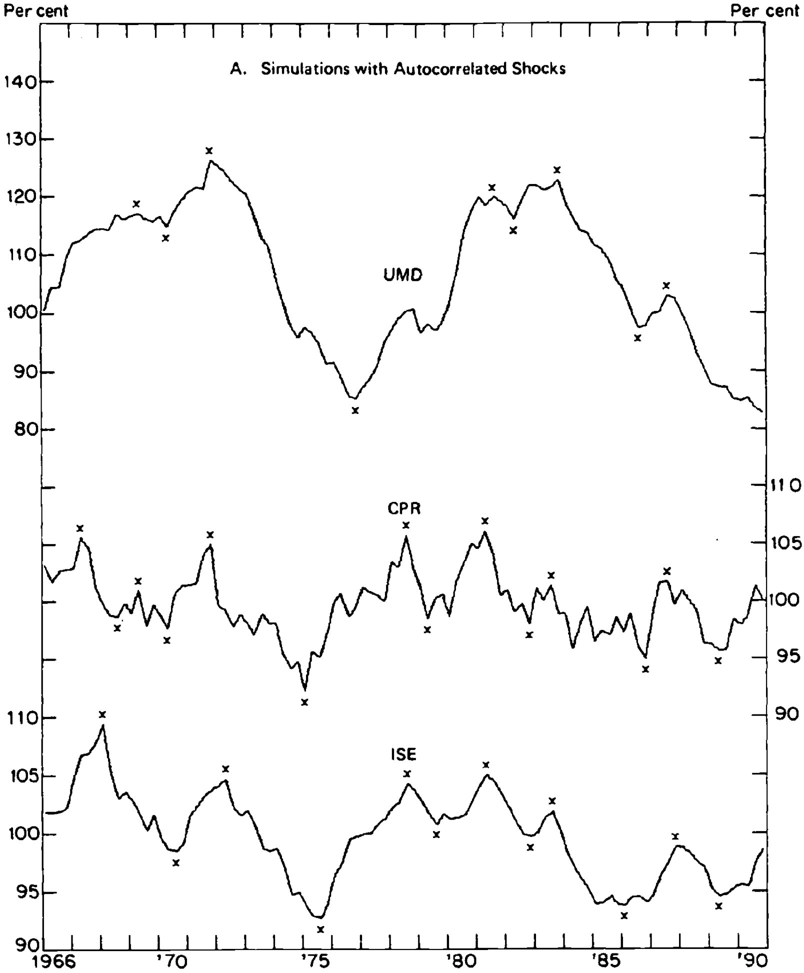
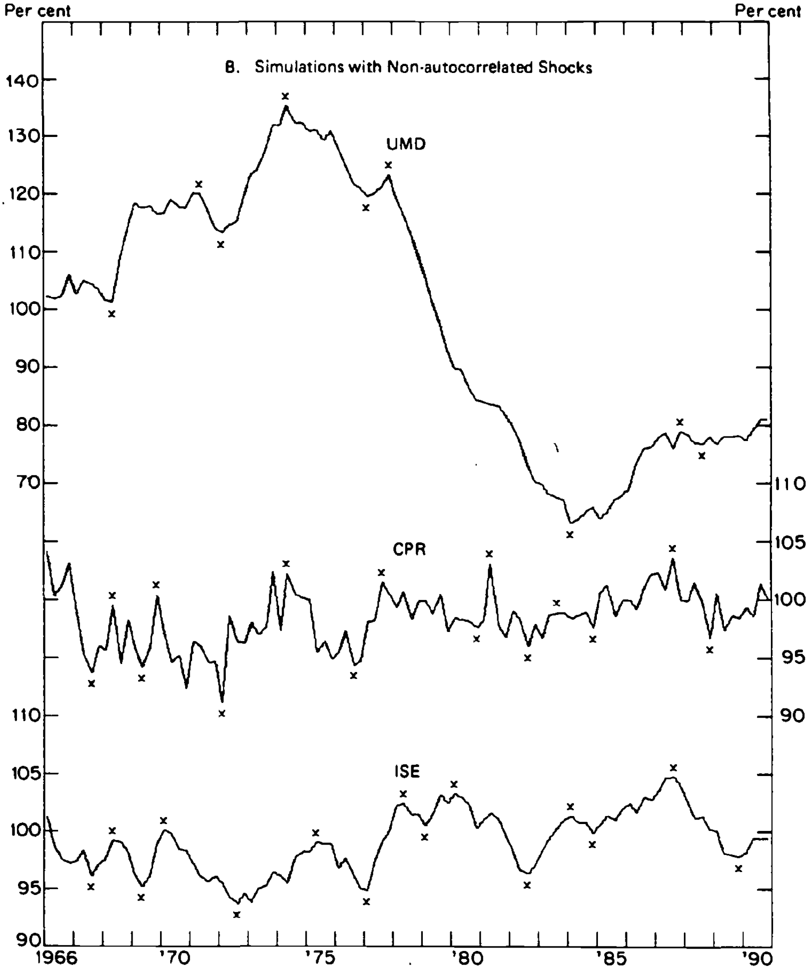


CHART 4.9 (*continued*)



(*continued*)

CHART 4.9 (*continued*)

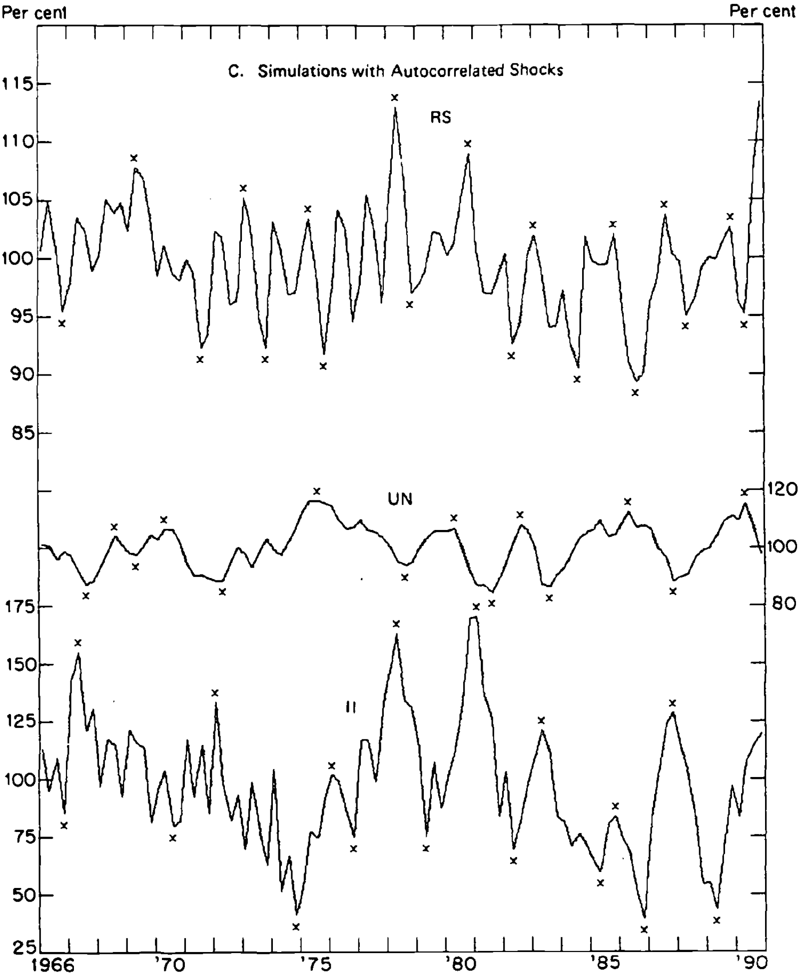


CHART 4.9 *(concluded)*

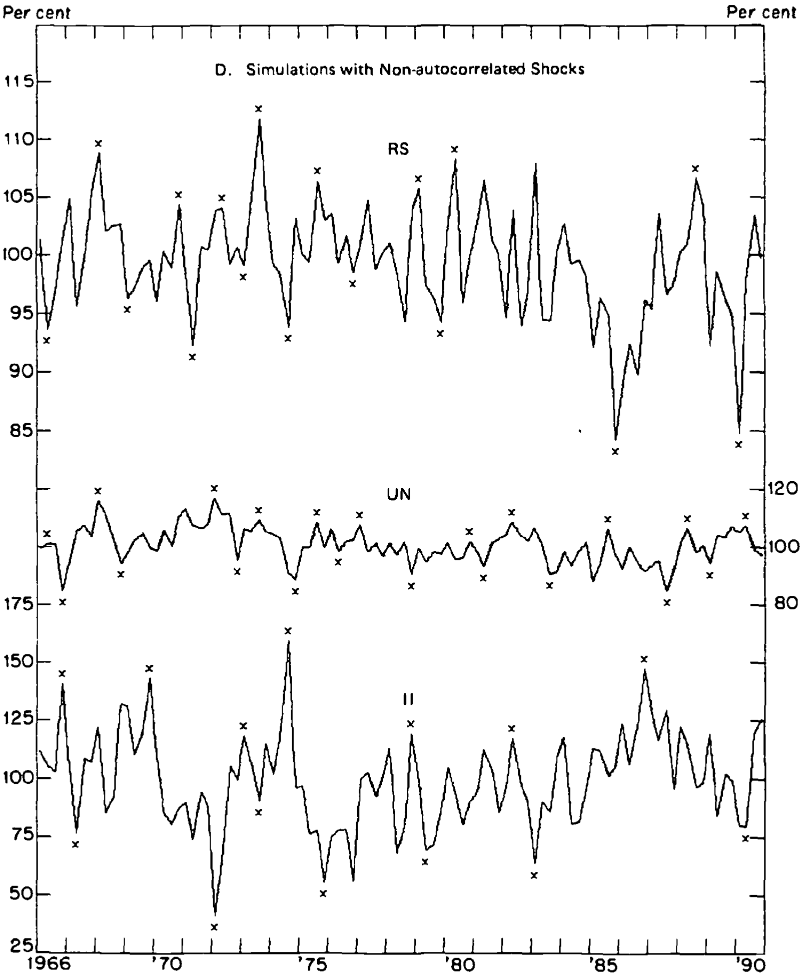


TABLE 4.13

Stochastic 100-Quarter Simulations, OBE Model: Average Duration of Rises and Declines in Relative Deviations from Trend in Quarters, Twenty-One Variables, Actuals and Three Simulation Runs

