Volume Title: Models of Income Determination

Volume Author/Editor: Conference on Research in Income and Wealth

Volume Publisher: Princeton University Press

Volume ISBN: 0-870-14184-8

Volume URL: http://www.nber.org/books/unkn64-2

Publication Date: 1964

Chapter Title: Determinants of Inventory Investment

Chapter Author: Michael C. Lovell

Chapter URL: http://www.nber.org/chapters/c1819

Chapter pages in book: (p. 177 - 231)

Determinants of Inventory Investment[1]

MICHAEL C. LOVELL

CARNEGIE INSTITUTE OF TECHNOLOGY

Introduction

THE crucial role of inventories in the generation of fluctuations in economic activity stands in marked contrast to the limited attention that economists have devoted in their empirical research to the study of inventory behavior. Of course, Jan Tinbergen [62], Lawrence Klein [33] [36], and Colin Clark [11] included inventory equations in their econometric models. Such studies as those of Edwin Mills [48] [50] [51], P. G. Darling [14] [15] [16], Franco Modigliani and Owen H. Sauerlander [52], Nestor E. Terleckyj [59], Jack Johnston [31], Murray Brown [8], and my own [40] [41] [42] have involved somewhat more extended econometric analysis of the behavior of inventories. Nevertheless, relative to the voluminous literature on consumption and fixed investment behavior, the area of inventory investment has barely been touched in econometric investigations.[2]

A convenient touchstone for appraising recent econometric investigations of aggregate inventory behavior is provided by the acceleration principle. In its most elementary form, the accelerator principle involves the assumption that entrepreneurs succeed in maintaining their stocks at an equilibrium level, H_t^e, which is linearly related to sales X_t.

[1] I am indebted to Richard Day, Ruth P. Mack, and Edward Mansfield for valuable suggestions and constructive criticism. Frederick Demming, James Keaten, Seong Y. Park of Yale, and E. Myles Standish of the Wesleyan University computation laboratory assisted with the computations. Research time for this paper was provided through the generosity of the Cowles Foundation for Research in Economics at Yale University and the National Science Foundation. The figures in brackets [ ] indicate references following the paper.

[2] Abramovitz [1, Chap. 21] presented a detailed analysis of the contribution of inventory investment to cylical fluctuations during the interwar period; for example, he contrasted peak-to-trough movements of GNP with the magnitude of inventory disinvestment during the downward half of the reference cycle; Thomas M. Stanback [58] presents a similar analysis of inventory movements during the post-World War II period. I have contrasted [40] the behavior of actual GNP with a hypothetical series derived by subtracting an estimate of the *gross* contribution of inventory investment to cyclical fluctuations, using the multiplier in order to compute the volume of consumption generated by inventory accumulation.

$$H_t^e = \alpha + \beta X_t \tag{1.1}$$

This assumption concerning the behavior of the inventory stock implies that actual inventory investment, ΔH_t, is proportional to changes in sales volume.

$$\Delta H_t = \beta \Delta X_t \tag{1.2}$$

Estimates of the parameters of this elementary model have been derived by D. J. Smyth [56] from annual deflated national income data for the United States covering the years 1948 through 1958.

$$\Delta H_t = -.86 + \underset{(.06)}{.30\Delta Y_t} + \underset{(0.24)}{0.07t} \qquad R^2 = .87 \tag{1.3}$$

The change in inventory is explained by ΔY_t, the change in gross national product; the coefficient of time is not significant.[3]

Complications have been introduced into the basic accelerator concept in an attempt to obtain a more adequate framework for the econometric investigation of inventory behavior. The simple accelerator model does not explain the *timing* of inventory investment. Moses Abramovitz [1] pointed out in his path-breaking study that in contrast to the implications of the simple accelerator hypothesis, actual inventory investment is *not* proportional to changes in output. Modifications of the basic accelerator model which provide an explanation of why inventory investment does not lead cyclical changes in gross national product are discussed in the second part of this paper.

Errors made by firms in anticipating future sales volume constitute another problem that must be considered in the econometric investigation of inventory behavior. The buffer-stock versions of the accelerator principle of Eric Lundberg [44] and Lloyd Metzler [47] incorporate expectational errors in the analysis of the inventory cycle. The difficulties involved in introducing either data on actual sales anticipations or suitable surrogates are discussed in detail in the third section of this paper.

Other factors in addition to sales and output may influence the volume of inventories that firms desire to hold. Several investigators

[3] The contrast between the small trend coefficient in the inventory investment regression with a trend parameter of 1.48, reported by Smyth for fixed investment, suggests that plant and equipment expenditures may be much more important than changes in business inventory as a generator of secular expansion and growth. On the other hand, the fixed investment accelerator coefficient is only 0.17, little more than half the estimated value of the inventory relation.

have attempted to explore within the context of accelerator models the possible effects of monetary policy upon inventory investment [7] [39] [42] [46]. The possible role of "speculative" or "price-hedging" purchases of stocks has also been considered [9] [36] [39] [41]. I have presented rough estimates of the impact of Department of Defense procurement upon inventory investment [42]. In the last part of this paper, I review these interesting questions concerning the structural determinants of inventory investment.

Equilibrium Inventory and Adjustment Lags

The lag of inventory investment behind changes in output might be taken into account by a slight change in the dating of variables. Kalecki [32] found that a closer fit was achieved with annual data for the United States for the period 1930 through 1940 by regressing inventory investment upon the change in output lagged six months; he reports a correlation of 0.913 for the lagged regression as opposed to 0.828 when the lag was not taken into account.[4]

An alternative procedure, frequently employed in econometric investigations of plant and equipment as well as inventory investment, involves the flexible accelerator complication suggested in a theoretical paper by Richard Goodwin [25]. With this approach it is assumed that the typical firm attempts only a partial adjustment of its inventory toward the equilibrium level within a single period. It is assumed that actual inventory investment is only a fraction of the discrepancy between last period's stock and the current equilibrium level.

$$\Delta H_t = \delta(H_t^e - H_{t-1}) + \epsilon_t \tag{2.1}$$

Here H_t^e represents the equilibrium level of stocks, an unobserved variable possibly determined by sales according to equation (1.1), but more likely influenced by additional variables as explained later in this paper. Only if δ, the reaction coefficient, is exactly equal to unity is an attempt made to adjust inventories fully to the equilibrium level. Consequently, an increase in sales volume or other determinant of equilibrium inventory may lead to a discrepancy between actual and equilibrium stocks which will only gradually be reduced with the passage of time.

[4] This evidence is not conclusive, however, for Smyth [56] reports that he achieved a closer fit with the unlagged rather than with the lagged regression.

As is well known, another expression equivalent to (2.1) is the Koyck [37] transformation

$$(2.2) \qquad H_t = \delta H_t^e + \epsilon_t + (1 - \delta)(\delta H_{t-1}^e + \epsilon_{t-1}) + (1 - \delta)^2(\delta H_{t-2}^e + \epsilon_{t-2}) + \cdots$$

Stocks are a weighted average of past equilibrium inventory levels. Robert Solow has explored an alternative scheme in which the weights are not restricted to simply successive terms in a geometric progression [57]. In his application of this procedure to inventory investment, which involves adding H_{t-2} as an additional explanatory variable to equation (2.1) above, only limited success was achieved, although quite interesting results were obtained with fixed investment [7]. It should be observed, however, that other investigators who have added lagged inventory investment to their inventory equation [16] [17] [39] have in effect followed the Solow rather than the Koyck procedure.

Several factors may account for the inertia of businessmen in adjusting inventories to equilibrium. Time may be required before orders placed to replenish stocks of purchased materials can be filled. Even if items are ordered promptly so as to maintain the sum of purchased materials inventory plus outstanding orders for additional items—what Ruth P. Mack calls "ownership position," adjusted to changes in sales volume—the physical magnitude of inventories actually on hand would still lag because of delays in delivery. Economies involved in large quantity orders may make it advisable for the cost-conscious firm to preserve only an imprecise relation between ownership position and sales volume. Because stocks are generally a conglomeration of heterogeneous items, the firm may find that considerable time is required in liquidating a surplus of a particular item, even though only a moderate excess in its aggregate inventory position is involved. When sales increase, a concomitant expansion of inventories may require enlarged warehouse capacity, and procuring this requires time. When sales of items produced to meet a seasonal pattern of demand prove disappointing, stocks may have to be carried over slack seasons before they can be liquidated. Such factors as these explain why firms are willing to suffice with a considerable departure of inventories from their equilibrium level.

Although most recent econometric investigations have involved a flexible accelerator principle, no attempt has been made to examine empirically possible determinants of the speed of adjustment. In studies of fixed investment behavior it is sometimes argued that there exists a maximum rate at which capital can be liquidated.[5] It might well be asked whether the speed with which inventories are adjusted toward the equilibrium level, the coefficient δ in equation (2.1), may not depend upon the sign as well as the magnitude of the discrepancy between actual and equilibrium inventory. The Goodwin formulation might be derived by assuming that the cost of adjusting inventories is related to the square of the discrepancy between equilibrium and actual inventories.[6] On the other hand, if costs of adjustment are simply proportional to the size of the discrepancy, firms may attempt an immediate adjustment to large departures from equilibrium but not respond at all when inventories are only slightly out of alignment.[7]

Several alternative formulations of equation (2.1) may be employed in econometric studies of inventory behavior. Instead of utilizing inventory investment as the dependent variable, one may fit an expression for the total stock of inventory

$$H_t = \delta H_t^e + (1 - \delta)H_{t-1} + \epsilon_t \tag{2.3}$$

This is obtained by adding H_{t-1} to both sides of (2.1). With this procedure, the method of least squares yields precisely the same parameter estimates as before, although the correlation coefficient may be expected to be somewhat larger. Another procedure, most appropriate in the study of finished goods inventory, is to utilize the definition of output $Q_t \equiv X_t + \Delta H_t$ in conjunction with (2.1) to obtain:

$$Q_t = X_t + \delta(H_t^e - H_{t-1}) + \epsilon_t \tag{2.4}$$

This approach has been employed by Modigliani and Sauerlander [52], by Edwin Mills [48] [50] [51], J. Johnston [31], and others in the analysis of the production decision. Observe that the error term ϵ enters equations 2.1, 2.3, and 2.4 in precisely the same form. This means that the application of least squares estimation procedures to any one of these three formulas will yield identical estimates of the

[5] Hicks [27] made the one-way accelerator play a prominent role in his model of the trade cycle. Leontief [38] also employed the construct in his generalization of the Hawkins multisector dynamic input-output model.

[6] In an interesting review by Charles Holt and Modigliani [29] of the contribution that the Carnegie quadratic decision rule approach can make to our understanding of inventory investment, the relationships between several alternative cost structures and the implied decision rule are considered.

[7] See Edwin Mills [48] and Martin Beckmann [4] concerning the details of this process.

parameters of the model and the same standard error of the estimate. On the other hand, the multiple correlation coefficient is sensitive to the particular form chosen for the regression; since generally $\sigma^2_{\Delta H} < \sigma^2_H$, we may expect

$$\frac{\sigma^2_{\Delta H} - \sigma^2_\epsilon}{\sigma^2_{\Delta H}} < \frac{\sigma^2_H - \sigma^2_\epsilon}{\sigma^2_H}$$

In certain other formulations of the same model the residual term enters in an essentially different form from the way it appears in (2.1). This is true of both the Koyck transformation and the expression for inventory investment obtained by first differencing equation (2.3).

$$\Delta H_t = \delta \Delta H^e_t + (1 - \delta)\Delta H_{t-1} + \Delta \epsilon_t \tag{2.5}$$

This procedure has been followed by Mills [48] [50] in an attempt to reduce problems created by autocorrelated error terms. A final possibility is to divide both sides of (2.3) by sales volume in order to have an expression for the inventory sales ratio

$$\frac{H_t}{X_t} = \delta\left(\frac{H^e_t}{X_t}\right) + (1 - \delta)\frac{H_{t-1}}{X_t} + \frac{\epsilon_t}{X_t} \tag{2.6}$$

For purposes of parameter estimation this last equation might be appropriately employed when one is concerned with the problem of heteroscedasticity, as when cross-section data can be utilized in the study of inventory behavior.

Whatever the form chosen for the regression, a problem is created by the fact that equilibrium inventory, H^e_t, is an unobserved variable.[8] If equilibrium inventory is regarded as a function of anticipated sales,

[8] Of course, the Munich business test surveys, the *Fortune* Business Roundup Survey, and the new Office of Business Economics survey of manufacturers' inventory and sales expectations provide some information on equilibrium inventory. But the data are often reported only in terms of the proportion of respondents reporting inventory "high," "low," or "about right." Even here, the validity of the response may be open to question. Thus, Murray Foss [21, p. 29] reports that "over the three-year period covered by the survey . . . relatively few firms have classified their stocks as 'low,' despite some sizable increases in inventories. At the moment it is too early to say whether the comparative absence of 'low' designations is an accurate portrayal of business sentiment regarding inventory conditions over this period, or whether it is the inevitable result of business thinking which always attempts to keep stocks as small as possible and thus classifies stocks as 'about right' so long as they are obviously not 'high.'" Foss also found it necessary to transform the raw anticipations series in order to obtain a relatively good predictor of actual inventory movements. Conceivably, an application of the "realization function" procedure, such as has been attempted by Murray Brown [8] on other data, would prove helpful here.

as with equation 1.1, substitution into (2.3) serves to eliminate the unobserved variable from the regression equation to obtain

$$(2.7) \qquad H_t = \delta(\alpha + \beta \hat{X}_t) + (1 - \delta)H_{t-1} + \epsilon_t$$

Since the coefficient of H_{t-1} provides an estimate of δ, the coefficient obtained for $\hat{X}_t$ may be unscrambled to obtain estimates of α and β, the parameters of the equilibrium inventory equation. Alternatively, the expression for equilibrium inventory could be substituted into (2.1) or (2.5) and utilized to explain investment in inventories. An estimate of the parameters of the equilibrium inventory equation can also be obtained by substituting into equation (2.4) the expression for production. In actual practice, of course, equilibrium inventory probably depends upon other variables in addition to sales, but this does not really introduce any new difficulties. Indeed, recognition of the distinction between equilibrium and observed inventory provides insight into the appropriate form in which additional variables should be introduced into the regression as well as a priori restrictions upon the magnitudes of parameters to be expected in empirical analysis.

