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CAPITAL AND LABOR IN PRODUCTION: SOME DIRECT ESTIMATES

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Introduction—New Data for Old and New Problems

ESTIMATES of critical parameters of the production function have been based frequently on special assumptions involving, most particularly, an equilibrium, perfectly competitive equivalence between wage rates and the marginal physical product of labor. They have also been bedeviled by difficulties in accounting for less-than-capacity utilization of capital. Further problems have originated in the uncertain biases introduced by the differing natures of variances and covariances in different sets of cross-sectional and time series observations. Disparate estimates have been obtained, for example, from cross sections of three-digit manufacturing industries, two-digit industries, states, and nations, and a variety of time series.

The data underlying the current analysis make possible, if not a head-on assault, a more direct attack upon a number of the problems which have become manifest in prior work. They may permit us eventually, with some effort, but without dependence upon assumptions of perfect competition and equality between the wage rate and the marginal product of labor, to estimate directly from output and capital and labor

NOTE: I am deeply indebted to a splendid group of young economists, research workers, and computer programers who have labored mightily with me in the collection and analysis of data and the preparation of this paper. They include: Margorie Bechtel, Betty Benson, Robert M. Coen, Joel Fried, Jon Joyce, Elsie Kurasch, Albert Morris, Hugh Pitcher, Judith Pitcher, Jon Rasmussen, Jay S. Salkin, Kenneth Smith, and Patricia Wishart. Particular credit should be given again to the McGraw-Hill Publishing Company Department of Economics and to Margaret K. Matulis of McGraw-Hill, who made available the original data. The research utilized the facilities of the Computing Center and the Econometrics Research Center of Northwestern University. It enjoyed the important financial support of a series of grants from the National Science Foundation.

inputs the parameters of constant elasticity of substitution production functions as formulated by Arrow, Chenery, Minhas, and Solow,[1] and by Brown and de Cani.[2] This paper, however, will constitute merely a tentative, preliminary report of results of analysis thus far.

Our basic data are literally thousands of individual-firm responses in McGraw-Hill Capital Expenditure Surveys over most of the period since World War II, especially from 1955 through 1962. These have been supplemented by accounting information collected to match the firms of the McGraw-Hill sample. Thus, while using code numbers to preserve anonymity of the respondents, it has been possible to combine the survey data with generally available financial statistics on an individual-firm basis over a substantial number of years.

The McGraw-Hill surveys themselves permit incorporation of variables reflecting explicit business evaluation of "per cent utilization of capacity" and per cent change in capacity. In addition, they offer a convenient compilation of year-by-year capital expenditures and the number of the firm's employees. In the current analysis, these data have been complemented by accounting statistics with regard to gross fixed assets, inventories, sales, and depreciation charges.

Price deflation of the basic data has been attempted where appropriate. Annual capital expenditures have been deflated uniformly by a single capital goods deflator, calculated as a weighted average of the implicit gross national product price deflators for "other new [nonresidential] construction" and "producers' durable equipment" weighted by the constant dollar volumes of these aggregates. Sales, inventories and, consequently, "output," defined as the sum of sales and inventory change, were deflated by one of eleven sets of price indexes constructed from Bureau of Labor Statistics price indexes and price relatives on the basis of the broad product or industry classes into which I was informed the McGraw-Hill firms could be categorized. Capital stock was in most instances measured as the "gross fixed assets" reported by "orginal cost" accounting. Attempts were also made, however, to deflate capital stock by a rather complicated scheme described briefly below.

[1] K. J. Arrow, H. B. Chenery, B. S. Minhas, and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics,* August 1961, pp. 225–50.

[2] Murray Brown and John S. de Cani, "Technological Change and the Distribution of Income," *International Economic Review*, September 1963, pp. 289–309.

The Variables and Methods of Analysis and Presentation

The results reported upon will stem from four sets of regressions involving individual firms: time series, cross sections, cross sections within industries, and over-all. Where there are sufficient observations, we shall also present three industry regressions: time series, cross sections, and over-all. In regard to the time series, which are pooled regressions of deviations about the means of observations for individual firms and for industries, respectively, a degree of freedom is lost for each firm or each industry included. In the case of the firm time series, where there was no more than one observation for a particular firm, that observation was excluded. Cross-section regressions were, in effect, pooled regressions of deviations about the means for each year, and a degree of freedom was hence lost for each year of observations.[3]

It should also be noted that, as in previous work, it has been deemed advisable to exclude observations containing extreme values of any of the variables. In variables requiring logarithmic transformations lower bounds were set to preclude the possibility of values relatively close or equal to zero. Upper and lower bounds of acceptable intervals were also established on the basis of preliminary analysis of means and standard deviations. Intervals were generally set so that inclusion of at least 99 per cent of the values on each variable might be expected.

No attempt was made to utilize information from incomplete observation vectors. A considerable number of observations were hence rejected because of missing information on only one or several variables. Table 1, "Definitions and Sources of Variables and Intervals for Acceptable Values," which follows shortly below, describes precisely the variables utilized and indicates the intervals for acceptable values. Subsequent tables report the number of observations (x) rejected because of extreme values of at least one of the variables in the observation vector.

The industry regressions are based on means of all of the observations included in the cross sections for each industry-year. There are eleven "industries" in all,[4] but in a number of cases lack of response or failure

[3] Appendix B states algebraically the precise nature of the deviations used in the various regressions.

[4] See Appendix Table A–1 for a tabulation of the total sample by industry.

to direct certain questions to an industry resulted in eliminating all observations for an industry for one or more years. Industry-year observations were in each case weighted by the number of individual firm observations. The regression results presented, therefore, involve several partitionings of the over-all sum of squares and cross products: firm cross section within industries plus industry cross section equal firm cross section; also, firm cross section within industries plus industry over-all equals firm over-all.

In the absence of information as to value added, "output" defined as sales plus inventory investment was frequently taken as the dependent variable. While this measure of "output" is generally conceptually more relevant to the production function than is sales itself, the absence of inventory statistics prior to 1957 in the body of data at our disposal caused us to use sales rather than output in several estimates over a longer period. It appears, however, that inventory investment, the difference between sales and output, is essentially a minor disturbance in the relation of sales to the arguments of the production function and does not markedly bias the estimates of parameters. In one set of relations the dependent variable was taken to be the change in the logarithm of "sales capacity," calculated as sales divided by the reported percentage utilization of capacity. In other relations the dependent variable was based upon firms' reports of their own "per cent change in capacity." More often, the reported figure for percentage utilization of capacity was introduced as an independent variable whose coefficient was estimated along with other variables introduced into the production function. The manner in which respondents' reports of utilization of capacity related to the production function was thus left open to estimation.

The data lend themselves obviously to estimates of "Cobb-Douglas-type" or log-linear production functions, and these have in most instances been estimated. The differing results stemming from utilization of differently structured sets of cross-sectional and time series data are presented below. In addition, however, direct estimates of constant elasticity-of-substitution production functions are being attempted, and a preliminary report on these attempts is offered in Appendix C.

TABLE 1

Definitions and Sources of Variables and Intervals for Acceptable Values

Variable[a]	Symbols and Definitions	Source[b]	Acceptable Interval[c]
Sales	S_t	FD	[20,000, .01]
Inventories (end of year)	H_t	FD	[∞, 0]
Output	$O_t = S_t + (H_t - H_{t-1})$	FD	[20,000, .01]
Utilization of capacity (end of year, ratio)	U_t	MH	[1.3, .3]
Sales capacity	$S_t^c = \frac{S_t}{U_t}$	FD/MH	[20,000, .01]
Change in capacity (ratio)	ΔC_t	MH	[.5, -.25]
Gross fixed assets (end of year)	K_t	FD	[20,000, .01]
Employees (end of year)	E_t	MH	[2,000, .01]
Capital expenditures	I_t	MH	[2,000, .01]
Capital expenditures as ratio of 1957 gross fixed assets	$i_t = \frac{I_t}{K_{57}}$	MH/FD	[.6, .001]
Depreciation charge ratio, 1953	$d_{53} = \frac{D_{53}}{K_{53}}$	FD	[.2, .001]
Time trend integer, beginning with zero for first year of dependent variable	T		[7, 0]

[a]Sales, output, and gross capital expenditure (*S, O,* and *I*) but not depreciation charges (*D*) are price deflated. A further description of the procedure is found in Robert Eisner, "Capital Expenditures, Profits and the Acceleration Principle," in *Models of Income Determination,* Princeton for NBER, 1964, pp. 141-42. Where price-deflated values of gross fixed assets were used, they were calculated as the sum of the previous five years of deflated capital expenditures plus an estimate of the deflated value of gross fixed assets not acquired in these five years. Thus,

$$K_{pt} = \sum_{j=0}^{4} I_{pt-j} + \frac{K_t - \sum_{j=0}^{4} I_{t-j}}{\frac{1}{M}\sum_{h=1}^{M} P_{t-h-4}},$$

Notes to Table 1 (concluded)

Where P denotes a price deflator or (as a lower-case subscript) a price-deflated variable and M is the lesser of an estimate of the firm's length of life of capital $\frac{1}{(D_{53}/K_{53})}$ and the number of years remaining back to 1946. (All gross fixed assets acquired prior to 1946 were thus arbitrarily assigned the post-1945 mean price deflator calculated above.)

[b]MH = McGraw-Hill surveys.
FD = financial data, generally from Moody's.
FD/MH = numerator from financial data and denominator from McGraw-Hill.
MH/FD = numerator from McGraw-Hill and denominator from financial data.

[c]All variables in millions of dollars except employees, which is in thousands; ratio variables, which are pure decimal numbers, and the time trend integer. Natural logarithms are used in all logarithmic transformations. $[U, L]$ = closed interval, including upper and lower bounds.

The Findings

A number of interesting results become apparent upon examination of Table 2. Here output is presented as a log-linear relation of utilization of capacity, gross fixed assets, and employment. Current and lagged values of each of the independent variables are introduced. This is important because the independent variables are defined at points of time as of the end of the year, while the dependent variable, output, is the integral of a rate over the entire year. It seems preferable to allow the regressions to indicate the relative weights to be attached to the two end-of-year points. Problems of collinearity for each couplet of variables can then be met by presenting the sums and standard errors of sums of coefficients.

