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Chapter Author: Michael K. Evans, Lawrence R. Klein, Mitsuo Saito, Michael D. McCarthy

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SHORT-RUN PREDICTION AND LONG-RUN SIMULATION OF THE WHARTON MODEL

MICHAEL K. EVANS · Chase Econometric Associates, Inc.
LAWRENCE R. KLEIN · Wharton School
MITSUO SAITO · Osaka University

ANALYSIS OF THE MODEL AT TURNING POINTS

THE version of the Wharton Model used for the sample-period simulations is the one contained in the first edition of the Wharton Econometric Forecasting Model; i.e., with the two-equation monetary sector. This model contains 47 stochastic equations and was estimated for the sample period 1948.1 to 1964.4, using two-stage least squares with twelve principal components. In performing these short-period simulations we used data including revisions of the July, 1967, national income accounts; the model was estimated with data revised through July, 1965. Thus, most of the 1963 and 1964 data are slightly changed between estimation and application; in addition, a few series were revised from earlier years. The general direction of the revision of the national income accounts since 1963 has been in an upward direction. Accordingly, the results of the performance of the model using different methods of constant adjustment are slightly biased against the no-adjustments forecasts ($b_k = 0$) at the 1966 peak. None of the other turning point comparisons is affected by the data revisions.

In using the Wharton-EFU Model for ex ante forecasting, we always adjust some of the constant terms of the stochastic equations to take into account revisions in the data, exogenous information not included in the equations (such as strikes), shifts in the institutional framework, or errors in the equations. However, for the sample period and ex post forecasts summarized here, no such adjustments were

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made. The only change from the published version of the model was the substitution of different tax and transfer equations during periods of divergent tax laws. These equations, used for the complete sample, are as follows:

Direct Business Taxes (T_b)

$$T_b = 14.02 + 0.0250NI + 0.3949t \qquad 1948.1\text{–}1953.4$$
$$T_b = -2.90 + 0.0721NI + 0.3839t \qquad 1954.1\text{–}1965.1$$
$$T_b = -3.40 + 0.0721NI + 0.3839t \qquad 1965.2$$
$$T_b = -4.65 + 0.0721NI + 0.3839t \qquad 1965.3\text{–}1967.4$$

Corporate Income Taxes (T_c)

$$T_c = -1.38 + 0.40(P_{cb} - IVA) \qquad 1948.1\text{–}1949.4$$
$$T_c = -1.38 + 0.45(P_{cb} - IVA) \qquad 1950.1\text{–}1950.4$$
$$T_c = -1.58 + 0.54(P_{cb} - IVA) \qquad 1951.1\text{–}1953.4$$
$$T_c = -2.25 + 0.50(P_{cb} - IVA) \qquad 1954.1\text{–}1961.4$$
$$T_c = -3.40 + 0.50(P_{cb} - IVA) \qquad 1962.1\text{–}1963.4$$
$$T_c = -3.50 + 0.48(P_{cb} - IVA) \qquad 1964.1\text{–}1967.4$$

Transfer Payments (T_r)

$$T_r = 9.84 + 0.400N_{Un} + 0.1369t \qquad 1948.1\text{–}1949.4,\ 1950.3\text{–}1953.4$$
$$T_r = 18.54 + 0.400N_{Un} + 0.1369t \qquad 1950.1$$
$$T_r = 12.54 + 0.400N_{Un} + 0.1369t \qquad 1950.2$$
$$T_r = -2.95 + 1.565N_{Un} + 0.5069t \qquad 1954.1\text{–}1965.2$$
$$T_r = -0.75 + 1.565N_{Un} + 0.5069t \qquad 1965.3\text{–}1966.2$$
$$T_r = 0.25 + 1.565N_{Un} + 0.5069t \qquad 1966.3$$
$$T_r = 3.25 + 1.565N_{Un} + 0.5069t \qquad 1966.4$$
$$T_r = 6.55 + 1.565N_{Un} + 0.5069t \qquad 1967.1\text{–}1967.4$$

Personal Income Tax (T_p)

$$T_p = -4.41 + .142(PI + SCI - T_r) \qquad 1948.1$$
$$T_p = -3.92 + .125(PI + SCI - T_r) \qquad 1948.2$$
$$T_p = -3.53 + .113(PI + SCI - T_r) \qquad 1948.3\text{–}1950.4$$
$$T_p = -22.26 + .210(PI + SCI - T_r) \qquad 1951.1\text{–}1953.4$$
$$T_p = -16.91 + .176(PI + SCI - T_r) \qquad 1954.1\text{–}1963.4$$
$$T_p = -16.02 + .167(PI + SCI - T_r) \qquad 1964.1$$
$$T_p = -14.91 + .155(PI + SCI - T_r) \qquad 1964.2\text{–}1967.4$$

where:

NI = national income, billions of current dollars

t = time trend, 1948.1 = 1

P_{cb} = corporate profits before taxes, billions of current dollars

IVA = inventory valuation adjustment, billions of current dollars

N_{Un} = number of unemployed, millions

PI = personal income, billions of current dollars

SCI = social insurance contributions by individuals, billions of current dollars

Several different methods were used to adjust the constant terms of the stochastic equations. In all cases, the rules were mechanical and no attempt was made to incorporate extraneous information, as is done in the actual ex ante forecasts. If the single-equation residual of the kth equation at time t is denoted by r_{kt}, then calculate the regression

$$r_{kt} = \rho_k r_{k,t-1} + u_{kt} \qquad k = 1, \ldots, 47$$

If the estimates of ρ_k are denoted by $\hat{\rho}_k$, then the constant adjustments were calculated in the following manner:

	Shorthand notation
(1) $r_{k,t+i} = 0$	$b_k = 0$
(2) $r_{k,t+i} = r_{kt}$	$b_k = 1$
(3) $r_{k,t+i} = (\hat{\rho}_k)^i r_{kt}$	$b_k = \hat{\rho}_k$
(4) $r_{k,t+i} = (\hat{\rho}_k)^i r_{kt}$ for $i = 1,2$; $r_{k,t+i} = 0$ for $i = 3, \ldots, 6$	$b_k = \hat{\rho}_k, 0$
(5) $r_{k,t+i} = r_{kt}$ for $i = 1,2$; $r_{k,t+i} = (\hat{\rho}_k)^i r_{k,t+i}$ for $i = 3, \ldots, 6$	$b_k = 1, \hat{\rho}_k$

The first two methods state that no constant adjustment was made at all, or that it was equal to the previous period's residual, respectively. Case (2) is, of course, the assumption made implicitly in first-difference forecasting for linear systems. Case (3) stems from a suggestion made by Goldberger;[1] it assumes that the serial correlation present in

[1] A. S. Goldberger, "Best Linear Unbiased Prediction in the Generalized Linear Regression Model," *Journal of the American Statistical Association*, Vol. 57, No. 2 (June, 1962), pp. 369–375.

the sample period will continue into the forecast period. Cases (4) and (5) were added to test the possibility that though errors for the first two periods were rather large, the model tends to get back on the track for spans of three periods or longer. They represent the natural extension of methods (1)-(3).

We now turn to the actual performance of the model at the turning points. If the figures for constant-dollar *GNP* (symbol X) are compared with the summary statistics given in Evans, Haitovsky, and Treyz, Table 3, it becomes apparent that when no mechanical constant adjustments are used, the model performs much more poorly at turning points than it does for the average of the sample period. The sample-period statistics for the period 1953.1–1964.4 show that with no constant adjustments, the average absolute errors of X for 1, 2, and 3 quarters from solution starting point are \$6.9, \$7.6, and \$7.3 billion, respectively. At the turning points, the figures are \$9.9, \$10.3, and \$10.2 billion. Furthermore, even if the "best" method of adjustment, $b_k = \hat{\rho}_k$, is used, the errors are still \$9.8, \$9.9, and \$9.9 billion, respectively.

In the post-sample period only one turning point observation is available, that of 1966.4. The errors in the ex post forecast of X for that quarter – 1, 2, and 3 periods ahead – are \$18.5, \$18.6, and \$18.6 billion; the comparable errors for the 1965.1–1967.4 period are \$8.5, \$9.8, and \$12.9 billion, respectively. In contrast, the ex ante forecasting record of the various Wharton models for the period 1965.1–1967.4 shows errors of \$2.8, \$4.9, and \$5.2 billion, respectively.[2] A further examination of the relevant tables reveals that this comparison is not limited to X but, in fact, extends to almost all of the components of aggregate demand and supply catalogued in these tables.

We now examine the individual tables for the various methods in order to see whether a particular method of constant adjustment yields improved results at peaks compared to troughs, or for certain variables. The over-all summary statistics are given in Tables 1–3. These tables show the number of times a given adjustment method (column designation) makes the best forecasts for the 17 variables being predicted; these 17 variables are the ones being studied for all the models at this

[2] This comparison is examined further in Evans, Haitovsky, and Treyz.

TABLE 1

Number of Best Forecasts for Different Adjustment Methods: 17 Variables (One Quarter Before Turning Points)

b_k	0	1	$\hat{\rho}_k$	$\hat{\rho}_k$,0	1,$\hat{\rho}_k$
All turning points	32	34	31	17	22
1949 T	2	6	0	5	4
1953	2	1	11	1	2
1954 T	6	1	6	2	2
1957	1	13	3	0	0
1958 T	4	1	2	4	6
1960	10	3	1	2	1
1961 T	7	1	6	1	2
1966	0	8	2	2	5
Troughs	19	9	14	12	14
Peaks	13	25	17	5	8

TABLE 2

Number of Best Forecasts for Different Adjustment Methods: 17 Variables (Two Quarters Before Turning Points)

b_k	0	1	$\hat{\rho}_k$	$\hat{\rho}_k$,0	1,$\hat{\rho}_k$
All turning points	29	19	48	21	19
1949 T	3	1	4	6	3
1953	6	3	7	0	1
1954 T	1	3	9	2	2
1957	2	1	9	3	2
1958 T	11	2	2	1	1
1960	4	0	7	3	3
1961 T	1	1	9	5	1
1966	1	8	1	1	6
Troughs	16	7	24	14	7
Peaks	13	12	24	7	12

TABLE 3

Number of Best Forecasts for Different Adjustment Methods: 17 Variables (Three Quarters Before Turning Points)

b_k	0	1	$\hat{\rho}_k$	$\hat{\rho}_k$,0	1,$\hat{\rho}_k$
All turning points	28	28	45	13	22
1949 T	2	2	12	1	0
1953	3	3	11	0	0
1954 T	2	3	11	1	0
1957	8	2	2	2	3
1958 T	6	5	1	3	2
1960	1	2	5	2	7
1961 T	4	5	2	2	4
1966	2	6	1	2	6
Troughs	14	15	26	7	6
Peaks	14	13	19	6	16

Conference.[3] The scores are given for each turning point (row designation). For the entire sample and ex post forecast period (hereafter referred to as the extended sample period), the methods $b_k = 0$, 1, or $\hat{\rho}_k$ appear to be roughly equal in their predictive efficacy and somewhat superior to methods 4 or 5. It might seem that the $b_k = \hat{\rho}_k$ method is somewhat superior for the forecasts 2 and 3 quarters ahead. However, its superiority is registered only for the first three turning points, which fall before, and immediately after, the Korean War. Since many of the equations of the latest version of the Wharton-EFU Model are estimated for the post-Korean War period alone, it might be reasonable to focus our attention on the post-1954 results.

For that period, it would seem that there is little difference between the first three methods, and that, in fact, only the fourth ($b_k = \hat{\rho}_k$,0) is definitely inferior. It is hard to apply much meaning to standard statistical tests for five observations; the statistics, in any case, show no significant deviation from normality in distributions of any of the summary totals. Similarly, categorizing the methods into performance at peaks and troughs does not yield clear-cut superiority

[3] Six-quarter simulations, starting 1, 2, and 3 quarters before each turning point, were made for each peak or trough. The "best" forecast is the one with the lowest root-mean-squared error in the turning point calculations.

for any of the methods. For the 1-quarter solutions, it appears that $b_k = 1$ does better at the peaks, but this is not supported by the other quarters. Similarly, the $1,\hat{\rho}_k$ method does well at troughs for the 1-quarter solutions but performs poorly for 2 and 3 quarters ahead. The tendency for the "best" method to shift over time is shown in Table 4, where it can be seen that the method $b_k = \hat{\rho}_k$ does better for almost every variable during the first three turning points but is somewhat worse than the no-adjustment assumption for the rest of the sample, and than the $b_k = 1$ assumption for the extended sample. This would suggest that choosing a given method on the basis of extended-sample-period results might lead to some difficulties when applying the material to ex ante forecasting. If there is any conclusion to be drawn from these results, it is that for the methods tried, no one mechanical method seemed to have much to recommend it over any other.

It should be stressed that these results are based on complete system solutions rather than on single-equation results. Thus, the large error for, say, consumption, might be due not to large errors in the consumption function itself but to poor predictions of disposable income. In the limiting case, there is no stochastic equation for X at all, since it

TABLE 4

Aggregation of Best Forecasts for All Three Periods Before Turning Points

b_k	0	1	$\hat{\rho}_k$	$\hat{\rho}_k$,0	1,$\hat{\rho}_k$
All turning points	89	81	124	51	63
First four turning points	38	39	85	23	19
Second four turning points	51	42	39	28	44
First three turning points	27	23	71	18	14
Next four turning points	59	36	49	28	32
Last turning point	3	22	4	5	17

NOTE: Two or more different methods are assumed to have done equally well if the difference in the error is less than 0.1 per cent of the actual value. Ties were calculated as 1/2, 1/3, 1/4, or 1/5 of a point, but in these tables, all results have been rounded to the nearest integer.

is merely identical to the sum of all components of aggregate demand. Yet, it may still be of interest to compare the size of the errors for individual variables, using different adjustment methods. In the first place, the statistics of Tables 1–4 may obscure the results for over-all important summary variables, such as output, prices, or unemployment. Second, some equations, such as those for interest rates or net foreign balance, are not so closely tied to the over-all structure; thus we might be able to improve over-all forecasts by adjusting the equations in these semi-exogenous sectors with a method different from that used in other equations. The evidence for this suggestion is given in Table 5. In order to conserve space, we list only the combined statistics for 1–3 periods ahead of each turning point. The results are not substantially changed by this aggregation.

The results presented in Table 5 suggest that for over-all summary variables (personal income, *GNP*, and unemployment) the method

TABLE 5

Best Adjustment Method for Each of 17 Variables: All Turning Points (All Three Periods Before Turning Points)

b_k	0	1	$\hat{\rho}_k$	$\hat{\rho}_k$,0	1,$\hat{\rho}_k$
i_s	7	7	4	4	2
i_L	6	7	8	1	2
I_p	5	4	9	1	5
p	4	7	6	5	2
U	10	2	6	4	2
I_h	2	3	4	13	2
PI	4	5	13	0	2
P_{cb}	5	4	8	2	5
$GNP\$$	7	2	10	1	4
X	7	3	7	2	5
Un	2	2	8	3	9
C	12	4	3	5	0
ΔI_i	4	4	9	3	4
B	1	8	9	1	5
E	3	6	8	3	4
H	6	5	6	1	6
w_r	4	8	6	2	4
	89	81	124	51	63

$b_k = \hat{\rho}_k$ gives somewhat superior results. However, among individual components, some of the other methods do much better. For example, unfilled orders – which category contains a large exogenous component determined mainly by spending needs of the military – is predicted best by using $b_k = 0$, and worst by $b_k = 1$, or $b_k = 1, \hat{\rho}_k$. Somewhat more surprising is the fact that consumption seems to be predicted best with $b_k = 0$, even though it is a relatively smooth series, and income is predicted best by the method $b_k = \hat{\rho}_k$. The $b_k = \hat{\rho}_k, 0$ method works poorly for all variables except residential construction, and this may be due to the built-in serial correlation for two periods in that series, caused by the method OBE uses to convert housing starts to actual investment.

It should be pointed out that while $b_k = \hat{\rho}_k$ does seem to perform better for the extended sample period, problems mentioned at the beginning of this section (such as data revision) may mean that methods closer to $b_k = 1$ could prove to be more reasonable. However, these results tentatively suggest that for those equations where there have been no noticeable shifts in data or structure, an adjustment based on the auto-correlation coefficients of the individual equations may improve forecast accuracy.

LONG-RUN SIMULATIONS

ALTHOUGH the Wharton-EFU Model is primarily designed for short-run forecasting (as are the other models being considered at this Conference), and is primarily subjected to the tests or applications considered in the previous section, it is both worthwhile and interesting to simulate the model over longer periods of time.