Lawrence Klein [33] pioneered the application of the flexible accelerator to inventory data. Least squares estimates derived from deflated annual data for the period 1921–40 are presented by Klein.

$$(2.8) \qquad H_t = 1.06 + \underset{(1.15)}{4.66p_t} + \underset{(0.02)}{0.13X_t} + \underset{(0.08)}{0.48H_{t-1}} + e_t,$$

where X_t represents final sales (GNP less inventory change) and p_t is a price index.[9]

Later in this paper I shall show that utilization of actual sales rather than anticipated sales in the regression is equivalent to assuming that errors made by firms in anticipating future sales volume are randomly distributed. Klein's reaction coefficient is approximately 0.5, rather than 1.0, the value implied by the Smyth regression pro-

[9] Klein also estimated the same equation by the method of limited information within the context of a simultaneous equation model. It is interesting to note that the two sets of parameter estimates are practically identical, differing less than alternative parameter estimates of the same equation calculated by Carl Christ [10] from data covering a longer sampling period. There remains some question concerning the accuracy of Christ's data. Nevertheless, in certain applications, parameter estimates may well be more sensitive to the particular years utilized in the regression than to the choice between a simultaneous equation versus a single-equation least squares approach. Klein presented a third set of parameter estimates based on quarterly rather than annual data; a transformation procedure revealed that these coefficients were quite consistent with those derived from annual data.

cedure, equation (1.1); firms attempt to adjust halfway toward equilibrium each year. His regression implies that equilibrium inventories are determined by

$$(2.9)\qquad H_t^e = 2.03 + 8.96p_t + 0.25X_t,$$

an equation suggesting that the level of prices, perhaps because of money illusion, has a pronounced influence on equilibrium stocks.

Paul G. Darling [14] considered the forecasting value of an equation explaining the behavior of the quarterly book value of manufacturing inventory investment.

$$(2.10)\qquad \Delta H = -.387 + \underset{(.044)}{.415X_{-1}} - \underset{(.022)}{.212H_{-2}} + \underset{(.054)}{.324\Delta U_{-1}} + e$$

$$R = .945$$

$$\frac{\delta^2}{S^2} = 1.85$$

The change in inventory, ΔH, is explained by lagged sales, X_{-1}; stocks lagged two periods, H_{-2}; and the previous quarter's change in unfilled orders, ΔU_{-1}. Data extending from the third quarter 1947 through the third quarter 1958 were utilized in the regression. Darling reports that the lag structure was empirically determined by trial and error. No attempt was made to incorporate explicitly within the regression the impact of errors in anticipating sales volume. In order to determine the equilibrium inventory equation implied by Darling's regression, values of the explanatory variables that would not have led to an attempt to change the level of inventories must be determined. Setting $\Delta H = 0$ and solving the implicit equation thus obtained for H yields

$$(2.11)\qquad H^e = -1.82 + 1.95X + 1.53\Delta U$$

A dollar increase in quarterly sales generates almost twice as large an increase in equilibrium inventory; for every dollar increase in the change in unfilled orders, equilibrium inventory increases by $1.53. The reaction coefficient is 0.212, implying that firms in manufacturing attempt to liquidate roughly one-fifth of the discrepancy between equilibrium and actual inventory each quarter.

Nestor E. Terleckyj [59] has presented an interesting study focused upon the behavior of total inventory holdings in manufacturing and trade combined. Although Terleckyj did not work with deflated data, he did in certain of his regressions subtract the inventory valuation adjustment from the change in book value inventories in order to eliminate the accounting effect of revaluating the existing inventory stock. The percentage quarterly change in the book value of trade and manufacturing inventory, less the inventory valuation adjustment, was explained by the lagged inventory sales ratio, I_{-1}/X_{-1}; the ratio of new orders to sales, N_{-1}/X_{-1}; and the unfilled orders-sales ratio, U_{-1}/X_{-1}.

$$\text{(2.12)} \qquad \frac{\Delta I}{I} = -14.59 - \underset{(2.23)}{11.26}\frac{I_{-1}}{X_{-1}} + \underset{(5.52)}{30.75}\frac{N_{-1}}{X_{-1}} + \underset{(.57)}{1.88}\frac{U_{-1}}{X_{-1}} + e$$

$$R^2 = .78$$

The adjustment mechanism implied by Terleckyj's analysis is somewhat more complicated than that usually utilized in most studies of inventory investment. In order to see exactly what is involved, it is first necessary to determine the equation for equilibrium inventory. The level of inventory implying zero investment for given levels of sales and new and unfilled orders is obtained by setting $\Delta I = 0$ in equation (2.12) and then solving the resulting implicit equation to obtain

$$\text{(2.13)} \qquad I^e = -1.3X + 2.7N + .17U$$

The coefficient of sales, $-1.3 = -14.59/11.26$, has the wrong sign; it is unfortunate that in every one of Terleckyj's regressions the intercept term is negative. It should be positive if the equilibrium level of inventory is to be positively associated with sales. In order to find the nature of the delayed adjustment mechanism, it is only necessary to observe that (2.13) may be rewritten in the form

$$\text{(2.14)} \qquad \Delta I = 11.26\left(\frac{I}{X_{-1}}\right)(I^e_{-1} - I_{-1}).$$

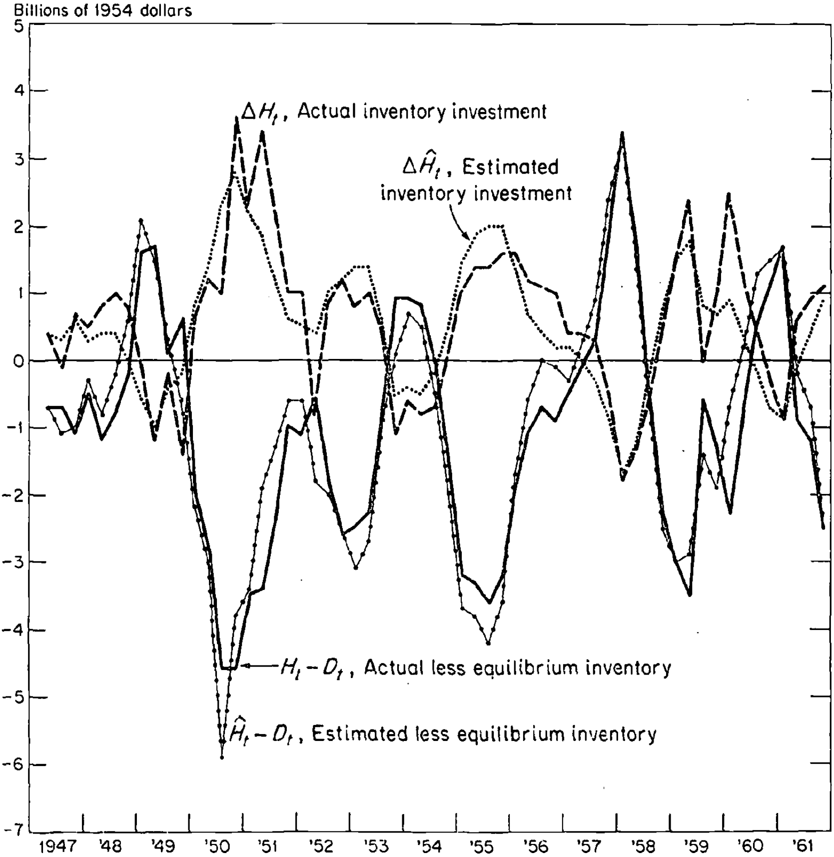
The speed of adjustment, $11.26(I/S)$, thus depends upon the current inventory sales ratio. Over the period of the regression the inventory-sales ratio averaged 1.56. Clearly, the parameter estimates presented by Terleckyj do not lend themselves to a simple interpretation in terms of the flexible accelerator concept.

As a final example, consider the following regression derived from deflated nonfarm inventory investment data for the period extending from the second quarter of 1947 through 1959. Nonfarm business inventory investment is explained by gross national product, the change in GNP, and the backlog of unfilled orders, all measured in constant 1954 dollars at quarterly rates (Table 1).

$$\text{(2.15)} \quad \Delta H_t = \underset{(2.9)}{2.49} + \underset{(.0405)}{.328X_t} - \underset{(.0485)}{.407H_{t-1}} - \underset{(.0925)}{.137\Delta X_t} + \underset{(.007)}{.043U_t} + e$$

$$R^2 = 0.736$$

In the chart, actual inventory investment is contrasted with the levels estimated by equation (2.15). The regression equation was computed in the summer of 1960. In order to illustrate how the model performs



outside the regression period, preliminary Commerce Department data on inventory change, estimates of inventory investment derived by equation (2.15), and the inventory discrepancy are recorded for all of 1960 and three quarters of 1961. Predicted inventory investment clearly tends systematically to fall short of actual inventory accumu-

TABLE 1

NONFARM INVENTORY INVESTMENT AND SURPLUS INVENTORIES, 1947–62
(billions of 1954 dollars at quarterly rates)

		Inventory Investment		*Surplus Inventory*	
		Actual ΔH_t	Estimated $\Delta \hat{H}_t$	$H_t - H_t^e$	$\hat{H}_t - H_t^e$
1947	I	0.4		1.1	
	II	0.4	0.4	−0.7	−0.7
	III	−0.1	0.3	−1.1	−0.7
	IV	0.7	0.6	−1.0	−1.1
1948	I	0.5	0.3	−0.3	−0.5
	II	0.8	0.4	−0.8	−1.2
	III	1.0	0.4	−0.2	−0.8
	IV	0.8	0.0	0.6	−0.2
1949	I	−0.1	−0.6	2.1	1.6
	II	−1.2	−1.0	1.5	1.7
	III	−0.2	−0.4	0.3	0.1
	IV	−1.4	−0.2	−0.6	0.6
1950	I	0.6	0.8	−2.2	−2.0
	II	1.2	1.4	−3.0	−2.8
	III	1.0	2.3	−5.9	−4.6
	IV	3.6	2.8	−3.8	−4.6
1951	I	2.3	2.2	−3.4	−3.5
	II	3.4	1.9	−1.9	−3.4
	III	2.3	1.2	−1.3	−2.4
	IV	1.0	0.6	−0.6	−1.0
1952	I	1.0	0.5	−0.6	−1.1
	II	−0.8	0.4	−1.8	−0.6
	III	0.8	1.0	−2.0	−1.8
	IV	1.2	1.2	−2.6	−2.6
1953	I	0.8	1.4	−3.1	−2.5
	II	1.0	1.4	−2.7	−2.3
	III	0.4	0.6	−0.9	−0.7
	IV	−1.1	−0.3	0.1	0.9
1954	I	−0.6	−0.4	0.7	0.9
	II	−0.8	−0.5	0.5	0.8
	III	−0.7	−0.2	−0.5	0.0
	IV	0.0	0.5	−2.0	−1.5
1955	I	1.0	1.5	−3.7	−3.2
	II	1.4	1.9	−3.8	−3.3
	III	1.4	2.0	−4.2	−3.6
	IV	1.6	2.0	−3.6	−3.2

(continued)

TABLE 1 (concluded)

		Inventory Investment		Surplus Inventory	
		Actual ΔH_t	Estimated $\Delta \hat{H}_t$	$H_t - H_t^e$	$\hat{H}_t - H_t^e$
1956	I	1.6	1.4	−1.7	−1.9
	II	1.2	0.7	−0.6	−1.1
	III	1.1	0.4	0.0	−0.7
	IV	1.0	0.2	−0.1	−0.9
1957	I	0.4	0.2	−0.3	−0.5
	II	0.4	0.0	0.3	−0.1
	III	0.3	−0.3	0.9	0.3
	IV	−0.5	−0.9	2.4	2.0
1958	I	−1.8	−1.7	3.3	3.4
	II	−1.3	−1.2	1.6	1.7
	III	−0.6	−0.2	−0.7	−0.3
	IV	0.5	0.8	−2.5	−2.2
1959	I	1.5	1.5	−3.0	−3.0
	II	2.4	1.8	−2.9	−3.5
	III	0.0	0.8	−1.4	−0.6
	IV	1.0	0.7	−1.0	−1.3
1960	I	2.5	0.9	−0.7	−2.3
	II	1.2	0.3	0.2	−0.7
	III	0.5	−0.2	1.3	0.6
	IV	−0.3	−0.7	1.5	1.1
1961	I	−0.9	−0.9	1.7	1.7
	II	0.6	−0.1	−0.2	−0.9
	III	0.9	0.4	−0.7	−1.2
	IV	1.1	0.9	−2.3	−2.5
1962	I	1.5	1.1	−1.6	−2.0
	II				

lation; much better predictions could have been made by taking advantage of the tendency toward autocorrelated disturbances.[10]

[10] Terleckyj has reported that his model did not perform too satisfactorily as a predictor of inventory investment during this same period. In the 1960–61 recession his equations indicated small amounts of inventory accumulation rather than the substantial disinvestment that actually took place [59, p. 161]. Of course, a test of the predictive ability of a model in this form is difficult at the current time because of the preliminary nature of data currently available on the 1960–61 recession. Judging by past experience, considerable revision in inventory data must be expected. An alternative test is to refit the equation over a subperiod and either observe the stability of the regression coefficients, a test reported by Terleckyj [59, p. 161], or examine the ability of the regression fitted to the subperiod to "predict" the observations excluded from the regression, a procedure I have applied in another connection [42, p. 131].

Regression (2.15) implies the following expression for equilibrium inventory:[11]

(2.16) $$H_t^e = 6.1 + .806X_t + .106U_t$$

The equilibrium inventory-to-final-sales ratio, obtained by dividing both sides of (2.16) by X_t, is

(2.17) $$\frac{H_t^e}{X_t} = \frac{6.1}{X_t} + .806 + .106\frac{U_t}{X_t},$$

not 0.806, the marginal equilibrium sales ratio. Two estimates of excess inventories, the discrepancy between equilibrium and actual inventory, are presented on the chart. The first of the estimated series, $H_t - H_t^e$, was obtained by application of equation (2.16). This series is obviously sensitive to the particular parameter estimates obtained in the regression analysis; it is also sensitive to the implicit assumption that the discrepancy between observed and estimated inventory investment may be attributed entirely to the stochastic term in (2.1), the inventory adjustment equation. There is no basis for assuming that (2.16) is nonstochastic; the observed residuals should be regarded as providing an estimate of the sum of stochastic disturbances in both (2.16) and (2.1). Although there is no obvious way of unscrambling the observed error in the surrogate measurement of excess inventory, a rough estimate of the magnitude of the problem is provided by $\hat{H}_t - H_t^e$; this second set of estimates of the discrepancy between equilibrium and actual inventory differs from the first by the observed residual.[12] The two estimates are quite similar, although $\hat{H}_t - H_t^e$ is subject to somewhat smaller fluctuations.

The provisional nature of the estimates of excess inventory cannot be too strongly emphasized. Single-equation least squares procedures were utilized in estimating the parameters of equation (2.15). Clearly,

[11] In an earlier study [41] I presented estimates of surplus inventory for durable manufacturing.

[12] If the only source of stochastic disturbance were the error made by firms in anticipating future sales volume, equation 2.17 could be regarded as nonstochastic. Such an approach suppresses the role of errors of observation and the possibility that variables have been omitted from (2.17). As long as the residuals of (2.17) and (2.3) are not negatively correlated, the standard error of the estimate may be utilized to obtain an upper bound on the variance of the residual of the equilibrium inventory equation.