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Actuals: Ratio to Exponential Trend 1953-II–1966-IV (1)	Stochastic Simulations: Ratio to Control Solution: Uncorrelated Shocks (Run 205) (2)	Serially Correlated Shocks (Run 107) (3)	Serially Correlated Shocks (Run 110) (4)
1	*GNP*	R	3.1	2.0	3.4	2.6
2		D	2.3	2.1	2.8	2.8
3	*GNP58*	R	2.3	2.0	3.3	2.4
4		D	2.2	1.7	2.9	2.8
5	*C*	R	2.1	1.7	2.0	1.9
6		D	1.9	1.5	2.1	2.0
7	*IH*	R	3.5	2.1	2.1	2.4
8		D	4.7	2.1	2.6	2.9
9	*ISE*	R	6.4	2.1	2.5	3.4
10		D	5.5	2.3	2.6	3.0
11	*II*	R	3.3	1.7	2.2	1.8
12		D	11.0	1.6	1.8	1.9
13	*NE*	R	2.0	1.6	2.2	2.0
14		D	2.3	1.6	1.8	1.8
15	*YP*	R	2.5	2.0	2.6	2.8
16		D	2.7	2.0	2.2	2.5

17	*P*	R	3.2	1.8	2.0	2.3
18		D	3.5	1.8	1.9	1.9
19	*LE*	R	4.1	1.8	1.8	2.9
20		D	3.6	1.7	2.4	2.8
21	*UN*	R	2.6	1.5	2.1	2.9
22		D	2.8	1.5	2.2	2.8
23	*CPR*	R	1.5	1.5	1.7	1.7
24		D	1.6	1.5	1.8	1.7
25	*AWW*	R	4.3	1.8	1.6	1.4
26		D	4.0	1.8	1.4	1.7
27	*OMD*	R	1.9	1.9	2.1	1.7
28		D	2.2	1.7	2.0	1.9
29	*UMD*	R	3.8	2.7	2.7	2.4
30		D	5.2	3.3	2.8	3.1
31	*HS*	R	3.9	1.5	1.6	1.7
32		D	4.5	1.5	1.7	1.8
33	*RS*	R	3.0	1.7	2.0	2.0
34		D	3.8	1.5	2.1	2.0
35	*RL*	R	4.4	1.8	2.1	2.0
36		D	3.3	2.0	2.3	2.3
37	*W*	R	2.6	1.6	1.8	1.7
38		D	2.5	1.6	1.7	1.7
39	*LC/O*	R	1.6	1.6	2.1	1.6
40		D	1.5	1.6	2.0	1.7
41	*M*	R	12.0	2.6	2.8	2.3
42		D	9.0	2.5	3.1	2.2

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[a] For meaning of symbols, see Table 1.1.

TABLE 4.14

Stochastic 100-Quarter Simulations, OBE Model: Average Percentage Amplitudes, Per Quarter, of Rises and Declines in Relative Deviations from Trend, Nineteen Variables, Actuals and Three Simulation Runs

Line	Variable Symbol[a]	Rise (R) or Decline (D)	Actuals: Ratio to Exponential Trend 1953-II–1966-IV (1)	Stochastic Simulations: Ratio to Control Solution: Uncorrelated Shocks (Run 205) (2)	Serially Correlated Shocks (Run 107) (3)	(Run 110) (4)
1	*GNP*	R	0.61	0.40	0.27	0.28
2		D	0.61	0.34	0.31	0.29
3	*GNP58*	R	0.60	0.30	0.25	0.24
4		D	0.50	0.36	0.22	0.23
5	*C*	R	0.50	0.33	0.34	0.30
6		D	0.41	0.38	0.31	0.31
7	*IH*	R	1.74	1.38	1.08	0.93
8		D	1.21	1.39	0.79	0.91
9	*ISE*	R	0.56	0.82	0.86	0.74
10		D	0.71	0.80	0.63	1.03
11	*YP*	R	0.44	0.34	0.22	0.25
12		D	0.30	0.35	0.24	0.26
13	*P*	R	0.16	0.20	0.15	0.13
14		D	0.08	0.19	0.16	0.17
15	*LE*	R	0.17	0.23	0.21	0.15
16		D	0.21	0.26	0.15	0.21
17	*UN*	R	3.98	5.75	2.82	3.44
18		D	2.54	5.60	3.00	3.15
19	*CPR*	R	2.91	2.19	1.83	1.62
20		D	2.16	2.00	1.68	1.49
21	*AWW*	R	0.06	0.35	0.37	0.44
22		D	0.08	0.36	0.42	0.40
23	*OMD*	R	4.42	2.88	2.33	2.73
24		D	3.14	3.05	2.43	1.92
25	*UMD*	R	1.43	1.90	1.48	1.70
26		D	1.62	1.43	1.55	1.35
27	*HS*	R	1.45	4.60	3.50	2.66
28		D	1.80	4.46	2.50	2.57
29	*RS*	R	8.65	4.96	4.62	4.41
30		D	4.05	4.70	3.70	3.43
31	*RL*	R	1.23	1.92	1.34	1.56
32		D	1.80	1.78	1.24	1.20
33	*W*	R	0.41	0.71	0.53	0.58
34		D	0.28	0.69	0.51	0.51
35	*LC/O*	R	0.41	0.45	0.28	0.24
36		D	0.31	0.46	0.28	0.23
37	*M*	R	0.33	0.16	0.16	0.12
38		D	0.43	0.13	0.16	0.13

[a] For meaning of symbols, see Table 1.1.

[b] All figures per quarter.

size of quarterly movements is smaller for the simulated ratio-series than for the corresponding actuals, and 46 instances in which the opposite applies. The sample data suggest that the outcome depends on the type of simulation. The ratio-series with uncorrelated shocks (S_u) show a strong tendency to have larger amplitudes than the ratio-series with autocorrelated shocks (S_c).[53] The average relative amplitudes of S_u are mostly larger—and those of S_c are mostly smaller—than the corresponding measures for the actuals (with majorities of about 60 per cent in either case).

4.2.6 Following the method applied to the Wharton series and described in Section 4.1.6 above, cumulated diffusion indexes based on specific-cycle movements in the ratios of shocked to control series, were constructed for the three randomly chosen simulation runs of the OBE Model. The identification and dating of the specific cycles presented greater difficulties for the ragged series with non-autocorrelated shocks (Run 205) than for the considerably smoother series with autocorrelated shocks (Runs 107 and 110). Accordingly, the results for the former set are probably less dependable than those for the latter sets.

The three cumulated diffusion indexes are displayed in Chart 4.10. Each of them includes twenty-one series; that is, all variables covered in the OBE simulations.

As would be expected, the indexes for the different runs differ greatly in timing and amplitude of fluctuations. As in the Wharton Model, these indexes show distinct movements of cyclical duration. The tabulation on p. 509 indicates that the expansions in the indexes averaged about 6 to 7 quarters, while the expansions in the relative deviations from trends of the postwar *GNP* and *GNP58* series averaged 11 to 12 quarters. For contractions, the mean durations are 6 to 11 quarters for the indexes and about 7 quarters for the *GNP* ratio-series. The differences between these measures are sizable but lie entirely within the range of historically observable variation. For example, without the single extra-long increase in the 1960's, the expansion averages for the *GNP* ratio-to-trend series would be reduced to 7 to 8 quarters. Mean duration figures for the *GNP* simulations in current and

