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On the Design of Consistent Output and Input Indexes for Productivity Measurement

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Introduction

A "CLASSICAL" PROBLEM

AN EFFECTIVE treatment of the assigned topic must rest on the application and extension of ideas identified with the classical index-number tradition rather than with the newer, so-called economic theory of index numbers. It requires acknowledgment of the conventional character of such measures, acquiescence in the strict demands of algebra, and acceptance of the operationalist view that the meaning of an index is determined by its composition and structure.

More than a generation ago, when price series were most highly developed and measures of production, man-hours, and labor productivity were hardly known or not yet established, index-number formulas were ranked for merit according to either their technical properties (e.g., by Fisher) or the pertinence of their operational meaning to the demands of a measurement situation (e.g., by King). The arbitrary nature of index numbers was widely recognized even though early writers often did refer hazily to output measures in terms of utility "totals." Data for different periods were taken out of presumed equilibrium contexts and, without a qualm, formally rearranged and juxtaposed like cut flowers. The theory of measurement was not patterned on any theory of personal or collective economic decision-making. It was made subject instead to algebraic conventions and restraints suggested by common sense and by the general equation of exchange (e.g., satisfaction of the factor-reversal and time-reversal tests). An active interest was shown in analysis of the difference between alternative formulas for the same variable and in the problem of consistent measurement of two or more variables, even though the available data permitted only a limited empirical exploration.

Historical index numbers comparing dates characterized by dissimilar tastes, population, and technology are much more numerous and more varied now than they were in Fisher's heyday. Little impact on current usage may be traced to Frisch's subsequent distinction between atomistic and functional indexes; and to the rise of consumer preference theory and welfare economics, which make ordinal use of weighted aggregates resembling those employed in cardinal historical index numbers. Indeed, there is ample cause to complain that today's makers and users of historical measures are not sufficiently interested in the arbitrary and nonunique character of these tools.

In any case, no rationale is offered for time series by the economic theory of index numbers, committed as it is to the ordinal comparison of two more situations by an individual or by a quasi-organismic collective having a fixed, well-defined preference map. All that this theory can do is declare historical measurement unsound by reason of incompatibility with its own limited premises.

But what of classical theory? Conceding the make-believe element in measurement and recognizing the multiplicity of conceivable indexes, this theory at least sanctions the act of making temporal comparisons. Indeed, if such comparisons are deemed useful even though they offend economic sense, then this theory offers criteria for discrimination among alternative measures, for design of consistent measures relating to coordinate variables, and for criticism and improvement of index-number practice.

VERBAL AND LITERAL ALGEBRA

With the increasing acceptance of economic time series as practical tools of analysis, interest in their theoretical underpinnings has diminished. For various reasons, some of which are suggested later in this paper, makers and users of index numbers nowadays commonly neglect certain basic classical tenets. In particular, they tend to overlook the strict demands and implications of literal algebra while meeting the easy requirements of verbal algebra. Both kinds of algebra must be taken into account, however, in the design of consistent output and input measures, the design of productivity measures that are compatible with index numbers of other associated economic variables (e.g., wages and prices), and the evaluation of indicators that actually are constructed or used.

Within the broad limits of the definition of "output per unit of (partial or total) input," many alternative specific indicators of productivity are conceivable for the economy or any division of it. Different analytical contexts call for constructions that sometimes closely, but more often vaguely, resemble the one or two measures that may already be available. Each conceivable alternative has a particular operational meaning that corresponds to the literal algebra of its make-up—to the basic output and input data, the weights, and the formula that it embodies.

Unfortunately, few alternative productivity measures are actually constructible from available materials, and the variety of practicable measures will not be quickly or appreciably extended by government and private endeavors to improve economic statistics. Furthermore, if all conceivable measures could be computed for a single industry or economic sector, they would not give identical results and might even disagree sharply for intervals of significant change in economic structure (e.g., transitions between war and peace and between good times and bad). Accordingly, the quotient of any available or constructible pair of output and input measures cannot be regarded as *the* productivity index without appeal to additional cogent criteria that rule out all rivals. The odds are overwhelming that this quotient is only *a* productivity index, rather than the best—the most pertinent—of the many conceivable specific measures. For the uses to which it is put, it may lack certain desirable properties and even possess some undesirable ones.

The customer, in other words, is not always right; nor is the producer. Neither of them is really at liberty to attach to a productivity quotient any economic or other meaning that he chooses or that an occasion demands. The meaning ascribed to a measure should be congruent with the operational meaning, with the implications of the literal algebra involved. Only those who would live by faith alone could risk accepting an official, general-purpose, or other authoritative measure as universally applicable, as appropriate for any problem situation, without reference to its literal meaning.

Literal Consistency within Productivity Definition

PROPERTIES OF "REASONABLE" INDEXES

NOTE: Some revisions have been made in the text and footnotes of the paper originally presented. John W. Kendrick offered many helpful comments as a member of the Reading Committee.

Let us now turn to the design of index numbers untrammeled by limitations of data, funds, time, and patience. What conditions of literal algebra might reasonably be imposed on measures satisfying the bald verbal definition of productivity as the quotient of output and input? In the first instance, we treat this question as involving no additional verbal identity. That is, the output, input, and productivity indexes are considered to describe a complete and closed system, to comprise the entire statistical apparatus needed for the universe of discourse.

Common sense—or, if one prefers, intuition—suggests that the three indexes should preferably (1) be similar in general form, (2) make equally good operational sense, and (3) be invariant to the order of their derivation. More explicitly, it seems reasonable to require (1) homologous structure of the measures of all three variables, (2) equivalence of each measure to an internal average of the corresponding relatives, and (3) derivability of each measure from the other two (as a quotient or product).

If the first requirement is interpreted somewhat generously, many formulas immediately come to mind as qualifying. It is not essential for our purposes to add criteria or to cite circumstances that would eliminate all eligible sets of measures but one. In the remainder of this paper, however, no notice will be given to geometric means of relatives. Attention will be confined to the types of indexes that are commonly approximated—weighted aggregative measures and weighted arithmetic and harmonic means of relatives. All these are ratios with dimensionally comparable numerators and denominators. Occasional reference will be made to combinations of two or more qualifying measures into Fisher-type "ideal" averages.[1]

The three requirements cannot be satisfied exactly with normally available data. The consistent theoretical measures, being mutually determined, are tethered one to the other by specially forged links of literal algebra. The particular concept of output, input, or productivity that enters explicitly into one measure must be retained in the companion measures. An output index, furthermore, may have to incorporate weights other than the usual pecuniary variety (e.g., man-hours). In short, the demands of consistency cannot be met precisely unless the basic data are available in proper kind, detail, and amount.

