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SHORT- AND LONG-TERM SIMULATIONS WITH THE OBE ECONOMETRIC MODEL

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1 INTRODUCTION

THIS paper reports procedures used and some results obtained from various simulations with versions of the OBE Econometric Model. The results presented are not comprehensive, some results being analyzed in another paper prepared for this Conference [21].

The section which follows provides a brief description of the OBE Model structure and a note on equation normalization for model solution. In Section 3, the procedures and results for ex post simulations over the sample period are considered. Modifications in model structure, and other procedures used for twenty-five-year simulations, are presented in Section 4. Also discussed are results obtained from simulations with stochastic shocks applied to endogenous behavioral equations. Included is a spectral analysis of real *GNP* series generated from these runs. A final section summarizes major results.

NOTE: All of the above are members of the Econometric Branch, Office of Business Economics, U.S. Department of Commerce. Although I [Mr. Green] assumed primary responsibility for the project and wrote this paper, I drew heavily upon the contributions of my two colleagues. We benefited greatly from the cooperation of fellow econometricians at other institutions: particularly, Lawrence Klein, Philip Howrey, and Michael McCarthy, of the University of Pennsylvania; and Gary Fromm and George Shink, of the Brookings Institution. Principal quality research assistance was provided by Judith K. Pritchard. Additional assistance was provided by Charles Alexander, Jr., and Fannie Hall. The views expressed are those of the author and do not necessarily agree with those of the U.S. Department of Commerce.

2 THE OBE ECONOMETRIC MODEL

2.1 THE STRUCTURAL EQUATIONS

The simulations reported in the next section use a quarterly econometric model of the U.S. economy developed in the Econometric Branch of the Office of Business Economics. The present model contains 56 stochastic equations and is the outgrowth of an earlier 36-equation model [17]. The present model structure follows the general scheme of the earlier version, but some parts of the model have been expanded, and many equations have been respecified. The present model includes endogenous equations for fixed nonresidential investment, an expanded financial sector, additional tax and transfer functions, and major respecification of price, wage rate, employment, and labor force equations. Appendix A defines all symbols used, and a complete list of structural equations is given in Appendix B. Alternative specifications are given for some equations. In such cases, the discussion below is confined to forms marked (*a*), which were used in the sample-period simulations. The (*b*) alternatives were used for the twenty-five-year stochastic simulations, and will be discussed in Section 4.

As in most other macroeconometric models, equations for components of GNP, on the product side, are estimated in constant dollar terms, while income items are estimated as current dollar values. The major exogenous variables are government purchases, government employment, gross exports, consumption of housing services, population, Federal Reserve member bank nonborrowed reserves, reserve requirements, the Federal Reserve discount rate, tax rates, and some transfer items.

The consumption equations relate components of consumption expenditures to relative prices, disposable income (sometimes disaggregated into transfer and nontransfer income), and measures of cyclical activity. Also, the equations for consumer durables expenditures include allowances for credit and liquidity effects and, for the automobile expenditures equation, strike effects. Taken as a whole, the consumption equations show a short-run marginal propensity to consume out of current disposable income of 0.464, which is somewhat lower than in other models [*cf.* 6, 7, 8, and 12], reflecting, in part, the exogenous treatment of housing services.

Equation 10, for the ratio of fixed investment in nonresidential structures and equipment (*ISE*) to capacity output, is an adaptation of research by Shirley Almon [2, 3], who estimated separately the lag between expenditures and appropriations, and that between investment determinants and appropriations. We have formed convolutions of her estimates of these lags, and have used the resulting relative weights in estimating the *ISE* equation shown. The explanatory variables in this equation—output, interest rates, and deflated cash flow—follow the determinants used by Almon.

Interest rates (with a shift of variable during the bills-only policy), a rent to cost-of-housing ratio, and the vacancy level (expressed as a deviation from a long-term trend) are used as explanatory variables in the equation for housing starts (*HS*). Housing investment (*IH*) is determined by a phase out of *HS* levels, where the phase weights are those used by the Census Bureau.

The change in business inventories (*II*) is split into two parts: change in auto inventory investment (*IIA*), and change in nonauto inventory investment (*IINA*). Each equation uses a stock adjustment mechanism to current and lagged sales levels. In addition, the lagged changes in unfilled orders and inventories are used as explanatory variables in the *IINA* equation. The implied adjustment of inventory stock to recent sales is much more rapid for autos than for the nonauto case.

Imports are divided into merchandise (*IMT*), military expenditures (*IMG*), and "other" services (*IMS*) imports. The second category is treated exogenously. *IMS* is made a function of current real disposable income and past levels of services imports. The cyclical sensitivity of *IMT* is introduced by a variable coefficient on output, the effect of which varies with domestic industrial capacity utilization (*CUW*). A relative price variable, and a dummy variable for dock strikes, are also included in this equation.

Since government purchases and housing services are treated exogenously, the broadest, essentially endogenous, measure of real productive activity in the OBE Model is real private *GNP*, excluding housing services (*X*). The basic price in the OBE Model is the implicit price deflator for *X* (*P*). The determinants in the *P* equation are unit labor costs, recent relative changes in final demand (modified by the level of capacity utilization), and a time trend. The interaction of capacity utilization and relative changes in final demand is an attempt to allow for demand effects, which are more evident at high levels of capacity utilization. Component deflators for the major categories of final demand are made dependent mainly on the over-all deflator and on the wage rate.

Since there is both an equation for *P* and equations, or exogenous values, for all components, the price sector is overdetermined. As a result, there are, initially, two estimates for *P*—one from the equation for *P*, and the other from a properly weighted sum of the component deflators. Any discrepancy between these two estimates of *P* is resolved by arbitrarily adjusting the component deflators. That is, the equation-determined over-all deflator serves as a control index to which component prices are adjusted.

The equation for wage per employee (*WR*) is of the Philips-curve type, in which the relative change in the wage rate is a function of the inverse of the unemployment rate. This equation is expressed as wage per employee rather than wage per man-hour because of substantial deficiencies of the aggregate hours series. In addition to the inverse of the unemployment rate, this equation includes the composition of unemployment, the rate of change in manufacturing hours, and recent rates of change in consumer prices as explanatory variables. Since unit labor costs are a key variable in the equation for *P*, the *WR* equation plays an important role in the determination of the over-all price level.

The equations for capacity output, labor force, employment, and hours are highly interrelated. The full rationale for these formulations is discussed in another paper [13]. First, a constrained Cobb-Douglas production function is estimated, along with the two equations for civilian labor force, and an equation for private weekly hours. Equations for potential private employment (*EC*), and for potential private weekly hours (*HC*), are derived from the above-mentioned equations by setting the capacity utilization index (*CUW*) equal to 1.0, and by setting unemployment rates at frictional levels. Values of *EC* and *HC*, together with lagged values of capital stock, are then used to solve for potential private *GNP*, excluding housing services (*XC*). The equations for private civilian employment (E) and private man-hours (H) are both partial adjustment mechanisms, constrained so that when X reaches XC, the desired employment equals EC, and the desired level of weekly hours is HC.

Corporate profits and inventory valuation adjustment (CPR) is made a multiplicative function of output, the wage share, and CUW. There are equations for two other income items: entrepreneurial income and dividends, and indirect business taxes. Other income components—interest, rent, and capital consumption allowances—are made exogenous. To avoid overdetermination, the statistical discrepancy (SD) is determined endogenously as the residual reconciliation item. However, when the full model is solved, SD is constrained to vary slowly and its absolute value is kept at a low level. This is achieved by adjusting, when necessary, other income components.

The tax and transfer equations relate tax items to tax rates and the tax base wherever possible. Most of the excise tax and transfer functions are the outgrowth of a study by Waldorf [20]. For Federal personal tax payments, different equations for sub-periods are used because of changes in tax legislation.

The set of eight stochastic equations in the monetary sector are broadly patterned after the work of the FED–MIT Model [4, 5]. Exogenous levels of nonborrowed reserves, the discount rate, and reserve requirements—together with disposable personal income—determine liquid assets, money supply, and various interest rates.

Finally, there are three equations—new orders, shipments, and unfilled orders—for manufacturing durables. The main impact of these equations on the rest of the model is through the nonauto inventory equation, which includes the lagged change in unfilled orders as an explanatory variable. We have used a direct estimate of ΔUMD, which is essentially a reduced-form equation, because of better performance over the sample period.

2.2 EQUATION NORMALIZATION FOR MODEL SOLUTION

The set of equations which comprise the OBE Model was solved using the Gauss-Seidel iterative solution method. A complete model solution required about one-fourth of a second of central processor time, using a Univac 1108 computer system. The solution method operates on normalized equation forms, with all normalized error terms set at expected values (zeros except for serial correlation adjustments).

In most cases, the normalized equation is a transformation of the estimated equation. This is most easily understood by considering some of the equations in Appendix B. Of the consumption equations, the equations for *CA*, *COD*, and *CN* are used without transformation, since the left-hand side of each equation consists of a single variable. The *CS* equation, however, is estimated with *CS*/*N* as the dependent variable; for model solution, the equation is rewritten by multiplying both sides of the equation by *N*, so that only *CS* is on the left-hand side of the equation. Similarly, the equation estimated as *ISE*/*XC* is normalized as an equation in *ISE*, and all equations estimated with dependent variables of the form ln (*x*) are converted to antilog form in the normalization process.

A discussion of constant-term procedures and the application of random shocks to the normalized model equations will be found below. It should be noted that the error properties of a normalized equation may be quite different from those of the corresponding estimated equation. In particular, if the normalization involves converting to antilogs, or multiplication of both sides of the equation by a variable, then the error term in the normalized form will be heteroscedastic if the error term in the estimated form was homoscedastic.

3 SAMPLE-PERIOD SIMULATIONS

THIS section discusses various simulations made with the model described in the previous section. All of these simulations used ex post data, revised through June, 1968, for "actual" values. (Subsequent data revisions are not reflected in the results presented.) The model equations were estimated using this same data base. For all simulations, exogenous variables were set at actual, ex post levels.

Six quarter ex post forecasts were made, using several adjustment procedures for serial correlation in the estimated endogenous equations. These procedures and brief results are presented in Section 3.1. Following this, we provide an analysis of single equation, and model forecast, errors for all endogenous variables. Section 3.3 considers, also briefly, the performance of the model as evidenced by short simulations around NBER reference-cycle turning points within the sample period. A final subsection comments on the results of a fifty-five-quarter simulation over the entire sample period.

3.1 ADJUSTMENTS FOR SERIAL CORRELATION

It has long been recognized that when serial correlation is present in a regression model, the pattern of equation residuals over prior observations contains information which is useful in prediction. Goldberger shows that in the single equation case with serially correlated residuals, the gain in predictive efficiency associated with such adjustments may be substantial [9]. We consider here appropriate adjustments for forecasts made with the OBE Model.

Six quarter ex post forecasts were made, using four different mechanical procedures for adjusting the constant terms of normalized stochastic equations.

All equations were stated in the normalized forms used to solve the model, and single equation residuals over the sample period were then calculated. A first-order serial correlation coefficient for each normalized equation was estimated from

$$e_t = b_0 e_{t-1} + v \tag{1}$$

and second-order serial correlation coefficients were estimated from

$$e_t = b_1 e_{t-1} + b_2 e_{t-2} + u \tag{2}$$

using least squares in both cases, where e_t refers to a residual value for a particular equation in time period t. For convenience, let t represent the jump-off quarter (i.e., one quarter before the first forecast period). Then $t + i$, $i = 1, 2, \ldots, 6$ represents one of the six forecast quarters.

Procedure 1 involves no constant-term adjustments of any kind.

Procedure 2 is a first-order serial correlation adjustment, using only the observed residual in the jump-off quarter. The adjustment for forecast period i is

(3) $$e_{t+i} = b_0^i e_t$$

Procedure 3 also employs a first-order serial correlation specification, but a weighted average of the last two residuals is used. This method guards against giving excessive weight to a large random element in the jump-off quarter residual.

The adjustment applied in the ith forecast period is

(4) $$e_{t+i} = b_0^i \left(\frac{e_t + b_0 e_{t-1}}{2}\right)$$

Procedure 4 uses a second-order serial correlation specification if the second-order serial correlation is significantly higher than the first-order serial correlation. In such cases, the adjustment for forecast period i is

(5) $$e_{t+i} = b_1 e_{t+i-1} + b_2 e_{t+i-2}$$

In equations where a first-order serial correlation was adequate, Procedure 2 was substituted.

For all procedures, no adjustment was made unless the serial correlation was significant at the 5 per cent level.

Selected summary results from using these four alternative procedures for nineteen different six-quarter model forecasts before NBER reference-cycle peaks and troughs, are presented in Tables 1 and 2. Any correction for serial correlation of residuals resulted in a substantial improvement in the average absolute errors (*AAE*) for the first-quarter forecasted values of the first three variables listed. Averages of forecast errors, without regard to sign, in the first four quarters of each forecast are nearly the same for all procedures, al-

TABLE 1

Average Absolute Errors in First Forecast Quarter of Nineteen Simulations, with Alternative Adjustment Procedures

Variable	No Adjustment	First-Order	First-Order Average	Second-Order
GNP	4.9	3.7	3.7	3.7
GNP58$	3.4	2.5	2.7	2.6
ISE	1.4	0.7	0.9	0.8
II	1.8	1.8	1.8	1.8

TABLE 2

Average of Quarterly Forecast Errors Without Regard for Sign in the First Four Quarters of Nineteen Simulations, with Alternative Adjustment Procedures

Variable	No Adjustment	First-Order	First-Order Average	Second-Order
GNP	5.6	5.2	5.2	5.3
GNP58$	4.2	4.3	4.3	4.3
ISE	1.6	1.3	1.3	1.3
II	2.4	2.5	2.5	2.3

though there is a slight improvement for *GNP* and *ISE* if an adjustment for serial correlation is made. It appears that any adjustment for serial correlation results in better model forecasts of real variables in the first quarter of the forecast, but somewhat larger errors in successive quarters, which to a large extent, cancel out the benefit of the smaller errors in the first forecast quarters. Also, adjustment for serial correlation results in better price forecasts over the entire forecast period.

The brief results presented above suggest that for our model, a first-order serial correlation adjustment is adequate. Of the two first-order procedures, we had a strong a priori preference for Procedure 3, which guards against large random residuals in the jump-off quarter, and which is closer to the adjustments we tend to make in ex ante forecasting. Two sets of short, six-quarter ex post forecasts were made, using first Procedure 1 (no adjustments), and then Procedure 3. All of the results for short forecasts presented below use Procedure 3. The relevant first-order serial correlation coefficients used are given in Table 3.

3.2 AN ANALYSIS OF SINGLE EQUATION AND MODEL FORECAST ERRORS

Econometric-model builders have devoted an overwhelming portion of their research efforts to the structural specification of single equations or small blocks of equations. Little attention has been given

TABLE 3

First-Order Serial Correlation Coefficients (Rho Values) for Endogenous Variables

Variable	Rho	Variable	Rho	Variable	Rho
CA	.438	*KC$*	.922	*RS*	.534
CN	.364	*LH*	–.214	*RT*	.345
COD	.552	*LFP*	.545	*RTB*	.525
CPR	–.120	*LFS*	.633	*TCF*	.603
CS	.483	*OMD*	.429	*TCRI*	.744
CUW	.153	*P*	.573	*TCSL*	.448
DD	.537	*PHS*	.822	*TD*	.465
DSE	.755	*PIE*	–.205	*TEXAV*	.577
EW	.381	*PIH*	.548	*TEXS*	.727
HM	.242	*PIS*	.433	*TISL*	.851
HS	.442	*PN*	–.095	*TPF*	.643
IH	.758	*POD*	.482	*TPSL*	.535
IINA	.191	*PRI*	.610	*TRU*	.729
IMS	.953	*PS*	.848	*TSSW*	.679
IMT	.799	*PWMD*	.411	*UMD*	.310
ISE	.905	*REM*	.953	*URP*	.647
IVA	.217	*RM*	.686	*WR*	.301

NOTE: All of the above coefficients are significant at the 5 per cent level. Variables with nonsignificant serial correlations were not adjusted.

to a comparison of error statistics from full model solutions with those of the component single equations, and their possible implications for model construction. This section represents a modest attempt at such a comparison for endogenous variables of the OBE Econometric Model.

Tables 4, 5, and 6 present average errors, average absolute errors, and root mean square errors for normalized, single-equation solutions and for one- to six-quarter full model ex post forecasts. *All averages shown cover the same forty-eight observations, from 1955-I through 1966-IV.*

The sample period extended over 55 quarters (starting in 1953-II), but the automatic constant-term adjustment procedure used requires data from two previous quarters, so that 1953-IV was the first quarter

TABLE 4

Average Errors for All Endogenous Variables from Single-Equation Solutions and from One Through Six Quarter Ex Post Model Forecast Solutions: 48 Observations, 1955-I–1966-IV

Type of Solution	Variable								
	C	*CA*	*CMP/MH*	*CN*	*COD*	*CPR*	*CS*	*CUW*	*DD*
Single equation	−.15	−.01	.0000	−.08	−.11	.01	.04	−.0008	−.01
First quarter forecasts	−.11	−.03	−.0016	−.05	−.08	−.09	.04	−.0010	.01
Second quarter forecasts	−.25	−.14	−.0039	−.09	−.07	−.23	.04	−.0024	−.02
Third quarter forecasts	−.43	−.23	−.0063	−.14	−.09	−.38	.03	−.0037	−.13
Fourth quarter forecasts	−.61	−.29	−.0083	−.19	−.13	−.59	.00	−.0050	−.21
Fifth quarter forecasts	−.82	−.37	−.0105	−.24	−.19	−.88	−.02	−.0064	−.27
Sixth quarter forecasts	−1.00	−.44	−.0124	−.29	−.25	−1.12	−.04	−.0077	−.31
	DIV	*DPI*	*DSE*	*EC*	*E*	*EW*	*FBF*	*FBSL*	*GNP*
Single equation	.01	−.16	.02	.00	−.03	−.03	.10	.01	−.11
First quarter forecasts	.01	−.36	.01	.01	−.03	−.03	−.16	−.01	−.42
Second quarter forecasts	−.01	−.73	.01	.01	−.05	−.05	−.30	−.03	−1.00
Third quarter forecasts	−.04	−1.10	.01	.01	−.08	−.08	−.46	−.05	−1.58
Fourth quarter forecasts	−.06	−1.44	.01	.01	−.10	−.10	−.64	−.08	−2.13
Fifth quarter forecasts	−.10	−1.81	.01	.01	−.12	−.12	−.88	−.12	−2.80
Sixth quarter forecasts	−.13	−2.14	.01	.01	−.14	−.14	−1.08	−.17	−3.41

(*continued*)

TABLE 4 (*continued*)

Type of Solution	Variable								
	GNP58$	*H*	*HM*	*HS*	*IE*	*IH*	*II*	*II$*	*IIA*
Single equation	.15	−.01	−.04	−8.	.00	.17	.08	.00	−.019
First quarter forecasts	.04	−.01	−.04	−5.	.00	.04	.06	−.02	−.022
Second quarter forecasts	−.19	−.02	−.06	−6.	.01	.00	−.01	−.09	−.040
Third quarter forecasts	−.51	−.02	−.08	−13.	−.01	−.06	−.10	−.17	−.053
Fourth quarter forecasts	−.90	−.03	−.11	−23.	−.04	−.15	−.19	−.27	−.062
Fifth quarter forecasts	−1.35	−.03	−.13	−33.	−.06	−.27	−.29	−.37	−.070
Sixth quarter forecasts	−1.77	−.03	−.15	−43.	−.08	−.40	−.38	−.48	−.078
	IINA	*IMS*	*IMT*	*IS*	*ISE*	*IVA*	*KA*	*KC$*	*KH*
Single equation	.14	.32	−.01	.33	.34	.12	.08	.14	−4.
First quarter forecasts	.12	.03	−.05	.00	.00	.20	.05	−.02	−4.
Second quarter forecasts	.07	.05	−.08	.01	.02	.22	−.04	−.03	−8.
Third quarter forecasts	.00	.07	−.12	.01	.00	.24	−.21	−.05	−15.
Fourth quarter forecasts	−.08	.10	−.15	.00	−.04	.25	−.39	−.08	−21.
Fifth quarter forecasts	−.18	.12	−.20	−.01	−.07	.26	−.60	−.09	−29.
Sixth quarter forecasts	−.26	.16	−.24	−.02	−.10	.26	−.75	−.07	−39.
	KI	*KIA*	*KINA*	*KSE*	*KSE$*	*LH*	*LFP*	*LFS*	*MONEY*
Single equation	.00	.000	−.01	.00	−.01	.21	.00	−.01	.00
First quarter forecasts	.02	−.005	.02	−.01	−.03	.16	.00	−.02	.01
Second quarter forecasts	.03	−.010	.04	−.01	−.06	.16	.01	−.02	−.02
Third quarter forecasts	.03	−.019	.05	−.01	−.11	.03	.01	−.02	−.13
Fourth quarter forecasts	.03	−.025	.05	−.02	−.17	−.17	.00	−.03	−.21
Fifth quarter forecasts	.02	−.032	.05	−.02	−.22	−.49	.00	−.03	−.27
Sixth quarter forecasts	.00	−.039	.04	−.02	−.22	−.77	.00	−.04	−.31

	NETEXP	*OMD*	*P*	*PADJ*	*PC*	*PERINC*	*PGNP*	*PHS*	*PIE*
Single equation	−.31	−.20	−.0007	—	−.0001	.00	.00	−.0897	.0001
First quarter forecasts	.02	−.27	−.0010	−.0002	−.0009	−.44	−.09	−.0211	−.0014
Second quarter forecasts	.03	−.46	−.0019	−.0005	−.0017	−.87	−.16	−.0387	−.0028
Third quarter forecasts	.04	−.62	−.0026	−.0005	−.0023	−1.32	−.21	−.0554	−.0041
Fourth quarter forecasts	.06	−.77	−.0030	−.0005	−.0027	−1.72	−.25	−.0680	−.0052
Fifth quarter forecasts	.07	−.97	−.0036	−.0005	−.0031	−2.16	−.29	−.0792	−.0064
Sixth quarter forecasts	.08	−1.14	−.0040	−.0005	−.0035	−2.56	−.32	−.0897	−.0072

	PIH	*PIS*	*PISE*	*PN*	*POD*	*PRI*	*PROD*	*PS*	*PWMD*
Single equation	−.0008	−.0011	.0001	−.0001	−.0005	−.08	−.10	.0004	−.0004
First quarter forecasts	−.0011	−.0020	−.0016	−.0011	−.0008	−.09	.11	−.0007	−.0008
Second quarter forecasts	−.0019	−.0037	−.0031	−.0023	−.0014	−.15	.13	−.0016	−.0015
Third quarter forecasts	−.0026	−.0049	−.0043	−.0033	−.0018	−.23	.17	−.0021	−.0024
Fourth quarter forecasts	−.0032	−.0058	−.0053	−.0041	−.0019	−.31	.25	−.0021	−.0031
Fifth quarter forecasts	−.0040	−.0070	−.0063	−.0051	−.0019	−.37	.35	−.0021	−.0034
Sixth quarter forecasts	−.0046	−.0079	−.0073	−.0060	−.0020	−.45	.43	−.0020	−.0034

	REM	*RESF*	*RL*	*RM*	*RS*	*RT*	*RTB*	*SD*	*SDADJ*
Single equation	.18	.0177	.01	−.01	−.01	.00	.00	−.10	—
First quarter forecasts	.00	.0156	.00	−.01	−.02	.00	−.01	.11	−.12
Second quarter forecasts	−.01	.0222	.01	−.02	−.02	−.01	−.01	.13	−.06
Third quarter forecasts	−.02	.0387	.01	−.03	−.03	−.01	−.01	.17	−.08
Fourth quarter forecasts	−.02	.0512	.02	−.02	−.03	−.02	−.02	.25	−.12
Fifth quarter forecasts	−.03	.0631	.03	−.01	−.03	−.02	−.02	.35	−.14
Sixth quarter forecasts	−.04	.0707	.03	.00	−.03	−.02	−.02	.43	−.12

(*continued*)

TABLE 4 (*concluded*)

Type of Solution	Variable								
	SIB	*SIP*	*SMD*	*SRATE*	*TCF*	*TCRI*	*TCSL*	*TD*	*TEXAV*
Single equation	.02	.02	.95	−.0001	.07	.0000	.01	.01	−.03
First quarter forecasts	.00	.00	.76	.0002	−.08	−.0014	.00	.01	−.02
Second quarter forecasts	.00	.00	1.50	.0003	−.14	−.0015	.00	−.03	−.02
Third quarter forecasts	.00	.00	2.17	.0003	−.20	−.0014	.00	−.07	−.03
Fourth quarter forecasts	.00	.00	2.68	.0003	−.28	−.0008	.00	−.11	−.04
Fifth quarter forecasts	−.01	−.01	3.17	.0003	−.41	.0006	−.01	−.22	−.04
Sixth quarter forecasts	−.02	−.02	3.52	.0003	−.51	.0026	−.02	−.29	−.05
	TEXS	*TIF*	*TISL*	*TPF*	*TPSL*	*TRP*	*TRU*	*TSSW*	*UMD*
Single equation	.00	−.03	.10	.02	.00	.02	.02	.05	−.17
First quarter forecasts	.01	−.01	.00	−.06	−.02	.01	.01	.01	−.13
Second quarter forecasts	.01	−.02	−.01	−.12	−.02	.03	.03	.01	−.38
Third quarter forecasts	.01	−.03	−.02	−.18	−.03	.05	.05	.00	−.67
Fourth quarter forecasts	.00	−.04	−.03	−.24	−.04	.07	.07	−.01	−.99
Fifth quarter forecasts	−.01	−.05	−.06	−.30	−.06	.09	.09	−.02	−1.26
Sixth quarter forecasts	−.02	−.07	−.09	−.36	−.07	.11	.11	−.04	−1.53

	UNI-TLC	UN-RATE	URP	V	W	WR	X	XC
Single equation	.0000	.00	.0000	−4.	−.06	.0017	.14	.00
First quarter forecasts	−.0008	.03	.0002	−4.	−.37	−.0041	.03	.03
Second quarter forecasts	−.0014	.06	.0005	−8.	−.74	−.0098	−.19	.03
Third quarter forecasts	−.0019	.09	.0008	−15.	−1.11	−.0154	−.51	.03
Fourth quarter forecasts	−.0022	.11	.0010	−21.	−1.43	−.0200	−.90	.03
Fifth quarter forecasts	−.0025	.14	.0012	−29.	1.79	−.0253	−1.35	.04
Sixth quarter forecasts	−.0027	.16	.0014	−39.	2.11	−.0298	−1.77	.06

NOTE: See Appendix A for full definitions of all symbols.

