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CHAPTER 3

NBER Indexes: Methods of Construction and Comparisons Among Them

CHAPTERS 1 and 2 summarize long-term trends in the foreign trade of the United States as they are described by the new NBER indexes in conjunction with data previously available for later years. The remaining chapters deal mainly with the NBER indexes themselves, and thus with the period they cover : 1879 to 1923. The process of studying the technical characteristics of the indexes uncovers additional substantive findings relevant to the earlier chapters.

How the NBER Indexes Were Constructed

The NBER price and quantity indexes used in Chapters 1 and 2 are Fisher "ideal" index numbers. Paasche and Laspeyres indexes, employed later in this chapter, were an intermediate product in the computation of the Fisher indexes.

All the indexes were constructed in four segments : 1913-23, 1899-1913, 1889-99, and 1879-89, using the final year of each as the base. The segments were then linked at the overlapping years. The use of a single base for a period of ten or fifteen years has great computational advantages over annual linking, and also simplifies the interpretation of changes extending over several years. While avoiding the arbitrary character of bases far removed from the period studied, it does introduce into year-to-year comparisons some elements extraneous to the years compared.

A change in the price of an article which is of negligible importance in both of two years being compared could cause a substantial change in the Laspeyres index if the article were important in the base year. The Paasche index comparing two years can change even when all individual prices have remained the same, if the importance of the commodities has altered. Neither of these somewhat odd phenomena could occur in a direct comparison between two years.[1]

[1] In a direct comparison between years 1 and 2 the Laspeyres price index is $\frac{\Sigma P_2Q_1}{\Sigma P_1Q_1}$. In an indirect comparison of years 1 and 2 with year 0 as a base, the Laspeyres index is $\frac{\Sigma P_2Q_0}{\Sigma P_1Q_0}$ The Paasche index under indirect comparison, is

$$\frac{\Sigma P_2Q_2}{\Sigma P_0Q_2} \Big/ \frac{\Sigma P_1Q_1}{\Sigma P_0Q_1} \left(\text{or } \frac{\Sigma P_2Q_2}{\Sigma P_1Q_1} \Big/ \frac{\Sigma P_0Q_2}{\Sigma P_0Q_1}\right),$$

instead of, as in direct comparison,

$$\frac{\Sigma P_2Q_2}{\Sigma P_1Q_2} \left(\text{or } \frac{\Sigma P_2Q_2}{\Sigma P_1Q_1} \Big/ \frac{\Sigma P_1Q_2}{\Sigma P_1Q_1}\right)$$

Comparison of years from different segments is conceptually quite complicated, since it involves different sets of base year weights. It may be thought of as implying the assumption that the index for one period, if extended, would be roughly parallel to the index of the adjoining period.

The main advantage of the backward-looking character of the index—the property that the base is the final year of a period rather than the initial year—is that it permits the fullest use of the steadily increasing detail in which trade data were published. In the first quarter of 1879, for example, there were slightly over 200 import commodities and 230 export commodities listed in the official trade returns; in 1923 there were more than 800 import and 1200 export commodities.

Indexes constructed with the terminal year instead of the initial year as the base have a number of peculiarities which must be kept in mind when the different types of indexes are compared. For example, the substitution effects which are expected on theoretical grounds (price and quantity changes negatively correlated), will cause our Paasche price indexes to rise relative to the Laspeyres indexes, the opposite of the usual case with initial-year weights. On the other hand, where quantity and price changes are positively correlated, the Laspeyres price index will rise in comparison with the Paasche, again the reverse of the results with initial-year weights.[2]

The commodity classification used here is the result of compromise among several objectives: comparability with other indexes, the isolation of economically significant classes of commodities, and reliability.

We constructed the classification to fit, with the proper combining of indexes, into the classifications used by the U.S. Department of Commerce. Thus, none of our minor groups were entered in more than one of the five economic classes or the eleven commodity groups of the Department of Commerce.[3] The distinction between agricultural and nonagri-

[2] This is the phenomenon of "weight bias." Mills suggests that it is characteristic of short and medium periods, including business cycles, while the substitution relationship may prevail over long periods. (Frederick C. Mills, *Statistical Methods*, 3rd Ed., New York, 1955, p. 452 n). It is, of course, the substitution relationship that is familiar from theoretical discussions of index numbers assuming constant tastes.

We could interpret these phenomena in another way. Substitution relationships are more likely to be observed when supply conditions are changing rapidly and demand is relatively stable; and "weight bias" when demand is shifting more rapidly (the 1913–23 period for many commodities).