In short-run testing, both ex post and ex ante forecasts have been considered. For the longer-run simulations, we have, in a sense, made ex post forecasts by simulating the model over the extended-sample period. In this case, we have actual performance data with which to check the model results. Equally interesting from the viewpoint of business-cycle analysis, however, is the simulation of the model over a hypothetical stretch of future time. Although it would be possible to view this long-run simulation as an ex ante forecast, we would prefer to regard it purely as a hypothetical simulation for the purpose of studying cyclical response characteristics of the system, inasmuch as the exogenous input was not carefully considered for true prediction purposes. It was simply extrapolated along reasonable trend paths from its own history, or so as to smooth approaches to targets for endogenous variables.

The major difference, of course, between the long- and short-run extrapolations is in the treatment of initial conditions. The short-run extrapolations (either ex post or ex ante) are *re-initialized* before every six- or eight-quarter extrapolation. Lagged values of endogenous variables are set at observed levels (ex post) or most recently observed levels (ex ante). Exogenous variables are retrospectively put at observed levels, and prospectively put at levels determined by judgments of future developments.

For the longer-run simulation over the historical sample period, lagged values are set initially at conditions prevailing before 1948.3, and are not adjusted during the course of the simulation exercise. Lagged inputs for subsequent periods are developed as needed by the solution of the system. As in the case of the short-run solutions over the sample period, exogenous variables are assigned observed values. Pre-1948.3 variables and exogenous variables are given; the model accounts for the rest of the solution. The end of the sample occurs in 1964.4, but the solution is extended until 1968.1.

The other longer-run simulation begins in 1968.3 and runs forward for one-hundred quarters. It is entirely outside sample experience, and is largely in the future, as we perceive it now. This solution is programmed for realistic initial (lagged) inputs as of 1968.3, and starts with initial exogenous variables that have realistic values. For the rest, exogenous inputs, mainly reflecting fiscal and monetary policy of central authorities, are fixed at values that attempt to keep the economy on a long-run growth path of approximately 4 per cent unemployment, with interest rates between 4 and 5 per cent.

Two special situations must be dealt with in the longer-run simulations. Over the sample period, there must be an attempt to deal with the dislocations caused by the Korean War; while over the future period, there must be some final settlement of the Vietnam War. Both these wars are major economic disturbances. To some extent, the

Korean episode is accounted for by special variables in the model, introduced for estimation purposes; but these are inadequate to handle the extreme movements that occurred in import demand and inventory investment. Stockpiling of basic materials and speculation distorted these magnitudes. We have, accordingly, adjusted inventory and trade equations upward and downward at strategic quarters, in order to account for the largest disturbances appearing as equation residuals. There is a similar adjustment needed for the export equation in late 1949 and 1950, because of the ineffectiveness of the devaluation of September, 1949.[4] There are no other adjustments to the model equations for the sample-period simulation, except to reflect changes in tax-transfer laws; or where explicitly introduced, through dummy variables that are listed with the model estimates.

The economic implications of a Vietnam settlement are more problematical. We have made the following assumptions: (1) A cease-fire and demobilization would begin in 1970.1. Over a period of six quarters, the military establishment would be cut back by 350,000 men. Military spending would be reduced by $11.1 billion at 1958 prices, spread over six quarters. Taxes would be reduced by the ending of the surcharge. Correspondingly, civilian expenditures would gradually increase, so that total spending falls only slightly for two quarters in real terms and never drops in current prices. This fiscal policy counteracts the decline in military expenditures, and monetary policy becomes easier through a drop in the discount rate of 0.5 percentage points. Net free reserves are held steady at $200 million.

The outcome produced by these assumptions is a pause in growth during the 1970 transition phase, allowing unemployment to reach 5.5 per cent, but an actual recession, such as the one that occurred after the Korean War, does not take place. The dimensions of the demobilization and peace settlement are nearly as great in absolute (not in percentage) terms as those that followed Korea, but it is assumed that wise government policy will enable us to avoid a similar recession. The temporary rise in unemployment is quickly corrected. Then there is a

[4] Our export equation, through relative price effects, suggests that U.S. exports should have dropped considerably after the devaluation, but this did not happen, because European nations could not increase supply at that time, and because U.S. exports were being used to reconstruct Europe.

movement toward the long-run growth trajectory of the economy. Steady taxes, steady monetary controls, growth in government spending, normal population growth, and normal growth in world trade all make a long-run growth in real *GNP* that keeps the economy at an unemployment rate of approximately 4 per cent.

In order to produce this result, however, we must go beyond our usual range of short-run assumptions about productivity, labor force growth, and length of the work week. Trends are introduced in the equations associated with these variables so that the unemployment rate does not fall below 3.77 per cent. Some of the equations explaining hours worked, or labor-force participation, would not produce reasonable long-term results unless they were adjusted throughout the solution period. The hours equations depend negatively on the wage rate, which rises steadily over the rest of the century. These equations are short-run equations in the model, and are not particularly well suited to such long-run exercises. The number of self-employed, farm and nonfarm, must be placed on a long-run time path (exogenously) so as to yield the desired unemployment rate. These are difficulties that arise in programming a short-run model for a long-run study. It is by no means a simple mechanical exercise.

These considerations are relevant for the deterministic solutions. In the hypothetical future simulation, we regard the deterministic case as a *base-line* solution. We then produced fifty replications of stochastic versions of this solution. Each equation of the model is written as

$$y_{it} = g_i(y_{1,t}, \ldots, y_{nt} y_{1,t-1}, \ldots, y_{n,t-p} x_{1t}, \ldots, x_{mt}) + e_{1t}$$

$$i = 1, 2, \ldots, n$$

There are n dependent (endogenous) variables and m independent (exogenous) variables. Lags of up to the pth order occur in the dependent variables. The system is written in a somewhat arbitrary way, with one (different) dependent variable isolated on the left-hand side. In general, the equations are nonlinear and are specified by parameters that have been estimated from sample data. The expected value of each e_{it} is zero, and the deterministic solutions are obtained by using point estimates of the parameters of the g_i functions, together with zero values for e_{it}. In stochastic solutions, we substitute random drawings for each of the e_{it}. The random numbers are normally distributed variates that have the same variance-covariance matrix as the sample residuals. This variance-covariance matrix is an estimate of

$$\Sigma = [Ee_{it}e_{jt}]$$

The method of drawing the random numbers is that suggested by Michael D. McCarthy.[5] It consists of forming the matrix

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1G} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ r_{T1} & \cdots & r_{TG} \end{bmatrix}$$

of residuals in each of G equations for a sample period of length T. Thus, each column of R is a T-element vector of residuals from one of the structural equations. The next step is to draw random numbers from a normal distribution with zero mean and unit variance.

$$N = \begin{bmatrix} n_{11} & \cdots & n_{1T} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ n_{S1} & \cdots & n_{ST} \end{bmatrix}$$

Each row of N is a vector of independent unit normal deviates of length equal to the sample span, T. There are as many rows in N as there are future periods of simulation. Thus we provide for one-hundred periods of stochastic simulation. The matrix product

$$V = \frac{1}{\sqrt{T}} NR$$

provides an $S \times G$ matrix of disturbances that have the same variance-covariance matrix as the sample estimate of Σ. This is the McCarthy

[5] See the Appendix by Michael D. McCarthy, pp. 185–191.

technique. It is essentially a scrambling of the sample residuals that preserves their variance-covariance matrix.

The random numbers in N are taken from the Rand Corporation table of random numbers.[6] In the fifty replications of this procedure, we enter the table (or tape) each time at a random position and draw ST successive numbers. For the Wharton-EFU Model, $G = 51$, and $T = 44$. We do not use the whole sample length, since some of the equations introduced at a later stage—in model estimation of an enlarged financial sector—were based on a shorter sample, from 1954.1 to 1964.4.[7]

Some of the equations are identities, and we did not shock these equations in the stochastic simulations. We shocked only the 51 behavioral, institutional, and technological equations.

Another detail requires explanation. In some of the equations, different normalizations were used for estimation and for solution. In the equation for consumer expenditures on nondurables and services, C_{ns}/Y is the normalized variable for *estimation*, while C_{ns} is the normalized variable for *solution*. Similarly, in the production functions, we normalized on $\log X_m$, or $\log X_n$, for estimation, but on N_m, or N_n, for solution. Since our solution program is written in such a way that it is easy to perturb, period by period, the constant term of each equation in the form that is normalized for solution, we have not shocked each equation in exactly the form that we assume random components for estimation theory. This has the effect of introducing some heteroscedasticity into the stochastic simulations.

If the random numbers are independently drawn, the successive elements within columns of V will be independent

$$Ev_{ti}v_{t-j,i} = 0 \qquad j \neq 0$$

The shock procedure preserves variances and unlagged correlations across equation residuals; it does not preserve serial properties, either within, or across, equation residuals. McCarthy has shown how serial properties can be preserved if we modify his procedure as follows: the rows of N should not be independent in this case. The first row will

[6] Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates* (Glencoe, The Free Press, 1955).

[7] In the short-run solutions, the revised and extended monetary equations were not used, as explained in the previous section.

consist of a T-element series of independent unit normal deviates. The second row will consist of the first T-1 elements of the first row shifted one place to the right, and a new independent drawing will be made for the vacant first position. The third row will consist of the first T-1 elements of the second row, shifted one place to the right and an independent drawing, and so on. The whole matrix will be

$$N^* = \begin{pmatrix} n_1 & n_2 & n_3 & \cdots & n_T \\ n_{T+1} & n_1 & n_2 & & n_{T-1} \\ n_{t+2} & n_{T+1} & n_1 & & n_{T-2} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ n_{T+S-1} & n_{T+S-2} & n_{T+S-3} & \cdots & n_S \end{pmatrix}$$

In this case

$$V^* = \frac{1}{\sqrt{T}} N^* R$$

will have the expected variances, covariances, and lag correlations (within and between disturbance series) equal to corresponding sample values obtained from the residual matrix.

Our stochastic simulation differs from the well-known stochastic simulation of the Klein-Goldberger Model by I. and F. Adelman in the following four respects:[8]

(*i*) Our simulations are quarterly; theirs are annual.
(*ii*) Our covariance matrix of errors has nonzero (sample) covariances; theirs has zero covariances.
(*iii*) Our covariance matrix of errors permits nonzero serial correlations; their errors are serially uncorrelated.
(*iv*) Our stochastic simulations are replicated (fifty times); theirs is a single run.

[8] Irma Adelman and Frank L. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica*, 27 (October, 1959), pp. 596–625.

DISCUSSION OF THE RESULTS

The first simulations from the longer-run solutions are like those from the first short-run solutions. The historical simulations begin in 1948.3 and continue beyond the end of the sample (1964.4) until 1968.1. The results are graphed in Charts 1-8 for some leading variables. The downturn of 1953 and the recovery of 1954 provide the first relevant cyclical test period for the model, beginning from initial conditions in 1948.3. The recession-recovery period of 1953 is generally well represented by the model. In the case of 1957-58 and 1960-61, many of the relevant calculated series do not actually turn down and then turn up. At best, they slow down, or pause, in these recession phases. Cyclical performance, however, is mixed. Many of the estimated series are smoother than observed series. Major deviations (either shocks or cyclical swings) are often missed in amplitude, and sometimes in direction. The computed series show steady growth, right through periods of wide actual movement up and down.

Apart from the behavior at turning points, many of the series start from approximately the correct position in 1948.3 and end at the right value for 1964.4. Some residual variables, such as net exports and corporate profits, are exceptions. They start to drift substantially at the end of the sample run and continue on a divergent course. The correspondence for personal income, *GNP*, and other aggregates is remarkably good. Although the computed general price level does not rise fast enough during the 1960's, the estimate of real *GNP* turns out to have the general drift of the actual series. It rises more than actually occurred after 1960, but the price deflator rises a few points below observed index values. These compensating errors leave *GNP* in current prices on the right trend path for the whole extended sample period. Although *GNP* and other very broad aggregates are projected fairly closely throughout the historical simulation, some of the components show bigger discrepancies. Total consumer expenditures, for example, are biased upward for the whole calculation.

Associated with the under-prediction of inflation is a low projection of wage rates. At the end of the sample and beyond, computed wage rates are below actual wage values. On the other hand, interest

CHART 1

GNP (Current Prices)

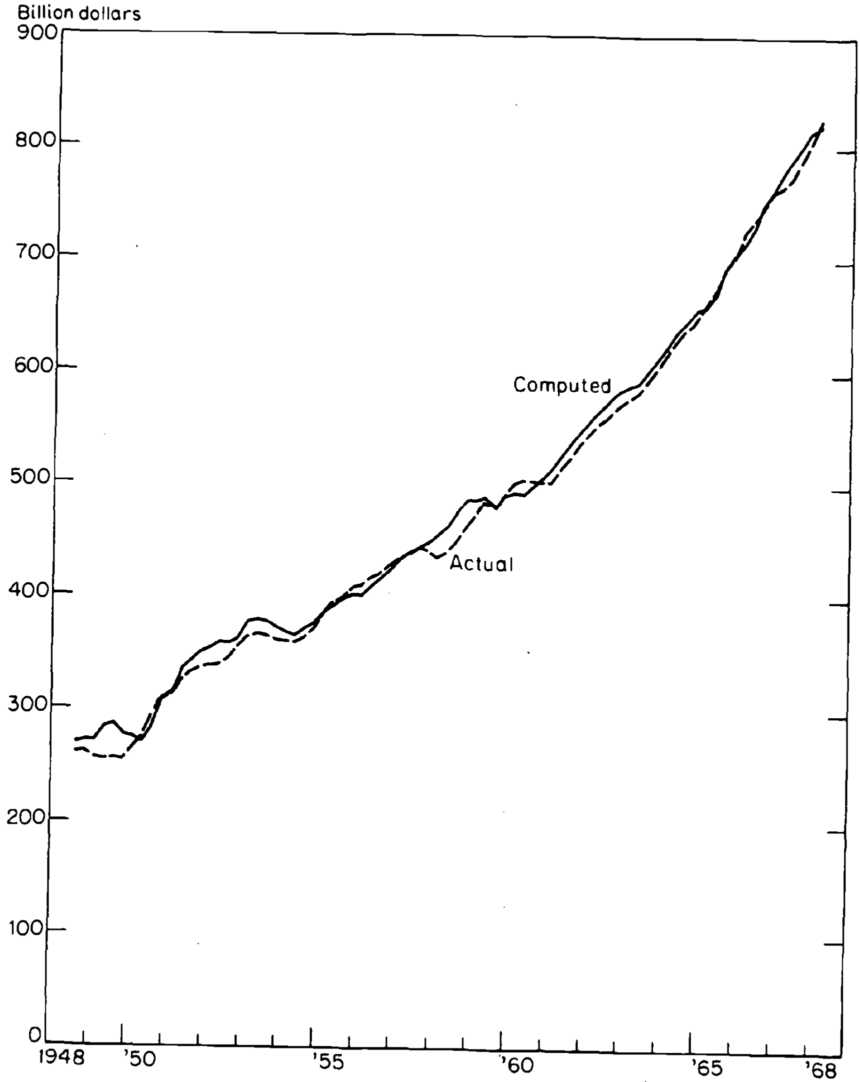
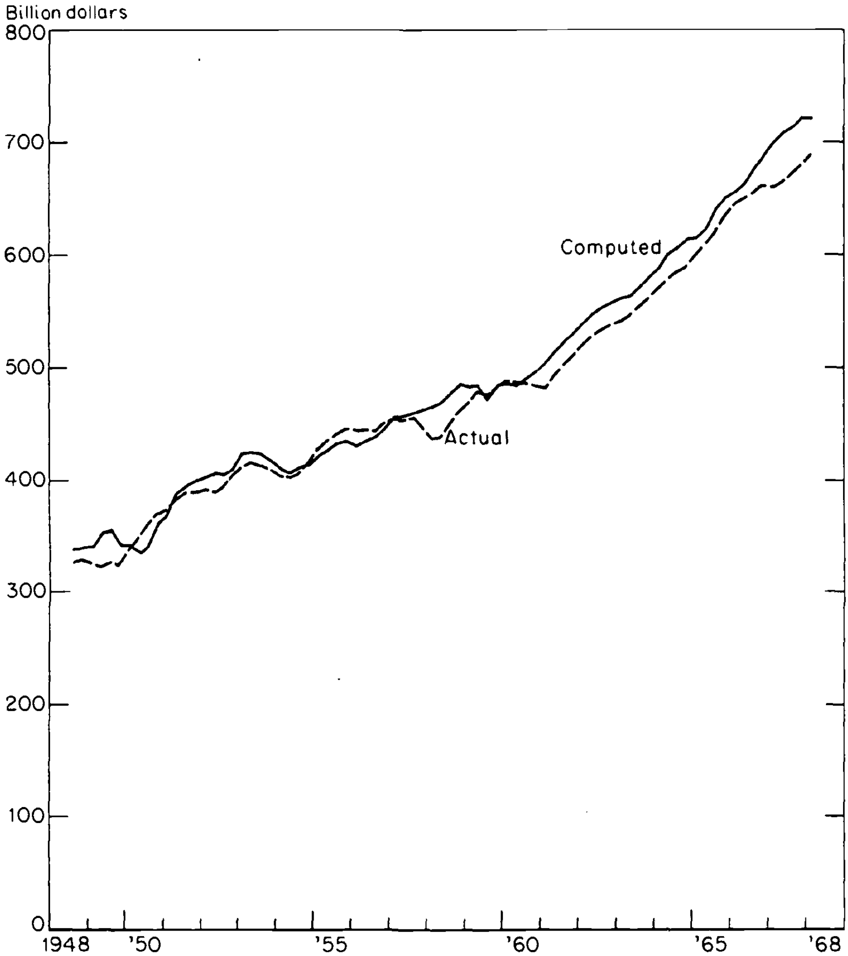


CHART 2

GNP (1958 Prices)



rates, which are strongly influenced by exogenous variables, are closely projected for the entire period.