TABLE 5

Average Absolute Errors for All Endogenous Variables from Single-Equation Solutions and from One Through Six Quarter Ex Post Model Forecast Solutions: 48 Observations, 1955-I–1966-IV

Type of Solution	Variable								
	C	*CA*	*CMP/MH*	*CN*	*COD*	*CPR*	*CS*	*CUW*	*DD*
Single equation	1.34	.85	.00	.81	.44	.72	.35	.0083	.52
First quarter forecasts	1.48	.82	.02	.85	.40	1.60	.35	.0117	.44
Second quarter forecasts	1.91	.94	.02	1.04	.43	2.00	.39	.0191	.76
Third quarter forecasts	2.48	1.05	.02	1.20	.50	2.48	.40	.0235	.93
Fourth quarter forecasts	2.75	1.12	.03	1.30	.60	2.85	.41	.0261	1.00
Fifth quarter forecasts	2.91	1.16	.02	1.33	.72	3.11	.42	.0285	1.09
Sixth quarter forecasts	3.06	1.21	.02	1.37	.79	3.18	.44	.0294	1.17
	DIV	*DPI*	*DSE*	*EC*	*E*	*EW*	*FBF*	*FBSL*	*GNP*
Single equation	.17	1.39	.07	.00	.16	.16	.45	.09	3.22
First quarter forecasts	.17	1.43	.04	.06	.17	.17	1.24	.25	2.84
Second quarter forecasts	.25	2.43	.05	.07	.28	.28	1.79	.36	4.54
Third quarter forecasts	.31	3.10	.06	.07	.36	.36	2.23	.45	5.93
Fourth quarter forecasts	.35	3.65	.06	.07	.44	.44	2.48	.48	6.62
Fifth quarter forecasts	.39	3.90	.07	.07	.48	.48	2.64	.51	7.01
Sixth quarter forecasts	.41	4.10	.07	.07	.50	.50	2.66	.55	7.19

	GNP58$	*H*	*HM*	*HS*	*IE*	*IH*	*II*	*II$*	*IIA*
Single equation	2.85	.12	.19	54.	.49	.42	1.51	1.51	.596
First quarter forecasts	2.35	.12	.19	48.	.87	.31	1.45	1.46	.609
Second quarter forecasts	3.58	.13	.27	52.	1.16	.43	1.75	1.77	.662
Third quarter forecasts	4.47	.14	.30	56.	1.40	.45	2.01	2.04	.670
Fourth quarter forecasts	4.92	.15	.31	62.	1.54	.52	2.06	2.12	.698
Fifth quarter forecasts	5.42	.15	.32	73.	1.63	.63	2.18	2.26	.706
Sixth quarter forecasts	5.82	.15	.34	85.	1.69	.79	2.44	2.51	.709
	IINA	*IMS*	*IMT*	*IS*	*ISE*	*IVA*	*KA*	*KC$*	*KH*
Single equation	1.21	.34	.54	1.80	1.75	.54	.25	.37	10.
First quarter forecasts	1.19	.10	.42	.48	.98	.67	.94	.31	10.
Second quarter forecasts	1.64	.13	.59	.56	1.34	.69	1.55	.60	18.
Third quarter forecasts	1.89	.17	.71	.66	1.69	.76	2.06	.94	29.
Fourth quarter forecasts	1.89	.20	.79	.72	1.96	.79	2.49	1.30	40.
Fifth quarter forecasts	1.98	.21	.85	.77	2.13	.79	2.98	1.67	53.
Sixth quarter forecasts	2.23	.24	.90	.83	2.30	.79	3.28	2.04	64.
	KI	*KIA*	*KINA*	*KSE*	*KSE$*	*LH*	*LFP*	*LFS*	*MONEY*
Single equation	.03	.000	.63	.03	.04	1.22	.07	.19	.00
First quarter forecasts	.38	.152	.87	.25	.26	1.32	.06	.15	.44
Second quarter forecasts	.64	.202	1.11	.52	.61	1.71	.07	.18	.76
Third quarter forecasts	.94	.222	1.40	.88	1.05	2.14	.07	.19	.93
Fourth quarter forecasts	1.19	.236	1.70	1.27	1.54	2.63	.07	.20	1.00
Fifth quarter forecasts	1.36	.246	1.89	1.65	2.05	3.16	.07	.21	1.09
Sixth quarter forecasts	1.45	.255	1.95	2.01	2.52	3.75	.07	.22	1.17

(*continued*)

TABLE 5 (*concluded*)

Type of Solution	Variable								
	NETEXP	*OMD*	*P*	*PADJ*	*PC*	*PERINC*	*PGNP*	*PHS*	*PIE*
Single equation	—	1.45	.0028	—	.0014	.00	.00	.2298	.0031
First quarter forecasts	.45	1.68	.0027	.0018	.0027	1.64	.22	.1507	.0047
Second quarter forecasts	.64	1.89	.0045	.0019	.0042	2.91	.37	.1973	.0084
Third quarter forecasts	.77	2.09	.0059	.0018	.0057	3.69	.49	.2295	.0113
Fourth quarter forecasts	.86	2.24	.0069	.0019	.0064	4.33	.57	.2522	.0140
Fifth quarter forecasts	.93	2.41	.0076	.0018	.0071	4.67	.62	.2699	.0160
Sixth quarter forecasts	.96	2.59	.0081	.0017	.0075	4.92	.66	.2802	.0178
	PIH	*PIS*	*PISE*	*PN*	*POD*	*PRI*	*PROD*	*PS*	*PWMD*
Single equation	.0043	.0049	.0031	.0023	.0022	.75	.00	.0048	.0032
First quarter forecasts	.0052	.0071	.0050	.0039	.0023	.59	.51	.0046	.0034
Second quarter forecasts	.0071	.0100	.0083	.0063	.0044	.77	.60	.0064	.0059
Third quarter forecasts	.0086	.0120	.0110	.0087	.0062	.89	.68	.0075	.0080
Fourth quarter forecasts	.0099	.0134	.0129	.0099	.0077	1.05	.71	.0088	.0101
Fifth quarter forecasts	.0108	.0141	.0144	.0111	.0094	1.16	.78	.0100	.0123
Sixth quarter forecasts	.0114	.0147	.0159	.0116	.0113	1.29	.88	.0114	.0145
	REM	*RESF*	*RL*	*RM*	*RS*	*RT*	*RTB*	*SD*	*SDADJ*
Single equation	.20	.0336	.05	.08	.08	.05	.20	4.04	—
First quarter forecasts	.04	.0766	.07	.07	.14	.05	.17	.98	.37
Second quarter forecasts	.06	.1130	.11	.13	.20	.08	.21	1.34	.28
Third quarter forecasts	.07	.1298	.13	.17	.21	.10	.22	1.43	.24
Fourth quarter forecasts	.09	.1425	.15	.20	.21	.12	.22	1.51	.22
Fifth quarter forecasts	.11	.1496	.16	.23	.21	.14	.22	1.54	.20
Sixth quarter forecasts	.13	.1572	.16	.25	.21	.14	.22	1.58	.18

	SIB	*SIP*	*SMD*	*SRATE*	*TCF*	*TCRI*	*TCSL*	*TD*	*TEXAV*
Single equation	.08	.08	1.52	.0006	.36	.0187	.05	.53	.16
First quarter forecasts	.06	.06	1.44	.0037	.83	.0223	.05	.50	.15
Second quarter forecasts	.08	.08	2.58	.0039	1.12	.0307	.06	.86	.17
Third quarter forecasts	.09	.09	3.33	.0042	1.38	.0365	.06	1.21	.17
Fourth quarter forecasts	.10	.10	3.98	.0045	1.56	.0430	.07	1.56	.17
Fifth quarter forecasts	.11	.11	4.80	.0048	1.64	.0524	.07	1.81	.17
Sixth quarter forecasts	.11	.11	5.25	.0049	1.64	.0614	.07	2.09	.17

	TEXS	*TIF*	*TISL*	*TPF*	*TPSL*	*TRP*	*TRU*	*TSSW*	*UMD*
Single equation	.24	.31	.36	.28	.14	.23	.23	.16	1.19
First quarter forecasts	.15	.23	.20	.35	.12	.26	.26	.13	1.13
Second quarter forecasts	.19	.27	.28	.51	.15	.34	.34	.16	2.15
Third quarter forecasts	.20	.28	.32	.60	.16	.42	.42	.18	3.10
Fourth quarter forecasts	.21	.29	.37	.66	.16	.48	.48	.20	4.05
Fifth quarter forecasts	.21	.28	.40	.67	.17	.49	.49	.21	4.91
Sixth quarter forecasts	.21	.28	.42	.72	.17	.49	.49	.22	5.34

	UNI-TLC	*UN-RATE*	*URP*	*V*	*W*	*WR*	*X*	*XC*
Single equation	.0000	.00	.0015	10.	1.15	.0213	2.86	.00
First quarter forecasts	.0028	.23	.0025	10.	1.73	.0280	2.34	.33
Second quarter forecasts	.0044	.37	.0035	18.	3.05	.0449	3.58	.35
Third quarter forecasts	.0055	.46	.0041	29.	3.77	.0530	4.48	.35
Fourth quarter forecasts	.0064	.49	.0044	40.	4.32	.0564	4.94	.36
Fifth quarter forecasts	.0069	.52	.0045	53.	4.43	.0579	5.43	.43
Sixth quarter forecasts	.0074	.52	.0046	64.	4.58	.0578	5.82	.54

NOTE: See Appendix A for full definitions of all symbols.

TABLE 6

Root Mean Square Errors for All Endogenous Variables from Single-Equation Solutions and from One Through Six Quarter Ex Post Model Forecast Solutions: 48 Observations, 1955-I–1966-IV

Type of Solution	Variable								
	C	*CA*	*CMP/MH*	*CN*	*COD*	*CPR*	*CS*	*CUW*	*DD*
Single equation	1.69	1.04	.0000	1.07	.54	.89	.44	.0102	.66
First quarter forecasts	1.93	1.00	.0186	1.14	.52	1.98	.43	.0154	.59
Second quarter forecasts	2.52	1.19	.0265	1.32	.58	2.53	.50	.0234	1.01
Third quarter forecasts	3.08	1.33	.0310	1.50	.68	3.03	.56	.0282	1.17
Fourth quarter forecasts	3.42	1.40	.0320	1.60	.78	3.29	.57	.0308	1.26
Fifth quarter forecasts	3.69	1.44	.0308	1.67	.96	3.52	.56	.0315	1.33
Sixth quarter forecasts	3.90	1.47	.0308	1.74	1.09	3.70	.57	.0325	1.39
	DIV	*DPI*	*DSE*	*EC*	*E*	*EW*	*FBF*	*FBSL*	*GNP*
Single equation	.24	1.76	.08	.00	.21	.21	.60	.11	3.79
First quarter forecasts	.24	1.84	.06	.07	.24	.24	1.61	.31	3.64
Second quarter forecasts	.34	2.94	.07	.08	.35	.35	2.26	.45	5.60
Third quarter forecasts	.40	3.74	.07	.08	.43	.43	2.74	.55	7.13
Fourth quarter forecasts	.45	4.30	.08	.08	.51	.51	2.94	.59	7.90
Fifth quarter forecasts	.49	4.67	.08	.08	.55	.55	3.02	.64	8.44
Sixth quarter forecasts	.52	5.08	.09	.08	.56	.56	3.11	.67	8.91

	GNP58$	*H*	*HM*	*HS*	*IE*	*IH*	*II*	*II$*	*IIA*
Single equation	3.41	.14	.23	67.	.57	.57	1.89	1.90	.764
First quarter forecasts	3.12	.15	.26	63.	1.09	.40	1.86	1.87	.782
Second quarter forecasts	4.49	.17	.35	65.	1.41	.54	2.24	2.27	.851
Third quarter forecasts	5.59	.17	.39	69.	1.64	.56	2.50	2.56	.864
Fourth quarter forecasts	6.20	.18	.39	77.	1.81	.62	2.60	2.68	.900
Fifth quarter forecasts	6.74	.18	.39	93.	1.95	.77	2.71	2.81	.902
Sixth quarter forecasts	7.34	.18	.41	109.	2.08	.96	2.94	3.05	.910
	IINA	*IMS*	*IMT*	*IS*	*ISE*	*IVA*	*KA*	*KC$*	*KH*
Single equation	1.72	.42	.67	2.32	2.24	.71	.38	.43	12.
First quarter forecasts	1.72	.14	.51	.62	1.26	.84	1.14	.39	12.
Second quarter forecasts	2.08	.18	.71	.70	1.75	.89	1.94	.77	22.
Third quarter forecasts	2.29	.21	.87	.80	2.13	.96	2.66	1.20	36.
Fourth quarter forecasts	2.37	.24	.98	.90	2.42	.98	3.28	1.68	49.
Fifth quarter forecasts	2.50	.26	1.07	.95	2.64	.99	3.78	2.15	62.
Sixth quarter forecasts	2.70	.28	1.14	1.00	2.82	.99	4.26	2.61	76.
	KI	*KIA*	*KINA*	*KSE*	*KSE$*	*LH*	*LFP*	*LFS*	*MONEY*
Single equation	.04	.000	2.90	.03	.06	1.82	.08	.25	.00
First quarter forecasts	.48	.195	2.83	.32	.33	1.85	.07	.19	.59
Second quarter forecasts	.82	.253	2.78	.69	.77	2.24	.08	.23	1.01
Third quarter forecasts	1.12	.288	2.85	1.12	1.30	2.64	.08	.25	1.17
Fourth quarter forecasts	1.35	.301	3.18	1.60	1.90	3.16	.08	.26	1.26
Fifth quarter forecasts	1.60	.312	3.34	2.08	2.50	3.81	.08	.26	1.33
Sixth quarter forecasts	1.78	.327	3.42	2.51	3.08	4.35	.08	.27	1.40

(*continued*)

TABLE 6 (*concluded*)

Type of Solution	Variable								
	NETEXP	*OMD*	*P*	*PADJ*	*PC*	*PERINC*	*PGNP*	*PHS*	*PIE*
Single equation	.00	1.80	.0033	—	.0019	.00	.00	.2887	.0044
First quarter forecasts	.54	2.10	.0035	.0022	.0036	2.15	.29	.1859	.0059
Second quarter forecasts	.78	2.37	.0057	.0024	.0055	3.50	.47	.2322	.0104
Third quarter forecasts	.97	2.61	.0074	.0024	.0071	4.44	.61	.2720	.0144
Fourth quarter forecasts	1.08	2.70	.0086	.0023	.0081	5.07	.70	.3008	.0176
Fifth quarter forecasts	1.13	2.95	.0095	.0022	.0089	5.49	.77	.3119	.0201
Sixth quarter forecasts	1.18	3.23	.0102	.0021	.0095	5.98	.83	.3251	.0228
	PIH	*PIS*	*PISE*	*PN*	*POD*	*PRI*	*PROD*	*PS*	*PWMD*
Single equation	.0052	.0064	.0041	.0027	.0027	.91	.02	.0059	.0045
First quarter forecasts	.0065	.0097	.0064	.0049	.0030	.75	.65	.0063	.0046
Second quarter forecasts	.0092	.0126	.0103	.0079	.0055	.95	.78	.0086	.0081
Third quarter forecasts	.0111	.0145	.0135	.0103	.0079	1.09	.88	.0101	.0110
Fourth quarter forecasts	.0123	.0158	.0160	.0120	.0097	1.23	.94	.0116	.0135
Fifth quarter forecasts	.0136	.0172	.0181	.0133	.0120	1.37	1.02	.0129	.0162
Sixth quarter forecasts	.0145	.0187	.0203	.0143	.0142	1.47	1.14	.0138	.0190
	REM	*RESF*	*RL*	*RM*	*RS*	*RT*	*RTB*	*SD*	*SDADJ*
Single equation	.28	.04	.07	.11	.11	.07	.26	4.76	—
First quarter forecasts	.08	.10	.09	.09	.20	.07	.23	1.15	.73
Second quarter forecasts	.10	.15	.13	.17	.26	.11	.27	1.59	.58
Third quarter forecasts	.12	.17	.15	.22	.28	.14	.28	1.78	.58
Fourth quarter forecasts	.14	.18	.18	.25	.28	.17	.28	1.80	.51
Fifth quarter forecasts	.16	.19	.20	.28	.28	.18	.28	1.84	.50
Sixth quarter forecasts	.18	.20	.20	.30	.28	.18	.28	1.91	.47

	SIB	*SIP*	*SMD*	*SRATE*	*TCF*	*TCRI*	*TCSL*	*TD*	*TEXAV*
Single equation	.12	.12	2.03	.0010	.48	.0355	.06	.67	.21
First quarter forecasts	.09	.09	1.93	.0046	1.03	.0395	.07	.67	.19
Second quarter forecasts	.11	.11	3.17	.0051	1.33	.0547	.07	1.06	.22
Third quarter forecasts	.12	.12	4.08	.0055	1.59	.0659	.08	1.42	.23
Fourth quarter forecasts	.13	.13	4.62	.0056	1.73	.0773	.08	1.85	.23
Fifth quarter forecasts	.14	.14	5.52	.0060	1.80	.0899	.09	2.13	.23
Sixth quarter forecasts	.14	.14	6.25	.0062	1.88	.1027	.09	2.42	.22

	TEXS	*TIF*	*TISL*	*TPF*	*TPSL*	*TRP*	*TRU*	*TSSW*	*UMD*
Single equation	.28	.36	.45	.37	.17	.31	.31	.24	1.59
First quarter forecasts	.20	.28	.24	.44	.16	.35	.35	.18	1.51
Second quarter forecasts	.23	.32	.33	.62	.19	.50	.50	.21	2.64
Third quarter forecasts	.24	.34	.39	.72	.21	.60	.60	.25	3.62
Fourth quarter forecasts	.25	.34	.43	.77	.21	.64	.64	.27	4.63
Fifth quarter forecasts	.25	.34	.47	.80	.22	.64	.64	.28	5.55
Sixth quarter forecasts	.25	.35	.50	.86	.23	.62	.62	.28	6.42

	UNI-TLC	*UN-RATE*	*URP*	*V*	*W*	*WR*	*X*	*XC*
Single equation	.0000	.00	.0019	12.	1.39	.0271	3.40	.00
First quarter forecasts	.0035	.31	.0035	12.	2.15	.0366	3.11	.39
Second quarter forecasts	.0054	.49	.0051	22.	3.71	.0571	4.50	.43
Third quarter forecasts	.0067	.60	.0060	36.	4.72	.0685	5.60	.42
Fourth quarter forecasts	.0079	.66	.0065	49.	5.28	.0722	6.21	.45
Fifth quarter forecasts	.0085	.66	.0065	62.	5.52	.0716	6.75	.53
Sixth quarter forecasts	.0093	.65	.0062	76.	5.74	.0725	7.36	.67

NOTE: See Appendix A for full definitions of all symbols.

for which a full model solution could be obtained, and 1955-I was the first quarter for which a sixth quarter forecast was available.

The single equation errors shown for each variable defined by an identity were derived by first substituting into the identity the calculated values from all stochastic variables, and then subtracting the actual value of the variable defined by the identity. The errors shown for first quarter forecasts are averages of errors (forecast minus actual values) in each of the 48 quarters. The errors shown for second quarter forecasts are average values of errors over these same 48 quarters; i.e., each quarter from 1955-I through 1966-IV is now the second quarter of an ex post forecast; and so on, through the sixth quarter. We will not attempt to discuss all of the results contained in these tables, but will concentrate on some of the more important aspects instead.

The extent and direction of bias in any variable can be ascertained from Table 4, which shows average errors. Nonzero values for single equation solutions can arise both from the normalization of equations and from the use of an analysis period which is not identical with the sample period used for equation estimation. The largest single-equation average errors for *GNP* components are slightly over 0.3 billion dollars for *IMS* and *ISE*; most single-equation average errors are quite small. However, the patterns of average forecast errors from full model solutions reveal, for some variables, persistent biases increasing in magnitude as the forecast period is lengthened. For example, first quarter forecasts of *GNP58$* are, on the average, virtually free of bias, but sixth quarter forecasts of *GNP58$* are 1.8 billion dollars low on the average. The biases in current dollar variables are even more striking. Forecasts of *GNP* show an average downward bias of 0.4 billion dollars for first quarter forecasts, but this bias is enlarged to 3.4 billion dollars for sixth quarter forecasts. An examination of the average errors for *P* reveals a downward bias of one-tenth of an index point for first quarter forecasts, mounting to four-tenths of an index point in sixth quarter forecasts.

We cannot give definitive answers to the questions raised by the above-noted biases without additional research, but a key element seems discernible. The downward bias for major current dollar variables is about twice as large as the downward bias in constant dollar counterparts in the sixth quarter forecasts. Moreover, there is noticeable bias for current dollar magnitudes and for prices, even in the first quarter forecasts. The level of wages is a main determinant in the over-all price equation, and wages are simply the product of private employment (E) and wage rates (WR). Both E and WR show a downward bias in the first forecast quarter, and an increasing downward bias as the forecast period is lengthened. The first quarter downward bias in E is the same as the average error from a single-equation solution for E. But the average error from a single-equation solution for WR is slightly positive, while the first quarter forecast solution average error is negative. Thus, it appears that the downward bias of the WR equation when placed in a model environment, leads to low forecasts of prices, and this in turn leads to forecast biases in other variables. When the forecast period is lengthened from one quarter to four or six quarters, these biases cumulate, becoming more prominent because of the under-prediction of lagged endogenous values. This does not necessarily mean that the wage rate and price equations are the only possible culprits. Other equations may also be contributors to the over-all bias. Nevertheless, as a practical aid in forecasting, it may be advisable to introduce adjustments in the WR and/or P equations so that biases in important magnitudes are eliminated.

Two explanatory forays were made in an effort to isolate the biases noted above. First, we tested to make sure that the automatic constant term adjustment procedure was not a culprit. We made simulation runs without automatic constant term adjustments and generated average error statistics. The biases noted above were still present, and the amounts of these biases were virtually unaltered. A second set of simulations was made, using the same procedures as used to generate Table 4, except that two equation parameters were altered slightly. The constant term in the estimated form of the WR equation was increased from 0.0076 to 0.00846, an effective increase of about four dollars per man per year. Also, the constant term in the price equation was increased from 0.263 to 0.264. These two changes eliminated about 97 per cent of the price bias noted earlier. The average error in P for six-quarter forecasts became −0.0001, compared with −0.004 registered in Table 4. Similarly, the GNP bias was cut from −3.41 to −1.84 for six-quarter forecasts. While the price bias was virtually eliminated, these two parameter changes had almost no effect upon the biases in constant dollar magnitudes. These results suggest that further research along these lines may prove fruitful.

The average absolute errors (*AAE*) shown in Table 5 and the root mean square errors (*RMSE*) shown in Table 6 show similar patterns. For nearly all variables, the *AAE* or *RMSE* for the sixth quarter forecasts is from one and a half, to two and a half, times the corresponding error measure for the first quarter forecasts. Some variables—for instance, other durables consumption, demand deposits, and all broad categories of investment—show lower *AAE* or *RMSE* for first quarter forecasts than for the single-equation solutions; this apparently reflects the use of a serial correlation adjustment procedure for the model forecasts. A comparison of single-equation errors with first quarter forecast errors also brings out quite clearly the difficulty of predicting certain variables. For instance, the first quarter *AAE* for each of the consumption variables is not far different from the single-equation *AAE*, but the first quarter *AAE* for corporate profits of 1.6 is more than twice as large as the single-equation counterpart value of 0.72. Profits are residual in nature, and thus are sensitive to errors in the determinants of the profits equation—private output, the wage share, and the industrial capacity utilization index—while the main determinant in the consumption equations is disposable income, which is much more stable and easier to predict.

The *AAE* for aggregates are smaller than sums of the *AAE* for components of the aggregates, reflecting the partial offsetting of errors of opposite sign when aggregates are formed. To illustrate, the sum of *AAE* for components of consumption (*C*)—*CA*, *CN*, *COD*, *CS*—is 2.45 for single-equation solutions, 2.42 for first quarter forecasts, and 3.81 for sixth quarter forecasts. The corresponding *AAE* values for *C* are 1.34, 1.48, and 3.06. The same holds true for the change in inventory investment (*II*), where the *AAE* for *II* is always smaller than the sum of the *AAE* for *IIA* and *IINA*. A similar benefit can be noted in even broader aggregates. The *AAE* for *GNP58$* is 2.35 for first quarter forecasts and 5.82 for sixth quarter forecasts, while the corresponding sums of *AAE* for broad components of *GNP58$*—*C*, *IH*, *II*, *ISE*, and *NETEXP*—are 4.67 and 9.55.

Values of *AAE* or *RMSE* for broad aggregates predicted by the model are fairly small. The *AAE* for first quarter forecasts of *X*, *GNP58$*, and *GNP* are 2.34, 2.35, and 2.84, respectively. Each *AAE* is only one-half of 1 per cent of the average value of the variable to which it refers. The *AAE* from sixth quarter forecasts for these same aggregates are about 1.3 per cent of mean values.

As an aid in appraising the general magnitude of these errors, a comparison can be made with results obtained using an auto-regressive equation for each variable. The *RMSE* (in billions of dollars) from first quarter model forecasts for *X*, *GNP58$*, and *GNP* are 3.11, 3.12, and 3.64, respectively. Comparable *RMSE* values for second-order auto-regressive equations are 4.64, 4.68, and 4.52, and fourth-order auto-regressive equations yield *RMSE* of 4.55, 4.58, and 4.42, respectively. The superior performance of the model is primarily due to better behavior at turning points.

3.3 SHORT SIMULATIONS AROUND TURNING POINTS

It has long been recognized that the most difficult and critical job for any forecaster is the correct indication of turning points in important series. For this reason, we will now consider the performance of short, ex post model forecasts over periods which contain NBER reference-cycle peaks and troughs. The sample period contained six of these critical periods: troughs in 1954-III, 1958-II, and 1961-I; and peaks in 1953-II, 1957-III, and 1960-II. The 1953-II peak is not included in our analysis, since its inclusion would have required solving the model for quarters prior to the sample period. Three simulations were made for each turning point, with first forecast quarters one, two, and three quarters before the one designated as a reference-cycle peak or trough. Each of these forecasts used actual data for all exogenous variables; mechanical constant term adjustments were made, based on the serial correlation in various equations, using the procedure explained above.