Murray Foss has suggested to me that the large discrepancy between desired and actual inventories during the early phases of the Korean War period may have been in part the consequence of governmental controls on the accumulation of strategic materials. The impact of such controls might be interpreted as a disturbance in the speed-of-adjustment mechanism, equation 2.1. A more complicated model might consider the effects of the availability of external funds upon the speed-of-adjustment coefficient.

this is an invalid procedure, for one would expect an increase in aggregate inventory investment, via the multiplier, to influence the level of final sales. Slight comfort with regard to the problem of simultaneity is provided by an examination of results achieved by other investigators who have compared single-equation least squares estimates of inventory-holding equations with those achieved by more complicated estimation procedures. Klein contrasts least square with limited information estimates [33]; Ta-Chung Liu, two-stage least squares with the single-equation results [39]. Although the parameter estimates were only moderately sensitive to the particular estimation procedure utilized, much greater credence could be given to the excess inventory equation if an estimation procedure recognizing the simultaneity problem had been utilized. Quite apart from the question of simultaneity, the presence of the lagged capital stock in the equation contributes to biased if consistent estimates of the reaction coefficient.

In addition to the question of interpretation of the residuals in estimating equilibrium inventory, a serious problem is created by the strong autocorrelation of the observed residuals of (2.15). The Durbin-Watson statistic is an embarrassingly low 0.68, and the serial correlation coefficient is 0.63. The estimates of the parameters of equations 2.16 and 2.17 are sensitive to whatever method is adopted in order to deal with this problem. If, for example, we follow a transformation procedure described by Klein [34] we obtain

$$(2.18)\qquad \begin{aligned} H - .63H_{-1} = {} & \underset{(16.5)}{0.9} + \underset{(.0653)}{.3118}(X - .63X_{-1}) \\ & + \underset{(.0733)}{.5723}(H_{-1} - .63H_{-2}) \\ & + \underset{(.0904)}{.0402}(X_{-1} - .63X_{-2}) + \underset{(.0150)}{.0377}(U - .63U_{-1}) \end{aligned}$$

This implies the following equation for equilibrium inventory,

$$2.17' \qquad H_t^e = 5.7 + .7290X + .0881U$$

where the residual term is again neglected. Comparison of these coefficients with (2.17) reveals that the estimates of the equilibrium inventory equation are moderately affected by the transformation.[13]

[13] Although the sign test does not suggest autocorrelated error terms, it would have been interesting to have attempted further iterations with Klein's procedure until the regression coefficients stabilized. Of course, the existence of autocorrelation of the residuals of equation (2.15) does not in itself mean that the parameter estimates of that

In other inventory studies involving the flexible accelerator the problem of autocorrelated error terms has not proved particularly serious. In deriving my earlier estimates of equilibrium inventory for durable manufacturing and component industries the Durbin-Watson statistics were considerably larger. Duesenberry, Eckstein, and Fromm did not encounter a serious autocorrelation problem in their study of aggregate nonfarm inventory based on a more complicated equation.[14]

Derived estimates of unplanned or excess inventories have been utilized as explanatory variables in several regression studies. Klein [33] interpreted the residuals of his inventory determination relation, equation 2.6, as an index of the impact upon inventories of errors made by firms in judging market conditions; these residuals proved to be significant in the equation explaining adjustment in output, where output was defined as final sales plus inventory accumulation. In a recent study, Darling [16] utilized estimates, derived within the framework of the flexible accelerator, of the excess of equilibrium over actual inventory in an equation explaining fluctuations in the manufacturing production index. His significant results are not surprising, for the equivalence between the study of the production decision and inventory investment, as revealed by equation 2.4, means that the flexible accelerator concept itself implies the existence of a relation between excess inventories and production levels. Liu [39] showed cognizance of this relation when he utilized estimates of excess inventories in a price determination equation. The gross national product deflator declines when inventories are excessive, for then producers cut prices as well as curtail production; the effort is frustrated under Liu's assumptions, for the aggregate volume of sales does not respond to the price reductions.

---

regression are biased or inconsistent; it would be possible to retain the original parameter estimates and apply Wold's correction procedure to their standard errors [63, Chap. 13]; Klein's procedure does contribute to efficiency.

14 The model considered by Duesenberry, Eckstein, and Fromm [17, p. 798] is a complicated equation containing a number of lagged variables. But their empirical results might be the consequence of a much simpler structure of the form

$$\Delta I = \alpha + \beta_1 X + \beta_2 I_{-1} + \beta_3 U_{-1} + \epsilon;$$

for simple calculations yield, for arbitrary ρ,

$$I_t = (1 + \rho)\alpha + (1 - \rho b_1)X + \rho\beta_1 \Delta X + [\beta_2(1 - \rho) + \rho]I_{-1} + \rho\beta_2 \Delta I_{-1} + \beta_3(1 - \rho)U_{-1} + \rho\beta_3 \Delta U_{-1} + \epsilon - \rho\epsilon_{-1},$$

the equation they considered. If the residuals of the first equation are autocorrelated, the second equation will yield a closer fit and perhaps be more satisfactory for prediction purposes; on the other hand, it will not necessarily give a more accurate representation of the determinants of inventory investment.

An explanation of the timing of finished goods inventory investment that is closely related to the flexible accelerator is the concept of production smoothing. This approach emphasizes the costs involved in changing output rather than inventory levels. A firm may systematically accumulate inventory of finished goods during periods of slack demand by having production exceed sales in order to run them off later during periods of peak demand. This practice serves to minimize costs involved in changing production levels and work force; it enables the firm to meet a larger peak demand with given plant capacity, thus economizing on capital. This approach has been employed in empirical studies by Modigliani and Sauerlander [52], Mills [48] [50] [51], and Johnston [31].

The production-smoothing argument implies that the seasonal pattern in inventories (or production) cannot be explained entirely by concomitant seasonal movements in sales. The complication may be suppressed by employing seasonally corrected data, one of several approaches utilized by Modigliani and Sauerlander. An alternative to working with deseasonalized data is to include seasonal dummy variables within the regression equation. This procedure, utilized by Johnston, could facilitate a statistical test of the production-smoothing hypothesis.[15] At a cost of additional degrees of freedom, the regression may be fitted separately for each season; this procedure has been employed by Modigliani and Sauerlander and by Johnston. An advantage of this practice, emphasized by Modigliani and Sauerlander, is provided by a theoretical demonstration that the extent to which changes in sales volume and other explanatory variables affect production levels and planned inventory depends upon whether the current quarter is typically one of seasonally high or low sales volume.

The production-smoothing hypothesis would not be of direct use in understanding cyclical movements in inventory investment if it only provided an explanation of a divergence of the seasonal pattern of inventory from that of sales volume. But the production-smoothing hypothesis may be invoked to explain the cyclical lag in inventory investment behind changes in sales volume that is to be observed in deseasonalized as well as uncorrected data. Mills introduced lagged

[15] The appropriate F-ratio for determining whether the addition of the set of seasonal dummies led to a significant improvement in fit was not provided by Johnston. For one model [31, p. 255] fourteen seasonal dummies out of thirty-two computed for eight industries were significant at the 5 per cent level; eight of these were significant at the 1 per cent level. For another model [p. 250], tested on the same data, twelve out of thirty-two were significant at the 5 per cent level; four of these, at the 1 per cent level.

production into equation 2.4; only a partial adjustment of production to the equilibrium level takes place within any one period. In this form, the production-smoothing hypothesis explains a lag of production behind changes in sales volume. Since sales differ from production by the change in inventory, the production-smoothing concept implies that the change in production, ΔQ, should appear in equation 2.1, explaining inventory investment, with a negative sign.[16] Since sales differ from production by the change in inventory, this leads to a lag of inventory behind sales changes over and above that which results from inventory-smoothing considerations. The evidence with regard to the cyclical form of the hypothesis is not conclusive; while lagged production has proved statistically significant in some regressions, in other applications the coefficient of lagged output has consistently had the wrong sign.[17] Additional evidence providing stronger support for the production-smoothing hypothesis is presented below in conjunction with an analysis of problems created by errors made by firms in anticipating sales volume.

Errors in Anticipating Sales Volume

Because production requires time, a firm selling its output in imperfect markets must have decided upon the current level of output on the basis of advance estimates rather than precise knowledge of demand conditions. When sales exceed the anticipated level, the buffer of finished goods inventory carried in order to prevent runouts is depleted; on the other hand, when sales forecasts are unduly opti-

[16] If Q_t^* is the level of output required to adjust inventories to the level prescribed by (2.2), the production-smoothing hypothesis implies that $Q_t = \gamma Q_t^* + (1 - \gamma)Q_{t-1}$. But this implies that actual inventory will fall short of the level suggested by (2.2) by

$$Q_t - Q_t^* = [1 - (1/\gamma)](Q_t - Q_{t-1})$$

where $1 - (1/\gamma) < 0$.

[17] When Edwin Mills ran his earlier tests [48] he analyzed his data in first-differenced form in order to avoid autocorrelated error terms. The production-smoothing coefficient inevitably had the wrong sign. In a more recent study [51], based on other data, Mills presents the results of regressions on non-first-differenced observations for four separate industries. In these regressions the lagged production terms generally have the right sign and are significant in terms of the customary tests; although the Durbin-Watson statistic indicates positive autocorrelation of residuals in two cases, it must be remembered that while inefficiency rather than bias is implied by autocorrelation, the standard tests of significance are not valid. I have considered [40, pp. 111–117] a flexibility-of-production term in regressions in which inventories serve as the dependent variable. The results were disappointing, perhaps because seasonally corrected data had to be employed. Approximately half the time the production-smoothing coefficient had the wrong sign; the coefficients were generally small relative to their standard errors.

mistic, unplanned inventory accumulation occurs. Only a firm fabricating goods to specific order escapes the problem. Two essential modifications of the basic accelerator model are required in order to take into account the complications created by anticipation errors in the analysis of finished goods inventory behavior. In the first place, anticipated sales ($\hat{X}_t$) rather than actual sales must be inserted into the equilibrium inventory equation when the planned level of inventory is being determined. In addition, the impact of errors in anticipating sales volume upon the level of inventory must be taken into account.

These considerations suggest that finished goods inventory behavior is determined by the following equation

$$(3.1)\qquad \begin{aligned} H_t &= \delta\alpha + \delta\beta_1\hat{X}_t + (1-\delta)H_{t-1} + \lambda(\hat{X}_t - X_t) + \epsilon_{1t} \\ &= \delta\alpha + \delta\beta_1 X_t + (1-\delta)H_{t-1} \\ &\qquad + (\delta\beta_1 + \lambda)(\hat{X}_t - X_t) + \epsilon_{1t} \end{aligned}$$

This modification of the basic accelerator may be further complicated if equilibrium inventories depend on other variables in addition to sales, or if production smoothing is introduced.[18] The surprise element $\hat{X}_t - X_t$, the excess of anticipated over actual sales, is preceded by λ, the "production adaptation coefficient," in order to take into account a complication introduced by Modigliani and Sauerlander [52]. If λ equals unity, the equation implies that the firm does not succeed in even partially compensating for errors made in anticipating sales during the period of observation; finished goods inventory falls below the planned level by the full extent of the forecast error. A λ less than unity implies that the firm manages at least partially to offset errors made in anticipating sales volume. At a possible cost of premium wage payments or, alternatively, losses due to idle time, production schedules may be revised on the basis of current sales experience. If $\lambda = 0$, the revision of the production plan is drastic enough to keep inventory at the planned level, a magnitude that may no longer be appropriate for current sales experience. If $\lambda = -\delta\beta$, the firm succeeds completely in compensating

[18] Furthermore, the anticipated sales variable determining equilibrium inventory, the $\beta_1\hat{X}_t$ term in (3.1), may most appropriately refer to moderately long-term expectations; in contrast, the error-of-expectations term, $\lambda(X_t - X_t)$, involves short-term anticipations of sales for the current period. This distinction, emphasized by Holt *et al.* [28], may be of but secondary importance for econometric studies if firms generally regard sales as having a stable seasonal pattern, so that short- and intermediate-range expectations are more or less proportional, particularly if seasonal dummy variables, deseasonalized data, or separate regressions for each season are employed.

for any errors made in anticipating sales volume.[19] It is apparent that the value of λ encountered in any empirical study would depend in part upon the length of the observation period involved in data collection relative to the duration of the production process; a smaller value of λ should be obtained when quarterly or annual rather than monthly or weekly data are utilized in the empirical study. Furthermore, the concept of adapting production so as partially to eliminate anticipation errors is the converse to the production-smoothing conjecture. If inventories are carried in order to iron out short-term fluctuations in sales, inventory rather than output levels may be expected to bear the brunt of the burden when sales anticipations prove to be incorrect.

UTILIZING ANTICIPATIONS DATA

When suitable data on expectations are available the parameters of the inventory equation may be estimated directly. This approach has been followed by Modigliani and Sauerlander [52], Mills [51], and in my own work [40] with Railroad Shippers' Forecast data. Murray Brown [8] and Peter Pashigian [54] have presented reports of studies based on *Fortune* magazine forecast data and annual Commerce Department–Securities and Exchange Commission anticipation series. T. Thonstad and D. B. Jochems [61] have utilized Munich business test data. None of these sets of data is entirely appropriate for the purpose, either because the data are presented in a form that requires transformation or because the number of observations is inadequate. Furthermore, a controversy continues as to whether the tendency of observed anticipations to regress toward former levels should be interpreted as implying that the data are subject to systematic measurement error or as revealing an important characteristic of actual anticipations.[20]

Consider the following regression derived from quarterly constant-dollar data on manufacturing finished goods inventory for 1948–55:

[19] As with δ, there is some question as to whether λ should be regarded as a parameter of the system unaffected by the magnitude or direction of the forecast error. Under the assumption of profit maximization the answer depends upon the costs involved in adjusting the work force and in sufficing with an inventory that is not at the equilibrium level; only if such costs are symmetric would λ be independent of the sign of the forecast error. If the costs are proportional to the square of the discrepancy, λ might be independent of the magnitude of the error.

[20] At one extreme, there is the argument of Albert Hart [26] that the expectations data must be reconstituted. On the other hand, Bossons [5] argues on the basis of cross-section evidence that expectations are actually regressive, and rightfully so!

$$(3.2a) \qquad H_t = -903.1 + \underset{(.0195)}{.0746X_t} + \underset{(.0262)}{.1283(\hat{X}_t - X_t)} + \underset{(.0599)}{.7591H_{t-1}}$$

$$R^2 = .970$$

These estimates may be transformed in order to obtain a marginal desired inventory coefficient, β, of 0.098; the production adaptation coefficient is 0.054, while the inventory reaction coefficient is 0.24. For total manufacturing durables

$$(3.2b) \qquad H_t = -515.5 + \underset{(.0144)}{.0906X_t} + \underset{(.0214)}{.1213(\hat{X}_t - X_t)} + \underset{(.0519)}{.6871H_{t-1}}$$

$$R^2 = .978$$
$$d = 1.33$$

For nondurables

$$(3.2c) \qquad H_t = 38.02 + \underset{(.0291)}{.0321X_t} + \underset{(.0310)}{.1035(\hat{X}_t - X_t)} + \underset{(.0811)}{.8785H_{t-1}}$$

$$R^2 = .953$$
$$d = 1.86$$

In every case the surprise element, $\hat{X}_t - X_t$, appears with a coefficient that is several times its estimated standard error. If these figures could be taken at face value, they would suggest that although manufacturing firms are not prompt about adjusting inventories to their equilibrium level, they are extremely agile in adapting production schedules to unanticipated changes in sales volume.