The correspondence is closer for most series during the sample period than in the ex post extrapolations from 1964.4 to 1968.1. It should be remarked that the better extrapolations in the sample period are not based on a resetting of the initial conditions, and that many of the basic series have undergone extensive revision for the period since 1964.4. Some of the series have been revised for the last three sample-years, as well.

Noteworthy features of the longer-run simulation starting from 1968.3 are the persistent, steady growth of the economy (real *GNP* grows from $703 billion in 1968.3 to $1,663 billion in 1993.2), just under 4 per cent annually; the divergent movement of prices (consumer durable goods come down in price after about six years, while services grow on a steady uptrend); and the eventual transition from government deficits to surpluses during the final third of the calculation. The economy is not cycle-free in this period. Capacity utilization, unemployment, inventory change, and other strongly cyclical variables show much rhythmic variation, but aggregate output stays close to its trend growth path. The implied management of the economy is responsible for the better results.

The stochastic solutions do, however, exhibit discernible cyclical

CHART 3

P (*GNP Deflator*)

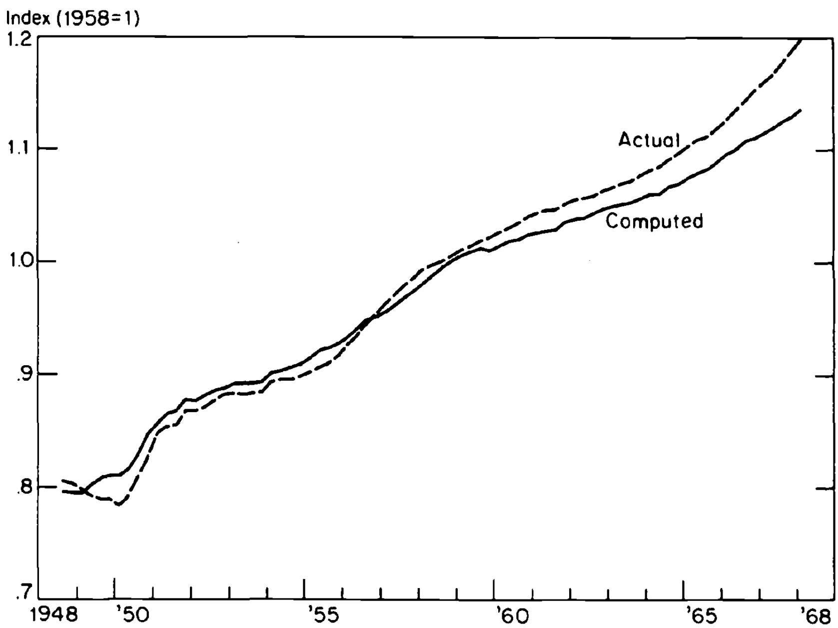
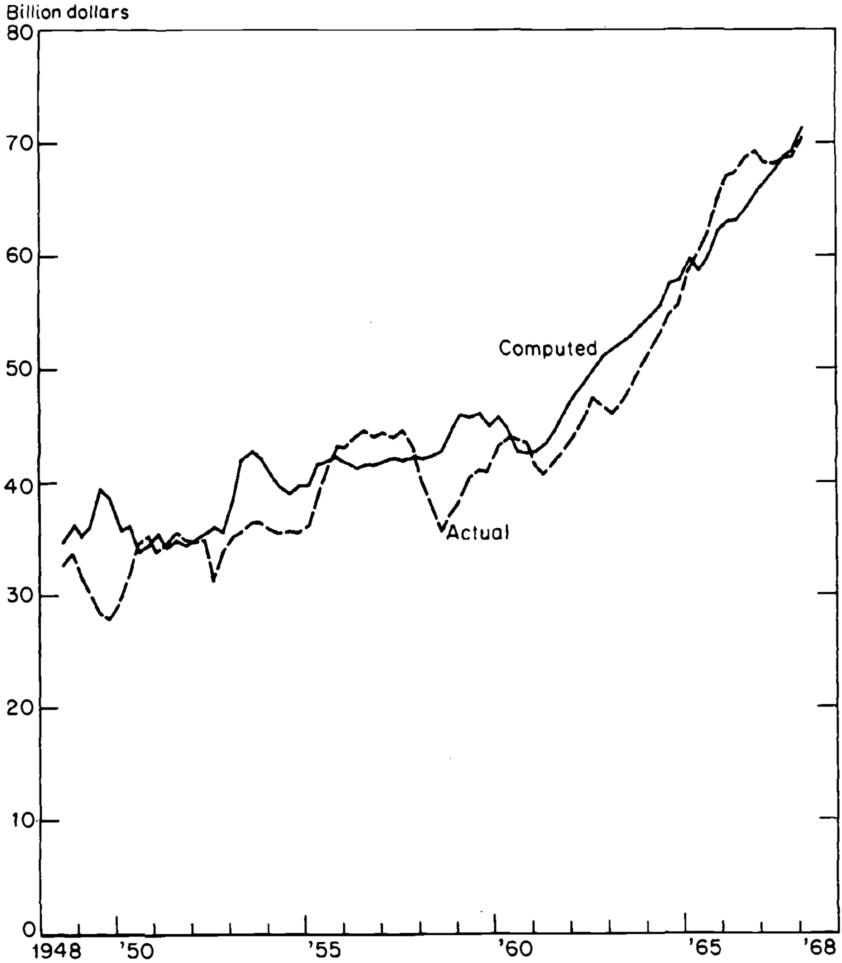


CHART 4

I_p *(Fixed Investment)*



behavior. In the fifty replications, there is much variability in cyclical performance, but the time chart of real *GNP* from a single one of the stochastic simulations of one-hundred quarters shows that there is cyclical movement (see Chart 9). This particular movement appears to oscillate about its trend approximately every ten quarters. This is a comparatively short cycle, but if the implied fiscal and monetary policies are being used to keep the economy on a 4 per cent unemployment course for twenty-five years, it is not unreasonable to find that the only evident cycle is the true inventory cycle of no more than two and a half years.

Two of the more cyclical variables of the model are the unemployment rate and residential construction expenditures; these are graphed in Charts 10 and 11. Even at the nonstochastic level, they show some cyclical variation, and this becomes more pronounced when random errors are introduced into the solution.

While the cyclical behavior of most variables in the system became more measurable in the stochastic, than in the nonstochastic, simulations, they seem to improve (in the sense of being more like the textbook case) when the random shocks to the system are programmed

CHART 5

I_s *(Short-Term Interest Rate)*

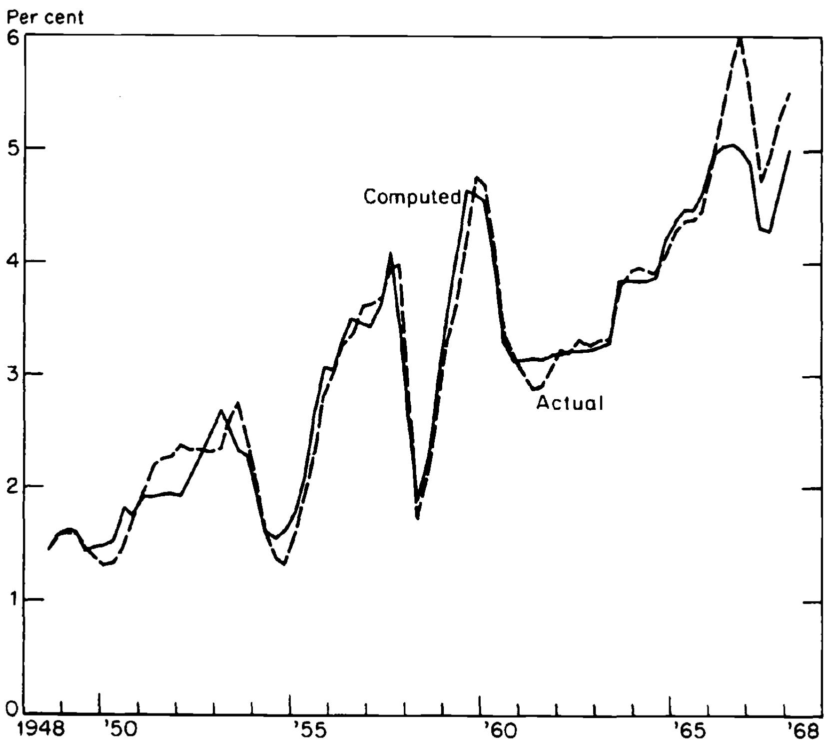


CHART 6

W_r (Wage Rate)

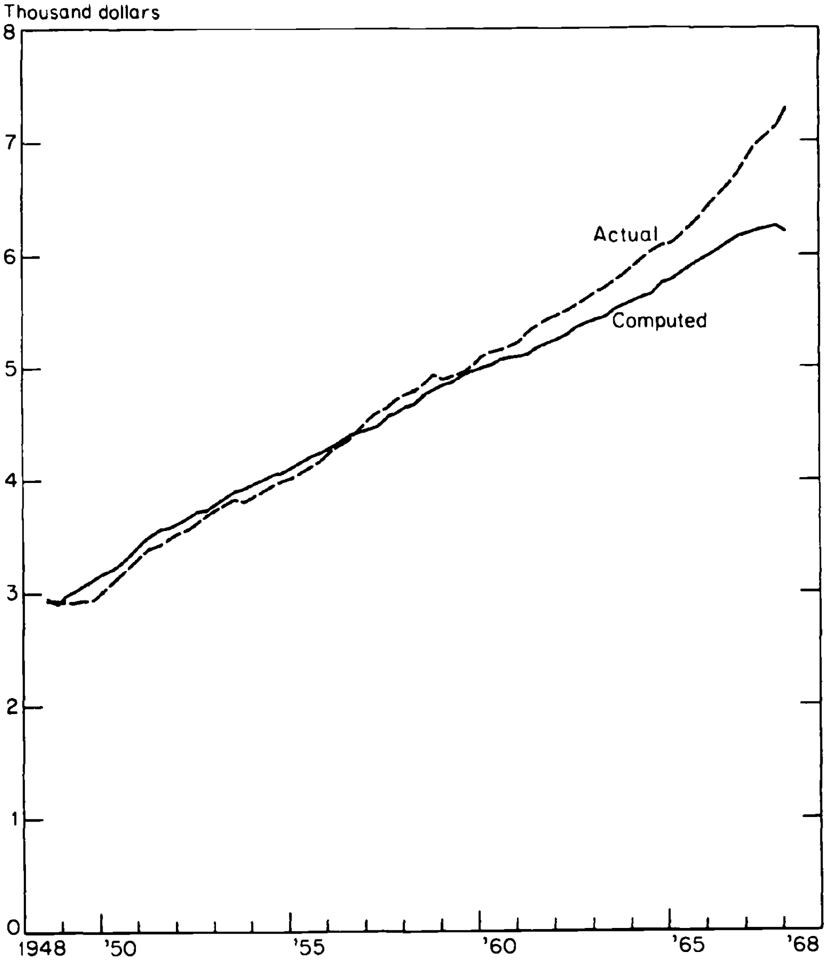


CHART 7

E (Employment)

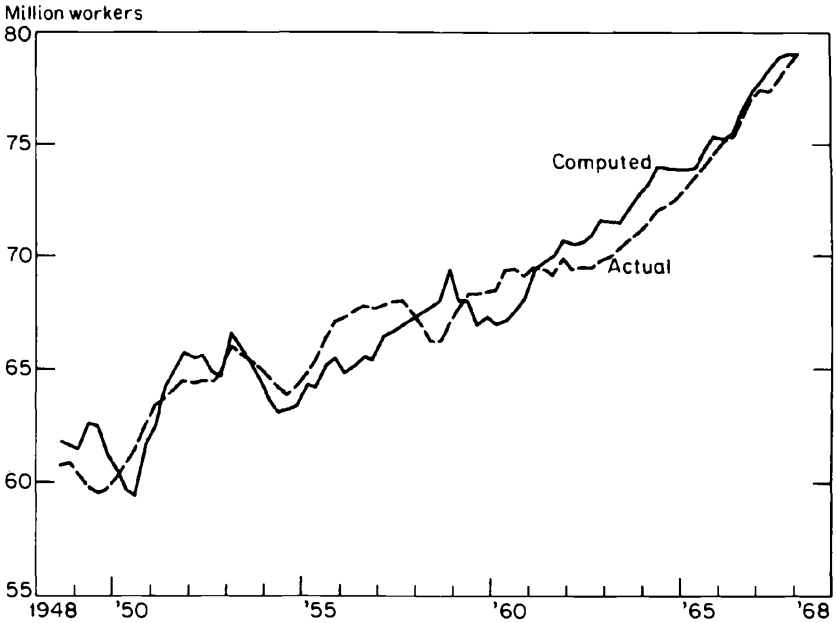


CHART 8

U_n (Unemployment Rate)

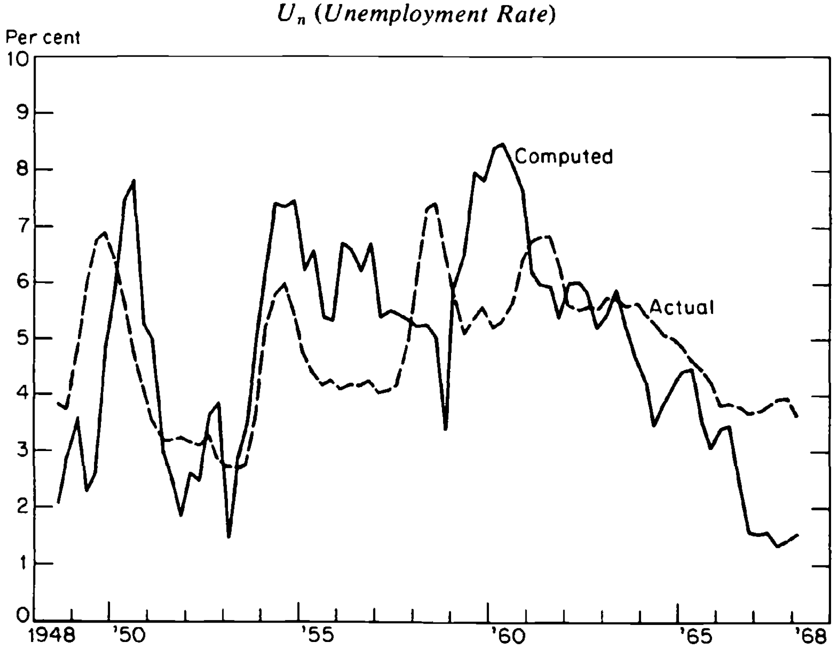
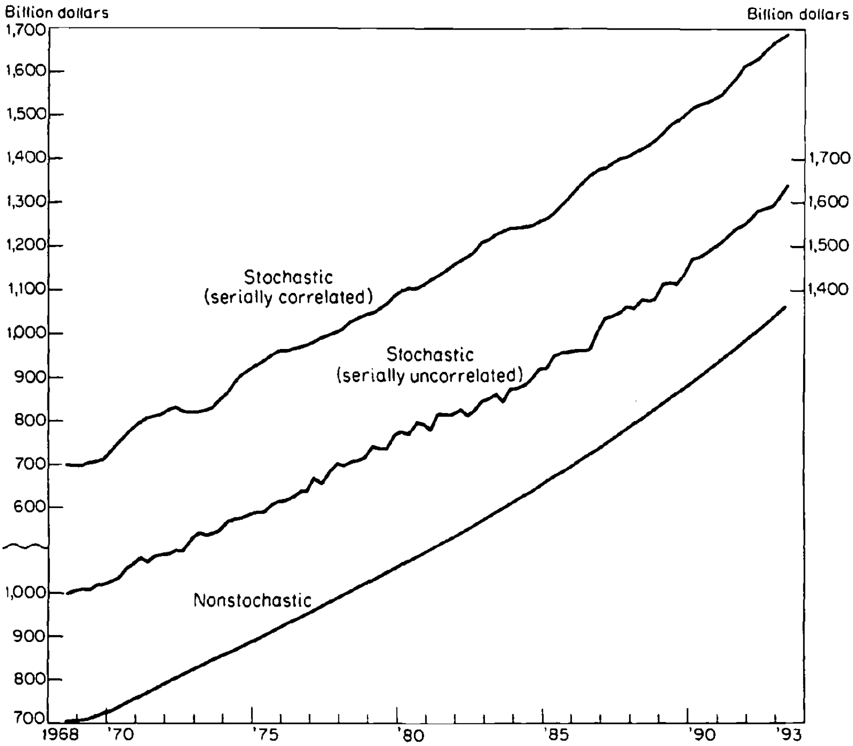


CHART 9

GNP: Stochastic Simulations (1958 Prices)



with the same serial dependence found in the sample residuals. For these variants of the stochastic simulations, the cycles are smoother and longer. In a corresponding simulation of one-hundred quarters, where we previously observed a ten-quarter cycle in real *GNP*, we now observe a more discernible cycle of close to sixteen quarters. This is the established average cycle-length put forward by the National Bureau of Economic Research.