Of special importance is the requirement of derivability of any measure from the other two. This property may be guaranteed by the design of the index-number systems in such a manner as to achieve a cancellation upon multiplication or division. The condensation of the product or quotient of two indexes to a ratio of only two compatible aggregates implies the conditional equivalence of weighted output

[1] The chain index is not explicitly considered. Even though discontinuities in data are conventionally handled by resort to this type of index, the author believes that it should be regarded as a crude alternative to a "free composition" aggregative measure, which is an extension of the ordinary Laspeyres or Paasche measure to cover *all* the products made at some time in the whole span under consideration. Thus, in the free composition index, fictitious weights and zero quantity entries may be required in the numerator or denominator for products not yet made or already obsolete. The conventions of this index, strange though they may seem, are much more sensible than those of the chain index, which never registers a rise from, or a fall to, zero for any product class and implies that nonexistent products show exactly the same changes over time as the products actually included in the measurement. See footnote 10.

and input. But this equivalence could not obtain in particular instances unless the mutually adapted weighting systems assure the direct dimensional comparability of output and input aggregates in the first place. In other words, the weights assigned to output and input quantity figures that are initially expressed in characteristic output or input units reduce all these figures to a single common denominator for all periods.

From the statements just made, useful corollaries flow. The cancellation criterion is as essential to thinking about consistent index numbers as the principle of conservation is to physics and the principle of exhaustiveness to accounting. It even provides a rule for the selection of output and input weights when productivity measurement does not embrace all factor inputs.[2] It offers the key to clean deflation, to the design of a suitable matching price index for literally as well as nominally converting a value index into a quantity measure. It permits expression of a productivity index as a ratio of appropriately constructed *price* measures for input and output.[3] The notion of direct comparability of output and input not only underlies cancellation but also permits (1) the expression of net output in any period as a difference between weighted gross output and weighted nonfactor input (e.g., materials and energy) and (2) the interpretation of an aggregative productivity index as a ratio between two expressions for weighted output or between two expressions for weighted input.

LABOR PRODUCTIVITY

In the thirties, the WPA National Research Project pioneered a program of consistent measurement of production, man-hours, and labor productivity in the manufacturing and mining industries.[4] The formulas were developed with reference to the primary aim of the Project—the study of unemployment and re-employment in a changing technological environment.

Consistent formulas were devised by WPA within the framework of

[2] See footnote 8 for an example of the application of the cancellation criterion to the case of productivity based on gross output and only one (labor) input.

[3] Let $QP=V=V'=Q'P'$, where Q and P represent quantity and price indexes for output; Q' and P', quantity and price indexes for input; and V, and V' value indexes for output and input. For productivity, we have

$$\Pi = \frac{Q}{Q'} = \frac{V/P}{V'/P'} = \frac{P'}{P}.$$

See footnote 15 for an explicit example.

[4] See H. Magdoff, I. H. Siegel, and M. B. Davis, *Production, Employment, and Productivity in 59 Manufacturing Industries, 1919–36*, WPA National Research Project Report No. S-1 (Philadelphia, 1939), 3 vols.; and V. E. Spencer, *Production, Employment, and Productivity in the Mineral Extractive Industries: 1880–1938*, WPA National Research Project Report No. S-2 (Philadelphia, 1940).

the verbal identity: Man-hours=Output × Unit man-hour requirements. Since unweighted man-hours were accepted for the input measure, two Paasche-Laspeyres sets were obtained for the other two variables. In the output formulas, the quantities were weighted by unit labor requirements; in the formulas for unit labor requirements, the weights were output quantities. The numerators and denominators of all these aggregative formulas were dimensionally equal to man-hours (which could also be interpreted as weighted output, if necessary). The reciprocals of the two indicators of unit man-hour requirements were internal means of productivity relatives.[5]

In practice, the WPA Project could seldom satisfy the strict demands of the literal algebra and accordingly had to resort to various degrees of approximation. In particular, it almost always had to use conventional output measures with unit-value weights for individual industries. But this very compromise could have defeated the WPA program objective: The quotients of such output measures and unweighted man-hours indexes need not be internal means of productivity relatives.[6]

If appropriately detailed data were available for weighting the man-hours expended on individual products,[7] WPA could have achieved

[5] One set of indexes satisfying the verbal identity is:

$$\frac{\sum q_1 l_1}{\sum q_0 l_0} = \frac{\sum q_1 l_0}{\sum q_0 l_0} \cdot \frac{\sum q_1 l_1}{\sum q_1 l_0},$$

where the q's and l's refer to output and unit man-hour requirements. The reciprocal of the second index on the right is an aggregative productivity measure, which, of course, may be rewritten as an internal mean of relatives.

[6] See I. H. Siegel, *Concepts and Measurement of Production and Productivity*, U.S. Bureau of Labor Statistics (March 1952), pp. 53–4. In general, an output index that is not designed for consistency with an input index yields a productivity quotient which may be factored into two terms, one being an internal mean of productivity relatives. The second term, however, may be such that the quotient falls outside the range of productivity relatives.

An egregious example of externality (and a troublesome one, too, in view of the increasing political importance of the rate of economic growth) is provided in the long-term productivity measures for our private economy and its agricultural and non-agricultural components. Thus, *Bulletin No. 1249* of the U.S. Department of Labor (published in December 1959, a year after this paper was presented) shows an average annual increase of 2.3 per cent in the real output per man-hour in the private sector during the period 1909–58, but smaller increases for both the agricultural and non-agricultural components—2.1 and 2.0 per cent, respectively. A variant set of measures for the same period shows corresponding rates of 2.4, 2.1 , and 2.1 per cent.