[3] The five economic classes are: crude materials; crude foodstuffs; manufactured foodstuffs and beverages; semimanufactures; and finished manufactures. The eleven commodity groups are: animals and animal products, edible; animals and animal products, inedible; vegetable food products and beverages; vegetable products, inedible, except fibers and wood; textile fibers and manufactures; wood and paper; nonmetallic minerals; metals and manufactures, except machinery and vehicles; machinery and

cultural products was also maintained. Classes were set up for groups that seemed interesting from an economic point of view, or that demanded separate treatment on sampling grounds. The latter groups would otherwise have been combined with others exhibiting substantially different price behavior. The separation of such groups both improves estimates of the price behavior of larger classes, and narrows the margins of doubt surrounding these estimates (see Chapter 5). The next step was the selection of "covered" commodities—those for which unit values were accepted as representing prices or for which price data could be obtained from other sources.[4] The other commodities are referred to as "uncovered."

The list of covered commodities rarely remained constant throughout a period. It was therefore often necessary for the index computation to have several base-year value totals [ΣP_0Q_0 (covered items)], each comparable in commodity composition to a different segment of the period.

When the selections had been made and matching base-year values computed, Paasche, Laspeyres, and Fisher "ideal" price indexes were calculated for the covered items in each minor class.

Following this, value indexes were calculated for each minor class, encompassing both covered and uncovered items. These indexes compare the total value of all commodities in the class with the base-year value for the same commodities. As was true of the covered items, the total list of commodities in a class changed during a period, mainly because items disappear from the published listing as one goes back in time. Such items were assumed to fall into the catchall class "all other articles."[5] As a result of these shifts, several base-year values—ΣP_0Q_0 (All items)—often were required for a minor-class value index, as well as for the price index.

Quantity indexes for minor groups were computed by dividing value indexes by the Fisher price indexes. The assumption underlying this operation was that changes in the prices of items not covered were parallel to those of covered items.[6]

vehicles; chemicals and related products; miscellaneous. See U.S. Department of Commerce, Bureau of the Census: *Schedule A, Statistical Classification of Commodities Imported into the United States*, January 1, 1954, pp. VII and XVII and *Schedule B, Statistical Classification of Domestic and Foreign Commodities Exported from the United States*, Part II, January 1, 1949, pp. XXIV and XXVII.

[4] The selection of covered commodities and the use of outside price data are discussed in Chapter 4.

[5] Sometimes a commodity disappeared by merger with another. In most such cases we placed them both in the same class during that period to minimize shifts in composition within periods.

[6] This is identical to the "coverage adjustment" used, for example, in Solomon Fabricant, *The Output of Manufacturing Industries, 1899–1937*, New York, NBER, 1940. See *ibid.* pp. 362–372 and Chapter 5, below, for a justification of this procedure in terms of the sampling assumptions used.

Price indexes for larger groups (the intermediate classes of Appendix B) were computed from data for the minor classes, giving each class the weight of both its covered and uncovered commodities. In effect, each minor class was treated as a commodity, with $\Sigma P_0 Q_0$ (All items) as its $P_0 Q_0$ and $\Sigma P_1 Q_1$ (All items) as its $P_1 Q_1$ The $P_1 Q_0$'s were calculated by multiplying $P_0 Q_0$'s by the Laspeyres price indexes, and $P_0 Q_1$'s were calculated by dividing $P_1 Q_1$'s by the Paasche price indexes. Those minor classes for which price indexes were computed were considered "covered" classes, analogous to covered items within minor classes. The calculated cross-products were summed across the minor classes to give the price and quantity indexes for intermediate classes, and these, in turn, were used to build the indexes for major classes and total exports and imports.

The base-year dates were selected on a number of grounds. The final year of the study—1923—was selected as the base year for the last period, because we felt that Cowden's indexes for exports[7] and an interpolation of the annual Department of Commerce series for imports could adequately fill the gap between that date and the beginning of the quarterly Department of Commerce series in 1929. The year 1913, the last year unaffected by the beginning of the European war, has been used as a base for many other prewar series. The years 1899 and 1889 which divided up the remaining period fairly evenly, were United States Census years, and therefore convenient for comparisons with domestic data.