Not only in the particular series plotted are smooth cycles exhibited in the serially correlated stochastic simulations, but throughout the whole model solution. The most satisfying simulation, from the point of view of business-cycle analysis, appears to be a stochastic simulation with serially dependent random errors.

A SPECTRAL ANALYSIS OF THE STOCHASTIC SIMULATIONS

A MORE careful analysis of cyclical characteristics can be evolved from the spectral density functions for such variables as real *GNP*, consumer expenditures, fixed capital formation, residential construction, inventory change, material imports, and the unemployment rate. For all fifty replications, we have made spectral analyses of leading variables, and will report, selectively, those listed above.

CHART 10

U_n *(Unemployment Rate)*

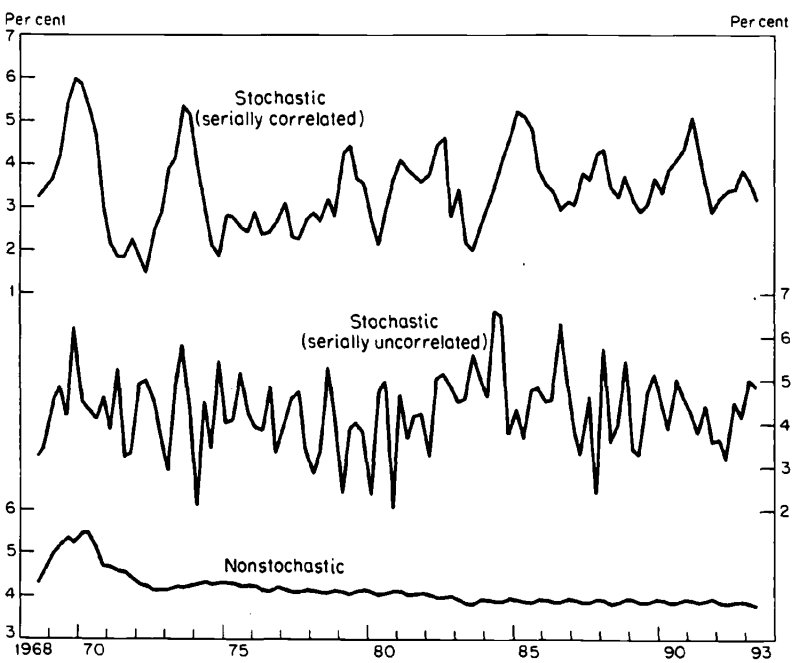
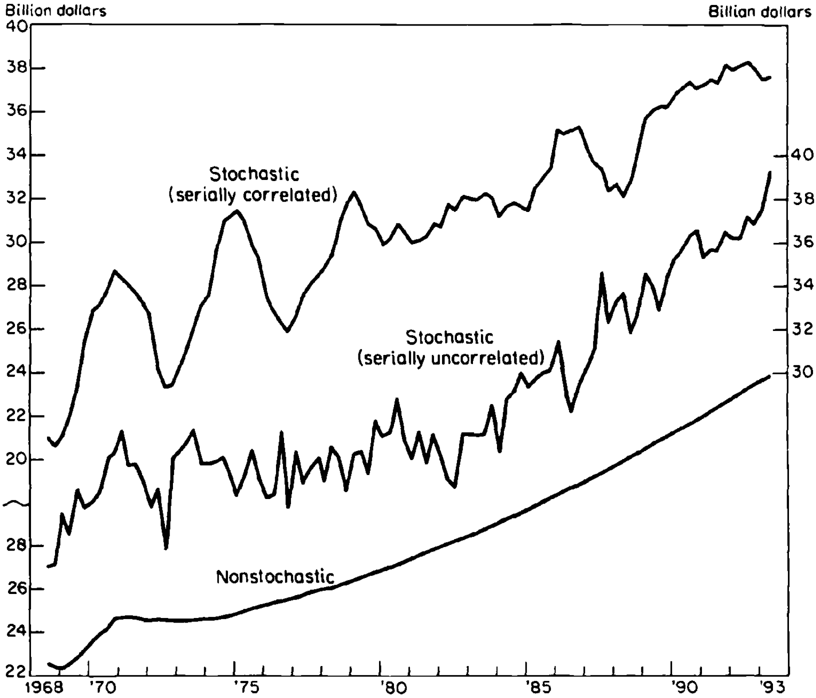


CHART 11

I_h (Residential Construction, 1958 Prices)



Before presenting the results, the procedure of calculation may be stated:

(1) All the spectra are estimated by

$$f(w_j) = \tfrac{1}{2}\lambda_0 R_0 + \sum_{k=1}^{m} \lambda_k R_k \cos(w_j k),$$

$$w_j = \frac{\pi j}{N} \qquad \text{and} \qquad j = 0,1, \ldots, N,$$

where R_k is the estimate of the auto-correlation coefficient, and λ_k is the weight of the filter using a Parzen window, i.e.,

$$\lambda_k = 1 - \frac{6k^2}{m^2}\left(1 - \frac{k}{m}\right), \qquad 0 \leqslant k \leqslant m/2,$$

$$= 2\left(1 - \frac{k}{m}\right)^3, \qquad m/2 \leqslant k \leqslant m$$

The m and N stand respectively for the number of lags and the number of points at which the spectrum is evaluated.

(2) The spectra are calculated for the deviations of the variables from fitted semilog trends, except for inventory investment. The presence of growth trends obscures the cyclical characteristics that would be revealed in the spectral density function. In the stochastic simulation, inventory investment sometimes takes a negative value; thus, an exponential trend cannot be estimated for this variable.

(3) The (nonstochastic) projected values of the first several quarters are obtained by taking into consideration the effect of demobilization, and provide relatively large fluctuations, compared with the values of the later period; therefore, the first ten quarters are dropped in the computation of the spectra. The spectrum is evaluated at 40 points.

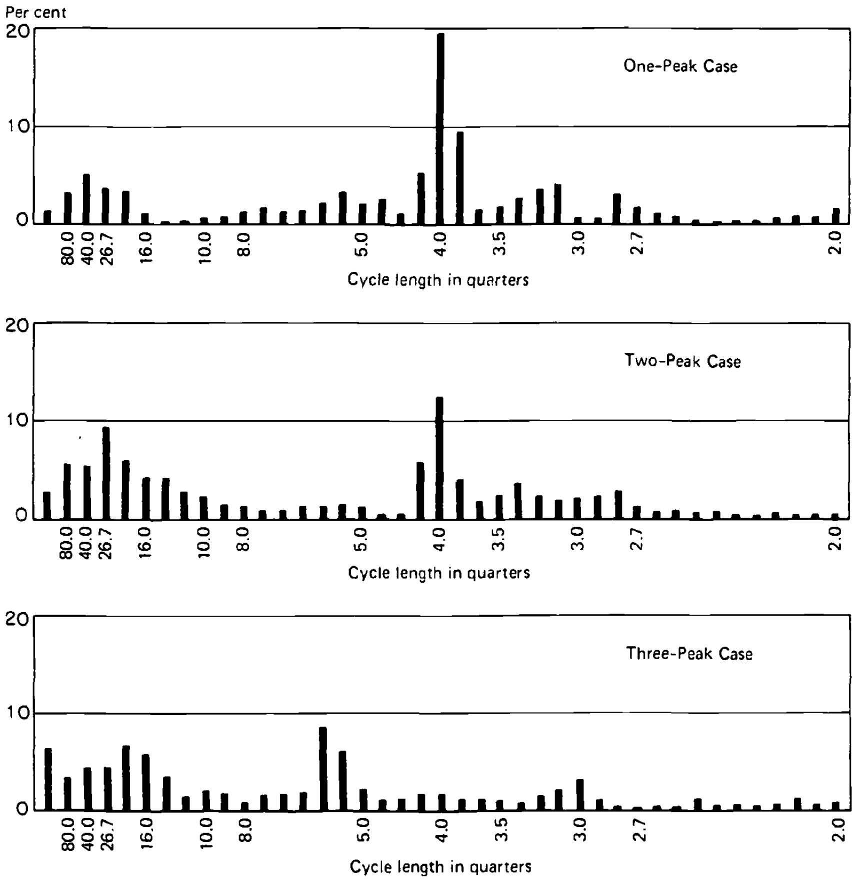
STOCHASTIC SIMULATION WITH SERIALLY UNCORRELATED ERROR TERMS

GNP IN CONSTANT DOLLARS (X)

Spectral density functions of real *GNP* in the fifty replications of stochastic simulations may be grouped by the number of peaks. For each case of one, two, and three peaks, typical examples of the spectral histogram are given in Chart 12, where the period is measured horizontally; and the percentage spectrum, vertically. In picking out peaks of the spectrum cycles, we may exclude periods in excess of forty quarters from consideration, since ninety observations are not enough to warrant definite decisions. With this consideration, the peak

CHART 12

Spectral Densities GNP (1958 Dollars)



of the first diagram occurs at 4.0 quarters only. Similarly, it is seen from the other two diagrams that peaks occur at 4.0 and 26.7 quarters in a two-peak example; and at 3.0, 5.7, and 20.0 quarters in a three-peak case. Although the procedure of counting the peaks is to some degree subjective, it is useful for describing the general features of the results. Table 6 presents the number of runs of each case. The two-peak case occurs in about 40 per cent of total runs; one- and three-peak cases each occur in about 20 per cent; while other cases occur in

TABLE 6

Frequency of Peaks
(Serially Uncorrelated)

Variable	Number of Peaks 0	1	2	3	Over 4
(1) X	6	10	19	10	5
(2) C	14	19	14	3	0
(3) I_p	19	15	11	5	0
(4) I_h	18	2	14	11	5
(5) ΔI_i	4	14	12	9	11
(6) F_{im}	10	9	16	11	4
(7) Un	2	18	13	14	3

the remaining 20 per cent. Chart 13 shows the frequency histogram of the position of the peak for each case. It is seen that there is a concentration of peaks at approximately 4.0 quarters in the one-peak and three-peak cases, and, though less distinct, near 16.0 and 4.0 quarters in the two-peak case.

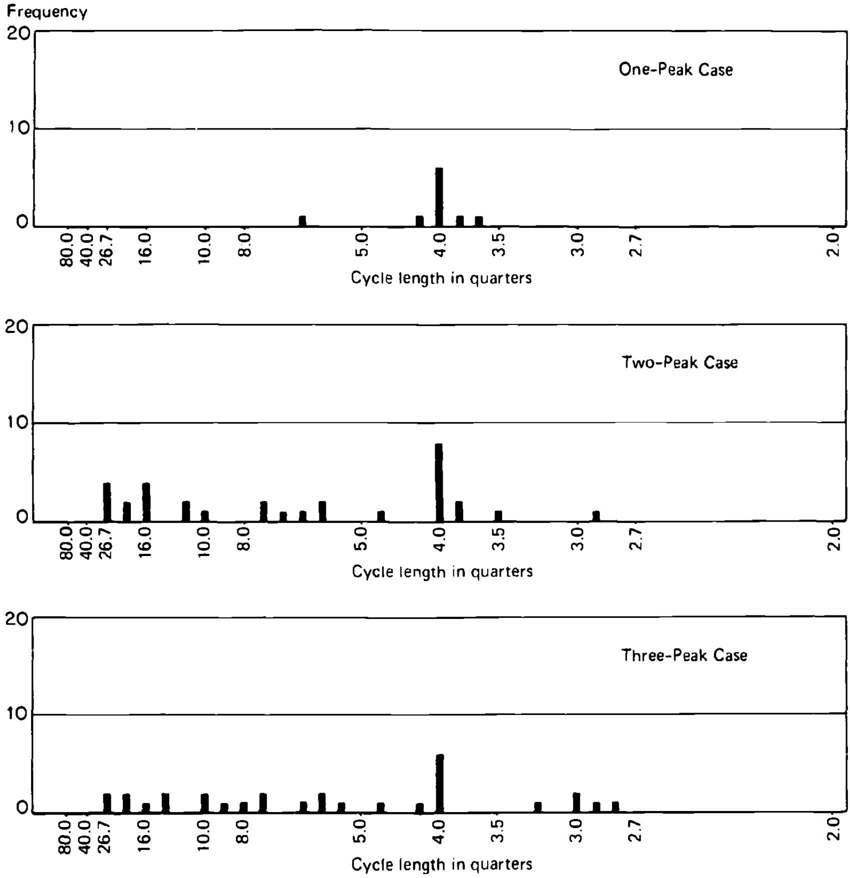
Chart 14*a* shows the frequency histogram of the *GNP* peak positions for all those included in the one- to four-peak cases. From the diagram we find four types of cycles in the behavior of *GNP*:

	Range of Period (Quarters)	Peak in Each Range (Quarters)	Frequency
(1)	26.7–10.0	26.7	29
(2)	8.9–5.3	5.7	20
(3)	5.0–3.5	4.0	38
(4)	3.3–2.0	3.0	11

Here, frequency implies the number of times that the *GNP* peak positions fall in each range. The 5.0–3.5 range has the highest frequency, and the 26.7–10.0 range the second highest. Chart 14*b* is the frequency histogram of the peak position for the highest peak of each run. Simi-

CHART 13

Spectrum Cycles GNP: One to Three Peaks



lar to Chart 14*a*, there are concentrations in the 26.7–10.0, and 5.0–3.5, ranges, while the highest peak appears less frequently in the 8.9–5.3, and 3.3–2.0, ranges.

I. and F. Adelman found the average length of a cycle in *one* stochastic simulation of 93 years to be 4.0 years.[9] In our *fifty* replications, the peaks of the spectrum in the range of 26.7–13.3 quarters (or 6.6–3.3 years) occur in 24 out of 50 cases (48 per cent of all trials), and 16 of these 24 peaks are the largest peak of each run. In addition, our results,

[9] Adelman and Adelman, *op. cit.*

based on the quarterly model, disclose that shorter cycles (11.4–6.2 quarters) occur with a lower probability, while 4-quarter cycles occur with a higher probability.

