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15 Some Simulation-Based Estimates of Commercial Bank Deposit Insurance Premiums

David Lane and Lawrence Golen

15.1 Introduction

Much of the empirical work in this volume is directed to a single question: What is the amount of risk in an insured financial intermediary? The chapter by Sharpe (chap. 8) shows that a useful way to view this question is in terms of the fair deposit insurance premium for an institution with a given initial asset and liability structure and capital position. The probability that an insurer will have to make a payout, and hence the size of the fair premium, varies with initial capital and depends upon a number of other factors. Among these are the value of operating earnings and expenses during the examination period; the extent of interest rate risk occasioned by imperfect alignment between time flows on assets and those on liabilities; the amount of undiversified default risk; and the covariances between these factors. Risk factors within an intermediary are not independent. Thus, for example, high interest rates tend to be associated with large default premiums. Both of these may have adverse effects on the value of a currently held portfolio.

To calculate fair insurance premiums, joint probability distributions of returns from each of the institution's activities must be specified. Given such distributions and the initial level of capital, the insurance premiums are determined by evaluating the expected end-period value of capital given that capital is negative and weighting this expected value by the probability that end-of-period capital is negative.

The problems of determining fair insurance premiums involve modeling all aspects of financial intermediaries and are difficult. It should come as no surprise, then, that none of the approaches in this volume is able to

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determine fair premiums based on all elements of risk contained within an institution. Each of them deals with certain aspects of financial intermediary risk and in so doing ignores other aspects. The value of employing multiple approaches is that each approach possesses some advantage for dealing with a particular aspect of the insurance problem.

The simulation approach utilized in this chapter involves sampling repeated end-of-period net values for a stylized intermediary operating in a simulated environment. For some of the sampled values net worth will be negative, and the deposit insurer will face payouts. The average amount of these payouts over all the drawings is the simulated fair insurance premium.

The steps in a simulation to determine fair insurance premiums are straightforward:

1. Utilizing an appropriate discounting technique, determine the current net market value of the institution. This is the institution's current capital.
2. Using a macroeconometric model, specify a joint probability distribution over all the variables exogenous to the intermediary that may affect the intermediary. We may call these environmental variables. These variables are of two types. The first are macro variables affecting the current or future levels of the intermediary's activities (asset or liability flows, operating earnings, etc.), and the second are discount variables affecting interest factors to be utilized in valuing these activities.
3. Specify quantitative relationships between the exogenous environmental variables and the levels of intermediary activities. This amounts to forming a micro model of the bank.
4. From the macroeconometric model obtain a drawing on the value of the relevant environmental variables.
5. Use the drawing to obtain a net market value of the intermediary. This is done by applying discount factors to the determined levels of the intermediary's activities.
6. Repeat steps (4) and (5) to obtain a frequency distribution of the intermediary's net market value.
7. Use this frequency distribution to calculate fair deposit insurance premiums for the institution, given its portfolio composition and initial capital. These premiums are equal to the summation of all negative simulated values of net worth divided by the number of simulations.

Fair insurance premiums appropriate to a particular institution can be calculated by using steps 1 through 7 above, based on the portfolio composition and capital value of the institution. By repeating the process for a variety of portfolio/capital assumptions, one can determine premiums appropriate for many different institutions.

Although the simulation procedure is conceptually clear, application is difficult for several reasons.

First, given our present state of knowledge, it is not possible with macroeconometric models or other techniques to adequately specify joint probability distributions over all the elements of the economic environment that might affect the levels or values of the intermediary's activities. Such distributions would have to provide accurate joint probability information on all major macroeconomic variables as well as on many regional and local variables. They would also have to specify relationships between measures of economic activity and a host of discount rates. Although such complete distributions could not be specified for this paper, the distributions utilized below do account for stochastic interrelationships between many government, municipal, and mortgage rates. No attempt is made, however, to deal with correlation between rates and other macro variables. Thus the interest rate distributions used below are not conditioned on the values of relevant macro variables, and these distributions have larger variance than would distributions correctly conditioned on appropriate macro variables.

The second major problem with the simulation approach is that comprehensive empirical models of financial intermediaries are not available. Thus in many cases it is not even possible to specify the relationship between the external economic environment and the level of bank activities. This problem is dealt with below principally by assuming nominal levels of portfolio items remaining fixed throughout the examination period.

