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A Survey of the Theory of Public Expenditure Criteria

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THE theory of expenditure criteria has received a lot of attention in recent years, stimulated by the practical needs of the world. In the United States, the evaluation of public works, particularly in the field of water resource development, has led to the evolution of techniques and criteria for project evaluation. This work was largely pioneered within the federal government. The need for devising development plans for underdeveloped countries has led to extensive theoretical study of investment criteria for that particular economic context.

This paper presents the elements of the theory. Rather than propose or defend specific criteria, I try to indicate the issues about which assumptions must be made. First, possible objective functions are discussed—What, if anything, is to be maximized? There follow sections on constraints, interest rates, repercussion effects, and the treatment of risk and uncertainty. Finally, with the taxonomy of the problem in hand, most of the more important decision-models that have been developed are surveyed and discussed.

Some limitations should also be mentioned. First, there is very little empirical work in this study, in particular, no real allocation-problems are presented or solved. In view of the scope of the problems to be covered at a theoretical level, intensive treatment of specific empirical situations was not attempted. Second, there is no treatment of the various technical maximization methods, such as Lagrangian techniques, linear and nonlinear programing, simulations,

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I have benefited from participation in the lively controversies of the Harvard University Seminar on Water Resource Planning. Professors Robert Dorfman, John Meyer, Arthur Maass, Peter Steiner, and Harold Thomas, Dean Fair, Messrs. Maynard Hufschmidt, Stephen Marglin, and Donald Farrar, and other some-time members of the Seminar have all left their imprint on this paper. Professors Bergson and Dorfman have read and commented on the paper and have improved it. Responsibility for anything said rests entirely with me, of course.

queuing theory, and game theory, since excellent treatments are now available. Third, the macro-economic decision models developed by Frisch, Tinbergen, and Theil are not discussed, even though they are closely related conceptually. Nor is the theory of public expenditures, advanced by Wicksell and others, and developed further by Samuelson and Musgrave, treated here. This theory concerns itself with those expenditures for which conventional value theory breaks down completely. My concern is largely confined to public works and development projects of a sort for which measures of value can be established empirically.

1. The Objective Function

A. INTRODUCTION

The most fundamental consideration in a decision model is the choice of an objective function. Should the model seek to maximize (or minimize) some operationally definable measures? And if so, what should the measure be?

Typically, in economics, the analysis presupposes that we seek to maximize economic welfare, however this may be defined. The notion of maximization is perhaps the central analytical concept of economics. Recently, Simon[1] has questioned this idea, and at least as a description of the real world has suggested that people and corporations merely seek to obtain a satisfactory state of affairs, rather than some optimum.

One could reconstruct prescriptive (or welfare) economics along Simon's lines, letting the analyst indicate what policies keep the state of affairs within the tolerance levels of the interest groups affected inside and outside government. The "putting out fires" approach to policy, which frequently characterizes American political leadership, certainly suggests that politicians are also "satisficers," in Simon's phrase. Nevertheless, the present study takes the view that economic analysis will play a more productive role if it seeks to maximize something. The extent to which policy makers decide to accept the economist's optimizing analyses will probably be decided by satisficing considerations.[2]

[1] Herbert Simon, *Models of Man*, 1957, Ch. 14, "A Behavioral Model of Rational Choice," reprinted from *Quarterly Journal of Economics*, February 1955, and *Administrative Behavior*, 2nd ed., 1957, Introduction and Chs. 4 and 5.

[2] For a different view, that politicians are maximizers of votes, see Anthony Downs, *An Economic Theory of Democracy*, Harper, 1957.

B. WHAT SHOULD BE MAXIMIZED? WELFARE ECONOMICS

Assuming a maximizing approach, a yardstick which will define the optimum must be specified. Economic welfare is the usual objective, but there are a number of alternative ways of defining this broad concept.

First, in Western economics, economic welfare is almost always related to individual welfare; it is postulated that there can be no welfare other than what accrues to individuals. This is a rejection of the organic theory of the state: the state as an entity enjoys no welfare, only the people that compose it.[3]

Following Bergson,[4] the function for social economic welfare at any point in time can be written formally:

$$W = W(W_1, \ldots, W_n),$$

where W_i is the economic welfare of individual i. A change in economic welfare can be written

$$\Delta W = \Delta W(\Delta W_1, \ldots, \Delta W_n). \tag{1.1}$$

When can the change in social economic welfare caused by a policy be said to be positive? A definition of this positive change is needed for to strive to maximize welfare, all changes which serve to increase it must be undertaken. An optimum point is defined as a situation in which no further positive changes in social welfare can be accomplished.

Clearly, if all individuals are made better off ($\Delta W_i > 0$ for all i) economic welfare is improved according to any nonmalevolent standard. A somewhat weaker, although still very strong, requirement is this: Let no one be made worse off and let at least one person be better off. ($\Delta W_i \geq 0$ for all i, with at least one $\Delta W_i > 0$.)

There are few (or no) economic changes which could pass this test. Usually, some person is affected adversely, which is sufficient to preclude this criterion from ruling on the desirability of the change.

[3] For the view that economics only encompasses the case of individually based welfare, see Howard Ellis, "The Economic Way of Thinking," *American Economic Review*, March 1950, pp. 1–12. For a reply, see Bushrod W. Allin, "Is Group Choice a Part of Economics?" *Quarterly Journal of Economics*, August 1953, pp. 362–79, and "Replies," *ibid.*, November 1953, pp. 605–14.

[4] A. Bergson, "A Reformulation of Certain Aspects of Welfare Economics," *Quarterly Journal of Economics*, February 1938.

A test which promised to yield an answer in a wider range of situations was introduced by Kaldor[5] and Hicks.[6] They did not require that no one be made worse off, only that the gainers of any economic change be able to compensate the losers, though the compensations need not necessarily be carried out. In this way, it was hoped that the production features of economic policy could be separated from their distributional implications. Presumably, pure lump-sum transfers of income can achieve any distribution of output that is desired. If the total value of output minus the value of factor services is increased, presumably the gainers can compensate the losers, and economic welfare is increased. If the economic change is so small that prices are unaffected, this is a simple and unambiguous test. Where prices change, Hicks suggested use of the new prices, Kaldor of the old.[7]

An implicit assumption of this approach is that the economic welfare of any individual (or family) only depends on the goods and services consumed and supplied by him; his welfare is not affected by the welfare of his neighbors. For if there were such external effects of consumption, even an increase in total net value which made more goods and services available to everyone might reduce economic welfare—by causing envy, for example.[8] Of course, if all individuals were so noble as to derive only happiness from an increase in other people's consumption, this result would be ruled out.[9]

The Kaldor-Hicks compensation criteria were subjected to much criticism. Scitovsky showed that if the economic change is large enough to cause prices to change, the criterion may become inconsistent; the gainers could compensate the losers after the change, yet the potential losers might be able to compensate the potential gainers prior to the change.[10] He found that an unambiguous increase in welfare required that the value of net output must increase both at the new and the old prices. Samuelson deepened this line of criticism, contending that all the potential distributions of welfare

[5] N. Kaldor, "Welfare Propositions and Interpersonal Comparisons of Utility," *Economic Journal*, 1939, pp. 549–52.

[6] J. R. Hicks, "The Valuation of Social Income," *Economica*, 1940, pp. 105–24.

[7] This is the interpretation of J. de V. Graaff, *Welfare Economics*. For a different view, see I. M. D. Little, *A Critique of Welfare Economics*, 2nd ed., Ch. 6.

[8] This phenomenon is stressed by W. J. Baumol in *Welfare Economics and the Theory of the State*, Harvard University Press, 1952, pp. 88, 127. Also see J. de V. Graaff, *op.cit.*, pp. 43–5.

[9] See Stefan Valavanis, "Schadenfreude and Freudeschaden," to be published.

[10] T. Scitovsky, "A Note on Welfare Propositions in Economics," *Review of Economic Studies*, 1942, pp. 98–110.

of any given production situation be considered relevant. One situation would be ruled superior to another only if every potential distribution of welfare possible under it is superior for at least one individual, and is inferior for none. This statement, which would be extremely difficult to implement empirically, eliminates some cases which would be ranked even under the Scitovsky double criterion.

Another line of criticism questions the use of potential compensation.[11] Can one situation be considered to yield greater economic welfare if everyone could be made better off even though, in fact, the necessary compensation payments are not made? Hypothetical payments, according to most later writers, are not an adequate device to remove the distribution issue. On the other hand, actual compensation payments have not been accepted either, since they would attach particular desirability to the income distribution before the economic change. The most widely accepted modern view insists that the redistribution of income of any economic change be evaluated separately on the basis of specific ethical judgments. As formulated by Little,[12] an appropriate criterion for the desirability of an economic change would be the following:

1. the gainers must be able to overcompensate the losers and/or the losers must not be able to overcompensate the gainers, and

2. the redistribution of income must be good. This assumes that the option of making pure redistributions is excluded; otherwise different criteria apply.

The above criterion permits the comparison of any two situations on the basis of what is probably rather close to a minimum of restrictions that must be imposed on the economic welfare function.

As a theory of economic policy, this formulation leaves much to be desired, however. First, as Graaff has argued persuasively, the prevalence of external effects in consumption contradicts a necessary assumption of the theory. Second, analysis of real-world situations is usually ill-suited to be couched in terms of choices among two alternatives. Third, since most policies involve a loss of welfare to someone, a formal basis for interpersonal comparisons is needed,

[11] W. J. Baumol, *Welfare Economics*, . . . , p. 123; I. M. D. Little, *A Critique of Welfare Economics*, Ch. 6; C. F. Kennedy, "The Economic Welfare Function and Dr. Little's Criterion," *Review of Economic Studies*, No. 52, 1952–53; R. Baldwin, "A Comparison of Welfare Criteria," *ibid.*, No. 55, 1953–54, p. 154.

[12] I. M. D. Little, *A Critique of Welfare Economics*, *op.cit.*, p. 105.

and since the economist has no particular right to attach social weights to individual welfare in the social welfare function, this is sufficient ground to rule out rigid prescriptions. Of course the economist can stop short of this judgment, leaving the evaluation of distributive effects to the politician. But even under this view the economist will find it very difficult to sidestep the distribution issue altogether. After all, he cannot conduct his analyses in terms of the names of the millions of people in his country, and grouping of population into categories—by income class or geography or anything else—already prejudges the distributive issue.[13]

The complexity of the criteria, their inability to resolve most practical issues, and the inherently ethical problem of judging the distribution of income has brought many of the leading students of welfare economics to very pessimistic conclusions. Baumol, in his "Epilogue: The Wreck of Welfare Economics?" stresses the prevalence of interdependence effects which invalidate the use of market prices and rejects the standard marginal optimum conditions. Historically, the main use of welfare economics has been the derivation of these conditions and the proof that laissez-faire is the best economic system;[14] thus, Baumol's stress on interdependence strikes at the heart of the theory. Other than as a means of exploding fallacious arguments, he writes, the fact that categories like "external economies" and "external diseconomies" remain largely empty economic boxes prevents any further applications of welfare theory as it now stands.

"Is there any hope of further progress based on empirical investigation and analysis of the problem of the interdependence of activities of economic units? I cannot pretend to offer even tentative answers. It seems to me, however, that if the subject is to achieve primary importance for practical men, this question must be faced and answered."[15]

Graaff's[16] pessimism rests on two grounds. First, he does not believe that there will be agreement on the ends of policy. He is concerned not only with the distribution of income, but also with

[13] For other difficulties of the concept of distribution of welfare, see K. J. Arrow, "Little's Critique of Welfare Economics," *American Economic Review*, December 1951, pp. 923–34, esp. 931–2.

[14] K. E. Boulding, "Welfare Economics" in *A Survey of Contemporary Economics*, Vol. 2, B. Haley, ed., p. 24.

[15] Baumol, *Welfare Economics . . . , op.cit.*, p. 167.

[16] Graaff, *Theoretical Welfare Economics*, pp. 170–1.

the attitude toward uncertainty, the time horizon, and the rate of progress. The external effects in consumption which he stresses also hopelessly complicate the problem. He concludes that economists had best devote themselves to factual studies of the functioning of the economic system, perhaps predicting the effects of policy on some index numbers, but attaching no prescriptive value.

C. A MORE MODEST ROLE FOR WELFARE ECONOMICS: THE OBJECTIVE FUNCTION

Its critics underestimate the usefulness of welfare economics. It is true that it has failed in the tasks which had been set for it: it has not (1) proved the superiority of laissez faire; (2) provided simple criteria for judging economic changes or economic optima, or (3) provided a method of isolating the economic aspects of policy from ethical considerations. But the failure to accomplish these objectives is due to their grandiose nature. There are more modest objectives of analysis for which welfare economics must play a crucial role.

What I propose is this. First, the rather casually dispensed advice of the critics of welfare economics should be taken seriously. I follow Baumol and seek to establish what interdependence effects should be measured, and to indicate the methods that may be appropriate. I follow Graaff by emphasizing measurement rather than absolutist advice. But this should be no senseless retreat into hypothesis-testing unrelated to potential action, nor the collection of random sets of facts; rather it should be the establishment of decision-models which will reveal explicitly what actions will maximize the achievement of specified objectives.

I do not insist that the economist be given the objectives in polished, formal manner. Rather, the economist must interpret the desires of the policy people whom he is serving and express them in an analytical form as an objective function. He then seeks to maximize this function, given the empirical relations in the economy and the institutional constraints that may be appropriate to the analysis. In this manner, the economist can play the role of technician, of bringing his technical equipment to bear on policy problems, with maximum effectiveness.

The specification of the objective function thus is not primarily meant to let the economist play omnipotent being; rather, it is a device for bridging the gap between the positive quantitative research

which is the main stock-in-trade of the economist, and the normative conclusions which policy requires.[17]

D. SINGLE *vs.* MULTIPLE INDICATORS

Individual and "Social" Welfare—The Problem of Income Distribution

In formulating the objective function so as to express our notion of economic welfare, there is a question about the number of variables to be employed. From a theoretical point of view, the ideal function would define at least one variable for each individual measuring his welfare, and probably more than one, say, a measure of expected gain in real income plus a measure of the probable dispersion. Thus the objective function might take the form

$$W = W(\bar{y}_1, \sigma_{y_1}, \bar{y}_2, \sigma_{y_2}, \ldots, \bar{y}_n, \sigma_{y_n}) \tag{1.1}$$

where $\bar{y}_i$ is expected gain in real income of individual y $(i = 1, \ldots, n)$ and σ_i is the standard deviation of that gain. Were we given individual preferences about risk, so that we could write $u_i = (\bar{y}_i, \sigma_{y_1})$, and perhaps of higher moments, (1.1) could be rewritten

$$W = W(u_1, u_2, \ldots, u_n). \tag{1.2}$$

But functions of this form are a counsel of perfection. Policy problems rarely present themselves in a form suitable for such ideal evaluation. Thus, W must be given some other form.

A particularly simple version weights a dollar of expected gain (or loss) of different individuals equally and ignores risk.