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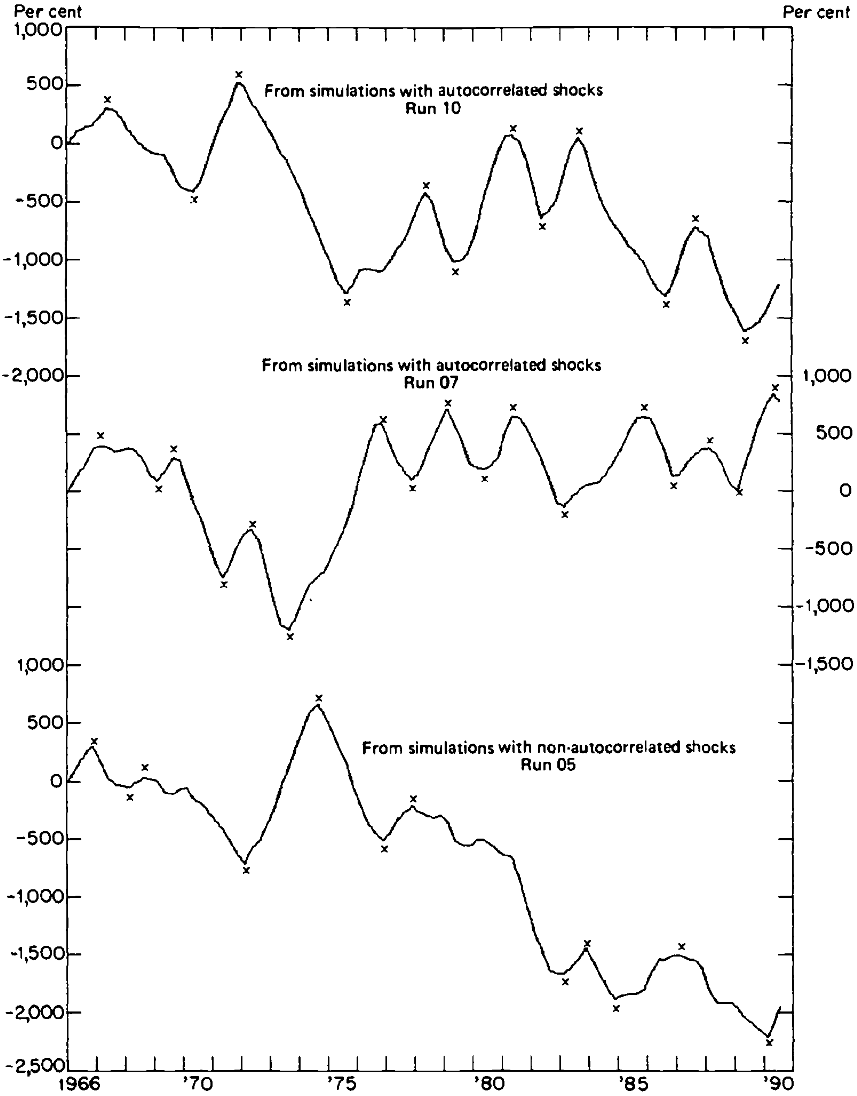
[53] The figures in column 2 of Table 4.14 exceed the corresponding entries in columns 3 and 4 in nearly 86 per cent of the comparisons.

CHART 4.10

Cumulated Historical Diffusion Indexes for Selected Sets of Stochastic Simulations, Ratios to Control Solutions, OBE Model (1966-I–1990-IV)



constant dollars are on the whole of the same general order of magnitude.[54]

	Ratios to Control Solution, OBE Model, Indexes of Cumulated Per Cent Expanding			Ratios to Exponential Trend, Actual Data, 1948–68	
	Run 107	Run 110	Run 205	*GNP*	*GNP58*
	(1)	(2)	(3)	(4)	(5)
	Average Duration of Movement, in Quarters				
Expansions	6.1	6.8	5.6	12.5	11.0
Contractions	5.5	9.0	10.8	7.0	6.5
Full cycle	11.6	15.8	16.4	19.5	17.5

4.2.7 Table 4.15 presents the conformity measures (frequencies of unmatched and extra turns) for the simulated ratio-series of the selected OBE runs. The measures result from timing comparisons between these series and the corresponding reference (cumulated diffusion) indexes, CDI. The table also shows the distributions of the leads and lags involved (columns 5 to 12). The average leads and lags are given in Table 4.16.[55]

The series in Run 205 have much higher proportions of extra turning points than the series in either of the other sets (Table 4.15, columns 3 and 4).[56] This is analogous to results shown in Table 4.7 for the Wharton simulations, and the reason is already familiar: the

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[54] For eight runs (including three with non-autocorrelated shocks, S_u, and five with autocorrelated shocks, S_c), the averages, weighted by the number of cycles per run, are as follows. (The figures refer to ratios of shocked- to control-series and are expressed in quarters.)

GNP: expansions, 9.7; contractions, 9.8; full cycle, 19.5.
GNP58: expansions, 9.5; contractions, 9.5; full cycle, 19.0.

The S_u runs tend to show considerably longer expansions and contractions (averaging 12 to 17 quarters). The S_c runs have somewhat shorter movements (averaging about 8 to 9 quarters in either direction).

[55] The format of these tables is the same as that of the Tables 4.7 and 4.8 for the Wharton Model (in Section 4.1.7 above).

[56] There are 20 series with extra specific-turns in Run 205, 14 in Run 107, and 15 in Run 110. The average percentages of such turns are 31.0, 16.0, and 21.6, respectively.

TABLE 4.15

Cyclical Conformity and Timing of Simulated Ratio-Series, with Reference Chronologies Based on Cumulated Diffusion Indexes, OBE Model, Three Runs

		Reference Turns Not Matched[b]		Extra Specific Turns[c]		Number of Timing Observations							
						At Reference Peaks				At Reference Troughs			
Line	Variable Symbols[a]	Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Coincidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Coincidences (10)	Lags (11)	Long Leads and Lags[d] (12)
						Run 205: with non-autocorrelated shocks[e]							
1	*GNP*	0	0	0	0	0	4	2	2	0	5	1	1
2	*GNP58*	0	0	4	25.0	1	3	2	3	1	3	2	2
3	*C*	0	0	6	33.3	1	4	1	2	4	2	0	1
4	*IH*	0	0	6	33.3	3	1	2	3	3	1	2	3
5	*ISE*	1	8.3	4	26.7	1	0	4	1	3	1	2	0
6	*II*	2	16.7	4	28.6	1	2	2	2	2	2	1	2
7	*NE*[f]	1	8.3	4	26.7	1	0	4	2	2	2	2	3
8	*YP*	0	0	4	25.0	1	3	2	2	0	5	1	1
9	*P*	1	8.3	2	15.4	0	0	5	2	2	1	3	4
10	*LE*	0	0	6	33.3	1	2	3	0	1	4	1	1
11	*UN*[f]	0	0	9	42.9	1	1	4	1	1	2	3	2
12	*CPR*	1	8.3	4	26.7	4	0	1	0	4	2	0	1
13	*AWW*	0	0	8	40.0	2	2	2	1	1	3	2	0
14	*OMD*	3	25.0	5	35.7	2	2	0	1	3	1	1	2

15	*UMD*	4	33.3	1	11.1	1	1	2	1	1	1	2	1
16	*HS*	3	25.0	5	35.7	2	1	2	3	4	0	0	3
17	*RS*	3	25.0	8	47.1	2	0	2	3	1	2	2	3
18	*RL*	3	25.0	3	25.0	2	1	2	4	1	1	2	3
19	*W*	2	16.7	8	44.4	3	1	1	2	2	0	3	3
20	*LC/O*	1	8.3	6	35.3	0	1	4	2	3	1	2	4
21	*M*	2	16.7	4	28.6	0	2	3	3	2	1	2	1
Run 107: with autocorrelated shocks^{g}													
22	*GNP*	0	0	0	0	1	5	3	0	1	6	1	0
23	*GNP58*	0	0	0	0	2	6	1	1	2	5	1	1
24	*C*	0	0	0	0	4	5	0	0	4	2	2	1
25	*IH*	3	17.6	4	21.1	1	2	4	1	2	2	3	0
26	*ISE*	5	29.4	0	0	0	4	2	1	2	1	3	2
27	*II*	0	0	2	11.8	4	3	2	0	3	2	3	2
28	*NE*^{f}	4	23.5	1	7.1	2	3	2	0	2	2	2	2
29	*YP*	0	0	0	0	2	6	1	1	2	6	0	0
30	*P*	3	17.6	4	22.2	1	2	4	2	0	3	4	1
31	*LE*	2	11.8	2	11.8	3	2	3	1	1	1	5	0
32	*UN*^{f}	1	5.9	4	20.0	2	2	4	1	2	2	4	0
33	*CPR*	0	0	2	10.5	4	2	3	1	1	4	3	1
34	*AWW*	1	5.9	0	0	4	2	2	3	7	0	1	4
35	*OMD*	2	11.8	2	11.8	5	2	1	3	4	1	2	2
36	*UMD*	8	47.1	0	0	1	3	1	1	4	0	0	1
37	*HS*	2	11.8	3	16.7	4	1	3	2	4	2	1	1
38	*RS*	2	11.8	3	16.7	4	0	4	3	2	3	2	0
39	*RL*	2	11.8	3	16.7	0	3	5	0	2	3	2	1
40	*W*	0	0	4	19.0	3	1	5	0	3	4	1	1
41	*LC/O*	3	17.6	5	26.3	1	0	6	1	1	3	3	0
42	*M*	3	17.6	2	12.5	4	2	1	0	4	2	1	0

(*continued*)

TABLE 4.15 (*continued*)

						Number of Timing Observations							
						At Reference Peaks				At Reference Troughs			
		Reference Turns Not Matched[b]		Extra Specific Turns[c]									
Line	Variable Symbols[a]	Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Co-incidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Co-incidences (10)	Lags (11)	Long Leads and Lags[d] (12)
						Run 110: with autocorrelated shocks[e]							
43	*GNP*	0	0	0	0	1	3	2	0	0	5	1	1
44	*GNP58*	0	0	0	0	1	5	0	0	0	4	2	0
45	*C*	0	0	0	0	1	4	1	0	2	2	2	0
46	*IH*	0	0	4	25.0	5	0	1	2	3	1	2	0
47	*ISE*	0	0	0	0	0	2	4	1	1	2	3	0
48	*II*	0	0	5	29.4	2	2	2	0	1	3	2	1
49	*NE*[f]	2	16.7	1	9.1	1	0	4	1	2	1	2	1
50	*YP*	0	0	0	0	1	2	3	0	2	3	1	0
51	*P*	2	16.7	3	23.1	0	0	5	3	0	0	5	3
52	*LE*	0	0	0	0	1	1	4	0	0	3	3	1
53	*UN*[f]	0	0	2	14.3	0	1	5	0	1	2	3	2
54	*CPR*	0	0	2	14.3	0	5	1	0	1	3	2	0
55	*AWW*	0	0	4	25.0	2	0	4	0	2	1	3	3
56	*OMD*	0	0	4	25.0	2	4	0	1	2	3	1	1
57	*UMD*	1	8.3	1	8.3	0	2	4	1	0	3	2	1
58	*HS*	1	8.3	6	35.3	5	0	0	3	5	0	1	0