CONSUMPTION (*C*)

As shown in Table 6, the one-peak case is more frequent in the spectral histogram of consumption than in that of *GNP*, while the case of three or more peaks occurs infrequently. Charts 15*a* and 15*b* present the histogram of the peak position of the consumption spectrum for all peaks, and for the highest peak, respectively. Comparison between consumption and *GNP* (or other variables, as shown later) reveals that concentration of the peak on 4.0 quarters is much more striking in consumption than in *GNP* (or other variables), while both

CHART 14

Spectrum Cycles GNP: One to Four Peaks

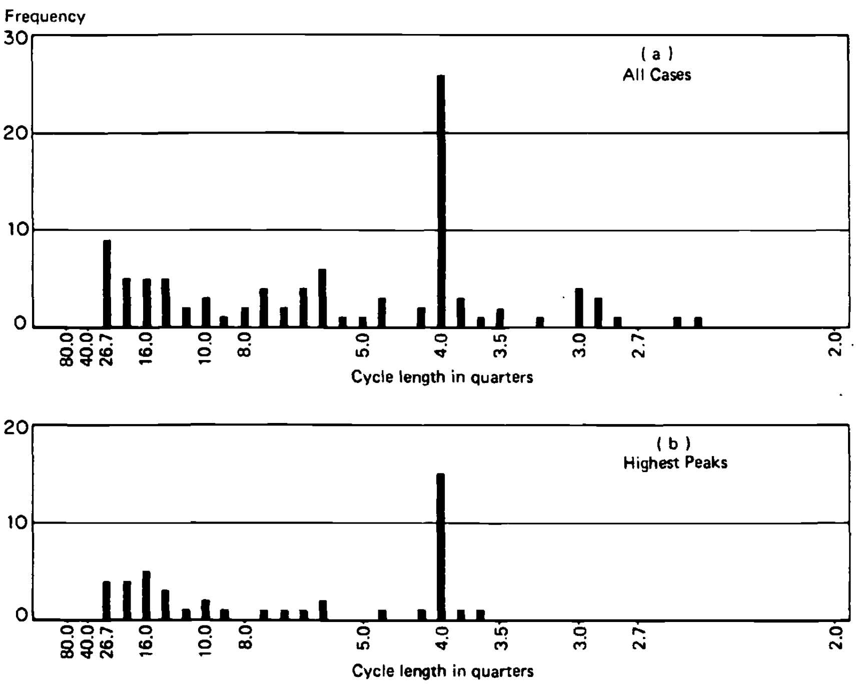
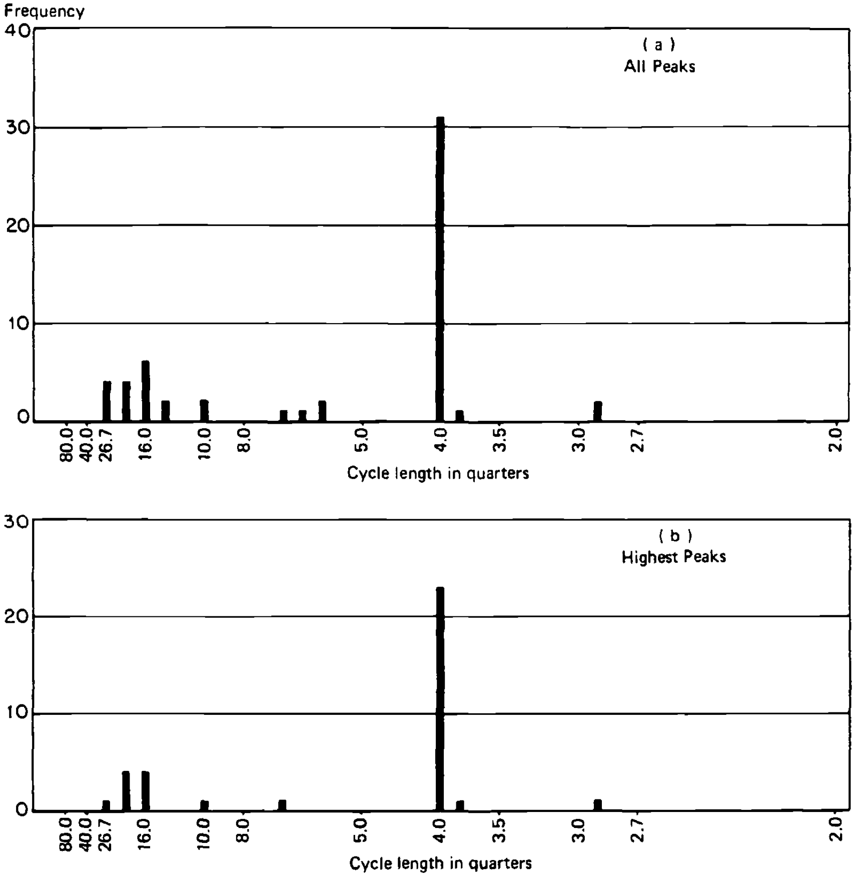


CHART 15

Consumption: Serially Uncorrelated Error Terms
(Histogram of Peaks)



indicate similar patterns in the distribution of cycles of the other quarters. This suggests that the consumption and related equations with stochastic terms tend to generate a cyclical movement of 4.0 quarters.[10] This cycle might be ascribed to the wage-determination

[10] Since seasonally adjusted series were used in estimating the equations, it is very unlikely that the seasonality of consumption causes the four-quarter cycle in our model. If the simulation path of each variable is seasonally adjusted, the spectral diagrams reveal no presence of the yearly cycle, but the computed seasonal patterns vary greatly for any variable among the fifty replications. This suggests that seasonal movement, in the usual sense, is not responsible for the four-quarter cycles found in our simulation.

process. Since the wage equations in the manufacturing and non-manufacturing sectors have a four-quarter lag, a four-quarter cycle may occur in wage rates and, thus, in income and consumption.

The basis for the yearly cycle may be deduced from the following considerations. In an approximation to our wage equation

$$w - w_{-4} = \alpha_1 Un + \alpha_0 + e$$

we may disregard fluctuations in Un, since the simulation is designed to keep the unemployment rate nearly steady at 4 per cent over the long-run equilibrium growth-path. The characteristic roots of the homogeneous equation are ± 1.0 and $\pm i$. The complex roots will provide a maintained cycle of four quarters. If we extend our equation to the form

$$w - w_{-4} = \alpha_1 Un + \alpha_2(w_{-4} - w_{-8}) + \alpha_0 + e,$$

the corresponding characteristic roots are

$$\pm 1, \pm i, \left(\frac{\alpha_2}{4}\right)^{1/4}(1 \pm i), > \left(\frac{\alpha_2}{4}\right)^{1/4}(1 \pm i)$$

If α_2 is small (0.2 in our model for the manufacturing sector), the dominant cyclical roots will still be $\pm i$.

It is interesting to note that the wage-price subsector of a model of the U.K., where this form of four-quarter wage adjustment was introduced some time ago, also has a yearly cycle.[11]

PRIVATE FIXED INVESTMENT (I_p)

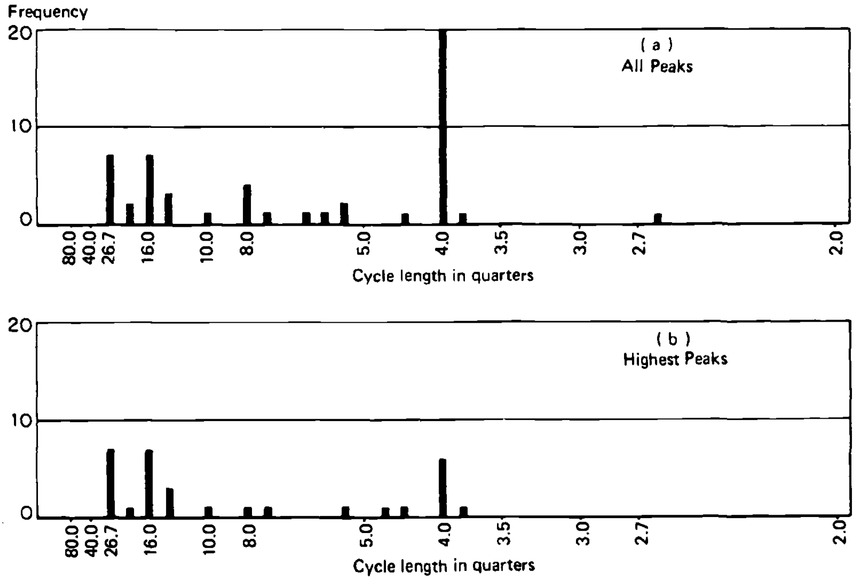
As shown in Table 6, there is no distinct peak in the investment spectrum in 19 out of 50 runs, while the one- and two-peak cases occur with frequencies of 15 and 11, respectively. Chart 16*a* provides the distribution of the peak position for all the peaks of 50 runs. This diagram bears a closer similarity to that of *GNP* than to any other variable, suggesting a close relationship between investment and general economic fluctuations.

Chart 16*b* describes the distribution of the largest peak. Here, we find a concentration in the region of 26.7–10.0 quarters (or 6.6–2.5 years), which corresponds to the standard business-cycle.

[11] See L. R. Klein, R. J. Ball, A. Hazlewood, and P. Vandome, *An Econometric Model of the U.K.* (Oxford, Blackwell, 1961). Appendix III, p. 269.

CHART 16

Fixed Investment (Histogram of Peaks)



RESIDENTIAL CONSTRUCTION (I_h)

Charts 17*a* and 17*b* present the histogram of the peak positions of the spectrum for all peaks, and for the largest peak, respectively. The latter shows that the greater part of the largest peaks are concentrated in the range in excess of 7.3 quarters, indicating a longer period for construction cycles. Further, it should be noted that the former, unlike the diagrams of the other variables, has very few peaks in the range 4.4–3.6 quarters.

INVENTORY INVESTMENT (ΔI_i)

Charts 18*a* and 18*b* refer to inventory investment. When compared with the corresponding figure of the other variables, Chart 18*a* indicates that the number of cases in which the peak of the spectrum falls in the range 10.0–5.0 quarters is relatively large in inventory

CHART 17

Residential Construction (Histogram of Peaks)

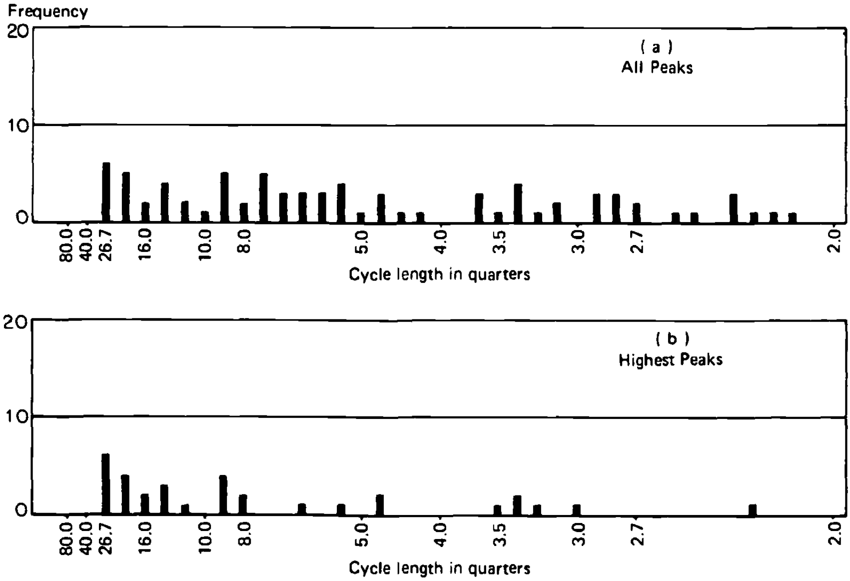
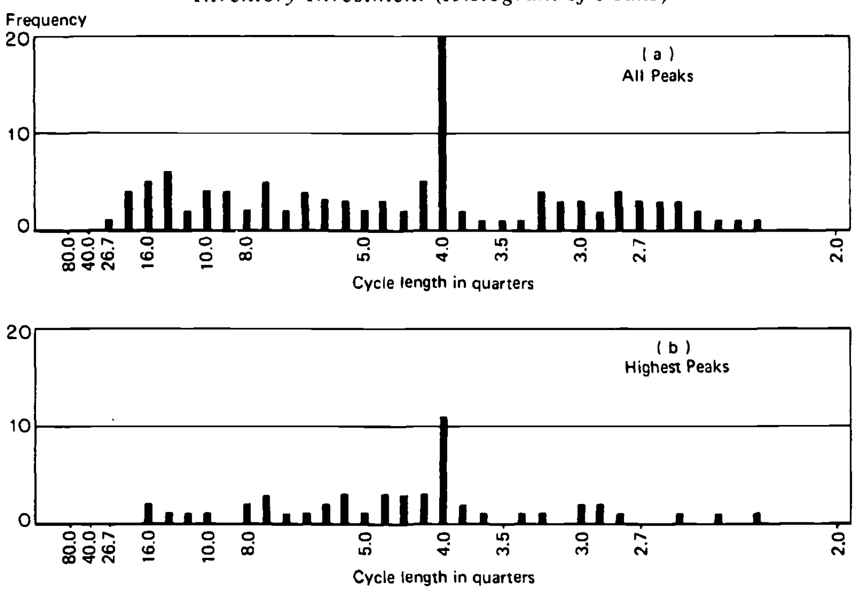


CHART 18

Inventory Investment (Histogram of Peaks)



investment. Chart 18*b*, relating to the largest peak, gives some support to this finding. This is the time-honored distinction in business-cycle theory between the *business cycle* (Juglar) and *inventory cycles.*

IMPORTS OF MATERIALS (F_{im})

Charts 19*a* and 19*b* suggest that the patterns of the spectral distribution of material imports, apart from the frequency of the 4-quarter cycle, are similar to those of inventory investment. The cycles of 8.9–5.0 periods are relatively frequent in both figures.

THE UNEMPLOYMENT RATE (Un)

It can be seen from Charts 20*a* and 20*b* that there is a large concentration at 4.0 quarters in the diagrams, both for all peaks, and for the highest peaks. Their patterns are similar to those for consumption.

CHART 19

Material Imports (Histogram of Peaks)

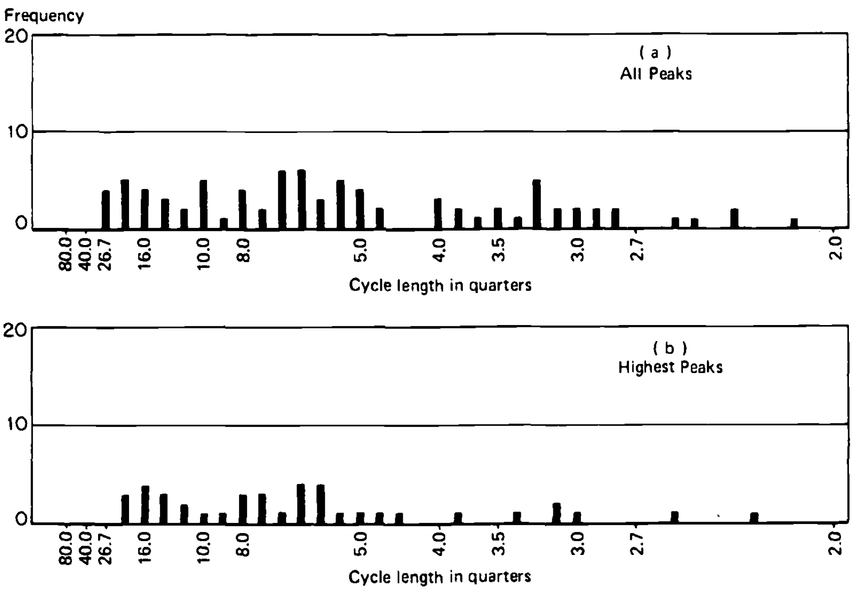
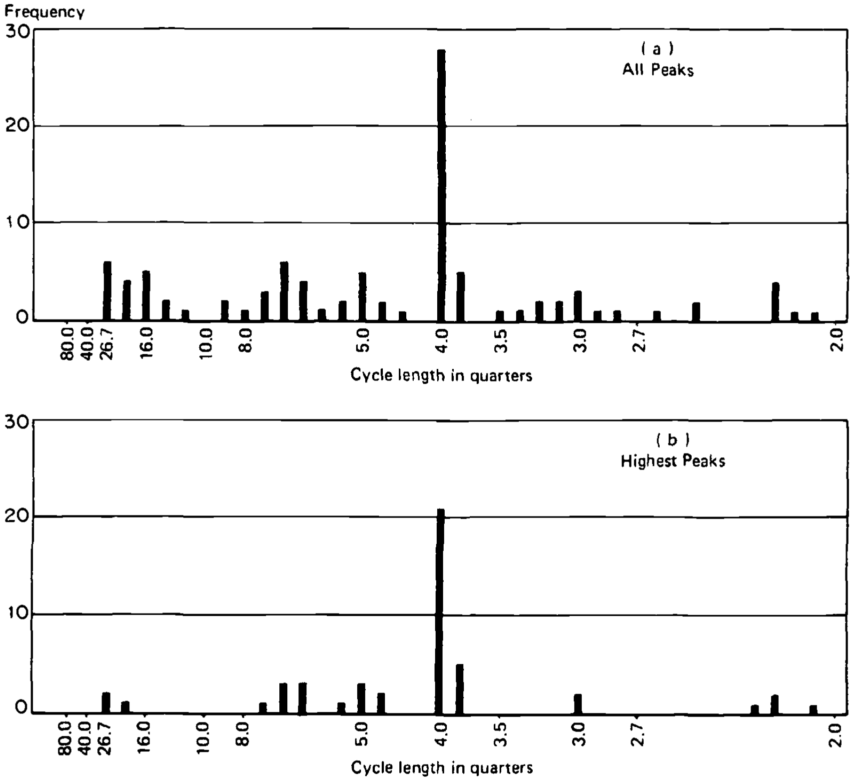


CHART 20

Unemployment Rate (Histogram of Peaks)



STOCHASTIC SIMULATIONS WITH SERIALLY CORRELATED ERROR TERMS

THE stochastic solutions with serially *correlated* random terms present outcomes substantially different from those with serially *uncorrelated* errors. First, the movements of each variable of the serially correlated scheme are much smoother. As a result, their cyclical patterns are much more distinct. Second, the period of the dominant cycle of all the variables treated here lies in the range of 26.7–10.0 quarters.