Thus

$$W = (\bar{y}_1 + \bar{y}_2 + \ldots + \bar{y}_n), \tag{1.3}$$

this is the form of the function which stresses economic efficiency to the exclusion of all else. The welfare theorists of the Kaldor-Hicks school sought to give strong normative significance to (1.3) through the compensation tests. In more recent literature, (1.3) plus an independent ethical judgment on the distribution of the

[17] This is not to say that an objective function must always be specified when economics is used for policy purposes. Perhaps in most cases, particularly where the analysis involves few steps, such as the mere marshalling of figures, it would be excess theoretical baggage. But once the analysis takes on some complexity, an explicit objective function becomes more important if normative recommendations are to be derived. At the least, the function is a means of forcing the technician to state his normative assumptions; at its best, it is a powerful analytical aid, eliminating uninteresting areas of exploration, and permitting the ranking of alternatives.

incomes $\bar{y}_i$ has found considerable favor and has been applied. This can be written as

$$W = W(\Sigma_i \bar{y}_i, \bar{y}_1, \ldots, \bar{y}_n), \tag{1.4}$$

where the detailed list of individual incomes permits judgment about the income distribution, a judgment to be rendered by the policy-maker. This information cannot, in fact, be specified for individuals since it would be an impossible statistical task. It can be presented for income classes however, either by size class or functional type of income, or the data can be developed by regions.[18] This specializes (1.4) to

$$W = W(\Sigma_m \bar{y}_i, \Sigma_{r_1} \bar{y}_i, \ldots, \Sigma_{r_k} y_i) \tag{1.5}$$

where i in m includes all individuals in the nation (or world), i in r includes all individuals in region (or income class) i, and so on.

When the policy-maker uses the objective function, he can attach any weights he wishes to the national and regional groupings of income. The economist *qua* economist has no right to attach these social utilities to the incomes of individuals. But he usually cannot escape the task of defining the groupings for which income distribution data are to be constructed. The efficiency minded economist will stress the national (world?) grouping and no other. The regionally oriented economist may stress the regional breakdown, and so on. Certain objective functions could be identified as bad economics if labeled as serving the public interest, e.g., the case where weight is only attached to the income of a specific pressure group.

While (1.4) and (1.5) have found most common application, they do not exhaust the possibilities of dealing with the distributional question. The policy-maker may specify more detailed rules. He may impose distributional side conditions, insisting that any policy produce a certain pattern of gain, or alternatively, that a certain minimum accrue to some group, or perhaps that no group suffer a net loss.

The economist can also feel free to perform experiments in policy evaluation using specific objective functions, treating the results as free of absolute normative significance. For example, he can assume a certain shape for the marginal utility of income functions. He may assume some elasticity to this curve, or he may choose to use a form

[18] For an example, see J. V. Krutilla and O. Eckstein, *Multiple Purpose River Development*, 1958, Chs. 7 and 8.

of the function that has been implicitly produced by the political process. The effective marginal rates of the personal income tax at different income levels can be interpreted as implying a marginal utility of income curve. If the government is assumed to act on the principle of equimarginal sacrifice, then marginal effective tax rates can be the basis for deriving a measure of the government's notion of marginal utilities of income.

The kind of question that could be posed when such a function is applied to the analysis of a policy is of the following form: assuming the values placed by the government on marginal income of different income classes in its personal tax legislation, will a policy raise total national economic welfare?[19]

Single vs. Multiple Objectives

Economic welfare can be viewed as a one-dimensional quantity for each individual or group, related presumably to the goods expected to be enjoyed, plus perhaps some allowance for the associated risk. The tools of economic analysis are not always designed to yield this type of answer; in practical work, the objective function has to be tailored to the analysis. For example, a public works program may produce certain outputs over a long period of time, generate a certain amount of economic growth, have a countercyclical potential, alleviate a pocket of local poverty, reduce some natural risks, create a potential for a future recreation facility, dot the landscape with beautiful monuments that have symbolic significance at home and abroad, and so on. Insistence on one-dimensional welfare indicators would either produce a meaningless hodge-podge, or a slighting of all objectives other than expected tangible output. In principle, the many outputs may be considered reducible to common units for each individual, assuming a scalar utility function to exist; in practice the many effects must be grouped into meaningful categories of objectives. These categories can be related to such factors as: (1) economic circumstance; for example, full employment benefits, which may be measurable from market data, can be considered a separate objective from depression benefits, which are more critically related to timing and to the employment and purchasing power generated; (2) the tangibility of the effect—is it measurable in some objective manner, or is it a rather arbitrary valuation? (3) reliability

[19] When the technique is applied to actual policies on an *ex post* basis, it yields a kind of consistency test of government attitudes.

of the estimates—with outputs meeting clear demands treated separately from more conditional benefits which may depend on various repercussion effects; and (4) the date of the benefits—with the usually more uncertain remote benefits treated as a different kind of benefit.

Policy people rarely view their problem as one-dimensional. A multiple objective function corresponds more closely to their view of the world. In particular, it leaves to them the all-important weighting of the various objectives, giving them the results of the technical analysis in the most useable form. The extent of elaboration of objectives is an issue that must be resolved between the policy-maker and the technician. But in no event should the technician arrogate the weighting of objectives to himself by presenting a one-dimensional answer after burying the weighting process in a welter of technical details.[20]

E. WHAT ARE THE BENEFITS?

Since the objective function must be suited to the problem and must often be multiple in nature, the definition of benefits is also a relative matter. On some assumptions, benefits are defined in a particularly simple way. For example, under full employment conditions, with the marginal utility of income the same for all individuals, and with perfect markets and no external economies or diseconomies in production and consumption, prices are perfect measures of benefit. If a project is so large as to affect the prices of its outputs, a simple result is obtainable if the marginal utility of income is assumed not only the same for all individuals, but also constant over the range of variation. The area under the output's demand curve then constitutes a measure of benefits, and if the curves are assumed linear, an arithmetic average of old and new prices multiplied by the number of units will measure benefit. Another interesting case is the following: if the underlying individual indifference curves are assumed hyperbolic, Fischer's "ideal" index number constitutes an indicator of benefit.[21]

In other cases, benefit cannot be defined so simply. While in principle it is always possible to measure the change in utility of individuals (assuming some cardinal concept that can be identified

[20] Examples of this practice abound in the evaluation practices in the water resource field. See my *Water Resource Development*, Harvard University Press, Cambridge, 1958, Ch. 7.

[21] This special case is discussed in R. L. Marris, "Professor Hicks' Index Number Theorem," *Review of Economic Studies*, October 1957, pp. 25–40.

with willingness to pay), in practice this is an enormous task and short cuts must be devised. Often there is the question of what chains of repercussion should be pursued in benefit estimation; this issue is treated in Section 4, below. And where public services are genuine collective goods, benefit estimation often becomes impossible.

F. A SPECIAL CASE: COST MINIMIZATION TO ACHIEVE FIXED OBJECTIVES

A case which has been found to have very wide applicability, particularly in the general field of operations research, is the case where the objectives are strictly fixed and the remaining economic problem is to minimize the real cost of accomplishing them. This is only an interesting economic problem where there are several alternative and quite different ways of achieving the objectives. The problem can be approached through the neoclassical theory of the firm, from which the theorem about marginal productivities can be drawn, through linear programing, through simulation, or through the other maximizing procedures. While, this paper does not elaborate on these techniques, the importance of the case for public expenditure analysis must be stressed, since it provides at least some role for economics even where the nature of the collective goods precludes benefit estimation.

2. The Constraints

A. INTRODUCTION

Economic policy is rarely concerned with the attainment of the best of all possible worlds. Rather, it seeks to improve economic welfare in the face of constraints. The economist, in devising a policy model, must decide how many of the constraints he will build into his analysis. Just as in the case of the objective function, there comes a point where the assumptions are so specific that they produce "bad" economics. Constraints can be assumed to rule out all solutions except one, which automatically is then justified. This procedure can be viewed as excluding the application of economics to the problem. On the other hand, to prohibit the use of constraints altogether is to confine economics to a very narrow—and usually utopian—range of problems.

B. SOME TYPES OF CONSTRAINTS

There are many different sorts of constraints, originating in various institutional or physical limitations. In a sense, they mold the

analysis, giving shape to the problem under study and determining the general nature of the solution.

For the kinds of public expenditures to which our analysis is meant to apply, several types of constraints can be distinguished. First, there are *physical* constraints. The most general of these is the production function, which relates physical inputs to outputs. There may also be absolute limits to the size of structures, or else such sharp discontinuities to the cost curves that any point beyond them can be considered beyond the domain of analysis.

Legal constraints also may need to be incorporated into the model. A program or project must be in accordance with laws, whether it be water laws, property laws, treaties, or whatever. In admitting legal constraints, care must be exercised not to assume laws as fixed which could be affected by the analysis. This is one of the areas where the economist is in peril of accepting so many constraints that he will exclude the interesting solutions.

Administrative constraints may be imposed by the capability of the agency. Limits on the rate of expansion of a program, caused by the need to expand personnel and to diffuse administrative know-how, is one example. Excessive complexity of the planning process, requiring consideration of too many variables, or perhaps requiring excessive centralization of decision-making, is another.

We have already considered *distributional constraints*, which may impose a fixed pattern on the distribution of benefits and costs, or which may impose side conditions of minimum benefits for different groups.

There can be constraints of many other forms. *Uncertainty* can be introduced via constraints; for example, the condition may be imposed that the net gain of a project be positive at some specified probability level. Political constraints can also be imposed, though the line between realism and bad economics is particularly hard to draw on this point.

The final type is *financial* or *budget* constraints. In general, they specify that the amount of money available from some source is limited. In deriving expenditure criteria, this is a critical matter because it is the limited kind of money which must be allocated optimally, and it is to the constrained kind of funds that expenditure criteria address themselves. Elsewhere,[22] I have explored the effect of alternative financial constraints on the form of expenditure criteria.

[22] *Water Resource Development*, *op.cit.*, pp. 47–80.

If there is only one constrained financial resource and one category of benefits, the criterion requires that the rate of net benefit per dollar of the constrained funds be maximized. This maximization is accomplished by computing ratios of benefit to constrained funds for each project (or smaller unit of choice where possible), ranking projects by these ratios and going down the ranked list to the point where the scarce funds are exhausted. Although the ranking is by ratios, it is not the maximization of the ratio which is the objective but rather the total net gain that is possible, given the constraint. Examples of the use of various constraints will be found in the discussion of various models in Section 6 below.

C. CONSTRAINTS AND THE THEORY OF BUDGETING

Constraints are rarely an accurate description of an institutional reality. Budgets are not rigidly fixed except over very short periods—and even then there can be supplemental appropriations. Financial requirements, e.g., that an operation be self-liquidating, are rarely followed if circumstances change. Particularly if a constraint severely interferes with the achievement of economic welfare, the constraint is likely to give way.

Nevertheless, the use of budget constraints is a powerful analytical device. It freezes one (or more) financial resource(s) and then permits an answer to the question: What is the best use of this scarce resource? The analysis then allocates the scarce kind of money in the optimal way. This is a meaningful procedure where, in fact, it is possible to identify the resource which serves to limit the over-all size of the program.[23] A government agency allocating a budget that has been determined at a higher level, or a planning commission in an underdeveloped country drawing up an investment plan subject to limited domestic capital and foreign exchange, can view its problem in these terms. Thus, in a fundamental sense, the theory of constraints is at the heart of the theory of budgeting.

D. CONSTRAINTS AND OPPORTUNITY COSTS

The acceptance of a budget constraint removes the possibility of reaching the *optimum optimorum* solution. In particular, it prohibits solution of the problem of determining the optimal level of

[23] If there were a high degree of substitutability among financial resources, no resource would serve as a limit, and it would make no sense to use a constraint. Thus the constraint approach presupposes that the unconstrained financial resources cannot be a perfect substitute for the constrained resource.

expenditure of the constrained financial resource. Thus, an analysis using a constraint is restricted to optimum allocation of a fixed "second-best" budget level, but it cannot determine the level itself.[24]

The latter problem requires some notion of the cost of budget money. What are the opportunity costs in other sectors of the economy and in fields of the budget outside the particular one under analysis? The extent to which these costs can be measured is still an open question, though some types of opportunity costs can definitely be estimated.

But whatever the difficulties of measurement, it is important to distinguish between two different problems. Where an undertaking must be assumed to be financed out of extra funds made possible by the political process, it is incorrect to compare it to projects within some budget constraint. The relevant comparison is between the project and the opportunity cost of the resources in the sector out of which the resources are drawn, whether by taxation, borrowing, or inflation. On the other hand, if a budget is accepted as fixed, the comparison must be made within that budget.

Wherever possible, constraints should not be accepted blindly. Even if there is an upper limit to expenditures in a particular budget, not only should the scarce funds be allocated in an optimal way, but also, a further test, which assures that the marginal expenditures yield a benefit as great as they would if spent outside the budget, must be performed.

3. *Interest Rates*

A particularly difficult problem in specifying an objective function is the choice of an interest rate. With outputs accruing at different points in time, it is necessary to place relative values on them, depending on the date at which they occur. Similarly, the dates at which costs are incurred may affect the value they represent. In this chapter, some of the possible approaches to specifying interest rates are examined.

A. THE INTEREST RATE AS A MEASURE OF VALUE OF OUTPUTS AT DIFFERENT POINTS IN TIME: PLANNERS' TIME PREFERENCE

There are several bases on which the interest rate for valuing outputs can be chosen. Acceptance of consumer sovereignty is, in

[24] For a general discussion of the theory of "second-best" see R. G. Lipsey and R. K. Lancaster, "The General Theory of Second-Best," *Review of Economic Studies*, Vol. 24 (1), No. 63, pp. 11–32.

one sense, most consistent with individualist welfare economics. It requires that the interest rate used by households in their saving-spending decisions be applied. Clearly the use of this particular rate (or rather rates) makes sense only if the consumers' decision about the amount of saving and investment is also accepted; with the time profile of future output dependent both on the interest rate used for planning and on the amount of investment, rejection of consumer sovereignty with regard to one of the two variables requires modification of the other, even if consumer sovereignty is given full weight. Thus consumer sovereignty must be judged with respect to both variables simultaneously.

There is a long literature of criticism of consumer sovereignty for intertemporal choices.[25] Pigou, Ramsey, Dobb, Baumol and others reject the rationality of time preferences which prefer consumption earlier rather than later simply by reason of the date. Strotz[26] has recently shown that a series of decisions made under pure time preference for the present lead to a total history of individual experience which contains less total satisfaction than would be possible in the absence of such "myopia." As the period of comparison lengthens, there is also the problem of comparing the welfare of future generations. And what assurance can there be that present consumers will make adequate provision for unborn generations?

An alternative approach has the public decision-maker, whether congressman, budgeteer, or central planner, exercise his own time preference. In a democratic society, the preferences on which he acts presumably bear some relation to the population's desires, though in practice, judging by the interest rates used in planning in most countries, there is also a good deal of concern with remote payoffs.

A theoretical foundation can be provided for planners' optimal time preferences, based on the notion of the diminishing marginal utility of individual income. If we assume that this marginal utility falls, then the value of marginal output falls as per capita income rises. Since the interest rate is designed to reflect the relative value of marginal output at different points in time, this rate should be lower the smaller the expected increase in per capita output, and where a decline in per capita income is in prospect, possibly due to excessive

[25] For a summary see F. D. Holzman, "Consumer Sovereignty and the Rate of Economic Development," *Economia Internazionale*, 1958, pp. 3–20.

[26] R. Strotz, "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 1955–56, pp. 165–80.