The firm cross-section regression shows a not unsurprising result. Output varies most with end-of-current-year utilization of capacity, capital stock, and employment. The sum of the capital stock and employment coefficients is .991, with a standard error of .012. This suggests virtually constant returns to scale; and the sum of the labor coefficients is .651, approximately twice the sum of the capital coefficients. The elasticity of output with respect to capital stock seems relatively a bit too high but not very far off what might be taken, on a priori grounds, to be a reasonable value. The sum of the utilization of capacity coefficients does not pass the null hypothesis test, and this

TABLE 2

Logarithms of Output, 1959-62, as a Function of Logarithms of Utilization of Capacity, Gross Fixed Assets and Employment, With and Without Time: Firm Time Series, Cross Section, and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors: Firm Time Series: With Time	Firm Time Series: Without Time	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All: With Time	Firm Over-All: Without Time	Means and Standard Deviations[a]
Constant term or ln O_t					2.156 (.063)	2.212 (.059)	4.947 (1.470)
ln U_t	.235 (.032)	.235 (.033)	.213 (.106)	.073 (.114)	.182 (.102)	.199 (.102)	-.257 (.183)
ln U_{t-1}	.179 (.035)	.174 (.036)	-.112 (.105)	-.165 (.111)	-.081 (.102)	-.087 (.102)	-.253 (.186)
ln K_t	.326 (.066)	.425 (.061)	.476 (.144)	.471 (.136)	.493 (.143)	.478 (.144)	4.404 (1.737)
ln K_{t-1}	-.061 (.058)	.021 (.054)	-.136 (.145)	-.097 (.139)	-.154 (.144)	-.135 (.145)	4.342 (1.730)
ln E_t	.188 (.031)	.178 (.032)	.390 (.098)	.367 (.093)	.375 (.095)	.378 (.096)	1.932 (1.342)
ln E_{t-1}	.037 (.041)	.063 (.041)	.261 (.100)	.262 (.096)	.276 (.097)	.270 (.098)	1.915 (1.343)
T	.019 (.005)				.039 (.015)		1.527 (1.074)
Σ ln U coeffs.	.414 (.053)	.409 (.054)	.101 (.101)	-.093 (.102)	.101 (.100)	.111 (.101)	
Σ ln K coeffs.	.266 (.067)	.446 (.046)	.340 (.017)	.374 (.025)	.339 (.017)	.343 (.017)	
Σ ln E coeffs.	.275 (.050)	.242 (.050)	.651 (.022)	.629 (.028)	.652 (.022)	.648 (.022)	
Σ ln U + Σ ln K coeffs.	.680 (.088)	.854 (.074)	.440 .100	.282 (.106)	.441 (.100)	.454 (.101)	

(continued)

TABLE 2 *(concluded)*

Variable or Statistic	Regression Coefficients and Standard Errors: Firm Time Series, With Time	Firm Time Series, Without Time	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All, With Time	Firm Over-All, Without Time
$\Sigma \ln K + \Sigma \ln E$ coeffs.	.541 (.071)	.687 (.059)	.991 (.012)	1.003 (.012)	.991 (.012)	.991 (.012)
$\Sigma \ln U + \Sigma \ln K + \Sigma \ln E$ coeffs.	.955 (.085)	1.096 (.077)	1.091 (.099)	.911 (.100)	1.092 (.099)	1.102 (.100)
n (−17)	606	606	674	674	674	674
r.d.f.	395	396	664	646	666	667
$\hat{R}^2$	.379	.359	.924	.926	.924	.923

n = number of observations; the figure following in parentheses is the number of individual firm observations eliminated because of extreme values for one of the variables.

r.d.f. = residual degrees of freedom.

$\hat{R}^2$ = adjusted or unbiased coefficient of determination.

[a]From firm over-all.

would suggest that differences between firms in reported utilization of capacity reflect rather interfirm differences in the measure than anything systematically related to output. Gross fixed assets and employment would seem to explain most of what there is to explain in differences in output between firms.

Partially contrasting results are found in the firm time series. Here we note, regarding the firm time series columns, that the employment coefficients are smaller than in the case of the cross section, but the utilization of capacity coefficients are high. The capital coefficients are somewhat higher, unless a time trend variance is included, but then are somewhat lower. It would appear that variation in output over time is accomplished in part by changing capital stock and changing the level of employment, but in part by varying their rates of utilization. This, of course, makes good sense in terms of our economic theory, old and new. Time series variations in output within the firm must be viewed in considerable part as short run or transitory in nature. They should not call for immediate equiproportionate changes in stocks of capital and labor.

Also of interest in Table 2 are the positive coefficients of the time trend variable. In the time series, the coefficient of .019 suggests that output tended to grow by 1.9 per cent per year during the period 1959–62, after taking into account changes in utilization of capacity, gross fixed assets, and employment. The even higher time trend coefficient of .039 in the firm over-all regression, in which cross sectional variance clearly dominated, similarly suggests substantial contributions to the growth of output not accounted for by the variables introduced into our relation.

What we have failed to include—increases in management skill, improvement in the quality of capital goods, greater productivity (or longer work weeks) of employees—we cannot presume to say. But it does appear that something is contributing to a growth in output beyond the factors we have introduced. And one may note that the higher capital coefficients when the time trend variable is excluded illustrate again the possibility that the true contribution of capital to output may be overestimated if a trend-like capital variable is allowed to act as a proxy for other, unspecified, trend-producing factors.

Table 3 offers some interesting confirmations of the findings of Table 2, along with information on the time profile of the capital and capital expenditures affecting output. Here, output is related to utilization of capacity, capital expenditures of the current and two previous years, the rate of depreciation, gross fixed assets in existence just prior to the beginning of the capital expenditure series, and employment. Each year's capital expenditures are found to be positively related to output in all of the regressions and the sums of the capital expenditure coefficients are in a statistical sense clearly significantly positive. On the other hand, the rate of depreciation, that is, the ratio of depreciation charges to gross fixed assets in 1953, before changing tax laws made this ratio a less reliable measure of replacement requirements, is negatively related to output.

As before, the contribution of the capital and labor variables is generally less in the time series than in the cross sections, except that the coefficient of ln K_{t-3} is high in the time series with the time trend

TABLE 3

Logarithms of Output, 1959-62, as a Function of Logarithms of Utilization of Capacity, Capital Expenditures, Depreciation Rate, Previous Gross Fixed Assets, and Employment, With and Without Time: Firm Time Series, Cross Sections and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors						Means and Standard Deviations
	Firm Time Series		Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All		
	With Time	Without Time			With Time	Without Time	
Constant term or ln O_t					2.363 (.154)	2.437 (.151)	5.273 (1.377)
ln U_t	.204 (.041)	.221 (.041)	.217 (.116)	.088 (.131)	.178 (.111)	.211 (.110)	-.252 (.175)
ln U_{t-1}	.170 (.043)	.180 (.044)	-.070 (.118)	-.166 (.129)	-.032 (.113)	-.047 (.113)	-.245 (.175)
ln I_t	.034 (.012)	.036 (.013)	.034 (.034)	.032 (.032)	.040 (.033)	.037 (.033)	1.745 (1.758)
ln I_{t-1}	.003 (.011)	.012 (.011)	.056 (.039)	.049 (.037)	.052 (.039)	.054 (.039)	1.722 (1.747)
ln I_{t-2}	.021 (.011)	.027 (.011)	.034 (.034)	.053 (.033)	.031 (.034)	.018 (.033)	1.787 (1.782)
ln d			-.093 (.049)	-.035 (.047)	-.093 (.049)	-.081 (.049)	-2.930 (.390)
ln K_{t-3}	.114 (.070)	.310 (.042)	.206 (.037)	.243 (.040)	.205 (.036)	.221 (.036)	4.557 (1.649)
ln E_t	.148 (.033)	.144 (.033)	.401 (.104)	.335 (.100)	.388 (.102)	.397 (.102)	2.219 (1.281)
ln E_{t-1}	.153 (.050)	.134 (.051)	.258 (.106)	.284 (.104)	.270 (.104)	.255 (.104)	2.202 (1.283)
T	.025 (.007)				.038 (.017)		1.712 (1.019)
Σ ln U coeffs.	.374 (.069)	.401 (.070)	.146 (.118)	-.077 (.118)	.146 (.118)	.165 (.118)	

(continued)

TABLE 3 *(concluded)*

	Regression Coefficients and Standard Errors					
	Firm Time Series			Firm Cross Section Within Industries	Firm Over-All	
Variable or Statistic	With Time	Without Time	Firm Cross Section		With Time	Without Time
$\Sigma \ln I$ coeffs.	.058 (.022)	.075 (.022)	.124 (.034)	.134 (.032)	.124 (.033)	.109 (.033)
$\Sigma \ln E$ coeffs.	.301 (.059)	.278 (.059)	.659 (.025)	.619 (.032)	.658 (.025)	.652 (.025)
$\Sigma \ln U + \Sigma \ln E$ coeffs.	.675 (.084)	.679 (.086)	.805 (.121)	.542 (.119)	.804 (.121)	.817 (.121)
$\Sigma \ln U + \ln K + \Sigma \ln E$ coeffs.	.790 (.111)	.989 (.096)	1.011 (.127)	.785 (.125)	1.010 (.127)	1.038 (.127)
n (−10)	425	425	500	500	500	500
r.d.f.	273	274	487	469	489	490
$\hat{R}^2$	.343	.317	.923	.929	.925	.925

variable excluded. And again, the coefficient of time is significantly positive.

Table 4 presents the results of regressions of changes in the logarithms of output on changes in the logarithms of utilization of capacity, gross fixed assets, and employment.[5] This, of course, makes most of the variance of our variables stem from changes over time in the cross sections as well as in the time series. One should now expect the coefficients of the various regressions to be less different; and, in fact, this is so. The coefficients of the change in the logarithms of gross fixed assets are indeed virtually identical in the time series and cross-section regressions. That the coefficients of $\Delta \ln U_t$ and $\Delta \ln E_t$ are somewhat higher in the time series may again involve a larger concentration of "noise" in the cross-section relation.