Model performance for selected variables over these critical periods is shown in the accompanying charts. Each chart plots actual data (revised through June, 1968) and three forecasted series. For instance, in the top panel of each chart, the dashed line shows forecasted results with 1953-IV as the first forecast quarter; the dotted line traces

CHART 1

Actual and Predicted Values for Gross National Product, Constant (1958) Dollars, Around Five Turning Points

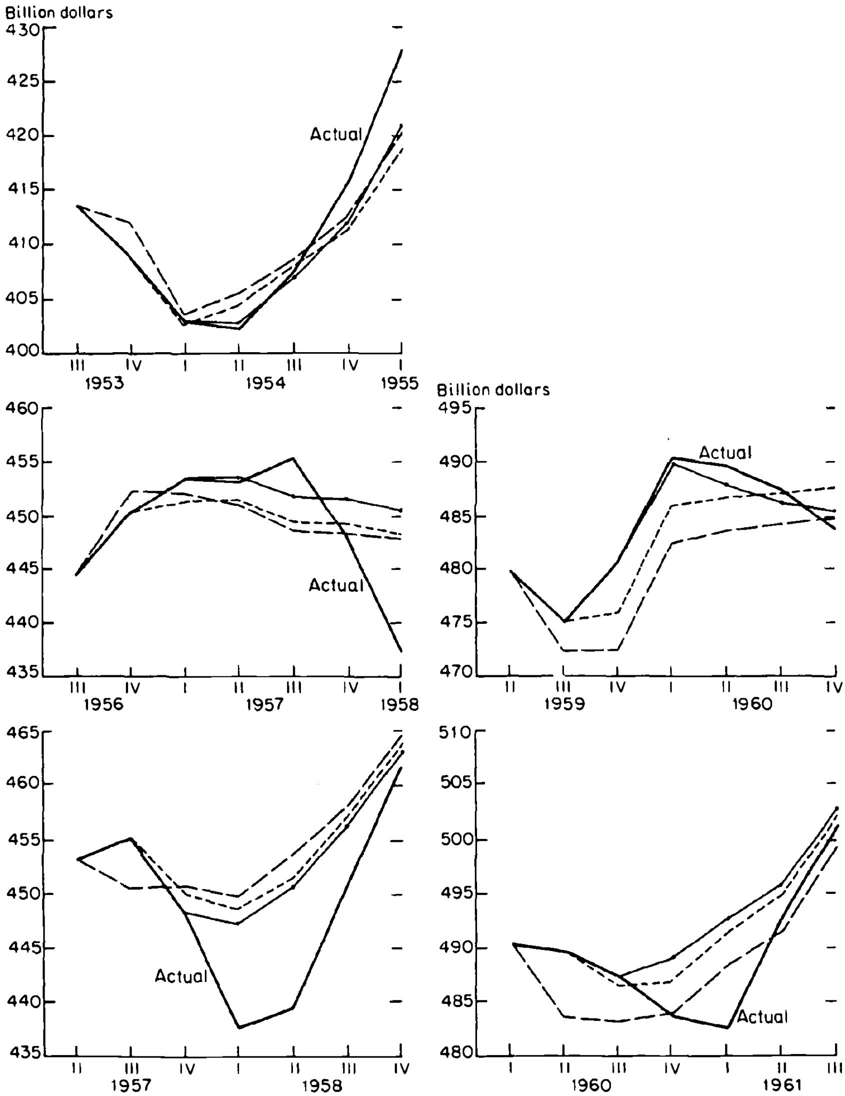


CHART 2

Actual and Predicted Values for Residential Fixed Investment, Constant (1958) Dollars, Around Five Turning Points

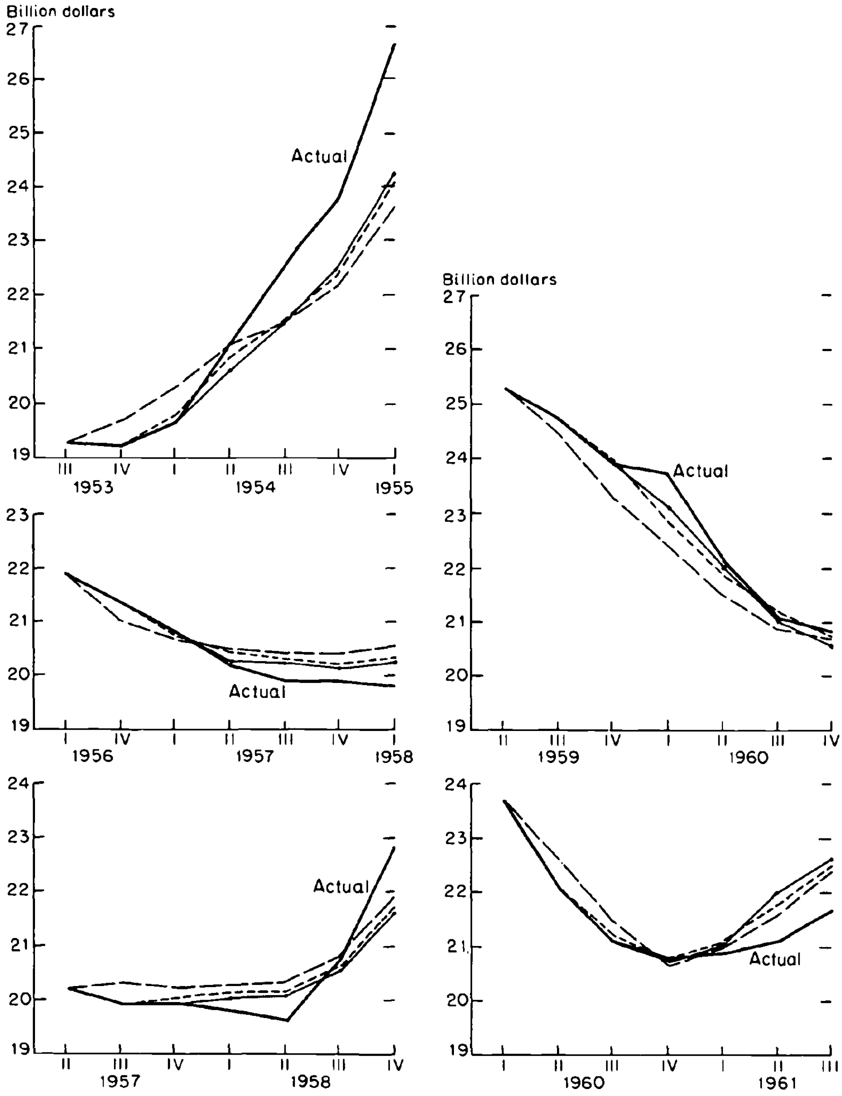


CHART 3

Actual and Predicted Values for Nonresidential Fixed Investment, Constant (1958) Dollars, Around Five Turning Points

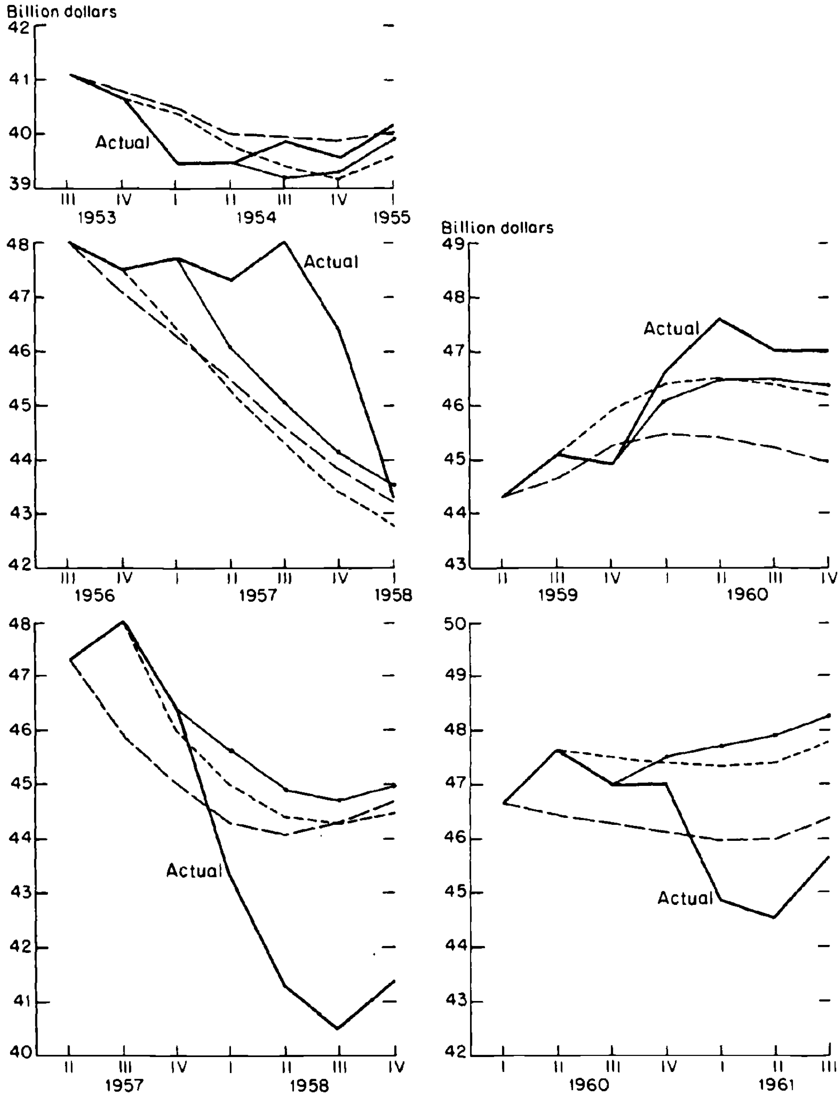


CHART 4

Actual and Predicted Values for Change in Business Inventories, Constant (1958) Dollars, Around Five Turning Points

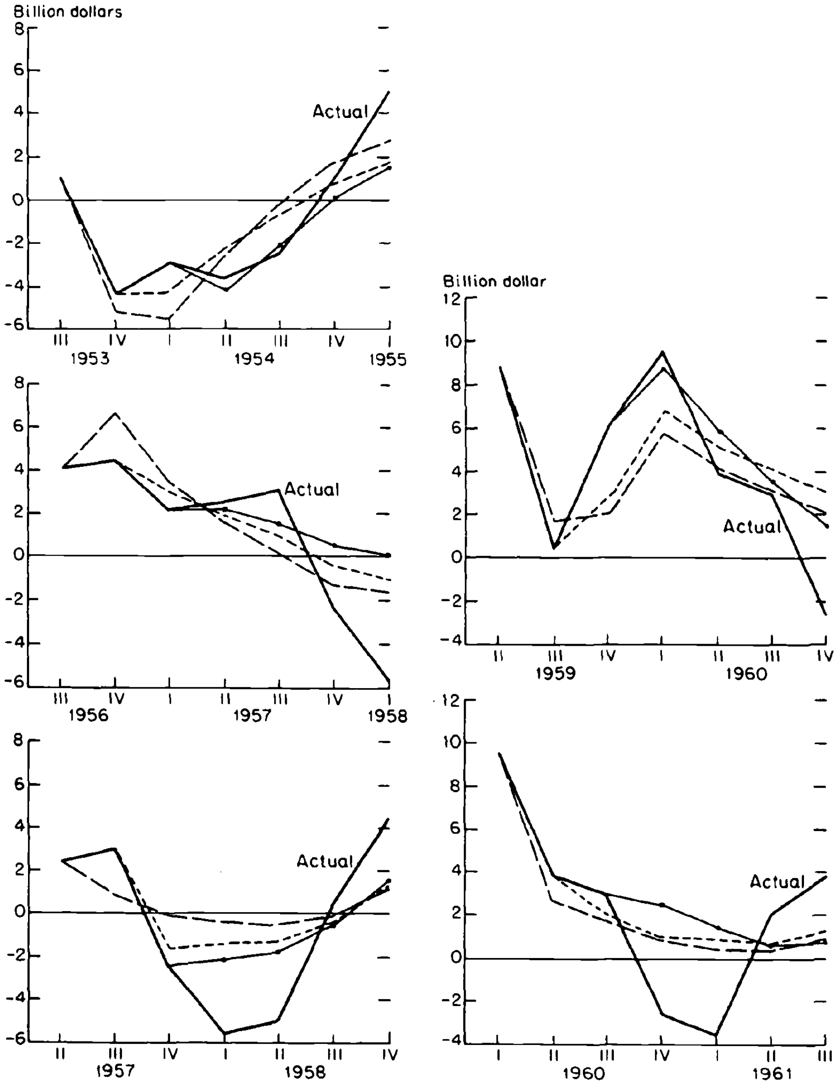


CHART 5

Actual and Predicted Values for Personal Consumption Expenditures, Constant (1958) Dollars, Around Five Turning Points

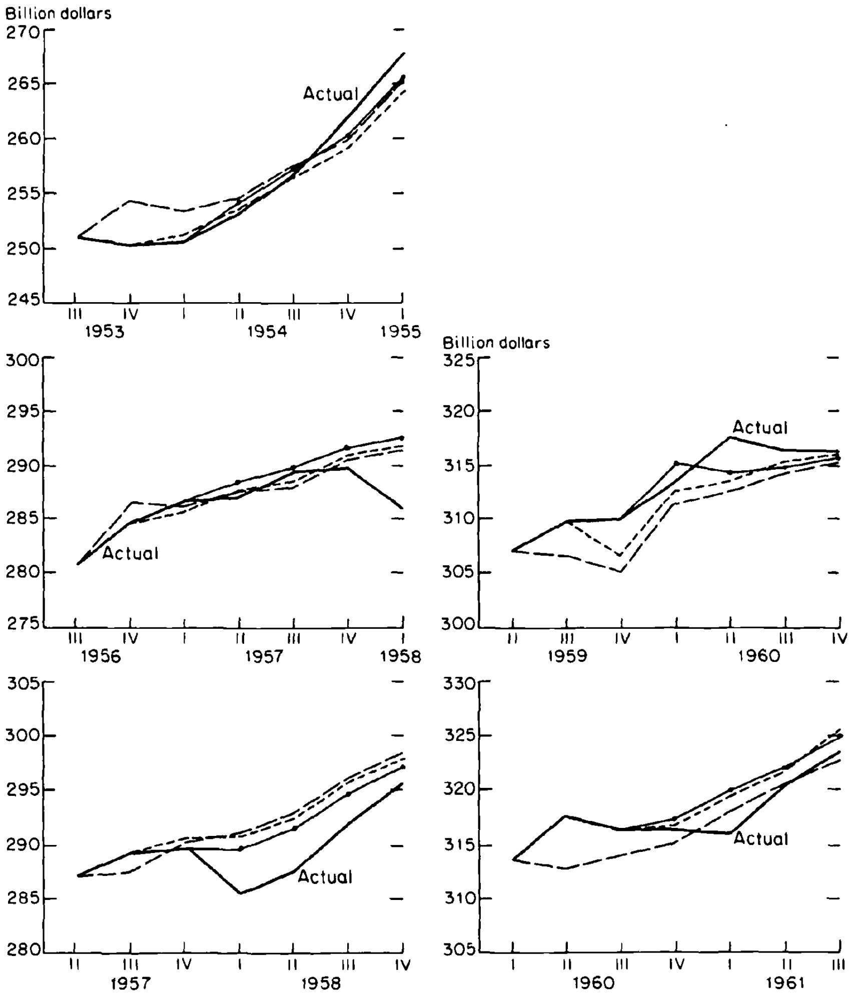


CHART 6

Actual and Predicted Values for Private Civilian Employment Around Five Turning Points

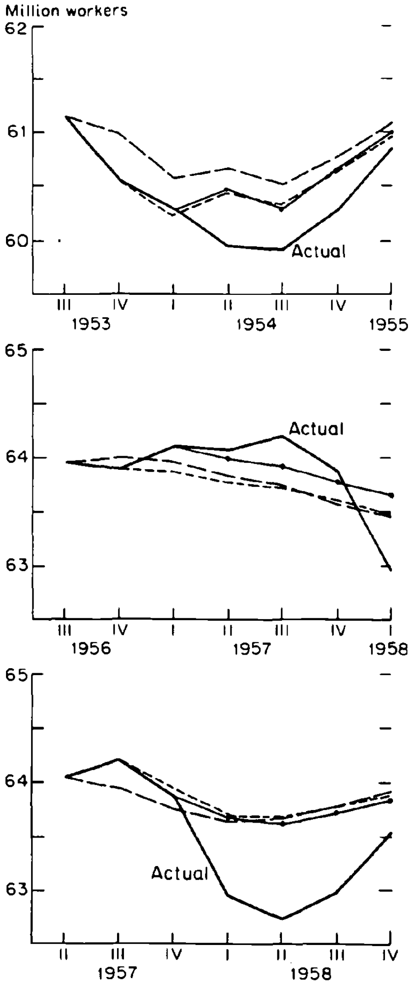


CHART 7

Actual and Predicted Values for Gross National Product, Current Dollars, Around Five Turning Points

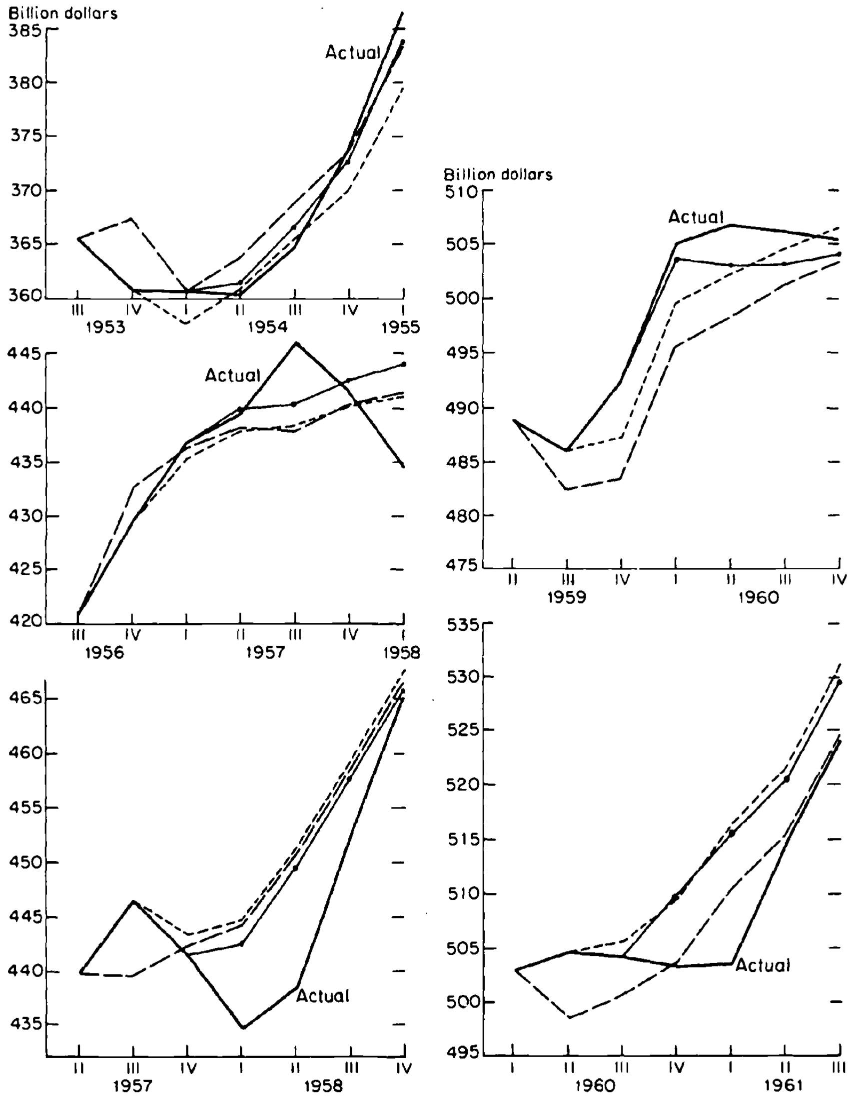


CHART 8

Actual and Predicted Values for Corporate Profits and Inventory Adjustment, Current Dollars, Around Five Turning Points

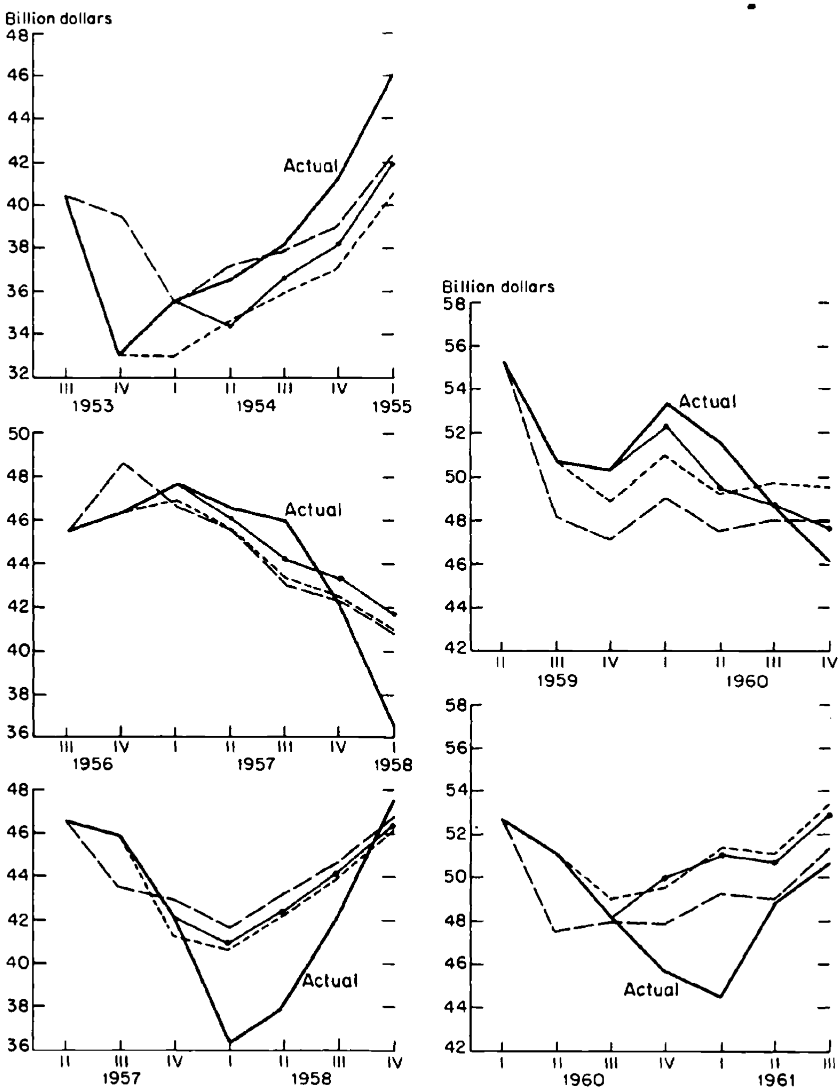


CHART 9

Actual and Predicted Values for Unemployment Rate Around Five Turning Points

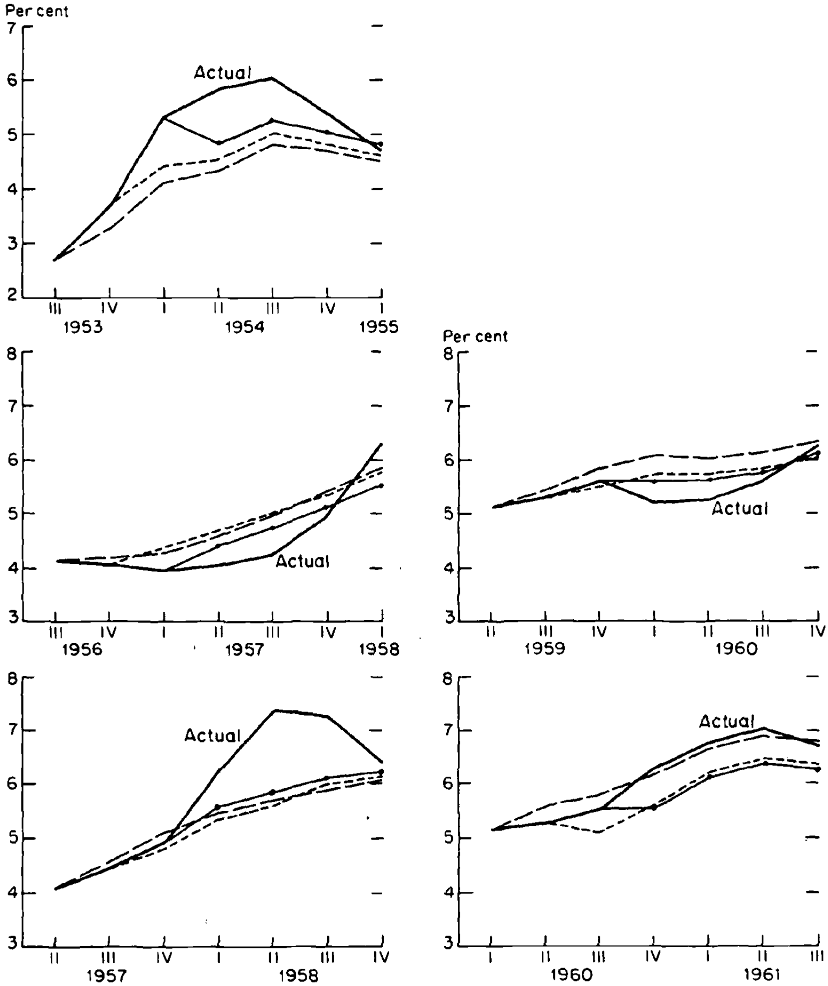


CHART 10

Actual and Predicted Values for Change in Money Supply, Current Dollars, Around Five Turning Points

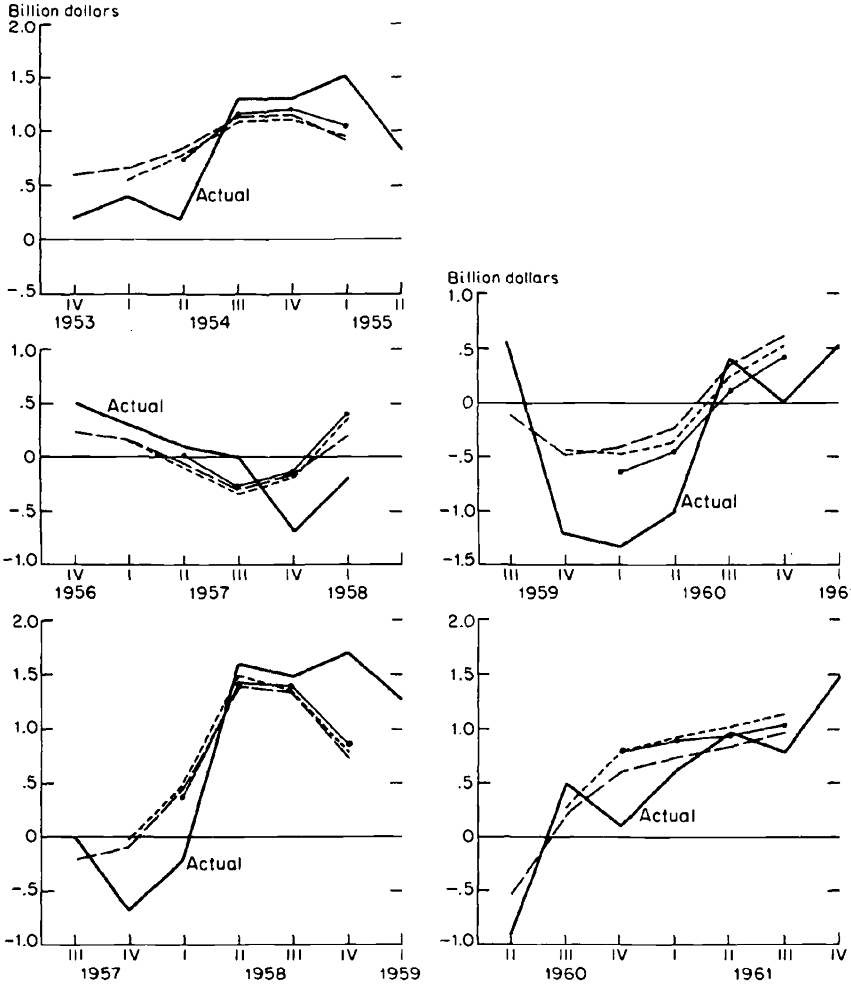
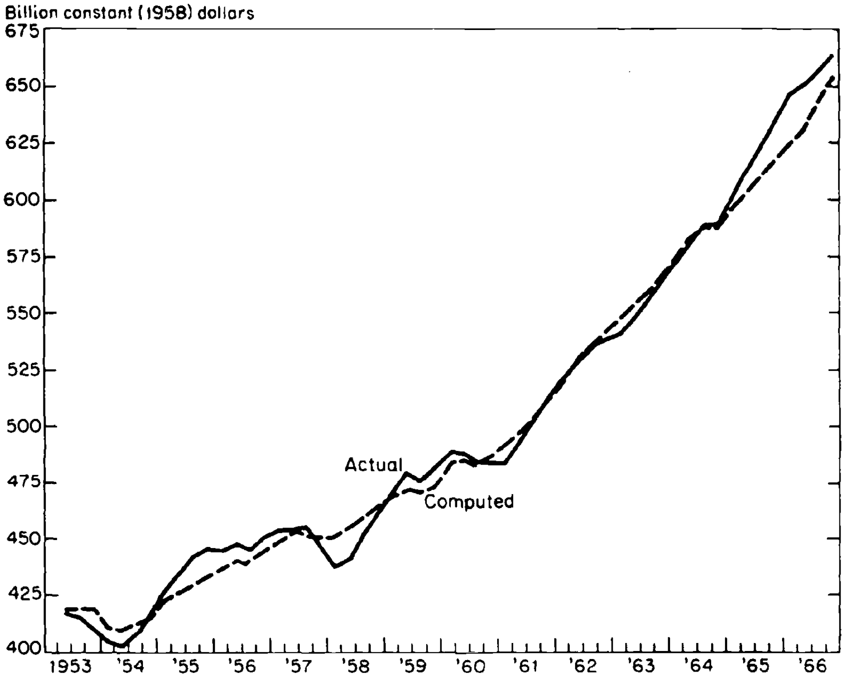


CHART 11

Actual and Computed Values for Real Gross National Product: Fifty-five Quarter Ex Post Simulation over Sample Period, 1953-II–1966-IV



a second simulation, using 1954-I as the first forecast quarter; and so on.