TABLE 1

Interest Rates Based on Diminishing Marginal Utility of Per Capita Consumption and Growth Rate of Per Capita Consumption[a]

ELASTICITY OF MARGINAL UTILITY OF INCOME FUNCTION	PER CAPITA GROWTH RATE OF CONSUMPTION			
	−2%	0	+2%	+4%
2.0	−4.0	0	4.0	8.0
1.5	−3.0	0	3.0	6.0
1.0	−2.0	0	2.0	4.0
0.7	−1.4	0	1.4	2.8
0.5	−1.0	0	1.0	2.0

[a] The underlying model is the following:
Let $W = W(y_1, \ldots, y_t, \ldots)$, where y_t is per capita consumption in year t.
Let $\frac{\partial W}{\partial y_t} = (y_t)^{-\epsilon}$, where ϵ is the elasticity of the marginal utility of consumption.
Then

$$\frac{\partial W}{\partial y_t} \Big/ \frac{\partial W}{\partial y_{t+1}} = \frac{y_t{}^{-\epsilon}}{y_{t+1}{}^{-\epsilon}} = \left(\frac{y_t}{y_{t+1}}\right)^{-\epsilon} = (1 + r)^{\epsilon},$$

where r is the growth rate of per capita consumption.
But $\frac{\partial W}{\partial y_t} \Big/ \frac{\partial W}{\partial y_{t+1}}$ is the ratio of marginal values of consumption, and thus equals the interest factor, $1 + i$. Therefore

$$1 + i = (1 + r)^{\epsilon}.$$

This is the formula used for the table. If we decompose the growth rate of per capita consumption into the growth rate of population, π, and of consumption, ρ, we get

$$1 + i = \frac{(1 + \rho)^{\epsilon}}{(1 + \pi)^{\epsilon}}.$$

Recently, Samuelson has examined a similar problem and pointed to the relation between population growth and the interest rate. P. A. Samuelson, "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, December 1958, pp. 467–82.

population growth, the interest rate can even be negative.[27] Given the kinds of empirical magnitudes that actually prevail in the world, the interest rates suggested by the model are relatively low, 4 per cent or less, even for fairly large elasticities of the marginal utility of income curves. Table 1, which is reproduced from an earlier study, summarizes these results.

One paradoxical result is suggested by the analysis: the lower the

[27] For a formal model reflecting these notions, see my "Investment Criteria . . . , *op.cit.*, pp. 76–8. An earlier model, which has some points of similarity, can be found in R. F. Harrod's *Toward a Dynamic Economics*, Macmillan, 1948, pp. 35–62. A model which derives the optimal rate of investment from utility functions is given in J. Tinbergen, "The Optimal Rate of Saving," *Economic Journal*, December 1956, pp. 603–10; but see the important comment by A. K. Sen, *ibid.*, December 1957, pp. 745–50.

rate of growth, the lower should be the interest rate. Since, typically, the lowest rates of growth of per capita income are found in the poorest countries, low interest rates should be used in these countries. Yet these are the places where the pressure for early consumption is greatest. The resolution of the paradox is simple: the interest rate relates not to the absolute level of consumption, but to the relative changes over time. Thus, high-growth countries, whatever the present levels, can afford the luxury of high valuation of present consumption versus future consumption, since they will have higher levels in the future. The association of high interest rates with low income levels is based on other phenomena, particularly the scarcity of capital and pure preference of present over future consumption. Pure time preference has often been believed to be greater at low levels of income.

Should the objective function of the planner allow for pure time preference? Or should it be above such "irrationality"? Even from the narrow point of view of economic efficiency, this question cannot be resolved without use of strong value judgments. Preference for experiences in the near future can be rational for individuals, given the uncertainty of the duration of life. A lifetime consumption plan which stresses early years is more certain of fulfillment than one which emphasizes later years, since the probability of survival to the expected consumption dates is greater. Even if rationality is defined to exclude aversion to risk there is room for pure time preference. The utility to be enjoyed at each future moment must be multiplied by the probability of being alive at the time, and since this probability falls with the remoteness of the period, a kind of pure discount factor emerges. This assumes individuals to be narrowly selfish, caring nothing about the wealth they leave behind when they die.

Numerical values for this discount factor can be computed from mortality statistics. For consumption one year after the present moment, the factor is equal to the probability of not surviving the next year; for longer intervals, it is a geometric average of annual rates.[28] Table 2 gives numerical values for the probability of surviving the next year at different ages. The figures, which are given for an advanced country, the United States, and an underdeveloped country, India, are based on mortality statistics compiled by the

[28] For the theory of deriving long-term interest rates from a structure of short-term rates, see F. A. Lutz, "The Structure of Interest Rates," *Quarterly Journal of Economics*, November 1950, pp. 36–63.

TABLE 2

"Rational Individual Time Preference," Based on Survival Probabilities, United States and India Both Sexes

Age	*United States (1950)* (in per cent)	*India (1941–50)* (in per cent)
5–9	0.04	1.50
10–14	0.04	1.10
15–19	0.07	0.85
20–24	0.10	0.95
25–29	0.10	1.25
30–34	0.15	1.60
35–39	0.25	1.90
40–44	0.40	2.15
45–49	0.65	2.50
50–54	1.00	3.10
55–59	1.45	3.80
60–64	2.10	4.90
65–69	2.90	6.15
70–74	4.50	7.50
75–79	5.85	8.95
80–84	7.45	10.55
Life expectancy:	68	32

SOURCE: United Nations, Department of Social Affairs, Population Branch, Population Studies No. 22, *Age and Sex Patterns of Mortality, Model Life Tables for Underdeveloped Countries*, New York, 1955, ST/TOA/Series A/22, pp. 30–1.

[a] The figures are the average values for the five-year interval.

United Nations; similar tables could be computed for many other countries, for either sex, and for the mortality experience at different points in history.

The figures for the United States turn out to be amazingly low. Up to age fifty the probability of not surviving the next year is less than 1 per cent, and below age forty-five less than 0.5 per cent. Thus, the pure time preference of the rational individual as I have defined him, should be less than 1 per cent a year up to age fifty. Or to cite a long-run figure, a twenty-year-old person looking ahead to a date fifty years away should discount at an average annual rate of 0.9 per cent. In old age the rate rises, of course, as the probability of survival diminishes.

The figures for India are considerably higher. Pure time preference based on rational mortality expectations never gets much below 1 per cent, and ranges up to 5 per cent even for moderate ages. For example, a rational Indian at age twenty, evaluating utility to be enjoyed fifty years hence, would discount it at an annual rate of 2.8 per cent. Thus, in underdeveloped countries, where life expectancy

is short, even "rational individuals" are governed by substantial pure time preference.

Two factors must be kept in mind in interpreting these figures. First, these pure time preference factors measure only one component of interest rates reflecting the intertemporal values of individuals. The time profile of expected incomes interacting with the shape of the marginal utility of consumption curve at different ages also affects the "rational" interest rate. A person with a rising income stream will find the marginal utility of early consumption greater and will use a higher interest rate in his intertemporal choices. Conversely, an older person with a falling income stream will find it worth while to postpone marginal consumption outlays to a time when the marginal utility of consumption will be greater; his valuation of present versus future marginal consumption may involve an implicit negative rate of interest. The second factor, trends in the marginal utility of consumption with age, probably works in the same direction; the cost of rearing children makes consumption expenditures in the younger, more active, years of greater utility than later on, i.e., the marginal utility of consumption function drifts downward in the conventional diagram.

Besides considering the other factors that enter into the "rational" marginal valuation of consumption at different points in time, the desire to leave an estate may modify intertemporal valuations. Thus the high time preference rates of old-age that would be derived from mortality expectations may be overruled by a desire to transfer wealth to a wife or to future generations.

Despite the significance of these motivations, our computations have some suggestive implications. First, for all countries, the uncertainty of survival leads to a purely rational preference of present consumption over future. Thus, the condemnation of all time preference as "due to weakness of the imagination" (Ramsey) is based on the unrealistic view that individuals live forever. Second, in advanced countries, the pure time preference discount rate is very low over a wide range of ages. But in underdeveloped countries, this pure discount rate is considerably higher, though even in a country with as bad a mortality experience as India, the factor ranges only from below 1 to 5 per cent over a range of ages which encompasses most of the population.

Granting then the existence of perfectly rational preference of earlier consumption for individuals, the question remains whether

a public planning body should take such preference into account. As has been pointed out in numerous places, the society goes on forever, and in the absence of thermonuclear war, there is almost certainty about the perpetual life of the population as an entity. If the social welfare function is a sum of utilities enjoyed by individuals, regardless of when they live, then mortality probabilities of specific individuals become irrelevant, and pure time preference is eliminated.

An interest rate can still be based on the diminishing marginal utility of income as per capita income rises, but rates based only on this mechanism are relatively low for plausible combinations of rates of increase of per capita income and elasticities of the utility curves.

A planner's welfare function which ignores individual time preference may well be in the long run interest of the society, but, strictly speaking, it is not a preference derived from individual desires, and hence falls into the category of dictatorial preference functions. Now a welfare function which adds up the utility of everybody, present and future, may be a defensible value judgment by which a policy-maker may choose to operate. Where government is democratic, the population may choose to operate by this value judgment through the political process; that is to say, its politicans may find decision-making according to low interest rates a successful component of their platform. But there is nothing in economic analysis *per se* which justifies this particular welfare function over all others.

What is more, there is growing and abundant evidence that the people of underdeveloped countries do place a considerable premium upon benefits in the early years of development projects. In India, for example, there has been some dissatisfaction with investment plans that yield little for quite a while, partly because of long periods of construction. Even in Russia there has been some revamping of investment plans, reducing the number of new projects being started for the sake of more rapid completion of the vast amount of work in progress;[29] there has also been some shift away from gigantic long-lived projects, such as huge hydroelectric dams, toward smaller, less durable and less capital-intensive projects, including steam plants.

The planner may feel that he is protecting the nation against its

[29] Reported in *The New York Times*, June 12, 1958, p. 14.

own shortsightedness by using low interest rates. But where this flies in the face of popular desire, he runs some risks that he may lose the chance of development on the Western model altogether. And even within the narrow perspective of economic analysis, early benefits may have morale effects which yield extra production and generally add momentum to the development effort.

Thus the choice of interest rates must remain a value judgment. I have discussed some of the elements that enter into the choice and have presented two models which derive elements of interest rates from empirical magnitudes, to show that there are some objective factors that can enter into the choice. But these must be combined with subjective judgments that cannot be value-free.

B. THE RATE OF RETURN *vs.* PRESENT VALUE COMPUTATIONS

Since the selection of an interest rate requires subjective judgment, the ranking of projects by means of rate of return comparisons has been an attractive alternative. The interest rate issue could be sidestepped, since each project has an internal rate of return which can be computed. Given a limited amount of capital, projects could be ranked by their rates of return, and the projects with the highest rates could be undertaken. The rate of the marginal project, the cut-off rate, could then serve for technological choices in project design.

This procedure is meaningful only under a regime of perfect competition, in which the capital market contains no rationing and is equated by the interest rate serving as the price.

Hirshleifer has recently shown, that only in very peculiar cases other than perfectly competitive capital markets does the use of the rate of return criterion result in optimal results.[30] Once the marginal returns inside the budget being planned differ from returns elsewhere in the economy and from the rates being offered to suppliers of capital, the internal rate of return loses any normative significance. Hirshleifer's neat and exhaustive analysis, which shows the inexorable relevance of the subjective time preference of the planning agent, disposes, once and for all, I think, of the rate of return criterion.

Other attempts to escape the necessity of specifying an interest rate have been made, and some of them are discussed in Section 7

[30] J. Hirshleifer, "On the Theory of Optimal Investment Decision," *Journal of Political Economy*, August 1958, pp. 329–52. Also see my "Investment Criteria . . . ," p. 64, where the same point is made.

below.[31] It will be argued there that none of these attempts is satisfactory. Relative values on the outputs of different periods must be established, and only an interest rate can do this. Once interest rates are used, present values of benefits and costs can be computed and utilized in decision criteria.

One of these present-value criteria is closely related to rates of return, and has sometimes not been sufficiently distinguished from it. This is the SMP, or social marginal product of capital criterion,[32] proposed by Chenery.[33] The SMP of a project is the rate of present value of net benefit per dollar of capital cost. It applies where capital is a constraining factor on a budget, the case where the rate of return criterion might appear to apply. But while focusing on a rate on capital, it differs from the rate of return in the crucial respect that it requires an interest rate to be specified for the computation of the present value of net benefit. Thus the SMP is one of a family of present value criteria, while the rate of return is not.

C. THE OPPORTUNITY COST OF CAPITAL AND THE INTEREST RATE

Just as some form of social time preference is required for planning within the expenditure field, it is also needed for measuring opportunity costs. For efficient resource allocation, the capital in a specific use must yield as much satisfaction as in the opportunity which is foregone. But this cost must be expressed in terms comparable to the benefits. The foregone flow of satisfaction must be reduced to a present-value concept by means of a social rate of discount.

Elsewhere,[34] I have measured the opportunity cost of capital raised by federal taxation in the United States, and expressed it as an interest rate. This rate proved to be 5 to 6 per cent. It is no more than an empirical approximation to the desired magnitude, since it does not employ a social discount rate.[35] To convert this opportunity cost rate to a present value concept, the chosen rate of social discount must be used in the following manner: suppose the opportunity cost

[31] See the discussion of the model of Sen below.

[32] For example, in *Water Resource Development*, p. 61, I mislabel the SMP criterion a rate of return criterion.

[33] H. B. Chenery, "The Application of Investment Criteria," *Quarterly Journal of Economics*, February 1953, pp. 76–96.

[34] J. V. Krutilla and Otto Eckstein, *Multiple Purpose River Basin Development*, 1958, Ch. 4.

[35] Peter O. Steiner first saw this point. See his "Choosing Among Alternative Public Investments," *American Economic Review*, December 1959.

rate is 6 per cent; as measured in my empirical study, this is a perpetual stream of .06 cents per dollar of capital. Assume that the chosen rate of social discount is 3 per cent, i.e., that the benefits of a project are reduced to a present value by discounting at that rate. Then in order to compare the benefits of the project with the foregone benefits of the opportunities in the private economy, the perpetual stream of .06 cents per dollar must also be valued at 3 per cent. In the present example, the present value of a dollar invested in the private opportunity is $2, since a perpetual stream of .06 discounted at 3 per cent has a present value of 2.[36] As an expenditure criterion, assuming the above numbers, this implies that public projects should only be undertaken if the ratio of present value of net benefits to capital cost is 2.0.

The particular notion of opportunity cost which was measured, the cost of capital raised by a particular tax system, is only of relevance in models which link expenditures to taxes, rather than to reduced other expenditures, inflation, foreign borrowing, tighter monetary policies, or whatever other method the government may devise to raise the capital. Each method has its own opportunity cost which must be measured, and valued with a social rate of time discount.

Thus both opportunity cost and an interest rate must be specified for expenditure models. For example, in conventional benefit-cost analysis, a present value of benefits must be computed, using some interest rate. The rate at which costs result in present value of benefit, that is, the marginal benefit-cost ratio, must then be compared with the rate at which present value is foregone elsewhere, i.e., the opportunity cost. A correctly constructed criterion will pass a project only if the rate of present value of benefit per dollar exceeds the rate of present value per dollar of opportunity cost. Even if a low rate of interest is chosen, this does not mean that projects which yield low rates of return can be built or that scales of development can be pushed to a point where increments yield a low return. It only means that the present values, both on the benefit and the cost side, are computed using a low interest rate.[37]

[36] If the opportunity cost is not expressed as a perpetual stream, for example, if the time profile of returns of the private opportunities can be identified, a present value has to be computed explicitly.

[37] The procedure proposed in my book, *Water Resource Development*, corresponds to this logic. A low interest rate is coupled with marginal benefit-cost ratios sufficiently in excess of 1.0 to assure that the project exceeds its opportunity cost.

The procedure of the Hell's Canyon study (in Krutilla and Eckstein, *Multiple Purpose River Development*) takes the short-cut of comparing opportunity cost with a rate of

The necessity for measuring opportunity costs springs from the fact that there is no perfect market mechanism which measures the cost of resources from the private sector. In the perfectly functioning market economy, opportunity cost of resources is fully measured by the price of factors of production purchased for a project, and there is no need to worry about the concept separately. It is because there are imperfections in the private economy, particularly in the capital market, that opportunity cost must be measured and utilized as a criterion in determining public budgets, and must be valued at a social rate of interest.