TABLE 4.15 (*concluded*)

						Number of Timing Observations							
		Reference Turns Not Matched[b]		Extra Specific Turns[c]		At Reference Peaks				At Reference Troughs			
Line	Variable Symbols[a]	Number (1)	Per Cent (2)	Number (3)	Per Cent (4)	Leads (5)	Coincidences (6)	Lags (7)	Long Leads and Lags[d] (8)	Leads (9)	Coincidences (10)	Lags (11)	Long Leads and Lags[d] (12)
						Run 110: with autocorrelated shocks[e] (*continued*)							
59	*RS*	0	0	7	36.8	2	2	2	2	2	2	2	2
60	*RL*	2	16.7	2	16.7	1	1	3	2	1	2	2	3
61	*W*	2	16.7	5	33.3	1	2	2	1	2	2	1	2
62	*LC/O*	0	0	1	7.7	0	0	6	0	2	1	3	3
63	*M*	0	0	3	20.0	1	3	2	0	3	1	2	2

[a] For meaning of symbols, see Table 1.1.

[b] Turns in the cumulated diffusion indexes (CDI) not matched by turns in the simulated series. For number of the reference turns (in CDI), see footnotes *d* and *e*.

[c] Turns in the simulated series that have no matching turns in CDI.

[d] Leads and lags of three or more quarters.

[e] Twelve reference turns (6 peaks and 6 troughs).

[f] Inverted (peaks matched with reference troughs; troughs, with reference peaks).

[g] Seventeen reference turns (9 peaks, 8 troughs).

TABLE 4.16

Average Timing of Simulated Series (Ratios to Control Solutions) at Reference Dates Based on Cumulated Diffusion Indexes, OBE Model, Three Runs
(Mean (M) and Median (Md) Leads (−) or Lags (+) in Months)

Line	Variable Symbol[a]	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns
		M (1)	Md (2)	M (3)	Md (4)	M (5)	M (6)	Md (7)	M (8)	Md (9)	M (10)	M (11)	Md (12)	M (13)	Md (14)	M (15)
		Run 205[b]					*Run 107*[c]					*Run 110*[c]				
1	*GNP*	+5.5	0	+1.5	0	+3.5	+1.0	0	0	0	+0.5	+1.0	0	+1.5	0	+1.2
2	*GNP58*	+5.5	0	+0.5	0	+1.5	−1.0	0	+0.7	0	−0.1	−0.5	0	+1.0	0	+0.2
3	*C*	+1.0	0	−4.0	−3	−1.5	−2.0	0	−0.7	−3	−1.4	0	0	0	0	0
4	*IH*	−3.0	−3	−3.0	−3	−3.0	+3.4	+3	+0.4	0	+1.9	−7.0	−4.5	−2.5	−1.5	−4.7
5	*ISE*	+4.2	+6	−1.0	−1.5	+1.4	+2.0	0	+0.5	+1.5	+1.2	+3.5	+3	+1.0	+1.5	+2.2
6	*II*	+4.8	0	−3.6	0	+0.6	−1.0	0	−0.3	0	−0.7	0	0	−0.5	0	−0.2
7	*NE*[d]	+13.8	+6	−4.5	0	+3.8	0	0	+1.5	0	+0.7	+3.6	+3	−1.2	0	+1.2
8	*YP*	+2.0	0	+1.5	0	+1.7	−1.0	0	−1.1	0	−1.1	+2.0	+1.5	−0.5	0	+0.7
9	*P*	+7.8	+7.5	0	+1.5	+3.6	+3.4	+3	+3.4	+3	+3.4	+10.8	+9	+15.0	+9	+12.9
10	*LE*	+1.5	+1.5	+0.5	0	+1.0	+1.5	0	+1.7	+3	+1.6	+2.0	+3	+3.5	+1.5	+2.7
11	*UN*[d]	+3.5	+3	+1.0	+1.5	+2.2	+0.4	+1.5	+0.4	+1.5	+0.4	+3.0	+3	+4.0	+1.5	+3.5
12	*CPR*	−1.2	−3	−5.0	−4.5	−3.2	−1.0	0	+2.2	0	+0.5	+0.5	0	+0.5	0	+0.5
13	*AWW*	+1.0	0	0	0	+0.5	−3.7	−1.5	−7.5	−6	−5.6	+0.5	+3	+2.5	+3	+1.5
14	*OMD*	−3.8	−3	−0.6	−3	−2.0	−4.1	−3	−4.7	−3	−4.4	−2.5	0	−1.5	0	−2.0
15	*UMD*	+9.7	+4.5	−0.8	+1.5	+4.5	−1.8	0	−6.7	−4.5	−4.0	+6.0	0	+3.6	+3	+4.0

TABLE 4.16 (*concluded*)

Line	Variable Symbol[a]	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns	Peaks		Troughs		All Turns
		M (1)	Md (2)	M (3)	Md (4)	M (5)	M (6)	Md (7)	M (8)	Md (9)	M (10)	M (11)	Md (12)	M (13)	Md (14)	M (15)
		Run 205[b]					*Run 107*[c]					*Run 110*[c]				
16	*HS*	+3.0	0	−21.0	−15	−7.7	−0.8	−1.5	−1.7	−3	−1.2	−10.8	−9	−3.0	−3	−6.5
17	*RS*	+3.8	+6	+3.0	0	+3.3	+1.5	0	−0.4	0	+0.6	+4.5	0	0	0	+2.2
18	*RL*	+1.8	0	+1.8	+3	+1.8	+2.6	+3	+3.9	0	+3.2	+10.2	+3	+1.8	0	+6.0
19	*W*	−1.2	−3	−1.2	+3	−1.2	+0.3	+3	−1.9	0	−0.7	+5.4	0	−4.8	0	+0.3
20	*LC/O*	+6.0	+6	−1.0	−1.5	+2.2	+4.3	+3	+0.8	0	+2.4	+5.0	+6	+2.0	+3	+3.5
21	*M*	+5.4	+9	−3.6	0	+0.9	−1.7	−3	−1.7	−3	−1.7	+1.0	0	−2.0	−1.5	−0.5

NOTE: The average leads and lags listed in this table cover the timing observations that are included in the frequency distributions of columns 5 to 12 of Table 4.15.

[a] For meaning of symbols, see Table 1.1.

[b] With non-autocorrelated shocks.

[c] With autocorrelated shocks.

[d] Inverted (*see footnote f in Table 4.15*).

S_u series are a good deal more volatile than the S_c series. Again, there is no systematic difference between S_u and S_c in matching the reference turns.[57]

Four of the simulations in Run 107, and six in Run 110, have perfect conformity scores (no unmatched reference or extra specific-turns), but there is only one such series (*GNP*) in Run 205. All of these cases of one-to-one correspondence between the cyclical fluctuations in CDI and the ratio-series relate to comprehensive aggregates that are expected to indicate closely the economy's broad movements. These include *GNP* in current and constant dollars, and total civilian employment; also, the cyclically sensitive, though lagging, business fixed-investment outlays, *ISE*, and *YP* and *C*, which recently have shown rather muted (but recurrent) reactions to cyclical developments.

The worst conformity-scores (highest totals of the percentage in columns 2 and 4 in Table 4.15) belong to the interest rates, price and wage levels, housing starts, and new and unfilled orders. Actual data, especially for the post-World War II period, do show *P* and *W* to conform poorly, and *HS* somewhat indifferently, but the interest rates and orders series should have performed substantially better according to their historical records.

4.2.8 According to the criteria specified in Section 4.1.8, the simulated ratio-series of the OBE Model can be classified by timing, with relatively little doubt for most variables.

(1) *GNP*, *GNP58*, *C*, *II*, *YP*, *LE*. These series belong in the *RC* (roughly coincident) group. For *GNP*, exact coincidences are most frequent and lags are somewhat more numerous than leads; the means are predominantly small positive ones (short lags), medians zero (coincidences). For *GNP58* the measures are similar, with somewhat more frequent leads at peaks. For *C*, there are a few more leads in two of the runs, but they are, on the whole, short. For *II*, leads are quite frequent—especially at peaks—in one run (107), and a few are long; but elsewhere, lags and coincidences are as frequent or more, and the medians are zero throughout. The *YP*

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[57] There are 13 series that fail to match all reference-turns in Run 205, 14 such series in Run 107, and 6 in Run 110. The average percentages of unmatched reference-turns are 17.3, 17.2. and 13.9 for the three runs, respectively (Table 4.15, columns 1 and 2).

series show a few longer lags in two runs, but once more the averages are all in the *RC* range, and all but one of the medians are zero. For *LE*, lags are more numerous than either leads or coincidences, and most of the timing averages are lags of one to three months. The *LE* series, then, could be classified as roughly coincident with some tendency to lag, *RC-Lg*.