Attention here will center on real *GNP*, with only incidental reference to other magnitudes. Timing and phase relationships between variables are analyzed in another paper prepared for this Conference [*cf.* 21].

Of the five critical periods selected for the simulations, it is evident that the model behavior over the 1954 recession is the most satisfactory. As can be seen from the top panel of the first chart for real *GNP*, the general contours of the actual series are fairly well depicted. There is an obvious tendency for the model to turn up prematurely, but the downturn is pretty well revealed in all of the early simulations. The forecast using 1954-I as a jump-off (which was one quarter before the actual trough in real *GNP*) predicted the trough and subsequent upturn correctly. In all the simulations for this period the strong rise after the trough is slightly underestimated.

The next period covered includes the peak in economic activity prior to the 1957–58 recession. (It is shown in the second panel of the charts.) In this instance, each of the three simulations reveals a peak in economic activity with subsequent recession, but the timing of the peak in *GNP58$* is incorrect. The first simulation, with 1956-III as the jump-off quarter, peaks fully three quarters prior to the actual high point in activity. The next simulation, with 1956-IV as the jump-off quarter, peaks one quarter early, as does the forecast using 1957-I as the jump-off. In each instance, the depth of the recession which followed is badly underestimated. It is apparent that such a series of forecasts could be used by decision-makers to detect basic weaknesses in the economy some time prior to their occurrence, but the estimated magnitude of the drop in activity could not be similarly relied upon. A glance at the charts for inventory investment and for nonresidential fixed investment (*ISE*) shows that, in general, the former was fairly well depicted. But in each instance, *ISE* shows an early downturn with values that depart markedly from the actual series, which was maintained at fairly high levels until 1957-III.

The next panel (bottom left on the charts) shows the period both before and after the 1957–58 recession. In this case, the most obvious point is that all the simulations badly underestimate the full extent of the recession. *ISE* is again the main culprit, but inventories are at least partly to blame. The model magnitudes are, of course, interrelated, and certain series cannot easily be isolated and labeled as the primary causes of model failure. But, had *ISE* been better predicted, inventory investment would have shown a larger drop than is reported. It is noteworthy, however, that each of the three simulations, begun at various periods prior to the trough, depict the timing of the trough in *GNP58$* correctly. The first simulation, that using 1957-II as a jump-off, turns down one quarter early, but by the end of the period covered by the chart, all simulations are roughly on target.

The peak in economic activity prior to the 1960 recession (shown in the upper right-hand panels of each chart) is reasonably well recorded. Despite obvious substantial departures from the series of actual real *GNP*, the general contours of activity during the period are depicted in the forecasted series. The simulation with 1960-I as the first forecast quarter shows good direction and exact timing of the peak, and the subsequent downturn is also fairly well approximated. However, simulations which start in earlier quarters—covering the same period—fail to reveal the correct peak in the first quarter of 1960, although a marked flattening does occur. Except for the flattening instead of a drop in activity after the peak, the first simulation, with 1959-II as a jump-off quarter, is noteworthy; it roughly follows the contours of real *GNP* throughout the entire period.

The last panel depicts the period around the 1960-I trough. The most conspicuous feature is that all the simulations show too-early recoveries. The model shows declines in activity during 1960, when such declines actually occurred, but in no case is the true trough correctly timed. Again, it should be noted that *ISE* fails to show the drop that actually occurred in this series, and that although the estimates of inventories decline, they sadly underestimate the full extent of the drop in the actual series.

In order to set in perspective the performance of the model around turning points, Tables 7 and 8 show, respectively, average absolute errors (*AAE*) and root mean square errors (*RMSE*) for selected variables over a set of 53 short forecasts started in consecutive quarters beginning in 1953-IV, and over a subset of 19 forecasts which started one, two, or three quarters before reference-cycle peaks or troughs. The initial forecast quarters for these 19 runs are: 1953-IV to 1954-II; 1956-IV to 1958-I; 1959-III to 1960-IV; and 1966-I to 1966-IV. All first quarter errors lie within the sample period, but some of the second (and subsequent) quarter errors involve post-sample-period observations.

For several variables shown, the *AAE* or *RMSE* from the 19 runs are larger than the comparable values from the 53 runs for most forecast quarters. The *AAE* or *RMSE* values from runs made near turning points are notably larger for *GNP* in real and current dollars, real consumption expenditures, inventories, and corporate profits. Substantial errors in real nonresidential fixed investment are evidenced for both the periods around turning points, and for other periods as well. The fourth forecast quarter *AAE* and *RMSE* values are more mixed, but this is not surprising, since for each of the 19 runs, the reference-cycle turning point occurs before the fourth quarter of forecast.

Any judgment regarding the performance of the model over the selected critical periods must depend on the stringency of the criteria imposed. If one requires exact timing and full movements in magnitudes, our model—at this stage of development—does not usually come up to such standards. The model does, however, reveal the rough contours of cyclical behavior, and in some instances, it does show correct timing.

TABLE 7

Average Absolute Errors from Short Ex Post Forecasts: 19 Runs Starting Before Cyclical Turning Points; Runs Starting in 53 Consecutive Quarters

Variable	First Forecast Quarter		Second Forecast Quarter		Third Forecast Quarter		Fourth Forecast Quarter	
	19 Runs	53 Runs	19 Runs	53 Runs	19 Runs	53 Runs	19 Runs	53 Runs
GNP	3.7	2.9	5.6	4.5	6.3	5.9	5.8	6.5
GNP58$	2.7	2.3	4.3	3.6	5.1	4.6	5.5	5.1
C	2.0	1.5	2.3	2.0	2.8	2.6	3.3	3.0
IH	.3	.3	.5	.4	.5	.5	.8	.6
ISE	.9	.9	1.2	1.3	1.6	1.6	1.7	1.8
II	1.8	1.4	2.6	1.8	2.6	2.1	3.0	2.1
NETEXP	.6	.4	.8	.6	.7	.7	.6	.8
PERINC	1.9	1.6	3.1	2.9	3.4	3.6	3.0	4.2
PGNP	.3	.2	.4	.4	.5	.5	.6	.6
E	.2	.2	.4	.3	.5	.4	.5	.4
UNRATE	.3	.3	.5	.4	.6	.5	.5	.5
CPR	2.2	1.7	2.6	2.1	3.2	2.7	3.6	3.1
HM	.2	.2	.3	.3	.3	.3	.3	.3
OMD	1.8	1.7	2.0	1.9	2.0	2.1	2.0	2.3
UMD	1.4	1.2	2.6	2.2	3.3	3.2	4.0	4.2
RS	.2	.1	.2	.2	.2	.2	.3	.2
RL	.1	.1	.1	.1	.1	.1	.1	.1
MONEY	.6	.4	1.0	.8	1.3	1.0	1.4	1.1

TABLE 8

Root Mean Square Errors from Short Ex Post Forecasts: 19 Runs Starting Before Cyclical Turning Points: Runs Starting in 53 Consecutive Quarters

Variable	First Forecast Quarter 19 Runs	First Forecast Quarter 53 Runs	Second Forecast Quarter 19 Runs	Second Forecast Quarter 53 Runs	Third Forecast Quarter 19 Runs	Third Forecast Quarter 53 Runs	Fourth Forecast Quarter 19 Runs	Fourth Forecast Quarter 53 Runs
GNP	4.5	3.6	6.9	5.6	7.5	7.1	6.9	7.7
GNP58$	3.7	3.1	5.7	4.6	6.5	5.8	7.1	6.5
C	2.5	2.0	3.0	2.6	3.5	3.2	4.1	3.7
IH	.4	.4	.6	.6	.7	.6	1.1	.8
ISE	1.1	1.2	1.6	1.7	2.0	2.0	2.1	2.3
II	2.3	1.8	3.0	2.2	3.3	2.6	3.6	2.7
NETEXP	.7	.5	.9	.8	1.0	.9	.9	1.0
PERINC	2.4	2.1	3.6	3.4	4.0	4.4	3.6	4.9
PGNP	.4	.3	.5	.5	.6	.6	.7	.7
E	.3	.2	.4	.4	.5	.4	.6	.5
UNRATE	.4	.3	.7	.5	.8	.6	.7	.7
CPR	2.7	2.1	3.5	2.8	4.0	3.4	4.4	3.7
HM	.3	.2	.4	.4	.4	.4	.4	.4
OMD	2.1	2.1	2.4	2.4	2.4	2.6	2.6	2.8
UMD	1.8	1.6	3.1	2.8	4.0	3.7	4.9	4.8
RS	.3	.2	.3	.2	.3	.3	.4	.3
RL	.1	.1	.2	.1	.2	.2	.2	.2
MONEY	.8	.6	1.3	1.0	1.7	1.3	1.8	1.5

3.4 SIMULATION OVER THE ENTIRE SAMPLE PERIOD

The sample period for the OBE Model was comprised of 55 observations, starting 1953-II and ending 1966-IV. The simulation run was made without constant term adjustments over the entire period, and all exogenous variables were set at ex post actual values. Results from this simulation are summarized in Table 9, which shows maximum errors and average absolute errors for selected variables, and in Chart 11, which shows actual and computed values for *GNP58$*. As in the short simulations, the actual values used are revised through the June, 1968, *Survey of Current Business*.

Since the OBE Model was designed for short-term use, a long-run simulation of this kind is a severe strain on the underlying assumptions and rationale used to justify the model structure. It would be expected that errors would accumulate in lagged variables and cause subsequent errors in later forecast periods. Even if the model were perfectly specified, the neglect of stochastic elements would in itself give rise to errors which, due to the presence of lagged terms, would be carried forward. Under such conditions, one might expect that model results would not follow the actual course of economic magnitudes very closely.

Despite such considerations, it is evident from an examination of the accompanying chart that the simulated values of *GNP58$* follow the general pattern of the actual data quite well. The general growth path over the period is fairly well predicted, although there is an evident drifting off of predicted values in the later quarters. The simulated

TABLE 9

Maximum Error and Average Absolute Error for Selected Variables— 55-Quarter Ex Post Simulations over Sample Period, 1953-II–1966-IV

Variable	Maximum Error	Quarter	Average Absolute Error
C	−11.70	1966 I	3.21
CPR	9.07	1953 IV	3.17
EMPLOY	1.14	1954 II	.58
GNP	−36.43	1966 II	10.11
GNP58$	−21.24	1966 I	6.40
II	−8.92	1966 IV	2.58
ISE	6.57	1963 II	2.66
PERINC	−26.16	1966 II	8.24
PGNP	2.75	1966 IV	1.35
UNRATE	−2.03	1954 II	.66

series depicts the 1954 recession quite well, although it fails to reveal the entire drop in output. In that recession, the trough occurs in the same quarter in both the simulated and actual series.

The 1958 recession is much less adequately portrayed. The simulated series flattens but fails to show the full drop noted in the actual data. The 1960 recession, which is small by any criteria, is depicted with a somewhat improper timing: the trough in the simulated series occurs two quarters before the trough in the actual data.

The maximum errors for most variables occur toward the end of the 55 quarter simulation period, when some series drift away from the actual data. This is especially true for prices (not shown), and may be a reflection of cumulative bias effects discussed earlier. However, the average absolute errors over the entire 55 quarter simulation are not vastly larger than the *AAE* from sixth quarter forecasts presented in Section 3.2. The *AAE* for *GNP58$* is 5.82 for 48 sixth quarter forecasts, and 6.40 for the 55 quarter simulation.

4 TWENTY-FIVE-YEAR STOCHASTIC SIMULATIONS

EACH of the model-builders participating in this Conference was expected to carry out simulations over a twenty-five-year period under reasonable assumptions of smooth growth in the exogenous variables. The resulting control solution was not to be regarded as a serious attempt at a model forecast over such a long period; such a "true forecast" would have demanded a realistic projection of all exogenous variables, and would have required a much larger effort. Moreover, the demands placed on a short-term model, which by and large neglects demographic and other long-run factors, would make any "true forecast" highly suspect. Instead, the control solution was meant to delineate a reasonable path for subsequent operations.

The later operations required that stochastic shocks be introduced into the model on a continuous basis, and that such shocked runs be repeated many times. The results obtained provide a large number of ready-made "observations" beyond the sample period, which permit analyses of model dynamics, including the timing and amplitudes of major component series. Presumably, if NBER studies of lead and lag series reveal real-world relationships between important magnitudes, then the same analyses applied to model simulated results would also reveal such relationships, providing yet another yardstick for judging the adequacy of a model structure. The task of carrying out such analyses was placed in the hands of the NBER [*cf.* 21].

Another, and perhaps equally important, purpose of introducing stochastic shocks over a long time period was to determine whether business cycles with realistic characteristics—e.g., amplitude, periodicity, and phase relationships—are found in the simulated results. Such an exercise bears directly on business-cycle theory. Specifically, it is addressed to the question of whether some of the major models in operation today yield business cycles as the result of interaction of model structure with stochastic elements.

The introduction of random shocks is, of course, not new. An early and noteworthy project was carried out by Irma and Frank Adelman [1]. Using the Klein-Goldberger Model, they found that random elements introduced in the endogenous system resulted in cyclical behavior not too unlike that observed in the real world. An annual model was used and only one time path was traced; moreover, the random shocks used were drawn under the assumption of no serial correlation and zero contemporaneous covariances. One can view the results presented here as a further development of their work, under conditions where the random shock procedure allows for nonzero covariances and, in some instances, for auto-correlation of residuals.

The reported simulations cover the 100 quarters from 1966-I to 1990-IV. The starting period was selected, in part, because behavior of the U.S. economy in 1964 and 1965 was very close to that depicted by the set of equations in the model. This minimized difficulties in the transition from actual past data to the model solutions.

Modifications in the model structure are discussed in Section 4.1. All exogenous variables over the simulation period were smoothed, trendlike series; the procedures used to generate these series are presented in Section 4.2. The nature of the resulting control (nonshocked) solution is treated in Section 4.3. Following this, Section 4.4 presents the methods used to generate two types of stochastic shocks. Finally, a spectral analysis of real *GNP* series obtained from the various stochastic simulations is presented in Section 4.5.

4.1 MODIFICATIONS IN MODEL STRUCTURE

This section discusses all changes made in the model structure for the twenty-five-year simulations. Initial attempts to solve the OBE Model far into the future, revealed deficiencies in the longer-term properties of a few equations in the model. The forms marked (*b*) in Appendix B document all alternative equations used for the twenty-five-year simulation runs.

The (*b*) form of the equation for nonresidential fixed investment (*ISE*) is still an adaptation from the work by Almon. Preliminary runs with the (*a*) alternative resulted in steeply rising capital/output ratios over time. The only difference between the (*b*) form and that used earlier is that now the coefficient of long-term interest rates varies with the level of capacity output when the equation is stated in normalized form. The (*b*) equation reflects more precisely the structure implied by the Almon work.

In simulations over the sample period, and for short-term forecasts, we treated total and corporate capital consumption allowances as exogenous. Over a long period of time, this is clearly not satisfactory, so we have made both of these magnitudes simple functions of capital stock.

Two equations which determine final demand variables yielded preliminary results which were judged to be somewhat low by the end of the twenty-five-year simulation period. Accordingly, we added a time trend of 24.8 thousand units per quarter to the housing starts equation, and a small trend of 0.17 billion dollars per quarter to the trade imports equation. Each of these equations was adjusted so that these trends started in the initial simulation quarter.

The price of government purchases from the private sector, normally exogenous for short-term forecasts, was made endogenous and set to grow at the same percentage rate as the price for private *GNP*, excluding housing services (*P*).

During the sample period, small negative trends in the primary labor-force participation rate and in hours worked at capacity output levels, plus a small positive trend for the frictional unemployment rate of secondary workers, were observed. We judged that these trends were unlikely to continue, and so they were not allowed to operate over the simulation period. This slightly alters the equations for the primary labor force, capacity hours, and capacity output.

The functions used during the sample period for Federal, and for state and local, personal tax and nontax payments, and for the investment tax credit, are empirical relationships devoid of longer-run considerations. All of these functions were changed for the twenty-five-year runs. The investment tax credit was made proportional to estimated nonresidential equipment investment (in current dollars). For state and local personal tax and nontax payments, we arbitrarily assumed both a rising marginal rate of taxation and an augmented time trend (to reflect rises in nontax payment rates). The parameters selected are based, in large part, on recent observations of these payments and the tax base.

In the case of Federal personal tax and nontax payments, we thought that the best approach would be to tie payments to liabilities. Payments are predicted in three recursive equations: the first derives taxable income from personal per capita income (per capita exemptions are held constant); the second derives tax liability based on the 1965 tax structure; the third is a simple empirical relation between liabilities and payments. The first two equations were adapted from the work of Waldorf [*cf.* 20, pp. 26–33]. This procedure is a considerable improvement over the equation forms used during the sample period, but it is somewhat deficient for purposes of the simulations with stochastic shocks, in that it fails to incorporate the varying short-term gap between liabilities and payments which would inevitably accompany any uneven growth in income.

The equation for the interest rate on savings deposits (*RT*) produced absurdly low values during preliminary runs; we held *RT* at its value in 1965-IV over the entire simulation period.

Finally, the reduced-form equation for the change in unfilled orders produced unacceptable negative values for the level of unfilled orders during preliminary runs. We had developed a better equation for shipments of manufacturers' durables (*SMD*) after the short simulation runs were initiated, and this newer equation for *SMD* was used, together with a near identity for the change in unfilled orders.

4.2 TREATMENT OF EXOGENOUS VARIABLES

All tax rates and exogenous interest rates—the discount rate and the time deposit rate—were held at constant levels, and most other variables were set to grow at constant rates of change. Usually, the average rate of change used was that observed over the sample period.

Various criteria were used to adjust the growth rates of a few series. In the course of preliminary runs, various magnitudes and ratios were examined for reasonableness. For instance, we examined ratios of final demand and income items to disposable income or to *GNP*. In addition, we scrutinized the paths of government deficits, net exports, and the growth of some exogenous categories relative to related endogenous elements. Where clearly unreasonable patterns were found, we adjusted growth rates of various exogenous variables until the results seemed plausible.

Table 10 shows values in the jump-off and final quarters for each exogenous variable, and an average annual rate of change over the twenty-five-year simulation period. While space prohibits a detailed description of procedures used for all variables, a few of them deserve special comment.

Population series used were based on projections provided by the U.S. Bureau of the Census. The paths of related series (for instance, Social Security payments) were made consistent with the population assumptions. Over the twenty-five-year simulation period, the population of males aged 25 to 54 (*NP*) grows at a slightly increasing rate of change, but the population of the remaining persons aged 16 to 64 (*NS*) grows at a decreasing rate. These projections are in large part a reflection of birthrate patterns over the 1930 to 1965 period.

Nonborrowed reserves of banks (*RESNB*) were determined by forcing free reserves to zero for all periods in the control run. In all of the runs with stochastic shocks, free reserves were not restricted, and the *RESNB* series from the control run was used. The resulting implied monetary policy is accommodating with respect to growth but unresponsive to cyclical movements.

TABLE 10

Values in 1965-IV and 1990-IV; with Average Annual Rates of Change for Exogenous Variables from Twenty-Five-Year Simulations

Variable	Value for 1965-IV	Value for 1990-IV	Average Annual Rate of Change
AM58$	33.1	33.1	.00
CH	59.5	193.2	4.82
CURR	36.1	60.3	2.07
D$	61.6	273.6	6.15
DC$	37.8	141.2	5.41
DH$	10.4	54.0	6.81
EE	8.3	6.3	−1.10
EG	8.6	27.2	4.71
EXP$	40.5	206.8	6.74
G58$	117.4	342.0	4.37
GFD$	52.4	193.5	5.36
GFND$	17.4	232.1	10.92
GIA	12.2	258.4	12.99
GSL$	72.5	601.0	8.83
HH	58,208.0	90,503.0	1.78
IHF	.5	.2	--3.60
IHR	5.0	11.9	3.53
IMG$	3.0	3.8	.95
INB	18.8	74.8	5.68
INC	11.7	46.5	5.67
INGF	8.9	8.9	.00
INGSL	.5	.5	.00
MAXSS	4,800.0	12,003.0	3.73
N	195.5	273.0	1.34
NP	32.6	51.9	1.88
NS	78.4	111.3	1.41
PA	.987	1.309	1.14
PE	55.4	170.7	4.60
PE1E	53.0	162.6	4.59
PE2E	54.8	163.9	4.48
PEX	1.041	1.259	.76
PF	.252	.269	.26

(*continued*)

TABLE 10 (*concluded*)

Variable	Value for 1965-IV	Value for 1990-IV	Average Annual Rate of Change
PH	1.098	1.613	1.55
PIM	1.033	1.248	.76
PR	1.093	1.582	1.49
PWG	1.364	4.323	4.72
RDIS	4.17	4.00	−.17
RENT	19.2	38.9	2.86
RESNB	21.6	169.4	8.59
RMBD	.884	.884	.00
RMBT	.826	.826	.00
RRD	.1465	.1465	.00
RRT	.04	.04	.00
RT	3.44	3.44	.00
RTCF	.48	.48	.00
RTEXAV	.791	.791	.00
RTEXS	1.098	1.098	.00
RTQ	4.0	4.0	.00
RTRU	51.5	144.1	4.20
SGF	4.1	14.1	5.06
SGSL	−3.2	−8.2	–
SIBOF	5.3	61.4	10.30
SIBSL	2.6	28.2	10.00
SIPOF	3.1	18.1	7.31
SIPSL	2.0	17.1	8.96
TIFO	4.98	5.20	.17
TRB	2.6	16.3	7.62
TRFF	2.0	3.3	2.02
TRFP	.7	1.1	1.82
TRPOF	28.9	162.5	7.15
TRPSL	7.0	39.4	7.16
TRUEX	1.0	1.0	.00
WG$	70.9	691.9	9.54

Series for government purchases and government employment were first set at reasonable trend-levels. But these preliminary levels were then raised or lowered to produce a desired path in the control solution. Forty per cent of any alteration in real government purchases was allocated to government employment and wages, and the remainder was assigned to government purchases from the private sector. It should be noted that the resulting series for the government variables exhibit very smooth and regular behavior over the entire period.

4.3 THE NATURE OF THE CONTROL SOLUTION

We wanted a control solution which exhibited a fairly smooth pattern for all major variables. As described in the previous section, we adjusted government purchases to achieve a stipulated path. At first, we had hoped to produce a control solution with a 4 per cent unemployment rate and a constant rate of growth in real *GNP*. These twin objectives were inconsistent, owing to the population patterns used. A constant rate of growth in real *GNP* resulted in unemployment rates which fell off sharply in the later simulation periods. Similarly, a control solution forced to a constant over-all unemployment rate exhibited a sharp decline in the rate of growth of *GNP*.

Table 11 lists annual levels and percentage changes for a few series taken from the final control solution. The final control solution has an unemployment rate of 4.2 per cent in the initial year of simulation, which gradually declines to 3.9 per cent by 1990. Real *GNP* grows at a declining rate of change, while prices and productivity vary within a fairly small range. Other variables produced by the control solution (not shown) show reasonable patterns.