4. *Repercussion Effects*

A. INTRODUCTION: PRICES *vs.* INTERDEPENDENCE RECOGNIZED

In devising an analytic framework for maximizing any given objective function, there is an important choice in the selection of the chain of effects which should be pursued, both on the benefit and on the cost sides. The proper circumscription of the analysis is one of the critical points in the economics of public expenditures. It is all too easy to find myriad favorable effects of a social nature, both tangible and intangible; it is equally easy to lapse into such rigid acceptance of the rationale of the perfect market mechanism that the broader public viewpoint is lost altogether.

So far in economic science, only one approach has been developed for defining the proper area of analysis. It uses the perfect competition scheme as a point of departure; as is well known, under perfect competition, with price ratios equal to marginal rates of transformation in production and marginal rates of substitution in consumption, prices are precise indicators of value, and there is no need to pursue any repercussion beyond the most immediate market;

return of alternative plans. None of the results would have been affected by going through the intermediate step of revaluing the alternatives and the opportunity costs at a social rate of interest and then comparing the present values.

An alternative interpretation of the latter study, the interpretation given in the theoretical derivation of the opportunity cost concept, is the following: the proper social rate of interest is stated to be the rate which the taxpayers who are forced to finance the project choose to be their marginal rate of time preference. That is, the time preference of individuals is accepted for the government. This model is certainly closer to the strict concept of economic efficiency based on individual tastes. The upshot of the present discussion is this, however: even if individual time preference is rejected for public decision-making, opportunity cost, including foregone consumption, is a critical parameter that must be in the model.

the market mechanism produces an efficient allocation of resources. Because the real world is not perfectly competitive, some repercussions ought to be pursued, but in this approach each instance is justified by showing how the specific situation fails to conform to the competitive ideal. As will be seen in what follows, the range of repercussion effects that can occur is very wide, and while the use of perfect competition as a point of departure may impart some conservative bias against measuring them, it is a small bias and a shrinking one, as economists concerned with the problems of underdeveloped countries discover more and more cases in which the repercussions count. Further, the need to justify inclusion of repercussions improves the quality of the analysis and provides a framework for empirical measurement; it gives specific meaning to the "social" effects within an individualist welfare economic point of view.

We shall assay no comprehensive treatment of repercussion effects, since they vary from case to case. But we shall list the major categories, and give some indication of the techniques which would measure them.

B. A LIST OF LOCAL EFFECTS

Local, including some regional, effects involve simpler considerations than over-all national repercussions; they are listed first, not necessarily in the order of their importance.

Physical Interdependence

If a project has off-the-site physical effects, a particularly common phenomenon in water resource projects, their economic implications clearly require inclusion in the analysis. Where off-the-site effects are small, it may be possible to make simple allowances for such benefits and costs. For example, when an upstream storage reservoir is added to a system, its incremental benefits to projects downstream can be computed directly. But as the number of projects increases above one and the interrelations become more complex, centralized planning must be applied to the river basin as a whole. The Harvard Seminar in Water Resource Planning is experimenting with various planning techniques, including elaborate simulations, programing, and marginal analysis; actual physical interdependence relations turn out to be very complex, involving mixtures of competitive and complementary relationships. From the researches of this

Seminar it is clear that the problem is soluble for the water resource case—which is perhaps the most complex—but that it tests the analytic and computing technology of modern engineering and economics.

Economic Interdependence: Investment Coordination

Where a project produces outputs that are producers' goods. the value depends on the existence of industrial markets. If the growth of industry in a region is rapid, additional outputs cannot be considered to be incremental, and must be evaluated as part of an overall investment plan.[38] In this field, too, planning techniques have not been perfected, but linear and nonlinear programing, utilizing input-output data, offer considerable hope of solution.[39]

Large Changes in Inputs and Outputs

If a project is so large that it changes the prices of its outputs and inputs, market prices cease to be unambiguous indicators of value. This is a matter we have touched upon in Section 1, where we saw that reasonable approximations, such as arithmetic or geometric means of old and new prices, are readily at hand. In some cases, such as large power projects, finer approximations are possible by analyzing separate segments of the demands.

Local Unemployment

Where part of the factors of production, including labor, are un- or underemployed, their market price overstates their opportunity costs. If their opportunity cost is zero, their real cost for purposes of project planning may also be zero. Should their employment generate other costs, however, such as large food consumption or urbanization costs, these need to be considered, of course.

Social Overhead

All productive enterprises require certain complementary investments in public facilities, such as police and fire protection, public

[38] For more detailed discussion of these matters, see R. Nurkse, *Problems of Capital Formation in Underdeveloped Countries*, pp. 1–24, where they are treated as part of the problem of balanced growth. Also see T. Scitovsky, "Two Concepts of External Economies," *Journal of Political Economy*, April 1954, pp. 143–51; and J. A. Stockfisch, "External Economies, Investment and Foresight," *ibid.*, October 1955, pp. 446–9.

[39] See the forthcoming book by Chenery, and H. B. Chenery and H. Uzawa, "Nonlinear Programing in Economic Development" in *Studies in Linear and Nonlinear Programing*, Arrow, *et al.*, eds., Stanford, 1958, pp. 203–29.

health services, schools, workers' housing, etc. These are all real costs, of course, whether the project is charged a price for them or not.

Local Monopoly and Monopsony

If the project deals with monopolists, either in selling its outputs or in purchasing its factors of production, market prices may not measure value; the project then generates monopoly profits, which after all, are also part of national income (though they might be given a weight of zero in some objective functions). For example, if the profits in the processing of output include a monopsony element, then the price is below the value of output. Measurement is very difficult in this case, since accounting profits include normal profits and the regular return on equity capital.

C. EFFECTS ON THE ECONOMY AS A WHOLE

Repercussion effects on the economy as a whole should properly be measured and included in the analysis to the extent that there are specific imperfections in the economic system. Unemployment, capital shortage, foreign exchange imbalance, and excessive population growth are some of the items that fall in this category. We treat a few of the more important.

Keynesian Unemployment

In a situation of general unemployment, the conventional multiplier measures the repercussion effects on national income which are caused by the increase in purchasing power. Recent empirical studies suggest that the total multiplier in the American economy is on the order of 1.4 or so,[40] when all the various leakages, including taxes and retained earnings, are taken into account.[41] Of course the multiplier differs from project to project, depending on the marginal propensities to consume of the income recipients; but the recent work of Strout, using input-output techniques, implies that the differences,

[40] J. S. Duesenberry, Otto Eckstein and Gary Fromm, "A Simulation of the U.S. Economy in Recession," paper presented at the December 1958 meetings of the Econometric Society (to be published).

[41] If it is assumed that government expenditures are limited by revenues, i.e., that the government has a marginal propensity to spend equal to 1.0, the multiplier becomes much larger. Also, if the impact on inventory fluctuations is included—an impact that has to come quickly because of the speed with which inventory fluctuations occur—the multiplier becomes larger.

at least as far as off-the-site purchases of goods and services are concerned, are extremely small.[42]

Whether multiplier effects should be incorporated in the analysis depends, in part, on administrative policy. Past experience with investment projects suggests that their time-table makes them rather ineffective in counteracting the swift disturbances that have characterized the postwar period in advanced economies. Some speed-up of work in progress does appear feasible in recession, but this does not require that original investment plans need reflect this possible repercussion; symmetry would also suggest that if multiplier effects in potential recessions be included in the analysis, similar effects in inflation also be measured. In underdeveloped economies, on the other hand, Keynesian lack of effective demand may not be the critical dimension of the unemployment problem.

Structural Unemployment and Underemployment

Where an underdeveloped economy simply has an inadequate number of jobs for its population, or where many people are in occupations in which their productivity is very low or zero, the expenditure decision model must take some cognizance of this state of affairs. From the point of view of maximizing national income, money-wages are not likely to be reasonable measures of opportunity costs, and so some "adjusted" wage, possibly equal to zero, may be needed. In addition, employment-creation may be an important part of the objective function; separate quantitative analysis may be needed to measure the performance of projects on this scale, including the employment generated in subsequent stages of production; input-output analysis appears the logical quantitative technique.

Capital and Foreign Exchange Scarcity

In many countries, the desired rate of economic development is limited by the scarcity of capital and of foreign exchange. This can be introduced into the expenditure model in at least two ways: first, the two sources of finance may be treated as budget constraints, determining the choice of criterion. Second, the repercussion effects of projects in this regard can be measured, including the indirect

[42] Alan Strout, "Primary Employment Effects of Alternative Spending Programs," *Review of Economics and Statistics*, November 1958, pp. 319–28. Strout analyzes employment multiplier effects, but since differences in marginal propensities to consume largely relate to differences between wage-earners and others, his conclusions carry over to income-multipliers.

effects caused in other industries and in the purchases of consumers. The generation of reinvestible capital out of the income payments of projects is a particular instance of these repercussion effects.

Population effects

If it can be shown that some expenditures change the environment of workers in such a manner as to reduce the rate of population growth, and if the objective function is expressed in terms of *per capita* income, expenditure analysis must include population repercussions. This factor is often cited as making investment in urban areas relatively more attractive.[43]

D. CONCLUDING COMMENT

These lists of possible repercussions that ought to be measured in certain cases are far from complete. Each situation has peculiarities of its own, which make different repercussions of relevance. The only analytically valid principle that has been advanced so far for determining their inclusion or exclusion is the technique of comparison of the actual case to the perfectly competitive model.

5. *The Treatment of Risk and Uncertainty*

A. INTRODUCTION

Expenditure criteria must take some cognizance of the risky and uncertain nature of the economic world. Unfortunately, welfare economics has no complete apparatus for dealing with risk, no applicable optimum conditions from which decision criteria can be derived.[44] Nevertheless, it can easily be shown that for many reasonable objective functions, some account must be taken of risk, and some approaches will be indicated.

In evaluating these various adjustments for risks and uncertainty, it must be borne in mind, however, that something is being given

[43] This factor, as well as the importance of generating reinvestible capital, was put into the center of discussion by Galenson and Leibenstein. They also stress the effect of projects on the skills of the workers. W. Galenson and H. Leibenstein, "Investment Criteria, Productivity and Economic Development," *Quarterly Journal of Economics*, August 1955, pp. 343–70.

[44] But see K. J. Arrow, "Le Rôle des Valeurs Boursières pour la Répartition la Meilleure des Risques," *International Colloquium on Econometrics*, 1952, pp. 41–47, Centre National de la Recherche Scientifique, Paris, 1953, reprinted as Cowles Commission Paper, N.S. 77, where an interesting model which seeks to incorporate risks, gambling, and insurance into the competitive model is presented. Also see, M. Allais, "L'Extension des Théories de l'Équilibre Économique Général et du Rendement Social au Cas du Risque," *Econometrica*, April 1953, pp. 269–90.

up in exchange for the greater security, and that from the point of view of the country in the long run, short-run adjustments may not prove to be optimal. For example, if there is an empirical foundation for the idea of a risk premium, i.e., that risky investments have to have a relatively higher expected gain, then the national income will rise more if risky investments are undertaken, even though more risk is being experienced. It is possible that a series of risky, high-return investments not only give the country a higher national income, but also will put it in a more secure position in the long run than safe, low-return investments. Thus, even though there is a strong case for various adjustments in the direction of secure actions, the sum of a lot of adjustments may have the opposite effect of what is desired.

B. SOME CRUDE ADJUSTMENTS

The traditional adjustment for risk is simply to be conservative. In expenditure analysis, this has often taken three forms: (1) contingency allowances, which arbitrarily raise certain categories of costs by a certain percentage or reduce benefits through price assumptions which are below expected prices; (2) a limit to economic life shorter than physical life but also shorter than expected economic life; and (3) a risk premium in the interest rate. The first of these adjustments, which in many instances is a part of standard engineering practice, may simply be an allowance for errors in forecasting which past experience suggests will recur. Contingency allowances for costs are particularly of this character, since some unexpected costs always occur in construction. Thus, in a sense, they simply improve the quality of forecasts by allowing for expected errors.

A limit on economic life, which is particularly significant for projects like dams which have no definite terminal date, partly serves to standardize analysis of different projects. It also is an adjustment for technological progress, since it implicitly assumes that the economic value of the project goes to zero at some future date; this is clearly a very crude adjustment. The risk premium in the interest rate accomplishes the same purpose more delicately, since it discounts remote benefits progressively more heavily. The risk premium can have a precise basis where the probability of failure is known and remains constant over time. For example, if the probability of failure, defined as an economic value of zero after some date, is equal to .04 per year, the risk premium should be

approximately[45] 4 per cent. But this precise application is only possible in connection with credit risks on securities with which there is lots of experience; for physical projects, there is, so far, no empirical method for determining the premium.

These crude adjustments are intellectually not very satisfying, and one should try to derive better adjustments from explicit objective functions and from the specific probabilistic nature of benefits. But where the probability distributions are unknown or based on very little information, or where it is difficult to specify an objective function that can fully value the effects of risk or uncertainty, the crude adjustments are appropriate and important; at least there is some cognizance of the problem.

C. THE CASE OF PURE RISK: PUTTING RISK ATTITUDES INTO THE OBJECTIVE FUNCTION

Following conventional terminology, we call "risk" the state of affairs in which a probability distribution can be specified without error. Strictly speaking, the evidence is never complete and the parameters of the probability distribution are given with some error. But when such error is small, good results can be obtained by treating the problem as one of pure risk. While events which can be characterized as risky are rarely encountered in the analysis of public expenditures, the water resource field is replete with them because of the dependence of projects on hydrology; flood control benefits depend on the probabilities of flood events; irrigation benefits depend on stream flow probabilities. Social insurance is another field subject to risk, actuarial risk in this case.

There are several different ways of handling risk in the analysis.[46] With probability distributions of outcomes known, means, variances, and possibly even higher moments can be put into the objective function.[47] In the most general form, the objective function can be written

$$W = (\mu, \sigma, \ldots). \tag{5.1}$$

[45] It is only an approximation because the proper risk premium is $\sum_t (1-p)^t$, where p is the probability of loss in a period. This expression is only approximated by $\sum_t \frac{1}{(1+p)^t}$, the expression in the text. I owe this point to Donald Farrar.

[46] The need to include risk attitudes in the objective function was impressed on me by Harold Thomas in the Harvard Water Resource Seminar. See his paper discussed below.

[47] For a survey of authors who have made this suggestion, see K. J. Arrow, "Alternative Approaches to the Theory of Choice in Risk-Taking Situations," *Econometrica*, October 1953, pp. 269–90.

If we confine ourselves to the first two moments, this can be represented by an indifference map with the mean on one axis and the standard deviation on the other.[48] If there is aversion to risk, the indifference curves will slope away from the origin, with an increase in expected gain offset by an increase in the standard deviation.

Maximization of Expected Utility

Following Bernoulli[49] a utility function can be specified for which the expected value is maximized. Unless that function is linear, implying a constant marginal utility, it will result in the utilization of the probability distribution of outcomes in the optimizing analysis. This maximization of expected utility has been argued widely to be the rational form of behavior under conditions of risk. As a descriptive hypothesis, it can be tested empirically,[50] and is certainly not a perfect description of human behavior. But in normative welfare economics, as a prescription of what the rational consumer ought to do, it may serve the same purpose as the conventional theory of the consumer under certainty.

A particular problem in applying the approach to public expenditures is the choice of the person(s) whose utility function is to apply. Is it the utility function of the planner or of the affected individuals? This is a problem analogous to the choice of interest rate. From a strictly individualist ethical point of view, the functions of the individuals should be used and weighted in some way. But in connection with the loss of utility due to risk, as with the loss of expected utility due to mortality risks, the group as a whole may suffer less than the individual. The variance of the total outcome may be relatively smaller than for each individual because of pooling.