Some other characteristics of the base years may be of interest. Three of them—1923, 1913, and 1899—are peak years in the NBER business cycle chronology, while the fourth, 1889, is roughly midway between a trough in April 1888 and a peak in July 1890. Against the more specific background of trade fluctuations, 1923 comes just after the trough in exports and imports following World War I, but is considerably above 1913. The latter comes at the end of a period of rising values, prices, and quantities for both imports and exports. The two decades from 1879 to 1899 mark something of an interruption in the very great rise in import and export values which characterized the post-Civil War period as a whole—an interruption resulting from a combination of increasing quantities and declining prices. The base year 1899 is situated just after the upturn in prices and import values, but several years after the upturn in export values. For import quantities, 1899 is in the middle of a fairly steady increase which covered the whole period 1879-1913; for export quantities it follows a period of very rapid growth and precedes a decade of retardation.

[7] Dudley J. Cowden, *Measures of Exports of the United States,* New York, Columbia University Press, 1931.

Comparison of Paasche and Laspeyres Indexes

The Paasche and Laspeyres indexes shown in Appendix A are of interest for two reasons. One is that they show the range of error arising from the comparison of our Fisher indexes with Paasche and Laspeyres price and quantity indexes from other sources. The second, discussed later in this chapter, is that the differences between the two types of indexes shed some light on relations between price and quantity changes.

Many series with which one might compare export or import prices (such as the GNP deflator, the wholesale price index, and most foreign indexes) are Paasche or Laspeyres price indexes. It is not immediately clear, therefore, to what extent the apparent differences between the U.S. indexes and other series, such as those discussed in Chapter 1, represent real divergences in behavior or only the results of comparing dissimilar types of index numbers. The computation of Paasche and Laspeyres indexes permitted us to judge, in any specific case, whether the latter was the case, and to note that fact in the text. Even where no specific comparisons are made, the extent of Paasche-Laspeyres differences indicates whether any relations found are strong enough to make this type of "formula error" unimportant.

Ratios of Paasche to Laspeyres indexes, which measure the percentage differences between them, fluctuated much more violently between 1913 and 1923 than before, according to the data for agricultural products, nonagricultural products, and the five Department of Commerce economic classes (Table 9). In ten out of sixteen cases the swings were wider in those ten years than in the previous thirty-four. All the cases in which the range was over 21 per cent occurred in the later period.

The most spectacular range was in manufactured goods exports, where the Paasche index reached a level 50 per cent higher than the Laspeyres in 1916 (on a 1913 base). Most of this great discrepancy can be traced to manufactured chemical products (Export Class 075 in Appendix C), in which the Paasche index soared to twice the Laspeyres in 1916 and remained almost as high in 1917. Within this class the responsibility can be placed on one commodity: smokeless and other powder (item 6 in class 075). Its price rose much more than the average for all chemicals, while the value of its exports, less than one million dollars in 1913 and 1923, reached 262 million in 1916 and 338 million in 1917.

This one commodity was thus of negligible importance in the 1913-23 comparison and in the Laspeyres index for 1916 (weighted by 1923 values), but was of overwhelming importance in the Paasche index for 1916. Its influence in total manufactured exports was reinforced by the weight of exports of fuses and explosive shells and projectiles (items 25, 26, and 28 in class 075), which were uncovered commodities in the same class. They amounted to only $652,000 in 1913 and $663,000 in 1923, but reached $394 million in 1916 and $256 million in 1917.

TABLE 9

RANGE OF VARIATION OF RATIOS (IN PER CENT) OF PAASCHE TO LASPEYRES PRICE INDEX (1913 = 100)

Major Class[a]	1879–1913	1913–1923	1879–1923
Exports, total	12.3	31.2	43.5
Agricultural prod. (209)	12.1	17.2	21.0
Nonagricultural prod. (222)	17.3	37.7	55.0
Crude foodstuffs (201)	20.6	11.4	27.3
Manuf. foodstuffs (203)	18.4	10.3	23.2
Crude materials (212)	4.8	8.6	12.9
Semimanufactures (213)	12.8	20.7	33.5
Manufactures (215)	13.2	50.5	63.7
Imports, total	11.4	12.3	23.7
Agricultural prod. (209)	8.8	20.4	29.2
Nonagricultural prod. (223)	10.4	7.9	15.3
Crude foodstuffs (201)	20.9	13.1	34.0
Manuf. foodstuffs (203)	8.6	7.0	10.1
Crude materials (212)	6.0	24.3	30.3
Semimanufactures (213)	11.3	19.9	19.9
Manufactures (220)	9.3	6.3	14.9

SOURCE: Tables A-20—A-23.

[a] Numbers following class titles are NBER major class designations as shown in Table A-30.