4.4 GENERATION OF STOCHASTIC SHOCKS

Fifty simulations were made with stochastic shocks applied to endogenous behavioral variables. Variables relating to taxes and transfers, and those explained by identities or near identities, were not shocked. The shocks were applied to the normalized equation forms.

TABLE 11

Annual Levels and Per Cent Changes for Selected Series from the Twenty-Five-Year Control Simulation

	Level			Per Cent Change			
Year	*GNP*	*GNP58$*	*UNRATE*	*GNP*	*GNP58$*	*PGNP*	*PROD*
1965[a]	684.1	616.6	4.5				
1966	735.6	649.3	4.2	7.52	5.31	2.11	3.03
1967	788.5	679.7	4.2	7.20	4.68	2.41	2.98
1968	847.5	713.5	4.2	7.47	4.97	2.38	3.32
1969	909.5	748.2	4.2	7.32	4.87	2.34	3.16
1970	974.7	782.9	4.2	7.17	4.63	2.43	2.97
1971	1,044.4	818.3	4.2	7.15	4.52	2.51	2.94
1972	1,119.4	855.1	4.1	7.18	4.50	2.57	2.95
1973	1,199.1	892.8	4.1	7.11	4.41	2.58	2.95
1974	1,283.4	931.5	4.1	7.04	4.34	2.59	2.87
1975	1,372.1	970.5	4.1	6.91	4.18	2.62	2.81
1976	1,466.2	1,010.4	4.1	6.85	4.11	2.63	2.81
1977	1,565.5	1,051.2	4.1	6.77	4.04	2.63	2.79
1978	1,670.1	1,092.8	4.1	6.68	3.96	2.62	2.77
1979	1,780.6	1,135.5	4.1	6.62	3.91	2.61	2.78
1980	1,898.3	1,179.9	4.0	6.61	3.91	2.59	2.83
1981	2,020.0	1,224.5	4.0	6.41	3.77	2.54	2.75
1982	2,148.0	1,269.8	4.0	6.33	3.70	2.53	2.75
1983	2,282.4	1,316.2	4.0	6.26	3.65	2.51	2.75
1984	2,423.2	1,363.6	4.0	6.17	3.60	2.48	2.76
1985	2,570.7	1,412.2	4.0	6.09	3.56	2.44	2.76
1986	2,725.1	1,461.9	3.9	6.00	3.52	2.40	2.77
1987	2,886.4	1,513.0	3.9	5.92	3.49	2.34	2.78
1988	3,054.9	1,565.5	3.9	5.84	3.47	2.29	2.80
1989	3,230.4	1,619.5	3.9	5.75	3.45	2.22	2.81
1990	3,413.4	1,675.1	3.9	5.66	3.43	2.16	2.83

[a] Actual values.

The forty-one variables subjected to shocks and the standard errors of estimate from normalized equations for these variables are shown in Table 12.

The procedures used for generating shocks were developed by Michael McCarthy, and are described in an Appendix to a paper prepared for this Conference [18]. The McCarthy procedures combine the sample period residuals with random normal deviates. The latter values were taken from a computer tape generated by the Rand Corporation, containing one-hundred-thousand random normal deviates. The McCarthy procedures are such that the expected value of the variance-covariance matrix of stochastic shocks over the simulation period equals the variance-covariance matrix of the observed residuals over the sample period. Moreover, one of these procedures allows for serial correlation of residuals.

These procedures differ in several respects from those applied to an annual model by Adelman and Adelman [1]. They generated shocks which assumed zero covariances and no serial correlation for all

TABLE 12

Standard Errors of Estimate over the Sample Period for Forty-One Variables Subjected to Shocks

Variable	$\bar{S}$	Variable	$\bar{S}$	Variable	$\bar{S}$
CA	1.0039	*IINA*	1.6825	*PN*	.0030
CN	1.0414	*IMS*	.4002	*POD*	.0034
COD	.5112	*IMT*	.6382	*PRI*	.9017
CPR	.9827	*ISE*	.9571	*PS*	.0058
CS	.4794	*IVA*	.6790	*PWMD*	.0047
CUW	.0108	*LH*	1.7452	*RL*	.0704
DD	.6415	*LFP*	.0778	*RM*	.1066
DIV	.2601	*LFS*	.2497	*RS*	.1190
EW	.2183	*OMD*	1.7628	*RTB*	.2527
H	.1476	*P*	.0034		
HM	.2168	*PHS*	.3789	*SMD*	1.2976
HS	70.7435	*PIE*	.0045	*TD*	.6618
IE	.5379	*PIH*	.0062	*URP*	.0018
IIA	.7452	*PIS*	.0064	*WR*	.0269

errors. Also, the Adelmans scaled their shocks so that the ratio of the standard deviation of residuals relative to the value of the dependent (normalized) variable observed in the sample period was maintained in the simulation period. To the extent that variances of the true normalized equation errors are heteroscedastic, with increasing size over time, the scaling aspect of the Adelmans' procedure seems preferable to that used for this Conference.

4.5 A SPECTRAL ANALYSIS OF REAL *GNP* SERIES FROM STOCHASTIC SIMULATIONS

Fifty stochastic simulations, starting in 1966-I and continuing for one-hundred quarters, were made. Twenty-five of these simulations used serially correlated random shocks, while the other twenty-five runs were made with non-serially correlated random shocks. The runs with stochastic shocks were designed to reveal the dynamic properties of the OBE Model and to determine whether the observed cyclical behavior of the economy could be replicated by the model through the interaction of model structure and stochastic elements applied to endogenous variables. We present here a summary analysis of the real *GNP* series from these fifty simulations as a supplement to the analysis presented by the NBER team [21].

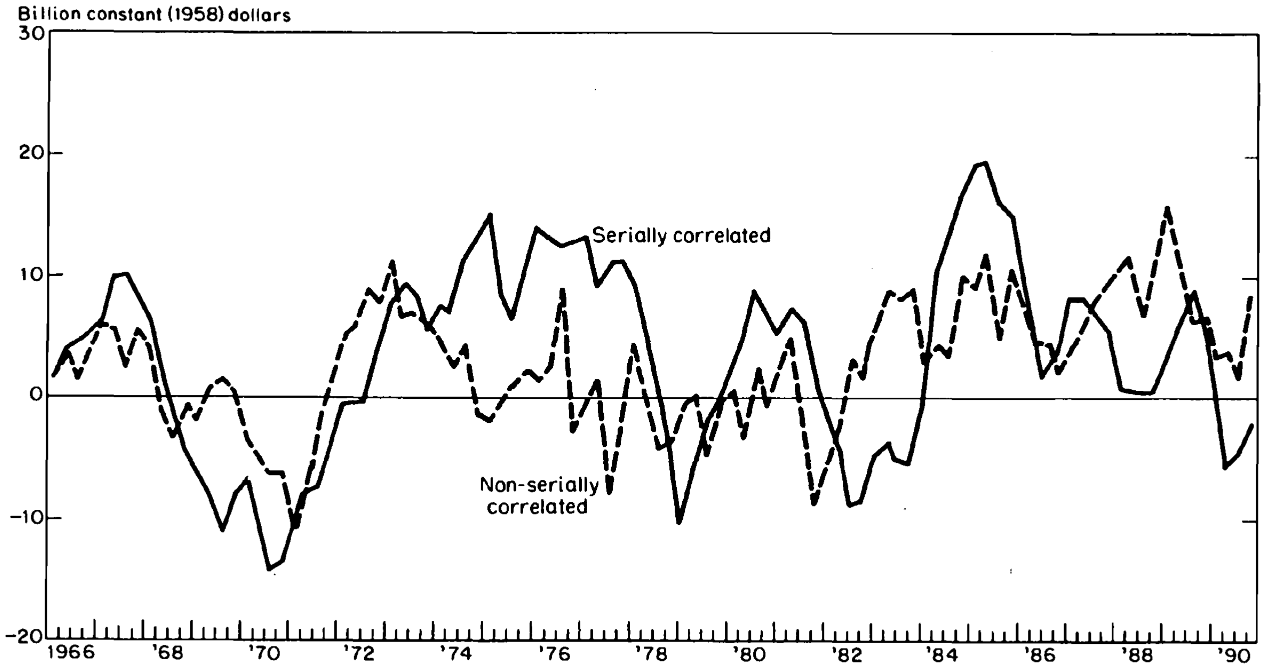
Chart 12 presents the time paths of real *GNP* series taken from two arbitrarily selected runs. The heavy line depicts the time path from a run which used serially correlated shocks, while the dashed line is a series from a run in which non-serially correlated shocks were introduced. Both series are given in terms of deviations from the control solution.

It is apparent that both of these time paths reveal cyclical movements. The maximum deviation from the control solution is not far different in the two series – 19.5 billion dollars for the serially correlated case; and 16.1 billion dollars for the non-serially correlated series.

While all of the resultant series in real *GNP* exhibited the same general character as the two presented, very few of the fifty series showed downturns in the real *GNP* series. When downturns were ob-

CHART 12

Deviations Between Shocked and Control Series for Real GNP: One Simulation with Serially Correlated Shocks, One with Non-serially Correlated Shocks



served, they were usually of very short duration with a strong tendency to rebound quickly. If the criterion for the presence of cycles is that protracted downturns must occur, then the present results do not depict cyclical behavior adequately. It should be emphasized, however, that these simulations incorporate very strong growth elements in the exogenous variables; such elements have to be overcome by the effects of stochastic shocks for actual downturns to occur. The importance of movements in exogenous variables during recent marked recessions in the United States economy is clear; both the 1953–54 and the 1957–58 recessions were accompanied by flattening or decreases in such variables as government purchases and gross exports.

The chart also shows that apart from slightly smaller amplitude, the non-serially correlated run produced a series which is quite ragged. This was, in general, true of all the runs made using this procedure. It is apparent that the ragged character of these series is not typical of recorded, real-world data. The serially correlated shocks generated series with much less ragged time paths, which are more in line with our expectations.

Spectral analysis was applied to the fifty real *GNP* series generated by the simulation runs in order to test for persistent periodicities in the revealed cyclical movements. The analysis presented here used forty-eight lags and a Parzen window. Since spectral analysis can only be applied to stationary series, it was first necessary to filter out any trend. Two filters were applied. Let X be the original, unfiltered series, and t be time; then the first filtered series, Y_1 is defined by

$$Y_{1,t} = X_t - X_{t-1}$$

and the second series, Y_2 is defined as

$$Y_{2,t} = X_t - \hat{X}_t$$

where $\ln(\hat{X}_t) = b_1 + b_2 t + b_3 t^2$, and b_1, b_2, and b_3 are determined by ordinary least squares. We shall call these filters a first-difference filter and a log-polynomial filter, respectively. Both filters are discussed in Jenkins and Watts [16], and in Granger [11]. The first-difference filter is applied to economic series in Howrey [14], and with some modification, in Nerlove [19]. The degree of the log-polynomial filter was chosen to allow for the declining rate of growth evidenced in the control solution.

Results from using each of the two detrending (filtering) methods are shown separately: for the series generated by serially correlated shocks; and for those obtained using non-serially correlated shocks.

Charts 13 and 14 show spectral power as a function of periodicity. In preparing each panel, spectral densities from twenty-five runs were averaged. The ordinate of each panel shows logarithms of spectra, which have identical confidence interval widths; and the periodicities, shown on the abscissa, are scaled as cycle lengths in quarters. Three of the four panels show highest power at the very long cycle lengths (low frequencies), with generally lower power for shorter cycles (higher frequencies). Granger [10] has noted similar patterns for several economic time series. The remaining panel, which depicts results from series generated using non-serially correlated shocks and using a first-difference filter, shows a high power at low frequencies, but a distinct minimum of power for 24-month cycles, followed by ever increasing power at the higher frequencies. This result is not too surprising, given the ragged character of the underlying series.

Each of these panels shows some evidence of peaks. The significance of any noted peak is ascertained by an F test, which is simply the ratio of the spectral value at the peak to the spectral value at least two spectrums distant. Each summed spectrum has approximately 193 degrees of freedom, so the significant values for an F test are 1.2, 1.28, and 1.41 for the 10, 5, and 1 per cent levels, respectively.

The summed spectra from runs which use data generated by the non-serially correlated shocks, using the log-polynomial filter, show a very weak peak at 6.6 quarters, which is not significant—even at the 10 per cent level. The spectra generated by the non-serially correlated shocks, when a first-difference filter is used, do reveal a peak between 12.0 and 13.7 quarters and another between 6.0 and 6.4 quarters. The latter peak is significant at the 10 per cent level, but the two peaks are not significantly different from one another. These two peaks may be a reflection of the same basic periodicity, since one is the first harmonic of the other.

The sums of spectra from data generated by serially correlated shocks show no peak at all when the log-polynomial filter is used, but the use of a first-difference filter yields two peaks, at 8.7 and 13.7 quarters. The peak at 8.7 quarters is significant at the 5 per cent level, and the two peaks are significantly different at the 10 per cent level.

CHART 13

Sum of Spectra for Twenty-five Real GNP Series Generated Using Non-serially Correlated Shocks

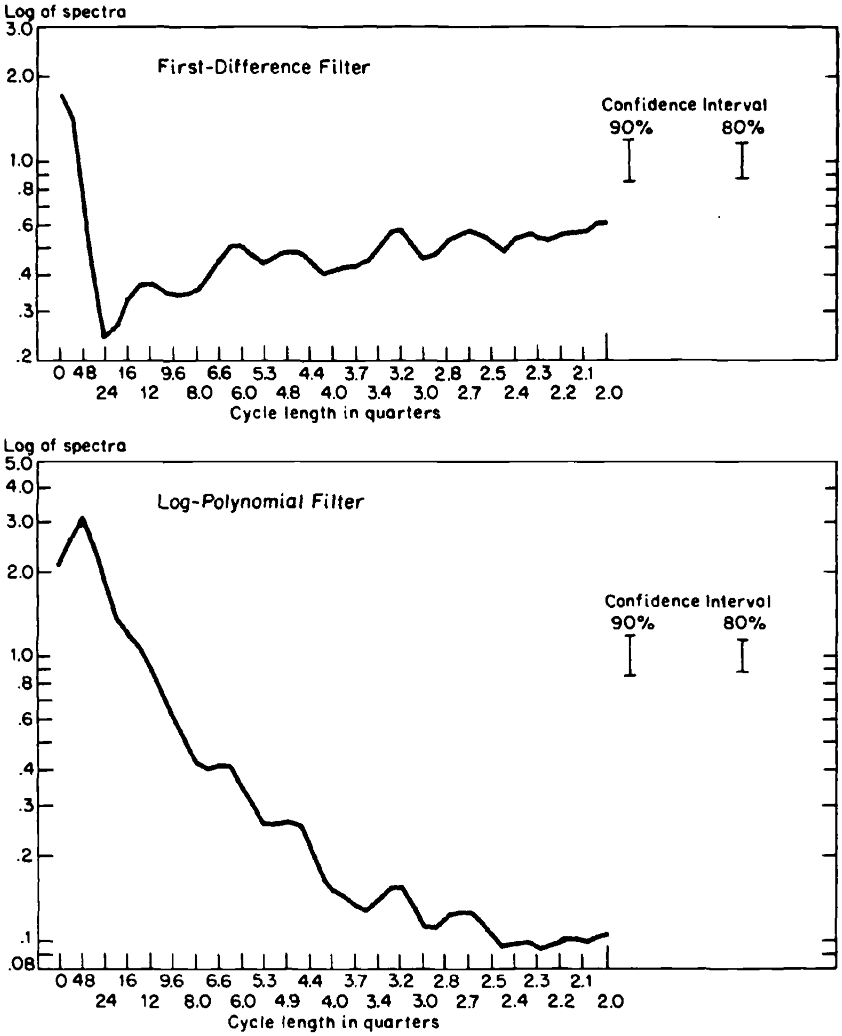
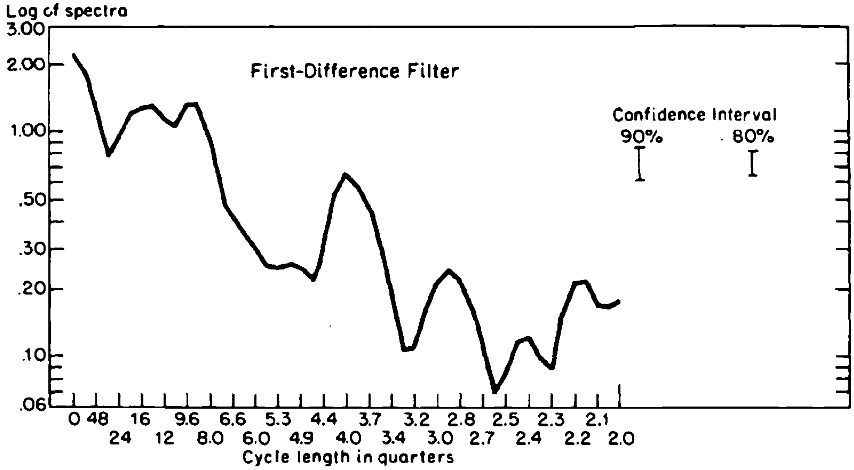
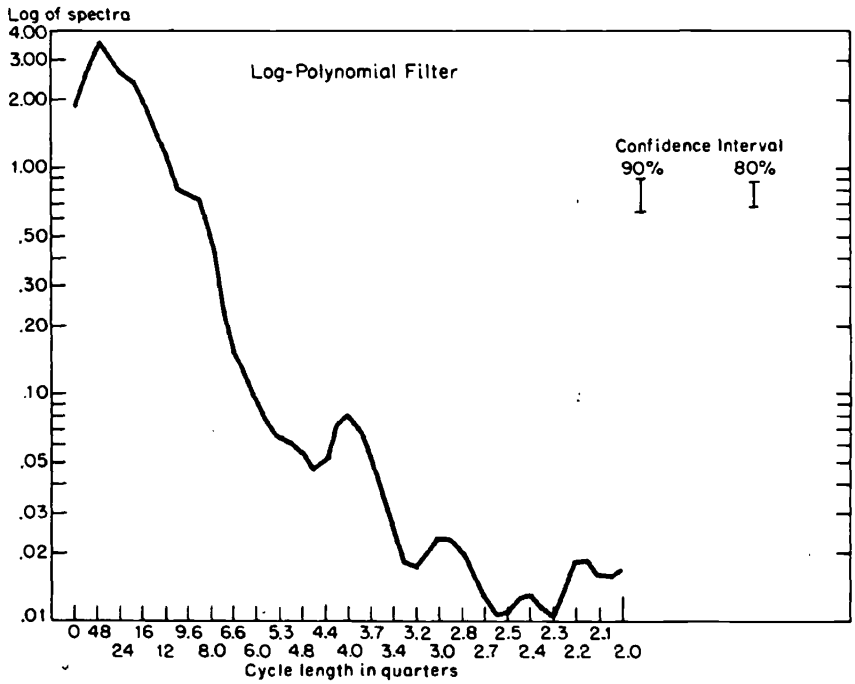


CHART 14

Sum of Spectra for Twenty-five Real GNP Series Generated Using Serially Correlated Shocks





Charts 15 and 16 show frequency bar charts of the most prominent spectral peaks in the range from six to thirty-two quarters, from each of the four sets of real *GNP* series. The choice of filter has a marked effect upon the results. The spectral peaks from data generated by the non-serially correlated shocks show a concentration at 13.7 quarters when a log-polynomial filter is used; and at 6.4 quarters when a first difference filter is used. In both of the above cases, a substantial number of spectral runs yielded no peak in the 6–32 quarter range. For the spectral runs generated by serially correlated shocks, using a log-polynomial filter, a concentration of peaks at 19.2 quarters is noted; but nine of the twenty-five spectral runs show no peak in the stipulated range. Use of the first-difference filter on the data generated from serially correlated shocks reveals a marked clustering of major peaks; here all spectral runs show major peaks in the stipulated range, with equal concentrations at 8.7, 9.6, and 13.7 quarters, and with four peaks in the 4 to 5 year range.

Despite the convenience and elegant nature of spectral analysis, the results presented here are not without ambiguity. The choice of filter has a dramatic effect upon the results. We tend to place more reliance on the results which employed a first-difference filter, since its use more successfully eliminated power at the very low frequencies.

The results presented above are considerably different from those obtained in the Adelmans' study [1]. Using a modified Klein-Goldberger Model, they found that random shocks applied to endogenous equations resulted in cyclical behavior similar to that observed in the real world. Also, they found that shocks applied to exogenous variables played a very minor role. The differences between the results presented here and those obtained by the Adelmans may be the result of differences in procedures: they used a small annual model, our model is quarterly and somewhat larger; they used random shocks drawn under the assumption of no serial correlation and zero contemporaneous covariances, while our shocks take into account the intercorrelation of equation residuals, and in some cases, serial correlation properties, as well.

A comparison of the size of the shocks on endogenous equations in the Adelmans' study with those used here reveals dramatic size differences. For instance, the coefficients of error variation (ratio of

CHART 15

Frequency Bar Charts of Most Prominent Spectral Peaks from Twenty-five Real GNP Series Generated Using Non-serially Correlated Shocks

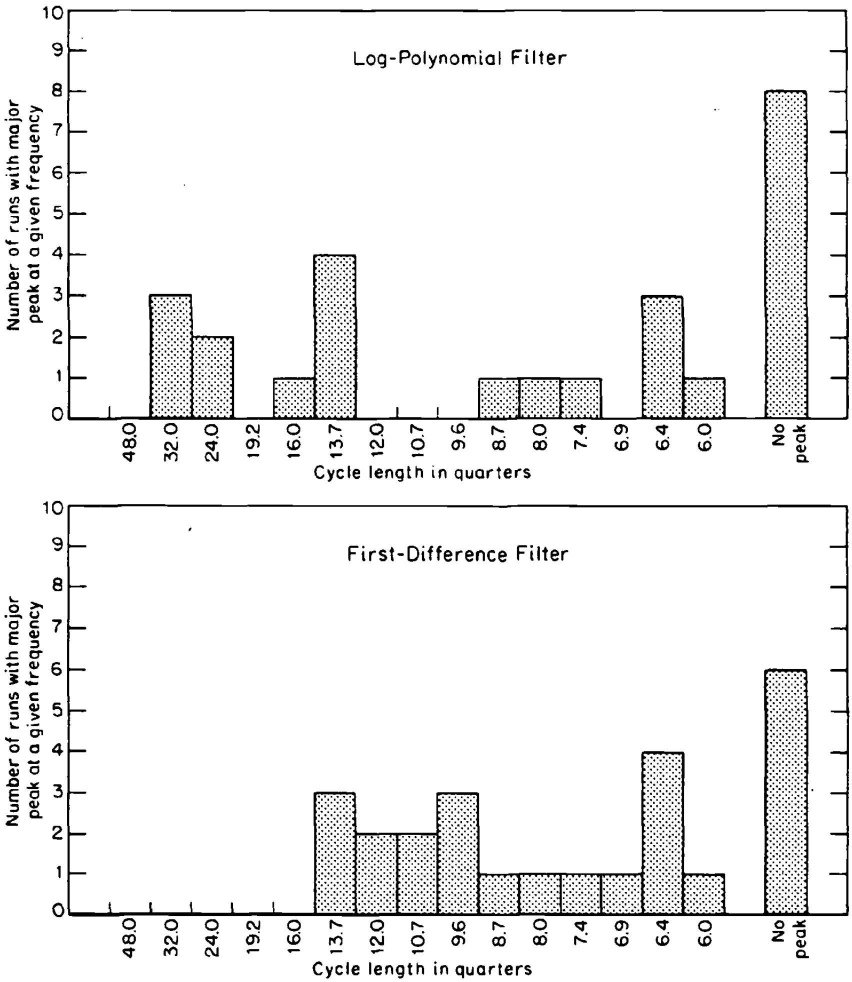
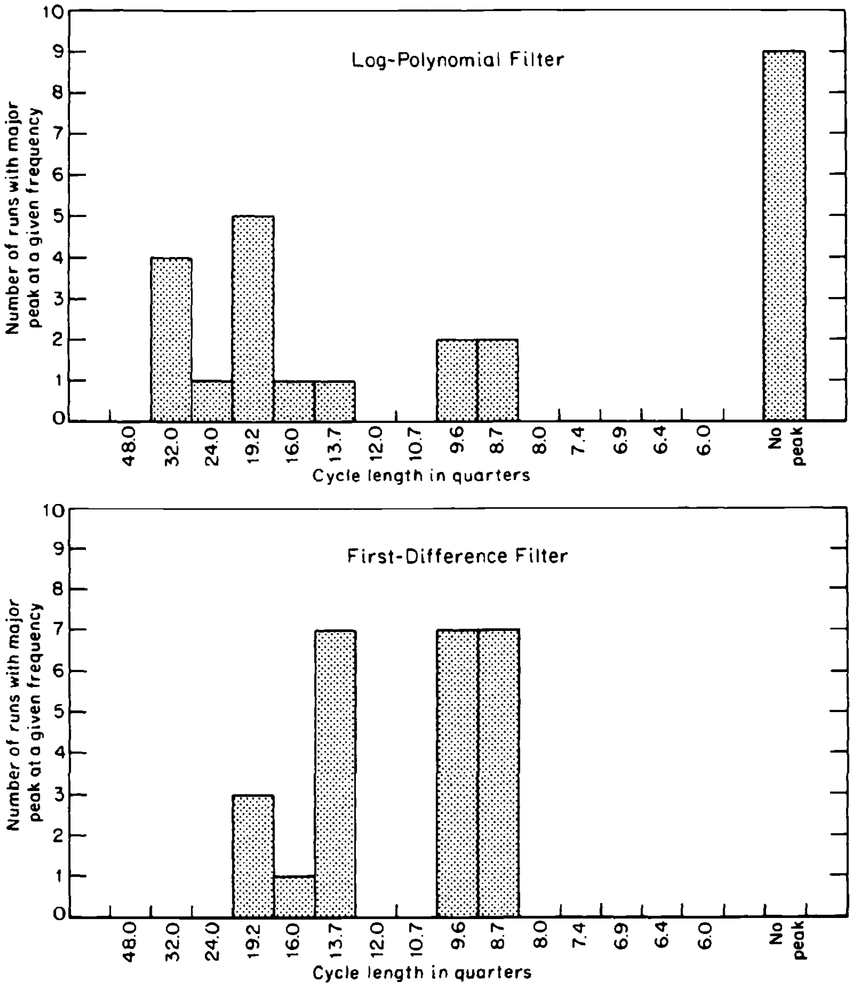


CHART 16

Frequency Bar Charts of Most Prominent Spectral Peaks from Twenty-five Real GNP Series Generated Using Serially Correlated Shocks



the standard error over the sample period to the mean value of the dependent variable in the simulated period) for total real investment and for corporate profits used in the Adelmans' study were 6.5 and 7.8 times as large, respectively, as those we used in this study. This suggests that exogenous variables may play a much more important role for our model, and that shocks on them may play an important role in cycle generation.