An Example: Flood Control Design

This example is a highly simplified illustration of the effect of introducing utility functions on flood control design. It will be shown that for a broad class of utility functions, utility maximization leads to

[48] This is drawn, for example, in F. Lutz and V. Lutz, *The Theory of Investment of the Firm*, 1951, pp. 190.

[49] See Daniel Bernoulli, "Exposition of a New Theory on the Measurement of Risk," (1738) translated by Louise Sommer in *Econometrica*, January 1954, pp. 23–46 for the original statement. For a summary of the literature see K. J. Arrow, "Alternative Approaches to the Theory of Choice in Risk-Taking Situations," *Econometrica*, October 1951, pp. 404–37.

[50] Early tests provide some support, but at this time the evidence must still be considered mixed. See K. J. Arrow, *ibid.*, p. 12, for a survey and further references.

more flood control than income maximization. The latter is equivalent to minimizing the expected cost of damages plus the cost of control works.[51]

Suppose utility is related to income by

$$U = F(y), \tag{5.2}$$

and let $F'(y)$ be strictly decreasing with increasing y. Let s be the height of floods in the absence of control, expressed in feet of flood stage, and let r be the number of feet by which flood stage is reduced through control works such as a dam. Let

$$x = s - r \tag{5.3}$$

be the number of feet of flood stage after control works are installed (s is determined by rainfall and other hydrological factors). The probability of occurrence of any particular value of s is described by the probability distribution $p(s)$, where s takes on only integral values.

Income is affected by a flood of x by an amount $g(x)$, measured by damages. Thus

$$\Delta U(x) = F(y_o + g(x)) - F(y_o) = h(x), \tag{5.4}$$

where y_o is the level of income without flood.

We wish to minimize the expected value of this loss of utility due to floods, where r is the policy variable. The loss comes from two sources, the flood damages and the cost of control works. Let $\delta(r)$ be the cost of r. Thus the function to be minimized is

$$\rho(r) = \bar{h}(r) + F'(y_o) \cdot \delta(r), \tag{5.5}$$

where $\bar{h}(r)$ is the expected value of $h(r)$ if control works serve to cut flood stage by r, and where $F'(y_o) \cdot \delta(r)$ is the marginal utility of income multiplied by the cost of r.

We must set

$$\rho'(r) = 0,$$

i.e.,

$$\bar{h}'(r) + F'(y_o) \cdot \delta'(r) = 0. \tag{5.6}$$

But

$$\bar{h}(r) = \sum_{s=r+1}^{\infty} p(s)h(s - r),$$

so that

$$\bar{h}'(r) = -\sum_{s=r+1}^{\infty} p(s)h'(s - r). \tag{5.7}$$

[51] The following argument is due to E. C. Schlesinger of the Department of Mathematics, Wesleyan University.

Hence the condition which minimizes the loss of utility is

$$\sum_{s=r+1}^{\infty} p(s)h'(s-r) = F'(y_o)\cdot\delta'(r), \tag{5.8}$$

which must be solved for r.

Next, suppose we seek to maximize income, i.e., minimize the loss of income caused by floods. We substitute $g(r)$ for $h(r)$. Then we must minimize

$$\psi(r) = \bar{g}(r) + \delta(r), \tag{5.9}$$

the expected damages plus the cost of control. This requires

$$\bar{g}'(r) + \delta'(r) = 0,$$

or

$$\sum_{s=r+1}^{\infty} p(s)g'(s-r) = \delta'(r), \tag{5.10}$$

which is to be solved for r.

We now contrast the two solutions (5.8) and (5.10). Suppose r_o satisfies (5.10), the minimum cost solution, and that (5.8) has only one solution. It will be shown that

$$\rho'(r_o) < 0,$$

from which we can conclude that the optimal value of r for (5.8) is larger than that for (5.10).

The proof is based on (5.4) and on the assumption of the decreasing nature of $F'(y)$. From (5.4) we obtain

$$h'(x) = F'(y_o + g(x))\cdot g'(x),$$

and from the assumption about the utility function we obtain that

$$h'(x) > F'(y_o)\cdot g'(x), \quad \text{since} \quad g(x) < 0 \quad \text{for} \quad x > 0. \tag{5.11}$$

We substitute (5.11) in (5.8). This yields

$$-\rho'(r_o) = \sum_{s=r+1}^{\infty} p(s)h'(s-r_o) - F'(y_o)\,\delta'(r_o).$$

The right side is greater than

$$F'(y_o)\left\{\sum_{s=r+1}^{\infty} p(s)g'(s-r_o) - \delta'(r_o)\right\},$$

which equals zero by our assumption that r_o satisfies (5.10). Hence $\rho'(r_o) < 0$, as asserted.

This shows that utility maximization leads to more flood control

than income maximization. This is no more than an application of the theory of insurance. It is interesting that the procedures actually applied in the design of flood control works reject income maximization, requiring substantially more control. The extent to which a utility-maximizing solution exceeds the income-maximizing result depends on the distribution of flood probabilities and the elasticity of the marginal utility of income curve. The more frequent and routine the flood losses, the smaller will be the deviation between the two solutions. An optimal set of flood-control design principles can be derived from a model which maximizes expected utility.

Models Utilizing Means and Standard Deviations of Outcomes[52]

In the general case, the maximization of expected utilities requires knowledge not only of the utility functions, but also of the complete probability distribution of outcomes. Decision criteria which require less knowledge have long had practical appeal. Several theoretical bases have been found for criteria that employ only the mean and standard deviation of outcomes.[53] Cramer[54] derived such a criterion in connection with insurance companies from the idea that the probability that income would fall below a certain level be minimized. A similar idea was applied by Roy.[55] He writes the objective function $W = f\left(\frac{\bar{B} - D}{\sigma_B}\right)$, where $\bar{B}$ is expected gain and D is the disaster level of outcome, the occurrence of which is to be minimized. The specification of the disaster level, which is a critical parameter for the ranking criterion, is a problem which appears to be of the same order of difficulty as specifying the utility functions. And the use of the standard deviation to measure the probability of failure is appropriate only if the probability distribution is normal, if it is of some other form that can be fully characterized by mean and standard deviation, or if all the alternatives have the same form of probability distribution.

[52] This section has benefited from my reading Don Farrar's, "The Investment Decision under Uncertainty," Harvard Water Resource Seminar Paper, September 1958, which discusses the models by Roy and Thomas, as well as an interesting model by Steindl.

[53] The indifference curves which correspond to this criterion are a series of parallel lines on a plane which has mean on one axis, standard deviation on the other.

[54] H. Cramer, "On the Mathematical Theory of Risk," Forsakringsaktiebolaget Skandias Festskrift, Stockholm, 1930, pp. 7–84, cited in Arrow, "Alternative Theories . . . ," *op.cit.*, p. 423.

[55] A. D. Roy, "Safety First and the Holding of Assets," *Econometrica*, July 1952, pp. 431–49.

A model of the same general type has been advanced by Thomas[56] for ranking water-resource projects. He argues that a project should have a positive pay-off over its life at some prespecified probability level. He suggests that an insurance fund be set up of such size that it is capable of making up the losses in any specific year, and that it have a positive balance at the end of the undertaking. It turns out that if the outcomes are normally distributed, the size of this insurance fund depends on the mean and standard deviation of outcomes and the specified probability level that the fund be adequate. With the fund considered part of costs, the objective function takes the form $W = \bar{B} + a\sigma$, where a depends on the probability level at which the success of the insurance fund is to be guaranteed. Where Roy minimizes the probability of disaster, Thomas maximizes net gain, including an insurance charge against failure. As in the previous model, the determination of this probability level is a problem akin to the specification of utility functions.[57]

The models of this type have a greater ring of concreteness than the maximization of expected utility. Yet except for those rare cases where there is an institutional basis for specifying the disaster level of outcome or the even rarer cases where the acceptable probability level of failure can be empirically determined, they are arbitrary adjustments to risk, perhaps not as crude as the "crude adjustments" discussed earlier, yet considerably removed from modern optimizing criteria. They stand in the same relation to expected utility maximization as the classical theory of statistical inference stands to modern decision theory[58].

Further Comment on Utility Maximization

If we insist on the specification of utility functions, we must be prepared to give some empirical implementation to this idea. While this is not an easy problem, it is an unavoidable one, since even in the classical models the specification of disaster levels of income or of confidence levels of probability presumably would need to be derived

[56] H. A. Thomas, Jr., "A Method for Accounting for Benefit and Cost Uncertainties in Water Resource Project Design," Harvard Water Resources Seminar Paper. For an empirical application see J. S. King, "A Method for Consideration of Risk and Uncertainty in Water Resource Project Evaluation," Harvard Water Resource Seminar Paper.

[57] L. Telser has employed the same model as Thomas' in an analysis of hedging behavior. See his "Safety First and Hedging," *Review of Economic Studies*, 1955–56, pp. 1–17.

[58] See R. D. Luce and H. Raiffa, *Games and Decisions*, Wiley, 1957, pp. 318–24.

from implicit estimates of utility functions. Certain elements of these functions can be derived from objective data. For example, in irrigation and hydroelectric power the money losses of shortage of stream flow are a crucial variable, and similarly in navigation and low flow control. Thus the derivation of these loss functions is a necessary and empirically feasible first step toward deriving optimal criteria. In addition, the shapes of individual utility of income functions must be specified, admittedly a heroic task. But reasonable assumptions about their general shape can be made, which, if not derived from experimental data, may be in the nature of value judgments (see Section 2 above). To fail to specify them is not to solve the problem, but simply to leave its resolution to the random process of picking the function which is implicit in the selection of values of other, more "pragmatic" parameters.

D. THE CASE OF UNCERTAINTY[59]

By uncertainty we mean the case where information about the probability distributions of outcomes is incomplete, that is, that their parameters are not known precisely. Strictly speaking, this includes all empirically derived probability distributions; but in this section we are concerned with that range of cases in which it is not a reasonable assumption for policy purposes to treat the distributions as known.

Even for individual action there are few settled conclusions about what constitutes rationality under conditions of uncertainty. In a few cases, considerable theoretical progress has been made. The most important of these are games of strategy involving at least two players. Here the Von Neumann-Morgenstern theory and subsequent developments (including bargaining theories) provide principles of decision. Public expenditures for national defense clearly require this type of analysis, as may expenditures which are primarily part of domestic political games.

Leaving genuine games aside, there still remain stubborn problems where decisions must be made with imperfect information. For example, no empirical probabilistic description can be given to the problem of price projection or to the forecasting of floods so extreme that the historical record contains only one or even no instance. Several principles have been advanced that might be applied. From the theory of games, the minimax principle has been drawn, which

[59] A critical survey of the relevant literature can be found in Luce and Raiffa, *ibid.*

would require that course of action which would minimize the losses which would occur if the worst possible circumstance arose. Where no rational opponent is involved, this is too conservative a principle. In flood control or irrigation design, for example, it would make decisions depend exclusively on the worst possible event that human imagination could visualize for the project, regardless of how remote the possibility. A somewhat different principle is the "minimax regret" criterion,[60] suggested by Savage. In choosing between two alternatives, it minimizes the difference between what would happen in the better outcome and in the worse outcome. Choices among more alternatives would be made by a series of comparisons among pairs. It is doubtful that this concept of regret is a desirable principle of action in the areas with which we are concerned; this criterion also suffers from excessive influence of very unlikely extreme values and has the additional fault that the optimum choice can be altered by the introduction of irrelevant alternatives. Hurwicz has suggested a third criterion, a weighted average of the best and the worst possible outcomes, with the weights left to the inherent pessimism or optimism of the decision-maker. This criterion, while it has the advantage of introducing both good and bad possible outcomes into the decision, still suffers from excessive influence of the extreme values.[61]

I can only echo Arrow's conclusion "that we do not really have a universally valid criterion for rational behavior under uncertainty. Probably the best thing that can be said is that different criteria are valid under different circumstances." For the range of decisions that is our concern, expenditure decisions that are not strictly strategies in a game, there is one important property that the decision criterion ought to reflect: while the probability distributions are not known, there is some experience, some knowledge which ought to aid in the decision. Typically the uncertainties are cases of difficult forecasting of prices, of rare floods, of industrial location patterns in the case of transportation facilities, and of other events the underlying mechanism of which is not fully understood.

The use of a priori probabilities is one possibility, in which the

[60] Besides Luce and Raiffa, there are several other interesting surveys of these criteria. See K. J. Arrow, "Utilities, Attitudes, Choices: A Review Note," *Econometrica*, January 1958, pp. 1–23; Roy Radner and Jacob Marschak, "Note on Some Proposed Decision Criteria," *Decision Processes* (1954); R. M. Thrall *et al.*, eds., pp. 61–8; and John Milnor, "Games against Nature," *ibid.*, pp. 49–59.

[61] Luce and Raiffa criticize this criterion on other grounds.

qualified "expert" attaches subjective probabilities on the basis of the evidence and of his intuition. Given these probabilities, including the joint probability distributions of the various dimensions of output, the problem can then be treated like a problem in risk. Since the subjective probability mechanism is no more than a method of utilizing a combination of evidence and intuition, there is a question as to whether it is the best method. This is a matter of personal taste; some may find the mechanism natural to their thinking processes; others may find it an encumbrance.

Another possibility is to use some sort of contingency approach, in which the major hazards are identified to which the undertaking is subject and which have some minimum a priori probability of occurrence. Strategies can then be devised which will reduce the maximum possible loss caused by each contingency to some bearable level. Within these constraints, some maximization of expected values might then be carried out. Alternatively, striking a completely defensive posture, the probabilities of certain loss levels might be minimized, with the weights given to the prevention of different contingencies determined from some preference function. The similarity of contingency planning to the models of Thomas, Roy and others discussed above will be seen.[62] It also has some strong similarities to Simon's "satisficing" analysis.[63]

I am sure enough has been said to indicate that this particular problem is far from a solution. In the meantime, judgment methods must be used, whether verbal or formal, with the identification of the major contingencies and some provision being made against them constituting a minimum program for the design of reasonable decision procedures in the face of uncertainty.

6. *A Survey of Some Recent Models*

A. INTRODUCTION

Having set up a taxonomy of the problem of public expenditure criteria, I shall now use it to classify the various models that have been advanced in recent years. No attempt will be made to present

[62] For discussion of contingency planning, see H. Kahn and I. Mann, *Techniques of System Analysis*, The RAND Corporation, RM-1829-1, ASTIA Doc. No. AD133012, June 1957, pp. 85–113. For a similar view, applied to research and development decisions, and stressing the resultant need for preserving flexibility, see Burton Klein and William Meckling, "Application of Operations Research to Development Decisions," *Operation Research*, May–June 1958, pp. 352–63.

[63] See Section 1, above.

each model in full detail; in particular, the algorithms that have been advanced for the numerical solution of some of them will not be given. But the taxonomy should permit us to give the essence of each model, and to show the interrelations between them. The models designed for projects of water resource development are presented first, followed by a model for transportation, and concluding with more general models for economic development planning.

B. U.S. GOVERNMENT PRACTICE IN EVALUATION OF WATER RESOURCE PROJECTS

The federal government evaluates water resource projects by means of benefit-cost analysis. There is no single model which is employed by all agencies;[64] one of the problems in evaluation practice has been the lack of uniform methods among agencies. But there are certain characteristics from which an "ideal" model of federal practice can be derived.[65]

The objective function of this model has two kinds of benefits; "direct" benefits which are largely net additions to individual incomes, and "indirect" benefits which are miscellaneous repercussion effects. At least in principle, the difference between benefits and costs is to be maximized in determining the scale of projects, while project ranking is to be based on the ratio of total benefits to total cost, the crude benefit-cost ratio. Except for the inconsistency between pursuing the scale of individual projects to a point where marginal benefit equals marginal cost while the benefit of marginal projects has to exceed costs at a rate equal to some benefit-cost ratio greater than one, this procedure corresponds to a model in which benefits minus costs are maximized subject to a constraint on cost. A dollar of benefit is given the same weight, no matter "to whom it may accrue," suggesting an objective function of the form (1.3) in Section 1.