(2) *HS*, *OMD*. Predominantly leaders, *L*. There are long average leads at troughs in Run 205, and at peaks in Run 110, for the housing starts, and short average leads in Run 107. (However, leads and lags balance each other at peaks in Run 205.) For new orders, all means and most medians are leads, but they are short, from two to five months. The leads tend to be somewhat longer at peaks than at troughs. In at least two of the three runs, the *OMD* series can be viewed as roughly coincident with a tendency to lead, *RC-L*.

(3) *ISE, UN, P, RL, LC/O*. Generally, laggers, *Lg*. Most of the individual observations for plant and equipment investment are lags, and so are most of the averages; but they are short, in the *RC* range. The same statements apply to the series for unemployment (inverted). Both *ISE* and *UN*, therefore, qualify as roughly coincident with a tendency to lag, *RC-Lg*. The lags are often much longer for the other variables in this group, particularly for *P* and *LC/O* at peaks in Runs 205 and 110.

The evidence is somewhat more ambiguous for the remaining variables, but it permits some further groupings and observations.

(4) *IH*, *CPR*, *AWW*. The timing of these series is mainly leading or roughly coincident, *L*, *RC*. Investment in housing leads, on the average, by three months in Run 205, and by longer intervals in Run 110 (especially at peaks), but it shows small mean lags elsewhere. For profits, leads prevail in the averages of one run; but for the rest, the medians are zero and the means are small lags. Much the same applies to *AWW*: there are sizable leads here in one run (107), and average coincidences and short lags in the other two runs.

(5) *RS*, *W*, *M*. These are all roughly coincident series, according to the over-all timing averages (for "all turns"), but leads and lags—some of them quite long—are here much more numerous than coincidences, and just about offset each other. There are a few

long lags at peaks in two of the short-term interest-rate series. For wages, lags somewhat outweigh leads; while for money, the opposite applies.

(6) *UMD, NE.* Here the timing is particularly mixed. For unfilled orders, lags prevail in two runs, and leads in one – yielding over-all averages of about +4 and –4 (months), respectively. Net exports, when treated on an inverted basis, conform fairly well (on a positive basis, very poorly).[58] Lags dominate the averages for *NE* at peaks in two runs; in one of which, they are rather long. Elsewhere the timing of *NE* is roughly coincident.

For the most part, the timing of the simulated ratio-series for the OBE Model agrees broadly with the timing of the corresponding variables, as established from historical data.[59] The agreement extends beyond the roughly coincident national product, income, and consumption aggregates (*GNP*, *GNP58*, *YP*, *C*) to some sensitive leading indicators (*HS*, *OMD*) and some laggards (*W*, *LC/O*, and probably also *P*). To be sure, there are deviations from this over-all correspondence in that the behavior of some of the *S* series is occasionally contrary to expectations (e.g., *HS* at peaks in Run 205), but the similarities do prevail.

In several cases, the differences are more quantitative than qualitative and not very large. Thus *LE* and *UN* are roughly coincident according to over-all timing averages for past data; in the OBE simulations, they also belong in the *RC* group but show distinct lagging tendencies.[60] *ISE* is recognized as a lagging indicator; in the simulations, it often lags but on the average by short intervals, and hence might be labeled *RC-Lg;* however, the average lag of *ISE* has been very short in the past, too. Something similar might be said about the interest rates, where *RS* also is coincident-lagging; here, the runs with

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[58] As noted in Section 4.1.8 (text and footnote 47), no meaningful timing comparisons could be made for this variable in dealing with the Wharton simulated series (on either basis). The rationale for the inverted treatment is that imports, which should conform positively, enter *NE* with a negative sign (*NE* = exports minus imports).

[59] See Section 4.1.8 above for a discussion of some of these historical timing patterns and related analytical problems.

[60] The unemployment rate (inverted) has often led at business-cycle peaks and lagged at troughs; but here, *UN* shows some average lags at peaks, too.

autocorrelated shocks must also be credited with having *RL* lag behind *RS*.

Investment in housing, *IH*, leads in two runs, and in one case by longer intervals at peaks: this agrees with its timing according to the postwar data. However, in one run (107) *IH* shows, perversely, some tendency to lag at peaks.

CPR—corporate profits after taxes—was included in the *RC* group in the 1950 list of NBER indicators, but ten years later it was shifted to the *L* group, in view of its longer leads in the postwar period. The average lead of *CPR* in Run 205 agrees approximately with the recent record, but the prevalence of coincidences and lags in Runs 107 and 110 does not.

There is evidence, too, that the timing since World War II has become earlier for another leader, the change in business inventories, *II*. Here the apparent discrepancies are quite considerable, since the simulations show *II* as roughly coincident, with leads about as frequent as either exact coincidences or lags.

The average workweek, *AWW*, belongs to the most dependable leaders; its median lead since 1921 has been 5 months. Only one of the simulated series (in Run 107) has a similar timing pattern, while the two others (in the *RC* or *RC-Lg* categories) definitely do not. The simulations for *UMD* seem rather inconsistent from run to run, and their timing is certainly quite different from that of the actuals (*RC*, with leads at peaks). For net exports, it is difficult to know what to expect: the series has not conformed well to domestic business cycles.[61]

The record for the money supply, *M*, is also easy to interpret. Total *M* has often shown retardations rather than absolute declines during relatively mild recessions; but where timing comparisons can be made, they suggest rough coincidence. The rate of change in *M* tends to lead by irregular, but frequently long, intervals. The *RC* patterns of most of the simulated series for this variable are probably not necessarily in conflict with the historical evidence, given the type of measurements here applied, but the long lags at peaks in one run (205) are.

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[61] See Ilse Mintz. *Cyclical Fluctuations in the Exports of the United States Since 1879*. New York, Columbia University Press for the National Bureau of Economic Research, 1967, Chapter 5. Also, see footnote 58 above.

In summary, the timing of the simulations for *II*, *AWW*, and *UMD* disagrees with the recorded timing for these variables; and there are also considerable partial discrepancies for a few other variables, notably *IH* and *CPR*. The results seem to be better here than for the Wharton Model with respect to the verisimilitude of timing for *LE*, *P*, and *RS*; but the reverse applies to *II*, *UMD*, and, perhaps, *AWW*.

4.3 STOCHASTIC SIMULATIONS: A SUMMARY

The main results of this part of our study are based on two sets of measures: (1) frequency, duration, and relative amplitude of rises and declines, and (2) conformity and timing of cyclical expansions and contractions. They will be summarized in this order for both the stochastic simulations proper (levels) and the relative deviations of these *S* series from their presumed trends (ratios of shocked- to control-series). The stochastic simulations must be seen against the background of the underlying control solutions and compared with sample-period realizations in some suitable form.

4.3.1 The *control solutions* for both the Wharton and OBE Models produce, for the most part, smooth series with upward trends. There are some mild effects of the start-up shock in the Wharton Model, but no recession develops. There are some fluctuations, downward trends, or trend reversals in one or both of the control series for eight variables (*II*, *NE*, *UN*, *CPR*, *AWW*, *OMD*, *RS*, and *RL*). The trendlike control series contrast sharply with the nonstochastic sample-period simulations, which do show recurrent fluctuations, although in markedly damped form. One probable reason for this contrast lies in the fluctuations of the exogenous variables, which are included in the sample-period simulations but not in the ex ante stochastic simulations; another reason (compatible with the first one) would be specification errors of the models.

The *stochastic simulations* proper are strongly trend-dominated for *GNP* in current and constant dollars, and for several other comprehensive aggregates, viz., personal income and consumption, employment, price and wage levels, and money supply. There are systematic differences between the series with non-autocorrelated shocks (S_u) and those with autocorrelated shocks (S_c): the latter are far smoother than the former, hence tend to have larger average durations (AD) and smaller average percentage amplitudes (APA) of rises and declines. The Wharton S_u series for *GNP* and *GNP58* show somewhat shorter and smaller declines than the historical data, while the S_c series show much fewer declines, which are all very short, and much too few rises, which are all very long. In the corresponding OBE simulations of either type, declines are altogether rare, short, and small.

For the other variables listed in the preceding paragraph, the AD of rises in the sample-period actuals are often overestimated by the S_c series and underestimated by the S_u series, particularly in the OBE runs; the AD of declines tend to be underestimated by both S_c and S_u figures. The series that have weaker trends and stronger fluctuations (relating to investment processes, unemployment, average workweek, orders, and interest rates) tend to have shorter movements than the actuals in either direction. The S_c series often underestimate the length of the movements recorded in the historical data less than do the S_u series.

For the Wharton simulations, the APA of quarterly changes tend to be too large in S_u and too small in S_c, when compared with the actuals. The OBE series have, for the most part, too small declines, and here the S_u series have the advantage of understating the APA of the actuals less than the S_c series do.

The criterion of duration is presumably more important than that of amplitude. (See p. 438.) When this is taken into account, the balance of our comparisons favors the S_c over the S_u simulations for most variables in both models. However, the S_u series yield results which are definitely better than those of the S_c series for *GNP* and *GNP58* in the Wharton Model. (In the OBE Model, neither type of simulation gives directly acceptable approximations to the historical behavior of these aggregates.)

Our charts and measures leave little doubt that the shocked simulations of both models can produce extremely diversified behavior-patterns in the different variables. Indeed, it appears that they accentuate—and often overstate by historical standards—the persistence of growth in national output, income, and employment aggregates on the one hand; and the frequency of short irregular fluctuations in the more sensitive series for investment and other partial indicators, on the other. It is the intermediate, cyclical movements that seem to become blurred. But this could be due, in large measure, to the inadequate handling or scaling of the shocks—in particular, to the neglect of disturbances in the exogenous variables. Hence the proposal to analyze the *relative deviations of shocked-series from control-series*, as such experimental data might be expected to be more sensitive to, and indicative of, the cyclical effects of relatively weak impulses.