[7] It should be recognized, of course, that the joint or overhead labor expended in, say, a plant or company on a group of products can be allocated only by procedures that are in some degree arbitrary. Value added cannot strictly be measured for individual products either. At least in principle, the subproduct approach (see footnote 10), which distinguishes the results of processing stages or operations instead of treating individual products as wholes, would minimize or avoid the need for arbitrary allocations of man-hours or value added. In other words, joint or overhead operations may be regarded as yielding subproducts meriting separate classification and measurement.

its technical objective by another route. Thus, if the conventional value-weighted output measures are considered satisfactory, it is possible to design weighted companion indexes yielding productivity quotients that must be internal averages of relatives. Corresponding to the Laspeyres-type output measure in this case are two sets of productivity and man-hours indexes rather than one. Similarly, for the output measure of the Paasche variety, two other sets of matching measures are derivable for the other variables (i.e., for man-hours and productivity).[8]

Economic analysts often wish to decompose the percentage change in man-hours between two years into particles reflecting percentage changes in output and in unit labor requirements. Actually, there are three particles rather than two, so the verbal identity is frequently stated incorrectly in the first place. The identity should read:

$$\left.\begin{array}{c}\text{Percentage change}\\ \text{in man-hours}\end{array}\right\} = \left\{\begin{array}{l}\text{Percentage change in output} + \text{per-}\\ \quad\text{centage change in unit man-hour}\\ \quad\text{requirements} + \text{joint percentage}\\ \quad\text{change in both output and unit}\\ \quad\text{man-hour requirements.}\end{array}\right.$$

The third particle is sometimes ignored; sometimes absorbed into one of the other two particles; sometimes distributed equally between the other two particles (with the result that the time reference of the "weights" differs from the time base for the computation of percentage changes); or sometimes incorrectly attributed to factors other than output and unit man-hour requirements. The joint particle has a distinct meaning in terms of algebra, geometry, or a Taylor expansion, and it should accordingly not be absorbed into either or both of the other particles. When it is not absorbed, it should not be interpreted in terms of variables extraneous to the identity. Furthermore, since

[8] To assure cancellation, we make one weighted output term equal to a (differently) weighted input term—say

$$\Sigma p_0 q_0 = \Sigma\left(\frac{p_0 q_0}{m_0} . m_0\right).$$

For the verbal identity shown in the text, we then have

$$\frac{\Sigma\left(\dfrac{p_0 q_0}{m_0} . m_i\right)}{\Sigma\left(\dfrac{p_0 q_0}{m_0} . m_0\right)} = \frac{\Sigma\, p_0 q_i}{\Sigma\, p_0 q_0} . L,$$

if we choose the Laspeyres production index; and L, an index of unit man-hour requirements, is equivalent to a weighted mean of relatives of the form $\left(\dfrac{m_i}{m_0} \div \dfrac{q_i}{q_0}\right)$. Three other sets of similar measures may be worked out.

See *Index Numbers of Industrial Production*, United Nations Statistical Office Studies in Methods No. 1 (New York, September 15, 1950), pp. 57–8.

partitioning is more like anatomy than physiology, it seems desirable to avoid causal language—to avoid labeling components as "due to", or showing the effects of, the particular variables involved.

Since output normally increases and unit labor requirements tend to decline over a span of years, the corresponding percentage changes are typically different in sign. In these circumstances, the distribution of the joint particle between the other two may have particularly awkward results. A difference in signs may often be avoided if the basic identity is rewritten so that output equals man-hours times productivity. Productivity, the reciprocal of unit labor requirements, commonly has an upward trend and, over long periods, man-hours too are likely to increase.

Beyond the verbal algebra of partitioning, there is still the matter of literal algebra. Percentage changes should be measured as differences between appropriate indexes and unity. If a Laspeyres (Paasche) index is used for production, the Paasche (Laspeyres) formula is indicated for unit man-hour requirements or productivity.[9]

A few additional points should be made before we turn our attention to productivity measures involving a broader input base:

1. The Fisher "ideal" index could be used to reduce the two sets of WPA measures to one. Similarly, geometric means could be taken of the two sets of formulas built around the Laspeyres value-weighted

[9] The partitioning identity may be written as:

$$(M-1) = (Q-1)+(L-1)+(Q-1)(L-1),$$

where M, Q, and L refer to indexes of man-hours, output, and unit man-hour requirements and, of course, $M=QL$. If M is an unweighted index, then, as in footnote 5, Q may represent a Laspeyres index and L a Paasche index. The identity is also satisfied by the Paasche formula for Q and the Laspeyres formula for L.

The particles are more likely to have the same sign if we write $Q=M\Pi$ and

$$(Q-1) = (M-1)+(\Pi-1)+(M-1)(\Pi-1).$$

If M and Q are defined as in the preceding paragraph, then Π is the reciprocal of the measure used for L.

The purely formal character of partitioning and the inappropriateness of the practice of distributing the joint particle may be effectively illustrated by a plausible case of $M=QL$, in which $Q-1=2$ and $L-1=-1/2$, so that $M-1=1/2$ and $(Q-1)(L-1)=-1$. The "effect" of the joint change in Q and L is a decline of 100 per cent in M, which is ridiculous; and if the joint particle is equally distributed between the other two, as is so often done, the change in M "due to" the change in L becomes minus 100 per cent!

On partition formulas including only two variables, see *Concepts and Measurement . . ., op. cit.*, pp. 86–90.

On the proper way to introduce extraneous variables into a discussion of partition identities, see a comment by I. H. Siegel in *Proceedings of the Business and Economic Statistics Section, American Statistical Association, 1957*, p. 309.

The position taken in this paper is not only at variance with common practice but is also opposed to the Divisia approach to index-number design and to the rationale more recently offered by Stuvel (*Econometrica*, January 1957 and July 1958).

output index, or of the two sets consistent with the Paasche value-weighted index.

2. Many different output and labor concepts may enter into the formulas intended literally to satisfy the verbal algebra. Thus, the output figures could refer to completed products, net output, or subproducts (which correspond to the arcs of production cycles). The man-hours could relate to hours worked or hours compensated, to wage workers or all employees. The verbal identity may also define a consistent set of generalized aggregative indexes that take account of the entry of new products and the exit of defunct products without chaining.[10]

3. In the interpretation of labor productivity measures, account must be taken of the special difficulties besetting the quantification of output and of the manner in which these difficulties are met. Among the problems frequently encountered are lags in reporting of quantities of new products, the heterogeneity of product classes, quality change, and the noncommodity character of many "services" performed by manufacturing company personnel (e.g., research,

[10] On the subproduct concept, see I. H. Siegel, "The Concept of Productive Activity," *Journal of the American Statistical Association*, June 1944, pp. 218–28. On extended aggregative indexes intended to replace chain indexes, see *Concepts and Measurement . . .*, *op. cit.*, pp. 70–4, and I. H. Siegel, "Aspects of Productivity Measurement and Meaning," in G. Deurinck, ed., *Productivity Measurement, I: Concepts*, Organization for European Economic Cooperation (Paris, August 1955), pp. 50–6.