The constraint is not applied to the same concept of cost by all agencies. *Proposed Practices* . . . suggests project costs as the proper denominator of the ratio, and hence implicitly as the proper constraint. These are all the costs incurred on the project itself and are contrasted with associated costs, the costs of associated enterprises.

[64] A detailed account of actual practices can be found in my *Water Resource Development.*

[65] The classical statement of the general approach of the government can be found in Federal Interagency River Basin Committee, Subcommittee on Benefits and Costs, *Proposed Practices of Economic Analysis of River Basin Projects,* May 1950.

The Corps of Engineers follows this concept. The Bureau of Reclamation uses federal cost, the cost borne by the federal government. The constraint is applied to costs occurring in all periods, present and future, discounted by the interest rate.

The interest rate needed to discount benefits and costs to derive present values[66] is specified independently and related to government borrowing costs. Since the funds for projects are rarely borrowed but rather raised by taxation, the government borrowing rate is irrelevant. Being rather low, it may be a reflection of social time preference however.

Repercussion effects are measured in the form of "indirect" benefits. These include profits created in processing and in sales to the project, in increased production and wage payments made possible by eliminating floods, and in several other ways. Most of these "indirect" benefits cannot be derived from any reasonable objective function unless a particularly heavy weight is attached to the income—and particularly to the profits—earned in the immediate proximity of the projects, and no weight at all is attached to the offsetting losses elsewhere in the economy.

There is relatively little adjustment for risk and uncertainty. Benefits that are particularly uncertain, usually only expected to begin to accrue in the future, are to be discounted at a higher rate of interest, injecting a slight risk premium. Also, there is general use of engineering contingency allowances in cost estimation. In flood control, some provision is made to stress control of rare "disaster" floods.

Without seeking to subject the federal techniques to systematic critique, five points should be made: (1) The objective function is consistent with the traditional individualist welfare economics and can fairly be interpreted as representing the national interest. (2) In the ranking procedure, there is recognition of the existence of a budget constraint, though the resultant implications for project design are not followed. Some ambiguity remains about the concept of cost to which the constraint is applied. (3) The opportunity cost of budget money is not brought into the analysis; no test is performed to assure that benefits on marginal outlays exceed these opportunity costs. (4) Measurement of repercussion effects largely seeks to measure irrelevant effects. Finally, (5) the model presented is an

[66] In actual practice annual equivalents are employed. These correspond to present value concepts, expressed as an annual average figure.

"ideal," with actual practice rarely utilizing the rankings by the benefit-cost ratio; the analysis is primarily used as a test by which projects with ratios less than 1.0 are rejected, with the scores above 1.0 having only a minor influence in project selection. Also, marginal principles are frequently not followed in project design, particularly in choice of scale.

C. A MODEL FOR BENEFIT-COST ANALYSIS

In my book, *Water Resource Development, the Economics of Project Evaluation*, I present a decision model which was designed to be appropriate to the budgeting problem of the federal water resource programs. This model maximizes the increase in real national income, assuming equal marginal utilities for individuals, subject to a constraint on federal cost. This constraint applies to both capital and operating and maintenance costs; in particular, it applies to the present value of these costs, measured with the interest rate of the analysis. This constraint was chosen over several others. A constraint only on capital was rejected because operating and maintenance costs represent a serious drain on the federal budget in several fields, particularly flood control and navigation; and in the others, e.g., irrigation, these costs are borne by local interests, and hence would automatically fall outside the constraint as far as appropriate. It was also chosen over separate constraints applicable to the funds used in different periods because there was no evidence to suggest drastic changes in the future pattern of availability of funds, and so a perennial constraint equal to present conditions was selected; this assumes that project opportunities are generated at the same rate as funds become available. Finally, while there is some exploration of constraints that include the funds generated by the reimbursable portions of a project, I reject this constraint, because, in actual federal practice, revenues of projects go into general treasury funds, and not into further expenditures for water resource programs.

The interest rate is to be chosen as an expression of social time preference. To bring the opportunity cost of budget money into the analysis, the marginal benefit-cost ratios which correspond to the opportunity costs of budget money raised by taxation are given. In the event that the benefit-cost ratio of marginal projects that can be undertaken within the budget constraint falls below this opportunity cost rate, the latter rate serves as a cutoff, and not all of the available budget money is to be spent.

Repercussion effects are limited in the model, because it is assumed to be applied in full employment conditions and in the mature market economy of the United States, where prices are on the whole, adequate indicators of value. Where there are genuine external economies, largely of a physical nature, these should be measured, of course. Also, in the case of decreasing cost transportation industries, marginal costs rather than actual freight rates measure value.

The treatment of risk and uncertainty is confined to "crude adjustments," particularly risk premiums in the interest rate. In connection with flood control, some recognition is taken of the effect of diminishing marginal utility of income, justifying departure from minimizing the expected total cost of floods in the direction of paying more attention to "disaster-type" floods, but no specific criteria are advanced.

D. MULTIPLE PURPOSE RIVER DEVELOPMENT

Some closely related models were used in a volume of empirical studies.[67] Four investigations were undertaken; (1) the opportunity cost of tax-raised budget money was estimated; (2) an economic analysis of alternative plans of development of the Hell's Canyon project was prepared, using the social cost of capital, as measured by opportunity cost; (3) the extent to which private development is likely to produce the potential nonmarketable outputs of multi-purpose projects was investigated through a case study of the Coosa River, Alabama, and (4), the income distribution effects of a project in the Pacific Northwest were measured under the alternative conditions of private and local, and federal development, with both costs and benefits allocated to regions and income classes. The fourth of these studies seeks to implement an objective function of the form (1.5) of Section 1, identifying distribution of gains and costs by region and of federal costs by income class. No effort is made to rank the alternatives, a task left to the political process; but the necessary data for judgment are presented. It turns out that federal development redistributes income toward the region, compared with other places. The distribution of federal costs by income class depends on the assumed tax changes, but under some likely assumptions falls heavily on the lower income groups. The third study determines the flood

[67] J. V. Krutilla and O. Eckstein, *Multiple Purpose River Development: Studies in Applied Economic Analysis*, Johns Hopkins, 1958. Also see G. L. Reuber and R. J. Wonnacott, "The Cost of Social Capital in Canada" (to be published), where the opportunity cost of funds raised by borrowing is estimated.

control and other nonmarketable benefits that could be produced by the project of the case study, compares them with the incremental costs, and then analyzes the private plan of development. It turns out that the private plan provides virtually none of the nonmarketable benefits.

The remaining two studies are interrelated. The opportunity cost of tax-raised funds is measured from what Musgrave calls the differential incidence of taxation; if the level of expenditures is changed, what tax changes would accompany it, assuming that stabilization policy requires some offset and that fiscal policy is the device chosen? Specific assumptions are made about these tax changes, based on judgment, the tax burden is traced to its ultimate incidence, and insofar as it falls on investment, the foregone rates of return are estimated. Foregone consumption is valued at the time preferences of the affected consumers, as revealed by their saving-borrowing behavior. The resultant average cost of marginal tax funds turns out to be on the order of 6 per cent. This rate is then applied to the Hell's Canyon case, and using it as a test, it turns out that a two-dam plan that costs less than the actual private three-dam plan of development but produces more output is the best choice. The incremental investment required for the one large dam, the public proposal, yields less than 4.5 per cent, assuming fully integrated operation. With opportunity costs at 6 per cent, this increment is rejected.

This model uses rate-of-return comparisons, though they are applied through a benefit-cost terminology. A strict efficiency point of view is taken, in which the interest rate of the analysis is based on individual time preference of the people who are taxed. Had a social time preference been used, perhaps including a lower interest rate, the results would have been the same. The opportunity cost would have had to be revalued into a present-worth concept at the preferred interest rate, and compared to the incremental benefit-cost ratios of the alternative plans. The empirical conclusions would have been identical, since the benefit-cost ratio of the foregone opportunities would have been on the order of 2.0, assuming an interest rate of 3 per cent, while the incremental investment of the large dam plan has a ratio of only 1.5; the two-dam plan would have continued preferable to the private plan, since it has lower costs and greater benefits.

This analysis assumes no budget constraints. This is justified because Hell's Canyon was not a question of choosing the best public projects, but rather to compare competing private and public plans. A victory for the public plan would have meant that the

additional budget money would have been voted; in fact, such a victory would have resulted in a general expansion of public power programs, since it was a symbolic showdown between public and private power advocates. To have used a budget constraint might have condemned the public plan on the grounds that it prevented other good public undertakings, a line of reasoning which was contradictory to the institutional reality of the situation.

There was no concern with risk and uncertainty, and repercussion effects were limited to physical downstream power benefits. Much of the difference in benefits among the plans proved to be in these repercussions, which are nonmarketable for a private developer and which therefore are not considered in private decisions.

E. THE STUDY BY McKEAN

A recent book by McKean devotes a great deal of attention to the theory of expenditure criteria.[68] I cannot summarize the entire discussion, much of which is devoted to practical problems of implementation, to saving the innocent from fallacy, and to setting out the fundamental principles of selecting criteria—a discussion which to some extent parallels this paper, but from a rather different conceptual point of view. I present only the bare outline of McKean's argument with regard to the criteria he considers appropriate. McKean stresses the many objectives of policy, and the limited weight that is to be attached to criteria that reflect only economic efficiency. In the economics, he seeks to maximize the expected gain of real income, though he also stresses the need for consideration of intangibles and of adjusting to uncertainty.

McKean takes the maximization of the difference of the present values of benefits and costs as the ultimate objective (p. 76). The interest rate he would use to compute present values is the marginal internal rate of return. As McKean points out, this is tantamount to a strict rate of return criterion, though there is still an open question about the interest rate to be used in the design of supramarginal projects. McKean makes clear the assumptions that are required for this to be the correct criterion. Either of two sets of assumptions suffices: (1) funds are available without constraint at an interest rate equal to the marginal rate of return—an assumption which makes many criteria, including benefit-cost ratios, come to the same result.

[68] Roland N. McKean, *Efficiency in Government Through Systems Analysis, With Emphasis on Water Resource Development*, A RAND Corporation Research Study, Wiley, 1958, esp. pp. 25–150.

(2) There is a constraint on investment funds, public and private, and the net returns can be reinvested at the marginal rate of return when they accrue (p. 85). McKean makes the necessity of the reinvestment assumption abundantly clear. He is concerned with the sensitivity of the results to the rate of return at which reinvestment occurs, and in view of the necessary arbitrariness on this matter, he proposes that supplementary data be submitted as part of the analysis which give some idea about the time profile of benefits and costs. By giving this profile, the need for any interest rate is eliminated, and the decision-maker, whether Congress or President, is forced to apply his own time preference. Recognizing the need for simple criteria, however, McKean ultimately does propose the internal rate of return as the best simple decision-rule.

McKean rejects benefit-cost ratios (pp. 113–8). He correctly seizes on the critical issue: What are the financial constraints which limit the program? McKean argues that the constraint only applies in the immediate future when the investment costs are incurred, that operating and maintenance costs are financed out of revenues generated by benefits—including the revenues recaptured through taxation. He also feels no need to distinguish between federal costs and other costs. Finally, he prefers to treat the benefits as being reinvested. In my own work, I have preferred other assumptions on these matters. First, I believe budget money will remain scarce for a long, long time, and operating costs a decade from now will prove as much a drain on a scarce financial resource as current investment outlays. Second, since it is the preparation of a federal program which is at stake, I prefer to treat only federal cost as the constrained financial resource. Third, I assume that there is no reinvestment, partly because the benefits of projects and the institutional arrangements in this particular field are such that there is very little direct revenue generated, and what there is does not return to the water resource field; as for benefits recaptured through taxation, in fields such as flood control and irrigation virtually no taxes are created, while in power and navigation it is not clear that the resultant taxes are more than the taxes that would have been paid by the alternative private investments that might have occurred.[69]

[69] This controversy repeats some of the issues of the Lutz-Hildreth exchange of the 1940's. Lutz rejected the internal rate of return in favor of a strict present-value concept, though he did not select the constraint issue as the critical one. Hildreth, in reply, used an illustration which had the reinvestment property which validates the internal rate of return. See F. A. Lutz, "The Criterion of Maximum Profits in the Theory of Investment,"

On analytical grounds, I believe there are no contradictions between the study of McKean and my own. Different assumptions are made, but these are matters on which reasonable men can disagree.

F. THE STEINER PREEMPTION MODEL

Steiner has extended models of this general type in an important way.[70] He employs the same general objective function as the models discussed above, maximizing the difference between present value of benefits and costs. He stresses the need for specifying an interest rate, not only to compute present values of benefits and costs of projects, but also to compute present values of opportunity costs. There is no treatment of risk and uncertainty. The novelty of his approach lies in a combination of constraints and of sectoral analysis which brings out some interesting features of public development in a predominantly private economy.

Steiner defines four sectors of the economy: (1) the public sector the budget of which is being allocated; (2) the private sector which would contain private alternative developments of the particular public projects being considered; (3) the broader public sector in which funds left over from the particular budget would be spent; and (4) the general private sector containing marginal opportunities into which private funds displaced by public projects are pushed.

The total outlay for projects in sector (1) is limited by a budget constraint. This outlay has certain direct benefits in sector (1) of course, but in addition, it leads to repercussions in the other sectors. Sector (3), the general public sector, may receive some funds from the budget of sector (1). This comes about in two ways: first, some funds may be diverted because the marginal returns in sector (1) fall below the opportunities in sector (3). Thus the introduction of public sector (3) assures that marginal projects yield benefits at a rate equal to the opportunities elsewhere in the public sector. Second, funds spill over into (3) because Steiner employs discrete projects and a fixed budget, and so a small amount of money is likely to be left over because the project costs do not exactly equal the constraint.

The other repercussion effect which emerges is the change in

[70] Peter O. Steiner, "Choosing Among Alternative Public Investments," *American Economic Review*, December 1959.

Quarterly Journal of Economics, November 1945, pp. 56–77 and C. G. Hildreth, "Note on Maximization Criteria," *ibid.*, November 1946, pp. 156–64. Also see the later and much extended discussion which resolves some of the issues in F. and V. Lutz, *The Theory of Investment of the Firm*, 1951, pp. 16–48.

benefits earned on private investments because an investment opportunity has been preempted by the government. This forces private funds from the preempted opportunity into a marginal investment, or in the terminology of Steiner, from sector (2) to sector (4). This creates a loss in the private economy.

Steiner also explores the case where there is no budget constraint, the case where funds are drawn from the private economy and where opportunity costs play a key role. He brings the preemption problem into this case as well.

To summarize his model, Steiner writes a general equation

$$y_{ij} = (G_{ij} - a_1 k_{ij}) - (G_j - a_2 1_j) - a_3 m_{ij},$$

where y_{ij} is the net gain from the ij^{th} project, G_{ij} is the present value of benefits minus costs of the project, a_1 is the opportunity cost in the general public sector (2), k_{ij} is the project's drain on the limited public budget, G_j is the present value of the preempted private opportunity, a_2 is the opportunity cost in marginal investments in the private sector (4), 1_j is the capital cost of the pre-empted private project, a_3 is the opportunity cost of funds transferred from the private sector by taxation, borrowing, inflation, or whatever method is actually employed, and m_{ij} is the amount of such funds actually transferred for project ij. This equation can assimilate combinations of budget constraints and transfers of funds from private to public sectors, can assure full recognition of opportunity costs elsewhere in the public and private sectors as far as this proves appropriate, and can reflect the losses caused by preemption of private opportunities.