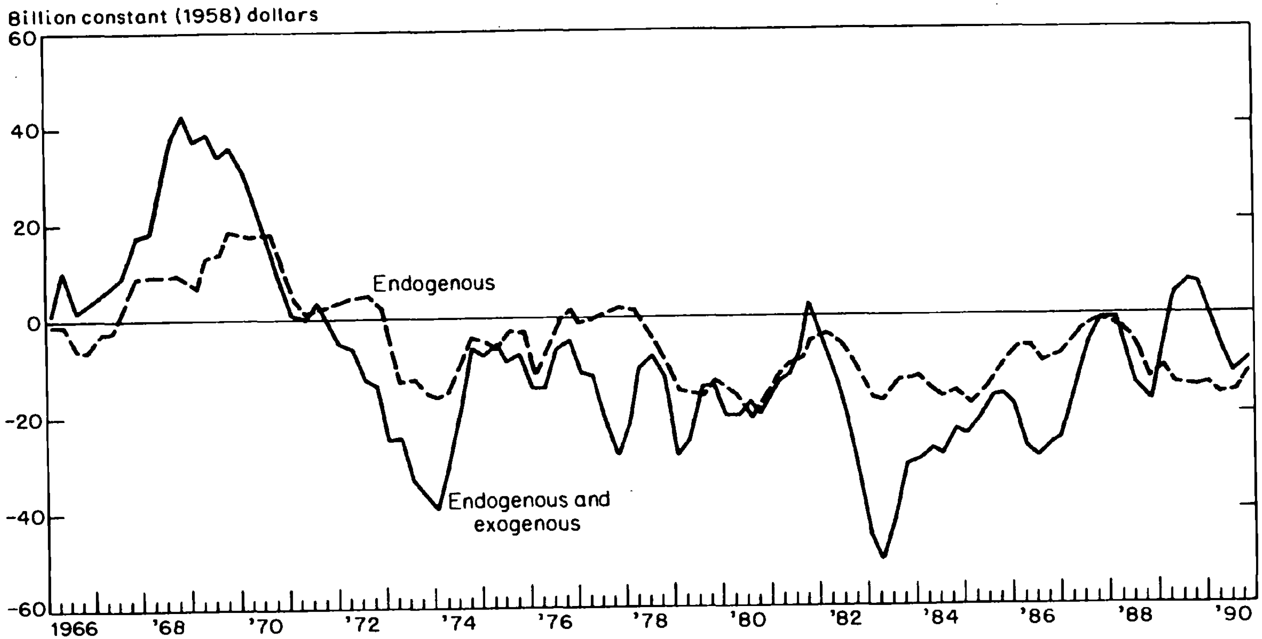
We performed some additional stochastic simulations with serially correlated shocks on the endogenous equations, and with shocks applied to five exogenous variables—government purchases from the private sector, government wages, government employment, exports, and nonborrowed reserves of the banking system. For pseudo-residuals for these exogenous variables, we used residuals from log linear trend equations, fitted over the sample period. The residuals for total real government purchases are substantial—$16.2 billion in 1953-II; and $11.3 billion in 1966-IV.

Chart 17 presents the time paths of real *GNP* series (shown as deviations from the control solution) taken from two arbitrarily selected runs. The same random normal deviates were used to generate the two time paths shown—but in one series, shocks were applied to endogenous equations only; while for the second series, shocks were applied to endogenous equations and to the five exogenous variables given above. The maximum deviation from the control solution is far different for the two series—21.4 billion dollars for the first series, but 51.4 billion dollars when selected exogenous variables are also shocked. Other simulation runs showed deviations of up to $60 billion when these selected exogenous variables were also shocked—a figure much larger than the $20 billion maximum deviation obtained from runs with shocks on endogenous variables only.

Moreover, unlike our earlier results, the simulations with shocked exogenous variables show sustained cycles in real *GNP levels*. In many cases, a peak in real *GNP* is followed by three to five quarters of lower real *GNP* values. These brief results suggest that movements in elements commonly considered exogenous in large-scale models may play a crucial role in the determination of business cycles.

CHART 17

Deviations Between Shocked and Control Series for Real GNP: One Simulation with Serially Correlated Shocks on Endogenous Variables, One with Serially Correlated Shocks on Both Endogenous Variables and Selected Exogenous Variables



5 CONCLUDING REMARKS

WE have here reported on various simulations with the OBE Quarterly Econometric Model. Major findings are:

(1) For short ex post forecasts during the sample period, mechanical adjustments of equation constants, based on serial correlation of estimated equation residuals, lead to improved forecasts of real magnitudes in initial quarters, and to improved price forecasts over at least a six-quarter period.

(2) Average errors from six-quarter ex post model forecasts revealed biases which were tentatively traced to the wage-rate equation. As a practical aid in model forecasting, it may be advisable to introduce adjustments into selected equations so that biases in important magnitudes are eliminated.

(3) The average absolute errors for *GNP* and real *GNP* from one-quarter ex post model forecasts during the sample period were about one-half of 1 per cent of the average value for each variable.

(4) Short, ex post model forecasts near peaks and troughs revealed the rough contours of actual movements in variables. However, pronounced movements in variables were typically underestimated, and precise timing of turning points was not generally achieved.

(5) A fifty-five quarter simulation over the entire sample period generally followed the patterns in actual data, but several simulated series drifted off from the actual series toward the end of the simulation period.

(6) Real *GNP* series from simulations with random shocks applied to endogenous behavioral equations rarely show downturns, presumably because of the strong growth elements in exogenous variables. However, when measured as deviations from the control solution values, real *GNP* series from runs with stochastic shocks show definite cycles.

(7) The use of serially correlated shocks yielded real *GNP* series much less jagged and more in line with expectations than comparable series generated using non-serially correlated shocks.

(8) Spectral analyses using a first-difference filter on real *GNP* series generated by the shocked simulations revealed spectral peaks in

the range usually assigned to business cycles. Each of the twenty-five runs on data generated using serially correlated shocks showed a spectral peak in the 2 to 5 year cycle-length range. However, spectral analyses using a log-polynomial filter on these same series did not reveal significant spectral peaks.

(9) When serially correlated shocks are applied to endogenous equations and to selected exogenous variables, the resulting real *GNP* series shows sustained cycles with prolonged downturns of from three to five quarters. Maximum deviations from the control solution are about three times as large as those observed when shocks are applied to endogenous equations only.

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APPENDIX A

DEFINITIONS OF SYMBOLS

**AM58$*	Prime contract awards, military, billions of 1958 dollars
C	Personal consumption expenditures, billions of 1958 dollars
CA	Personal consumption expenditures, automobiles and parts, billions of 1958 dollars
**CH*	Personal consumption expenditures, housing, billions of 1958 dollars
CMP/MH	Private employee compensation per man-hour, dollars
CN	Personal consumption expenditures, nondurables, billions of 1958 dollars
COD	Personal consumption expenditures, durables other than automobiles and parts, billions of 1958 dollars
CPR	Corporate profits and inventory valuation adjustment, billions of dollars
CS	Personal consumption expenditures, services except housing, billions of 1958 dollars
**CURR*	Currency outside banks, daily average of quarter, billions of dollars

NOTE: Asterisk indicates exogenous variable.

CUW Utilization rate of industrial capacity for manufacturing, mining, and utilities, Wharton School index, decimal

**D$* Capital consumption allowances, billions of dollars

**DC$* Capital consumption allowances, corporate, billions of dollars

**DCA1* Strike dummy used in auto consumption function: 1953-I to 1956-II = 0.0; 1956-III = −1.0; 1956-IV = 1.0; 1957-I to 1959-III = 0.0; 1959-IV = −1.0; 1960-I = 1.0; 1960-II to 1964-III = 0.0; 1964-IV = −1.0; 1965-I = 1.0; 1965-II to 1966-I = 0.0; 1966-II = −1.0; 1966-III and on = 0.0

**DCA2* Dummy for change in auto installment credit regulations used in auto consumption function: 1953-I to 1954-IV = 0.0; 1955-I and on = 1.0

DD Demand deposits, adjusted daily average of quarter, billions of dollars

**DH$* Capital consumption allowances, residential, billions of historical dollars

**DHS1* Dummy for shift in interest rate policy used in housing starts equation: 1953-I to 1962-I = 0.0; 1962-II to 1966-I = 1.0; 1966-II and on = 0.0

**DHS2* Dummy for interest rate shift used in housing starts equation: 1953-I to 1962-I = 1.0; 1962-II to 1966-I = 0.0; 1966-II and on = 1.0

**DII* Strike dummy used in investment equations: 1953-I to 1959-I = 0.0; 1959-II = 1.0; 1959-III = −2.0; 1959-IV = 0.0; 1960-I = 1.0; 1960-II to 1964-IV = 0.0; 1965-I = 1.0; 1965-II and on = 0.0

**DIM* Strike effects dummy used in import functions: 1953-I to 1959-II = 0.0; 1959-III = 1.0; 1959-IV to 1964-III = 0.0; 1964-IV = 0.5; 1965-I = −1.0; 1965-II = 0.5; 1965-III and on = 0.0

DIV Dividends, billions of dollars

**DLFS* Dummy for secondary labor force transition after the Korean Conflict, used in secondary labor force and employment capacity functions: 1953-I to 1955-I = 1.0; 1955-II = 0.7; 1955-III = 0.3; 1955-IV and on = 0.0

**DPHS* Dummy for discontinuity in *PHS* series: 1953-I to 1954-III = 1.0; 1954-IV = 0.5; 1955-I and on = 0.0

DPI	Disposable personal income, billions of dollars
**DRS*	Dummy for introduction of certificates of deposit used in short-term interest rate equation: 1953-I to 1962-I = 0.0; 1962-II and on = 1.0
DSE	Depreciation, structures and equipment, billions of 1958 dollars
**DSMD*	Strike dummy for shipments of manufacturers' durables: 1953-I to 1959-I = 0.0; 1959-II = 1.0; 1959-III to 1959-IV = −1.0; 1960-I = 1.0; 1960-II and on = 0.0
**DTRU*	Dummy for supplemental benefits for state unemployment transfer payments function: 1953-I to 1957-IV = 0.0; 1958-I to 1958-IV = 1.0; 1959-I and on = 0.0
**DTSSW*	Dummy for additional coverage used in Social Security tax function: 1953-I to 1954-III = 0.0; 1954-IV and on = 1.0
**DUMD*	Dummy for Korean Conflict effects used in new and unfilled manufacturers' durables orders equations: 1953-I = 0.0; 1953-II to 1953-III = 1.0; 1953-IV and on = 0.0
E	Total private civilian employment, millions
EC	Employment, potential private (capacity), millions
**EE*	Self-employed, millions
**EG*	Employment, general civilian government, millions
EW	Employment, civilian wage and salary, millions
**EXP$*	Exports, billions of dollars
FBF	Fiscal balance, Federal net surplus (*NIA* basis), billions of dollars
FBSL	Fiscal balance, state and local net surplus (*NIA* basis), billions of dollars
**G58$*	Total government purchases of goods and services, billions of 1958 dollars
**GFD$*	Federal government defense purchases of goods and services, billions of dollars
**GFND$*	Federal government nondefense purchases of goods and services, billions of dollars
**GIA*	Federal grants-in-aid to state and local governments, billions of dollars
GNP	Gross national product, billions of dollars
GNP58$	Gross national product, billions of 1958 dollars

**GSL$* State and local government purchases of goods and services, billions of dollars

H Average weekly hours, private employment

HC Average weekly hours, capacity

**HH* Households at end of quarter, thousands

HM Average weekly hours, production workers in manufacturing establishments

HS Private nonfarm housing-starts, annual rate, thousands of units

IE Fixed investment, nonresidential, producers' durable equipment, billions of 1958 dollars

IH Fixed investment, residential structures, billions of 1958 dollars

**IHF* Fixed investment, residential structures, farm, billions of 1958 dollars

**IHR* Fixed investment, residential construction on other than new units (additions, alterations, etc.), billions of 1958 dollars

II Change in business inventories, billions of 1958 dollars

II$ Change in business inventories, billions of dollars

IIA Change in auto inventory investment, domestic new cars, billions of 1958 dollars

IINA Change in nonauto inventory investment, billions of 1958 dollars

**IMG$* Imports, military, goods and services, billions of dollars

IMS Imports, other nonmilitary (mainly services), billions of 1958 dollars

IMT Imports, merchandise, billions of 1958 dollars

**INB* Net interest, business, billions of dollars

**INC* Interest paid by consumers, billions of dollars

**INGF* Net interest payments, Federal, billions of dollars

**INGSL* Net interest payments, state and local, billions of dollars

IS Fixed investment, nonresidential structures, billions of 1958 dollars

ISE Fixed investment, nonresidential, billions of 1958 dollars

ITER Number of iterations required for model convergence (used only during forecast periods)

IVA Inventory valuation adjustment, billions of dollars

KA	Stock, automobiles and parts, billions of 1958 dollars
KC$	Net stock of nonfarm, nonfinancial corporate plant and equipment at end of quarter, billions of historical dollars
KH	Stock of dwelling units at end of quarter, thousands
KI	Stock of inventories at end of quarter, billions of 1958 dollars
KIA	Stock of auto inventories, billions of 1958 dollars
KINA	Stock of nonauto inventories, billions of 1958 dollars
KSE	Net stock of plant and equipment at end of quarter, billions of 1958 dollars
KSE$	Net stock of plant and equipment at end of quarter, billions of historical dollars
LFP	Labor force, civilian, prime, males aged 25–54, millions
LFS	Labor force, civilian, secondary (excludes prime males), millions
LH	Liquid assets held by households at end of quarter (currency plus demand and bank savings deposits plus savings and loan shares), billions of dollars
**MAXSS*	Maximum salary subject to Social Security deductions, dollars
MONEY	Total money supply, demand deposits plus currency, billions of dollars
**N*	Total population, millions
NETEXP	Net exports, billions of 1958 dollars
**NP*	Population, civilian resident, males aged 25–54, millions
**NS*	Population, other civilian resident, aged 16–64, millions
OMD	New manufacturers' orders, durable goods, billions of dollars, deflated by *PWMD*
P	Implicit price deflator, gross private output except housing services, 1958 = 1.000
**PA*	Implicit price deflator, personal consumption expenditures, automobiles and parts, 1958 = 1.000
PADJ	Implicit price deflator adjustment (nonzero only during forecast periods)
PC	Implicit price deflator, personal consumption expenditures, 1958 = 1.000
PERINC	Personal income, billions of dollars

**PEX*	Implicit price deflator, exports, 1958 = 1.000
**PF*	Index, prices received by farmers for all farm products, 1910–1914 = 1.000
PGG	Implicit price deflator, government purchases of goods, 1958 = 1.000
PGNP	Implicit price deflator, gross national product, 1958 = 1.000
**PH*	Implicit price deflator, personal consumption expenditures, housing, 1958 = 1.000
PHS	Average cost per new private nonfarm housing unit started, thousands of dollars
PIE	Implicit price deflator, fixed investment, nonresidential, producers' durable equipment, 1958 = 1.000
PIH	Implicit price deflator, fixed investment, residential structures, 1958 = 1.000
**PIM*	Implicit price deflator, imports, 1958 = 1.000
PIS	Implicit price deflator, fixed investment, nonresidential structures, 1958 = 1.000
PISE	Implicit price deflator, fixed investment, nonresidential, 1958 = 1.000
PN	Implicit price deflator, personal consumption expenditures, nondurables, 1958 = 1.000
POD	Implicit price deflator, personal consumption expenditures, durables other than automobiles and parts, 1958 = 1.000
**PR*	BLS consumer rent index, 1957–1959 = 1.000
PRI	Proprietors' income, billions of dollars
PROD	Index of real private *GNP* per man-hour, 1958 = 100.0
PS	Implicit price deflator, personal consumption expenditures, services except housing, 1958 = 1.000
**PWG*	Implicit price deflator, compensation of general government employees, 1958 = 1.000
PWMD	Wholesale price index, durable manufactures, 1957–1959 = 1.000
**RDIS*	Federal Reserve average discount rate, per cent
REM	Net removal of private housing during quarter, thousands
**RENT*	Rental income of persons, billions of dollars

RESF	Free reserves, daily average of quarter, billions of dollars
**RESNB*	Nonborrowed reserves, daily average of quarter, billions of dollars
RL	Per cent yield, corporate bonds (Moody's)
RM	Per cent yield, secondary market, FHA-insured homes
**RMBD*	Ratio, Federal Reserve member bank demand deposits to money supply component of demand deposits, decimal
**RMBT*	Ratio, Federal Reserve member bank time deposits to total time deposits, decimal
**RRD*	Average reserve requirement against member bank demand deposits, decimal
**RRT*	Average reserve requirement against member bank time and savings deposits, decimal
RS	Short-run interest rate, average yield on 4–6 month commercial paper, per cent
RT	Interest rate on commercial bank savings deposits, per cent
RTB	Three-month Treasury bill yield, per cent
**RTCF*	Rate, Federal corporate profits tax, decimal
**RTEXA*	Index of average rates, Federal ad valorem excise taxes, 1958 = 1.00
**RTEXS*	Index of average rates, Federal specific excise taxes, 1958 = 1.00
**RTQ*	Maximum interest rate on Federal Reserve member bank savings deposits under Regulation Q, per cent
**RTRU*	Maximum weekly benefit rate, unemployment insurance, dollars
**RTSSW*	Combined employer-employee contribution rate for OASI, decimal
SD	Statistical discrepancy, billions of dollars
SDADJ	Statistical discrepancy adjustment (nonzero only during forecast periods)
**SGF*	Federal subsidies less current surplus of Federal government enterprises, billions of dollars
**SGSL*	Subsidies less current surplus of state and local government enterprises, billions of dollars

SIB	Social insurance, employer contributions, billions of dollars
**SIBOF*	Federal social insurance programs, employer contributions, excluding OASI, billions of dollars
**SIBSL*	State and local social insurance programs, employer contributions, billions of dollars
SIP	Social insurance, personal contributions, billions of dollars
**SIPOF*	Federal social insurance programs, personal contributions, excluding OASI except for self-employed contributions to OASI, billions of dollars
**SIPSL*	State and local social insurance programs, personal contributions, billions of dollars
SMD	Manufacturers' shipments, durable goods, billions of dollars, deflated by *PWMD*
SRATE	Personal savings as a proportion of disposable personal income, decimal
**T531*	Time in quarters, 1953-I = 1.0
**T581*	Time in quarters, 1958-I = 1.0; 1953-I to 1957-IV = 0.0
TCF	Corporate profits tax liability, Federal, billions of dollars
TCRI	Investment tax credit, billions of dollars
TCSL	Corporate profits tax liability, state and local, billions of dollars
TD	Time deposits, commercial banks, billions of dollars
TEXAV	Ad valorem excise tax receipts, Federal, billions of dollars
TEXS	Specific excise tax receipts, Federal, billions of dollars
TIF	Indirect business tax and nontax receipts, Federal, billions of dollars
**TIFO*	Indirect tax receipts, business other than excise, Federal, billions of dollars
TISL	Indirect business tax receipts, state and local, billions of dollars
TPF	Personal tax and nontax payments, Federal, billions of dollars
**TPFS*	Final personal tax and nontax settlements plus estate and gift taxes, Federal, billions of dollars

TPSL	Personal tax and nontax payments, state and local, billions of dollars
**TRB*	Transfer payments, business, billions of dollars
**TRFF*	Net transfer payments to foreigners, Federal, billions of dollars
**TRFP*	Transfer payments to foreigners, personal, billions of dollars
TRP	Transfer payments to persons, billions of dollars
**TRPOF*	Transfer payments to persons, Federal, except unemployment insurance benefits, billions of dollars
**TRPSL*	Transfer payments to persons, state and local, billions of dollars
TRU	Transfers, state unemployment insurance benefits, billions of dollars
**TRUEX*	Unemployment insurance factor for supplementary unemployment programs
TSSW	Personal and employer contributions for old-age and survivors' insurance (OASI) excluding self-employed and personal medical payments, but including hospital insurance, billions of dollars
UMD	Unfilled manufacturers' orders, durable goods, at end of quarter, billions of dollars, deflated by *PWMD*
UNITLC	Private employee compensation per unit of real private *GNP*, dollars
UNRATE	Unemployment rate, civilian labor force, per cent
UR	Unemployment rate, civilian labor force, decimal
URP	Unemployment rate, prime males, decimal
V	Vacant nonfarm housing units at end of quarter, thousands
W	Wages and salaries plus other labor income, billions of dollars
**WAMD*	Wage accruals less disbursements, billions of dollars
**WG$*	Wages, compensation of general government employees, billions of dollars
WR	Annual wage per private employee, thousands of dollars
X	Gross private output, except housing services, billions of 1958 dollars
XC	Gross private output, except housing services, capacity, billions of 1958 dollars

APPENDIX B

LIST OF STRUCTURAL EQUATIONS

I. *GNP* COMPONENTS

(1) Gross National Product (Current Dollars)

$$GNP = PC \times C + PIH \times IH + PISE \times ISE + II\$ + EXP\$ - PIM(IMT + IMS) - IMG\$ + GFD\$ + GFND\$ + GSL\$$$

$$GNP = P \times X + PH \times CH + WG\$$$

(2) Gross National Product (Constant Dollars)

$$GNP58\$ = X + CH + \left(\frac{WG\$}{PWG}\right)$$

(3) Gross Private Output, Except Housing Services

$$X = C - CH + IH + ISE + II + \left(\frac{EXP\$}{PEX}\right) - \left[IMT + IMS + \frac{IMG\$}{PIM}\right] + \left(\frac{GFD\$ + GFND\$ + GSL\$ - WG\$}{PGG}\right)$$

Consumption functions.

(4) Personal Consumption Expenditures, Autos and Parts

$$CA = -20.9 - \underset{(8.7)}{41.3}\frac{PA}{PC} + \underset{(.029)}{.117}\frac{DPI - TRP}{PC} + \underset{(.340)}{1.392}(HM)_{-1} - \underset{(.0269)}{.0623}(KA)_{-1} + \underset{(.4)}{1.9}DCA1 + \underset{(.7)}{3.1}DCA2$$

$$TSLS \qquad \bar{R}^2 = .95 \qquad \bar{S} = 1.0 \qquad DW = 1.2$$

(5) Stock of Autos

$$KA = \sum_{i=0}^{40} (.929)^i CA_{t-i}$$

(6) Personal Consumption Expenditures, Durables Other Than Autos and Parts

$$COD = 43.95 - \underset{(19.7)}{53.1}\frac{POD}{PC} + \underset{(.02)}{.056}\frac{DPI}{PC} + \underset{(.015)}{.103}\left(\frac{LH}{PC}\right)_{-1}^{\mathrm{dev}_t}$$

$$+ \underset{(.00007)}{.00148}(\overline{HS})_{-2:3}$$

$$TSLS \qquad \bar{R}^2 = .99 \qquad \bar{S} = .6 \qquad DW = .96$$

where $\left(\frac{LH}{PC}\right)_{-1}^{\mathrm{dev}_t} = \left(\frac{LH}{PC}\right)_{-1} - 129.6 - 3.39T_{531}$

(7) Personal Consumption Expenditures, Nondurables

$$CN = 16.4 + \underset{(.04)}{.216}\left(\frac{DPI - TRP}{PC}\right) + \underset{(.247)}{.282}\left(\frac{TRP}{PC}\right)$$

$$+ \underset{(.207)}{.393}(\overline{CN})_{-1:8}$$

$$TSLS \qquad \bar{R}^2 = .994 \qquad \bar{S} = 1.5 \qquad DW = .97$$

(8) Personal Consumption Expenditures, Services Except Housing

$$\frac{CS}{N} = .158 + \underset{(.014)}{.075}\left(\frac{DPI}{N \times PC}\right) + \underset{(.131)}{.555}\left(\frac{\overline{CS}}{N}\right)_{-1:8} - \underset{(7.5)}{19.2}\left(\frac{1}{N}\right)$$

$$TSLS \qquad \bar{R}^2 = .994 \qquad \bar{S} = .0021 \qquad DW = .98$$

(9) Personal Consumption Expenditures

$$C = CA + COD + CN + CS + CH$$

Fixed investment functions.

(10) Fixed Investment, Nonresidential Structures and Equipment

a. $$\frac{ISE}{XC} = .02897\left[\frac{\sum_{i=1}^{11} WT1_i X_{t-i+1}}{XC}\right]$$

$$+ .5310 \left[\frac{\sum_{i=1}^{11} WT2_i FV_{t-i+1}}{XC} \right]$$

$$+ 3.4698 \left[\frac{\sum_{i=1}^{11} WT3_i RL_{t-i+1}}{XC} \right] + .05218$$

$$\text{where } FV = \frac{(CPR + DC\$ - TCF - TCSL) \times ISE}{PISE \times ISE - TCRI}.$$

$$OLS \qquad \bar{S} = \left(\frac{ISE}{XC}\right) = .0048$$

i	$WT1$	$WT2$	$WT3$
1	.592	.024	−.135
2	1.043	.065	−.257
3	1.050	.112	−.297
4	.671	.150	−.255
5	.272	.166	−.168
6	−.104	.161	−.113
7	−.435	.137	−.028
8	−.678	.100	.049
9	−.780	.056	.105
10	−.476	.024	.072
11	−.155	.006	.028

$$\text{b. } L\left(\frac{ISE}{XC}\right) = \underset{(.00423)}{.01608} + \underset{(.00403)}{.03151L} \left[\frac{\sum_{i=1}^{11} WT1_i X_{t-i+1}}{XC} \right]$$

$$+ \underset{(.0362)}{.7022L} \left[\frac{\sum_{i=1}^{11} WT2_i FV_{t-i+1}}{XC} \right]$$

$$+ \underset{(.000944)}{.004214L} \left[\sum_{i=1}^{11} WT3_i RL_{t-i+1} \right]$$

where $L(X) = X_t - .8432X_{t-1}$

$$FV = \frac{(CPR + DC\$ - TCF - TCSL) \times ISE}{PISE \times ISE - TCRI}.$$

$$\bar{S}\left(\frac{ISE}{XC}\right) = .0022$$

$WT1$, $WT2$, $WT3$ same as weights in version (a).