As already suggested in footnote 7, subproducts are the results of the discrete activities that lead to the end products normally measured. According to the subproduct approach, automobile production is much more than final assembly of the completed product; it should also include the manufacture of components, which in turn may be visualized as a series of discrete steps. Production measures based on subproducts would have many advantages; they would reflect the structure of productive activity more accurately, provide better indicators for industries making heterogeneous end products, and avoid distortions associated with changes in the degree of technical integration.

The following equation illustrates the satisfaction of the verbal identity considered in footnote 5 with free composition indexes:

$$\frac{\overline{\sum} q_1l_1+\overline{\overline{\sum}} q_1l_1+\overset{\equiv}{\sum}\cancel{q_1l_1}}{\overline{\sum} q_0l_0+\overline{\overline{\sum}}\cancel{q_0l_0}+\overset{\equiv}{\sum} q_0l_0}=\frac{\overline{\sum} q_1l_0+\overline{\overline{\sum}} q_1l_0+\overset{\equiv}{\sum}\cancel{q_1l_0}}{\overline{\sum} q_0l_0+\overline{\overline{\sum}}\cancel{q_0l_0}+\overset{\equiv}{\sum} q_0l_0}\cdot\frac{\overline{\sum} q_1l_1+\overline{\overline{\sum}} q_1l_1+\overset{\equiv}{\sum}\cancel{q_1l_1}}{\overline{\sum} q_1l_0+\overline{\overline{\sum}}\cancel{q_1l_0}+\overset{\equiv}{\sum} q_1l_0}.$$

The index on the left refers to man-hours; those on the right, to output (Laspeyres) and unit man-hour requirements (Paasche). In each of the aggregates, a single bar designates the partial sum for products common to both of the compared periods; a double bar, the partial sum for products not made in the base period; and a triple bar, the partial sum for products not available in the comparison period. A strike-through indicates that a particular partial sum equals zero because the output quantities equal zero. The l_0's for the partial sums with the double bars and the l_1's for the partial sums with the triple bars are fictitious; they may be estimated as the lowest unit labor requirements consistent with zero output in the given period. The numerator of the output index and the denominator of the input index are identical, but cancellation marks are not shown as in other footnotes in order to avoid confusion with the strike-throughs.

starting-up of new facilities, office work, and minor construction). Although some of these services represent quasi-investments and the corresponding labor often is included in input, their economic contributions are not recorded as discounted equivalent current product.

4. The true character of an output measure derived by deflation is determined by the concepts, composition, structure, and compatibility of the two indexes employed in this process. If the same value index may be written in several meaningful ways (as in the case of net value of output), each form calls for a different matching price deflator and implies a different production quotient. If an appropriate unit of measurement for a particular type of output (e.g., research) cannot be directly visualized, it certainly cannot be established by deflation of the corresponding value (or man-hours) by a vaguely pertinent price (or unit man-hour requirement) measure. The literal sense of an output measure obtained by deflation must be understood before a subsequently derived productivity index may be interpreted.

5. Manufacturing and other output measures with pecuniary weights are not necessarily superior to, or more "economic" than, measures incorporating labor weights. They are easier to make, and their weights are more comprehensive, but they need not be the most suitable measures for any particular analysis. Furthermore, since all indexes are artifices, their construction and use must be reckoned as closer to accounting than to economics—a point suggested in the introductory section of this paper.

6. The incorporation of labor weights in production measures does not imply acceptance of a labor theory of value, and the construction of labor productivity indexes does not imply that labor is the only productive input. Index computations in terms of labor units or any other units (including money) are a species of accounting divorced from the process of market valuation. Since historical productivity measures refer to average ratios for changing technologies, rather than to marginal ratios for a given technology, they have no imputational significance that is obvious.

7. A productivity index which is not an internal mean of relatives cannot be properly understood until it is rewritten as, say, the product of an internal productivity average and a factor representing everything else.[11]

[11] When an output index with unit value weights is divided by the index of unweighted man-hours, the quotient may be factored into a WPA-type productivity index and another term. See footnote 6.

8. Despite increases in the amount, and improvements in the scope, of government and private statistics since the 1930's, the task to which the WPA National Research Project addressed itself remains unmanageable. The compilation of detailed establishment information on a product or subproduct basis is essential for the development of literally consistent output and input measures. With the end of the biennial census system in 1939, this prospect, perhaps never brilliant, perceptibly dimmed for manufacturing. At any rate, extant government programs to revitalize productivity measurement contemplate no fundamental or extensive change in the basic information system for manufacturing. Beyond a certain point, ingenuity cannot substitute for limited or nonexistent detailed coordinate data for production and other variables.[11a]

MULTIFACTOR PRODUCTIVITY

It is much easier to write consistent formulas for multifactor productivity than actually to measure nonlabor factor inputs in suitable characteristic units. The process of setting down formulas is simplified by the introduction of a second summation sign, to cover (factor or nonfactor) inputs entering into each gross product. As for assuring that the productivity formulas are internal means of relatives, we again invoke the cancellation rule. Specifically, we employ a concept of net output that is identical in scope with factor input. We also use weights that make output and input dimensionally comparable and that equalize them (i.e., cause cancellation) for one of the two compared times. If the inputs refer, say, to labor and capital, the net output concept should ideally be restricted to the value added by these factors.

The verbal identity, Multifactor input=Net output×Unit factor requirements, leads to at least two kinds of consistent formulas. In one case, the net output measure is really a Paasche or Laspeyres aggregative index of gross quantities with "nettifying" unit-value-added weights. Since the input concept corresponds to net output in scope, valuation at cost permits us to write a Paasche or Laspeyres input index that yields a productivity measure containing only two weighted aggregates. This productivity measure may be rewritten as a weighted mean of relatives, the numerators of which refer to gross

[11a] Man-hour series adjusted for presumed productivity changes account for about half of the 1957 weighted aggregate in the revised FR production index for manufacturing and mining and for about 54 per cent of the aggregate relating only to manufacturing.