Because of the growth of these commodities, manufactured chemicals accounted for over a third of the total weight of covered classes in "manufactured products of mineral origin and rubber" (Export Class 147 in Appendix B) in 1916 and over 30 per cent in 1917, as compared with roughly 4 per cent in 1913 and 4½ per cent in 1923. The wide fluctuations in the Paasche-Laspeyres ratio, illustrated by this extreme case, are the direct result of wartime changes. At no other time does an insignificant commodity became a staple of international trade in a few months.

The Paasche-Laspeyres ratios were higher in 1923 than in 1879 for every major export and import class; for all but two of forty-five classes, they were higher in 1913 than in 1879 and higher in 1923 than in 1913 (Table 10). The only exceptions in the prewar period were two closely related export classes : agricultural products (Class 209) and products of animal or vegetable origin, except printed matter and rubber products (Class 210). The two exceptions in 1913-23 were Import Classes 203 and 204 (manufactured foods, including and excluding tobacco products).

Column 3 of Table 10 shows how different the changes in price would appear if measured by a Paasche instead of a Laspeyres index.[8] The Paasche index always shows a larger change, ranging from 2.7 per cent greater for Import Class 203 (manufactured foodstuffs) to more than 40 per cent for Import Class 202 (crude foodstuffs, including tobacco products).

Differences between Paasche and Laspeyres indexes not only shed light on the range of possible "errors" in comparisons of one type of index with another, but provide economic information as well. Both indexes are averages of identical price relatives for individual commodities, differing only in the weights they assign to each. In the Laspeyres indexes the price relatives are weighted by base-year values—those of the last year of each period in our indexes. (Alternatively, one could say that the prices are weighted by base-year quantities.) In the Paasche indexes each price relative is weighted by P_0Q_1, the base-year price multiplied by the given (earlier) year quantity; each price is weighted by the given-year quantity. The Paasche index thus gives more weight than the Laspeyres to those commodities which have declined in quantity relative to the average—those for which Q_1/Q_0 was greater than the average.

What does this difference in weighting imply as to the meaning of discrepancies between the two indexes?[9] Suppose, for example, that the Laspeyres index for a class is higher than the Paasche. Since the base year in our indexes is at the end, this means that the Laspeyres index has declined relative to the Paasche. It follows that the base-year weights were heavier than given-year weights for those commodities with the highest P_1/P_0—those for which prices fell the most or rose the least. There was a shift in quantity terms toward those commodities that fell relatively in price.[10] If, on the other hand, the Paasche index is higher, the base-year weights were lower for commodities with high P_1/P_0, that is, there was a shift in quantity terms toward those commodities that rose most in price or fell least.

It is clear, then, that a higher (relatively falling) Laspeyres index suggests that substitution (or changes in supply conditions) was of predomi-

[8] The Fisher index is of course closer to each of them than they are to each other.

[9] It is simplest, in this connection, to think in terms of the original indexes for the four periods prior to linking.

[10] This does not imply a shift in value terms. Evidence of such a shift could be found by comparing the Laspeyres index to an index with given-year value weights.

TABLE 10
RELATION OF PAASCHE TO LASPEYRES PRICE INDEXES
1879 AND 1923, MAJOR CLASSES
(1913 = 100)