Another example is provided by regressions obtained with quarterly data on the cement industry, covering 1947–56. Because the data are not deseasonalized, it is possible to subject the production-smoothing hypothesis to further test. It is to be observed that when the seasonal dummy variables[21] are excluded from the regressions (3.3a) and (3.3c), the current sales term has the wrong sign; furthermore, the introduction of the dummy variables leads to a reduction in the unexplained variance which is significant at the 5 per cent level, providing further support for Johnston's formulation of the production-smoothing hypothesis.

[21] $d_1 = 1$ in first quarter, zero otherwise;
$d_2 = 1$ in second quarter, zero otherwise; and
$d_3 = 1$ in third quarter, zero otherwise.

$$\text{(3.3a)}\quad \Delta H = \underset{(.0912)}{-.4596X} + \underset{(.4313)}{.2429(\hat{X} - X)} - \underset{(.1698)}{.2131H_{-1}} + \underset{(4447.8)}{34{,}439} + e \qquad R^2 = .643 \quad \bar{S}_e = 6{,}944 \quad n = 39$$

$$\text{(3.3b)}\quad \Delta H = \underset{(.0037)}{.0047X} + \underset{(.1855)}{.5753(\hat{X} - X)} - \underset{(.1124)}{.2322H_{-1}} + \underset{(1{,}585)}{11{,}446d_1} - \underset{(2{,}687)}{8{,}215d_2} - \underset{(1{,}914)}{14{,}213d_3} + \underset{(1{,}824)}{9{,}340} + e \qquad R^2 = .9408 \quad \bar{S}_e = 2{,}958 \quad n = 39$$

$$\text{(3.3c)}\quad \Delta H = \underset{(.0794)}{-.2615X} + \underset{(.3281)}{.2203(\hat{X} - X)} - \underset{(.1294)}{.1705H_{-1}} - \underset{(.2264)}{1.1652\Delta Q} + \underset{(4{,}096)}{22{,}551} \qquad R^2 = .799 \quad \bar{S}_e = 5{,}282$$

$$\text{(3.3d)}\quad \Delta H = \underset{(.0035)}{.0040X} + \underset{(.1781)}{.5217(\hat{X} - X)} - \underset{(.1085)}{.2713H_{-1}} - \underset{(.1717)}{.3598\Delta Q} + \underset{(1{,}711)}{9{,}751d_1} - \underset{(2{,}834)}{5{,}646d_2} - \underset{(2{,}020)}{12{,}376d_3} + \underset{(1{,}763)}{10{,}008} \qquad \bar{S}_e = 2{,}813 \quad R^2 = .948$$

The change-in-quantity variable appears significant and with the correct sign in (3.3d), suggesting that in the cement industry production-smoothing has more than a seasonal influence upon inventory and production decisions.

Unfortunately, the expectational data utilized in all these regressions are an inaccurate synthetic series constructed from suspect Railroad Shippers' Forecast data. A description of the procedure utilized in deriving the $\hat{X}_t$ series may be relegated to the Appendix of this paper. It is necessary to emphasize at this point that since the discrepancy between anticipated and actual sales volume $(\hat{X}_t - X_t)$ is not observed with precision, the least squares procedure may be expected to yield a biased estimate of the production adaptation coefficient. This danger is enhanced if there is a systematic element in the observation error. Suppose that the actual mistake made by the firm in anticipating future sales is proportional to the observed error:

$$\hat{X}_t^a - X_t = b(\hat{X}_t^o - X_t) + \epsilon_{4t}, \tag{3.4}$$

where $\hat{X}_t^a$ is the actual level of anticipated sales and $\hat{X}_t^o$ observed anticipations. It has been argued by Albert Hart [26], in connection with the Railroad Shippers' Forecast anticipatory data, that the observed sales anticipations suggest that firms are unbelievably inaccurate forecasters. If this equation is substituted into (3.1), one obtains

$$H_t = \delta\alpha + \delta\beta_1 X_t + (1 - S)H_{t-1} + (\delta\beta_1 + \lambda)b(\hat{X}_t^o - X_t) + (\delta\beta_1 + \lambda)\epsilon_{4t} + \epsilon_{1t},$$

an equation of the same form as (3.1). Clearly, if observed expectational data systematically overstate errors made by firms in anticipating future sales volume, the regression will tend to suggest an excessive degree of flexibility in production plans.[22]

Although considerable improvement in the availability and accuracy of expectational data is currently being made, a number of studies testify to the extreme inaccuracies present in the *ex ante* data currently available. In his recent investigation of inventory investment, utilizing expectational data compiled by the Business Roundup staff of *Fortune* magazine, Murray Brown concluded with the comment that ". . . the *Fortune* ex ante variables provide only marginal gains to the prediction of inventory behavior. However, the anticipations data may become more useful in the future as observation error is reduced." Undoubtedly, expectational data in time series form will prove of increasing usefulness as additional observations become available.[23] While in principle additional degrees of freedom might be obtained by utilizing observations on individual firms, it has not yet proved possible to obtain data in the cross-section form most useful for econometric investigation of inventory behavior.

[22] When, in the summer of 1958, I originally computed the coefficients of equation (3.2), I assumed that production plans were completely inflexible; so the error of anticipations, $\hat{X}_t - X_t$, should enter the equation with a coefficient of unity. When the regression was run with the coefficient of the prediction error forced equal to unity, the fit was grossly unsatisfactory. I was then led, as a result of learning of Albert Hart's [26] attempts to "reconstitute" the basic Railroad Shippers' Forecast data, to the conjecture summarized by equation (3.4). Some time later, I was reminded by Arthur Okun of the Modigliani-Sauerlander [52] point that production plans might have an element of flexibility, permitting their revision when actual sales proved to be developing in a different direction from that anticipated.

[23] The new Office of Business Economics Anticipation Survey [21] should prove particularly useful.

One way of judging both the validity and the usefulness of actual anticipations data is to compare the results obtained when the data are utilized to explain inventory behavior with those provided when proxies for actual anticipations data are used. It is necessary to review various procedures that have been devised in order to circumvent the difficulties created by the meagerness of data on actual sales expectations.

STRUCTURAL PROXIES FOR ANTICIPATIONS

One alternative to the utilization of actual anticipations data is to make some particular assumption about the structure by which anticipations of future sales volume are actually generated. It might be assumed, for example, that the structure explaining the generation of expectations takes the form

$$\hat{X}_t = \nu_0 + \nu_1 U_t + \nu_2 \Delta U_t + \nu_3 X_{t-1} + \epsilon_{5t}, \tag{3.5}$$

where U_t is the backlog of unfilled orders, and $\Delta U_t = U_t - U_{t-1}$. If we substitute into equation (3.1) because $\hat{X}_t$ is unobserved, we obtain the equation

$$\begin{aligned} H_t = \delta\alpha + (\delta\beta_1 + \lambda)(\nu_0 + \nu_1 U_t + \nu_2 \Delta U_t + \nu_3 X_{t-1}) \\ + (1 - \delta)H_{t-1} - \lambda X_t + (\delta\beta_1 + \lambda)\epsilon_{5t} + \epsilon_{1t} \end{aligned} \tag{3.1'}$$

More generally, this procedure involves the assumption that actual anticipations are some specified linear function of exogenous or predetermined variables.

$$\hat{X}_t = \sum \nu_i E_{i,t-1} + \epsilon_{5't} \tag{3.5'}$$

Here the ν_i are unknown structural coefficients and the $E_{i,t-1}$ specified predetermined or exogenous variables; $\epsilon_{5't}$ is a stochastic disturbance. If this equation is substituted into (3.1) we obtain

$$\begin{aligned} H_t = \delta\alpha + (\delta\beta_1 + \lambda) \sum \nu_i E_{i,t-1} + (1 - \delta)H_{t-1} \\ - \lambda X_t + (\delta\beta_1 + \lambda)\epsilon_{5t} + \epsilon_{1t}. \end{aligned} \tag{3.1''}$$

Once more there is an error of observation connected with an explanatory variable, raising the danger of biased estimates of the parameters of the equation. An additional difficulty with this technique is that its application does not yield an estimate of the marginal desired inventory coefficient, β_1, as the ν_i are unknown. Although the application of this procedure might provide some indication about the relative importance of various determinants of expectations, some of the variables thought to be determinants of expectations may actually have a direct influence upon equilibrium inventory, leading to a lack of identification. For example, the backlog of unfilled orders might influence sales anticipations; it might also have a direct influence upon desired inventory, and would therefore belong in equation (3.1) as well as (3.5′). Such influences cannot be disentangled when an expression relating to the structure of expectations is substituted into the inventory determination equation. Consequently, this procedure yields less information concerning the structure of the inventory determination equation than might potentially be gained if good data on actual anticipations were available.

The investigator who wishes to employ this procedure has a host of alternative specifications of equation (3.5′) to consider. Alain Enthoven [18] attributed naïve expectations to entrepreneurs in an interesting study of inventory behavior. If $\hat{X}_t = X_{t-1}$, and if it is assumed that both the reaction and production adaptation coefficients are unity, equation (3.1) may be written

$$H_t + X_t - X_{t-1} = \alpha + \beta_1 X_{t-1} + \epsilon_{1t}. \tag{3.6}$$

By making the total inventory stock plus the change in sales the dependent variable, Enthoven ensured that the reaction coefficient would be unity.[24] Johnston [31] has suggested that for nondeseasonalized quarterly data the expectations function may take the more complicated form

$$\hat{X}_t = X_{t-4} + \nu_1 X_{t-4}\left(\frac{X_{t-1} - X_{t-5}}{X_{t-5}}\right) + \epsilon_{7t}. \tag{3.7}$$

If this expression is substituted into (3.1), estimates of the marginal desired inventory coefficient may be obtained. Needless to say, valid results will be provided by this procedure only if the structure by which expectations are generated as well as the inventory equation have been correctly specified. Johnston himself has doubts about this particular formula. For one thing, the parameter estimates are not too satisfactory. He considers an alternative, more flexible expectations-generating equation, due to Charles Holt [28], that is

[24] When this equation was fitted to GNP data and to manufacturing and trade figures, an extreme problem of serially correlated disturbances was encountered. Although Enthoven ingeniously applied a correction procedure of Herman Wold [63] in order to test the significance of the marginal desired inventory coefficient, the fact remains that the highly autocorrelated disturbances imply that the lag of inventory investment behind changes in sales is not adequately explained by the assumption of naïve anticipations. A delayed pattern of response rather than a reaction coefficient of unity may be more appropriate.

related to the "adaptive expectations" concept of Marc Nerlove [53] and McGee [45], but more complicated in that both seasonal and trend terms are assumed to be determined by a distributed lag. Johnston proceeds to compute artificial $\hat{X}_t$ series for alternative sets of possible values of the parameters of this second expectations-generating equation, and then fits a production adjustment equation, contrasting the closeness of the fits obtained with alternative values of the expectation parameters.[25] Potentially, the use of such surrogate procedures may eventually yield information concerning the way in which expectations are generated as well as an understanding of the production and inventory decisions.[26]

ACTUAL SALES AS A SURROGATE MEASURE OF ANTICIPATIONS

Edwin Mills [48] [49] [50] [51] has argued that a second alternative to the utilization of anticipations data is to employ *actual* sales (X_t) as a proxy for the anticipated sales volume. This procedure was implicit in the pioneering Klein study [33] based on data for the interwar period. Mills has spelled out its rationale in detail. It is not supposed that firms are clairvoyant. It is assumed that whatever the procedure utilized by the firm in predicting demand, it is not biased and that the errors of prediction are random;[27] hence,

$$\hat{X}_t = X_t + \epsilon_{8t}; \quad \mathbf{E}(\epsilon_8) = 0 \tag{3.8}$$

Substitution of this equation into (3.1) yields an equation equivalent to (2.3):

$$H_t = \delta\alpha + \delta\beta_1 X_t + (1 - \delta)H_{t-1} + (\lambda + \delta\beta)\epsilon_{8t} + \epsilon_{1t} \tag{3.9}$$

Klein calls the residuals "undesired inventory"; he presents numerica estimates of the disturbances for the sample period [33, p. 111].

[25] Johnston cites Ferber's study of the Shippers' Forecasts [19] as partial support for the assumptions he makes concerning the structure of the equation generating expectations; one interesting use of whatever expectations data is available would be in exploring the most fruitful assumptions to make concerning the structure of expectations in studying inventory behavior.

[26] Johnston's evidence is not decisive, as five out of eight regressions yielded a closer fit as measured by the multiple correlation coefficient with functions of form (3.7); this may be seen by comparing Tables IV and V in Johnston's study; on the other hand, the estimated values of the parameters appear somewhat more reasonable with the more complicated regression function. Johnston also makes comparisons in terms of predictive ability, and here again the evidence is not decisive.

[27] It must be noted that this assumption does not involve any particular restriction upon the actual structure by which expectations are generated, equation 3.5′. It does imply, however, certain similarities between the structure generating actual sales volume and the way in which expectations are formed.

Mills [50], who worked with a production determination equation, was interested in transforming the observed residuals in order to obtain an estimate of the actual error made in anticipating future sales; the analysis was based on the assumption that production schedules are completely inflexible, i.e., that $\lambda = 1$. Interpretation of the observed residuals of a regression of the form (3.9) in this way involves the implicit assumption that observation errors and the effects of variables omitted from the equation are small relative to the impact of erroneous sales anticipations. The straightforward application of estimation procedures to an equation of the form of (3.9) under the assumptions embodied in (3.8) involves certain other difficulties. The limited information estimation procedure employed by Klein relies on the assumption that the residuals (undesired inventory) of successive time periods are independent. When the procedure of least squares is applied, parameter estimates are necessarily inefficient if the residuals are autocorrelated; customary tests of significance are not valid. Furthermore, the sum of excess inventories over the sample period will necessarily be zero when the least squares procedure is employed. Mills [50] circumvented these difficulties by applying least squares to the equation obtained by first differencing (2.4), a procedure that is appropriate if *changes* in errors made by firms in predicting sales are independent. In order to obtain estimates outside the sample period of errors made by firms in anticipating sales volume, he substituted the parameter estimates obtained with the first-differenced regression back into equation 3.9. There is a difficulty with this procedure: owing to the stochastic element of 3.8, biased estimates will be yielded by the application of least squares to equation (3.9).[28] Furthermore, an inspection of equations (3.2) and (3.3) reveals that the observed forecast error term was significant in those regressions, suggesting that considerable precision may be sacrificed when that variable is excluded.