The results obtained in this paper represent the fruits of a limited application of the simulation technique. In what follows, the simulation technique is applied only to simple, stylized financial intermediaries. It is assumed that such institutions hold fixed asset portfolios containing specified amounts of government securities, municipal securities, and mortgages. These assets are financed by capital and Treasury bill rate borrowing. For ease of exposition the technique is initially developed for government securities alone, then subsequently extended to more complex portfolios.

15.2 Insurance Premiums for Portfolios of Government Assets

In this section empirical estimates of fair insurance premiums are calculated for financial intermediaries holding only government assets. The approach is one of simulating term structures of interest rates and valuing hypothetical portfolios under the simulated structures. Through repeated simulation, distributions of portfolio values can be obtained and fair insurance premiums determined.

Future term structures are simulated by utilizing implicit rates contained in current structures, adjusting for systematic relationship between adjacent term structures, and adding noise. The noise is introduced to reflect the fact that forecasts of future rates based on implicit rates often show substantial deviation from realizations.

Techniques for dealing with shifting term structures often assume that changes in intermediate and long rates are determined exclusively by changes in short rates. The technique developed here allows for substantial variation of the former rates independent of the latter. However, to account for the tendency of adjacent forward rates to move together, much of the simulation is done in terms of the principal components of interest rate changes rather than the raw changes themselves.

Each simulation drawing consists of a sequence of 24 consecutive monthly observations, where each observation is composed of a one-period spot rate and 149 one-period forward rates. The program generates 700 such drawings. As noted, each consecutive observation on a structure in a sequence is derived from the previous one by adjusting term premiums and adding noise. Simulated observations from two different sequences on the term structure corresponding to the same date may differ owing to differences in current random drawings in the noise terms and owing to differences in the term premium adjustment. The premium adjustments differ because they are based in part on earlier noise drawings that differ across sequences.

The capital valuation element of the technique generally assumes that for the entire insurance period the financial intermediary holds a fixed portfolio of assets with known cash flows. The portfolio is financed by capital and borrowing at six-month intervals at the then-existing simulated six-month Treasury bill rate. For each drawing an observation on the value of the portfolio at the end of a specified insurance period is obtained by applying simulated six-month Treasury bill rates at semiannual intervals to flows received within the period and by evaluating anticipated flows remaining at the end end of the period by the simulated end of insurance period term structure.

For each simulation drawing the value of a portfolio may change owing to changes in borrowing costs or owing to changes in end-period asset values. The evaluation process is repeated for each of 700 simulation drawings, and an end-of-period frequency distribution of the intermediary's net worth is obtained. From such a distribution fair insurance premiums may be calculated given the initial characteristics of the intermediary. For each of four hypothetical insurance periods—six, twelve, eighteen, and twenty-four months—distributions are calculated for assumptions regarding asset holdings and initial capitalization, and corresponding insurance premiums are obtained. Some asset maturities are less than twenty-four months long. For these the simulated term structures that correspond to dates after the end of the period are not used.

Insurance premiums are specified as the expected value of the deposit insurer's liability per dollar of assets. This is calculated as the average amount by which end-of-period value of capital is negative, given that it is negative, times the probability that end-of-period capital is negative. For given portfolio and capital assumptions, this is simply the average amount per dollar of assets paid out by the deposit insurer per simulation drawing.

Since capital acts as a buffer against insolvency, the fair insurance premium will increase as the initial level of capital decreases. The premium will also increase with the length of time between inspections.

15.2.1 Simulating the Riskless Term Structure

Techniques for evaluating term structure changes are not new (see Bradley and Crane 1975 and chapters 13 and 14 above); however, many of these techniques share the characteristic that changes in intermediate and long-term rates bear a deterministic relationship to changes in spot rates. Often sensitivity of portfolios to interest rate changes is evaluated under the assumption that longer rates change by the same absolute amount as short rates.

This approach is at odds with the observed tendency of long rates to vary less than shorts and may lead to unduly pessimistic views of the riskiness of long-term assets. Chapter 13 drops the assumption of equal absolute changes but continues to assume a fixed deterministic relationship between changes in short and long rates. Morrison finds, however, that changes in the current one-period spot rate account for very little of the variability of current long rates. The simulation technique developed in this paper allows for changes in short rates to be correlated with changes in intermediate and long rates; however, it also allows for substantial variability of intermediate and long rates independent of short rates.