See *Industrial Production: 1959 Revision*, Board of Governors of the Federal Reserve System (Washington, July 1960).

output of individual ratios and the denominators of which refer to a narrower concept of factor input.[12]

A second system of aggregative formulas satisfying the above verbal identity is built around the Fabricant-Geary measure of net output. This measure has two aggregates, a minuend and subtrahend, in both numerator and denominator. In the common version, one aggregate refers to weighted gross output, the other to weighted nonfactor input (e.g., materials and energy). For exact correspondence with factor input in scope, however, the measure must either be interpreted in terms of subproducts or altered in some other manner to allow for net change in the inventories of goods in various stages of processing. If the measure is interpreted in terms of subproducts, then the value added in each of the compared periods is determined incrementally; intermediate subproducts made and consumed in the same period appear in both minuend and subtrahend and hence cancel. If, instead, the minuends are reserved for finished products and the subtrahends refer either to the corresponding nonfactor input or to all the nonfactor input of the period, adjustment terms have to be added to the formula to assure coextensiveness with the factor-input index.[13]

The Fabricant-Geary output formula has Laspeyres and Paasche variants, both of which are equivalent to aggregative indexes of net output. Using the same weighted factor-input indexes as we did in conjunction with the output measures weighted by value added, we again effect the desired cancellations in deriving productivity expressions. In this case, however, the productivity relatives, rather than simply the weights, are net. That is, the numerator of each productivity relative actually refers to the net output of an individual

[12] The verbal identity becomes, for the Laspeyres input and output variants,

$$\frac{\sum Sw_o f_t}{\sum Sw_o f_o} = \frac{\sum q_t\left(p_o - \frac{SP_oQ_o}{q_o}\right)}{\sum q_o\left(p_o - \frac{SP_oQ_o}{q_o}\right)} \cdot L';$$

and L' may be rewritten as a weighted mean of relatives of the form $\frac{Sw_o f_t}{Sw_o f_o} \Big/ \frac{q_t}{q_o}$. S signifies a summation corresponding to a gross product, and the w's stand for the remuneration of the factor inputs, the f's. In the expression for gross output with unit-value-added weights, the q's stand for gross outputs, the p's for gross prices; and the Q's and P's refer to nonfactor inputs (e.g., purchased materials and energy).

The approach discussed in this section should well satisfy a measurement need mentioned in Christ's comments: It not only recognizes the difference between gross and net product but also permits the derivation of consistent measures for both of these concepts, factor inputs, other inputs (e.g., materials and energy), and gross and net productivity.

[13] See *Concepts and Measurement . . .*, *op. cit.*, pp. 61–2.

product. In the earlier case, it will be recalled, the relatives had gross output numerators.[14]

In both systems, productivity change may be viewed *sub specie pretii* as well as *sub specie quantitatis*. Thus, the ratio of Laspeyres (Paasche) net output and factor input measures equals a ratio of Paasche (Laspeyres) price measures. This conversion is based on the identity of values of net output and factor input when the price and quantity subscripts match. It also implies the equivalence of value indexes for net output and factor input and their factorability in turn into compatible quantity and net price indexes.[15]

Whichever net output formula is taken as the starting point, the appropriate Laspeyres and Paasche product price measures are net, not gross. This important fact is typically overlooked in the deflation of national product, of the nongovernment component, and of the value added in industries for which output is not measured directly.

For students interested in the anatomy of multifactor productivity increase over time, a close study of the expressions already discussed will prove rewarding. Economists, for example, may obtain new insights by viewing productivity advance as *the asymmetrical change of prices for the same resources defined as net output and as factor input.* Equality obtains when these resources are valued as output and as input in prices of the same period, but the productivity aggregates that are left after cancellation show that the equality no longer holds when

[14] The verbal identity for the Laspeyres situation is

$$\frac{\sum Sw_0 f_1}{\cancel{\sum Sw_0 f_0}} = \frac{\sum p_0 q - \sum SP_0 Q_1}{\cancel{\sum p_0 q_0 - \sum SP_0 Q_0}} . L'';$$

and L'' may be written as a weighted mean of relatives of the form

$$\frac{Sw_0 f_1}{Sw_0 f_0} \div \frac{\left(q_1 - \frac{SP_0 Q_1}{p_0}\right)}{\left(q_0 - \frac{SP_0 Q_0}{p_0}\right)}.$$

[15] The Laspeyres productivity index derived in footnote 12 (i.e., I/L') is equivalent to the quotient of two Paasche price measures:

$$\Pi' = \frac{\cancel{\sum Sw_1 f_1}}{\sum Sw_0 f_1} \div \frac{\cancel{\sum q_1 \left(p_1 - \frac{SP_1 Q_1}{q_1}\right)}}{\sum q_1 \left(p_0 - \frac{SP_0 Q_0}{q_0}\right)}.$$

The productivity index derived in footnote 14 (i.e., I/L'') is equivalent to:

$$\Pi'' = \frac{\cancel{\sum Sw_1 f_1}}{\sum Sw_0 f_1} \Big/ \frac{\cancel{\sum p_1 q_1 - \sum SP_1 Q_1}}{\sum p_0 q_1 - \sum SP_0 Q_1}.$$

In both of these cases, we have simply inserted $\sum Sw_1 f_1 = \sum q_1 \left(p_1 - \frac{SP_1 Q_1}{q_1}\right) = \Sigma p_1 q - \Sigma SP_1 Q_1$, which states the equivalence of weighted factor input and weighted net output in period i. See *Concepts and Measurement . . ., op. cit.*, pp. 74–5.

the prices refer to some other period. Furthermore, the price indexes show that a productivity rise implies the fall of the net-output price with respect to the factor-input price of the same resources.[16]

The theory underlying static production functions and marginal productivity determination must be distinguished from, although it has some relation to, the basic idea stressed here that input and output time series are measures of the "same" resources from different viewpoints. A production function, which is not expected to remain a literally valid description of technological relationships over any length of time, mathematically connects small changes in input and output as though they were cause and effect. For the limited period to which the function applies, it may be thought to have a live economic meaning, an imputational significance. But no similar claim is justifiable for indexes that report historical changes in average output per unit of composite factor input—changes reflecting, among other things, the transition from one static production function to another.[17]

Acknowledgment should be made of some of the challenges presented by multifactor productivity measurement in addition to that of designing consistent indexes. One basic problem is the expression of factors other than labor, such as capital (which changes in quality over time) and enterprise in significant quantitative units. Another problem is the proper representation of certain kinds of output, such as the current production equivalent of current research input, in indexes conventionally restricted to commodity measurement. Finally, we should note the difficulty of completely enumerating the factors relevant to output, including factors that are not compensated. Thus,

[16] This paragraph carries a message concerning efforts, mentioned by Christ, to identify completely the sources of observed changes in the output. It suggests that, even if all factors could be taken into account and properly weighted, a productivity "gap" or residual would still arise in time comparisons.