It took some working familiarity with these simulated ratio-series to recognize that the method can, and does, bring out errors of measurement, as well. Short erratic movements, often of relatively large amplitude, are a feature of many of the ratio-series, and the presence of longer cyclical movements that are not mere statistical artifacts is not always clear. Some of the control-series are probably rather arbitrary, and the procedure can, perhaps, reduce errors from this source.

It is particularly the ratios of S_u to the control-series that are highly erratic in many cases; the ratios for S_c are much smoother. The S_c ratio-series have larger AD than the S_u ratio-series in about 80 per cent of all cases, for both models; also, the former series generally have smaller relative movements (APA) than the latter. (These observations for the ratios, it will be noted, parallel those for the levels of the S series.) The simulated ratios tend to understate the AD of the corresponding sample-period actuals (relative deviations from trend), but often by much smaller margins for the S_c than for the S_u runs; indeed, the deviations are very small for the former series in a substantial proportion of the comparisons. The APA of the S_u ratios are often larger, while those of the S_c ratios are generally smaller, than the corresponding amplitude measures for the actuals; and here, the differences are frequently smaller for S_u than for S_c.

The trend-adjusted postwar *GNP* series in current and constant dollars are, in most cases, better approximated by the S_c than by the S_u ratios, in terms of the frequency and average durations of rises and declines. Again, giving more weight to the duration than to the amplitude criterion, the results for the ratio-series generally favor the S_c over the S_u simulations, and do so rather more strongly than do those findings based on the level comparisons.

4.3.2 In their original form, many of the stochastic simulations show only short declines—isolated or more frequent—but no recurrent fluctuations in the nature of specific cycles. This applies particularly to the comprehensive income, production, and employment aggregates; and still more to the OBE than to the Wharton series (as is evident from the charts). Accordingly, the cyclical conformity and timing analysis could be carried out fully only for the ratios of shocked-series to control-series, not for the shocked-series proper.

Using the ratio-series, cumulative diffusion indexes (CDI) were constructed for three randomly chosen runs of the Wharton Model and for three of the OBE Model. For either model, the selection includes one set of series based on the S_u simulations and two based on the S_c simulations. Each of the CDI shows reasonably well-defined cyclical movements, whose turning point dates can be used as a reference chronology with which to compare the timing of the simulated series in the given set. The average durations of the specific cycles in the CDI are of the same general order of magnitude as the average durations of cycles in the relative deviations from trends of the postwar *GNP* and *GNP58* series.

The lower the proportions of those turns in CDI and the simulated ratio-series that cannot be matched, the higher the *cyclical conformity* of the series. In general, the series involving autocorrelated shocks show fewer "extra" turns and, therefore, have better conformity scores than the more erratic series with serially uncorrelated shocks. Among the best conformers, in the models as in the historical data, are the national product and income aggregates in current and constant dollars, and also some of the largest real-expenditure components (*C*, *ISE*); among the poorest are the price and wage levels, and net exports. The interest rate simulations show relatively poor conformity in both models, and so do the Wharton series for profits, and the OBE series for new and unfilled orders—all contrary to the actual records.

Measures of *relative timing* (based on comparisons at the reference or CDI turns) show *GNP*, *GNP58*, *YP*, *C*, and *UN* all to be roughly coincident, in the simulated as in the actual data. Both models agree broadly with historical records in regard to *IH* and *CPR*, which are predominantly leading, and *ISE*, *RL*, and *W*, which tend to lag. In the Wharton Model, the average timing measures for *II* and *UMD* (leads

TABLE 4.17

Stochastic 100-Quarter Simulations (Ratio-Series) for Two Models, Absolute and Relative Frequency Distributions of Leads and Lags at Turns in Cumulated Diffusion Indexes (number and per cent)

		Timing Observations at Business Cycle Turns			
Line	Grouped Variables[a]	Total (1)	Leads (2)	Exact Coincidences (3)	Lags (4)
			Wharton Model[b]		
	Leading (5)				
1	Number	159	76	42	41
2	Per cent	100.0	47.8	26.4	25.8
	Coincident (6)				
3	Number	192	45	82	65
4	Per cent	100.0	23.4	42.7	33.9
	Lagging (3)				
5	Number	89	25	19	45
6	Per cent	100.0	28.1	21.3	50.6
			OBE Model[c]		
	Leading (7)				
7	Number	256	111	72	73
8	Per cent	100.0	43.4	28.1	28.5
	Coincident (7)				
9	Number	279	62	132	85
10	Per cent	100.0	22.2	47.3	30.5
	Lagging (4)				
11	Number	142	34	36	72
12	Per cent	100.0	23.9	25.4	50.7

[a] Classified according to the timing of actuals (historical series). The variables included in each group are those used in Table 3.14, lines 1 to 6 and 7 to 12 (based on Tables 3.4 and 3.8 for the Wharton and OBE Models, respectively).

[b] Based on Table 4.7. The count includes all observations at reference (CDI) peaks and troughs for the three runs (31, 14, and 26) combined.

[c] Based on Table 4.15. The count includes all observations at reference (CDI) peaks and troughs for the three runs (205, 107, and 110) combined.

and *RC*) are also correct in terms of past behavior, but this is not so in the OBE Model. On the other hand, the timing patterns of *LE*, *P*, and *RS* are reproduced better in the OBE than in the Wharton simulations. The leading tendency of *AWW* is largely missed in both models. No major inconsistencies prevail in the relative timing of the OBE simulations for *OMD*, *HS*, *LC/O*, and *M* (variables not included in the Wharton Model).

Both models score relatively well on timing according to these comparisons, and neither appears clearly superior to the other. From the timing measures alone, it would not be possible to say that the S_u runs are systematically worse (or better) than the S_c runs. However, the quality of these measures seems particularly uncertain for S_u, because these series have greater frequencies of turns (all and extra) and, hence, conform worse than do the S_c series.

Table 4.17 sums up the evidence on cyclical timing of the simulated ratio-series. Like Table 3.14 for the sample-period simulations, it attempts to answer the question: How well do the models differentiate between the groups of historically leading, coinciding, and lagging indicators? These distributions classify the observations by model and timing group only, combining the individual runs and the measures at peaks and at troughs within these categories. The results are reasonably satisfactory in that, in both models, leads are more frequent than either lags or coincidences for the group of leading indicators; and, similarly, coincidences represent the modal class for the roughly coincident group, while lags represent the modal class for the lagging group. Indeed, these distributions appear to be better than those based on the sample-period simulations for Wharton and OBE in discriminating between the timing categories, because of a superior performance with respect to the group of coinciders (compare Table 4.17 with Table 3.14, lines 1 to 12, columns 6 to 9). However, the differences between the leaders and the laggers are still less pronounced here than in the distributions for the sample-period actuals (Table 3.14, columns 2 to 5).

5 CONCLUSION AND SUGGESTIONS FOR FURTHER RESEARCH

TO COMPLETE this report, three inter-related tasks remain to be done. We shall now sum up the main findings of this study, identify its principal limitations, and consider its implications for future work.

5.1 SURVEYING THE RESULTS

(1) The nonstochastic simulations analyzed in Parts 2 and 3 refer to the periods to which the models were fit and use the correct ex post values of the exogenous variables; hence, they do not provide tests of the predictive powers of the models. They do, however, subject the models to rather demanding tests of a different kind, since, in simultaneous estimation, errors are liable to cumulate across a model and over time. There is evidence that the calculated values do tend to drift away, though not necessarily continuously, in simulations that cover more than one or two business cycles. The drift is easy to spot visually on some charts for trend-dominated variables such as *GNP*, where it takes the form of increasing underestimation of growth. Generally, the discrepancies between the levels of the simulated and actual series are much greater than those between the corresponding quarterly changes. The reason lies in autocorrelated errors, which cumulate, thus throwing off base the long multiperiod predictions that are involved here.

(2) Simulation of turning points presents a particularly difficult test for the models. Missed turns, large discrepancies in timing, and drastically reduced amplitudes of fluctuation are all major sources of error in the simulated series that are associated with directional shifts in the actuals. For more cyclical and volatile variables, such timing and amplitude differences result in especially large errors.

(3) The nonstochastic sample-period simulations indicate that models such as the Wharton and OBE produce a progressively—and heavily—damped time-path of aggregate output (real income). Only the first one or two recessions covered have found some reflection in the declines of the simulated *GNP58* series for these models. The FMP series, being quite short, allow no examination of whether this

model would have simulated another contraction in *GNP58* beyond the first two rounds.

(4) It is consistent with these results that the six-quarter simulations, which cover only one business-cycle turn each, disclose no dampening or other systematic changes over time. Since each of these short simulations starts from new (correctly measured) initial conditions, any one of the included episodes has an approximately equal chance to be replicated. Small shifts in the base have rather little effect; the simulations are not significantly better when they start one quarter ahead of a reference peak or trough than when they start two or three quarters ahead. About 75 per cent of the specific-cycle turns in the actual series are matched in these short simulations when the differences in timing are disregarded, whereas the corresponding proportion for the long sample-period simulations is close to 65 per cent.[62]

(5) Common to both short and long nonstochastic simulations is a strong tendency to underestimate the amplitudes of the observed cyclical movements. Since these simulations exclude the component of random disturbances which is present in the actuals, the total variance of any of them must be smaller than the variance of the corresponding historical series. However, the six-quarter and reference-cycle amplitudes refer to separate cyclical episodes, as reflected in the complete-model simulations; underestimation could well show up much less consistently in such measures than in the over-all changes in the *S* series, and it does. To the extent that the simulations underpredict the longer cyclical movements and not just the short irregular variations in the actuals, errors of this kind acquire a systematic and undesirable element.[63]

(6) The simulated series are, for the most part, classifiable according to their timing at business-cycle turns; but some of them are not,

[62] The gain from reducing the span of the calculations is considerably larger for real *GNP*, where short simulations still reproduce about 70 per cent of the turning points, while long simulations match only 55 per cent. There may be some bias in these comparisons in favor of the long simulations to the extent that *S* and *A* have corresponding turns that either lead or lag at the reference dates by long intervals, for such observations are included in the counts for the sample-period series but may not be included in those for the six-quarter runs. (However, the admission of the "inferred prior turns" in the latter measures—see Table 2.1 and text—should counteract some of this bias.)