A simulated observation on a term structure for a particular period is produced by utilizing the implicit rates from the immediately preceding period's term structure, adjusting for forecast changes in term premiums, and adding noise terms. The noise terms are necessary because forecasts based on implicit rates generally exhibit substantial deviation from subsequent realizations.

Before proceeding further, it is useful to define terms:

$F_t(j)$ – is the forward rate at the start of period t for a one-period loan to begin $j - 1$ periods after the start of t.

$r_t = F_t(1)$ – is the spot rate for a one-period loan to begin at the start of t.

$P_t(j)$ – is the term premium at the start of t for a one-period loan to begin $j - 1$ periods later.

Under a wide variety of term-structure hypotheses:

$$F_t(j) = E_t(r_{t+j-1}) + P_t(j) \qquad j = 2 \text{ to } M.$$

Simple manipulations yield:

$$(1) \qquad F_t(j) - F_{t+1}(j-1) = E_t(r_{t+j-1}) - E_{t+1}(r_{t+j-1}) + P_t(j) - P_{t+1}(j-1)\,, \qquad j = 2, \ldots, M$$

or

$$(2) \qquad F_t(j) - F_{t+1}(j-1) = \Delta P_{t+1}(j-1) + \mu_{t+1}(j-1)\,,$$

where

$$\Delta P_{t+1}(j-1) = P_t(j) - P_{t+1}(j-1)$$

and

$$\mu_{t+1}(j-1) = E_t(r_{t+j-1}) - E_{t+1}(r_{t+j-1}).$$

Rewriting (2), we obtain:

$$(3) \qquad \Delta F_{t+1}(j-1) = \Delta P_{t+1}(j-1) + \mu_{t+1}(j-1).$$

If the market is efficient, $\mu_{t+1}(j-1)$ has zero mean, since if one had systematic information that the future expectation would differ from the current expectation, that information would be incorporated into E_t, bringing it into line with E_{t+1} (cf. Roll 1970).

Equation (2) suggests an approach to simulating the term structure. Rearranging the equation and using double tildes for simulated values and "hats" for forecast values we obtain:

$$(4) \qquad \overline{\overline{F_{\tau+1}(j-1)}} = F_\tau(j) - \widehat{\Delta P_{\tau+1}(j-1)} - \overline{\overline{\mu_{\tau+1}(j-1)}}.$$

If the premiums vary over time in some systematic manner, an estimation technique is needed to capture this variation. Structural modeling on equation (3) would be appropriate; however, to facilitate forecasting and simulation, Box-Jenkins methods of time-series analysis are used below.

If, as is often assumed, term premiums are constant, and past ΔF contain no information about future ΔF, then simulation can be accomplished by merely drawing on the error term (after adjusting for its mean).

Simulations based on both these approaches are presented in this paper. The first of these approaches is considered immediately below, and the second is considered subsequently.

15.2.2 Simulation Based on Box-Jenkins Analysis of the Systematic Element of the ΔF

Box-Jenkins techniques could have been applied individually to each forward rate in equation (3). However, this approach would have ignored the tendency of adjacent forward rates to move together. To capture this tendency, we first calculated the principal components of the standardized ΔF, then applied time-series techniques to these components, which formed the basis of subsequent analysis. These components were simulated and their simulated values combined to simulate the individual ΔF. For ease of exposition, the discussion below refers to ΔF; the reader should keep in mind that analysis was done on the standardized rather than the raw ΔF.

By the nature of principal component analysis, each of the ΔF can be written as:

$$\Delta F_t(j) = \sum_{i=1}^{N} \alpha_i(j)\, C_{it} + \eta_{jt}\,, \qquad j = 2, \ldots, M-1 \tag{5}$$

where

C_{it} is the ith principal component at time t,
N is the number of components selected for analysis,
$\alpha_i(j)$ is the loading of the jth ΔF on component i, and
η_{jt} is the portion of ΔF that is left unexplained by the N principal components.

In what follows we ignore the η_{jt}.

The principal components simply linear combinations of the ΔF.

$$C_{it} = \sum_{j=2}^{M-1} \beta_{ij} \Delta F_t(j)\,, \qquad i = 1, \ldots, N \tag{6}$$

where β_{ij} is the weight of $\Delta F_t(j)$ in component i.