We may also note the substantial and statistically significant positive constant term in the firm over-all regression. This implies that output

[5] Acceptable intervals of [.5, −.5] were established for $\Delta \ln U$ and $\Delta \ln E$ and of [.693, −.693] for $\Delta \ln K$. These permit corresponding arithmetic ranges of [+65 per cent, −39 per cent] and [+100 per cent, −50 per cent].

TABLE 4

Changes in Logarithms of Output, 1959-62, as a Function of Changes in Logarithms of Utilization of Capacity, Gross Fixed Assets and Employment: Firm Time Series, Cross Sections and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors					Means and Standard Deviations
	Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Firm Over-All with Constant Term Constrained to Zero	
Constant term or $\Delta \ln O_t$				.031 (.006)		.063 (.134)
$\Delta \ln U_t$	.148 (.042)	.110 (.032)	.119 (.033)	.124 (.033)	.137 (.034)	.007 (.151)
$\Delta \ln K_t$	.384 (.105)	.407 (.061)	.408 (.061)	.416 (.064)	.621 (.051)	.062 (.076)
$\Delta \ln E_t$	.388 (.057)	.250 (.038)	.258 (.038)	.302 (.039)	.289 (.039)	.016 (.132)
$\Delta \ln U + \Delta \ln E$ coeffs.	.537 (.061)	.360 (.043)	.376 (.045)	.426 (.043)	.426 (.044)	
$\Delta \ln K + \Delta \ln E$ coeffs.	.772 (.111)	.657 (.063)	.665 (.064)	.718 (.066)	.910 (.056)	
$\Delta \ln U + \Delta \ln K$ coeffs.	.532 (.118)	.516 (.072)	.526 (.072)	.540 (.076)	.759 (.063)	
$\Delta \ln U + \Delta \ln K + \Delta \ln E$ coeffs.	.421 (.117)	.767 (.069)	.784 (.070)	.842 (.072)	1.047 (.061)	
n (−37)	555	622	622	622	622	
r.d.f.	357	615	597	618	618	
$\hat{R}^2$	.210	.185	.190	.214	.330	

would grow at some 3.1 per cent per year with utilization of capacity, gross fixed assets, and employment all held constant. The results of a firm over-all regression with time and with a constant term constrained to zero is a markedly higher coefficient for the change in the logarithm of gross fixed assets. It would appear that there are increases in output essentially independent of capital stock and the other variables that are attributed to increases in gross fixed assets when it is impossible to measure their causes elsewhere.

Table 5 differs from the previous tables in covering all the years from 1955 to 1962 but presenting the relation between the logarithms of sales and the logarithms of utilization of capacity and gross fixed assets. (Neither inventory investment nor employment figures were available for this study in the earlier years.) Again we note the coefficients of the factor input variables are distinctly lower in the firm time series than in the firm cross sections. We note also, however, that the role of utilization of capacity is apparently higher in the firm cross sections within industries than indicated in previous tables. This is perhaps accountable to the exclusion of employment from these regressions. The utilization-of-capacity variable within industries (where interfirm differences in measurement might account for less error than in the firm cross section between all firms) may be picking up some of the effect properly attributable to variance in employment. It may also be presumed that larger quantities of labor were associated with larger quantities of capital so that the "capital coefficient" in this regression reflects the effect of indeterminate covariances of capital and labor and of labor and output.

In Table 5, for the first time, we have a sufficient number of industry-year observations to warrant presentation of results of regressions on industry-year means. One might expect the variance over time in the experience of broad industry groups to be more "permanent" or long run in character than variance over time in the experience of individual firms. The regression coefficients in the industry time series are thus higher, as might be expected, although in fact a bit too high for plausible explanation. It would appear that variance in capital is more closely related to current and immediately subsequent output in the industry time series than in individual firm time series. The coefficients of utilization, however, are also higher than we should anticipate.

The introduction of a time trend variable did little to clarify the matter. Its coefficient was slightly negative (−.021) in the industry time series, trivially positive (.006) in the firm time series regression, in which the sum of the capital coefficients was then reduced from .520 to .469, but slightly negative (−.014) in the over-all regression.

Thus far all of the regressions discussed have included utilization of capacity as independent variables with coefficients to be estimated. We have also constructed a "sales capacity" variable, defined as actual sales

TABLE 5

Logarithms of Sales, 1955-62, as a Function of Logarithms of Utilization of Capacity and Gross Fixed Assets: Firm and Industry Time Series, Cross-Section, and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors: Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Industry Time Series	Industry Cross Section	Industry Over-All	Means and Standard Deviations
Constant term or ln S_t				1.751			2.757	5.074
				(.061)			(.298)	(1.392)
ln U_t	.235	.265	0.324	0.322	0.814	.174	0.571	−0.223
	(.028)	(.117)	(.109)	(.110)	(.191)	(.493)	(.400)	(.181)
ln U_{t-1}	.260	−.108	0.072	−0.056	0.572	−.493	−0.026	−.217
	(.029)	(.116)	(.105)	(.110)	(.275)	(.571)	(.508)	(.182)
ln K_t	.341	.793	0.762	0.889	0.928	.090	3.002	4.533
	(.043)	(.162)	(.135)	(.160)	(.645)	(1.644)	(1.228)	(1.670)
ln K_{t-1}	.179	−.054	0.101	−0.146	0.009	.408	−2.507	4.456
	(.040)	(.162)	(.136)	(.160)	(.656)	(1.647)	(1.227)	(1.662)
Σ ln U coeffs.	.516	.157	0.396	0.266	1.386	−.318	0.545	
	(.041)	(.113)	(.100)	(.109)	(.364)	(.625)	(.555)	
Σ ln K coeffs.	.520	.739	0.863	0.744	0.937	.498	0.495	
	(.019)	(.010)	(.010)	(.010)	(.053)	(.038)	(.040)	

(continued)

TABLE 5 *(concluded)*

Variable or Statistic	Regression Coefficients and Standard Errors: Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Industry Time Series	Industry Cross Section	Industry Over-All	Means and Standard Deviations
$\ln U_t$	.596	1.058	1.086	1.211	1.742	.264	3.573	
$+ \ln K_t$ coeffs.	(.052)	(.196)	(.169)	(.189)	(.605)	(1.754)	(1.263)	
$\ln U_{t-1}$	.439	-.162	0.173	-0.202	0.581	-.085	-2.533	
$+ \ln K_{t-1}$ coeffs.	(.054)	(.208)	(.180)	(.207)	(.851)	(1.715)	(1.510)	
$\Sigma \ln U$	1.036	.896	1.259	1.009	2.323	.180	1.039	
$+ \Sigma \ln K$ coeffs.	(.051)	(.111)	(.099)	(.107)	(.356)	(.609)	(.538)	
n (-38)	1,299	1,352	1,352	1,352	48	48	48	
r.d.f.	1,012	1,340	1,300	1,347	38	36	43	
$\hat{R}^2$	.422	.803	.852	.809	.932	.835	.838	

divided by reported end-of-year utilization of capacity. The changes in the logarithm of this variable are then regressed on the changes in logarithms of gross fixed assets, current and lagged. In effect, therefore, the coefficient of the logarithm of utilization of capacity is constrained to unity. If reported utilization of capacity figures could be taken literally, we would thus be relating changes of capital stock to measures of changes in the rate of output which they are capable of producing.

As might be expected, with implicit constraints on the utilization-of-capacity coefficient and with first differences of the logarithms of our variables, the coefficients of determination are generally low. The large number of observations reported upon in Table 6 (1,210 in the firm cross section, even after eliminating 61 observations for extreme values [6]) permits estimates with reasonably low standard errors. It is to be noted, therefore, that the sum of the coefficients of the $\Delta \ln K_t$ coefficients ranges from .679 to .709 in the various individual firm regressions. Relative changes in sales capacity were thus some two-thirds of relative changes in gross fixed assets of the current and preceding years. This, of course, is not to argue that the elasticity of capacity with respect to capital stock is two-thirds. For we have not, in this regression, allowed for the effect of changes in employment. Employment now, perhaps like other factors in the other regressions, has been "embodied" in capital stock.

It will be recalled that we have included another variable from the McGraw-Hill surveys bearing on capacity. This is the annual change in capacity reported by respondents at the end of the year. The logarithm of this capacity change ratio (plus unity) has been regressed on the ratio of capital expenditures to 1957 gross fixed assets. Coefficients of current and lagged values of the logarithm of the capital expenditure ratios are positive, as shown in Table 7, but their magnitudes are deceptive. Since the geometric mean of the capital expenditure ratio was some 6.3 or 6.4 per cent, the sum of coefficients of, for example, 0.03, would imply that a 10 per cent increase in capital stock would bring about an increase in capacity of just under 5 per cent, which would be roughly consistent with our other results involving capital stock directly.

We have also introduced the depreciation ratio in Table 7. Its

[6] Intervals of [.47, −.23] and [.531, −.92] were established for $\Delta \ln K$ and $\Delta \ln S_t^c$, respectively. These permit corresponding arithmetic ranges of [+60 per cent, −20 per cent] and [+70 per cent, −60 per cent].