BIASED EXPECTATIONS

It is possible to examine empirically a more general assumption about the nature of expectations that includes as special cases both Mills' hypothesis, equation (3.8), and the alternative assumption of naïve expectations employed by Alain Enthoven in his empirical work. In

[28] If $\sigma_{\epsilon_1} = 0$, it might be appropriate to utilize X_t as the dependent variable rather than I_t, and then translate the equation back into the form of (3.9).

earlier work [40] [41], I hypothesized that expectations, however formed, turn out to be a linear combination of lagged sales and actual developments:

(3.10) $$\hat{X}_t = \rho X_{t-1} + (1 - \rho)X_t + \epsilon_{10t};\ \mathrm{E}(\epsilon_{10t}) = 0$$

If the "coefficient of anticipations" ρ equals zero, we have the case considered by Mills. On the other hand, $\rho = 1$ corresponds to the assumption of naive expectations invoked by Enthoven. A value of ρ between these two extremes implies that on the average firms anticipate a definite fraction of actual changes in sales:

(3.11) $$X_t - X_{t-1} = (1 - \rho)(\hat{X}_t - X_{t-1}) + \epsilon_{10t}$$

Theil's empirical studies suggest that anticipations data have a systematic tendency to understate changes [60]. Since the error made by the firm is

(3.12) $$\hat{X}_t - X_t = -\rho(\hat{X}_t - X_{t-1}) + \epsilon_{10t},$$

the coefficient of anticipations is a measure of the bias of forecasts toward last period's sales. $\rho = 1$ implies that firms have no success in anticipating the direction of changes in sales volume; expected sales are randomly distributed about last period's sales. A negative ρ implies that firms have a systematic tendency to overstate changes; $\rho > 1$, on the other hand, corresponds to the perverse case in which the direction of change in sales is generally misjudged, an extreme form of regressive anticipations.[29]

The conjecture underlying equation 3.10 implies nothing about how expectations are actually formed; it says nothing about the structure of anticipations. In the study of inventory behavior, the conjecture does permit the study of a possible systematic tendency for firms to underestimate average changes in sales volume. Substitution of equation (3.10) into (3.1) yields

(3.13) $$H_t = \delta\alpha + \delta\beta X_t - (\delta\beta + \lambda)\rho\Delta X_t + (1 - \delta)H_{t-1} + (\lambda + \delta\beta)\epsilon_{10t} + \epsilon_{1t}$$

I have reported [41] the following estimates of the coefficients of this equation for quarterly deflated seasonally adjusted data for finished goods inventory of all manufacturers, for 1948–55:[30]

[29] If $X_{t-1} = 10$ and $X_t = 15$, then $\rho = -0.5$ implies $\mathrm{E}(\hat{X}_t) = 17.5$, $\rho = 0$ yields 15, $\rho = 0.5$ yields 12.5, $\rho = 1$ yields 10, and $\rho = 2$ yields 5.

[30] When the ΔQ production-smoothing term is included in these aggregative regressions it inevitably has the wrong sign.

$$(3.14a) \qquad H_t = -258.2 + \underset{(.0203)}{.0419X_t} - \underset{(.0417)}{.1315\Delta X_t} + \underset{(.0649)}{.8479H_{t-1}}$$

$$R^2 = .958$$
$$d = 1.39$$

Data for total durables yield

$$(3.14b) \qquad H_t = -325.8 + \underset{(.0143)}{.0550X_t} - \underset{(.0283)}{.0970\Delta X_t} + \underset{(.0523)}{.8171H_{t-1}}$$

$$R^2 = .966$$
$$d = 1.33$$

Estimates obtained for the nondurable sector are

$$(3.14c) \qquad H_t = 418.7 + \underset{(.0292)}{.0058X_t} - \underset{(.0685)}{.1695\Delta X_t} + \underset{(.0858)}{.9351H_{t-1}}$$

$$R^2 = .946$$
$$d = 1.57$$

The coefficients for total manufacturing imply that $\delta = .1521$ and $\beta = .2755$. It is not possible to unscramble the regression coefficients in order to obtain estimates of ρ; the effects of flexibility of production cannot be segregated from the measure of degree of bias of expectations. If it could be assumed that production plans are completely inflexible, i.e., $\lambda = 1$, then the estimates imply that $\rho = 0.1262$ in manufacturing; this figure may be interpreted as the *effective bias* of expectations; although expectations may be much more strongly biased toward last period's sales than this figure suggests, the value of ρ obtained under the assumption of $\lambda = 1$ does indicate the net prediction bias after reductions for a partial readjustment of production plans.[31] Even if a fair degree of flexibility of the production plan is admitted, say $\lambda = 0.5$, then the total manufacturing $\rho = 0.24$, a figure still implying that expectations are, on the average, quite precise. The estimates are consistent with an anticipations coefficient greater than unity only if the flexibility coefficient is less than 9 per cent.

An imprecise check upon the validity of the assumption that actual expectations may be described by equation 3.10 is provided by the

[31] I have also obtained regressions over the same period with inventory data for a number of durable goods industries [41]; purchased materials and goods-in-process inventories are not published separately from finished goods inventory with the industry breakdown. The "effective bias" coefficients range from a low of 0.0283 for transportation equipment to a high of 0.2114 for the stone, clay, and glass industry.

Railroad Shippers' Forecast data. A comparison of the regressions obtained with the observed railroad expectations series, equations 3.2 above, with those obtained under assumption 3.10, indicates discrepancies in the estimates of the underlying parameters of the model. The estimated speed of adjustment is always larger when the anticipations series derived from the Railroad Shippers' Forecast data is employed. The nondurable marginal desired inventory coefficient is particularly sensitive. On the other hand, the differences are no greater than should be anticipated on the basis of a casual interpretation of the standard errors of the regression coefficients. Furthermore, the railroad anticipations series itself involves considerable measurement error, particularly at this level of aggregation. A comparison of the multiple correlation coefficients suggests that the labors involved in compiling the anticipations series are not rewarded by a substantial improvement in fit, although they do provide a rough estimate of λ, the production flexibility coefficient.

A second more direct check of the validity of the assumption that the expectations error is proportional to the change in sales is to regress the observed prediction error $\hat{X}_t - X_t$ upon the change in sales from the preceding quarter in accordance with equation 3.12. The calculations were performed for the cement anticipations data, both with and without seasonal dummy variables. For contrast, the annual change, $X_t - X_{t-4}$, was also utilized as an alternative explanation of the prediction error.

$$(3.15a)\quad \hat{X}_t - X_t = \underset{(.0218)}{-.0180}(X_t - X_{t-1}) - \underset{(421.9)}{1{,}158} + e_t \qquad R^2 = .0466,\ \bar{S}_e = 2{,}631,\ n = 39$$

$$(3.15b)\quad \hat{X}_t - X_t = \underset{(.0817)}{-.6777}(X_t - X_{t-4}) + \underset{(381.3)}{1{,}172.4} + e_t \qquad R^2 = .6692,\ \bar{S}_e = 1{,}539,\ n = 36$$

$$(3.15c)\quad \hat{X}_t - X_t = \underset{(.0988)}{-.2644}(X_t - X_{t-1}) \underset{(1{,}152.2)}{-660.0d_1} + \underset{(4{,}705.9)}{11{,}421.d_2} + \underset{(2{,}571.6)}{5{,}734d_3} - \underset{(1{,}864.)}{5{,}130.} + e \qquad R^2 = .2165,\ \bar{S}_e = 2{,}488,\ n = 39$$

$$(3.15d)\quad \hat{X}_t - X_t = \underset{(.0794)}{-.7242}(X_t - X_{t-4}) - \underset{(673.2)}{1{,}114.5}d_1 + \underset{(694.9)}{378.5}d_2 + \underset{(681.2)}{792.8}d_3 + \underset{(518.4)}{1{,}319.0} + e_t \qquad R^2 = .7402,\ \bar{S}_e = 1{,}428,\ n = 36$$

Inspection of the regressions suggests that the forecast error in the cement industry is best explained by the change in sales from the preceding year; even when dummy variables are added in order to net out the effects of stable seasonal influences, it is the annual change that contributes most to an explanation of the prediction error. Equation (3.12) should be modified

$$(3.12')\qquad \hat{X}_t - X_t = -\rho(X_t - X_{t-4}) + \epsilon_t$$

The superiority of this equation, which might well have been anticipated on the basis of Hart's work on the Railroad Shippers' Forecasts [26], may stem from difficulties encountered by firms in correctly judging seasonal movements. Donald J. Daly reports [13, p. 258]: ". . . the practice of using year-to-year changes dated at the end of the period appears to be widely followed by businessmen. Insofar as this practice is widely used, it will contribute to belated recognition of economic changes and perhaps contribute to a distorted view of the recent rates of change with inevitable effects on company expectations." The dummy variable procedure for correcting seasonals utilized *ex post* data not available to the firm at the time that anticipations were formed.

A pragmatic test of the most appropriate proxy to utilize for the error made by firms in predicting future sales is provided by contrasting their effectiveness in explaining inventory investment in the cement industry. In Table 2 each column represents a separate regression. The first four regressions involve the assumption that the anticipations error is proportional to the change in sales from the preceding quarter; the last four utilize $X_t - X_{t-4}$ as the proxy for the forecast error. The four-quarter change in sales again proves to be the best proxy for the error in anticipating sales volume. The estimated marginal desired inventory coefficient has an incorrect negative sign whenever the one-quarter change is utilized as the proxy.[32] Furthermore, the signs of the various parameters of the

[32] It might be interesting to rerun the aggregative regressions for total manufacturing and the durable and nondurable components with $X_t - X_{t-4}$, rather than $X_t - X_{t-1}$, as the proxy for the forecast error. Pending such an investigation, it is hard to explain

TABLE 2

INVENTORY INVESTMENT IN THE CEMENT INDUSTRY, 1947–56

X_t	−.4571 (.0548)	−.0805 (.0711)	−.4322 (.0739)	−.0545 (.0771)	−.4454 (.0943)	.1760 (.0823)	−0.2670 (0.0847)	.2151 (.0831)
$X_t - X_{t-1}$	−.5128 (.0648)	−.5637 (.1009)	−.4707 (.1053)	−.5268 (.1094)				
$X_t - X_{t-4}$					−.2673 (.3808)	−.5119 (.1403)	−0.0751 (0.3048)	−.5051 (.1360)
I_{t-1}	.7447 (.1586)	.1037 (.1589)	.6711 (.2154)	.0271 (.1810)	−.2522 (.1718)	−.3933 (.1596)	−0.2098 (0.1363)	−.4987 (.1667)
ΔQ			−.1486 (.2909)	−.1510 (.1690)			−1.072 (0.240)	−.2404 (.1419)
d_1		8,102 (2,224)		8,140 (2,231)		13,719 (2,404)		13,793 (2,330)
d_2		11,281 (4,922)		11,597 (4,951)		−8,641 (2,731)		−5,587 (3,209)
d_3		−3,988 (2,549)		−3,822 (2,564)		−15,979 (1,381)		−14,510 (1,593)
c	11,539 (3,925)	−214.2 (3,381)	11,896 (4,029)	24.93 (3,402)	35,692 (4,457)	3,447 (3,033)	23,986	2,647
R^2	.871		.872	.961			0.807	.973
S_e	4,175	2,439			6,737	2,165		
n	39	39	39	39	36	36	36	36

model are correct only when the seasonal dummy variables are included; as with the Railroad Forecast data, the proxy procedure supports Johnston's seasonal form of the production-smoothing hypothesis. On the other hand, the ΔQ form of the production-smoothing hypothesis, while of correct sign, is significant only when the rail anticipations data are employed.

Although it appears that suitable proxies for the errors made by firms in anticipating future sales may be employed when accurate expectational data are not available, it is necessary to emphasize certain limitations of the procedure. It is obvious that the presence of the stochastic term means that biased parameter estimates should

why the ΔX_t term is satisfactory at the higher level of aggregation but inappropriate for cement; this may result from the more complex deseasonalizing procedure to which the aggregative series were subjected or from the offsetting of conflicting errors in the aggregation process itself.

be expected whenever the proxy procedure summarized by equation (3.12) or its modified form (3.12′) is employed. This in itself is reason for suspecting that even under the assumption of complete inflexibility, the effective bias should not be interpreted in terms of equation (3.13) as an accurate measure of any systematic tendency for firms to underestimate changes in demand.

An additional reason is provided by an identification problem similar to that involved with one of the surrogate procedures discussed earlier. This problem arises once it is admitted that other variables in addition to sales may influence the equilibrium level of inventories. Suppose, for a moment, that equation 3.5 does indeed constitute a correct specification of the structure by which anticipations are generated, so that (3.1′) offers a valid description of actual inventory behavior. Comparing this equation with (3.13) we see that certain coefficients of the latter equation are identified only because the backlog of unfilled orders and its change were not included in the inventory-determining equation. Consider next the following nondurable manufacturing regression based on quarterly deflated data extending from the second quarter of 1948 through 1960, where total stocks, H_t, had to be utilized rather than just finished goods inventories because of restrictions on the availability of deflated data stratified by stage of fabrication.

$$(3.16)\qquad \Delta H_t = \underset{(.5434)}{-.1885} - \underset{(.0522)}{.0755H_{-1}} + \underset{(.0216)}{.0362X_t} + \underset{(.0595)}{.1950U} - \underset{(.0420)}{.0922\Delta X_t} + e_t \qquad R^2 = .378$$

In contrast, when the change in unfilled orders is added to the regression, we have

$$(3.16')\qquad \Delta H = \underset{(.4090)}{-.5084} - \underset{(.0390)}{.0823H_{-1}} + \underset{(.0161)}{.0426X_t} + \underset{(.0456)}{.2541U} - \underset{(.0331)}{.0285\Delta X_t} - \underset{(.0581)}{.3557\Delta U} + e_t \qquad R^2 = .574$$

The order terms were included because earlier empirical work suggested that they have a direct influence upon stocks of purchased materials and goods in process. To maintain the assumption that the change in orders does not influence stocks directly, the first of the reported regressions is identified under the assumption that $v_2 \neq 0$ in (3.5); the estimate of the effective bias is 0.089. If the change in unfilled orders is regarded as having a direct influence upon stocks, certain coefficients of (3.13) are no longer identified because there is now no variable in (3.5) excluded from the structural stock equation. The second set of estimates, which implies a much lower effective bias of expectations, 0.032, would be identified only if (3.5) were replaced by an equation involving the maintained hypothesis that expectations are influenced by other variables in addition to current and lagged orders and sales. The choice between these two alternative estimates of expectational bias can be made only on the basis of a priori knowledge as to the actual structure generating expectations; it cannot be made on the basis of the statistical evidence summarized by equations (3.16) and (3.16′).

The employment of surrogate procedures rather than actual data on expectations has not at this stage provided decisive results. On the one hand, Johnston's analysis has not yet established the preferred assumption, among the alternatives he considers, concerning the structure by which expectations are actually generated. While I have found the ΔX_t term significant in my earlier regressions [41] covering 1948–55, suggesting a bias in manufacturers' forecasts, subsequent regressions [42], using more recent data on manufacturers' inventory holdings as well as equations (2.16), (3.16), and (3.16′) do not yield such strong results. The evidence for the cement industry suggests that $X_t - X_{t-4}$ may be appropriately employed as a proxy when accurate observations are not available on the actual error made by firms in anticipating sales volume.