By substitution from (3):

$$C_{it} = \sum_{j=2}^{M-1} \beta_{ij}[\Delta P_t(j) + \mu_t(j)]. \tag{7}$$

Rewriting and separating terms:

$$C_{it} = \sum_{j=2}^{M-1} \beta_{ij}\, \Delta P_t(j) + \sum_{j=2}^{M-1} \beta_{ij}\mu_t(j). \tag{8}$$

The last term is the sum of error terms, which by the assumption of efficiency have zero mean and are serially independent. Hence this term must have these same properties.

Equation (8), like equation (3), is in a form appropriate for time-series analysis. However, inspection of either of these equations shows that ΔP is measured (as ΔF) with considerable error. Thus, if one were interested in premiums alone, these errors in measurement would probably impair the optimal forecast properties of the Box-Jenkins technique. However, the underlying purpose in the instant paper is to capture the systematic element of the ΔF and then utilize this element in subsequently simulating the ΔF. For this purpose time-series analysis should retain its optimal properties. From the time-series analysis, one can obtain forecasts of the systematic portion of each of the principal components:

$$\text{(9)} \qquad \widehat{\sum_{j=2}^{M-1} \beta_{ij}\, \Delta P_{\tau+1}(j)}\,. \qquad i = 1, \ldots, N$$

Adding a random drawing on a serially independent error term, one can obtain a simulated value for each component:

$$\text{(10)} \qquad \overline{\overline{C_{i\tau+1}}} = \widehat{\sum_{j=2}^{M-1} \beta_{ij}\, \Delta P_{\tau+1}(j)} + \overline{\overline{\varepsilon_{i\tau+1}}}\,, \qquad i = 1, \ldots, N$$

where $\overline{\overline{\varepsilon_{i\tau+1}}}$ is the drawing on the error term. For all simulations in this paper error terms are drawn from normal distributions.

Substituting $\overline{\overline{C}}$ for C in equation (5) yields

$$\text{(11)} \qquad \overline{\overline{\Delta F_{\tau+1}(j)}} = \sum_{i=1}^{N} \alpha_i(j)\, \overline{\overline{C_{i\tau+1}}}\,. \qquad j = 2, \ldots, M-1$$

Equation (11) yields the simulated changes from one term structure to the immediately following term structure in terms of the simulated changes in the principal components.

Using the definition of ΔF in equation (3) and substituting into equation (4) yields:

$$\text{(12)} \qquad \overline{\overline{F_{\tau+1}(j-1)}} = F_\tau(j) + \overline{\overline{\Delta F_{\tau+1}(j-1)}}\,. \qquad j = 2, \ldots, M$$

Equation (12) provides one complete simulated observation on the term structure at time $\tau + 1$. Equation (12), like equation (4), suggests that a simulated term structure can be obtained by "aging" the implicit forecasts contained in the previous period's term structure through adjustment for premium changes and then adding the error term.

A simulated term structure for period $\tau + 2$, dependent on the structure for $\tau + 1$, can be obtained by repeating the process above. $F_\tau(j)$ in equation (12) is replaced by $\overline{\overline{F_{\tau+1}(j)}}$ and a simulated value of $\overline{\overline{\Delta F_{t+2}(j-1)}}$ obtained from equation (11) is added to yield $\overline{\overline{F_{t+2}(j-1)}}$. This technique is repeated until the desired number of related consecutive simulated monthly structures is obtained. The entire sequence of such related monthly structures is referred to here as a simulation drawing.

There is no condition imposed on the simulation that satisfies the efficient markets equilibrium condition of ex ante portfolio equilibrium. Thus, for example, securities with large simulated price variance may yield lower returns than those with small variance. In fact, since such behavior is characteristic of the sample period, it is also likely to be characteristic of the simulation.

15.2.3 Results of Estimation and Term Structure Simulation

Term structure data were supplied by J. Huston McCulloch. They were derived by applying a cubic spline term structure fitting technique to mean monthly bid-ask prices for specified government securities. The estimation period is from 1 January 1960 to 1 June 1975. Each observed term structure contains 150 rates; the current one-month spot rate plus 149 one-month forward rates (McCulloch 1971).