TABLE 6

Changes in Logarithms of Sales Capacity, 1955-62, as a Function of Changes in Logarithms of Gross Fixed Assets: Firm and Industry Time Series, Cross-Section, and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors							Means and Standard Deviations
	Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Industry Time Series	Industry Cross Section	Industry Over-All	
Constant term or $\Delta \ln S_t^c$				-.014 (.008)			.027 (.034)	.042 (.177)
$\Delta \ln K_t$	.519 (.081)	.456 (.065)	.433 (.064)	.536 (.066)	1.489 (.472)	.944 (.564)	1.482 (.428)	.079 (.079)
$\Delta \ln K_{t-1}$	.190 (.077)	.222 (.064)	.255 (.063)	.172 (.064)	-0.607 (.430)	-.291 (.482)	-0.599 (.402)	.080 (.081)
$\Sigma \Delta \ln K$ coeffs.	.709 (.109)	.679 (.076)	.688 (.075)	.708 (.075)	0.882 (.476)	.653 (.570)	0.882 (.412)	
n (-61)	1,165	1,210	1,210	1,210	48	48	48	
r.d.f.	896	1,200	1,160	1,207	40	38	45	
$\hat{R}^2$	.050	.064	.068	.074	.259	.020	.176	

TABLE 7

Logarithms of 1 Plus Change in Capacity, 1955-62, as a Function of Logarithms of Capital Expenditure Ratios and Depreciation Ratios and of Time: Firm and Industry Time Series, Cross-Section and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors							Means and Standard Deviations
	Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Industry Time Series	Industry Cross Section	Industry Over-All	
Constant term or $\ln(1+\Delta C)$				.150 (.016)			.255 (.046)	0.042 (.061)
$\ln i_t$	.031 (.004)	.025 (.004)	.024 (.004)	.027 (.004)	.050 (.008)	.049 (.019)	.047 (.011)	-2.767 (.690)
$\ln i_{t-1}$	.006 (.004)	.004 (.004)	.006 (.004)	.002 (.004)	-.012 (.008)	-.026 (.020)	-.018 (.011)	-2.745 (.652)
$\ln d_{53}$		.005 (.005)	.000 (.006)	.005 (.005)		.045 (.019)	.041 (.017)	-2.931 (.398)
T	-.0042 (.0010)			-.0033 (.0009)	-.0031 (.0007)		-.0029 (.0010)	4.000 (2.260)
$\Sigma \ln i$ coeffs.	.038 (.005)	.029 (.004)	.030 (.004)	.029 (.004)	.038 (.009)	.023 (.017)	.029 (.013)	
n (-15)	762	814	814	814	46	46	46	
r.d.f.	593	803	765	809	37	35	41	
$\hat{R}^2$	.140	.102	.093	.131	.670	.299	.511	

coefficient is, perhaps surprisingly, either zero or positive. Apparently, industries with more rapid rates of depreciation were those which, other things equal, reported more rapid growth in capacity. The absolutely small but negative coefficients of the time trend variable suggest that increases in capacity have been harder to come by as the years have progressed from 1955 to 1962.

In Table 8 we show the results of work with price-deflated capital stock and with output, capital stock, and employment all expressed as ratios of average sales of the 1956–58 period. While the latter transformation was intended to eliminate heteroscedasticity associated with the considerable differences in size of firms in the sample, it also had the effect of changing considerably the nature of the variance in the cross-section and over-all regressions. With each firm's observations normalized on its own sales base, the variance became essentially variance over time, as in the first-difference relations already discussed. The coefficient of lagged employment is strangely negative in the cross-section and over-all regressions, but the sum of the employment coefficients remains positive, if somewhat small. The coefficients of deflated, lagged capital stock are positive but small in the cross sections but quite high in the time series where the trend variable is excluded. The coefficient of the trend variable is positive and substantial, implying again a 3 to 5 per cent per annum increase in output with employment, capital, and utilization all held constant. Inclusion of trend, however, again reduces sharply the positive coefficient of lagged and deflated capital stock in the time series. This implies that the positive relation between capital stock and output involves a common trend element or that our relation again forces "embodiment" in capital of some other unspecified factors of production.

Summary and Conclusions

By way of summary we may indicate the following:

1. Log-linear relations of output with utilization of capacity, gross fixed assets, and the number of employees in individual firm cross sections prove consistent with many estimates suggesting relatively constant returns to scale. The elasticity of output with respect to capital stock is about one-third. Utilization of capacity does not enter significantly.

TABLE 8

Logarithms of Output/Sales,[a] 1959-62, as a Function of Logarithms of Utilization of Capacity, Price-Deflated-Capital-Stock/Sales[a] and Employment/Sales,[a] With and Without Time: Firm Time Series, Cross-Section and Over-All Regressions

Variable or Statistic	Regression Coefficients and Standard Errors: Firm Time Series, With Time	Firm Time Series, Without Time	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All, With Time	Firm Over-All, Without Time	Means and Standard Deviations
Constant term or $\ln O_t^s$					.609 (.058)	.682 (.059)	.118 (.222)
$\ln U_t$	.078 (.036)	.089 (.038)	.075 (.062)	.013 (.074)	.057 (.059)	.083 (.061)	−.254 (.191)
$\ln U_{t-1}$	.196 (.039)	.215 (.041)	.240 (.065)	.234 (.076)	.261 (.061)	.249 (.064)	−.252 (.183)
$\ln k_{pt-1}^s$	.088 (.072)	.381 (.057)	.061 (.012)	.064 (.019)	.061 (.012)	.059 (.013)	−.379 (.730)
$\ln e_t^s$	.469 (.046)	.455 (.049)	.472 (.078)	.498 (.080)	.471 (.077)	.476 (.080)	−2.933 (.515)
$\ln e_{t-1}^s$	.052 (.046)	−.001 (.048)	−.313 (.080)	−.309 (.081)	−.311 (.079)	−.319 (.082)	−2.941 (.501)
T	.030 (.005)				.051 (.009)		1.539 (1.063)
$\Sigma \ln U$ coeffs.	.274 (.059)	.305 (.063)	.315 (.056)	.247 (.062)	.317 (.056)	.332 (.058)	
$\Sigma \ln e$ coeffs.	.521 (.060)	.454 (.063)	.159 (.018)	.189 (.023)	.160 (.018)	.157 (.019)	
$\Sigma \ln U +$ $\ln k$ coeffs.	.362 (.102)	.686 (.093)	.376 (.057)	.311 (.067)	.378 (.057)	.392 (.059)	
$\Sigma \ln U +$ $\ln k +$ $\Sigma \ln e$ coeffs.	.883 (.096)	1.140 (.092)	.535 (.060)	.500 (.065)	.538 (.060)	.549 (.063)	
n (-10)	391	391	423	423	423	423	
r.d.f.	256	257	414	396	416	417	
$\hat{R}^2$	.490	.419	.262	.276	.308	.250	

[a]Average sales, 1956-58.

2. Pooled time series regressions of the same individual firm data for the years 1959–62 offer partially contrasting estimates. The utilization-of-capacity variable here looms large. Current changes in output of the firm are apparently accomplished partly by changing the "stocks" of employment and capital, and partly by altering their rates of utilization.

3. There is an upward movement of output of some 2 per cent to 5 per cent per annum not explained by the utilization, capital, and employment variables.

4. Current output is related to capital expenditures over a succession of previous years, and output is positively associated in time series with capital stock lagged several years. Much of this latter association is apparently accounted for by a linear trend in the 1959–62 period. The rate of depreciation is found to be negatively related to output.

5. Time series and cross-section regressions involving first differences (of logarithms) are more similar to each other, as might be expected from the nature of the variance and covariance involved. The sums of capital and labor coefficients are less than unity in all regressions. A positive constant term confirms again the contribution to growth of output of variables excluded from our relation. When the constant term is constrained to zero a large role is ascribed to changes in capital.

6. Exclusion of employment from relations over the years 1955–62 leads to generally higher coefficients for utilization of capacity and capital stock.

7. A positive relation is found between current and lagged ratios of capital expenditures to gross fixed assets and reported changes in capacity. One might infer, perhaps not too dangerously, that capital expenditures contribute to production because they contribute to the capacity to produce, and also that the McGraw-Hill data on year-to-year changes in capacity make some sense.

8. Comparison of firm time series regressions involving price-deflated capital stock, with and without inclusion of a time trend variable, suggests that capital can be constrained to "embody" the growth-producing effects of factors of production excluded from the relation estimated.

This is indeed a preliminary report. Much in the tables may merit further scrutiny and critical analysis. As new data are added, permitting more years of observations on several of the key variables, it should prove fruitful to devote further consideration to relations involving industry-year means. In particular, parameters should be estimated separately for different industries or industry groups. I am all too painfully aware of the specification errors and problems of aggregation involved in estimates of pooled regressions of a mythically unique production function for hundreds of different firms in many different industries with countless differences in technology.

Some broad outlines should, however, already be clear. Estimates will differ as between cross sections and time series of the same body of data, reflecting in large part, I would argue, differences in the nature of the variances which we seek to relate. Firms are not always, if ever, in equilibrium. They adjust differently to short-run changes in output, which dominate the time series variance, and long-run differences in output measured in cross sections, with short-run reactions involving significant alterations in the utilization of existing capacity.

These considerations would seem important to evaluation of estimates of critical parameters of the production function. Perhaps keeping them firmly in mind in further work will lead us a bit closer to Truth.

References

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Appendix A

TABLE A-1

Classification of Firms by Industry and Response on Utilization-of-Capacity Questions

Industry	Number of Firms: Total	With Information on Utilization-of-Capacity Variable, 1955–62: Min.	Max.
Primary metals	38	15	32
Metalworking	230	113	188
Chemical processing	100	48	83
All other manufacturing	166	60	122
Mining	30	1	6
Utilities	43	0	0
Petroleum	33	10	16
Insurance and banks	44	0	0
Stores	71	0	0
Railroads	28	0	0
Transportation and communications other than railroads	19	0	0
All Industries	802	252	445

Appendix B

Algebraic Statement of Deviations Used in the Various Regressions

Let X_{fnt} denote the observation vector of firm f in industry n for the year t.
Let F_{nt} denote the number of firms with observations in industry n in the year t.
Let τ_{fn} denote the number of years of observations for firm f in industry n.
Let N_t denote the number of industries containing observations in the year t.
Let τ denote the number of years for which observations are available.