CONCLUSION

The direct forecasting value of sales anticipations data has frequently been questioned in such studies as that of Modigliani and Sauerlander [52] and, most recently, by Peter Pashigian [54]. It has been argued at the same time that *ex ante* sales observations are chiefly useful in helping to explain changes in such other variables as inventory investment. In terms of this criterion, the reconstituted railroad forecast anticipation data appear to make a significant contribution in (3.2) and (3.3) in explaining the behavior of manufacturers' aggregate inventory holdings and the behavior of cement. Certainly, the regressions offer a substantial improvement in closeness of fit over what would have been achieved if actual sales were employed as a proxy for anticipations. A comparison with those regressions that utilized either the quarterly or annual change in actual sales as an approximation for the forecast error reveals that the reconstituted Railroad Forecast anticipations series offers only a marginal improvement over what can be obtained with suitable proxies. The importance of considering surrogate alternatives in appraising the contribution that data purporting to measure actual anticipations can make toward an understanding of actual inventory movements should not be underestimated.[33]

The review presented here of alternative procedures for analyzing the impact of sales expectations upon inventory behavior suggests that making correct inferences concerning the structural determinants of inventories is extremely difficult. If data purporting to measure actual expectations have a systematic tendency to overstate forecast errors, production plans will appear excessively flexible. Procedures derived by Enthoven and Johnston for circumventing the use of actual anticipations data require strong a-priori judgments concerning the structure by which anticipations are actually generated. On the surface, both Mills' suggestion that actual sales provide a good surrogate measure of anticipations and my generalization that the *change* in sales may be proportional to the error made by firms in anticipating sales appear to circumvent the problem of specifying the structure by which anticipations are actually generated. On closer inspection, however, it becomes apparent that the issue is clouded unless it is assumed that production plans are completely inflexible; furthermore, the unspecified structure of the equation explaining the actual generation of expectations might conceivably be such as to imply that other parameters of the inventory equation are unidentified.

It seems clear from all this that only a limited amount of information about the structure of anticipations may be gleaned from the study of inventories. In particular, the two sets of regressions summarized by equations (3.2) and (3.14) are both compatible with either (1) quite inaccurate, perhaps regressive, anticipations but extremely flexible production plans *or* (2) rather accurate expectations but not much flexibility in production scheduling. A reconstitution of the Railroad Shippers' Forecast data for the cement industry, discussed

[33] Although Modigliani and Sauerlander [52] observed that the Railroad Forecasts assisted in predicting cement inventories, they failed to consider possible surrogate measures of anticipations as alternatives. For a more elementary model with output as the dependent variable, Mills found that under the assumption of production inflexibility current sales provided a much better fit then the Shippers' forecasts [51, p. 12a].

in the Appendix of this paper, offers some support for the conjecture that expectations are not as inaccurate as *ex ante* data sometimes imply. However, the investigation of inventory behavior has not established that the expectations of future sales held by individual firms do not have a "regressive" tendency to forecast a reversion toward former sales levels.

Research on inventories and anticipations are clearly complementary rather than competing efforts. It definitely would be worthwhile to test the various alternative assumptions developed by Johnston concerning the structure of anticipations upon actual *ex ante* data as well as to investigate further the extent to which actual anticipations are approximated by my conjecture and that of Mills, equations (3.10) or (3.8). In addition to providing a potential check for determining whether data on anticipations actually help in describing inventory behavior, surrogate procedures facilitate the study of inventory behavior, surrogate procedures facilitate the study of inventory and production movements when concomitant series on sales anticipations are unavailable.

Determinants of Equilibrium Inventory

Other variables in addition to sales influence equilibrium inventory. The role of orders has already been mentioned in this paper. A more detailed study has revealed their importance in explaining stocks of purchased materials or goods in process, although they may have a negligible effect upon finished goods inventory.[34] A tightening of credit conditions might be expected to lead to a reduction in the equilibrium level of inventories. The impact of military procurement upon inventory accumulation has been subjected to preliminary investigation [42]. The influence of speculative considerations upon inventory movements also bears consideration. Here, the conflicting evidence with regard to the possible influence of credit conditions

[34] In [41], where durable and nondurable manufacturing inventories were stratified by stage of fabrication, the orders variable was included in the purchased materials and goods-in-process equation, but excluded from the finished goods regressions. The coefficient of the orders variable in the equation explaining total inventory behavior was only moderately changed from its value in the purchased materials and goods-in-process equation, suggesting that role of orders in the aggregative equation reflects its influence upon inventory in the first two stages of fabrication. When new orders are included in the inventory equation, a problem of collinearity is created because new orders are essentially the sum of sales plus the change in unfilled orders; a more reliable estimate of the role of sales is obtained when the change in unfilled orders rather than in new orders is utilized as an explanatory variable.

and speculative forces will be reviewed within the context of the flexible accelerator principle.

SPECULATIVE INVENTORY HOLDINGS

To the extent that firms accumulate additional inventory in periods of inflation in an attempt to hedge against rising prices they contribute to the inflationary spiral. The evidence with regard to such "price-hedging" or "speculative" behavior is mixed. Klein [36] reports a significant positive association between aggregate inventory investment and the change in the GNP deflator; T. M. Brown [9] also obtains a positive relation in his study of Canadian inventory behavior. On the other hand, two investigations involving a less aggregative approach have not provided strong support for the speculation hypothesis. In a study of manufacturers' holdings of stocks of purchased materials and goods in process, I found [41] the relationship was insignificant in both durable and nondurable regressions, and had the wrong sign for total manufacturing stocks. Darling [15] found that price change, while of correct sign, was insignificant at the 5 per cent level in the equation explaining manufacturers' holdings of purchased materials and goods in process and in regressions for wholesale and retail trade; in other regressions, which constituted the majority, the sign was incorrect. This evidence is compatible with the null hypothesis that firms do not speculate in stocks. Of course, the test is not conclusive; for one thing, firms may simply change the composition rather than the magnitude of their holdings; in addition, they may seriously misjudge price movements. Nevertheless, the negative conclusion is not a complete surprise, for the literature describing current inventory practice does contain some indications that price-hedging is discouraged in most firms.[35]

INFLUENCE OF CREDIT STRINGENCY

Because fluctuations in inventory investment play such a pronounced role in the business cycle, the extent to which the monetary authorities can successfully exert countercyclical pressure depends in part upon the responsiveness of inventory investment to changes in credit conditions. The evidence that has accumulated at this date is not

[35] Baumes [3, p. 22] reports that "while most companies say that they do not speculate in the commodity markets, some companies have a policy of allowing forward buying when the price is right. Companies that allow forward buying usually stipulate that purchases above normal requirements be approved by top management."

decisive. Certain regressions have indicated a perverse relationship of incorrect sign between monetary variables and the equilibrium level of inventories, a difficulty that stems in part from problems of simultaneity.

Terleckyj [59] reports on an attempt to include the interest rate on four to six months' prime commercial paper in an equation explaining the percentage change in inventory book value for trade and manufacturing. Although the variable has the correct sign, it is less than one-tenth the magnitude of its estimated standard error. Consequently, Terleckyj excluded this variable from later regressions. He found that corporate liquidity was not significantly correlated with the residuals from his equations.

The study by Brown, Robert Solow, Albert Ando, and John Kareken for the Commission on Money and Credit [7] contains estimates of the effect upon manufacturers' inventory holdings of the interest rate charged on short-term bank loans to business. Undeflated data were utilized; ΔX served as a proxy for errors in anticipating sales volume. One regression suggests that a 1 percentage point rise in the interest rate reduces inventory investment by $1.15 billion in the following quarter; the ultimate impact is a reduction in inventory of $4.86 billion. The authors are rightly cautious about the imprecise nature of their estimates. Although the interest variable was significant at the 95 per cent level in that regression, it was only roughly equal to its standard error in a second equation involving a more complicated lag structure. Furthermore, an attempt to determine a direct link between Federal Reserve policy and inventory investment revealed a perverse negative relation between an availability index (the maximum potential earning assets of commercial banks) and the equilibrium level of inventories.

Three other investigations have failed to yield decisive evidence of a negative relation between credit availability and inventory investment. Paul F. McGouldrick, of the Board of Governors, Federal Reserve System, obtained rather disappointing results in an attempt to determine the influence upon inventory holdings of the ratio of liquid assets to current liabilities, the loan-deposit ratio of commercial banks, and the bank rate on short-term business loans [46]. The interest rate variable had the correct sign in durable manufacturing, but was not significant; the loan-deposit ratio for commercial banks, the measure of availability, was perverse in sign. In trade, either the interest rate variable or the loan-deposit ratio had an incorrect sign.

I have also found [42] incorrect signs in regressions involving deflated durable and nondurable manufacturing inventory. Darling [15] reveals in a preliminary report that the coefficient of the bank rate on business loans had a perverse sign in regressions for manufacturing and trade combined and for durable and nondurable manufacturing. Although Darling obtained an appropriate negative relationship between the rate of interest and finished goods inventory as well as various components of wholesale and retail trade stocks, none of the coefficients were significant at the 5 per cent level.

Ta-Chung Liu [39] found the appropriate negative relationship in a study of deflated nonfarm business inventory. He utilized the real rate of interest, the average rate on prime commercial paper less the lagged rate of change in the GNP implicit deflator. Liu reports both single- and two-stage least square parameter estimates; in both cases, the coefficient of the interest rate term is roughly twice its estimated standard error. Liu's regression also contains nonfarm nonfinancial holdings of monetary assets, measured in constant dollars; this term has a positive coefficient, as would be expected, but is not significant.[36]

The evidence accumulated in these studies is conflicting rather than reinforcing. Application of the flexible accelerator to this problem has not established the magnitude of the impact of monetary policy upon inventory investment.

Summary

Although the literature reporting on econometric studies of inventory behavior is quite small relative to the numerous studies on the determinants of other components of effective demand, this neglect may be at least partially explained by the difficulties of the subject. The distinction between actual versus desired inventory and the problem of measuring anticipated sales are but two of a host of hurdles that have confronted the investigator. Techniques have been developed for circumventing the problem created by the fact that both equilibrium inventory and sales anticipations are, for the most part, unobserved variables. But, at this stage, they have not provided decisive evidence concerning the influence of such factors as credit conditions and speculative forces upon inventory investment.

[36] It is interesting to observe that Liu includes in his regression several lagged inventory terms, the complication that created trouble for Solow *et al.* Unfilled orders and the rate of change in the wage rate also appear in the regression.

Part of the difficulty may arise from certain weaknesses in the accelerator principle. The model assumes that the impact of erroneous anticipations falls either upon output or inventory, making no allowance for the possibility that adjustments in either price or advertising expenditures may shoulder part of the burden. Although price adjustment models are available, the choice has been between one extreme or the other as typified by the two alternative approaches compared by Mills [51] and by Shozaburo Fujino [23], rather than a successful blend of the two extremes; either a price or a quantity adjustment model rather than a blend of the two polar models is required to do the work. The approach of Liu is a first step in remedying this problem.[37]

A second source of difficulty involves the form in which variables enter the regression equation. Whether or not the desired results are obtained is in part a matter of the persistence of the investigator as well as the validity of the hypothesis. The assumption of profit-maximizing behavior, emphasized by Mills [48] [49] [50] [51], by Modigliani and Sauerlander [52], and by Holt and Modigliani [29] still leaves the empiricist with a wide range of choice. Several alternative modes of behavior have been shown to be consistent with the assumption of profit maximization; what types of behavior are incompatible with it? A second source of a priori knowledge, the assumption that the economy has reasonable dynamic properties, may place further restrictions upon the range of models to be considered. I have argued [43] that the assumption of immediate adjustment is incompatible with stability for reasonable values of the parameters of a multisector model. Further theoretical research may serve to narrow the range of choice that now confronts the empirical investigator.

A final and most serious difficulty is created by the current unavailability of adequate monthly or quarterly cross-section data on the movement of inventories, sales, and related variables at the level of the individual firm. Cross-section data expose movements that are concealed in the process of aggregation. A more complete understanding of the structure of inventory behavior, a prerequisite for successful prediction and hypothesis-testing, will be obtained when suitable cross-section data, a possible byproduct of current statistical

[37] It will be remembered that although Liu allowed for an impact of excess inventory upon prices, he omitted the influence of the resulting fall in prices upon demand and, hence, upon inventory.

collection activities of the federal government, become available on a confidential basis for research purposes.

Appendix: A Reconstitution of the Railroad Shippers' Forecasts

Because problems of interpretation arise with any attempt to circumvent the utilization of data on actual sales anticipations in the study of inventory behavior, it is important to interpret correctly whatever information is available on expectations. Here one body of expectational data, the Railroad Shippers' Forecasts, will be considered. This is the set of data utilized in the empirical study of inventory behavior reported in the body of this paper.

The data concern anticipated quarterly shipments by rail broken down into thirty-two commodity groups. A sample of firms contributing a sizable portion of railway freight traffic has provided the data published since 1927 in the *National Forecast* of the Regional Shippers Advisory Boards under the auspices of the Association of American Railroads.[38] The forecasts have proved to be quite inaccurate predictors of actual railroad carloadings, being frequently less accurate than simple naïve projections of the previous quarter's shipments.[39] Nevertheless, they still constitute an important body of anticipations data which has been subjected to repeated analysis.

Albert G. Hart [26] attempted a reconstitution of the Railroad Shippers' Forecast data for the interwar period in order to obtain a series of more accurate carload anticipations, one in closer conformity with the type of expectations entrepreneurs might be expected to hold. Hart found it hard to believe that the actual anticipations held by businessmen could have the "regressive" property of the Shippers' Forecasts, a systematic tendency to predict a movement back toward earlier levels in the face of opposing trends. But arguments concerning the validity of a revised anticipations series based upon their conformity with the way anticipations are expected to behave is inherently a most subjective process. Here, a second attempt to reconstitute the Railroad Shippers' Forecasts, based on post-World War II data, will be described.

Although the traffic manager generally completes the return utilized in preparing the Railroad Shippers' Forecasts of carload uti-

[38] For a detailed discussion of the sampling procedures and other aspects of the survey see [19].

[39] Thor Hultgren [30, pp. 364–371, 374–378].

lization, this does not imply that the estimate is derived independently of the firm's sales anticipations. The respondent is asked to state the anticipated percentage increase in carloadings over the corresponding quarter of the preceding year, actual shipments for that quarter of last year, and an anticipations figure in carload units. The respondent may simply assume that carloadings will increase by the same percentage as the increase anticipated by the firm for total sales by all modes of transportation. Firms frequently utilize comparisons with the same quarter of the preceding year as an implicit form of seasonal adjustment. Even if this procedure is not followed explicitly, it seems reasonable to assume that the traffic manager must be aware of the sales forecast and that this figure influences both his planning and the figures he submits in completing the questionnaire on carload shipments. If $\hat{X}_t$ represents anticipated total sales volume and X_{t-4} actual sales in the corresponding quarter of the preceding year, while C_{t-4} stands for actual shipments by rail in carload units for the same quarter of the preceding year, the hypothesis implies that anticipated carload shipments $\hat{X}_t$ were formulated by the respondents by utilizing the equation:

$$\hat{C}_t = C_{t-4}\left(\frac{\hat{X}_t}{X_{t-4}}\right) \tag{A.1}$$

This hypothesis cannot be tested directly, for the variable $\hat{X}_t$ is not observed. Furthermore, the other variables are observed at best only in aggregative form. Inaccuracies may result not only from sampling errors but also because the reports of the various firms are weighted by the number of carloadings shipped by the firm in corresponding quarters of the preceding year. Consequently, carload forecasts of firms which ship a relatively large portion of their total output by rail will be overweighted when sales anticipations, the unobserved $\hat{X}_t$, are derived by equation A.1.