A constant composite productivity index of unity, however, may be obtained by design, and without an exhaustive accounting of inputs. Thus, the measure of every recognized factor input could be "adjusted" for the most significant "quality" change of all—the very productivity change indicated for that factor. Such adjustments, which compensate for the omission of various inputs as well as for changes in the character of the acknowledged inputs, could lead to constant productivity series for labor, capital, and so forth, and for the multifactor composite too.

[17] Reference is made, here as elsewhere in this paper, to the static production function because it is usually assumed in discussions of marginal productivity and the theory of income distribution. The text could be suitably modified, of course, to reflect the increasing interest in empirical dynamic production functions, from which time series are derivable for marginal as well as average productivity. Comments made earlier on efforts to determine the contributions of recognized variables to changes in an aggregate also apply to attempts to isolate the roles of labor, capital, and so-called "technological change" (or "organization," in Aukrust's terminology) in explaining the difference between two values of a dynamic production function.

government services might be overlooked; and, in the case of agriculture, better-than-normal amounts of rain and sunshine should be reckoned as technical inputs even if normal amounts are taken for granted as part of the environment of production.

Literal Consistency beyond Productivity Definition

So far, we have discussed the design of consistent index-number systems for only those situations in which productivity, output, and input have the stage to themselves. But economists, statisticians, government officials, business and labor leaders, news commentators, editorial writers, and assorted speechmakers are often concerned with problems in which one or more of these variables must share attention with others of coordinate importance. In such instances, the verbal identities may be much more complex than the kind already considered, and the aggregates may involve three or more variables, rather than two as in the familiar Laspeyres and Paasche formulas.

The problem of achieving consistency beyond the productivity definition arises, for example, when wages and employment are discussed in conjunction with productivity. For such situations, it seems reasonable to require homologous, aggregative measures satisfying one of these two verbal identities:

Payrolls = Unit labor cost × Man-hour productivity × Man-hours
= Average hourly earnings × Unit man-hour requirements × Output.

In either case, many sets of consistent aggregative formulas may be written. A particular formula for any variable may occur in more than one set, in different combinations with others. If the geometric mean is applied to all possible sets, generalized ideal indexes are obtained. These indexes have such desirable properties as meeting the time-reversal and (generalized) factor-reversal tests in addition to remaining internal means of relatives.[18]

The number of sets of formulas satisfying the above verbal identities may be reduced by the introduction of additional subsidiary requirements. Thus, we may stipulate that indexes of productivity and man-hours entering into the first of these identities should meet the

[18] All aggregates entering into the first identity would have the form $\Sigma c\Pi m$ while differing in their time subscripts. All the aggregates entering into the second identity would have the form Σelq. Here, c stands for unit labor cost and e for average hourly earnings. Note that $\sum c_0\Pi_0 m_0 = \Sigma e_0 l_0 q_0$ and $\Sigma c_i \Pi_i m_i = \Sigma e_i l_i q_i$. See *Concepts and Measurement . . .*, *op. cit.*, 84–5; and I. H. Siegel, "The Generalized 'Ideal' Index-Number Formula," *Journal of the American Statistical Association*, December 1945, pp. 520–3.

cancellation criterion upon multiplication—in other words, that the resulting index of output should be a ratio of two aggregates and an unequivocal internal mean of output relatives. Or we may invoke the cancellation criterion for the man-hours index formed as the product of measures of unit man-hour requirements and output in the second identity. The introduction of these supplementary cancellation conditions in effect simplifies the original verbal identities to

$$\begin{aligned}\text{Payrolls} &= \text{Unit labor cost} \times \text{Output}\\ &= \text{Average hourly earnings} \times \text{Man-hours};\end{aligned}$$

and these statements are satisfied by only two sets of simplified measures for the indicated variables.[19]

Another verbal identity involving payrolls should be mentioned because it is especially appropriate to the discussion of the productivity-wage-price connection:

$$\text{Payrolls} = \frac{\text{Average hourly earnings}}{\text{Output per man-hour}} \times \frac{\text{Value of output}}{\text{Price}}.$$

Since a number of the variables are of coordinate importance, it is reasonable to require that the measures be conceptually compatible, have a certain structural similarity, and reduce to internal means of relatives. So crude are existing quantitative tools compared to the ones required that clamor for more detailed and more complete basic statistics would surely seem as appropriate as the babel of diagnosis and prescription heard throughout the land.

It is a curious fact that at least two distinct aggregative formula systems may be designed to satisfy the last payroll identity; and, within each system, many sets of compatible measures may be written down.[20] Again, if subsidiary cancellation conditions are introduced, fewer aggregative measures are eligible, the number of sets of consistent measures is reduced, and the identity is telescoped. Thus, we may wish to stipulate that the first quotient on the right-hand side of the identity be a guaranteed internal mean of unit-labor-cost relatives, and that the second quotient be an internal mean of output quantity relatives.

One other point should be mentioned before this section is concluded. Indexes for the same variable are not necessarily equal if they arise in connection with different verbal identities. Similarly, an index

[19] The eligible aggregates in footnote 16 are limited to those that may, by virtue of the compatibility of time subscripts, be rewritten as $\Sigma\, c\Pi m = \Sigma\, cq$ and $\Sigma\, elq = \Sigma\, em$.

[20] According to one system, all of the aggregates would have the form $\sum elr$ while differing in time subscripts. According to the other, all aggregates would have the form $\sum pelq$. Here, r is the ratio of value of output to wages for each product. Note that $e_i l_i r_i = p_i$; and that various partial products of $pelq$ (which itself is dimensionally exceptionable!) also have pertinent meanings (e.g., pq = value of output).

of unit man-hour requirements satisfying one identity need not be the reciprocal of an index of man-hour productivity entering into another identity. Thus, the productivity measures derived for the first payroll identity cited above are not the same as those implied by the last payroll identity. Furthermore, only two of the four productivity measures literally satisfying the first payroll identity are exactly equal to the reciprocals of two of the four measures of unit man-hour requirements satisfying the second payroll identity.[21]

Conclusion

MORE HONORED IN THE BREACH

As we look about, we find little current awareness of, or interest in, the basic ideas that inform this paper.[22] One reason, of course, is the paucity of data. Another is the enshrinement of various indexes alleged to be suitable for general purposes. Still another is optimism, the belief that only in one's own specialty can the difference between tweedledum and tweedledee ever really matter. Without pretending to exhaust this fascinating topic, we must also mention the reluctance of sophisticated employees and consultants to footnote their findings with skull and crossbones, to risk undermining with candor the complacency of their "practical" employers and clients.