TABLE 7

Frequency of Peaks
(Serially Correlated)

	Number of Peaks				
Variable	0	1	2	3	Over 4
(1) X	3	25	16	6	0
(2) C	8	17	12	13	0
(3) I_p	3	32	11	3	1
(4) I_h	2	42	5	1	0
(5) ΔI_i	1	15	15	12	7
(6) F_{im}	0	8	22	13	7
(7) Un	0	8	26	14	2

In most of 50 replications, the spectra of real *GNP* present either the one-peak case, with a peak around 16.0 quarters; or the two-peak case, with two peaks around 16.0 and 4.0 quarters (see Table 7). As shown in Charts 21*a* and 21*b*, the position of the highest peak of each run occurs in the range of 26.7–10.0 quarters in 46 out of 50 replications; another concentration of the peak positions, though less frequent and less strong, is found in the range of 5.0–3.5 quarters. The results for consumption, fixed investment, and residential construction, as shown in Charts 22 to 24, are quite similar to those for real *GNP*. However, the peak positions of fixed investment and residential construction reveal more distinct concentration in 16.0 quarters; in particular, the highest peaks for residential construction—in all cases but one—occur in the range 20.0–13.3 quarters.

The highest peak of inventory investment is also strongly concentrated at 26.7–10.0 quarters, while the frequency diagram covering all peaks shows some concentration in the range of 8.9–5.0 quarters, with a peak of 7.3 quarters. Finally, it should be noted that the mode of the frequency diagrams of material imports and of the unemployment rate are found, respectively, at 7.3 and 11.4 quarters, instead of the 16.0 quarters for real *GNP* and other variables mentioned above (see Charts 25–27).

Another way of looking at the spectral density functions in fifty replications is to graph the averages of the density functions at each frequency. In contrast to the diagrams of Figure 12, which are presented for single cases of one, two, and three peaks, in Chart 28 we show the average spectral densities for seven variables in the case of serially correlated disturbances. The results portray a preponderance of distinct peaks for all variables. The corresponding "average" spectral density functions for the case of serially independent errors does not have distinct peaks, except for some accumulation at 4 quarters.

These findings suggest that the stochastic simulations with serially correlated disturbances are more consistent with the historical facts on business cycles than those with serially uncorrelated disturbances.

CHART 21

Gross National Product (Histogram of Peaks)

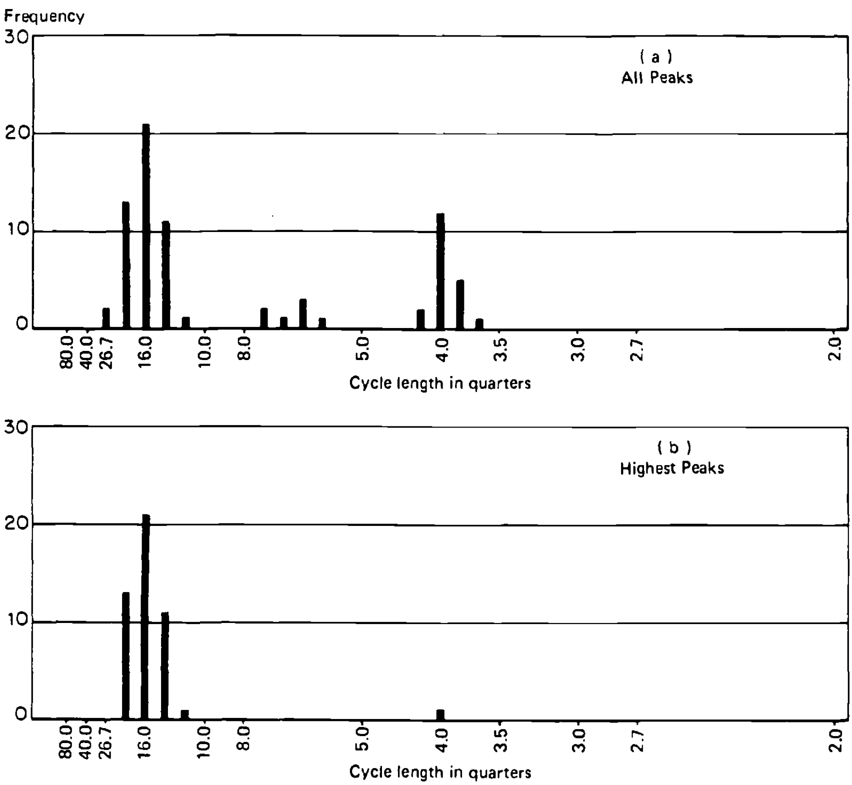


CHART 22

Consumption: Serially Correlated Error Terms
(Histogram of Peaks)

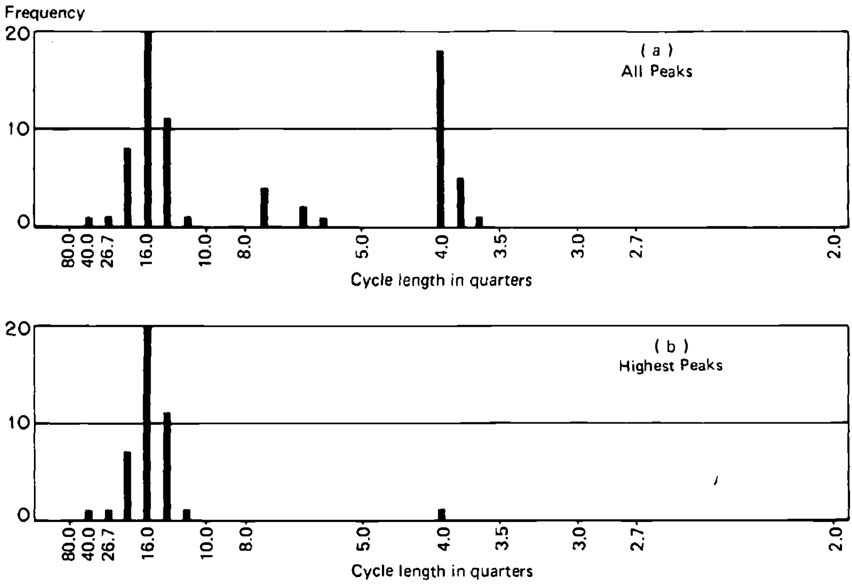


CHART 23

Fixed Investment (Histogram of Peaks)

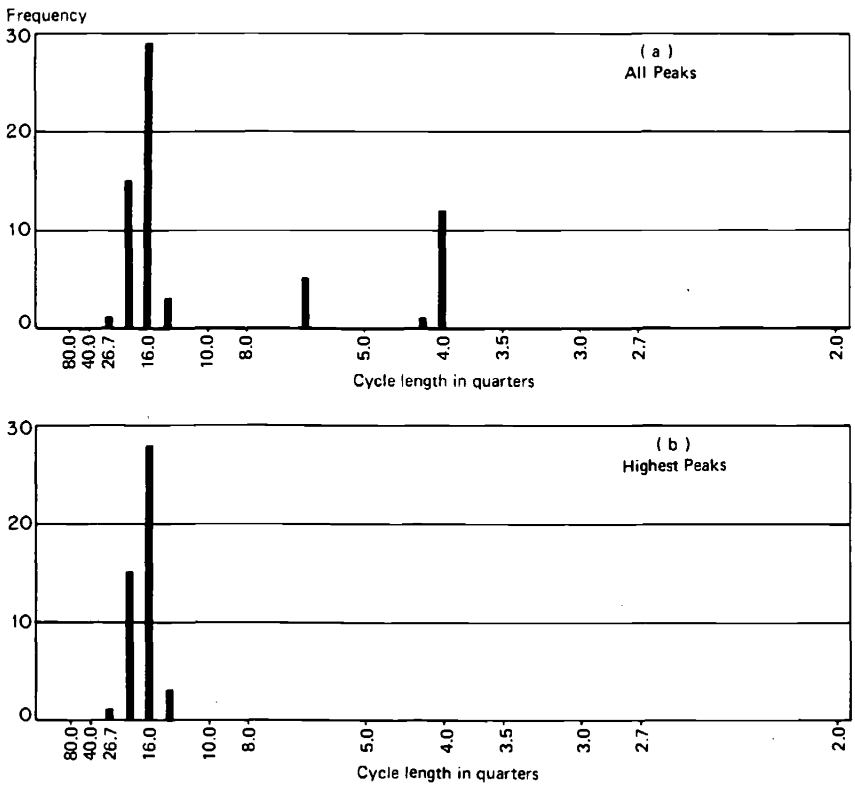


CHART 24

Residential Construction (Histogram of Peaks)

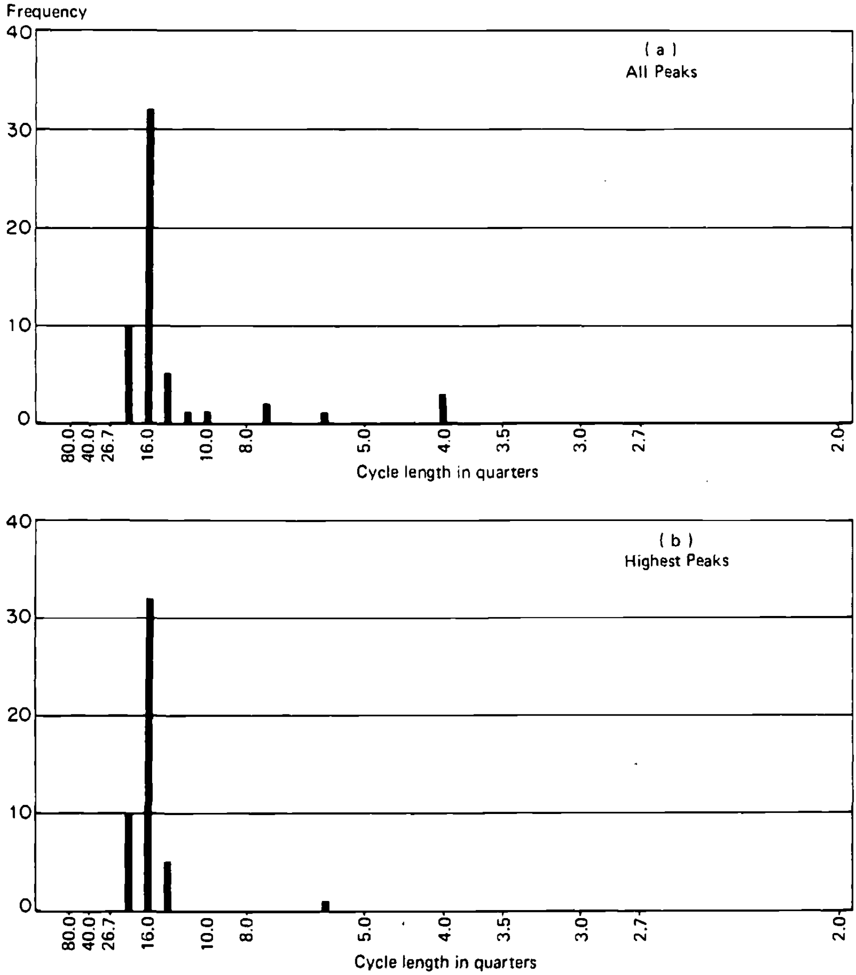


CHART 25

Inventory Investment (Histogram of Peaks)

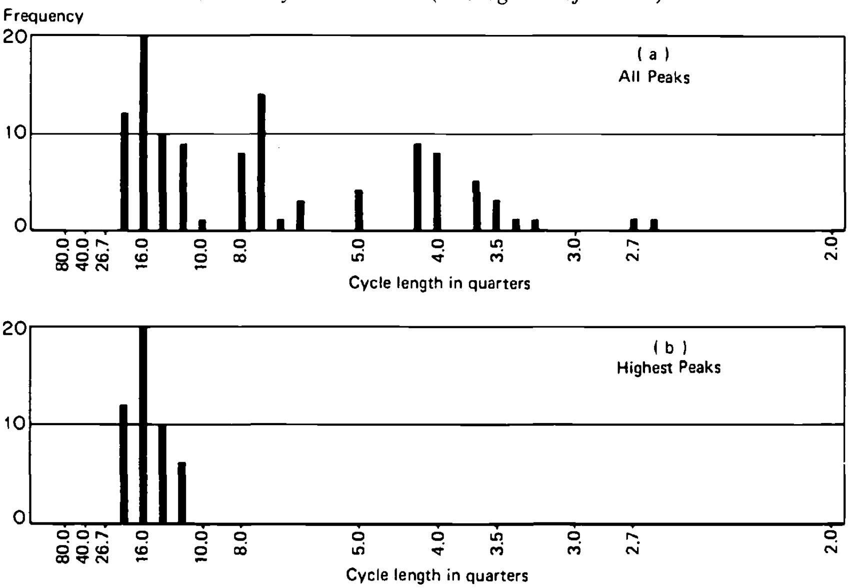


CHART 26

Material Imports (Histogram of Peaks)

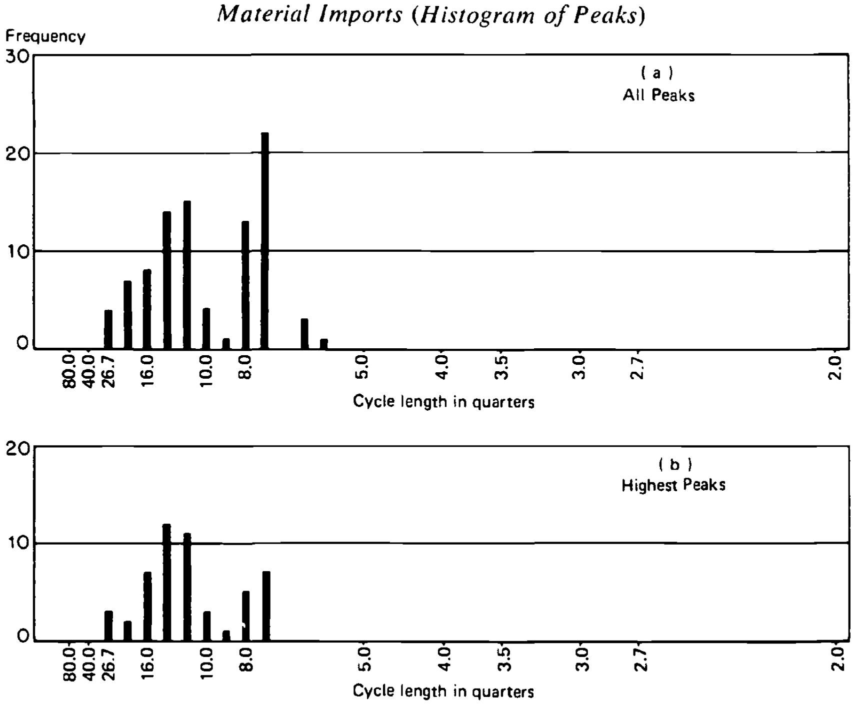


CHART 27

Unemployment Rate (Histogram of Peaks)