A possible test of the validity of the hypotheses is provided by the fact that the Railroad Shippers' Forecasts of carload shipments are not accurate predictors. An inspection of equation A.1 reveals that a sales anticipations series derived from the published rail forecast data could be either more or less accurate than the carload anticipations.[40] If the sales anticipations derived by equation A.1 are

[40] This is the essential difference between the conversion procedure proposed here and that utilized by Modigliani and Sauerlander in a study of the value of the Shippers' Forecasts in the prediction of output in the cement industry. In their study, which covered only the output of firms in the first two quarters of each year, they at first

in fact more accurate, it would offer support for the hypothesis that firms derive their carload anticipations on the basis of this equation and that the derived sales anticipations obtained by solving for the unobserved variable $\hat{X}_t$ is a valid representation of actual sales anticipations. Conversely, if the derived sales anticipations series are less accurate than the carload forecasts it would suggest that the former are not as precise as might reasonably be expected of actual anticipations.

For the cement industry, data on sales in real terms as well as the forecast data in terms of carloadings are easily obtained. A pilot study testing the hypothesis of equation A.1 was made. Although a

TABLE A-1

ACCURACY OF RAIL FORECASTS AND DERIVED SALES ANTICIPATIONS: CEMENT INDUSTRY, 1947–56

		Correlation Coefficients		
Period	*Number of Observations*	Sales and Carloadings	Forecast and Actual Carloadings	Anticipated and Actual Sales
All quarters	40	.8895	.9605	.9866
1st quarter	10	.9589	.4159	.9121
2nd quarter	10	.8345	.4568	.9688
3rd quarter	10	.8523	.6259	.9847
4th quarter	10	.0042	−.0753	.9538

relatively large portion of cement is shipped by rail, an inspection of the first column of correlation coefficients in Table A-1 reveals that for the postwar period the relation between carloadings and sales is not too close and varies considerably for different quarters of the year. The second and third columns of the table present correlation coefficients measuring the closeness of the relation between forecast and actual shipments and between derived anticipations and actual sales. For every quarter of the year as well as for an overall comparison, the sales anticipations series is a much closer predictor than the rail forecasts. For the fourth quarter, the correlation

converted carloadings into barrel figures by assuming that firms correctly estimated the number of barrels of cement loaded into a freight car for the particular quarter. Later, deciding this was unrealistic, they in effect averaged the figure given by the above formula with the one obtained by their original assumption. They did not discuss the effects of this procedure upon the accuracy of the anticipatory series. If it is assumed that entrepreneurs derive the carload anticipations by correctly forecasting the ratio of barrels to freight cars, the carload and sales anticipations will be equally accurate when measured in terms of the variance of the percentage error in the forecast [cf. 52, p. 335].

between actual sales and carloadings is extremely poor; the correspondingly poor predictive power of the railroad forecasts for this quarter is to be expected under the hypothesis formulated in equation A.1.[41]

It seems safe to conclude that the raw Railroad Shippers' Forecast data constitute a most tenuous form of evidence for judging the accuracy with which business firms actually forecast demand. The conjecture summarized by equation A.1 offers an alternative explanation. While it cannot be concluded with great confidence that expectations are not regressive, the validity of the raw Shippers' Forecast evidence seems open to serious question.

One test of the usefulness of the derived sales anticipation series is obtained by contrasting their ability to predict cement sales with a naïve projection of the sales level realized in the preceding period. The correlation between lagged and current sales is only 0.117; while the fit is improved to $r = 0.979$ when seasonal dummy variables are added, the derived sales anticipations series still provides a somewhat better prediction than that obtained by a naïve projection of last quarter's experience.

Another test concerns the contribution that the anticipations series derived from the Shippers' Forecast can make in predicting the behavior of other operating variables.[42] A preliminary test on the cement industry involved predicting output over the 1947–56 period in terms of anticipated sales and lagged inventory, this is a special case of the model discussed earlier in which it is assumed that there is no production flexibility. With the assumption of static expectations, $X_t = X_{t-1}$, a multiple correlation of 0.646 was obtained; although the addition of seasonal dummies served to raise the multiple correlation coefficient to 0.914, the inventory and lagged sales terms were no longer significant, the dummies carrying the brunt of the explanatory burden. The derived anticipations sales

[41] The same results are apparent when the accuracy of the forecasts is measured in terms of the variance of the percentage error.

[42] Except for the cement industry, a prime difficulty arises from classification complications. The commodity classifications utilized in the preparation of the Railroad Shippers' Forecasts had to be reconciled with the grosser categories of sales data published in the *Survey of Current Business*. This was accomplished by constructing indexes combining the various categories of the rail data with weights in proportion to the value of sales; the ratio of the current figure in the expected shipments index to the value of the index of weighted actual shipments in the corresponding quarter of the preceding year was utilized as the estimate of anticipated change in sales in accordance with equation A.1. Needless to say, the usual problems encountered in index number construction are involved.

series and the alternative provided by Edwin Mills' suggestion that actual sales be utilized as a proxy did equally well, both yielding a multiple correlation coefficient of 0.933; when seasonal dummies were added, the correlation coefficient was raised to 0.951 for the Mills proxy procedure versus 0.946 for the derived sales anticipations series. These studies were conducted under the assumption that the three-month period between successive observations coincides with the length of the planning period; the more promising results reported in the text allowed for a partial revision of production plans within the three-month observation period.

References

[1] ABRAMOVITZ, MOSES, *Inventories and Business Cycles, with Special Reference to Manufacturers' Inventories*, New York, NBER, 1950.

[2] BASSIE, V. LEWIS, *Economic Forecasting*, New York, McGraw-Hill, 1958.

[3] BAUMES, CARL G., *Inventory Management in Industry*, National Industrial Conference Board Studies in Business Policy No. 88, 1958.

[4] BECKMANN, MARTIN J., "Production Smoothing and Inventory Control," *Operations Research*, 1961.

[5] BOSSONS, JOHN D., "Regressive Expectations," paper delivered at August 1960 Econometric Society meetings, as abstracted in *Econometrica*, July 1961, p. 458.

[6] BRATT, ELMER C., "Availability and Reliability of Inventory Data Needed to Study Economic Change," Office of Statistical Standards, Bureau of the Budget, 1961.

[7] BROWN, E. CARY, SOLOW, ROBERT, M. ANDO, ALBERT, and KAREKEN, JOHN, "Lags in Fiscal and Monetary Policy," unpublished paper prepared for Commission on Money and Credit, 1961.

[8] BROWN, MURRAY, "Ex Ante and Ex Post Data in Inventory Investment," *Journal of the American Statistical Association*, September 1961, pp. 518–554.

[9] BROWN, T. M., "A Forecast Determination of National Product, Employment, and Price Level in Canada, from an Econometric Model," above.

[10] CHRIST, CARL, "A Test of an Econometric Model for the United States, 1921–1947," in *Conference on Business Cycles*, New York, NBER, 1951.

[11] CLARK, COLIN, "A System of Equations Explaining the United States Trade Cycle, 1921 to 1941," *Econometrica*, April 1949.

[12] CRAWFORD, C. M., *Sales Forecasting: Methods of Selected Firms*, Urbana, Bureau of Economic and Business Research, University of Illinois, 1955.

[13] DALY, DONALD J., "Seasonal Variations and Business Expectations," *Journal of Business of the University of Chicago*, July 1959, pp. 258–270.

[14] DARLING, PAUL G., "Manufacturers' Inventory Investment, 1947–1958," *American Economic Review*, December 1959, pp. 950–963.

[15] ———, "Tabulations of Quantitative Materials for Report on Regression Analysis of Fluctuations in Inventory Investment," Conference on Social Science Research Council Econometric Models, August 21, 1961 (hectographed).

[16] ———, "Evidence on Postwar Inventory Cycles," paper delivered at December 1961 American Statistical Association meetings.

[17] DUESENBERRY, JAMES S., ECKSTEIN, OTTO, and FROMM, GARY, "A Simulation of the United States Economy in Recession," *Econometrica*, October 1960.

[18] ENTHOVEN, ALAIN CHARLES, "Studies in the Theory of Inflation," Unpublished Ph.D. dissertation, Massachusetts Institute of Technology, May 1956.

[19] FERBER, ROBERT, *The Railroad Shippers' Forecasts*, Urbana, Bureau of Economic and Business Research, University of Illinois, 1953.

[20] ———, "The Accuracy and Structure of Industry Expectations in Relation to Those of Individual Firms," *Journal of the American Statistical Association*, June 1958, pp. 317–336.

[21] FOSS, MURRAY F., "Manufacturers' Inventory and Sales Expectations: A Progress Report on a New Survey," *Survey of Current Business*, August 1961.

[22] FRIEND, IRWIN, and JONES, ROBERT, "Short-Run Forecasting Models Incorporating Anticipatory Data," below.

[23] FUJINO, SHOZABURO, "Some Aspects of Inventory Cycles," *Review of Economics and Statistics*, May 1960.

[24] GOLDE, ROGER A., "Square Root Behavior of Inventories and the Production Distribution Cycle of the Firm," unpublished Senior thesis, Harvard College, 1956.

[25] GOODWIN, RICHARD M., "Secular and Cyclical Aspects of the Multiplier and Accelerator," *Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen*, New York, Norton, 1948.

[26] HART, ALBERT G., "Quantitative Evidence for the Interwar Period on the Course of Business Expectations: A Revaluation of the Railroad Shippers' Forecast," *The Quality and Economic Significance of Anticipations Data*, Princeton for NBER, 1960, pp. 205–239.

[27] HICKS, JOHN R., *Contribution to a Theory of the Trade Cycle*, Oxford, Clarendon Press, 1950.

[28] HOLT, CHARLES *et al.*, *Planning Production, Inventories and Work Force*, Englewood Cliffs, N.J., Prentice-Hall, 1960.

[29] HOLT, CHARLES, and MODIGLIANI, FRANCO, "Firm Cost Structures and the Dynamic Responses of Inventories, Production, Work Force, and Orders to Sales Fluctuations," *Inventory Fluctuations and Economic Stabilization*, Part II, Joint Economic Committee, 87th Cong., 1st sess., December 1961.

[30] HULTGREN, THOR, "Forecast of Railroad Traffic," *Short-Term Economic Forecasting*, Princeton for NBER, 1955.

[31] JOHNSTON, JACK, "An Econometric Study of the Production Decision," *Quarterly Journal of Economics*, May 1961.

[32] KALECKI, MICHAEL, *Theory of Economic Dynamics*, London, George Allen and Unwin, 1954.

[33] KLEIN, LAWRENCE R., *Economic Fluctuations in the United States, 1921–41*, Cowles Commission Monograph No. 11, New York, Wiley, 1950.

[34] ———, *A Textbook of Econometrics*, Evanston, Ill., Row, Peterson, 1953.

[35] KLEIN, LAWRENCE R., and POPKIN, JOEL, "An Econometric Analysis of the Post-War Relationship Between Inventory Fluctuations and Changes in Aggregate Economic Activity," *Inventory Fluctuations and Economic Stabilization*, Joint Economic Committee, 87th Cong., 1st sess., December 1961, Part III.

[36] KLEIN, LAWRENCE R., "A Postwar Quarterly Model: Description and Applications," above.

[37] KOYCK, L. M., *Distributed Lags and Investment Behavior*, Amsterdam, North-Holland, 1954.

[38] LEONTIEF, WASSILY, *Studies in the Structure of the American Economy*, New York, Oxford University Press, 1953.

[39] LIU, TA-CHUNG, "An Exploratory Quarterly Model of Effective Demand in the Postwar U.S. Economy," mimeographed, undated.

[40] LOVELL, MICHAEL C., "Inventories and Stability: An Interindustry Analysis," unpublished Ph.D. dissertation, Harvard University, March 1959.

[41] ———, "Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle," *Econometrica*, July 1961.

[42] ———, "Factors Determining Manufacturing Inventory Investment," *Inventory Fluctuations and Economic Stabilization*, Joint Economic Committee, 87th Cong., 1st sess., December 1961, Part II.

[43] ———, "Buffer Stocks, Sales Expectations, and Stability: A Multi-Sector Analysis of the Inventory Cycle," *Econometrica*, April 1962.

[44] LUNDBERG, ERIC, *Studies in the Theory of Economic Expansion*, London, P. S. King, 1957.

[45] MCGEE, JOHN F., *Production Planning and Inventory Control*, New York, McGraw-Hill, 1958.

[46] MCGOULDRICK, PAUL F., "The Impact of Credit Cost and Availability on Inventory Investment," *Inventory Fluctuations and Economic Stabilization*, Joint Economic Committee, 87th Cong., 1st sess., December 1961, Part II.

[47] METZLER, LLOYD A., "The Nature and Stability of Inventory Cycles," *Review of Economics and Statistics*, August 1941, pp. 113–129.

[48] MILLS, EDWIN S., "The Theory of Inventory Decisions," *Econometrica*, April 1957, pp. 222–239.

[49] ———, "Expectations, Uncertainty, and Inventory Fluctuations," *Review of Economic Studies*, 1954–1955, pp. 15–23.

[50] ———, "Expectations and Undesired Inventory," *Management Science*, October 1957, pp. 105–110.

[51] ———, *Some Empirical Estimates of Short-Run Price and Output Policies*, Cowles Foundation Discussion Paper No. 123, July 27, 1961.

[52] MODIGLIANI, FRANCO, and SAUERLANDER, OWEN H., "Economic Expectations and Plans in Relation to Short-Term Economic Forecasting," *Short-Term Economic Forecasting*, Princeton for NBER, 1955, pp. 261–351.

[53] NERLOVE, MARK, *The Dynamics of Supply: Estimation of Farmers' Response to Price*, Baltimore, Johns Hopkins, 1958.

[54] PASHIGIAN, PETER, "The Accuracy and Determinants of Sales Anticipations," 1961 (hectographed).

[55] ———, "Sales Anticipations as a Determinant of Inventory Investment," 1961 (hectographed).

[56] SMYTH, D. J., "The Inventory and Fixed Capital Accelerators," *Economic Record*, August 1960.

[57] SOLOW, R. M., "On a Family of Lag Distributions," *Econometrica*, April 1960, pp. 393–406.

[58] STANBACK, THOMAS M., JR., "Cyclical Behavior of Manufacturers' Inventories 1945–1955," Proceedings, Business and Economic Statistics Section, American Statistical Association, Washington, D.C., 1957, pp. 87–95; abstracted in *Journal of the American Statistical Association*, June 1958, p. 592.