[63] Underestimation of changes is not per se undesirable—indeed, it is a property of unbiased and efficient forecasts—but it can also occur in grossly incorrect predictions. See [24, p. 18] and [29, p. 43].

because they have too few turning points. The series in this subset consist mainly of comprehensive aggregates for *GNP*, employment, personal income, and consumption—series that should have shown good cyclical conformity and roughly coincident timing. Although the simulations do differentiate broadly between the groups of leading, coincident, and lagging indicators, these distinctions are much less sharp here than in the actual data. This applies to both the short, and the long, nonstochastic simulations. In particular, for the coincident indicators, the simulations show a preponderance of leads and lags that balance each other, rather than the large percentages of exact coincidences (in quarterly terms) that typify the recorded timing distributions for these series.

(7) The ex ante simulations (control-solutions), by reaching far into the unknown future, confront the models with difficult problems of internal consistency. They include, for both models examined here (Wharton and OBE), some series that are either made to behave in a more-or-less arbitrarily predetermined fashion or are permitted to behave in ways that would seem difficult to rationalize (as illustrated by the simulations for unemployment and interest rates). For the comprehensive indicators of over-all economic activity, the nonstochastic simulations for future periods, unlike those for the sample periods, produce smooth trend-dominated series rather than series with recurrent, if damped, fluctuations. Thus these models do not generate cyclical movement endogenously.[64]

(8) In the stochastic ex ante simulations many fluctuations do occur, but they are in large part too short to qualify as cyclical movements. The series with autocorrelated shocks are much smoother than those with non-autocorrelated shocks; that is, they have longer, but also smaller, declines, which interrupt their upward trends less frequently. The use of autocorrelated shocks is helpful in many—but not in all—cases: it works better for the more volatile series than for the comprehensive aggregates with dominant growth trends and sub-

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[64] It is important to recall that here the models are unaided by fluctuations in the exogenous variables, which in reality—as reproduced in the sample-period simulations—are often pronounced. The projections for these variables are essentially monotonic growth trends, and the models evidently contain no mechanisms that would cause the simulated system to undergo fluctuations in the absence of any shocks (either in the exogenous quantitites or in the relationship with the endogenous variables).

dued fluctuations. In general, the cyclical aspects of the simulated series are much weaker than those observed in the historical series, in contrast to the long trends and short erratic variations that are often considerably stronger.

(9) Since the shocks used may not be adequately scaled, ratios of the stochastically simulated to the control series were also analyzed, in the expectation that they would show greater cyclical sensitivity. This expectation was confirmed, but the ratio-series are also much more erratic than the shocked-series proper, reflecting not only greater over-all susceptibility to the effects of the shocks but presumably, also, a telescoping of measurement errors. The ratios based on simulations with serially uncorrelated shocks are particularly volatile; those with autocorrelated shocks are substantially smoother and generally more plausible.

(10) Cumulated diffusion indexes constructed from the ratio-series exhibit specific cycles whose average duration is similar to that of cycles in trend-adjusted *GNP*, as recorded in the postwar period. Series that incorporate autocorrelated shocks conform better to these reference indexes than do those with non-autocorrelated shocks. The comprehensive indicators of national product, income, and expenditures, which historically rank high on conformity, also score relatively well according to these comparisons.

(11) There is considerable correspondence between the relative timing of the ex ante stochastic simulations and of the historical data for the same variables, as indicated by the average leads, coincidences, and lags of the ratio-series at reference-cycle dates—that is, at the major peaks and troughs in the appropriate diffusion indexes. The distributions of the timing observations for these series are at least as good as those for the sample-period simulations in differentiating between the groups of typical leaders and laggers—and appreciably better in identifying the coinciders. However, the total picture is less favorable than these measures alone would imply, for many turns in the more volatile ratio-series cannot be matched with the reference turns; and some that can be, are difficult to date, so that the timing comparisons involved are rather uncertain.

5.2 SOME PROBLEMS AND AGENDA

(1) One of the basic questions raised at the outset of this study has been answered in the negative by our results for the Wharton and OBE simulations: neither of these models endogenously produces movements corresponding to the historical business cycles. To answer this question for the FMP Model, nonstochastic post-sample-period simulations would be needed.

(2) The absence of any "shocks" or fluctuations in the projected exogenous variables is an unrealistic feature that could, to a large extent, be responsible for the weakness of the cyclical elements in the stochastic simulations here examined. Further experiments should test whether this weakness can be remedied, or reduced, by imposing more-or-less sporadic disturbances on the exogenous factors.

(3) More-standardized simulations for the different models are required in dealing with a comparison of the models regarding their ability to approximate the main characteristics of major short-term fluctuations of the economy. The need here is, at least, for a suitable common sample-period for the different models. Such standardization would also help to solve some analytical problems. For example, it should then be possible to learn more about the relative "damping" properties of the models.

(4) Every econometric model embodies a set of tentative hypotheses, and these theoretical frameworks can differ in important respects without any one of them being obviously unreasonable or inferior to the others: economic theory is not so well developed—and anyhow, cannot be as specific—as to preclude this situation. To the extent that this is so, the more differentiated the models are, the greater should be the potential gains from empirical studies of such models. From this point of view, it is of major interest to obtain and examine the ex ante stochastic simulations for the FMP Model, which differs importantly from other models. Comprehensive simulation studies are needed, as well, for the large Brookings system and for some other more modest, but interesting, models.

(5) This leads directly to the contributions that simulation analysis can make to a comparative study of specification errors in different

models. While we believe this to be a promising area that should be explored systematically, the subject is as vast and difficult as it is important and it was largely left outside the scope of the present report.

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DISCUSSION

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Zarnowitz et al. are to be congratulated on their unusually careful analysis of the dynamic properties of the recent econometric models. On the whole, their tests offer confirmation of the findings of my husband and myself in our earlier paper on the dynamic properties of the Klein-Goldberger model.[1] All the models tested by them were noncyclical in their behavior in the absence of shocks;[2] the amplitudes and frequencies of oscillation in the presence of shocks were rather similar to those in the U.S. economy. Nevertheless, the basic issue raised by our paper (namely: Are the cycles induced by stochastic forces exogenous to the models, or are the models a poor representation of the actual economy?) remains unsolved. This is so because all of the models tested strongly resemble the Klein-Goldberger Model in their basic structural specification of the economy. The only model whose economic and mathematical structure is somewhat different (the MIT-FRB Model) could not be tested, since the simulation results were not made available to Zarnowitz et al. while their paper was being prepared. If the analysis of the simulation results with the MIT-FRB Model leads to conclusions similar to those derived from the other models, this will, to my mind, tip the scale in favor of the hypothesis that the origin of business cycles in the real economy is truly stochastic.

Naturally, even if the dynamic simulations—whether shocked or nonshocked—indicate that the dynamic properties of a model resemble those of the U.S. economy exactly, this cannot be taken as a sufficient test of the validity of the model. The reason for this is that the dynamic simulations are based on simultaneous solutions of the *reduced forms*

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[1] I. Adelman and F. L. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica* (Oct., 1959), 596–625.

[2] In this connection, it is only the stochastic simulations for 25 years, using the extrapolated values of the exogenous variables, which are truly free of shocks over the sample period. Both the nonstochastic simulations using actual values of the exogenous variables and the nonstochastic simulations over six quarters—starting from actual values—contain shocks. The first set of simulations includes shocks in exogenous variables, while the second set incorporates shocks in initial conditions.

of the models, with the lagged endogenous variables treated as endogenous; but to any given reduced form there can correspond an infinite number of differently specified structural models, even when the model is identified in the statistical sense. Identification of structural parameters requires equating certain coefficients of the structural model with specific combinations of coefficients of the reduced forms; in the absence of both the structural specification and the reduced form estimates, one cannot infer a particular model structure from a specific set of reduced forms. Therefore, an exploration of the dynamic properties of a model is not a substitute for an equation-by-equation validation of the structural specification of the model, as carried out, for example, in the Griliches review of the Brookings Model,[3] or in the Christ review of the Klein-Goldberger Model.[4] At a minimum, models must pass the analytic-structural tests, the dynamic simulation tests, and the forecasting tests before one can have some confidence in their validity as good approximations to the behavioral relationships of a real economy.