Then, $\bar{X}_{fn} = \frac{1}{\tau_{fn}} \sum_{t=1}^{\tau_{fn}} X_{fnt}$ = the mean of observations of all years for firm f in industry n

and $X_{fnt} - \bar{X}_{fn}$ = the deviations used in firm time series, including only firms for which $\tau_{fn} > 1$.

$$\bar{X}_t = \frac{1}{\sum_{n=1}^{N_t} F_{nt}} \sum_{n=1}^{N_t} \sum_{f=1}^{F_{nt}} X_{fnt}$$ = the mean of observations of all firms in all industries in year t

and $X_{fnt} - \bar{X}_t$ = the deviations used in firm cross sections.

$$\bar{X}_{nt} = \frac{1}{F_{nt}} \sum_{f=1}^{F_{nt}} X_{fnt}$$ = the mean of observations of all firms in industry n in year t (industry-year-mean)

and $X_{fnt} - \bar{X}_{nt}$ = the deviations used in firm cross sections within industries.

$$\bar{X} = \frac{1}{\sum_{t=1}^{T} \sum_{n=1}^{N_t} F_{nt}} \sum_{t=1}^{T} \sum_{n=1}^{N_t} \sum_{f=1}^{F_{nt}} X_{fnt}$$ = the mean of all observations of all industries in all years

and $X_{fnt} - \bar{X}$ = the deviations used in firm over-all regressions.

$$\bar{X}_n = \frac{1}{\sum_{t=1}^{\tau} F_{nt}} \sum_{t=1}^{T} \sum_{f=1}^{F_{nt}} X_{fnt}$$ = the mean of all observations in industry n,

$\bar{X}_{nt} - \bar{X}_n$ = the deviations used in industry time series,

$\bar{X}_{nt} - \bar{X}_t$ = the deviations used in industry cross sections,

$\bar{X}_{nt} - \bar{X}$ = the deviations used in industry over-all regressions.

and $$\frac{F_{nt} \sum_{t=1}^{T} N_t}{\sum_{n=1}^{N_t} \sum_{t=1}^{T} F_{nt}}$$ = the weight attached to the observation for industry n in the year t.

Appendix C
On CES Functions

It is hoped that the data available to us will permit direct estimates of parameters of the currently popular constant-elasticity-of-substitution production functions. Results obtained thus far do not appear to warrant more than a brief tentative report of what is being attempted.[1]

[1] I am particularly indebted to Jon Rasmussen for assistance in the preparation of this appendix.

Taking the somewhat more general form of Brown and de Cani, we may write the CES function as

$$(A1) \qquad Y = A[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-v/\rho},$$

where A is an "efficiency" or scale parameter, δ is the "distribution" parameter, ρ is the substitution parameter—the elasticity of substitution $= 1/(1+\rho)$ —and v defines the degree of the function, so that $v = 1$ implies homogeneity of the first degree and constant returns to scale and $v \gtrless 1$ imply increasing or decreasing returns to scale respectively.

The CES function, obviously, is not linear in either capital and labor or their logarithms. Following a suggestion of Murray Brown, however, it should be possible to form least-squares estimators of the CES function parameters by utilizing first-order Taylor series approximations.

Thus, letting A, δ, ρ, and v be designated respectively by C_i, $i = 1,2,3,4$, the CES function may be written:

$$(A2) \qquad Y = F(K, L, C_i),$$

and its first-order Taylor series approximation for assumed initial values, C_i^0, of the parameters, is

$$(A3) \qquad Y = F(K, L, C_i^0) + \sum_{i=1}^{4} F_i(K, L, C_i^0)(C_i - C_i^0),$$

where the F_i denote derivatives with respect to the C_i, and Y, K, and L are output, capital, and labor respectively. Transposing terms and letting the subscript j denote the jth observation, we may then form the statistical relation,

$$(A4) \quad Y_j - F(K_j, L_j, C_i^0) + \sum_{i=1}^{4} C_i^0 F_i(K_j, L_j, C_i^0) = \sum_{i=1}^{4} C_i F_i(K_j, L_j, C_i^0) + U_j.$$

Thus for each observation, j, for any assumed set of C_i^0, all of the terms are known values except the C_i on the right side of the equation, which are the parameters to be estimated, and the disturbance, U_j. We have a linear relation in which the unknown parameters can be estimated by direct least squares.

Initial values of the unknown parameters may be chosen on the basis of a priori knowledge or on the basis of prior estimates of the parameters of related linear or log-linear functions. Estimates of parameters secured from the Taylor-series formulation can then be used as assumed parameters in a second iteration. Successive iterations will reduce the variance of the dependent variable and, it is hoped, yield converging estimates of the parameters themselves.

The probability of achieving convergence would appear to depend considerably on initially assuming parameters not too far from the ultimate results of a possible convergence. This is made difficult, however, by the sensitivity of two of the parameters, A and δ, to the units of measurement of capital, labor and output.

For the estimates on which we are now able to report we used price deflated capital stock and divided each of our variables by average sales of the firm from 1956 to 1958, so that the variables would not be dominated by inter-firm size differences. We then defined

$$Y = 0_t^s$$

$$K = .5\,(k^s_{pt} + k^s_{p,t-1})$$

and

$$L = 10\,(e_t^s + e^s_{t-1})$$

thus setting a unit of employment as 50 employees, and assumed initial values of $A_0 = 1.5$, $\delta_0 = .25$, $\rho_0 = .20$, and $v_0 = 1.1$.

Attempts to achieve convergence in cross sectional estimates were unsuccessful. As indicated in Table C-1, after obtaining an acceptable first set of estimates we quickly exploded. By the fifth iteration not only had the estimates of ρ and v gone sky-high but, rather embarrassingly, those of A and δ turned negative. When we attempted to improve matters by constraining v at its estimated value after the first iteration, estimates still got out of

TABLE C-1

First-Order Taylor Series Estimates of Parameters, Firm Cross Sections Within Industries[a]

Parameter or Statistic	Initial Assumed Value	Iterations				
		1	2	3	4	5
A	1.5	0.379	.377	0.269	0.243	-1.862
		(.073)	(.079)	(.480)	(.024)	(2.227)
δ	0.25	0.263	.312	0.288	0.369	-.745
		(.011)	(.047)	(.144)	(.031)	(.618)
ρ	0.20	0.178	-.171	-2.132	-1.453	8.149
		(.035)	(.569)	(1.784)	(1.520)	(20.362)
v	1.1	1.050	.369	1.535	0.154	9.222
		(.030)	(.504)	(1.266)	(.193)	(5.818)
$\hat{R}^2$		0.955	.296	0.202	0.692	0.181
σ dependent variable		1.080	.274	0.257	0.406	0.253

[a]646 Observations, 1959-62.

hand, with that of ρ going to -10.503 in the fifth iteration and 202.642 in the sixth. At this latter point δ was estimated as 1.319, thus implying a negative marginal product of labor!

We were able to secure apparent convergence in time-series estimates, however. It seemed clear within a dozen iterations, as may be seen in Table C-2, that all of our estimates were settling down. With a couple of "guided leaps" we were finally able to bring estimates of all of our parameters except A to within one unit at the fourth decimal place of their values at the previous iteration, at which point we terminated the laborious computations. And the estimates of A, albeit with large standard error, were apparently oscillating within a relatively narrow range.

Estimates of δ and ρ seemed plausible, the former not too far from the share of capital implied by our log-linear regressions and the latter suggesting an elasticity of substitution of 1.51. The time trend coefficient of about .025 is also consistent with our other estimates. The results of log-linear regressions with variables and observations identical to those employed here also argue for the reasonableness of the CES estimates of δ and the time trend, as seen in Table C-3.

What is troublesome, however, is the estimate of .266 for v, suggesting sharply decreasing returns to scale. It seemed that this might relate to the high covariance of F_A and F_v along with the substantial difference between the estimate of A and our a priori expectations based on the dimensions of the variables. We therefore undertook to constrain v at the estimate of .865 produced by the initial iteration. Apparent convergence was again attained as shown in Table C-4, with the estimates of .311 and $-.254$ for δ and ρ neither unreasonable nor far from those secured without constraining v. The estimate of .918 for A is sharper and more plausible, but the coefficient of determination is much lower and the standard error of the estimate of ρ is much higher. With v constrained at unity, iterations were continued to the point where the estimates of δ seemed to be settling about .25 and those of ρ were in à fairly narrow interval about zero.

The low unconstrained estimate of v from these data is perhaps not justly to be dismissed. It may well reflect the absence of a variable measuring utilization of capacity. (Attempts to introduce one raised formidable computational problems and convergence was not obtained, but the effort will be resumed in later work.) For if changes in inputs of capital and labor are associated with opposite movements in the rate of utilization, as would appear likely, output would move less than proportionately with capital and labor.

It was not feasible to iterate separately, but results of using as initial values in other regressions the penultimate estimates obtained in the unconstrained firm time series are fairly similar, as may be seen in Table C-5. This closeness of estimates of δ, ρ and v obtained from firm and industry cross-section and over-all regressions to those of the firm time series offers

TABLE C-2

First-Order Taylor Series Estimates of Parameters, Firm Time Series[a]

Parameter or Statistic	Initial Assumed Value	Iteration						Assumed Value	Iteration		Assumed Value	Iteration	
		1	2	3	4	11	12		3	4		1	2
A	1.5	1.358	1.644	2.141	2.740	3.935	3.962	4.093	4.017	4.018	4.020	4.019	4.026
		(.213)	(.302)	(.516)	(.759)	(1.959)	(2.057)		(2.425)	(2.436)		(2.465)	(2.466)
δ	.25	.239	.256	.323	.369	.408	.408	.4105	.4110	.4110	.4111	.4112	.4112
		(.038)	(.060)	(.073)	(.063)	(.054)	(.055)		(.059)	(.057)		(.059)	(.059)
ρ	.20	.203	.132	-.029	-.130	-.296	-.305	-.3492	-.3442	-.3432	-.3396	-.3397	-.3397
		(.117)	(.192)	(.175)	(.105)	(.046)	(.046)		(.049)	(.047)		(.049)	(.049)
υ	1.1	.865	.621	.503	.430	.293	.287	.2705	.2682	.2677	.2665	.2664	.2663
		(.074)	(.113)	(.106)	(.078)	(.039)	(.039)		(.040)	(.039)		(.041)	(.041)
T		.033	.032	.029	.027	.025	.025		.0248	.0248		.0248	.0248
		(.005)	(.005)	(.005)	(.005)	(.005)	(.005)		(.005)	(.005)		(.005)	(.005)
$\hat{R}^2$		.892	.792	.782	.834	.929	.928		.924	.928		.923	.923
σ dependent variable		.295	.212	.207	.237	.360	.359		.349	.359		.347	.346

[a]609 Observations.