Everywhere we see the distinction between literal and verbal algebra blurred. "Any old" index with a suitable name is frequently used as though the details of its construction are of no moment and the purpose to which it is put is irrelevant. The mismatching of concepts, like deflation of the value of net product by gross rather than net price, is routine. The libelous anachronistic label of "nonproduction" employees is pinned on research workers and other personnel engaged in quasi-investment activities; and the failure to reckon their discounted product in current output is hardly taken into account in discussions of such phenomena as productivity "slowdown" or profit "squeeze". A price-deflated index of output value is commonly said to

[21] This sentence corrects too sweeping a statement made in *Concepts and Measurement . . ., op. cit.*, p. 85.

The equivalent indexes of the first and second payroll identities are:

$$\frac{\sum c_0 \Pi_i m_i}{\sum c_0 \Pi_0 m_i} = \frac{\sum e_0 l_0 q_i}{\sum e_0 l_i q_i}$$

and

$$\frac{\sum c_i \Pi_i m_0}{\sum c_i \Pi_0 m_0} = \frac{\sum e_i l_0 q_0}{\sum e_i l_i q_0}.$$

[22] It is pleasant to record, however, that works like R. Stone's *Quantity and Price Indexes in National Accounts*, Organization for European Economic Cooperation (Paris, November 1956), still appear occasionally and, of course, that conferences such as the present one are held too.

represent output in constant dollars of the base period even though the story told by the literal algebra is different and more complex. Chain production and price indexes are usually interpreted as though they were of the ordinary Paasche or Laspeyres variety. The changes shown by the leading index of manufacturing production are chronicled as though they convey message without noise, as though the huge gaps filled with data synthesized by verbal algebra were actually filled by the real thing. Changes indicated by, say, a man-hours (or production) index, are often broken arbitrarily into two parts, one said to be "due to" a change in production (or man-hours) and the other "due to" a change in productivity. Analysts of the productivity-wage-price relationship typically overlook the incompatibility of the concepts, scope, and weighting systems of the aggregate measures that they actually use. Users of available productivity indexes often do not seem to know or care whether these measures are internal averages of productivity relatives or reflect something more. In short, on the contemporary scene, we find too little vigilance exercised to distinguish a properly weighted index from a loaded one.

A LITTLE ONWARD

The history of productivity measurement, especially the recent record, does not encourage hope for rapid or cumulative improvement of practice. Perhaps, leadership in developing and utilizing data more nearly in accord with theoretical requirements will pass to the younger industrial nations, which have a clear need to raise productivity and are less encumbered by statistical custom. In our own country, the renewed vigorous advance of productivity measurement must not be expected to result from intermittent flurries of government support induced by labor-management controversy, fears of inflation, or failures of measures to accord with expectations and beliefs. Instead, fundamental progress will probably come about, if at all, as companies, remaining preoccupied with cost control and profit-making and having adopted electronic data processing systems, gradually introduce private programs of productivity measurement and generally strengthen the establishment basis of statistical reporting.[23]

No powerful lobby may be expected to arise in support of more and better productivity statistics, but movement toward these twin objectives could be encouraged by the development and display of a more critical attitude by makers and users of productivity and related

[23] For other remarks by the author on the same subject, see *Concepts and Measurement . . .*, *op. cit.*, pp. 99–103; and "Next Tasks in the Measurement of Production and Productivity," *Estadística*, September–December 1955, pp. 390–2.

measures. In particular, it seems desirable to keep a constant eye on the divergence between practical and theoretically preferred indexes. More attention should accordingly be given to methods of investigating this divergence and of estimating the numerical consequences of compromise. These methods employ elementary, vector, and matrix algebra. They make effective use of the Pearsonian, rank, and von Bortkiewicz correlation coefficients; the generalized Lagrange identity; and correct partition formulas, the terms of which correspond to those of Taylor expansions.[24] Finally, it is hoped this paper has demonstrated the value of multiplicative identities as frameworks for consistent formula design and for the appraisal of available data and of current practice in measuring.[25]

COMMENT

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Siegel's paper deals mainly with the choice of index-number formulas to be used in measuring input, output, and productivity. While we were both at Johns Hopkins University I learned a good deal from him on this subject. I have very little to add to what he has said in his paper, except to second most of it firmly. I agree with the main points that he makes, which I take the liberty of selecting as follows:

1. Each index should be a ratio of two aggregates or, in other words, a weighted arithmetic or harmonic mean of corresponding relatives.

2. The data for the three indexes for any economy or sector should have the same coverage. That is, they should apply to the same economy or the same sector, and if (as is almost always the case) they are based on a sample of the goods and services in the economy or sector, they should all be based on the same sample.

3. The indexes should be consistently designed, so that the productivity index is the ratio of the output index to the input index, i.e., so that any one of the three can be obtained directly from the other two.

4. The three foregoing requirements can be satisfied by more than one formula (e.g., Laspeyres and Paasche), and numerical results obtained will be different for the different acceptable formulas.

[24] See *Concepts and Measurement . . . op. cit.*, Chapter 3 and pp. 90–2, and various articles by the author in *Journal of the American Statistical Association* on index-number differences (September 1941, December 1941, June 1943).

[25] For comments by the author on two productivity publications appearing after this paper was presented, see reviews in *Journal of Business*, January 1960, pp. 63–4, and *Personnel and Guidance Journal*, May 1960, pp. 764–5.

5. The choice of the formula which satisfies conditions 1 to 3, as well as the choice of the procedures to be used in collecting data for the formula, should depend on the purpose for which the indexes are wanted.

6. In many, if not most cases, the data that have already been collected by somebody else are not those that one would want for constructing the indexes. In such a case it is necessary either to collect the desired data or to fashion some kind of approximation of the readily available data.

I can object to only two points that Siegel makes. First, he suggests that the classical theory of index numbers is sufficient for our purpose and that the so-called economic theory of index numbers, dealing with indifference curves and production functions, is of no help at all in designing productivity indexes. Of course he is right if he means that there are terrible problems involved in trying to aggregate indifference curves and production possibility curves, and that without quite severe simplifying assumptions we cannot easily interpret the results of such aggregation. Nevertheless, I feel that shifts of production functions are what productivity indexes are really about, and that in trying to measure productivity we will be ahead if we remember that production functions are in the theoretical background of what we are doing.