As noted above, the first step in the simulation process was to compute the principal components of the correlation matrix of the ΔF. The component analysis indicates that most of the historical variation of the 149 ΔF can be accounted for by relatively few factors. The first principal component accounts for roughly half the variation of ΔF. The first seven components account for 99 percent of the variation. The contributions of the principal components to the variation of the ΔF are summarized in table 15.1. For purposes of time-series analysis and subsequent simulation, we used only the first seven components. This cut-off corresponds to a widely used convention of retaining only those components that account for a portion of the variance larger than one over the number of raw variables (Kaiser 1960). The individual factors did not correspond in any obvious ways to particular identifiable variables or to particular interest rates.

As we had expected, the components were less closely associated with the $\Delta F(j)$ for the first three or four values of j than they were for higher values of j. The first seven components accounted for 99 percent of the variance of most ΔF, but for only 80 to 85 percentof the variance of the

Table 15.1 Contribution of Principal Components to Total Variation of 149 ΔF

Component	Contribution	Cumulative Contribution
1	.418	.418
2	.291	.709
3	.128	.837
4	.084	.921
5	.039	.960
6	.015	.975
7	.011	.986

first three or four ΔF. Thus some of the variation in very short rates is not shared by other rates.

Table 15.2 presents the results of the Box-Jenkins analysis on the standardized components. In all cases the time-series analysis yielded at least one autoregressive term. For components four, five, and six there were also moving average terms.

While the time-series analysis did not produce a constant term, this does not imply that the ΔF do not have a constant element; the constant element of the ΔF is suppressed because the principal components are standardized.

For most of the principal components, tests for white noise yielded the result that at roughly the 10 percent level we could not accept the hypothesis that the residuals were nonwhite.

For simulation purposes, we drew error terms for each component from normal distributions. There is considerable interest and controversy in the literature as to the proper distribution of the error terms for financial assets. Some observers (chap. 9) have suggested using stable Paretian distributions for financial models. Compared with normal distributions, such distributions have more of their mass in the tails and less near the mean. Because of this characteristic, use of such distributions would yield larger values for fair insurance premiums than those presented below.

Graphic presentations of term structures are presented in figure 15.1, which shows simulated structures for June 1976 that produce quartile values for an average bank portfolio of government securities. The median June 1976 structure reflects forward rates implicit in the June 1975 structure as well as changes in term premiums. Seven hundred simulated sequences of term structures were produced. Each sequence consisted of observations for twenty-four consecutive months on the spot rate and 149 forward rates.

Table 15.2 Time-Series Analysis on Standardized Principal Components

Component	First-Order Autoregressive Coefficient	Second-Order Autoregressive Coefficient	First-Order Moving Average Coefficient	Second-Order Moving Average Coefficient	Probability That Residuals Are White Noise
1	−.3897				.9105
2	−.2900				.2270
3	−.2240				.0774
4	.1162		.4814	.1998	.0989
5	−.3925	−.3845	−.2459	−.0877	.0578
6	−.0359		.2309	.1002	.0649
7	−.2669				.1038

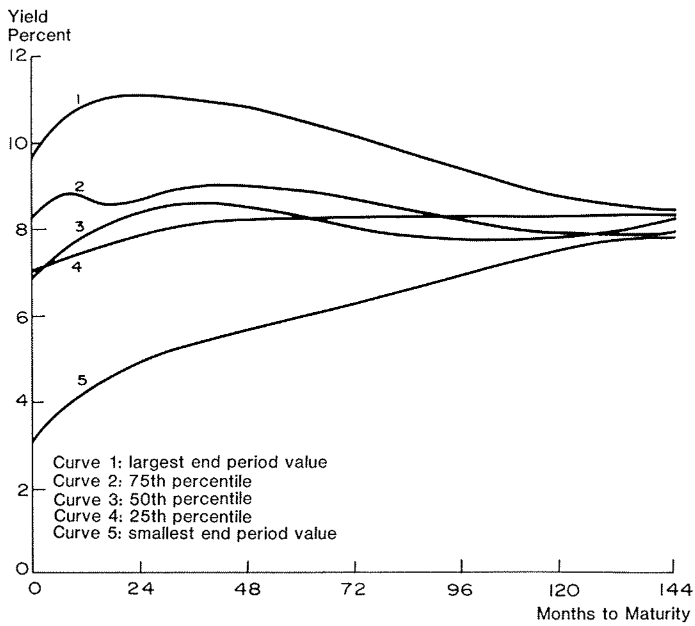

Fig. 15.1 Simulated term structures, June 1976.