TABLE C-3

Log-Linear Estimates for Observations Used in CES First-Order Taylor Series Approximations

Variable or Statistic	Regression Coefficients and Standard Errors								Means and Standard Deviations	
	Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Firm Over-All, Time Series Observations Only	Industry Time Series	Industry Cross Section	Industry Over-All	Firm Time Series	Firm Over-All
Constant term or dependent variable				.028 (.016)	.036 (.017)			.033 (.023)	.128 (.101)	.126 (.236)
ln K	.232 (.061)	.016 (.010)	.041 (.017)	.017 (.010)	.015 (.010)	−.092 (.138)	−.009 (.017)	−.008 (.017)	−.407 (.084)	−.383 (.849)
ln L	.517 (.054)	.152 (.016)	.191 (.019)	.153 (.016)	.157 (.016)	.152 (.136)	.020 (.044)	.024 (.043)	.053 (.077)	.062 (.525)
T	.0212 (.0044)			.0574 (.0079)	.0538 (.0082)	.0507 (.0071)		.0534 (.0111)	1.672 (1.145)	1.638 (1.063)
Σ ln K + ln L	.749 (.073)	.168 (.018)	.231 (.022)	.169 (.018)	.173 (.019)	.060 (.168)	.010 (.050)	.015 (.049)		
n	609	646	644	646	609	34	34	34		
r.d.f.	410	640	610	642	605	22	28	30		
$\hat{R}^2$	.336	.125	.175	.175	.178	.699	−.049	.384		

TABLE C-4

First-Order Taylor Series Estimates of Parameters, Firm Time Series, v Constrained to .865

Parameter or Statistic	Assumed Value	Iteration 1	Iteration 2	Iteration 3	Iteration 6	Iteration 7	Assumed Value	Iteration	Assumed Value	Iteration
A	1.358	.695 (.106)	.953 (.106)	.898 (.105)	.919 (.105)	.918 (.105)	.9177	.9189 (.105)	.9187	.9185 (.105)
δ	.239	.242 (.061)	.271 (.116)	.300 (.078)	.310 (.075)	.311 (.075)	.3116	.3113 (.075)	.3113	.3112 (.075)
ρ	.203	.118 (.195)	−.140 (.719)	−.185 (.337)	−.250 (.328)	−.253 (.325)	−.2566	−.2550 (.325)	−.2550	−.2544 (.325)
v	.865									
T		.032 (.005)	.031 (.005)	.030 (.005)	.030 (.005)	.030 (.005)		.0295 (.005)		.0295 (.005)
$\hat{R}^2$		.312	.347	.334	.344	.344		.345		.345
σ dependent variable		.118	.121	.120	.121	.121		.121		.121

TABLE C-5

First-Order Taylor Series Estimates of Parameters, Assumed Values from Firm Time Series, Penultimate Iteration, Firm and Industry Time Series, Cross-Section, and Over-All Regressions

		Estimates and Standard Errors								Means and Standard Deviations	
Parameter or Statistic	Assumed Value	Firm Time Series	Firm Cross Section	Firm Cross Section Within Industries	Firm Over-All	Firm Over-All, Time Series Observations Only	Industry Time Series	Industry Cross Section	Industry Over-All	Firm Time Series	Firm Over-All
Constant term or dependent variable					.877 (.763)	.209 (.800)			−.244 (2.639)	1.030 (.346)	1.115 (2.195)
A	4.019	4.026 (2.466)	.194 (.771)	−1.066 (.890)	.140 (.770)	.820 (.806)	−15.426 (10.7)	1.402 (2.677)	1.275 (2.648)	.983 (.016)	.988 (.123)
δ	.412	.411 (.059)	.371 (.011)	.376 (.019)	.370 (.011)	.365 (.011)	.268 (.144)	.385 (.024)	.384 (.024)	−.411 (.103)	−.394 (1.002)
ρ	−.340	−.340 (.049)	−.377 (.016)	−.355 (.020)	−.376 (.016)	−.374 (.016)	−.284 (.126)	−.386 (.060)	−.387 (.059)	−.542 (.106)	−.542 (.647)
υ	.266	.266 (.041)	.275 (.013)	.301 (.015)	.276 (.013)	.265 (.014)	.520 (.171)	.249 (.042)	.252 (.042)	−.567 (.962)	−.266 (7.374)
T		.0248 (.0054)			.0705 (.0089)	.0671 (.0090)	.0723 (.0120)		.0662 (.0143)	1.672 (1.145)	1.638 (1.063)
n		609	646	644	646	609	34	34	34	Y 1.166 (.124)	1.164 (.267)
r.d.f.		408	638	608	640	603	20	26	28	K .931 (.084)	.974 (.908)
$\hat{R}^2$		.923	.988	.982	.988	.988	.951	.997	.996	L 1.201 (.092)	1.214 (.685)
σ dependent variable		.346	2.194	1.699	2.195	2.077	.227	1.524	1.462		

some hope for convergence to roughly common values in most of the regressions.

Finally, some attention may be given to the problem of our estimates of A, which vary so much from regression to regression, with high standard errors. The derivative of the CES function with respect to A is, of course,

$$F_A = [\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-v/\rho}, \tag{A5}$$

and another estimate of A may thus be obtained by constraining δ, ρ and v at their final estimated values and regressing Y on F_A with the constant term constrained to zero. Employing the time series observations only, we thus estimate A as 1.178 (with a standard error of .088). This is, in a sense, the weighted average efficiency or scale parameter of the various firms in the sample. It may perhaps be compared with the value of 1.037, the anti-log of the constant term of .036 in the firm over-all log-linear regression of Table C-3 (column 6). Combining it with our estimates of the other parameters, we have

$$Y = 1.178[.411K^{.340} + .589L^{.340}]^{.784} + u \tag{A6}$$

for which the coefficient of determination, relating back to deviations from the mean for each individual firm, is .158. Where the constant term is not constrained to zero, the estimate of A is again a high 4.686, but the coefficient of determination is .363, which compares favorably with the analogous $\hat{R}^2$ of .336 shown in the log-linear relation of Table C-3.

A number of refinements, including appropriate application of utilization variables, are called for. I hope that it will be possible to incorporate some of them in ongoing research. But it does appear now that there is a possibility of progress in direct estimation of CES production functions along the lines we have attempted.

COMMENT

BERT G. HICKMAN, The Brookings Institution

In this preliminary report on important research in progress, Robert Eisner uses micro data to estimate production functions of the Cobb-Douglas and CES types. The McGraw-Hill Capital Expenditure Surveys provide estimates of capacity utilization, employment, and capital expenditures, and these observations have been supplemented by matched accounting data on sales, inventories, gross fixed assets, and depreciation charges. The unit of observation is the individual firm, and annual observations are available for a broad range of industrial activities and for periods of four or more years. This body of data affords some unusual advantages in the estimation of production functions, and Eisner has exploited many of them in this report of his preliminary findings. I believe that he has not gone far enough in this direction, however, and I will have some, I hope constructive, suggestions to offer during my discussion of his preliminary results.

The basic Cobb-Douglas estimates are contained in Table 2 of the paper. The logarithm of output is regressed on the logarithms of capacity utilization, gross fixed assets, and employment. Results are presented for both cross sections and time series. The time series results are the pooled regressions of deviations about the means of observations for individual firms over periods of two to four years. The cross sections refer to deviations about the means of observations for all firms in each year, although the deviations for all years are pooled in the regressions.

Eisner notes in his introduction that estimates of the production function have been "bedeviled by difficulties in accounting for less-than-capacity utilization of capital," and one of the presumptive advantages of the McGraw-Hill data is the availability of a utilization variable. As they stand, Eisner's results suggest that this may be an illusory advantage insofar as the cross-section estimates are concerned, since the utilization variable is insignificant and therefore of little help in accounting for interfirm differences in the utilization of capital. As Eisner points out, this finding suggests that "differences between firms in reported utilization of capacity reflect rather interfirm differences in the measure than anything systematically related to output." It is hoped, however, that further work with the utilization variable may lead to better results.

One problem is that the utilization rates are reported on an end-of-year basis, whereas output is measured for the year as a whole. Thus the output differences among firms are free of seasonal influences, but this is not true of the utilization rates, which should be adjusted to an annual basis for comparability with the output variables. I have shown elsewhere that the disparity between the McGraw-Hill and other estimates of capacity utilization based on annual data can be substantially reduced by converting the former estimates to an annual basis. The correction is based on the ratio of, say, December production to that for the entire year, and while this information probably is not known for the individual firms in the McGraw-Hill sample, a uniform adjustment could be made for all firms in a given industry on the basis of independent production data.

Another problem is that the optimum or cost-minimizing rate of utilization may vary widely among firms, so that if two firms report, say, 80 per cent utilization of peak capacity, one may be operating to the right of the minimum point on its short-run cost curve, whereas the other is to the left of that point. The first firm would therefore possess less than the equilibrium amount of capital for the given output and the second firm, more than the equilibrium amount. Now, the firms in the McGraw-Hill sample are asked at least occasionally to report their preferred as well as their actual operating rates, and it seems reasonable to assume that the preferred rate is the cost-minimizing rate. This suggests that the ratio of the actual to the preferred utilization rate—which ratio implicitly expresses output as a percentage of optimum instead of peak capacity—would be a better indicator than the actual rate of the extent to which the existing capital stock falls short or exceeds the equilibrium stock corresponding to the existing level of output. The preferred rate for each firm may not be available for every year, but it probably does not change much over time and could be extrapolated or interpolated for the missing years.

I have discussed these measurement problems at some length because of the importance of the utilization adjustment for Eisner's avowed purpose of obtaining direct estimates of production function parameters from data on output, capital, and employment. He correctly observes that firms are not always, or ever, in equilibrium. Thus the measured labor and capital inputs of a given year are unlikely to be the equilibrium values corresponding to the measured output of the firm. Whereas I agree that cross sections are better able than short time series to reflect long-run adjustments of capital and labor to output, it is nonetheless likely that the capital stock variable in particular will be subject to large errors owing to disequilibrium conditions in each firm. The resulting bias in the estimate of the capital parameter could be reduced or eliminated by an adequate adjustment for under- or overutilization of capital in the various firms.