(11) Fixed Investment, Nonresidential Equipment

$$\frac{IE}{ISE} = \underset{(.028)}{.563} + \underset{(.0164)}{.033225}[(CUW)_{-1} + (CUW)_{-2}]$$

$$+ \underset{(.0962)}{1.308}\left(\frac{TCRI}{IE \times PIE}\right)_{-1}$$

$OLS \qquad \bar{R}^2 = .784 \qquad \bar{S} = .011 \qquad DW = .80$

(12) Fixed Investment, Nonresidential Structures

$$IS = ISE - IE$$

(13) Net Stock of Plant and Equipment

$$KSE = (KSE)_{-1} + .25ISE - DSE$$

(14) Depreciation, Fixed Nonresidential Investment

$$DSE = -.186 + \underset{(.0007)}{.0269}(KSE)_{-1}$$

$OLS \qquad \bar{R}^2 = .974 \qquad \bar{S} = .22 \qquad DW = .111$

(15) Capital Consumption Allowances

a. Exogenous

b. $D\$ = .1995(KSE \times PIE)_{-1}$

(16) Capital Consumption Allowances, Corporate

a. Exogenous

b. $DC\$ = 7.83 + .0973(KSE \times PIE)_{-1}$

Housing functions.

(17) Private Investment, Residential Structures

$$IH = \underset{(.000003)}{.00102}\left(\frac{1}{PIH}\right)[.41(PHS \times HS) + .49(PHS \times HS)_{-1}$$

$$+ .10(PHS \times HS)_{-2}] + IHR + IHF$$

$$OLS \qquad \bar{R}^2 = .93 \qquad \bar{S} = .5 \qquad DW = .81$$

(18) Private Nonfarm Housing Starts

$$\text{a. } HS = 309. + \underset{(1186.)}{3846.DHS1} + \underset{(570.)}{1493.}\left(\frac{PR}{PIH}\right)_{-1}$$

$$- 118.6DHS2 \times (\overline{RTB})_{-2:4} - \underset{(234.2)}{726.5DHS1(RM)_{-2}}$$

$$- \underset{(.072)}{.356(V - .03HH)_{-2}}$$

$$OLS \qquad \bar{R}^2 = .82 \qquad \bar{S} = 74.7 \qquad DW = 1.10$$

$$\text{b. } HS = -896.6 + 3846.DHS1 + 1493.\left(\frac{PR}{PIH}\right)_{-1}$$

$$- 118.6DHS2 \times (\overline{RTB})_{-2:4}$$

$$- 726.5DHS1 \times (RM)_{-2} - .356(V - .03HH)_{-2}$$

$$+ 356.9DHS2 + 24.8T_{531}$$

(19) Stock of Dwelling Units (End of Quarter)

$$KH = (KH)_{-1} + .25(HS)_{-2} - REM$$

(20) Vacant Nonfarm Housing Units (End of Quarter)

$$V = (V)_{-1} + .25(HS)_{-2} - \Delta HH - REM$$

(21) Net Removal of Private Housing During Quarter

$$REM = 17.5 + .0018(KH)_{-1}$$

Inventory investment functions.

(22) Change in Business Inventories (Current Dollars)

$$II\$ = PWMD \times II$$

(23) Change in Business Inventories (Constant Dollars)

$$II = IIA + IINA$$

(24) Change in Auto Inventory Investment

$$\underset{}{IIA} = \underset{(.5177)}{-1.46906} + \underset{(.0456)}{.21144CA} - \underset{(.1054)}{.13171\Delta CA}$$

$$- \underset{(.321)}{1.363(KIA)_{-1}} + \underset{(.378)}{.52405DCA1}$$

$OLS \quad \bar{R}^2 = .315 \quad \bar{S} = .782 \quad DW = 1.91$

(25) Change in Inventory Investment, Nonauto

$$IINA = \underset{(3.14)}{1.684} + \underset{(.039)}{.0811}(X - II - CS - CA)$$

$$- \underset{(.0010)}{.1759}\Delta(X - II - CS - CA)$$

$$+ \underset{(.111)}{.3529(IINA)_{-1}} - \underset{(.104)}{.1512(KINA)_{-1}}$$

$$+ \underset{(.118)}{.6675\Delta(UMD)_{-1}} + \underset{(.690)}{1.958DII}$$

$OLS \quad \bar{R}^2 = .828 \quad \bar{S} = 1.8 \quad DW = 1.54$

(26) Stock of Inventories (End of Quarter)

$$KI = (KI)_{-1} + .25II$$

(27) Stock of Auto Inventories (End of Quarter)

$$KIA = (KIA)_{-1} + .25IIA$$

(28) Stock of Nonauto Inventories (End of Quarter)

$$KINA = (KINA)_{-1} + .25IINA$$

Imports.

(29) Merchandise Imports

a. $$IMT = \underset{(5.43)}{8.88} + (\underset{(.0054)}{.01006} + \underset{(.0040)}{.01578}CUW) \times [X + (X)_{-1}]$$

$$- \underset{(3.72)}{12.955}\left(\frac{PIM}{P}\right)_{-1} + \underset{(.3948)}{1.5053}DIM$$

$OLS \quad \bar{R}^2 = .975 \quad \bar{S} = .618 \quad DW = .56$

b. $$IMT = .04 + (.01006 + .01578CUW) \times [X + (X)_{-1}]$$

$$- 12.955\left(\frac{PIM}{P}\right)_{-1} + 1.5053DIM + .17T_{531}$$

(30) Nonmilitary Services Imports

$$IMS = -2.8 + \underset{(.004)}{.019}\frac{DPI}{PIM} + \underset{(.196)}{.328}(\overline{IMS})_{-1:4}$$

$TSLS \quad \bar{R}^2 = .991 \quad \bar{S} = .1 \quad DW = 1.97$

Government purchases.

(31) Government Purchases of Goods and Services, Total (Constant Dollars)

$$G58\$ = \left(\frac{GFD\$ + GFND\$ + GSL\$ - WG\$}{PGG}\right) + \left(\frac{WG\$}{PWG}\right)$$

NOTE:

$$(\bar{Z})_{-j:k} = \frac{1}{k - j + 1}\sum_{i=j}^{k} Z_{t-i}$$

II. PRICES AND WAGE RATES

(1) Implicit Price Deflator, Gross Private Output (Except Housing Services)

$$P = .263 + \underset{(.053)}{1.230} \sum_{i=0}^{3} W_i \left(\frac{W - WG\$}{X}\right)_{t-1}$$

$$+ \underset{(.084)}{.456} CUW^{*7.1} \left(\frac{(X - II) - (\overline{X - II})_{-1:4}}{(\overline{X - II})_{-1:4}}\right) + \underset{(.00013)}{.00137} T_{531}$$

where $CUW^* = .87$ if $CUW \leqslant .87$ $\quad W_0 = .4 \quad W_1 = .3$
$W_2 = .2 \quad W_3 = .1$

$TSLS \quad \bar{R}^2 = .998 \quad \bar{S} = .0032 \quad DW = 1.02$

(2) Implicit Price Deflator, Consumer Durables (Excluding Autos)

$$\Delta POD = -.007 + \underset{(.12)}{.32}\Delta P + \underset{(.000043)}{.00011}(UMD)_{-1}$$

$OLS \quad \bar{R}^2 = .210 \quad \bar{S} = .003 \quad DW = 1.12$

(3) Implicit Price Deflator, Consumer Nondurables

$$\Delta PN = \underset{(.08)}{.72}\Delta P + \underset{(.07)}{.45}\Delta PF$$

$TSLS \quad \bar{R}^2 = .58 \quad \bar{S} = .0028 \quad DW = 1.66$

(4) Implicit Price Deflator, Consumer Services (Except Housing)

$$\frac{PS - (\overline{PS})_{-1:4}}{(PS)_{-1:4}} = .0117 + \underset{(.053)}{.258}\left[\frac{WR - (\overline{WR})_{-1:4}}{(\overline{WR})_{-1:4}}\right]$$

$$- \underset{(.075)}{.177}\left[\frac{HM - (\overline{HM})_{-1:4}}{(\overline{HM})_{-1:4}}\right]$$

$TSLS \quad \bar{R}^2 = .15 \quad \bar{S} = .006 \quad DW = .35$

(5) Implicit Price Deflator, Residential Investment

$$PIH = -.075 + \underset{(.022)}{.687}\left(\frac{W - WG\$}{X}\right) + \underset{(.00024)}{.00230}(IH + ISE)$$

$$+ \underset{(.070)}{.540}(\overline{PIH})_{-1:4}$$

$$TSLS \qquad \bar{R}^2 = .994 \qquad \bar{S} = .007 \qquad DW = .81$$

(6) Price Per New Dwelling Unit Started

$$PHS = \underset{(.499)}{1.9066} + \underset{(.471)}{10.472}PIH - \underset{(.131)}{1.2198}DPHS$$

$$OLS \qquad \bar{R}^2 = .955 \qquad \bar{S} = .257 \qquad DW = .73$$

(7) Implicit Price Deflator, Nonresidential Structures

$$PIS = -.173 + \underset{(.131)}{.944}\left(\frac{W - WG\$}{X}\right) + \underset{(.00016)}{.00118}(IH + ISE)$$

$$+ \underset{(.053)}{.565}(\overline{PIS})_{-1:4}$$

$$TSLS \qquad \bar{R}^2 = .996 \qquad \bar{S} = .0065 \qquad DW = 1.34$$

(8) Implicit Price Deflator, Equipment

$$\frac{PIE}{(PIE)_{-1}} = \underset{(.255)}{-.5005} + \underset{(.308)}{.80003}\frac{P}{(P)_{-1}} + \underset{(.191)}{.69885}\frac{PWMD}{(PWMD)_{-1}}$$

$$TSLS \qquad \bar{R}^2 = .449 \qquad \bar{S} = .0054 \qquad DW = 1.91$$

(9) Price Index, Wholesale Durables, Manufacturing

$$\frac{PWMD}{(PWMD)_{-1}} = \underset{(.269)}{.33272} + \underset{(.0117)}{.02750}\left(\frac{OMD}{SMD}\right)_{-1} + \underset{(.272)}{.64169}\frac{P}{(P)_{-1}}$$

$$TSLS \qquad \bar{R}^2 = .227 \qquad \bar{S} = .0054 \qquad DW = .95$$

(10) Implicit Price Deflator, Government Purchases of Goods and Services Other Than Employment Compensation

a. Exogenous

b. $\frac{PGG}{(PGG)_{-1}} = \frac{P}{(P)_{-1}}$

(11) Implicit Price Deflator, Personal Consumption Expenditures

$$PC = \left(\frac{PA \times CA + POD \times COD + PN \times CN + PS \times CS + PH \times CH}{C}\right)$$

(12) Implicit Price Deflator, Nonresidential Fixed Investment

$$PISE = \left(\frac{PIS \times IS + PE \times IE}{ISE}\right)$$

(13) Wage Per Employee, Private Sector

$$\frac{WR - (\overline{WR})_{-1:4}}{(\overline{WR})_{-1:4}} = \underset{(.000139)}{.0076 + .000982}\left[\frac{1}{\sum_{i=0}^{3} W_i(UR)_{t-i}}\right]$$

$$- \underset{(.0196)}{.0385}\left[\frac{URP \times LFP}{LFP + LFS - EW - EG}\right]$$

$$- \underset{(.144)}{.529}[UR - (\overline{UR})_{-1:4}]$$

$$+ \underset{(.103)}{.622}\left[\frac{HM - (\overline{HM})_{-1:4}}{(\overline{HM})_{-1:4}}\right]$$

$$+ \underset{(.179)}{.869}\left[\frac{(PC)_{-1} - (\overline{PC})_{-2:5}}{(\overline{PC})_{-2:5}}\right]$$

where $W_0 = .4 \quad W_1 = .3 \quad W_2 = .2 \quad W_3 = .1$

$OLS \quad \bar{R}^2 = .757 \quad \bar{S} = .0061 \quad DW = 1.39$

NOTE:

Any discrepancy between the value of P given by Equation 1 of this section and that given by a properly weighted sum of the component deflators (P') is resolved as follows:

Let $PADJ = P - P'$

Then the component deflators are adjusted as follows:

PN adjustment $= PADJ \times .3 \times X/CN$
PS adjustment $= PADJ \times .4 \times X/CS$
PIH adjustment $= PADJ \times .07 \times X/IH$

$$PIS \text{ adjustment} = PADJ \times .15 \times X/IS$$
$$PIE \text{ adjustment} = PADJ \times .08 \times X/IE$$

After adjustment $P' = P$.

III. CAPACITY, CAPACITY UTILIZATION, AND PRODUCTIVITY

(1) Potential Private Employment

a. $EC = .985LFP + (.97 - .000191T_{531})[NS(.2991 + .05285 \ln (T_{531} + 100) - .0203DLFS)] - EG$

b. $EC = .985LFP + .9593[NS(.2991 + .05285 \ln (T_{531} + 100) - .0203DLFS)] - EG$

(2) Potential Private Weekly Hours

a. $\log HC = 1.6319 - .000654T_{531} + .000466T_{581}$

b. $HC = 40.9261$

derived from estimated equation

$$\log H = 1.6319 + \underset{(.012)}{.0855} \log CUW - \underset{(.000066)}{.000654}T_{531} + \underset{(.000085)}{.000466}T_{581}$$

$$OLS \qquad \bar{R}^2 = .90 \qquad \bar{S} = .00169 \qquad DW = 1.656$$

(3) Potential Private GNP (Except Housing Services)

$$\log XC = -.5506 + .002056T_{531} + .3 \log (\overline{KSE})_{-1:4} + .7 \log (EC \times HC)$$

derived from estimated equation

$$\log \left(\frac{X}{E \times H}\right) - .3 \log \left[\frac{.96CUW \times (\overline{KSE})_{-1:4}}{\frac{PROD}{PROD_{TR}} \times E \times H}\right] = -.5453 + \underset{(.000030)}{.0002056}T_{531}$$

where $PROD_{TR}$ = the trend in productivity.

$$OLS \qquad \bar{S} = .0032 \qquad DW = .36$$

(4) Industrial Capacity Utilization, Wharton Index

$$\ln CUW - \rho(\ln CUW_{-1})$$

$$= \underset{(.076)}{-.127203} + \underset{(.2096)}{1.43398}\left[\ln\left(\frac{X}{XC}\right) - \rho \ln\left(\frac{X}{XC}\right)_{-1}\right]$$

$$\underset{(.224)}{-.389164}\left[\ln\left(\frac{CS}{X}\right) - \rho \ln\left(\frac{CS}{X}\right)_{-1}\right]$$

where $\rho = .8058$.

$$OLS \qquad \bar{R}^2 = .789 \qquad \bar{S} = .0129 \qquad DW = 1.7$$

(5) Private GNP (1958 dollars) Per Man-Hour

$$PROD = \left[\frac{X + CH}{H(EW + EE - EG)}\right] \times 563.7$$

IV. LABOR FORCE, EMPLOYMENT, AND HOURS

(1) Civilian Labor Force, Males (25–54)

a. $$\frac{LFP}{NP} = .956 - \underset{(.000020)}{.000068T_{531}}$$

$$OLS \qquad \bar{R}^2 = .22 \qquad \bar{S} = .0025 \qquad DW = .91$$

b. $$\frac{LFP}{NP} = .952$$

(2) Civilian Labor Force Except Males (25–54)

for $URP < 0.045$:

$$\frac{LFS}{NS} = .3123 - \underset{(.299)}{1.068URP} + \underset{(3.78)}{12.53URP^2} - \underset{(.002)}{.0203DLFS}$$

$$+ .05285 \ln (T_{531} + 100)$$

$$TSLS \qquad \bar{R}^2 = .924 \qquad \bar{S} = .00375 \qquad DW = .765$$

for $URP \geqslant 0.045$:

$$\frac{LFS}{NS} = .28635 + .07222URP - .0203DLFS + .05285 \ln (T_{531} + 100)$$

(3) Private Civilian Employment

$$\Delta \log E = b \left[\log EC + \underset{(.021)}{.418} \log \left(\frac{X}{XC} \right) - \log (E)_{-1} \right]$$

$$OLS \qquad \bar{R}^2 = .651 \qquad \bar{S} = .0017 \qquad DW = 1.07$$

where $E = EW + EE - EG$

$$b = \begin{cases} .33422, \; URP \geqslant .06 \\ (.5627 - 3.808URP)(1 - e^{-224.0(URP - .01)}), \\ \quad URP < .06 \end{cases}$$

(4) Civilian Unemployment Rate

$$UR = \left(\frac{LFP + LFS - EW - EE}{LFP + LFS} \right)$$

(5) Unemployment Rate, Males (25–54)

$$URP = \underset{(.0081)}{-.0629} + \underset{(.1761)}{.2816}UR + \underset{(1.714)}{6.482}(UR)^2 + \underset{(.0132)}{.1528} \left(\frac{LFP}{LFP + LFS} \right)$$

$$OLS \qquad \bar{R}^2 = .971 \qquad \bar{S} = .0018 \qquad DW = .806$$

where $URP \leqslant .85(UR)$

(6) Private Man-Hours

$$\Delta \log H = \underset{(.105)}{.661} \left[\log HC + \underset{(.020)}{.130} \log \left(\frac{X}{XC} \right) - \log (H)_{-1} \right]$$

$$OLS \qquad \bar{R}^2 = .423 \qquad \bar{S} = .0016 \qquad DW = 1.96$$

(7) Average Weekly Hours, Manufacturing

$$HM = 17.35 + \underset{(.0048)}{.0450}[X - (X)_{-2}] + \underset{(1.49)}{4.27}CUW + \underset{(.119)}{.469}(\overline{HM})_{-1:4}$$

$$TSLS \qquad \bar{R}^2 = .803 \qquad \bar{S} = .0073 \qquad DW = 1.24$$

V. INCOME

(1) Corporate Profits and Inventory Valuation Adjustment

$$\rho \log (CPR + DC\$) = \underset{(.00199)}{-.3480} + \underset{(.0032)}{1.092}\rho \log (P \times X) - \underset{(.0339)}{2.165}\rho \log \left(\frac{W - WG\$}{P \times X}\right) + \underset{(.1014)}{.563}\rho \log CUW$$

where $\rho = \underset{(.0014)}{.762}$

$$\bar{S} = .00659$$

(2) Dividends

$$DIV = -.19 + \underset{(.012)}{.034}CPR + \underset{(.099)}{.899}(DIV)_{-1}$$

$$TSLS \qquad \bar{R}^2 = .994 \qquad \bar{S} = .3 \qquad DW = 1.64$$

(3) Inventory Valuation Adjustment

$$IVA = .14 - \underset{(17.5)}{134.5}\Delta PWMD - \underset{(.0230)}{.0628}(II)_{-1}$$

$$OLS \qquad \bar{R}^2 = .592 \qquad \bar{S} = .7 \qquad DW = 1.52$$

(4) Proprietors' Income

$$PRI - (\overline{PRI})_{-1:4} = -.3 + \underset{(.051)}{.221}[(PN \times CN + PS \times CS) - (\overline{PN \times CN + PS \times CS})_{-1:4}]$$

$$- 4.98[WR - (\overline{WR})_{-1:4}]$$
$$(3.36)$$

$$TSLS \qquad \bar{R}^2 = .24 \qquad \bar{S} = 1.0 \qquad DW = .70$$

(5) Wages and Salaries Plus Other Labor Income

$$W = WR(EW - EG) + WG\$$$

(6) Personal Income

$$PI = W - WAMD + PRI + DIV + RENT + INB + INC$$
$$+ INGF + INGSL + TRP - SIP$$

(7) Disposable Personal Income

$$DPI = PI - TPF - TPSL$$

(8) Savings Rate

$$SRATE = \left[\frac{DPI - (PC \times C - INC + TRFP)}{DPI}\right] \times 100$$

(9) Statistical Discrepancy

$$SD = GNP - W - RENT - INB - PRI - CPR - TRB$$
$$- WAMD - SIB - D\$ - TIF - TISL$$
$$+ SGF + SGSL$$

NOTE:

During forecast periods, restrictions are placed on the level and change in SD:

(1) The absolute level of SD must be less than, or equal to, the larger of \$4.0 billion or 0.00522 × GNP_{-1}.

(2) The change in SD from the previous period must be no larger than \$1.0 billion.

The model is first solved without taking into account the above restrictions. Then tests are made to see whether or not the calculated value of SD from identity (9) meets the above restrictions. If the restrictions are met, then no further calculations are necessary.

If the restrictions are not met, then $SDADJ$, an amount just sufficient to bring SD into line, is calculated. Additive adjustments are then made to the equations for three income items as follows:

$$WR \text{ adjustment} = SDADJ \times .7574(EW_{-1} - EG_{-1})$$

$$CPR \text{ adjustment} = SDADJ \times .5065$$

$$PRI \text{ adjustment} = SDADJ \times .2440$$

Then the entire model is re-solved, and the above tests repeated until the calculated value of SD from identity (9) meets the two restrictions.

VI. TAXES, TRANSFERS, AND FISCAL BALANCE

(1) Personal Tax Payments, Federal

a. for 531–534:

$$(TPF - TPFS) = \underset{(10.2)}{1.8095} + \underset{(.037)}{.10031}BASE$$

$$OLS \qquad \bar{R}^2 = .682 \qquad \bar{S} = .122$$

where $BASE = W + DIV + PRI + RENT + INB + INC + INGF + INGSL.$

for 541–634:

$$(TPF - TPFS) = \underset{(.522)}{-9.2113} + \underset{(.0014)}{.13047}BASE$$

$$TSLS \qquad \bar{R}^2 = .995 \qquad \bar{S} = .465$$

for 641:

$$(TPF - TPFS) = -12.45 + .133BASE$$

for 642–661:

$$(TPF - TPFS) = \underset{(1.85)}{-20.3324} + \underset{(.0037)}{.13599}BASE$$

$$TSLS \qquad \bar{R}^2 = .995 \qquad \bar{S} = .231$$

for 662:

$$(TPF - TPFS) = -35.31 + .165BASE$$

for 663–674:

$$\underset{}{(TPF - TPFS)} = \underset{(5.33)}{-42.824} + \underset{(.0091)}{.17951}BASE$$

$OLS \qquad \bar{R}^2 = .987 \qquad \bar{S} = .347$

b. TPF is found by recursively solving the three equations below:

$$(1)\ \log\left(1 - \frac{TI}{PI}\right) = .0619 - .3431 \log\left(\frac{PI}{N}\right) + .3466 \log[.656(1.004)(T_{531} - 52.0)]$$

where TI = taxable personal income.

$$(2)\ \log TPFL = -1.0038 + 1.124 \log (TI)$$

where $TPFL$ = personal Federal income tax liabilities.

$$(3)\ TPF = -1.004 + 1.01TPFL + .052T_{531}$$

(2) Personal Tax and Nontax Payments, State and Local

a. $TPSL = \underset{(.496)}{3.5579} + \underset{(.0018)}{.02527}BASE + \underset{(.00739)}{.01122}T_{531} + \underset{(.00901)}{.06498}T_{581}$

[$BASE$ is defined as in the TPF equation given above.]

$TSLS \qquad \bar{R}^2 = .997 \qquad \bar{S} = .174 \qquad DW = .883$

b. $TPSL = -10.292 + [.146 + .0002T_{531}]BASE + .176T_{531}$

(3) Corporate Tax Liability, Federal

$$\log (TCF + TCRI) = -.1056 + \log RTCF + \underset{(.0092)}{1.0150} \log (CPR - IVA)$$

$OLS \qquad \bar{R}^2 = .996 \qquad \bar{S} = .0065 \qquad DW = .36$

(4) Corporate Tax Liability, State and Local

$$TCSL = -.13203 + .02052(CPR - IVA) + .01286T_{531}$$
$$\quad (.0629) \quad (.00177) \qquad\qquad (.00144)$$

$$TSLS \qquad \bar{R}^2 = .973 \qquad \bar{S} = .077 \qquad DW = 1.06$$

(5) Investment Tax Credit

a. $TCRI = 0$; prior to 1962-I

$$= -.97505 + .057734(IE \times PIE); \text{ after 1962-I}$$
$$(.0729) \quad (.00175)$$

$$OLS \qquad \bar{R}^2 = .983 \qquad \bar{S} = .055 \qquad DW = .57$$

b. $TCRI = .02554(IE \times PIE)$

(6) Federal Specific Excise Tax Liability

$$\log TEXS = -1.8521 + 1.5511 \log RTEXS$$
$$(.1675) \quad (.1072)$$
$$+ 1.047 \log (X - II - CS)$$
$$(.0669)$$

$$OLS \qquad \bar{R}^2 = .977 \qquad \bar{S} = .0171 \qquad DW = .720$$

(7) Federal Ad Valorem Excise Tax Liability

$$\log TEXAV = -2.8341 + 1.1641 \log RTEXAV$$
$$(.2436) \quad (.2310)$$
$$+ 1.2275 \log [P(X - II)]$$
$$(.0938)$$

$$OLS \qquad \bar{R}^2 = .799 \qquad \bar{S} = .0412 \qquad DW = .829$$

(8) Indirect Business Tax and Nontax Receipts, Federal

$$TIF = TEXS + TEXAV + TIFO$$

where $TIFO$ = other indirect business tax receipts.