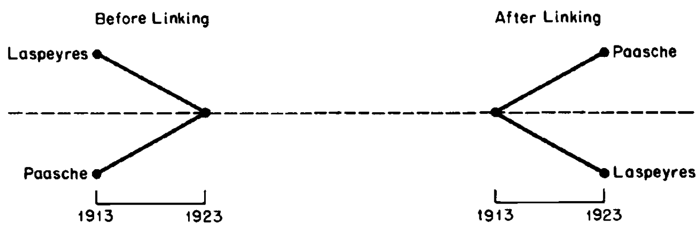
Class	Paasche Index as % of Laspeyres 1879 (1)	Paasche Index as % of Laspeyres 1923 (2)	1923 Ratio As % of 1879 Ratio (Col. 2 ÷ Col. 1) (3)	1923 Ratio Minus 1879 Ratio (Col.2 minus Col.1) (4)
		EXPORTS		
201	95.0	100.6	105.9	5.6
202	98.4	101.6	103.3	3.2
203	82.7	104.8	126.7	22.1
204	82.1	105.4	128.4	23.3
205	97.1	103.3	106.4	6.2
206	97.3	104.9	107.8	7.6
207	92.3	103.6	112.2	11.3
208	92.4	105.5	114.2	13.1
209	108.3	113.1	104.4	4.8
210	104.2	112.5	108.0	8.3
211	95.0	105.1	110.6	10.1
212	96.0	103.7	108.0	7.7
213	96.6	105.6	109.3	9.0
214	91.3	123.4	135.2	32.1
215	91.0	123.6	135.8	32.6
216	95.7	108.4	113.3	12.7
217	95.9	107.1	111.7	11.2
218	97.8	111.0	113.5	13.2
219	92.2	116.7	126.6	24.5
220	92.2	116.7	126.6	24.5
221	91.3	117.5	128.7	26.2
222	84.9	116.4	137.1	31.5
		IMPORTS		
201	82.7	106.2	128.4	23.5
202	77.4	108.5	140.2	31.1
203	95.9	98.5	102.7	2.6
204	93.7	99.0	105.7	5.3
205	91.5	103.5	113.1	12.0
206	88.8	103.9	117.0	15.1
207	89.8	104.8	116.7	15.0
208	86.2	105.2	122.0	19.0
209	93.8	117.2	124.9	23.4
210	89.7	114.4	127.5	24.7
211	99.0	120.0	121.2	21.0
212	97.9	120.6	123.2	22.7
213	91.0	104.7	115.1	13.7
214	95.1	115.2	121.1	20.1
215	94.5	115.5	122.2	21.0
216	90.3	113.1	125.2	22.8
217	91.7	105.5	115.0	13.8
218	91.1	105.6	115.9	14.5
219	89.7	112.0	124.9	22.3
220	91.0	105.6	116.0	14.6
221	89.7	112.0	124.9	22.3
222	90.9	104.7	115.2	13.8
223	89.6	104.9	117.1	15.3

SOURCE: Tables A-20—A-23.

nant importance, while the higher Paasche index implies that changes on the demand side were a stronger influence.[11]

The difference between these indexes and conventional formulations (with the base year at the beginning) should be kept in mind. In both cases a higher Laspeyres index implies substitution in response to relative price changes. But in our indexes, Laspeyres>Paasche means that the Laspeyres index is declining relative to the Paasche, while in the usual formulation, the reverse is true.

The interpretation of the Paasche-Laspeyres ratios is more complicated when the indexes are placed on a 1913 base by linking (Tables 9 and 10). For example, in the 1913-23 period the shifting of the base to 1913 transforms the situation as follows.



The interpretation must be reversed : the higher Paasche index implies substitution and the higher Laspeyres index, changes in demand. A preferable procedure is to concentrate attention on changes in the Paasche-Laspeyres ratio between any year and its matching base year. A relatively declining Laspeyres index, or a rising Paasche-Laspeyres ratio, implies shifts toward commodities becoming relatively cheaper. A decreasing ratio implies a shift in the opposite direction.

These relationships suggest that the upward drift of the Paasche-Laspeyres ratios, evident in most of the series (Table 10), is the result of substitution in favor of commodities with relatively falling prices. Although, strictly speaking, each year can be compared only with the base year of its period, a steady drift in the ratio can be identified with a gradual change in composition.

Several of the exceptions to the upward trend are associated with changes in demand. Most of the substantial declines in the Paasche-Laspeyres ratio occurred between 1916-18 and the 1923 base year (see basic tables, Appendix A). This means that high relative prices in 1916-18

[11] The observed price predominance does not necessarily imply a larger shift in the schedule, since the slopes of the supply and demand curves also influence the direction of the price-quantity relation.

were positively correlated with high relative quantities; many of the highest wartime prices were for those commodities (for example, gunpowder) which experienced spectacular increases in demand.

The cause of the downward trend in the Paasche-Laspeyres ratio for agricultural exports before 1913 is less clear. The relative increase in tobacco exports may be responsible. Tobacco was one of the few commodities whose prices increased even between 1879 and 1899, and one of the few to show a strong positive correlation between price and quantity relatives. These relations, together with the rapidly increasing consumption per capita, particularly of cigarettes, suggest that there were large increases in demand for tobacco products. A similar explanation can account for the fall in the Paasche-Laspeyres ratio for imports of manufactured foodstuffs. Here the main influence was the relative growth of sugar imports in the face of relatively increasing prices.