Incidentally, the utilization variable cannot properly be used to adjust both the capital and labor inputs. Despite the growing recognition of the fact that labor as well as capital inputs may adjust with a lag to short-term output movements, it would be surprising indeed if the speed of adjustment were not faster and the amount of disequilibrium smaller for labor than capital inputs. Ambiguous though it may be to employ a capacity utilization variable to adjust capital alone, the error is apt to be smaller than to neglect the adjustment entirely or to apply it to labor instead of capital.

This line of thought suggests that the utilization variable should not enter the regressions independently, but rather should be constrained so that its coefficient equals that of the capital variable to which it is conceptually attached; or, what amounts to the same thing, that the utilization ratio should simply be used to adjust the capital stock data prior to fitting the regressions. Conceptually, this adjustment would amount to converting the actual stock data into estimates of the equilibrium capital requirements corresponding to the observed outputs and labor inputs.

It may be objected that the foregoing procedure would place too much trust in the utilization variable as an indicator of the divergence between the actual and equilibrium capital stock. Eisner's practice of including the utilization variable separately could be regarded as a test of whether the variable is indeed a valid one for the purpose, in which case its independently estimated coefficient should differ insignificantly from that of the unadjusted capital variable. The two coefficients do appear to differ insignificantly in the time series results without trend, though not in the trend-adjusted results.

To turn to other aspects of the Cobb-Douglas estimates in Table 2, Eisner notes that the estimated cross-section elasticities of output with respect to capital and labor are respectively about one-third and two-thirds and that their sum is virtually equal to one, implying constant returns to scale. There are several reasons why these economic implications should not be taken at face value, however. (1) A radically smaller estimate of the labor coefficient is obtained when the variables are transformed to logarithmic first differences in the regressions presented by Eisner in Table 4, even though the parameters to be estimated are conceptually the same as in Table 2. (2) The dependent variable is output rather than value added, so that even if the capital and labor coefficients were equal to the true values, the unknown coefficient of the missing third factor—materials input—should properly enter the calculation of the degree of returns to scale. (3) The estimated coefficients are biased downward because of errors in the independent variables.

It should be possible to reduce some of the bias stemming from errors in the independent variables. Capacity utilization, capital stock, and employment are all measured on an end-of-year basis, whereas output refers to the entire year. Eisner therefore includes both the current- and preceding-year values of the independent variables in his regressions, and then sums the two coefficients for each variable to estimate its elasticity. This procedure, however, makes no allowance for different seasonal patterns in the labor and utilization variables as among firms in widely differing industries. Moreover, labor input is measured by number of employees rather than man-hours; so no account is taken of differences in hours of work between firms or over time. It should be feasible to devise correction factors to mitigate some of these observational errors, along the lines suggested above in my discussion of the utilization variable.

Capital stock in the regressions under discussion is measured by the book value of gross fixed assets at original cost. Price deflation to reduce the various vintages of capital goods to a constant-cost basis would be desirable and is attempted by Eisner in some additional regressions presented in Tables 3 and 8. How much difference this makes for the estimated elasticities is impossible to state, however, since the latter regressions differ from the earlier ones in other characteristics than the deflation of gross fixed assets.

Even when deflated to a constant-cost basis, the stock of gross fixed assets may comprise diverse vintages of capital goods with widely varying productivities. The simplest way to allow for the differing productivities of differing vintages is to measure the stock net of depreciation, on the assumption that technical change occurs smoothly and that the depreciation rate includes an obsolescence factor. If net stock data were not available for the firms in the sample, probably little could be done through other methods of allowing for embodied technical change, since these other methods would require knowledge of the past history of capital accumulation in each firm in order to weight the several vintages by the appropriate productivity improvement factors, even assuming that independent information existed about the latter.

So much for detailed comments on the data and the basic Cobb-Douglas results. Eisner also presents several regression experiments for which no theoretical justification is provided. The regressions in Table 3 are still avowedly production functions, in that they involve capital and labor inputs as explanatory variables, but they also include the depreciation rate as an independent variable, and the capital variable is furthermore broken into several vintages which enter independently. Neither of these departures from the conventional in production function specification is explained. Table 5 relates output to capacity utilization and capital stock, but includes no labor input. In Table 6 the utilization variable is used to convert observed output into a measure of the maximum output which the capital stock is presumptively capable of producing, and the corrected output is then regressed on capital stock alone. In Table 7, McGraw-Hill data on reported changes in capacity are regressed on the ratio of capital expenditures to gross fixed assets, the depreciation rate, and time. Whatever they may be, the regressions in Tables 5 to 7 are not complete production functions, and the reader may reasonably ask what meaning can be attributed to them as structural relationships and what purpose they are intended to serve.

In conclusion, I strongly endorse Eisner's proposal to make parameter estimates separately for different industries in future work. One of the principal advantages of the firm as the unit of observation is the opportunity it affords to allow for differing technologies among industries and to obtain greater homogeneity in the data, and thus far Eisner has neglected this opportunity. Another characteristic of his sample, which distinguishes it from most other cross-sectional data that has been used to estimate production functions, is the fact that time series observations are available for the firms. Thus it becomes possible to estimate a separate set of cross-section parameters for each year and to study the changes in the parameters over successive years, instead of pooling the cross-section observations as in the present work. A series of cross-section parameter estimates, preferably within well-defined industry groups, would permit study of changes in the constant term and input coefficients over time, possibly throwing important light on questions about the rate and neutrality of technical progress and about cyclical variations in apparent productivity.

DALE W. JORGENSON

I have only one objection to Eisner's very interesting paper. It seems to me that the paper lacks an explicit theoretical framework for interpreting the empirical results. We have some forty-eight regressions, yielding a wide variety of estimates. The coefficients associated with employment range from .157 to .659. Those associated with capital stock range from .266 to .937. It is difficult to see just what one is supposed to make of all these estimates.

The purpose of this comment is to supply a theoretical interpretation for Eisner's empirical findings. The basic theoretical model is only partly implicit in Eisner's regressions. When the model is made explicit, it becomes apparent that Eisner's statistical models could be improved. Nevertheless, the results obtained provide a clear and convincing picture of the underlying reality. I hope that the familiarity of the picture will not prevent a proper appreciation of it.

In constructing a theoretical framework for production one would like to have a single model that could be applied to all the different sets of data considered by Eisner—time series and cross section, firm and industry. Implicitly, Eisner takes the position that for the present this is asking too much. We are presented instead with a set of results based on a less comprehensive model. To fit this simpler model to the data a certain amount of legitimate doctoring of the data—taking out the variation associated with differences in firm or year or industry means—is required.

The basic theoretical model consists of a neoclassical production function,

$$Q^* = F(K^*, L^*),$$

where Q^* is the flow of output, K^* the flow of capital services, and L^* the flow of labor services. We may write each of these flows as the product of a stock and a rate of utilization of the stock:

$$Q^* = Q \cdot W,$$
$$K^* = K \cdot U,$$
$$L^* = L \cdot V,$$

where, for example, K is capital stock and U is the rate of utilization of that stock.

It may be noted that this model is essentially technological rather than economic. A model based on the economic theory of production would include not only a production function but also marginal productivity conditions for capital and labor services. Eisner disregards the possibility of obtaining information about the production function from the covariation of outputs and inputs on the one hand and price ratios on the other.

If the production function has Cobb-Douglas form we may write (ignoring the constant term):

$$\ln Q + \ln W = \alpha[\ln W + \ln U] + \beta[\ln L + \ln V],$$

where α and β are the elasticities of output with respect to capital and labor services, respectively. It is here that objections to Eisner's statistical model arise. Eisner has direct observations on the flow of output. He identifies K with the stock of capital and U with its rate of utilization. By analogy we must identify L with employment (the stock of labor). But V, the rate of utilization of labor, measured as man-hours per man employed, is missing from Eisner's initial empirical specification. Later in the paper the stock of labor itself is omitted from the empirical specification. One might also object to Eisner's omission of inventories and financial assets from capital and to errors of aggregation in both labor and capital.

In view of all the objections that can be raised to Eisner's implementation of the basic statistical model, it may be surprising to find that the model "works" at all. But it does work, as Eisner's empirical findings reveal.

Starting with the statistical model Eisner calls "firm over-all" in Tables 2 and 3 we find that the sum of coefficients associated with capital stock (Table 2) and lagged capital stock together with investment (Table 3) is .339 and .329, respectively, with a time variable included, and .343 and .330, respectively, with no time variable included. Turning to the coefficients associated with the stock of labor we find coefficients of .648 and .650 without time and .652 and .658 with time.

The disturbing feature of the firm over-all results is that the sum of coefficients associated with rates of utilization of capital is essentially zero, with or without a time variable. From the basic theoretical model it is clear that this variable should have exactly the same coefficient as capital stock, namely, .35 or thereabouts. As a statistical hypothesis, the equality of the coefficients of capital and its rate of utilization would be rejected at almost any level of significance.

The story told by Eisner's results for "firm cross section within industries" and "firm cross section" models is much the same. In these models "dummy variables" for years and industries and for years alone are included along with the explanatory variables. The coefficients of the explanatory variables are essentially unaffected. The results in Table 4 are similar to those in Tables 2 and 3 even though employment is excluded, provided that a time variable is included. Table 6 presents a somewhat different picture, but this is the result of Eisner's assumption for the statistical model underlying this table that the coefficient of the rate of utilization is unity. This assumption is so clearly contradicted by the evidence that these results may be disregarded.

The statistical model underlying the results labeled "firm time series" is different from Eisner's other statistical models in that dummy variables for individual firms are included along with the explanatory variables. No dummies for year or industry are included in these regressions. The results for firm time series are dramatically different from those for the other models. The sum of coefficients associated with the rate of utilization is .414 and .374 in Tables 2 and 3 with time included in the regression. With time excluded the sums are .409 and .401, respectively. On the other hand, the coefficients associated with capital stock are .266 and .172 with time included and .446 and .385 without time.