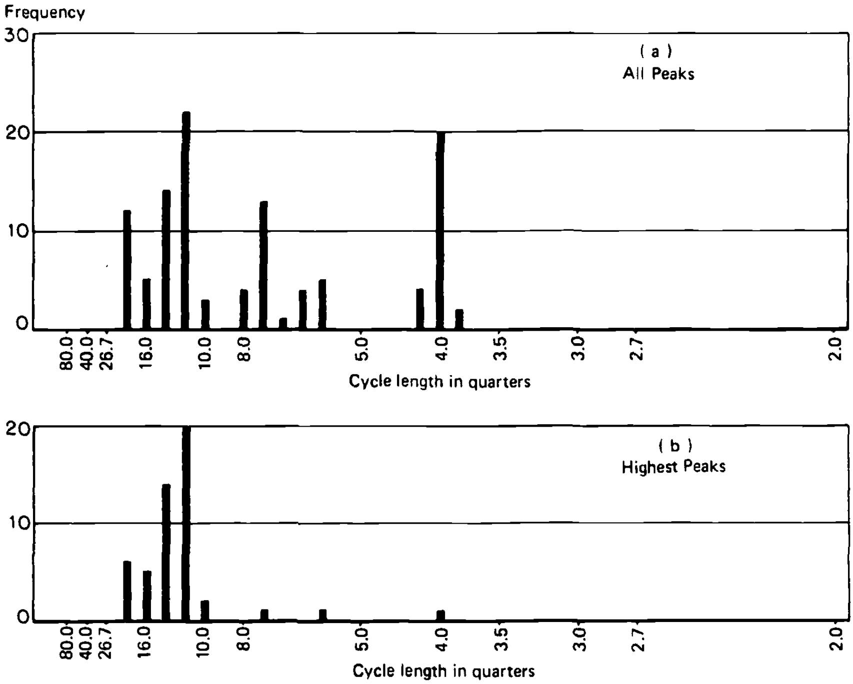
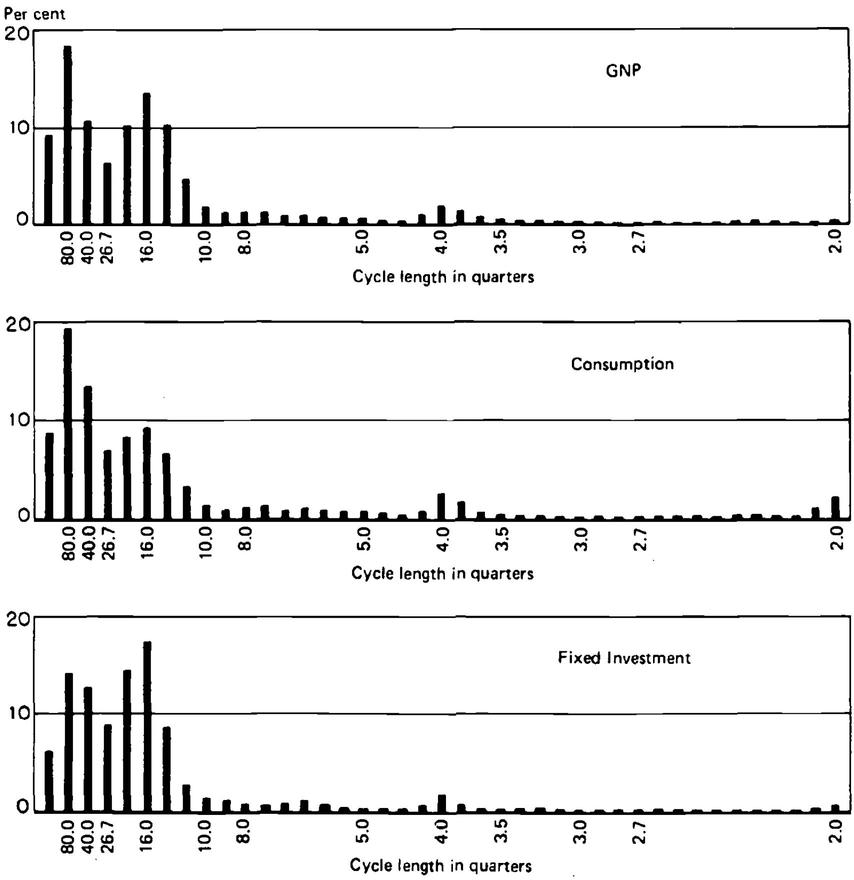
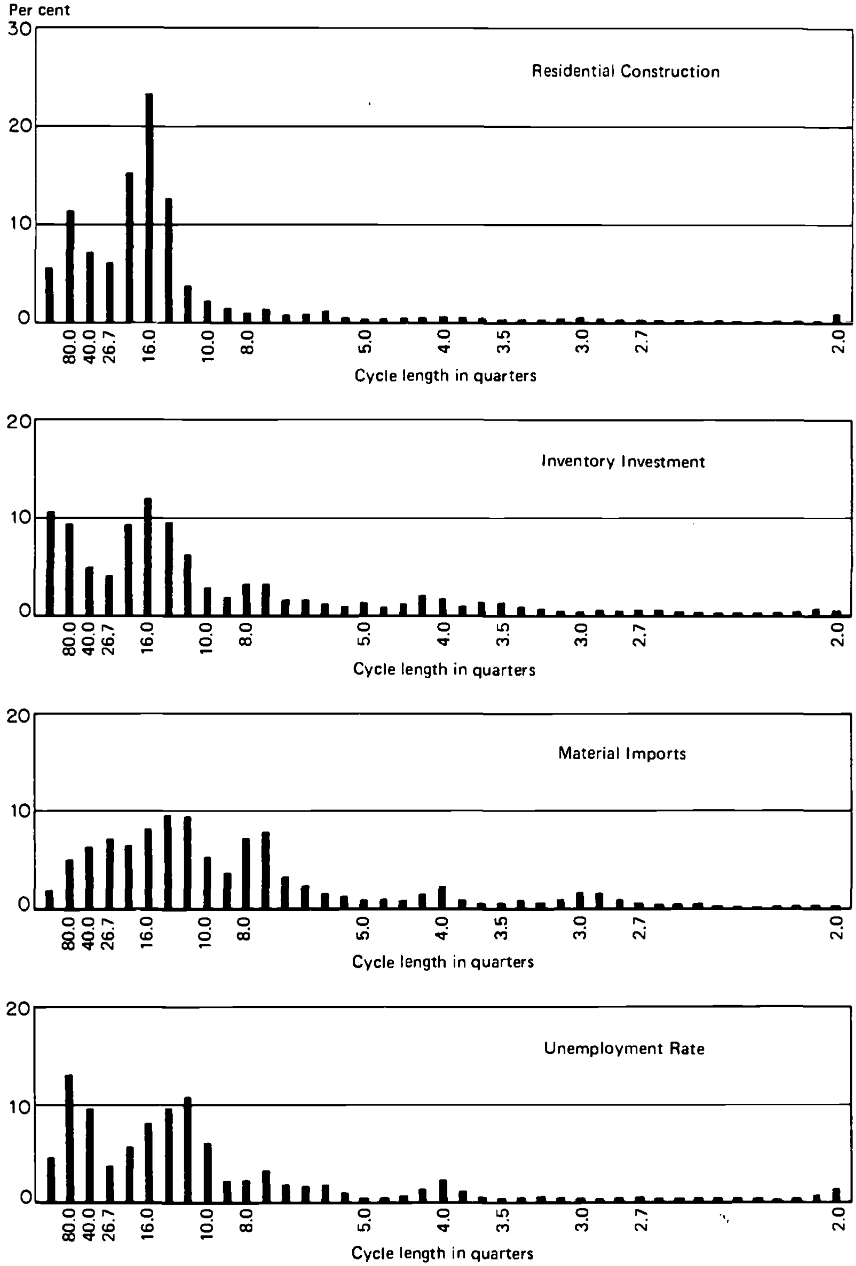


CHART 28

Average Spectral Densities: Serially Correlated Errors



Average Spectral Densities: Serially Correlated Errors (concluded)



Thus, we conclude that the results of our research are favorable for the former scheme, not only in the efficacy of short-run predictions by non-stochastic simulations, but also in the experimental generation of cyclical movements by the stochastic simulations.

APPENDIX

SOME NOTES ON THE GENERATION OF PSEUDO-STRUCTURAL ERRORS FOR USE IN STOCHASTIC SIMULATION STUDIES

MICHAEL D. McCARTHY
WHARTON SCHOOL

It has been claimed that in order to achieve a realistic simulation of an econometric model of a society—a simulation in which business cycles might be exhibited—it is necessary to simulate the model in such a way that its equations are subject to random stochastic errors (or shocks).[1] The idea here, put forward by Slutzky,[2] is that economic fluctuations may be due simply to random shocks. Another, perhaps more appropriate, reason for undertaking stochastic simulation is that the device provides a way of studying the statistical distribution of the endogenous variables and of policy predictions.[3]

Whatever the reason for simulating econometric models subject to stochastic shocks, one would presumably want to undertake the simulation using the "appropriate" types of shocks—shocks which exhibited the same statistical properties as those which affect the economic system.

In the material that follows, a method is described for generating disturbances for use in the stochastic simulation of an estimated econometric model. One advantage of the method is that it can be used

[1] See Irma Adelman and Frank L. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica*, XXVII (October, 1959), pp. 596–625. For a more recent study, see A. L. Nagar, "Simulation of the Brookings Econometric Model," presented at the December, 1966 meetings of the Econometric Society, San Francisco.

[2] E. Slutzky, "The Summation of Random Causes as the Source of Cyclical Processes," *Econometrica*, V (1937), p. 105.

[3] See Nagar, *op. cit.*

regardless of the number of observations available for estimating the model. However, the statistical properties of the generated errors will, of course, depend on this number. It will be shown that for a large number of observations, if the true structural errors are jointly normally distributed, and have expected values of zero, the generated structural disturbances will be distributed in the same manner as the structural disturbances of the model. A method is also described for deriving disturbances with suitable auto-regressive properties, assuming that the number of observations is sufficiently large. It will be assumed that the true disturbances are generated by a stationary stochastic process.

1. Let S be a $1 \times M$ matrix of pseudo-structural disturbances.
2. Let r be a $1 \times T$ matrix of random errors, assumed to be distributed $N(0,1)$.
3. Let U be any $T \times M$ matrix of disturbances from T observations of M true structural equations. We shall derive S as follows:

$$S = T^{-1/2} rU$$

4. Let $1/T\, EU'U = \Sigma$, where Σ is the $M \times M$ covariance matrix of the system, which is assumed constant over time. The term E is an expectations operator.
5. Let r be independent of U and consider the expected value of the covariance matrix of S

$$\Sigma_S = T^{-1} EU'r'rU = ES'S$$

The typical element in Σ_S is given by

$$\Sigma_{S_{ij}} = T^{-1} EU_i' r' r U_j \tag{1}$$

where U_i is the ith column of U and U_i' is its transpose. The term U_j is similarly defined.

Equation (1) may be rewritten:

$$\Sigma_{S_{ij}} = T^{-1} E \sum_{b=1}^{T} \sum_{a=1}^{T} (r_a r_b)(U_{ia} U_{jb}) \tag{1a}$$

where r_a is the ath element in the row vector r, and U_{ia} is the ath element in the column vector U_i. Again, the terms r_b and U_{jb} are similarly defined.

Since r is independent of U and $Er_ar_b = 0$, $a \neq b$ and $Er_ar_b = 1$, $a = b$, it follows that

$$\begin{aligned}\Sigma_{S_{ij}} &= T^{-1}E \sum_{a=1}^{T} r_a^2 U_{ia} U_{ja} \\ &= T^{-1}E \sum_{a=1}^{T} U_{ia} U_{ja} \\ &= \Sigma_{ij}\end{aligned}$$

Thus, $\Sigma_S = \Sigma$.

This property holds as long as U is derived from a known structural form. This result suggests that if we are given a sample set of errors, $\hat{U}$, derived from an estimated model, we might use as pseudo-disturbances the following:

(2) $$\hat{S} = T^{-1/2} r \hat{U}$$

For models estimated by consistent methods, the covariance matrix of $\hat{S}$ will be asymptotically equal to the true covariance matrix of the system.

For small samples, if we take the $\hat{U}$ as given, it will be true that the conditional expected value of $\hat{S}$ will be equal to the sample variance-covariance matrix of the system, calculated as

$$\hat{\Sigma} = T^{-1} \hat{U}' \hat{U}$$

To obtain pseudo-errors that have suitable auto-regressive properties, we might define

$$S_t = T^{-1/2} \bar{r}_t U, \text{ and } S_{t-1} = T^{-1/2} \bar{r}_{t-1} U$$

where $$\bar{r}_t = [r_t r_{t-1} r_{t-2} \ldots r_{t-T+1}]$$

and $$\bar{r}_{t-1} = [r_{t-1} r_{t-2} r_{t-3} \ldots r_{t-T}]$$

and t is a time subscript.

In what follows, we examine the first-order auto-regressive properties of S. Form

(3) $$ES_t' S_{t-1} = T^{-1} EU' \bar{r}_t' \bar{r}_{t-1} U$$

The typical element in (3) is given by

$$Z_{ij} = T^{-1}EU_i\bar{r}_t'\bar{r}_{t-1}U_j$$

$$= T^{-1}EU_i'\left[\begin{bmatrix} r_t \\ r_{t-1} \\ r_{t-2} \\ \cdot \\ \cdot \\ \cdot \\ r_{t-T+1} \end{bmatrix} [r_{t-1}r_{t-2} \quad \cdots \quad r_{t-T}] \ U_j\right]$$

$$= T^{-1}EU_i'\begin{bmatrix} r_tr_{t-1}r_tr_{t-2} & & r_tr_{t-T} \\ (r_{t-1})^2r_{t-1}r_{t-2} & & r_{t-1}r_{t-T} \\ r_{t-2}r_{t-1}(r_{t-2})^2 & & r_{t-2}r_{t-T} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ r_{t-T+1}r_{t-1}r_{t-T+1}r_{t-2} & \cdots & (r_{t-T+1})^2r_{t-T+1}r_{t-T} \end{bmatrix} U_j$$

Since $\bar{r}$ is independent of U, and $Er_ar_b = 0$, $a \neq b$, and finally $Er_a^2 = 1$, we have:

$$Z_{ij} = T^{-1}E \sum_{k=t-T+1}^{t-1} U_{ik}U_{jk+1}$$

$$= \frac{T-1}{T} EU_{jt}U_{it-1}$$

where U_{jt} is the tth error of the jth equation, and $U_{i_{t-1}}$ is the $t-1$th error for the ith equation.

Returning to equation (3), we have:

$$Z = ES_t'S_{t-1} = \frac{T-1}{T} EU^{t'}U^{t-1}$$

where $U^{t'}$ is the transpose of the tth row of U, and U^{t-1} is similarly defined.

In the case of kth-order auto-regression, we would find

$$ES_t'S_{t-k} = \frac{T-k}{T} EU^{t'}U^{t-k}$$

If T is sufficiently large, this suggests that we use as pseudo-structural errors:

$$\hat{S}_t = T^{-1/2}\bar{r}_t\hat{U} \text{ and } \hat{S}_{t-1} = T^{-1/2}\bar{r}_{t-1}\hat{U}$$

where $\hat{U}$ is a set of sample structural errors and $\bar{r}_t$ and $\bar{r}_{t-1}$ are defined as above. Hopefully, $\hat{U}$ will be derived from a model estimated by consistent methods.

In what follows, we examine the asymptotic distribution of S in the absence of auto-regression. It will be shown that if r is a vector of random numbers distributed $N(0,1)$, and if the structural disturbances are jointly normally distributed, the asymptotic distribution of S will be the same as the distribution of the structural errors.

PROOF: Let U^i be the ith observation of the structural disturbances. We assume that U^i is distributed $N(0,V)$ for all i; that is, the frequency function is given by

$$F(U^i) = (2_\pi)^{-M/2}|V|^{-1/2}e^{-1/2U^iV^{-1}U^{i'}}$$

for all i. (Note that U^i is a row vector containing M elements.)

The joint moment generating function of U^i is

$$M(U^i) = e^{1/2t'Vt} \tag{4}$$

where t is now a $M \times 1$ column vector of constants, with elements t_i, $i = 1, 2, \ldots, M$.

In the following demonstration we show that as the number of observations approaches ∞, the joint moment generating function of S is the same as (4).

The moment generating function of S is given by

$$M(S) = (2_\pi)^{-MT/2}|V|^{-T/2} \int \cdots \int e^{T^{-1/2} \sum_{j=1}^{M} t_j r U_j - \frac{1}{2}\sum_{i=1}^{T} U^iV^{-1}U^{i'} - \frac{1}{2}\sum_{i=1}^{T} r_i^2} \cdot \prod_i \prod_j dU_j^i \cdot \prod_i dr_i$$

where U_j is the jth column of U.

This may be rewritten

$$M(S) = (2_\pi)^{-MT/2}|V|^{-T/2} \int \cdots \int e^{T^{-1/2} \sum_{i=1}^{T} r_i U^i t - \frac{1}{2} \sum_{i=1}^{T} U^i V^{-1} U^{i'}}$$

$$- \frac{1}{2} \sum_{i=1}^{T} r_i^2 \cdot \prod_i \prod_j dU_j^i \prod_i dr_i$$

The term $T^{-1/2} r_i U^i t$ can be interpreted as a linear combination of the terms in U^i. It can be verified that $M(S)$ may be written

$$M(S) = (2_\pi)^{-MT/2}|V|^{-T/2} \int \cdots \int e^{1/2} \sum_{i=1}^{T} r_i^2/T\ t'Vt$$

$$\cdot\ e^{-1/2} \sum_{i=1}^{T} (U^i - r_i T^{-1/2} t'V) V^{-1} (U^i - r_i T^{-1/2} t'V)'$$

$$- \frac{1}{2} \sum_{i=1}^{T} r_i^2 \cdot \prod_i \prod_j dU_j^i \prod dr_i$$

$$= (2_\pi)^{-MT/2}|V|^{-T/2} \int \cdots \int e^{1/2} \sum_{i=1}^{T} r_i^2/T\ t'Vt - \frac{1}{2} \sum_{i=1}^{T} Z^i V^{-1} Z^{i'}$$

$$\cdot\ e^{-1/2} \sum_{i=1}^{T} r_i^2 \cdot \prod_i \prod_j dZ_j^i \prod_i dr_i$$

where $Z^i = U^i - r_i T^{-1/2} t'V$. The Jacobian of the transformation equals one.

We first integrate $M(S)$ with respect to Z^i, taking the r_i as given. The result is

$$(5) \quad M(S) = (2_\pi)^{-T/2} \int \cdots \int e^{1/2} \sum_{i=1}^{T} r_i^2/T\ t'Vt - \frac{1}{2} \sum_{i=1}^{T} r_i^2 \cdot \prod_i dr_i$$

$$= (2_\pi)^{-T/2} \int \cdots \int e^{1/2t'Vt} \sum_{i=1}^{T} r_i^2/T - \frac{1}{2} \sum_{i=1}^{T} r_i^2 \cdot \prod_i dr_i$$

Next, integrate (5), taking T as given, and then take the limit. The result is

$$M(S) = \left(1 - \frac{t'Vt}{T}\right)^{-T/2}, \text{ and}$$

$$\lim_{T \to \infty} M(S) = e^{1/2t'Vt}$$

In the case of consistently estimated models, the pseudo-structural errors

$$\hat{S} = T^{-1/2} r \hat{U}$$

will have the same asymptotic distribution as U^i, if r and U^i are distributed in the manner assumed above. Generalization of the proof about asymptotic distribution to the case where errors exhibit autocorrelation is not difficult.