[59] TERLECKYJ, NESTOR E., *Measures of Inventory Conditions*, National Industrial Conference Board Technical Paper No. 8, 1960, as reprinted in *Inventory Fluctuations and Economic Stabilization*, Joint Economic Committee, 87th Cong., 1st sess., December 1961, Part II.

[60] THEIL, H., *Economic Forecasts and Public Policy*, Amsterdam, North-Holland, 1958.

[61] THONSTAD, T., and JOCHEMS, D. B., "The Influence of Entrepreneurial Expectations and Appraisals on Production Planning: An Econometric Study of the German Leather and Shoe Industries," *International Economic Review*, May 1961.

[62] TINBERGEN, JAN, *Statistical Testing of Business-Cycle Theories*, Vol. II, *Business Cycles in the United States of America, 1919–1932*, Geneva, League of Nations, 1939.

[63] WOLD, HERMAN, *Demand Analysis; A Study in Econometrics*, New York, Wiley, 1953.

COMMENT

RUTH P. MACK, Institute of Public Administration

Michael Lovell's reviews of the efforts of econometricians to build up and "test" inventory models is skillful and to the point. The analysis is ingenious. The material is organized around a systematic progression of important questions.

On many issues Lovell finds results inconclusive because of intrinsic difficulties in salting the tail of *ex ante* concepts, because variables elude econometric identification, or because business adjustments are delayed and incomplete. But much also has been achieved. What is needed, Lovell concludes, is more persevering work, perhaps fewer either-or questions and more combination packages, a better basis for restricting hypotheses, cross-section studies.

Yet though I second his dissatisfactions and find his prescriptions unexceptionable, they miss some of the broad implications of the information that the paper spreads out for examination.

The figures that Lovell assembles and analyzes seem to point to two striking, however highly tentative, conclusions. The first concerns the scope of the sales-linked inventory objective. The data show that sales are a far more ambiguous and less important determinant of inventory investment than generally supposed. The second concerns the role of unfilled orders. It is much too forceful to be explained as a modifying or forecasting adjunct of the sales-linked inventory objective.

These notions involve a judgment about matters of degree. The problem is not, of course, whether businessmen look to other things than sales in formulating their inventory objectives; obviously, they do. The point is rather that as these other matters start to account for as much or more of inventory investment and disinvestment as do sales, the analytic formulation primarily in terms of an accelerator mechanism, however modified, starts to creak and strain. Strained far enough, it falsifies the *essential* dynamics. Feedbacks become innovators of change and innovators become feedbacks.

The evidence that econometric models yield on this matter of relative importance is not discussed by Lovell nor, for that matter, by most of the authors of the investigations to which he refers. It consists of the relative sizes of the contribution of the several measured variables and their relationship to the theoretical requirement; it concerns the economic elements which, paralleling the measured variables, may, in fact, get picked up by them, though anonymously; it involves the likely influence on measured parameters of causality which in fact moves *from* the "dependent" *to* the "independent" variables as well as among the latter. It may be useful to review such evidence as the paper presents on each of the two points in turn—the weak sales-linked inventory objective and the strong role of unfilled orders.

The Sales-Linked Inventory Objective

The studies do not seem to support the notion, central to the accelerator dynamics, that the volume of sales is the primary determinant of inventory investment. Though the subject requires explicit study, the following observations bear on the point:

1. The *wide variety among coefficients* linking inventories to sales, as developed by the several investigators, clouds the significance of each result. Lovell reformulates the analyses to ask what "values of the explanatory variables . . . would not have led to an attempt to change the level of inventories." Three postwar studies of quarterly inventory investment respectively show that an increase in sales of one dollar per month generates changes in equilibrium inventories of the following multiples over a three-month period: −1.30, 1.95, 2.42. An interwar analysis of annual data yields a figure of 0.25, which in terms of monthly average sales would be about 3.0.[1]

[1] The quarterly studies are those of Terleckyj, Darling, and Lovell, respectively (see Lovell's paper). As far as I can judge, though all calculations are quarterly, the first two authors use figures in monthly averages for everything except change in stock, which is the total during the quarter. Lovell uses quarterly units. Since the units affect the size of the sales coefficient—the marginal stock-sales ratio—I have multiplied Lovell's coefficient by three to make it comparable. The coverage of the calculation includes deflated data for all manufacturing in Darling, plus trade for Lovell and Terleckyj (not deflated), and all stocks and GNP for Klein.

The size of the equilibrium marginal coefficients also seems to require justification. The size of the actual *average* stock-sales relationships might be one criterion, on the assumption that a situation that would lead to no attempt to change the level of inventories should be one in which that level is somewhat near, though doubtless smaller than, the usual over-all relation. If widely different, it would seem that businessmen were resigned to abject defeat in controlling the size of stocks.[2]

2. *Distributed lags are so large that the notion of a stock intention takes on an equivocal light.* This applies to lags attributed to delayed response, or to the choice to smooth production rather than to adjust stocks, or to failure to predict sales. A "flexible accelerator" implies delayed and often incomplete adjustment of stock, the extent of which is measured by the "reaction coefficient," which was 0.5 for Klein's annual calculations and 0.21 per quarter, or about 0.4 per year,[3] according to Darling's quarterly calculations. But what is the significance of an intention about so volatile a matter as stocks if a business moves only half-way toward its validation in the course of a whole year and only one-fifth of the way in the course of three months? The question answers itself when interest focuses on cyclical dynamics.

In reviewing his own calculations, Lovell does not specifically mention a reaction interval which he has used elsewhere. Nevertheless, the need to anticipate sales causes positive or negative "surplus inventories," and it is notable that they tend quarter by quarter to be larger than either actual or predicted inventory investment (see Table 1). Also, they tend to have the opposite sign (note the inverse pattern in the chart). Again, then, inventory response is pictured as perennially way too little or too late.

3. *Contribution of the sales parameter, which is small relative to the theoretical requirement.* Unfortunately, it is not usual to give partial correlation or beta-coefficients or to graph the contribution of each variable, for this omission makes it awkward to see how the parts of an econometric investigation fit together. However, it seems

[2] Post-World War II stocks averaged about 1.9 times monthly sales for manufacturing, and about 1.6 when trade is included. Darling's figure comes close to passing this test, though it is high—the marginal desired ratio should, I would expect, be substantially smaller (because of the inevitable slow-moving items), not larger, than the actual average of 1.6 for all stocks. Lovell's figure is higher still (compare with 1.9 above).

[3] I use the formula $1 - (1 - \delta)^4$, following Lovell in *Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle*, Cowles Foundation Paper No. 169, 1962, p. 300, n. 10 (a reprint of [41] in Lovell's paper).

clear that, stated very conservatively, sales explain less than half of the explained quarterly changes in stocks in the three investigations mentioned before. (The figures discussed below indicate that change in unfilled orders alone accounts for close to or over half.) Moreover, some portion of this gross association in highly aggregative models must reflect the multiplier impact of inventory investment on sales rather than the acceleration impact of sales on inventory investment.

The Forceful Role of Unfilled Orders

Unfilled orders or changes in unfilled orders have been used to explain stocks by Lovell, Darling, and Terleckyj. Beta coefficients that the first two authors very kindly supplied me, some while ago, show that in the first case the unfilled order parameter accounted for more variation in inventory investment in purchased and in-process stock of manufacturers than did any other factor, including sales. In the case of Darling's analysis they were almost but not quite as important as sales; the simple correlation of change in unfilled orders and change in stock is 0.82.[4] Introducing the extreme values in Terleckyj's equations suggests that here, too, the unfilled orders term was the most powerful of the independent variables. Simple correlation with change in stocks in the following three- and six-month period was 0.81 and 0.84, respectively.[5]

Now, if we look at the actual time series, we find that total unfilled orders are dominated by those in the machinery and transportation equipment industries. These two industry groups constitute on the average over 70 per cent of total outstanding orders and also dominate rates of change. Is it then meaningful to say that investment in in-process and materials stocks of all manufacturers (Lovell), in all stocks of all manufacturers (Darling), and in all stocks of all manufacturers and distributors (Terleckyj) are thus heavily influenced by unfilled orders or their rates of change largely in the machinery and transportation industries?

Lovell, in another paper, explains their impact in the following terms:

[4] "Manufacturers' Inventory Investment, 1947–58: An Application of Acceleration Analysis," *American Economic Review*, December 1959, p. 952.

[5] Terleckyj used the ratio of new orders to sales, which must of course virtually parallel change in unfilled orders. The correlation coefficients are quoted from Thomas M. Stanback, Jr., "A Critique of Inventory Forecasting Techniques," in American Statistical Association, *1960 Proceedings of the Business and Economic Statistics Section.*

> If unfilled orders represent an established demand, indeed a possible committal to deliver at some future date, entrepreneurs may well consider it advisable to carry additional stocks when unfilled orders are large as a hedge against possible shortage and price commitments. In addition, a rise in the backlog of unfilled orders may be expected to lead to an acceleration of production that is felt first in terms of an increase of goods in process rather than a rise in the output of completed commodities. These considerations suggest that stocks of purchased materials and goods in process should be positively related to the backlog of unfilled orders. Conversely, if unfilled orders were only a surrogate measure of the tightness of the markets on which firms purchase their inputs, a negative relationship between orders and stocks would be revealed . . .[6]

Terleckyj says much the same thing: "One would expect that when new orders are running above sales, and the reservoir of future business is built up, an accumulation of inventories becomes desirable, as the planned production rate rises to fill these orders. The subsequent increase in the actual production rate entails a rise in inventories concentrated in the in-process stocks."[7] Darling originally placed more emphasis on the expectational aspect. He now focuses on industries in which goods are made largely to order. Here "inventory investment is more closely associated in time with receipt of the order, or more accurately with changes in the 'unfilled order' backlog than with the delivery (sale) of the goods to the buyers."[8]

Certainly, influences of the sorts described are at work. The point at issue is merely whether, particularly in view of the overpowering emphasis in the actual data of two groups of industries alone, unfilled orders can reasonably be expected to account for such strong modification of the basic sales-linked inventory objective. The modification, like the camel's head, appears to have taken over the

[6] Michael Lovell, "Factors Determining Manufacturing Inventory Investment," *Inventory Fluctuations and Economic Stabilization*, Joint Economic Committee, 87th Cong. 1st sess., December 1961, Part II, pp. 140–141.

[7] Nestor E. Terleckyj, "Measures of Inventory Conditions," in *Inventory Fluctuations and Economic Stabilization*, Part II, p. 185.

[8] "Inventory Fluctuations and Economic Instability" (in *Inventory Fluctuations and Economic Stabilization*, Part III, p. 30). When the theory is incorporated in a regression, the impact of unfilled orders and their rate of change cuts down the impact of sales on *all* stocks, not merely those in made-to-order industries, by about one-half (p. 37). The theory suggests that unfilled orders would be a more important determinant in durable than in nondurable goods. But Lovell's computations suggest the opposite (pp. 129, 143).

premises. It seems likely that backlogs are actually pushing the estimates around with muscle belonging to attributes not recognized by the theory. What might these attributes be?

One set of candidates must be buying prices and other factors that reflect changing short-term patterns of supply and demand. In a different section of the paper from that in which the influence of orders is shown, Lovell summarizes the evidence on "price-hedging" or "speculative" behavior yielded by four studies. His own study yields the only clearly adverse finding. In view of the fact that the price change that he uses involves accurate forecasts of the next quarter's prices, the failure to find it significant is not surprising. A similar requirement for clairvoyance with respect to changes in sales would have shown, no doubt, similarly negative results.

In any event, it seems clear that changes in unfilled orders must reflect a substantial part of the eventual impact of price expectations or other market expectations on stocks. The point is clarified if unfilled orders (reports are for unfilled sales orders) are thought of as outstanding purchase orders of the customer. Also, restrict consideration for a moment to orders for materials rather than for complicated goods including machines. Then, it stands to reason that a large part of the influence of expectations about changing buying prices will be reflected, in the first instance, in a lengthening of the number of weeks' supply on order. By buying more, and thereby fixing prices on the additional supply at an earlier date, the purchaser forestalls the rise. The result is an increase in his outstanding purchase orders or, precisely, the unfilled sales orders of his supplier. But outstanding purchase orders become, in due course, additions to stocks of purchased materials. Thus, changes in outstanding purchase orders act as a vestibule for changes in stock.

But if so, how can a theory that purports to explain change in stocks do so in terms of changes in outstanding purchase orders? One might as well "explain" the number of people just inside the door of a department store by the number outside of it trying to get in. *Obviously, change in outstanding orders must itself be explained,* if any real insight concerning related inventory change is to be achieved.

A second hat that unfilled orders may be wearing is that of the impact of stock on the economy—the feedback unrecognized in the single-equation system. And this may be one reason why a series that is so heavily weighted with machinery helps so materially to "explain" total stock. Change in unfilled orders applies to an earlier date than does the change in stock which it explains. There is some evidence, and certainly it is reasonable to expect, that production schedules will respond to the rate at which backlogs change (or perhaps to the active element in this change, the rate of flow of new sales orders). The association is between change in unfilled orders and change in production, and it may well be almost immediate. The rate of change in production (or its reflection in the rate of change in wages or other income) is presumably a chief determinant of inventory investment. Thus, the causal association could run from changes in unfilled orders to changes in production and income more or less immediately; changes in stocks would then reflect, a bit later, *both* the change in orders and the change in output.

Purport

My difficulty, then, with Lovell's paper is that he has done a better job of review than he is willing to admit. He has arrayed empirical results inconsistent with one another and with the theory. He has uncovered a challenging mystery: unfilled orders and their rates of change explain too much and sales too little of inventory investment.

If I am right in believing that the relative magnitudes rest uncomfortably in the accelerator model, then the theory requires reformulation. The solo theme of sales, however enriched by accompaniment, needs to be recast as a duet in which expectations about market conditions and the entire complex of business choices may have an equal voice.

This will not be easy. For open-end study is required of how businessmen formulate, as well as solve, problems that result directly or indirectly in inventory investment. Economizing inventories have their opportunity costs elsewhere in a business. Does it, for example, make sense to think of the flexible accelerator, production-smoothing, and sales forecast errors as *competing* hypotheses. Are not all necessarily present and substantial? (Lovell himself raises at least part of this question and shows brilliantly how econometric distinction between two of them is virtually impossible.) Are there not, characteristically, cyclical patterns in the relation among the several opportunity costs of changes in stocks? Are errors in forecasts of sales the only ones that motivate changes in stocks or unfilled orders; how about errors in forecasting delivery periods, selling or buying prices, material requirements? Changes in backlogs of unfilled sales orders and in outstanding orders for materials are critically *interrelated* with all these matters.

In short, rather than a better basis for placing "further restrictions upon the range of models to be considered" for which Lovell asks (p. 215) we require, I fear, a better basis for expanding them. Obviously, intense simplification is required, but it must contain rather than amputate the essential bone in business choice. To do so the model will have to penetrate far more deeply into the economic meaning of expectations than any we have used heretofore. It will have to cope with the cumulative social process of the spread of opinion, action, feedbacks, and, particularly, feedins. These are tough assignments, but only tough enough to excite their own solution.