Turning to the results presented in Table 8, both output and capital stock are divided by sales for the individual firm. Not too surprisingly, this has essentially the same effect as removing "firm effects" in the firm time series model. The sum of coefficients associated with rates of utilization is .332 for the firm over-all model with no time variable included in the regression. Taking out year effects as well changes the coefficient to .315; removing both year and industry effects results in a coefficient of .247. Taking out firm effects yields a coefficient of .305 with no time variable included and .274 with time included. The sum of coefficients associated with capital stock is relatively small in most regressions— −.088, .381, .061, .064, .061, .059.

The switch in relative importance of employment and capital stock and their rate of utilization enables us to conclude with Eisner that interfirm variations take the form of variations in "factor stock" while intrafirm variations take the form of variations in rates of utilization. However, the equality of the sum of coefficients of factor stock and utilization in the firm over-all regressions to the sum of coefficients of factor stocks and rates of utilization with firm effects removed enables us to draw a much stronger conclusion. The basic technological model with the flow of output as a function of flows of labor and capital inputs is strongly confirmed by the data.

We can suggest a further test of the basic theoretical model. Just as interfirm variations in capital input are largely variations in capital stock rather than its rate of utilization, so interfirm variations in labor input should be largely variations in employment. Turning to the empirical results we find this hypothesis strongly confirmed. With no firm effects removed the sum of coefficients of employment is always in the neighborhood of .65. The estimates from Tables 2 and 3 are .651, .659, .629, .619, .652, .658, .648 and .652. With firm effects removed this sum of coefficients drops to .275, .301, .242, and .278. Deflating output and employment by sales in Table 8, Eisner obtains further estimates of .521, .454, .159, .189, .160, and .157.

To test the basic theoretical model further, observations on the rate of utilization of labor would be required. The expected result would be a sum of coefficients of rates of utilization equal to approximately .65 with firm effects removed. The use of dummy variables for firms or deflation of output, employment, and capital stock by sales should produce approximately the same results.

My over-all conclusion is that the basic theoretical model is strongly confirmed by Eisner's empirical results. Interfirm variations in output are explained largely by interfirm variations in capital stock and employment. Variations in rates of utilization of productive capacity and stocks of capital and labor wash out because of errors of measurement. Intrafirm variations in output are explained largely by intrafirm variations in rates of utilization. Variations in productive capacity and stocks of inputs wash out. Both interfirm and intrafirm variations may be represented as movements along a production function of Cobb-Douglas form with an elasticity of labor services of .65 and an elasticity of capital services of .35. These results are, of course, precisely the results obtained by Douglas in the course of his research over the twenty-year period, 1927–47.

To sum up: Eisner has successfully extended the applicability of the basic Cobb-Douglas model to the level of the individual firm. By introducing the rate of utilization of capital explicitly, a new feature of the model has been uncovered. Eisner's findings should be followed up as quickly as possible by testing the implications of the model for the stock of labor and its rate of utilization. The implications of the model for both capital and labor should be further tested by direct measurement of capital and labor services.

EVSEY D. DOMAR

My question here is addressed to Mr. Eisner in particular, but also applies as well to several other authors. I wonder what has happened in all these studies to material inputs? If they are omitted because of the lack of required data, we have an answer, even if, to my mind, a regrettable one. But usually an author begins his paper with the model that he *would like to fit;* then he apologizes for the lack of data and fits a different one. I have not found any apologies for omitting material inputs from both sides of the equation and thus working with value added on the one side and with only labor and capital on the other. Is this then the desired method? And yet it seems to me that a production function is supposed to explain a productive process, such as the making of potato chips from potatoes (and other ingredients), labor and capital. It must take some ingenuity to make potato chips without potatoes. I do not mean that the omission of material inputs is necessarily wrong. Rather that it is not at all obvious that it is the preferred method. Among other things, it results in a larger residual, at least with a Cobb-Douglas function, and this remains true even when output and value added grow at exactly the same relative rate.

REPLY by Eisner

I may respond briefly to several related themes in the very useful comments which have been presented.

First, on the failure to include materials as an argument of the production function—the creation of potato chips without potatoes, as Domar has so aptly phrased it—I must of course plead guilty. I have boasted frequently about the nature of the data which we have built about the McGraw-Hill capital expenditure surveys; but these were put together originally in connection with work on the investment function and whether or not information as to materials purchased might have been solicited, I did not think of it at the time.

At some point it may indeed be well to graft on to this body of data individual-firm figures regarding materials, but I do wonder whether their use is likely to alter significantly our estimates of the relative contributions to output of labor and fixed capital. As matters stand, the omission of the materials variable may be expected to add considerably to the error term in our relations. But it is not clear that subtraction of materials purchased from our measure of "output," so that the dependent variable could be value added, would give us different parameter estimates for "labor" and "capital." [1]

[1] This may smack a bit of the Marxian notion of "constant capital," but is that likely to be far wrong in the case of materials? Fixed capital may well have some

Both Hickman and Jorgenson seem to believe that the coefficient of "utilization of capacity" should be the same as that of capital. I am not persuaded, either on conceptual grounds or by consideration of the nature of the data. First, I fail to see why, for example, a 10 per cent increase in utilization of existing capital stock should have the same effect upon output as a 10 per cent increase in the amount of capital utilized at the existing rate. Indeed, if this were so one might wonder why firms ever did alter their capital stock. It would appear that one could do just as well by utilizing existing capital more intensively, and think of the money one would save!

But further, it is hardly clear that the McGraw-Hill question, "At what rate of capacity were you operating?" relates exclusively to utilization of capital. "Capacity" means different things to different firms, but one might well imagine the concept to relate to what can be produced, without prohibitive cost, when utilizing fully *all* of the productive factors available to the firm. As has become increasingly well recognized in recent work, labor itself is a somewhat less than completely variable stock, the utilization of which varies markedly as the firm maintains a relatively stable level of employment in the face of fluctuating demand and output. Indeed, Jorgenson allows explicitly for varied utilization of labor in the "explicit model" which he provided for my paper. I fail to see why, in terms of *his* model, he then proceeds to interpret the utilization-of-capacity variable as relating only to capital.

I could not know a priori precisely what the utilization-of-capacity variable might reflect. If it did relate to the capacity provided by the stocks of both capital and labor and if diminishing returns to increased utilization of existing stocks were not of major importance over the range of variation of utilization experienced, the coefficient of the rate of utilization might even be unity.

In fact, as Jorgenson observes, the utilization-of-capacity variables in the firm time series (with firm effects removed, as Jorgenson puts it) do tend to have sums of coefficients of the same order of magnitude as the sums of the capital coefficients in the firm cross sections, which are based upon firm effects, and in the firm over-all regressions, where firm effects dominate. I would not carry the point as far as he, however. The

substantial "productivity" associated with the magnitude of its time dimension, as the socialist planners of Eastern Europe seem now to be perceiving more clearly, but should relatively short-lived inputs of materials be expected to contribute much more (or less) to output than their own value?

sum of the utilization coefficients of .414 shown in the "firm time series with time" in Table 2 is somewhat higher than the .340 estimate of the capital coefficients suggested in the cross-section and over-all regressions. What with the errors in variables due to seasonal factors of which Hickman reminds us and the general impreciseness of this utilization variable I remain skeptical of Jorgenson's inference that variations in capital stock and in utilization of capacity have precisely the same effect upon output.

I do probably owe the reader some further explanation, as Hickman has indicated, of the results reported in Tables 3 through 7. In Table 3, I was attempting to offer some measure of the time profile or lags relating capital and output. This was in part, I believe, successful. Rates of depreciation were included as an offset to the gross figures entering into the capital expenditure variables.

Table 4 involves the same theoretical relation as Table 2 but, put in first-difference form, removes any common trend effect from the parameter estimates. It also almost certainly reflects a relatively greater "noise" component in the variance of the independent variables, which tends to bias their parameter estimates downward. Table 5 seems worth noting because it includes a considerably larger body of data, relating to eight years instead of four. Unavailability of employment data for the early years forces the obvious misspecification in the exclusion of the labor variables.

Tables 6 and 7, while of only marginal value and again involving misspecifications because of the unavailability of employment data, might merit some consideration for the light they cast on the role and nature of the capacity variables in the McGraw-Hill series. It might of course also be observed that, to the extent that labor and capital stock are kept in a fairly stable equilibrium relation, the coefficients of capital variables will offer some at least rough measure of the joint contribution of capital and labor to output.

I am inclined to defend myself somewhat against the charge of presenting "a wide variety of estimates" without "an explicit theoretical framework for interpreting the empirical results." For one thing, Solow's paper is devoted to a major survey of the theoretical literature, and the implicit theory of the Cobb-Douglas function (or the CES function discussed in my Appendix B) is certainly well known to participants in this conference. As I have suggested above, Jorgenson's attempt to be more "explicit" than I felt I should or could be runs into difficulties both at the conceptual level, in the assumption that quantities of factors and their rates of utilization enter multiplicatively in the production function with identical parameters, and in the further interpretation of "capacity" as relating only to the stock of capital. (I may add that I am puzzled by Jorgenson's apparent view of output as the product of a stock of output and the rate of utilization of that stock.)

But the complaint of the wide variety of estimates is by now a familiar one. Nerlove has, I hope, made us all shudder with the weight of evidence of just how disparate many of the estimates are. That variety bears some consideration precisely in the light of my paper. For aside from certain differences which may be ignored because of the high standard errors attached to industry regressions based on relatively few observations, and further discounting differences relating to alternate specifications where data on all variables were not available, a point of stress in my findings may well be the explanation of the systematic differences in estimates which emerge. These, it may be reiterated, as both Jorgenson and I have stated them, relate to the different results to be obtained from cross sections and from time series. And these differences, I have stressed, are illuminated by the role of the utilization-of-capacity data fortunately available from the McGraw-Hill surveys. What can now be seen fairly clearly is that variation over time reflects in considerable part disequilibrium positions which, at least without appropriate adjustment, will give us biased and misleading estimates of the production surface. In the short run, firms alter their utilization of capacity, and changes in capital stock have relatively little to do with output, while the effects of changes in employment remain significant but are also sharply reduced. Interfirm variance, on the other hand, is apparently dominated by differences in equilibrium levels of capital stock and employment.

Exposure of these differences in estimates, and of their nature, would seem of considerable moment in explorations of the production function, and elsewhere as well.