(9) Indirect Tax and Nontax Liability, State and Local

$$TISL = -2.0 + .0646P(X - II) + .250T_{531}$$
$$(.0038) \qquad\qquad (.021)$$

$$TSLS \qquad \bar{R}^2 = .998 \qquad \bar{S} = .5 \qquad DW = .26$$

(10) Transfer Payments to Persons

$$TRP = TRU + TRPOF + TRPSL + TRB$$

(11) OASDI Contributions, Employer-Employee

$$\log TSSW - \log RTSSW = \underset{}{-1.5471} + \underset{(.035)}{.53062} \log MAXSS + \underset{(.0041)}{.005744} DTSSW + \underset{(.024)}{.78495} \log (W - WG)$$

$$OLS \qquad \bar{R}^2 = .995 \qquad \bar{S} = .00725 \qquad DW = .67$$

(12) State Unemployment Insurance Benefits

$$\log TRU - \log(1 + TRUEX) = -.9644 + \underset{(.0944)}{.33551} \log RTRU + \underset{(.0714)}{1.4238} \log (LFP + LFS - EW - EE) + \underset{(.0243)}{.066773} DTRU$$

$$OLS \qquad \bar{R}^2 = .942 \qquad \bar{S} = .041 \qquad DW = .832$$

(13) Social Insurance, Personal Contributions

$$SIP = .5TSSW + SIPOF + SIPSL$$

(14) Social Insurance, Employer Contributions

$$SIB = .5TSSW + SIBOF + SIBSL$$

(15) Fiscal Balance, Federal Net Surplus or Deficit

$$FBF = TPF + TCF + TIF + TSSW + SIPOF + SIBOF - GFD\$ - GFND\$ - TRU - TRPOF - TRFF - GIA - INGF - SGF$$

(16) Fiscal Balance, State and Local Net Surplus or Deficit

$$FBSL = TPSL + TCSL + TISL + SIPSL + SIBSL + GIA - GSL\$ - TRPSL - INGSL - SGSL$$

VII. INTEREST RATES AND MONEY SUPPLY

(1) Free Reserves Identity

$$RESF = RESNB - (RMBD \times RRD \times DD + RMBT \times RRT \times TD)$$

(2) Liquid Assets, Households

$$LH = -6.34 + \underset{(.0633)}{.8599}(LH)_{-1} + \underset{(.057)}{.142}DPI - \underset{(.95)}{3.82}(RL - RT)$$

$$OLS \qquad \bar{R}^2 = .999 \qquad \bar{S} = 1.82 \qquad DW = 2.433$$

(3) Demand Deposits (Adjusted) Plus Currency (Money Stock)

$$DD + CURR = .9 + \underset{(.0923)}{.9617}(DD + CURR)_{-1} - \underset{(.5460)}{.9122}RT - \underset{(.1787)}{.7180}RTB + \underset{(.0170)}{.0269}DPI$$

$$OLS \qquad \bar{R}^2 = .997 \qquad \bar{S} = .67 \qquad DW = .914$$

(4) Time Deposits

$$TD = -1.5 + \underset{(.0191)}{.9935}(TD)_{-1} - \underset{(.1734)}{.7140}(RTB - RT) - \underset{(.3594)}{.8808}(RL - RT) + \underset{(.0079)}{.0170}DPI$$

$$OLS \qquad \bar{R}^2 = 1.00 \qquad \bar{S} = .7 \qquad DW = 1.02$$

(5) Interest Rate, 3 Month Treasury Bills

$$RTB = \underset{(.1582)}{-.35163} + \underset{(.0495)}{1.0723}RDIS$$

$$- \underset{(23.67)}{126.12} \left[\frac{RESF}{(DD + CURR)_{-1}}\right] - \underset{(3.865)}{7.06} \left[\frac{FBF}{(GNP\$)_{-1}}\right]$$

$$OLS \qquad \bar{R}^2 = .94 \qquad \bar{S} = .262 \qquad DW = .95$$

(6) Interest Rate, 4–6 Month Commercial Paper

$$RS = .45 + \underset{(.044)}{.750}RTB + \underset{(.045)}{.317}(RTB)_{-1} - \underset{(.050)}{.189}DRS$$

$$OLS \qquad \bar{R}^2 = .985 \qquad \bar{S} = .12 \qquad DW = .88$$

(7) Interest Rate, Savings Deposits

a. $$RT = \underset{(.096)}{-.189} + \underset{(.0488)}{.9322}(RT)_{-1} + \underset{(.0559)}{.128}(RTQ)$$

$$- \underset{(.0008)}{.00168}(TD)_{-1} + \underset{(.0315)}{.0686} \times [(1.0 - RRD)(-.57)$$

$$+ (1.0 - RRT)] \times RS$$

$$OLS \qquad \bar{R}^2 = .992 \qquad \bar{S} = .07 \qquad DW = 1.3$$

b. $RT = 3.44$

(8) Interest Rate, Corporate Bonds (Term Structure)

$$(RL - RS) = \underset{(.0849)}{.05891} - \underset{(.0329)}{.67679}RS + \underset{(.0381)}{.68793}(RS)_{-1}$$

$$+ \underset{(.0433)}{1.13338}(RL - RS)_{-1} - \underset{(.0421)}{.20604}(RL - RS)_{-2}$$

$$OLS \qquad \bar{R}^2 = .98 \qquad \bar{S} = .0878 \qquad DW = 1.828$$

(9) Mortgage Yield, FHA Secondary Market

$$RM = .59 + \underset{(.070)}{.198}RL + \underset{(.077)}{.739}(RM)_{-1}$$

$$OLS \qquad \bar{R}^2 = .96 \qquad \bar{S} = .10 \qquad DW = .97$$

VIII. NEW ORDERS, SHIPMENTS, AND UNFILLED ORDERS

(1) New Orders, Manufacturing Durables

$$OMD = \underset{}{42.3} + \underset{(.123)}{1.101}(CA + COD) + \underset{(.067)}{.134}\Delta X$$

$$+ \underset{(.0266)}{.0647}[(X)_{-1} - (X)_{-3}] - \underset{(.0843)}{.3951}(KI)_{-1}$$

$$+ \underset{(.0762)}{.2515}(AM58\$)_{-1} - \underset{(1.58)}{4.96}DUMD$$

$$OLS \qquad \bar{R}^2 = .957 \qquad \bar{S} = 1.9 \qquad DW = 1.14$$

(2) Shipments, Manufacturing Durables

a. $$\frac{SMD}{(UMD)_{-1}} = \underset{(.0896)}{.419} - \underset{(.0958)}{.3973}CUW + \underset{(.0236)}{.9271}\frac{(SMD)_{-1}}{(UMD)_{-2}}$$

$$OLS \qquad \bar{R}^2 = .97 \qquad \bar{S} = .034 \qquad DW = 1.73$$

b. $$SMD = \left[.637 - .153\left(\frac{UMD}{SMD}\right)_{-1}\right]OMD$$

$$+ \left[.324 - .078\left(\frac{UMD}{SMD}\right)_{-2}\right](OMD)_{-1}$$

$$+ \left[.104 + .002\left(\frac{UMD}{SMD}\right)_{-3}\right](OMD)_{-2}$$

$$+ \left[-.023 + .079\left(\frac{UMD}{SMD}\right)_{-4}\right](OMD)_{-3}$$

$$+ \left[-.058 + .157\left(\frac{UMD}{SMD}\right)_{-5}\right](OMD)_{-4}$$

$$- 1.52DSMD$$

$$OLS \qquad \bar{R}^2 = .972 \qquad \bar{S} = .132 \qquad DW = 1.264$$

(3) Unfilled Orders, Manufacturing Durables

a. $$\Delta UMD = \underset{(8.214)}{22.187} + \underset{(.064)}{.145}(AM58\$)_{-1} - \underset{(1.426)}{6.564}DUMD$$

$$+\sum_{i=1}^{5} WT1_i(KI)_{t-i} + \sum_{i=1}^{5} WT2_i X_{t-i+1}$$

where

i	$WT1$	$WT2$
1	−.9285	.06143
2	−.4035	.07257
3	−.0485	.07200
4	1.375	.05972
5	.1536	.03572

$$OLS \qquad \bar{R}^2 = .656 \qquad \bar{S} = 1.79 \qquad DW = 1.36$$

b. $\Delta UMD = -.299 + .949(OMD - SMD)$
$\qquad\qquad (.0798) \quad (.0255)$

$$TSLS \qquad \bar{R}^2 = .962 \qquad \bar{S} = .592 \qquad DW = 1.85$$

DISCUSSION

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1. INTRODUCTION

My task is to comment on the paper prepared by George Green and his associates. This I am pleased to do, since they have provided us with an excellent report on an interesting body of simulations, carried out with an important and sizeable econometric model (over 90 equations). Interest in their study is heightened by the fact that both the model, and the simulations with it, were developed and carried out by government employees within the U.S. Department of Commerce. Except for what are, perhaps, minor points, I will not find fault with what they have done. Rather, my primary contribution—if I have one to make—will be to point out additional simulation studies that they or others might find useful in complementing what has already been achieved.

In thinking about simulation studies that should be done with models of this type, I believe it helpful to focus attention on the objectives which simulation studies might help us reach. To this end, I have grouped my remarks under the following headings: forecasting, prediction of policy implications, evaluation of predictive ability, and research guidance. I shall treat these topics in turn, concluding with some general observations.

2. FORECASTING

By forecasting, I mean use of a model in predicting future—and thus, unobserved—values of endogenous variables of the model. This I take to be the primary objective of the model. In practice, it must be carried out employing only that information available at the time of prediction. Two points cause me some concern.

First, how are values to be assigned to unlagged input variables? In the sample-period simulations of this paper, actual values have been assigned, but where are the values of these variables to come from in real forecasting? Should not whatever procedures are to be used in practice be presented and tested to see how they work? A concern with values of predetermined variables which—although they have already occurred at the point of forecasting—have not yet resulted in available measurements also seems reasonable.

A second, and perhaps negligible, point which I would like to raise, relates to the use of—or, rather, failure to use—stochastic simulation in forecasting applications of the model. Except for special cases, which do not appear to include this model, even the expected values of endogenous variables will depend on the stochastic specification of a model. Setting all error terms equal to zero and running a model *may* produce reasonably close approximations to expected values, but this is not guaranteed, in general. Thus, some concern on this score is warranted; particularly so, since there is a reasonably straightforward way to explore the matter.

Stochastic shocks could be introduced, much as they are in the twenty-five-year simulations for the spectral analysis found in Part 4 of Green's paper. Repeated simulations could be carried out over the periods to be forecast. For any given variable and time period, the mean for these repeated simulations with independent sets of shocks would provide estimates of expected values which could be compared with results obtained by setting all error terms equal to zero when simulating. The sampling variances of means could be estimated, of course, from the samples of values averaged to obtain the means. These would facilitate the provision of interval forecasts, as well as serving as a check to see whether or not neglecting stochastic terms resulted in significant biases.

3. PREDICTION OF POLICY IMPLICATIONS

An important use of models is in exploring the implications of hypothetical uses of policy instruments. Prediction is involved here, as well as in forecasting, but the focus of attention is on predicting the *dependence* of endogenous variables on the level of policy variables, rather than on predicting the level of endogenous variables.

Forecasts of the level of endogenous variables have an obvious utility, since policy-makers need warning if the effects of their actions are to be timely. Nevertheless, knowledge about how the future depends on policy choices is even more important than forecasting ability. After all, many physical control systems work very well by adjusting corrective actions to observed past discrepancies between actual and desired. These systems count on limited continuity and have no built-in forecasting devices. Their design, nevertheless, did require an approximate knowledge of the way endogenous variables respond to control or policy variables.

In my opinion, policy implications of the OBE Model could, and should, be explored by simulation techniques. Perhaps this has been done adequately elsewhere. If so, it would have been helpful to have such a fact footnoted, at least, in the paper under discussion. If such simulations are carried out, special attention should be given to the treatment of stochastic inputs. It would be desirable to explore policy implications with different sets of stochastic shocks. Nevertheless, in comparing the implication of different policies, whatever set of stochastic shocks are used should be held constant. A policy-maker does not know what shocks are in store for him—but he does not expect the error terms of a model to be systematically dependent on his actions.

4. EVALUATION OF PREDICTIVE ABILITY

One of the many fine features of Green's paper is the extensive investigation of how the OBE Model works in the neighborhood of turning points which it reports. Another of its fine features is the exploration of the effects on forecast errors of the alternative adjustment procedures aimed at taking advantage of the substantial auto-correlation found in residuals. For me, this part struck a particularly responsive note. In some early papers in the forties, I pointed out that many models generated residuals which implied highly auto-correlated error terms, that this fact could be used in improving forecasts, and, in addition, might be used in improving parameter estimation.

The primary weakness of the approach used by Green and his associates in exploring predictive ability is the fact that the same data is used both for estimating the model and for judging the model's forecasting utility. Unfortunately, experience shows that residuals obtained from within-sample forecasting may be poor guides as to how well a model will predict beyond the time span used in estimation. In addition to the excellent paper prepared for this Conference by Ronald Cooper, which uses forecasting outside of the sample period to good effect in comparing models, I would like to draw attention to a paper by John Edwards and myself. It is entitled "The Reliability of Statistical Indicators of Forecasting Ability," and is available on request to me. Our paper presents some evidence of how misleading within-sample fits can be, both in guiding selection of variables to be retained, and in anticipating errors to be made.

5. RESEARCH GUIDANCE

The paper by Green and his associates uses two simulation approaches to obtain results which would be useful in planning research strategy. In the first, an analysis of single-equation forecast errors is made; this would be useful in localizing the origin of forecast errors down to the level of particular equations. In this approach, actual rather than generated values of all except a single output variable are used for each equation. This would not be very useful in actual forecasting, but it does help to sort out errors resulting from incorrect values of equation inputs from errors which arise as a consequence of poor equations.

The second use of simulation in bringing forth results that might help guide subsequent research lies in obtaining the set of 50 twenty-five-year stochastic simulations for a hypothetical specification of policy and exogenous variables. These, along with the spectral analysis of outputs, might throw some useful light on whether or not the model needs to be adjusted to achieve outputs of acceptable time-series properties. Of course, to be useful in this connection, some information about the spectral properties of real economic time-series would be essential. In addition, there are some difficult problems of estimation and testing which require careful consideration.

Another type of simulation analysis which might be useful in providing research guidance is sensitivity experimentation. By finding out how sensitive results were to parameter specification, the researcher could better determine where to direct his efforts.

Still another type of simulation study which might be informative in guiding research effort is the following: having estimated a model, treat it as though it were an exact representation of the world. Use it to generate repeated sets of data, and then see how well the estimating techniques (used in obtaining the model from real-world data) would work in estimating the model from the data obtained by running it. The objective would be to find out how well the estimating techniques used would work if the world were really like the model. If the techniques do not work well under these circumstances, it is hard to see why they should be expected to work well when applied to real-world data.

6. CONCLUDING COMMENTS

I do not know whether or not to accept fully Cooper's remark at the end of his paper: "It is as true now as it was at the time of Christ's study that mechanical forecasting models can be constructed which predict economic variables about as well as econometric models." In any case, I find it very sobering. Nor was I greatly reassured by Daly's paper on the forecasting value of statistical indicators.

Gains made in improving the quality and timeliness of measures describing past behavior of the economy are important. Ascertainment of expectations, intentions, and plans of consumers and businessmen are helpful in peering a little way into the future. However, on two fronts the situation is disappointing. Not only does our ability to predict the future seem inadequate relative to our perceived needs, but I find no evidence, in the papers presented at this Conference, of how successful econometric models are in predicting policy implications. Since I believe that models for predicting policy implications are far more necessary than models for forecasting the future, I am sorry that the problem of building and testing policy-response models has not received more attention.

If econometric models had shown more spectacular results in terms of forecasting, I am sure that this success would have led to their wider use in predicting policy implications. Nevertheless, any connection between success in these two types of prediction may be extremely weak. Econometric models have policy implications, and the predictive value of the models in this area may be more, or may be less, than the predictive value of these models for forecasting the level of economic variables. But, at present, we don't seem to know. My own view is that we never will know with even modest assurance if we restrict our analyses to data of the United States national-accounts type.

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INTRODUCTION

A carefully designed computer simulation experiment with a model of an economic system requires that special attention be given to the following activities: (1) definition of the problem, (2) formulation of an econometric model, (3) formulation of a computer program,

(4) validation, (5) experimental design, and (6) data analysis [9]. In evaluating Dr. Green's simulation experiments with the OBE Model, we shall focus on these six activities.

DEFINITION OF THE PROBLEM

I found Dr. Green's paper particularly difficult to evaluate, because his procedure, generally, was a simple description of his simulation results and his conclusions. Nowhere did he state explicitly why he conducted the simulation experiments in the first place. However, from his concluding remarks [3, pp. 89–90], it is possible to work backward and gain some insight into what the experimental objectives may have been. As near as I can tell, the objectives were:

(1) To test the effects of four different mechanical procedures for adjusting the constant terms of the model on the historical performance of the model over the sample period [3, pp. 31–33].

(2) To compare the error statistics of the complete model solutions with the error statistics of the single equation components [3, pp. 33–51].

(3) To evaluate the short-run historical performance of ex post simulations over periods which contain NBER reference-cycle peaks and troughs [3, pp. 51–66].

(4) To compare the simulated time paths of the endogenous variables of the model with the actual observed values of these variables over the entire sample period [3, pp. 66–68].

(5) To determine the cyclical properties of the model over the twenty-five-year period beginning in the first quarter of 1966 [3, pp. 68–88].

FORMULATION OF AN ECONOMETRIC MODEL

Since the structure of the model described in Dr. Green's paper "follows the general scheme of the earlier version" of the OBE Model—which is well known to this audience—I shall limit my comments on the model per se and focus attention on the simulation experiments. It goes without saying that the performance of the OBE Model might benefit from the expansion of some of its sectors along the lines of the more recent versions of the Brookings and Wharton models. Furthermore, I question the treatment of population as an exogenous variable for a twenty-five-year period. One could certainly argue that over a twenty-five-year period, population size and the behavior of the economy of the United States may be jointly determined, and that, therefore, population should be treated endogenously.

FORMULATION OF A COMPUTER PROGRAM

I have only one comment regarding the computer programming techniques used in conducting the simulation experiments. It seems incongruous to use such an "old-fashioned" technique as the Rand Table to generate random variables with such a sophisticated model. To be sure, there is nothing wrong with the numbers in the Rand Table, but reading numbers from a magnetic tape is not a particularly efficient way to use a third-generation computer, when there exist a number of fully tested computer sub-routines [6, 7] for generating pseudo-random numbers internally, and these can be easily transformed into normal deviates by suitable transformations. Perhaps, unlike the situation at most universities today, computer time is still a free gift of nature at the Department of Commerce. If that is the case, I wonder if I could have an hour or two of time on your computer next month?

VALIDATION

The validity of an econometric model depends on the ability of the model to predict the behavior of the actual economic system on which the model is based. To test the degree to which data generated by simulation experiments with econometric models conform to observed data, two alternatives are available: historical verification, and verification by forecasting. The essence of these procedures is prediction, for historical verification is concerned with retrospective predictions (ex post simulations over the sample period), while forecasting is concerned with prospective predictions (ex ante simulations beyond the sample period).

Sections 3.3 and 3.4 of Dr. Green's paper describe his attempts to validate the OBE Model through the use of ex post simulations over the sample period. Although Dr. Green conducted ex ante simulations beyond the sample period, he did not use these simulations for validation purposes. Therefore, I shall restrict my comments to the ex post simulations over the sample period.

In Section 3.3, Dr. Green describes short-run ex post simulations over periods which contain NBER reference-cycle peaks and troughs. In Section 3.4, he describes ex post simulations over the entire sample period. In comparing the simulated time paths of the endogenous variables of the model with the actual time paths, Dr. Green makes use of average absolute errors, root mean square errors, maximum errors, and graphical observation. On the basis of these criteria, Dr. Green concludes that

> it is evident . . . that the simulated values of *GNP* follow the general pattern of the actual data quite well. The general growth path over the period is fairly well predicted, although there is an evident drifting off of predicted values in later quarters [3, p. 67].

In the paper by Naylor and Finger [10] several other criteria are suggested for deciding when the time paths generated by a simulation experiment agree sufficiently with the observed time paths so that agreement cannot be attributed merely to chance. Several specific measures and techniques are suggested for testing the "goodness-of-fit" of simulation results, i.e., the degree of conformity of simulated time series to observed data.

EXPERIMENTAL DESIGN

In a computer simulation experiment, as in any experiment, careful thought should be given to the problem of experimental design. Among the important considerations in the design of computer simulation experiments are: (1) factor selection, (2) method of randomization, (3) number of replications, (4) length of simulation runs, and (5) multiple responses [9].

Factor selection. In a factorial design for several factors, the number of design points required is the product of the number of levels for each of the factors in the experiment. It is clear that a full factorial design can require an unmanageably large number of design points if more than a few factors are to be investigated. Given the limited number of design points considered by Dr. Green in his experiments, the problem of factor selection is not a relevant consideration in the evaluation of his work.

Method of randomization. Two different types of simulation experiments are described by Dr. Green: deterministic simulations and stochastic simulations. The ex post simulations reported in Section 3 are all deterministic. The ex ante simulations in Section 4 are stochastic. The reasons given by Dr. Green [3, pp. 69] for using stochastic simulations are somewhat obscure.

There are at least three reasons why one might want to include stochastic shocks in simulation experiments with simultaneous, nonlinear, difference-equation models. First, as Philip Howrey has pointed out in an unpublished paper entitled "Dynamic Properties of Stochastic Linear Econometric Models," if the long-term properties of an econometric model are to be investigated

> . . . it may not be reasonable to disregard the impact of the disturbance terms on the time paths of the endogenous variables. Neither the characteristic roots nor the dynamic multipliers provide information about the magnitude or correlation properties of deviations from the expected value of the time path.

Secondly, Howrey and Kelejian [4] have demonstrated that "the application of nonstochastic simulation procedures to econometric models that contain nonlinearities in the endogenous variables yields results that are not consistent with the properties of the reduced form of the model."

Thirdly, by including stochastic error terms, one can then replicate the simulation experiment and make statistical inferences and test hypotheses about the behavior of the system being simulated, based on the output data generated by the simulation experiment.

Number of replications. If one is going to make inferences of the type made by Dr. Green in his concluding remarks [3, pp. 89–90], then the optimal sample size (number of replications) depends on the answers one gives to the following questions: (1) How large a shift in population parameters do you wish to detect? (2) How much variability is present in the population? (3) What size risks are you willing to take? Dr. Green has arbitrarily used a sample size of fifty replications without providing us with a clue as to how he would answer these questions. The paper by Gilman [2] describes several rules for determining the number of replications of a simulation experiment when the observations are independent. (Observations obtained by replicating a simulation experiment will be independent, provided that one uses a random-number generator which yields independent random numbers.)

Length of simulation runs. Another consideration in the design of simulation experiments is the length of a given simulation run. This problem is more complicated than the question of the number of replications, because the observations generated by a given simulation rule will, typically, be auto-correlated, and the application of "stopping rules" based on classical statistical techniques may underestimate the variance substantially, leading to incorrect inferences about the behavior of the system being simulated.

> In the large majority of current simulations, the required sample record length is guessed at by using some rule such as "stop sampling when the parameter to be estimated does not change in the second decimal place when 1000 more samples are taken." The analyst must realize that makeshift rules such as this are very dangerous, since he may be dealing with a parameter whose sample values converge to a steady state solution very slowly. Indeed, his estimate may be several hundred per cent in error. Therefore it is necessary that adequate stopping rules be used in all simulations [2, p. 1].

The paper by Gilman [2] describes several "stopping rules" for determining the length of simulation runs with auto-correlated output data.

Dr. Green's paper does not provide any information on how he decided on the particular simulation-run lengths used in his experiments.

Multiple-response problem. The multiple-response problem arises when we wish to observe and evaluate many different response variables in a given experiment. Dr. Green's simulation experiment contains approximately one hundred response variables. A question arises as to how one goes about validating multiple-response simulation experiments, and how one evaluates the results of the use of alternative policies in the case of policy-simulation experiments. To solve the validation problem, the analyst must devise some technique for assigning weights to the different response variables before applying specific "goodness-of-fit" tests. Gary Fromm [1, p. 8] has proposed the use of utility theory to evaluate the results of policy-simulation experiments with the Brookings Model. Dr. Green's approach to the multiple-response problem is simply to present the results of his experiments, letting the policy-maker assign his own weights to the different output variables. Given the practical and theoretical problems involved in assigning weights or utilities to different response variables, Dr. Green's approach is likely to remain the most popular answer to the multiple-response problem.

DATA ANALYSIS

Given the fact that Dr. Green's ex post simulations: (1) consist of a single replication, (2) involve a small number of observations, and (3) yield output data with a high degree of auto-correlation present, not a great deal more can be said about the analysis of the data generated by these simulation experiments. About all one can do is to observe the graphical output of the simulation experiment—and, perhaps, calculate average errors, average absolute errors, and root mean square errors.

If Dr. Green had chosen to replicate his ex post simulation experiments, several other options would have been open to him. First, he could have given statistical precision to some of the inferences which he made in his concluding remarks [3, p. 89]. Second, he could have applied a conventional analysis of variance to test errors. Third, he could have used multiple-comparison procedures to show how average errors and root mean square errors differ among alternative simulation runs. Fourth, he could have used multiple-ranking procedures to rank the sample means of the errors associated with a single output variable, like *GNP*, for different simulation runs. For example, Dr. Green might have used multiple-ranking procedures to rank the average quarterly forecast errors of *GNP* associated with the four procedures for adjusting the constant terms. With what probability can we say that a ranking of sample means represents the true ranking of the population means? Basically, it is this question which multiple-ranking procedures attempt to answer [5].

The paper by Naylor, Wertz, and Wonnacott [11] describes the application of the F test, multiple comparisons, and multiple-ranking procedures to the analysis of national income data generated by policy-simulation experiments with an econometric model.

Finally, Dr. Green has computed the power spectra for the *GNP* series generated by his ex ante, stochastic-simulation experiments, using serially correlated shocks, non-serially correlated shocks, and two different filters. He then tested the statistical significance of the spectral peaks of these series. Having gone so far as to calculate the power spectra of these *GNP* series with different types of shocks and filters, he could have said rather more about his results than he reported in his concluding remarks. For example, with spectral analysis it is relatively easy to construct confidence bands and to test hypotheses for the purpose of comparing the simulated output for two or more series. The paper by Naylor, Wertz, and Wonnacott [12] describes several procedures of this type and applies them to national income series generated by an econometric model.

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REPLY

GREEN

Both discussants suggest many additional simulations which one might undertake. Some of these projects are indeed worthy of study, while others are not directly relevant to the main theme of this Conference. Some of the suggested projects have been completed and are reported elsewhere. An analysis of ex ante forecasting performance is contained in the paper by Evans, Haitovsky, and Treyz, prepared for this Conference. I reported on some policy simulations at last winter's meetings of the Econometric Society.

Two of the suggestions made by Professor Orcutt seem of special interest to us, and we hope to undertake efforts in this direction in the future. One is the use of stochastic shocks in short-term forecasts to see if the mean of these results agrees with results obtained when shocks are not introduced. In the case of the twenty-five-year simulations reported here, the mean of the stochastic runs is very close to the control solution.

A second suggestion concerns use of the results from the twenty-five-year shocked simulations with different estimating techniques, to try to recapture the "true" parameters. The payoff from this study might well be quite high. Our present estimation methods leave open a lot of questions, especially where the model is to be used for multiperiod forecasts.

Concerning the use of the Rand tape of random normal deviates instead of a pseudo-random number generator, I disagree with most of Professor Naylor's comments. First, several pseudo-random number generators do exist, but most of them are too "pseudo"—i.e., tests by Fromm and Nagar at Brookings, and by the staff at the National Bureau of Standards, have shown that the generated series are not very random in several respects. Relatively speaking, the Rand numbers are much better. Second, the same random number generator, used on different computers, can result in far different degrees of accuracy and adequacy. Third, some other participants in this Conference had elected to use the Rand tape, and we conformed—in part, to standardize procedures. Finally, the amount of computer time required is not, as Professor Naylor suggests, a matter of hours, but a matter of a few minutes. The computer time required to generate shocks for twenty-five stochastic runs over twenty-five years, plus twenty-five stochastic simulations (each solving the model for one-hundred quarters) was about fifteen minutes.