The empirical magnitudes necessary to implement the model, other than the usual benefit and cost data for each project, include the three constants a_1, a_2, and a_3 (the three opportunity costs) and a rate of interest. Steiner does not advocate any particular interest rate, nor does he propose any specific method of measurement of the opportunity costs. As an empirical matter, in the general public sector, where many outlays do not produce outputs that can be measured with prices, it is extremely difficult to place a value on alternatives which would be comparable to the values attached to the projects being analyzed. The private opportunity cost of marginal investments could presumably be valued; in fact, in a market economy, money costs should be such a measure and no explicit treatment needed. The opportunity cost of funds transferred from the

private sector to augment the public budget is measured by computations of the sort discussed above in connection with the Hell's Canyon study, or by similar computations applied to funds raised by public borrowing, or perhaps even by inflation. Thus Steiner's emphasis on the opportunity costs in the general public sector, sector (3), is likely to remain a counsel of perfection, but the rest of the analysis could probably be implemented empirically.

G. TINBERGEN'S TRANSPORTATION MODEL

Tinbergen has devised a model designed to measure the change in national income due to projects which improve the transportation system of a country.[71] This model consists of a set of geographical points in which production and consumption are carried on. For each product, supply and demand equations are determined, as well as the transportation costs for each product among all points. Each supply function contains the price of the product and of the other products in the geographical point; the demand functions contain the product's delivered price, and hence reflect transportation costs. Given these functions, it is possible to determine what will be produced in each place, and hence what its total production and income will be.

A transportation project will change some of the transportation costs in the model. The equations can be solved again assuming the new, lower transportation costs, and the change in total production and income can be seen from the difference between the two solutions.

This model is a technique for estimating benefits of transportation projects. It allows for the repercussions on production caused by broadening the markets in which the output of a place can compete. This increase in production and of income leads to further increases in demand and production. The resultant estimate of the impact on national income is greater than the estimate produced by conventional benefit-cost analysis, where the impact on national income is limited to the savings in transportation cost. The extent of the difference depends particularly on the supply elasticities, high elasticities implying large increases in production.

To apply the model as an expenditure criterion, a symmetrical analysis must also be carried out for the cost side. Presumably,

[71] J. Tinbergen, "The Appraisal of Road Construction: Two Calculation Schemes," *Review of Economics and Statistics*, August 1957. Also see H. C. Bos and L. M. Koyck, "The Appraisal of Investments in Transportation Projects: a Practical Example" (to be published).

repercussion effects on production and income would also result from alternative uses of the resources. Some assumptions would also have to be made about interest rates and budget constraints.

H. CHENERY'S SMP MODEL

H. B. Chenery has advanced an expenditure model designed to aid in the planning of investment budgets for economic development.[72] The objective is to maximize the present value of benefits minus costs, i.e., to maximize the present value of the real national income. In the closed-economy model, a constraint is applied to capital funds, with the resultant criterion, the Social Marginal Product (SMP), consisting of incremental ratios of present values of benefits minus operating costs divided by the requisite increment of capital. This criterion can be applied to the design of projects, and to project selection, with individual projects treated as increments in the determination of a program. Thus the technique is similar to the use of incremental benefit-cost ratios, except that the denominator contains only capital costs.

The criterion requires an interest rate. Chenery avoided this issue by confining his criterion to projects within the same field and with very similar capital intensities, so that the rankings of projects would be unaffected.

Chenery also applied the model to an open economy where foreign exchange has a higher opportunity cost than the nominal exchange rate. The SMP in this case consists of two terms.

$$\text{SMP} = \frac{B - M}{K} + f\frac{E}{K},$$

where B is present value of benefits, M of operating costs, f is the premium on foreign exchange, E the total effect of the project on the balance of payments, and K is the capital cost. Chenery has a very sophisticated repercussion analysis to estimate the balance of payment effect, including direct foreign exchange needs of the project, import savings made possible, as well as the import demands generated by the increase in the national money income caused by the multiplier effects of the project. These models are applied to development planning in several countries.

[72] H. B. Chenery, "The Application of Investment Criteria," *Quarterly Journal of Economics*, February 1953, pp. 76–96. There is an earlier literature by N. S. Buchanan, A. E. Kahn, and J. J. Polak, which is discussed by Chenery. Kahn introduced the SMP criterion.

I. CHENERY'S PROGRAMING MODELS

More recently, Chenery, in collaboration with others, has used programing techniques to solve the same type of problem. The practical advantage of programing is the great potential of empirical implementation. While some simplifying assumptions must be made to make the problem fit the apparatus of linear (and nonlinear) programing, complete solutions of the investment allocation problem of rapidly changing economics are possible. The marginalist approach, based on Lagrangean multipliers, is fundamentally a partial equilibrium approach (though in principle it could of course be applied to centralized planning of an economy as well).[73] When applied to expenditure decisions, it usually requires, at the least, that prices be projected. In advanced economies, particularly where the programs being planned are a small part of the economy, such projections can be made and are likely to be more accurate than prices which emerge from a programing computation. But where an economy is being transformed by rapid development, the supply and demand relations are so strongly modified by the development program itself that prices cannot be assumed. Even prices for planning must emerge from the planning computations; the programing technique produces such prices, in addition to solving for the over-all quantities.

Without seeking to present the results of the programing approach, the key characteristics of these models will be presented, particularly the assumptions made about objective functions, constraints, interest rates and the other matters with which we have dealt above.

In a study of development planning for Southern Italy, Chenery and Kretschmer[74] employed the following model: the economy is divided into 14 sectors, each of which is an industry aggregate. The sectors are divided into subsectors which have the property that they have the same input-output structure except for differences in capital inputs. A set of targets is specified, a list of goods, which is derived from demand projections based on income elasticities. The objective of the program is to meet these targets at a minimum total investment, with the total availability of labor and of foreign exchange acting as constraints. The production relations of the economy consist of two

[73] At a high level of abstraction, linear programing and the quadratic programing problems used by Chenery are logically equivalent to a Lagrangean problem, following the Kuhn-Tucker Theorem. See Chs. 1, 3, 4 and 5 of *Studies on Linear and Nonlinear Programing*, K. J. Arrow, L. Hurwicz and H. Uzawa, eds.

[74] H. B. Chenery and K. S. Kretschmer, "Resource Allocation for Economic Development," *Econometrica*, October 1956, pp. 365–99.

parts: first there is the input-output matrix of the 14 sectors (applied also to the subsectors). This matrix, together with the capital coefficients, defines one method of production for each subsector. Purchase from abroad, at a given import price, is an alternative method. The good of each subsector also has an export demand curve, relating the price the good can command abroad to the amount being sold. This foreign demand curve, assumed to be a declining straight line, introduces a nonlinearity into the model and makes it a case of quadratic programming.[75] When solved, the model reveals what demands should be met by production in domestic subsectors as well as their total outputs, what and how much should be imported, and how much of various goods should be exported. It also reveals the total amount of investment that is required and in what subsectors it has to be placed. In the event more capital is available than is needed, the targets can be raised, of course.[76]

Models of this type clearly have an enormous potential for expenditure analysis in many areas. In water resource planning, for example, the most efficient program of meeting specified needs could be derived. Similarly in planning regional development, the most economic means of raising, say, the average income of substandard regions could be approximated.

J. REINVESTMENT MODELS

Galenson and Leibenstein[77] proposed that several sets of repercussion effects which had not been considered previously in formal analysis ought to be given an important place in decision models. They stress three effects: first, education of the labor force on the job is considered a benefit of some projects; second, if per capita growth of income is in the objective function, differential effects of projects on population growth should be included in the criteria.

[75] In a subsequent paper, this is generalized to declining demand curves both at home and abroad. See H. B. Chenery and H. Uzawa, "Nonlinear Programing in Economic Development," in *Studies in Linear and Nonlinear Programing*, K. J. Arrow, L. Hurwicz, and H. Uzawa, eds., 1958, Ch. 15.

[76] In a paper to be published in the *Essays in Honor of E. S. Mason*, Chenery applies a similar model to illustrate several problems in development planning. He shows, with realistic empirical magnitudes, how much is gained (1) by using cost figures that reflect real costs rather than money costs, (2) by using a changing price structure suggested by the programing solution rather than constant prices, and (3) by including urbanization costs in the analysis. He also shows (4) how programing can be used to measure the value of generating reinvestible funds in a dynamic (three-period) program.

[77] W. Galenson and H. Leibenstein, "Investment Criteria, Productivity and Economic Development," *Quarterly Journal of Economics*, August 1955, pp. 343–70.

Finally, if a government finds it impossible to achieve an optimal level of investment, the capability of projects to generate further capital out of benefits should be considered, and a marginal reinvestment coefficient is advanced as a measure. All three of these repercussion effects, it is argued, would favor industrial projects in urban locations as opposed to agricultural or handicraft investments in the countryside. Galenson and Leibenstein do not propose a formal criterion;[78] they make their points by illustrative example.

In a subsequent model, I sought to incorporate the reinvestment factor in a formal decision model. The present value of benefits minus costs, or real national income, is maximized subject to a capital constraint. Each alternative has a reinvestment coefficient which states what fraction of its benefits is reinvested, either through private saving or through taxation. The resultant criterion has two components: an efficiency term indicating the present value of benefit minus operating cost per marginal dollar of investment, plus a term which places a premium on that portion of the output which is to be reinvested. This premium has to be derived from the productivity of the reinvestible capital. Because of the long perspective over time, the resultant criterion is very sensitive to the choice of interest rate, and it was in this connection that the analysis of planner's time preference in Section 3 above was worked out.

K. THE MODELS OF A. K. SEN[79]

A. K. Sen has advanced a series of theoretical models designed to illustrate the problem of development planning in an underdeveloped country. These models are not meant to be used in practical planning, but to provide the theoretical underpinning for rules-of-thumb that are empirically feasible.

Sen is particularly interested in exploring the right degree of capital intensity for development, particularly when viewed in relation to the level of reinvestment that might be generated and to balance-of-payments effects. He sets up a simple sectoral model for an underdeveloped country, and by means of it evaluates the alternative strategies of development.

There are two sectors, a backward sector containing lots of

[78] Subsequent criticism interprets the reinvestment coefficient as a decision criterion, and shows it to be wrong or incomplete. But I think this interprets their position too broadly.

[79] A. K. Sen, "Some Notes on the Choice of Capital-Intensity," *Quarterly Journal of Economics*, November 1957, pp. 561–84.

unemployment, which can supply labor in any amount without loss of output, and an advanced sector which contains two departments, one producing capital goods, the other "corn." Two techniques can be employed to add to the output of "corn," one requiring relatively little capital, with labor having relatively low productivity, the other being more capital intensive but having a higher productivity. Following Ricardo, all of wages are consumed, all of profits constitute a surplus and are reinvested. In order to maximize the rate of growth, the rate of reinvestment per dollar of original investment is to be maximized, and this requires that the technique be chosen which produces the greatest surplus. With labor productivity greater under the capital-intensive technique, the rate of surplus per unit of output will also be greater. But there will be less output per unit of investment. The empirical question, which can only be answered by getting magnitudes for the parameters of the model, then becomes this: is the extra surplus per worker made possible by the more capital-intensive technique sufficiently great to offset the loss of total surplus caused by the smaller output which results from sinking the capital into intensive uses?

In a second model, Sen adds foreign trade to this scheme. He assumes that the capital-intensive technique requires imports of foreign machinery, which can be purchased by means of the export of some of the corn being produced. The rate of surplus of corn still needs to be maximized, but in addition to the corn going into wages, the corn absorbed by exports must be subtracted from the total to derive the reinvestible surplus.

Maximization of the rate of growth of output is an odd objective function, and in realistic cases with alternative time profiles, it is ambiguous. However, in Sen's model, if the parameters are assumed to remain unchanged, the growth rate remains constant unless there is a switch in technique. And so, assuming the target date is chosen far enough in the future, the higher growth rate will always dominate short-run losses of output. Maximization of the rate of growth is considered by Sen to be a polar case in which only the economic situation at a remote point in time is considered.[80] Sen views simple turnover criteria, which only takes the first period into account, as the other polar case.

To bring time discount back into the analysis, Sen employs the concept of a "recovery" period. If it is true that the capital-intensive

[80] He identifies Galenson-Leibenstein with this particular case.

technique produces less output in the early years but more later on, the only case in which there is a real problem of choice, then there must be some number of years over which both techniques produce the same amount of output. It is up to the government to decide how many years of output it wishes to consider in its objective function, and by comparing the "recovery" period of the capital-intensive technique with the government's time horizon, a choice of technique can be made.

These models allow the analyst to bring certain important empirical features of underdeveloped countries into the analysis. Particularly where broad strategic choices are concerned, such as the concentration on urban industry or rural cottage industries, empirical evaluation of models of the type proposed by Sen may prove valuable. It is my feeling, however, that whatever can be done by means of these explicit sectoral models, which must simplify reality enormously in order to keep the mathematics from getting out of hand, can be done more easily and more completely by means of programing techniques.

In Sen's particular illustrative models there is a weakness, I think, in the choice of objective function. Maximizing the rate of growth will, among interesting choices, bury more detailed time preferences of the objective function; the decision-maker will not be applying sufficient judgment to the issue, and will essentially leave it to chance. Similarly, the "recovery" period, which is the same as the "pay-out" period of private investment criteria, is arbitrary, placing equal value on output at any time within the period, and a zero value on any output thereafter.

7. *Concluding Comments*

Since this paper is a commentary on the problem of expenditure criteria and models, little further remains to be said. I have tried to bring out the major issues on which the choice of economic criteria turn. A deliberately narrow economic point of view has been taken, not because noneconomic factors are unimportant, but rather because we ought to be clear about things about which we can be clear. I would also pass the judgment that there is no excessive preoccupation with the economic aspect in public expenditures decisions, and that improvement of the economics of government activities can be justified by higher criteria.

COMMENTS

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Professor Eckstein has given us the benefits of his thinking on a wide variety of topics covering the field of public expenditure decisions. His over-all conceptual organization of the subject I find unexceptionable, and about a great deal of his content all that I could do would be to record agreement. To avoid such a dull proceeding, I shall center my remarks about one general subject on which Eckstein does, I feel, stray somewhat from the true path—the question of what interest or discount rate to use in making decisions on adoption or rejection of public expenditure alternatives. This discussion will, therefore, concentrate on Eckstein's Section 3 ("Interest Rates"), though there are certain spillovers (to use our private jargon) to a number of other Sections—most especially to Section 5 ("The Treatment of Risk and Uncertainty").

The first topic I shall discuss under this heading is "The Interest Rate as a Measure of Value of Outputs at Different Points in Time." Eckstein's argument here may be summarized as follows, I believe. The acceptance of the market interest rate for public decisions involves the acceptance of consumers' saving-spending decisions. But these decisions based on personal time-preference rates have been criticized as representing "myopia" or intertemporal selfishness. Eckstein provides a theoretical foundation for planners' time-preference free of "myopia"—based on expected growth of income together with diminishing marginal utility of income. That is, given the growth rate and the schedule of diminishing marginal utility, the relative utility value of marginal units of income in different periods, and consequently the interest rate, can be inferred. If marginal utility of income declines 2 per cent for each per cent rise in income, and we expect a 4 per cent higher per capita income next year, a planners' interest rate of 8 per cent is implied (Table 1). Next, Eckstein shows (as did Rae and Fisher long before) that on the purely individual level, a certain amount of time-preference is "rational" because of the risk of dying, an eventuality known to impair enjoyment of deferred consumption. Using plausible numbers, Eckstein finds that both of these considerations under present world conditions indicate positive but low interest rates. His conclusion is that the planners'

decision as to what "social" rate of discount to use remains necessarily a value judgment.