My second disagreement with Siegel is probably not a disagreement in substance at all. He says that an increase of efficiency in an economy which is the result merely of transfer of resources from less productive to more productive industries, with no change of productivity within any industry, should not be called an increase in productivity. This statement is made in connection with his plea that every productivity index should be a weighted mean of productivity relativities, which I referred to as item 1 above. I believe, as George Stigler said in his paper for this conference, that social organization is a kind of factor of production, and that any improvement in social organization which permits the transfer of resources from a less productive to a more productive industry ought to be regarded in one sense as an increase in productivity. I would agree with Siegel that this kind of increase in productivity ought to be clearly separated from the kind of increase which occurs within individual industries.

Siegel's brief discussion of partioning and eight "additional points" on labor productivity (see Section II) is particularly noteworthy. Some of these touch on many of the important issues of productivity measurement.

The paper contains a large number of references to Siegel's "Concepts and Measurement of Production and Productivity," a 108-page mimeographed document of the Bureau of Labor Statistics dated March 1952. This document, I am very sorry to say, is not readily available, but those who take Siegel's message to heart will find a great deal more detail there.[1]

I should now like to return to a discussion of three broader comments on productivity measurement.

First, I believe that the distribution of knowledge concerning technological possibilities is important. It is worth distinguishing two kinds of lags in the use of new knowledge. First, there are lags because capital equipment embodies the knowledge available when the equipment was designed, and capital equipment lasts quite a long time, so that at any moment there is a large amount of capital in use which is inferior to currently designed capital equipment based on better knowledge. Second, there is a lag in the use of new knowledge by people who are currently investing in new capital, because some designers of capital equipment have not yet learned about recent advances that would cause them to improve their designs.

These remarks suggest that three kinds of empirical study might be useful. First, a time series study of the productivity, when new, of plants constructed in successive years in a single industry, in order to discover the rate of progress of knowledge about how to build plants. An example of a study of this type is that by Anne Grosse (Carter) entitled "The Technological Structure of the Cotton Textile Industry," in Leontief's volume, *Studies in the Structure of the American Economy*. Second, a cross-section study of the productivity, when new, of a number of plants constructed in the same year in a single industry, in order to discover the dispersion of knowledge about how to build a plant at any point of time. Third, a time-series study of the productivity of each of several given plants, beginning when they were new and extending over their lifetime, to determine the extent to which the original design of a plant "freezes in" the knowledge available when the plant was designed, and to what extent new discoveries can be used to improve the productivity of existing plants. (I was glad to be told by Professor Leontief, in a comment from the floor following these remarks, that he believes he will be able to obtain data to perform some studies of these three types.)

The second of my three broader comments on productivity measurement deals with the relationship between materials and fuels on the one hand, and indexes of output, input, and productivity on

[1] Siegel advises that arrangements were made for another run (the third) in the summer of 1960.

the other. Except for input-output analysis of the Leontief type, most studies of production functions explicitly mention as input only things like labor and capital, ignoring materials and fuels. It seems to me that this is a theoretical oversight, or at best a failure to make explicit the assumptions used. Surely the better approach is to begin with a production function that explicitly makes gross output depend on labor, capital, and materials inputs. Net output or value added is then gross output minus materials input.

When dealing with a closed economy as a whole, of course, the distinction between gross and net output is not quantitatively important because a closed economy has no materials input except for things like minerals, fish, air, water, and sunshine, and hence nearly all of the value of gross output is value added. However, for any economy with substantial imports or for any sector of an economy the distinction becomes important.

It is not hard to show that if material input is a stable function of gross output, and if the production function just mentioned is stable, then gross output can be expressed as a stable function of labor and capital input alone, or alternatively, net output can be expressed as a function of labor and capital input alone. However, there is a big "if" involved, because materials input need not be a stable function of gross output. Indeed, if the price of materials changes relative to the price of labor and capital, then one can expect a substitution between materials on the one hand and labor and capital on the other. Thus, for example, the ratio of gross output to labor and capital input can rise in response to a reduction in the relative price of materials, without any change in any production function. It can also happen, of course, that the fall in the relative price of materials can be the result of a technological improvement somewhere in the economy, either a material-saving improvement in the material-using industry or an increase in productivity in the material-producing industry; or it can be the result of a reduction in the demand for the material on the part of some other user.

These considerations suggest that it is important to use productivity indexes in which attention is given to material input.

The last of my three general comments concerns the objectives of productivity research. Many authors of papers for this conference have said, I think rightly, that the kind of productivity measure we want depends on the question we want to answer. Let me try to sketch a framework within which I think we might agree on what kinds of things we are trying to measure. I think we have a twofold objective in productivity research. One objective is to make it possible to push forward our production frontier more rapidly or more cheaply than we now know how to do. This is a practical sort of objective. The other objective, I would say, is to understand both past and future increases in the money value of gross output, looked at from the supply side. This is a seeking-after-truth-for-its-own-sake sort of objective, though, of course, if we attain it we will be better able to attain the practical objective too.

Concerning the attempt to understand increases of money value of gross output, we as a profession have already proceeded as follows.

1. We have devised measurements of price change, and when their effect is taken account of, we are left with a change in *real* gross output.

2. We have subtracted real materials inputs to obtain a measure of real *net* output or real *value added.*

3. We have tried to measure inputs in the form of labor and capital, and we have divided the real value added measured by this real labor-and-capital input measure. The result is the now familiar *index of output over input*, i.e., real net output divided by real labor-and-capital input, of the type put forth by Abramovitz, Kendrick, Schultz, and others.

At first it seemed enough to compute such an index, to note that it appears to increase at about 1 per cent per year, and to attribute this growth to increases in technological knowledge. This is no longer sufficient. It is now necessary to try to get independent measurements of things that we believe are components of the index of real net output divided by real labor-and-capital input and see whether they account for observed rate of growth of that index. In other words, we should try to force to zero the residual or unexplained part of the increase in the money value of gross output. The following possibilities have been suggested (I continue to number them in series with the preceding steps):

4. Changes in the quality of the labor force.

5. Changes in the quality of the stock of capital, or in the ratio of the quantity of services produced by capital to the stock of capital.

6. Increasing returns to scale.

7. The fruits of investment in the search for new knowledge about production possibilities. Here I imagine a conventional production function, in which output depends on inputs of labor, capital, and materials, and also on a parameter describing the current state of technology. Also I envision a second production function, whose output consists of improvement in the technology that enters into the

conventional production function, and whose input consists of resources devoted to research activities.

I believe that if we can succeed in getting the residual down to zero, we will have a much better understanding of economic growth than we have now. There will then be plenty of time to discuss what factors to include in the definition of productivity and what factors to call by some other name, such as improvements in the quality of inputs, increasing returns to scale, and the like.