One of the sharpest declines in the ratio occurred in exports of crude foodstuffs after 1880-81. The high level during the first three years was clearly a demand phenomenon, when "a failure during the years 1879, 1880, and 1881, of the cereal crops of Europe and most other countries of the world, with the exception of the United States—a failure for which, in respect to duration and extent, there has been no parallel in four centuries—occasioned a remarkable demand on the latter country for all the food products it could supply at extraordinary prices."[12]

The information on price-quantity relations provided by the NBER indexes can be put in more formal terms. The Paasche-Laspeyres ratio, since it involves the extent and direction of responses of quantity changes to price changes, could be expected to bear some relation to the covariance between the two. And, in fact, a weighted covariance can be calculated from the two indexes.

The weighted covariance between price and quantity relatives for any year "1" is

$$Cov_w = \sum \left[\frac{P_0Q_0}{\Sigma P_0Q_0} \left(\frac{P_1}{P_0} - \frac{\Sigma P_1Q_0}{\Sigma P_0Q_0} \right) \left(\frac{Q_1}{Q_0} - \frac{\Sigma P_0Q_1}{\Sigma P_0Q_0} \right) \right]$$

If we use the following abbreviations:

Value index	$(\Sigma P_1Q_1/\Sigma P_0Q_0)$	=	V
Laspeyres price index	$(\Sigma P_1Q_0/\Sigma P_0Q_0)$	=	L_p
Laspeyres quantity index	$(\Sigma P_0Q_1/\Sigma P_0Q_0)$	=	L_q
Paasche price index	$(\Sigma P_1Q_1/\Sigma P_0Q_1)$	=	P_p
Paasche quantity index	$(\Sigma P_1Q_1/\Sigma P_1Q_0)$	=	P_q

[12] David A. Wells, *Recent Economic Changes*, New York, 1890, p. 6.

Then,

$$Cov_w = \sum \left[\frac{P_0Q_0}{\Sigma P_0Q_0} \left(V - \frac{P_1}{P_0} L_q - \frac{Q_1}{Q_0} L_p + L_pL_q \right) \right]$$

$$= V \sum \frac{P_0Q_0}{\Sigma P_0Q_0} - L_q \sum \frac{P_1Q_0}{\Sigma P_0Q_0} - L_p \sum \frac{P_0Q_1}{\Sigma P_0Q_0} + L_pL_q \sum \frac{P_0Q_0}{\Sigma P_0Q_0}$$

$$= V - L_qL_p - L_pL_q + L_pL_q$$

$Cov_w = V - L_pL_q$[13] or $\Sigma P_1Q_1/\Sigma P_0Q_0 - (\Sigma P_1Q_0/\Sigma P_0Q_0)(\Sigma P_0Q_1/\Sigma P_0Q_0)$[14]

The weighted covariance, then, is the value index minus the product of the Laspeyres price and quantity indexes. Since we do not list the Laspeyres quantity indexes in Appendix A, the covariances can be computed for the NBER indexes as $Cov_w = V(1 - L_p/P_p)$.

The covariances are related to the Paasche-Laspeyres ratios as follows:

$$\frac{P_p}{L_p} = \frac{V}{V - Cov_w}$$

We have not computed covariances for many of the classes in Appendix A. From the Paasche-Laspeyres ratios, it can be inferred that those for the major classes, at least, were almost all negative once the effect of linking to a 1913 base is removed. The covariances, in combination with the variances among price ratios calculated in Chapter 5 and Appendix E, permit one to estimate the slope of the relationship between price and quantity relatives, comparing each year with the corresponding base year. Thus

$$\text{Slope} = \frac{Cov_w}{\sigma_w}$$

where σ_w is the weighted variance of the price relatives.

To summarize, this chapter gives further evidence of the pervasiveness of negative relations between price and quantity changes. To the comparisons among countries and among major classes in Chapters 1 and 2, it adds indirectly derived information on price-quantity relations within

[13] This expression is Irving Fisher's factor-reversal test. The Laspeyres index passes this test (the expression is equal to zero) only when the covariance of price and quantity relatives (weighted by base year values) is zero, that is, when there is no correlation between price and quantity changes.

[14] A recent paper by Victor Zarnowitz, "Index Numbers and the Seasonality of Quantities and Prices," in *The Price Statistics of the Federal Government,* New York, NBER, 1961, points out that these relationships between the Paasche and Laspeyres price indexes and the covariance of price and quantity changes were originally derived by Ladislaus von Bortkiewicz in *Nordisk Statistisk Tidskrift,* II, 1922, pp. 374–379, and III, 1924, p. 218.

major classes. It suggests, furthermore, that these indirect methods, using the differences between Paasche and Laspeyres indexes, could reveal more information on these relationships within intermediate and minor classes, and could, in addition, be applied to problems outside the area of international trade wherever the two types of indexes are available.