This approach to the question of what discount rate to use is defective, I believe, in overemphasizing one element of the problem, time preference. Following Fisher's analysis, we know that the interest rate ruling in the market represents the interaction of the factors of *time preference* and *time productivity* of savings, given the initial distribution of consumable income as between individuals and also over time. Eckstein is of course aware of the influence of these other elements of the problem, but his analysis brings them in only through a side door—for example, postulating a rate of growth of income in Table 1 may be regarded as representing either an initial distribution of income over time or alternatively the product of a quantum of savings and the average productivity thereof. It is clear, however, that this is not the correct form of analysis: the rate of discount adopted for investment decisions will affect the rate of growth of income, which cannot therefore be taken as a datum in determining a "social" rate of discount.

The important practical implication of this theoretical consideration is that the choice of a rate of discount for a particular public project, or for the public sphere in general, is not quite as unconstrained a value judgment as Eckstein indicates. Since the interest rate represents a marginal balance of time preference and productivity, a 4 per cent market rate implies not merely a 4 per cent rate of marginal time preference but also a marginal productivity of investment equal to 4 per cent, setting aside for the moment market imperfections. In such a case the use of a 2 per cent rate in public investment decisions will be inefficient—public investments yielding just over 2 per cent will displace private investments yielding 4 per cent. Now, of course, value judgments can still enter; for example, there may be a political preference for "socialist" versus "capitalist" projects. But it is not true that a planner can correctly reject the market 4 per cent rate and use his own value-judgment 2 per cent rate instead unless he is prepared to accept a loss of economic efficiency.

I cannot forbear from making two additional comments, though they are not centrally relevant here. There is much confusion between time preference as a particular rate and time preference as a *schedule*, akin to the elementary confusion between the demand schedule and a particular quantity demanded. A positive interest rate need not

imply time preference "myopia" in the schedule sense: that is, the time-preference utility isoquant may be perfectly symmetrical when diagrammed on the axes of present income versus future income. On the other hand, the productive opportunity locus for current versus future incomes may be (and usually is) of such a shape as to indicate positive marginal time preference *at the optimal point.*

The second comment relates to Eckstein's "paradoxical" result that the lower the rate of growth, the lower should be the interest rate. (This is his planner's optimal time preference interest rate, based on diminishing marginal utility of income combined with a postulated rate of growth of income.) The paradox is that poor countries having low rates of growth should then use a low public discount rate, but in these countries there is pressure for early consumption (that is, high market interest rates). Eckstein explains the high interest rates there as being based on "pure time-preference" at low levels of income. A more satisfactory analysis of this phenomenon would separate different possible reasons for low growth of income in poor countries. First, the productivity of available investments may be low (Fisher's "hard-tack" illustration), in which case the real rate of interest will necessarily also be low, regardless of time preference. More frequently, perhaps, highly productive investments will be available and the market interest rate consequently high even in the absence of "pure" time preference (in the schedule sense), though this possibility need not be excluded. The low growth nevertheless observed of per capita income may be due largely to population expansion, or else may be low only in an absolute sense while still large relative to the initial level. Perhaps the most important explanation of all is that the observed high interest rates are probably not the real riskless rates we have been talking about, but are nominal monetary rates incorporating adjustment for the very high default and confiscation risks faced by potential investors in such countries as well as a very high inflation risk. I will turn again to this matter of risk and uncertainty later.

The next major topic I will discuss is Eckstein's attempt to reconcile his "social" discount rate with the opportunity cost of capital. His basic error, I believe, is in arbitrarily establishing the social rate independently of the time productivity of savings. If it is intertemporal selfishness that we are worried about, what we want to do is to increase current sacrifice for the benefit of the future. Once we do this, by a system of taxes and subsidies for example, the

interest yield on projects will be driven down to lower real values over the economy in general. No separate "social rate of discount" is necessary, and if one is used in the public but not the private sphere inefficiency will result.

This error vitiates, I believe, Eckstein's analysis of the opportunity cost of capital, which is based upon measuring opportunity cost and converting it to a present-value dimension by dividing through by the social rate of discount. Thus, if opportunity cost is 6 per cent, and the social rate is 3 per cent (for investments yielding perpetual streams), present value of $2 is foregone elsewhere for each dollar invested. This leads to Eckstein's prescription, for this case, that project cost and benefits should be discounted at 3 per cent, but that a benefit-cost ratio of 2 to 1 should also be required. The purpose of this peculiar device is to incorporate into the criterion the social time preference rate (3 per cent) while still precluding adoption of projects inferior in yield to alternatives foregone. The first of these aims is misguided, I have maintained. The second will not in general be achieved, since Eckstein's rule is biased in favor of alternatives with higher futurity of yield. For example, his rule would prefer (discounting at 3 per cent) an investment yielding 12 per cent after a lapse of two years (cash-flow sequence: $-1, 0, 1.12$) to one yielding 6 per cent in one year (cash-flow sequence: $-1, 1.06, \epsilon$—the terminal ϵ is an infinitesimal added to give the projects the same life). But if the marginal opportunity rate remains 6 per cent compounding of the quicker-yielding investment will dominate the slower one—whatever the rate of discount used in the comparison.

Eckstein at times implies that the per cent yield of a project cannot in general even be measured except by first postulating a discount rate—e.g., "his social rate of discount." Of course, if the marginal yield on alternative projects cannot be unambiguously determined, we cannot speak of the opportunity cost of capital as an interest rate appropriate for discounting public projects and Eckstein's concentration on time preference as the source of the discount rate becomes understandable.

While Eckstein seems to have been influenced by an article of mine[1] he cites in this connection, that article did not prove or assert inability to measure project yields independently of interest rates. While showing the limitations, as an investment criterion, of project

[1] "On the Theory of Optimal Investment Decision," *Journal of Political Economy*, XVI (August 1958), 329–52.

"rate of return" as familiarly defined—that discount rate which equates the present value of the cost-revenue stream to zero—the article went on to attempt a reconstruction of the important concept of yield of a project. The basic idea of the reconstruction is that project yield for any investment *can* be defined independently of the interest rate, but only as a vector or sequence of two-period yields. This conception has been improved and generalized in a paper by Martin Bailey.[2] In sum, we can continue to speak of opportunity yield foregone as a per cent rate, and the market rate of interest—setting aside market imperfections—as equating marginal time preference *and* marginal productivity.

A possible counterargument to my position on opportunity cost of capital could be based upon imperfection of the capital market—for example, that marginal time preference is not equal to marginal time productivity. In such a case, it might be possible to justify low-yield projects if the funds for them were diverted from low interest-rate sectors of the imperfect market. All that can be here said is that the possibility may be worth empirical study.

Eckstein refers in this paper to his empirical work with Krutilla which estimated the opportunity cost of capital as in the neighborhood of 5 to 6 per cent. His own criticism here of this work, that it fails to incorporate a social rate of discount, I do not accept. It is vulnerable to criticism on another ground, however. While I cannot go into the details here, I believe that the procedure used was faulty in crudely averaging nominal interest rates rather than expected yields. In considering alternative uses of funds secured via personal taxes, for example, a weighted average was made of the earnings of funds in such items as government bonds (3 per cent), stocks (6 per cent?), and reduction of consumer debts (12 per cent). Obviously, these rates incorporate varying risk allowances, and in general the over-all 5 to 6 per cent arrived at is an average of nominal market rates representing some undetermined average riskiness, and thus exceeds the expected yields. My own belief is that the riskless money interest rate for any given term can be estimated by the corresponding rate on United States government obligations, now about 4 per cent for long terms. Yields on such obligations, while (nearly) free of default risk, still represent some allowance for price-level or inflation risk. I think that presently $3\frac{1}{2}$ per cent would be a closer approximation

[2] "Formal Criteria for Investment Decisions," *Journal of Political Economy*, LXVII, October 1959, pp. 476–88.

of the real rate of interest—the rate that a riskless purchasing-power bond would yield. On the other hand, incorporating a risk adjustment comparable to that implicit in the market evaluations of private utilities can be shown, I believe, to lead to the prescription to use rates in the neighborhood of 9 to 10 per cent for government investments—the assumption being that these are of comparable riskiness with those of private utilities.

This discussion has now brought us to the next main topic—risk and uncertainty. While my previous comments on the discount rate have been designed to correct what I believe to be errors in Eckstein's analysis, here my remarks will be mainly of a clarifying nature. First, it is important to distinguish between two logically separate types of "adjustment" for risk. The first or "expected-value adjustment" would correct a nominal or quoted interest rate to allow for the probability of partial or complete default. For example, a nominal 12 per cent rate on consumer debt is not inconsistent with an expected yield to creditors and expected interest cost to debtors of 6 per cent or even 3 per cent. The second or "risk-aversion adjustment" starts from the expected value of a risky (high standard deviation) interest yield and adjusts it downward to allow for risk aversion (or upward for risk preference) as compared with a security with equal expected value but smaller standard deviation of yield. The first adjustment, then, says that a 4 per cent bond of Fly-By-Night, Incorporated really has an expected yield of but 3 per cent, after allowing for default probability. The second says that a particular individual will evaluate Fly-By-Night stock yielding 8 per cent on an expectational basis, but with a high dispersion of outcomes, as no better than the stock of Safe-and-Sure, Incorporated, yielding an expectation of 7 per cent with small dispersion of outcomes. An interest rate correction designed to allow for the notorious cost underestimates of government agencies would be an expected-value adjustment. Incidentally, as Eckstein points out, it is neither necessary nor always convenient to allow for risk through a high discount rate in planning projects; however, this is a familiar way in which the capital market reflects risk.

In the earlier part of Section 5, where Eckstein quotes an example using a specific probability of failure, he is clearly speaking of an expected-value adjustment. Later on, however, and in particular for the discussions of the Roy and Thomas criteria, it is risk aversion which is under discussion, the question being how high an expected

value is required to compensate for higher dispersion of outcomes.

This distinction clarifies the familiar argument about the advantages of "pooling" of risks—that government has a natural advantage in undertaking risky projects because it can pool the risks of a great many such together. Eckstein has some remarks on this subject, whose point I did not grasp. The contention about the advantage of pooling is correct insofar as the argument relates to private risk aversion, assuming that risk aversion rather than risk preference dominates in the private sphere (since the law of large numbers reduces the dispersion of the average outcome). It is not correct insofar as it relates to expected-value adjustment: projects will fail for governments as well as for private investors, and this must be allowed for. The 9 to 10 per cent figure mentioned above incorporated only an expected-value adjustment, and is just as applicable for pooled as for unpooled investment alternatives.

As a minor comment, I think that the main lesson to draw from Eckstein's flood-control example in this Section is that risk aversion follows from an assumption of diminishing marginal utility of income. Of course, the same point was made by Marshall in "proving" the irrationality of gambling.

In conclusion, let me say that Eckstein's abstract formulation of the problem of social decision in this field is admirable, despite my criticism of particular points, in its logical organization, its many original touches, and its attempt to boldly measure parameters that others have merely hypothesized about. I must admit to an uneasy feeling, however, that all our logical improvement of the theory of decision may be getting us no closer to stopping, for example, the federal reclamation scandal. Perhaps the fruitful topic for research in this field is "Imperfections of the Political Decision Process"; if we knew more about this, I would feel more confident of the usefulness of theories of optimal public expenditure criteria.

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Both Mr. Eckstein and Mr. Hirshleifer[1] present a very strong case in favor of the present value approach (or cost benefit analysis), rather than the internal rate of return approach (or profitability

[1] Hirshleifer presents his case in a paper cited by Eckstein, "On the Theory of Optimal Investment Decision," *Journal of Political Economy*, August 1958.

analysis). This preference is based on a rather theoretical argument—as more limiting assumptions are needed in the case of the internal rate of return to lead to ideal allocation of resources than in the case of the present value approach.

We do not know how much more actual limitation is introduced into the analysis by the rate of return approach, and to what extent resources may be misallocated by following this efficiency criterion. The only thing we do know is that a highly elaborate and complicated procedure is recommended.

The rate of return analysis uses as its basic tool a detailed profit and loss estimate, with adjustments on income and expenditure sides. The data is on an annual basis, as in any financial report of any business. And it goes without saying that to the public whatever in government looks like business, looks sound.

The present value argument embraces 20–50–100 years in its analysis and can be understood only by the quite complex notion of present value for future money flows, computed by using a compound rate of discount.

There is no difference whatsoever in the two approaches, neither in the process of price adjustments, i.e., replacing market prices by "real" accounting prices, nor in the process of introducing indirect benefits or costs into the analysis.

There are further the enormous statistical problems inherent in all of these studies, which make one worry much less about the perfection of the model, than about the availability of reliable data. The internal rate of return approach makes it possible to do the analysis without specifying the rate of interest. It is precisely this problem of specifying the applicable price of capital which is the least clear and most disputed element in the whole benefit cost analysis, as the discussion of Eckstein, Hirshleifer, Vickerey, and Buchanan clearly indicates.

In arranging a list of investment projects according to priority, the rate of interest does not enter the picture. It does enter the picture only if instead of a list of priorities, a classification between "good" and "bad" projects, is necessary, because then a borderline of minimum efficiency has to be fixed. The rate of interest may also be a necessary consideration, when we compare projects with different pay-off intervals.

Public opinion in Congress, government, business or elsewhere is very much inclined to reject rational efficiency evaluations as the

sole basis for decision making, as it narrows the sphere where "mutual partisan adjustment" rules.

Congress recently has had a very good opportunity to prove its fullest interest in the regional allocation of water programs, and its fullest disinterest in the application of a rational way to evaluate these programs. In hearings before the Committees of Interior and of Public Works, the Bureau of the Budget and its circular A-47 were sharply criticized. The benefit cost approach in this circular was denounced as a political device to fight against federal water resource development.[2]

Nothing would be easier to rationalize an opposition to efficiency evaluation of government programs, than the mere fact that nobody understands the tool used, or that experts cannot agree on the right prices involved.

We should be cautious not to trade too much clarity, public acceptance, and statistical needs for an elaborate model with conceptual perfection.

Reply by Mr. Eckstein

The fundamental issue between Professor Hirshleifer and myself is quite simple: the interest rate in the model underlying his discussion serves its classical function, of equating the marginal rates of substitution in production and consumption. The profusion of interest rates in the economy is considered to be due to differing risk premiums. My model assumes the capital market to be imperfect, to be rife with rationing, ignorance, differential tax treatments, reluctance to finance investment from external funds, slow adjustment processes, etc., which destroy the normative significance of actual rates found in the market. The enormous gaps between the investing and savings rates in the system, with savers typically receiving 3 per cent and investment decisions being made at 20 per cent and more (and yielding similar *ex post* returns), is strong evidence against the pure, classical view.

Once the interest rates in the markets are denied their normative role, the rate for public decision-making must be derived from other considerations. It can be derived from individual revealed preference, from a planner's preference model, or from a vision in a dream; it is a value judgment, pure and simple.

[2] Report No. 2686, Senate, 84th Cong., 2nd sess., on S. Res. 281.

To assure a correct allocation of funds, capital or otherwise, marginal yields must be equated—the yield being present value, measured at the social rate of discount. The productivity of capital enters into the analysis as the rate at which present value can be created in alternative employment.

I agree that the determination of the social discount rate cannot be divorced from the over-all investment plan, and have tried to show the interrelation elsewhere.[1] The notion of schedules of rates, related to growth rates of output per capita or elasticities of the marginal utility functions is part of the model, of course.

Finally, I want to echo Professor Hirshleifer's call for getting on with the job of applying the criteria to public expenditures. The potential of this type of economic analysis is large and it is important that our theoretical quarrels not interfere with our empirical performance in this area.

[1] "Investment Criteria . . . ," *op. cit.* pp. 78–82.