DISCUSSION

FRANK DE LEEUW

URBAN INSTITUTE

One way to classify theories of economic fluctuations is by which forces they identify as the underlying ones setting an economy's dynamic responses in motion. The Slutsky-Frisch theory emphasizes stochastic disturbances—forces of the kind we often cannot measure directly or pinpoint in time. Many textbooks on macroeconomics emphasize measurable and relatively autonomous injections, or withdrawals, from the income-expenditure process: government spending, exports, tax laws, and so on. Monetarists emphasize exogenous disturbances affecting the quantity of money: changes in monetary policy, gold discoveries, bank runs, and so on. The theories have to do with which one, or ones, of these forces have been the prime movers in the past, or are likely to be dominant in the future.

The classification is useful here, since the Conference appears to have been designed with the stochastic-disturbance view exclusively in mind. There has been much attention as to how the models behave under different short-run error-term adjustments, and how they respond to simulated "typical" stochastic shocks. These are characteristics concerning which the Slutsky-Frisch theory has strong implications. In contrast, no attention has been given to which kinds of exogenous variables the models are sensitive, or to what kinds of fluctuations "typical" patterns of change in exogenous variables produce.[1] Outside of this Conference, the emphasis is largely reversed; current discussion of economic fluctuations centers mainly around which exogenous forces matter, and how they work, to the virtual exclusion of discussion of the role of stochastic disturbances.

What I would like to do here is look at the Evans-Klein-Saito paper, and some of the other papers, from the point of view of the light they shed on these alternative explanations of economic fluctuations. First, I shall discuss the question of whether the simulation results tell us anything about the validity or strength of the stochastic-disturbance theory. My conclusion is that the simulation results tell us very little, since we would expect the same kind of results if: (*a*) the stochastic-disturbance theory is valid; or (*b*) the theory isn't valid, but the models are mis-specified. The next section takes up the question of whether the models—mainly, the Wharton and OBE Models—are mis-specified, and attempts to use some comparisons of model results with single-equation "reduced-form" explanations of *GNP* fluctuations to make headway on this question. The conclusion stresses the likelihood that there are important mis-specifications in the models.

1. THE SIMULATION RESULTS

From the many simulations of the models under investigation at this Conference, three generalizations find support (though not unanimous support by each investigator of each model). First, both within and beyond the sample period, forecasts one quarter ahead are better than forecasts two or more quarters ahead. Second, forecasts which take some account of serial correlation in error terms are better than forecasts which do not. Third, simulations of the model when it is subjected to serially correlated stochastic shocks whose distribution is based on equation errors during the sample period, succeed in reproducing mild fluctuations; exactly how mild seems to be a matter of controversy. This third generalization is somewhat ambiguous in the case of the Wharton Model simulations, because the exogenous variables are not set at perfectly smooth growth rates for the stochastic simulation runs but include at least half a cycle due to an assumed

[1] In his oral presentation, George Green referred to some simulation results produced by "typical" patterns of change in exogenous variables in the OBE Model. I hope that these interesting results will be made available in written form.

Vietnam War settlement. For the OBE Model simulations, in which exogenous variables *are* set at perfectly smooth growth rates, the cycles generated by stochastic simulations are very mild. The first two generalizations hold for the OBE Model, and also for a version of the FRB-MIT Model that I worked with in late 1968.

Now, if economic fluctuations are largely the economy's response to stochastic disturbances which interact over time, then we would expect all of these generalizations to hold true for a well-specified econometric model. Predictions should improve, the shorter the prediction period; because a short prediction-period enables us to capture, in the initial conditions, the effect of past stochastic forces. Predictions should improve when account is taken of intercorrelations of error terms, because the error terms, including their intercorrelation properties, are the basic force driving the economy. Finally, stochastic simulation should reproduce the characteristics of historical fluctuations, because they reproduce the kind of impulses which, in fact, give rise to historical fluctuations. Thus, the reported simulation results might seem at first glance to provide strong support for the stochastic-disturbance theory—at least, as far as mild cycles are concerned.

Unfortunately, another set of conditions can just as easily account for these reported simulation results. Suppose that an economy responds to the forces which drive it by effects which develop gradually over time; but suppose, further, that a particular model mis-specifies the path by which these effects develop. This mis-specification could result from incorrect lag distributions, from bias in estimating the initial impact effect of some exogenous variable, or from other problems. Whatever the cause, one result ought to be better predictions one quarter ahead than two or more quarters ahead, since much of the effect of the mis-specification ought to be undone by inserting actual initial conditions of the endogenous variables in one-quarter forecasts. Another result ought to be serially correlated residuals in the sample period; taking account of this correlation ought to correct for some of the model's error and improve forecasts.

As for stochastic simulation, it might, on first thought, seem unlikely that random shocks could result in anything like historical fluctuations unless the stochastic-disturbance theory has validity. But when we remember that the simulations are based on the model's actual errors during the historical sample-period, I think success in matching the characteristics of historical fluctuations is not so surprising. The actual errors from a poorly specified model will tend to be larger than the true magnitude of stochastic forces. These large actual errors would, in themselves, lead to exaggeration of the power of stochastic forces to generate fluctuations. But if a model happens to understate the economy's response to stochastic forces, this understatement will work in the opposite direction. It is, therefore, hard to form any clear expectation of how stochastic simulation results should turn out if a model is mis-specified.

In summary, the principal simulation results reported in the Evans-Klein-Saito paper and in some of the other Conference papers could, it seems to me, just as easily result from mis-specification as from the historical validity of the Slutsky-Frisch theory.

2. COMPARISONS WITH REDUCED-FORM *GNP* EQUATIONS

Another way of stating the conclusion of the previous analysis is to remark that much of what we can say about the simulations prepared for this Conference depends on what we are willing to assume about the specification accuracy of the models. One way to form an impression of the specification accuracy of the models is to compare them with simpler, reduced-form relationships between *GNP* and major exogenous variables, such as the recently publicized relationship of *GNP* to monetary and fiscal policy variables. If these simple relationships catch certain features of historical fluctuations that a model misses, then, I think, there is some support for the proposition that the model is mis-specified. These reduced-form relationships are crude in their fixed-weight distributed lags, and in their ignoring of initial conditions; but it seems to me that these and other shortcomings put the reduced-form method under a handicap, making any indication of superiority over models all the more impressive.

The reduced-form equations I will use are those estimated by Edward Gramlich and reported in his paper, "The Usefulness of Monetary and Fiscal Policy as Discretionary Stabilization Tools."[2] He has

[2] Gramlich's paper was presented at the Conference of University Professors, sponsored by the American Bankers' Association, Milwaukee, September, 1969.

improved on earlier reduced-form equations: (1) by separating Federal government transactions into those which affect final demand directly (purchases plus grants-in-aid); those which affect household income directly; and those which operate through other channels (representing these by dummy variables)—instead of employing the usual classification into disbursements and receipts; (2) by adding exports to Federal purchases and grants-in-aid; (3) by adjusting defense expenditures to a value-added basis (that is, adding in changes in inventories of defense products); and (4) by adding a variable to represent major strikes.

Like previous investigators, Gramlich has estimated his equations in first-difference form, using the Almon polynomial-weight technique to reduce the lag problem to manageable proportions. He has tried out several monetary policy variables; I shall report on results using the monetary base, and on those using unborrowed reserves—the residuals of which he has kindly made available to me.

Since these equations take no account of initial conditions—that is, initial capacity utilization, initial unemployment rates, recent residuals, and so forth—they are most directly comparable to the "long-period" historical simulations for the Wharton and OBE Models, which also make *GNP* depend only on current and lagged exogenous variables. However, I will, in addition, compare the equations with the shorter-span simulations for the OBE Model. Unfortunately, the shorter-period simulations of the Wharton Model have not been reported in detail, so I cannot comment on them. The summary statistics presented indicate that the Wharton Model is less satisfactory than the OBE Model in these short-period simulations.

In the two recession-recovery periods of the 1950's—the 1953–54 cycle and the 1957–58 cycle—reduced-form equations do no better than the models. In fact, in the 1953–54 cycle, the models mirror the actual *GNP* path quite closely, proving slightly better than the reduced-form equations. The Wharton Model misses the 1957–58 cycle completely, and the OBE Model (both long-period and short-period simulations) indicates only the mildest of slowdowns; the reduced-form equations likewise indicate a very mild slowdown for this period.

In the 1960's, however, the reduced-form equations do capture some fluctuations that the models tend to miss. The models do not capture the 1960 decline, apart from slight declines in the second quarter of 1960, which followed the post-steel-strike inventory jump. The reduced-form equations do better – especially the one using unborrowed reserves, which correctly estimates practically no change in *GNP* from the second through the fourth quarter of 1960. The Wharton Model completely misses the 1967 slowdown; the reduced-form equations indicate, at least, a reduction in *GNP* growth, though not as much of a reduction as actually took place. Model calculations for 1968, as reported in the Evans-Haitovsky-Treyz paper, indicate a severe understatement in the second half of that year. Reduced-form equations also understate that period, but I would guess that their understatement of about $2 billion per quarterly change in *GNP* is not so severe. (Perhaps, however, the inclusion of 1968 in the reduced-form sample period heavily affects these results.)

Thus, there does seem to be some evidence in the 1960's supporting the view that the models are mis-specified. The evidence is not dramatic but, I think, it is enough to raise serious doubts as to how to interpret the simulations prepared for this Conference. Furthermore, the periods in which the reduced-form equations do better than the models suggest that it may be in the representation of the effects of monetary policy that these models are weak.

As a final point, let me note that these comments bear on only one possible contribution of this Conference; namely, its assistance in understanding the underlying causes of economic fluctuations. There are other contributions, as well; not the least of which is simple tabulation of the average errors models make when unaided by forecaster judgment. Any negative remarks about the area I have chosen to discuss are not meant to belittle these additional contributions.

BRIDGER M. MITCHELL

STANFORD UNIVERSITY

For purposes of long-run simulation, the structure of the Wharton Model can be considered as a set of 47 stochastic equations

$$y_i = G(y,x) + \epsilon_i, \quad i = 1, \ldots, 47$$

where the vectors of the endogenous variables y and exogenous variables x include both current and lagged values.

If G is a set of linear difference equations in the endogenous variables, the model can be solved to yield the values of the endogenous variables as rational lag functions of both the exogenous variables and the disturbances. The stochastic behavior of current y, given the values of the exogenous x, is then determined by the distribution of the errors ϵ_i and the estimated lag structure. In particular, for fixed exogenous values, such a model can be viewed as a linear filter operating on the random errors. In the frequency domain, the spectral densities of the output of this filter completely characterize the model's response to random shocks, obeying the assumed probability distribution, and the presence of peaks in the spectra of the outputs indicates the tendency of endogenous variables to reflect a cycle at the corresponding frequencies.

When the model contains *nonlinear* functions of the endogenous variables, this approach is not directly available. Since virtually all interesting macroeconomic models will include both real and monetary variables with endogenously determined prices, nonlinearity is the general case. One line of attack has been taken in another paper prepared for this Conference by Howrey; namely, to linearize the model around sample means and to evaluate the spectra of the endogenous variables as computed from this linear approximation.

The present paper by Evans, Klein, and Saito preserves the nonlinear features of the model by fixing the exogenous variables at the values of a control solution, drawing random numbers for values of the disturbances, and solving the model for 100 successive quarters. Fifty such experiments yield an equal number of time series for each of the endogenous variables. The authors then *estimate* the spectra of the nonlinear filter applied to the random process, treating each experiment as a random sample from that process.

Unfortunately, the paper is marred by a technical deficiency at this point. The authors have produced 50 separate estimates of the power spectrum of each of the important endogenous variables. However, as the experiments are designed to generate independent drawings from the same stochastic process with fixed initial conditions, it will be efficient to average the estimated auto-covariances from the 50 experiments, each of about 100 observations, and then compute a single estimate of the spectrum. For estimation purposes this must approach the ideal large sample in economics, and will certainly allow asymptotic confidence limits to be used to test hypotheses regarding spectral peaks at business-cycle frequencies.[1]

In lieu of this, the authors have tabulated the relative frequency of occurrence in the 50 experiments of various characteristics of the estimated spectra, such as the number of experiments having one, two, or three peaks. It is difficult to use these statistics to draw inferences about the probability of occurrence of periodic fluctuations of different frequencies in the model, since the spectrum of a particular endogenous variable has a determinate number of peaks, whereas the occurrence of estimated spectra with different numbers of peaks, or peaks at different frequencies, is an indication of the sampling variation of the estimate. In any event, the probability of a cycle of a given period occurring in a 100-quarter run of the model is not related in a simple way to the existence of significant power at the corresponding frequency. A direct estimate, of course, would be to count the number of occurrences in the time domain of cycles of the given period over the 50 experimental series.

Turning to the evidence which is tabulated for these runs, there appears to be substantial variance in the *GNP* series at a three-and-one-half- to seven-year band of frequencies. This is in striking contrast to the results from Howrey's linearized condensed version of the model, since in the latter, the longest period is only a year and a half (apart from a very low frequency component).[2]

The makings of an equally interesting difference between the linearized and simulated results appear when one is considering the stability of the model for constant values of the exogenous variables. The simulated nonlinear model's stability—in the range of appropriate values of the variables—is somewhat uncertain and calls for further investigation. To achieve a desirable base line, or nonstochastic path, for the endogenous variables required that equations relating to labor markets be more-or-less continually adjusted to keep the control solu-

[1] In estimating the spectra, the particular trend removal technique can have a major effect on the final results. In stochastic simulations of this sort, a natural procedure would be to take deviations from the nonstochastic control solution.

[2] The quantitative results from Howrey's study are based on his first-circulated computations, whose numerical accuracy he later questioned.

tion close to the target values for long-run steady growth.[3] On the other hand, the solutions as plotted on the charts, which show only one of the 50 stochastic simulation runs, do not appear to exhibit increasing variance with time. Not readjusting the labor equations for the different shocks in different experiments may account for the rather large variability seen in the chart, as compared with the nonstochastic run. The fact that this does not lead to explosive behavior may be due to damping effects of some nonlinear relations, when values depart sufficiently far from the base-line solution.

By way of comparison, Howrey has analyzed the linearized model both with, and without, the monetary sector. Both versions display explosive behavior, but the presence of the monetary equations has a decided effect in making the model more nearly stable. This, too, suggests that the stabilizing role of prices and interest rates at values away from their sample means may be stronger than one measured by purely linear equations.

I have suggested the possible role of nonlinearities as the key to the different dynamics found by these two papers, for uncovering such a structural explanation would be most interesting. However, other differences between the papers obscure such a finding. First, the linearized model is a somewhat reduced version of the full Wharton Model. Secondly, Howrey appears to assume that the shocks are independent both over time and across equations, whereas the Evans-Klein-Saito results I have referred to are based on disturbances independent over time, but having the contemporary covariances estimated from the sample. The effects of this relatively greater interdependence of the model on the cyclical behavior of the endogenous variables are uncertain a priori but could be established.

I would urge the authors of these two papers to collaborate in order to obtain comparative results from their different approaches. By use of the same model and parameter estimates, constant values for all exogenous variables, and identical assumptions about the distribution of disturbances, any differences due to nonlinearities can be isolated, and the adequacy of linear approximations in analyzing dynamic responses assessed.

[3] The emphasis on the short-run nature of the model suggests that the authors would not draw policy implications from this behavior.

McCarthy's suggested method of calculating Monte Carlo shocks, having the covariance structure estimated from the sample, provides a useful addition to our computational techniques, as one simply uses disturbances calculated as linear combinations of independent random variables, with the sample residuals as weights. I question, however, whether one wants to employ this method for generating auto-correlated disturbances in the manner that Evans, Klein, and Saito have adopted in their second set of stochastic experiments. In those runs, the generating mechanism reproduces auto-correlations up to the forty-fourth order. Most of the smoothing effects and longer cyclical periods are likely to be obtained from merely first- or second-order auto-correlation, and it is straining the data rather fine to estimate auto-correlation coefficients of all orders, as well as the structural parameters of the model.

The authors found no clear gain from using first-order auto-correlation corrections in making their short-run forecasts of the historical period. However, in view of the abundant evidence of auto-correlated data in quarterly models, it would be interesting to experiment further with this approach, employing asymptotically more efficient estimators of both the structural and auto-correlation parameters.