There is some evidence in the Zarnowitz results that a combination of Type I and Type II shocks would perform better than either kind of shock taken in isolation. The tests also offer some ground for the belief that a combination of correlated and uncorrelated shocks would be superior to either purely random (both across variables and across equations), or purely correlated, shocks. In this connection, it would be interesting to create a set of mixed correlated and uncorrelated shocks by using the output of a control-model (in the engineering sense) to generate shocks upon some of the basic input variables. The model would be used to forecast the future levels of some of the target variables (e.g., *GNP*, price levels, and unemployment) and then the shocks upon specific instrument variables (money supply, interest rates, government expenditures, taxes) could be determined by specifying a set of control functions. The control function for each variable would have the form

$$\delta_i^t = F_i(y_1^{ft} - y_1^{dt}, y_2^{ft} - y_2^{dt}, \ldots)$$

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[3] Zvi Griliches, "The Brookings Model: A Review Article," *Review of Economics and Statistics*, Vol. L, No. 2 (1968), 215–234.

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where δ_i^t is the shock imposed by the control authorities (Federal Reserve, Bureau of the Budget, and Congress) upon instrumental variable i at time t; y_j^{ft} is the level of the jth target variable forecast for time t; and y^{dt} is the desired level of that variable for that point of time. The sign of $\frac{\partial fi}{\partial\,(y_j^f - y_j^d)}$ should be set by Keynesian conventional wisdom, and the order of magnitude of the shocks would be fixed by reference to the variance of such shocks in the past. The function F_i would probably be quadratic. Each variable in the model could then be subjected to a shock: $\mu_i^t = \lambda_i \rho_i^t + (1 - \lambda_i)\ \delta_i^t$; where ρ_i^t is an uncorrelated shock with zero mean and a fixed variance; δ_i^t is the shock calculated from the control function; and λ_i is a weighting factor $1 \geqslant \lambda_i \geqslant 0$. Such an approach to the generation of shocks would appear to be more realistic than either of the two extreme specifications employed by Zarnowitz et al. By varying λ_i towards unity one could also, incidentally, have a test of the Friedman hypothesis.

Tests of the type performed by Zarnowitz et al. are important inputs, aiding insight into both the properties of econometric models and the dynamics of a real economy. Unfortunately, the results of the careful tests performed upon the existing quarterly econometric model in the present paper suggest that we have not progressed substantially along either front during the decade since publication of the original paper by my husband and myself.

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1. INTRODUCTION

The main task of this Conference should be to assess the existing econometric evidence as it bears on the causes of business cycles. An alternative view—and one that I tend to reject—is that we are gathered to assess various econometric models. None of the models before us is a bad model; after all, each was built by competent economists who were then willing to publish the model, to use it, and to submit it to the scrutiny of both members of the profession and the *Survey of Current Business*. Each model has had moments of glory—even forecasting real *GNP* correctly for two quarters in succession is enough to warm the heart of a model-builder. And, at times, each model will surely be very wrong. It takes very little to remember 1968.

One of the significant findings of this Conference has been the fact that the record of ex ante forecasting by a particular model has generally been superior to the model's ex post forecasting performance over the same period. The reason for this is immediately obvious to anyone who operates a model. No operator—at least, not one with much success as a forecaster—lets the computer center run his model. Rather, the operator considers the model to be nothing better than the best statement of the internal logic of the economy which he happens to have available. While he rarely tampers with the model's interactive logic, he recognizes that there are relevant factors which he *thinks* he knows, and which he is *sure* the model does not know, about current realities in the economy. In some way, he attempts to communicate this information to the model. The value of an operation like Wharton-EFA is that someone who really understands the interactions in that model will be the one to phase in the removal of the investment tax credit, or to take account of a strike, or to tell the model that it simply does not understand the state of expectations in the business sector. And what is most important, much of the relevant information which has to be communicated to the model is simply not contained in the values of the exogenous variables. That is why an outsider who does no more than feed in the exogenous data is really only testing whether the model possesses the necessary property of a dynamic structure which keeps its endogenous motion *within* the extreme limits of reality.

2. SAMPLE-PERIOD TESTS

What, then, do we make of the performance of these models in the ex post tests run by Zarnowitz, Boschan, and Moore? Specific peculiarities aside, the over-all performance was fairly successful. The major intermodel discrepancy seems to have been that FMP was capable of picking up the cyclical peak—though not the succeeding amplitude—

in 1957. In the section on sample-period simulation, Zarnowitz, Boschan, and Moore speculate that the superior performance of FMP in 1957 may be due to its having been initiated in 1956, a period far closer to the 1957 peak than the initiation period of either Wharton or OBE. However, in the section on six-quarter simulations, the authors point out that FMP's superior ability to pick up turns is maintained even when the comparison is restricted to the 1957–61 period – in which case, all the models would have been identically initiated.

Since full data were not provided, one can only speculate about the reason for this difference in behavior. One obvious possibility is that FMP does have better structural equations – at least in the particular aspects which were, at the margin, critical in reproducing the 1957 peak. A look at the available charts, the actual data, and the FMP structure does, however, suggest a plausible alternative. In real terms, *GNP* fell by $7 billion in the final quarter of 1957. Final sales, on the other hand, declined by only $1½ billion, while inventory investment experienced a sharp drop of $5½ billion. FMP completely misses the decline in inventory investment, and projects real *GNP* to rise from third to fourth quarter. The sense in which FMP does pick up the cycle is in the simulation of a drop in real *GNP* of $6–7 billion over the two-quarter period, 1957.4 to 1958.2, concurrent with a very small initial drop in inventory investment from fourth to first quarter, and a larger decline in the following quarter. The Wharton Model actually does a much better job than this regarding the direction and timing of simulated inventory investment, but it fails to show any decline in real *GNP*. Wharton, of course, is simulating all of net exports endogenously – and probably poorly enough to miss the $3½ billion decline which followed the artificial boost from the Suez Crisis – while FMP is being fed an exogenous $3.3 billion decline in exports. It is therefore quite possible that the discrepancy between the Wharton and the FMP simulations in late 1957 rests largely on the differences in endogeneity of the two models.

All three models produce distinct biases in their sample-period simulations. This may not be of grave concern in short-term applications of the models, though it can serve to point up weaker elements in the structure. For example, the real expenditure sectors in the three models seem to be superior in performance to the wage-price sectors, by and large. The models in question treat all, or most of, government spending exogenously, and determine endogenous interest rates largely via term-structure equations based on an exogenous short-rate or discount rate. These major inputs to the expenditure sector are, therefore, well determined in ex post simulations. Since the wage-price sector is treated as only a minor input to the expenditure sector, it is hardly surprising that the latter performs creditably. On the other hand, small errors in output-expenditure determination are capable of resulting in serious distortion of the wage-price-productivity configuration. The relative sector performances are thus not difficult to trace down.

3. STOCHASTIC SIMULATIONS

The sample-period tests revealed that under a regime of fixed-parameter simulation, the models would respond to the true values of the exogenous variables by cycling—but well within the limits of reality. Under these circumstances, the noncyclical path in response to twenty-five years of smooth exogenous variables is readily understood. This situation is about as close as we can come in practice to the textbook ideal of investigating the properties of the pure endogenous system. It corresponds to a laboratory experiment free of external shocks, free of differential policy errors, and free of changing expectations. All these results, combined with the subsequent stochastic simulations, lead me to pose the following alternative inferences.

(*i*) The models contain extreme specification errors. A more nearly correct specification would produce endogenous cycles even with smooth exogenous variables.

or (*ii*) The business cycle is not endogenous; rather it is the result of a normally stable, or damped, system reacting to external influences.

I suggest that the time has come to admit that the weight of reasoned evidence is on the side of the latter. There is simply no clear evidence to support the view that the business cycle results from the endogenous interaction of consumption and investment spending as they are *normally* determined in an industrialized market economy free of external shocks. Any stock-adjustment model which exhibits endogenous cycles, clearly rests on a gross denial of the ability of the business sector to understand the realities of an aggregative natural growth-rate. This seems particularly inappropriate as the *general* description of a highly industrialized economy with concentrated market structures.

The parameters which we estimate in our models are surely not correct—nor is the structure correct. But within the general structure, it requires only minor changes in particular parameter values to get a model to reproduce closely any cyclical episode which it does not duplicate under a fixed-parameter regime. The altered parameters cannot, however, be expected to work well in the majority of time which lies outside the turning point areas; nor will the parameter alterations adequate for one episode be those required for the next. In an important sense, then, we live with variable parameter sets. Most of the time, one set serves well to represent the system. At other times, the normal set is a poor approximation.

At some junctures, the effective parameter set may change for reasons which are not immediately clear to any observer. Such episodes are more apt to produce data outliers than anything else. They correspond in spirit to the uncorrelated shocks which—in the Type I ratio simulations—generally failed to produce a path with marked similarity to the cycles of experience.

The effective parameter set is almost sure to change when external shocks conspire to push the economy steadily away from the path on which normal expectations are fulfilled, and normal decision criteria are suitably rewarded. Such episodes are quite likely to result in a cyclical response pattern. They correspond in spirit to the correlated shocks which—in the Type II ratio simulations—succeeded in producing a path of alternating activity with duration and timing patterns remarkably close to those in the observed data.

We are as certain as we can be that throughout the past twenty-five years, expectations changed markedly at certain critical times: external factors of monetary policy, fiscal policy, and world-trade circumstances impinged on the economy; and production technology and demographic patterns changed substantially. And this is only an abbreviated list. Given the mildness of the business cycle over the same period, how can we expect the data to reveal anything other than a system which would be stable or damped in the absence of such factors?

In an obvious sense, this returns us to Frisch and his emphasis on external shocks. But more specifically, it suggests that the cycle itself arises *after* the economy has already been displaced from its normal path. The process by which the economy gropes its way back from unfamiliar events to a self-justifying set of decisions constitutes the cycle, as we know it. And that, in fact, returns us to Schumpeter. Maybe that is not such a bad place to be after all.