15.2.4 Insurance Premiums and Portfolio Values

Using the simulated riskless term structures, we evaluated simple balance sheets of government assets at the end of hypothetical examination periods ranging in six-month intervals from six months to two years. We considered simple financial intermediaries whose assets consisted entirely of hypothetical par coupon bonds.

For these intermediaries, we calculated for each hypothetical examination period the average end-of-period value of capital and the fair insurance premiums.

For these calculations we assumed that assets were financed by six-month Treasury bill rate borrowing plus various amounts of equity. Net cash inflows, whether from coupons or principal, that were received during the examination period were applied to reduce borrowing; any surpluses were then invested at the six-month Treasury bill rate. For assets of maturity shorter than the examination period, this assumption means that principal repayments are reinvested at the six-month Treasury bill rate.

Changes in the value of capital derive either from changes in borrowing costs or from changes in asset values brought about by unexpected changes in discount rates. For assets of maturity shorter than the examination period the latter factor is not operative, since the par value of the asset will be received during the examination period. The simulation allows capital changes to accumulate by prohibiting dividend payments and capital contributions during the examination period.

We assume that the same instrument or instruments are held at the end of the period that were held at the beginning. Thus the maturity of the portfolio shortens by the length of the examination period.

Results are presented for intermediaries holding bonds of a single maturity and also for an intermediary holding an average portfolio of governments. The former intermediaries may be viewed as being composed of two activities. The first activity is lending in a specified maturity range, and the second is borrowing at the six-month Treasury bill rate. The latter intermediary may be viewed as engaging in many activities—lending in various maturity ranges and borrowing at the six-month bill rate. The average government portfolio weights, displayed in table 15.3, were obtained from May 1975 commercial bank holdings of government securities as published in the *Treasury Bulletin*.

Table 15.4 shows the end-of-examination-period value of equity for combinations of initial capital, portfolio composition, for a one-year examination period ending June 1976. (Similar tables for different examination periods are available from the authors.) Although only par bond portfolios are displayed, the technique used in generating the tables is capable of dealing with government portfolios of mixed maturities and with discount instruments. The mean values shown are based on the initial capital, on the earnings of that capital at the simulated interest rates during the year, on the difference between the yield on the portfolio of a particular maturity and the cost of funds at the six-month Treasury

Table 15.3 **Weights in Average Government Portfolio**

Par Bond Maturity in Years	Weight
1	.35
2	.26
3	.12
4	.10
5	.05
6	.04
7	.04
8	.01
9	.01
10	.01
11	.005
12	.005

Source: Bulletin of the U.S. Treasury Department, June 1975.
Note: Weights are calculated from reported par values, not market values.

Table 15.4 Mean Value at End of Twelve-Month Examination Period of Government Par Bond Portfolio with Stated Maturity (700 ARIMA Simulations)

Percentage Capital	Average Government Portfolio	Maturity in Years 1	2	3	5	7	10	12
0	.11	.04	.22	.33	−.12	−.18	.30	.52
1	1.17	1.10	1.28	1.39	.94	.88	1.36	1.58
3	3.30	3.23	3.41	3.52	3.07	3.00	3.49	3.71
5	5.42	5.35	5.54	5.65	5.45	5.13	5.62	5.83
7	7.55	7.48	7.66	7.77	7.57	7.26	7.74	7.96
10	10.74	10.67	10.85	10.96	10.76	10.45	10.93	10.09

bill rate, and on any simulated changes in end-of-period capital values for securities one year shorter than held initially.

Since the simulation drawings were for a period of positive capital gains and an upward sloping term structure, most entries show positive expected increases in capital for the simulated year. The pattern of earnings in the tables is consistent with habitat or segmentation theories of the term structure. The pattern is probably not consistent with a capital asset pricing model concept of ex ante portfolio equilibrium, since long-term securities (which generally evidence greater price variability) show smaller returns than short-term securities.

The low return–high total variance pattern implicit in our results might be consistent with capital asset pricing models if the market correlation of long-term securities were smaller than that of the short securities. However, an attempt to test this hypothesis showed that correlation with the market was generally an increasing function of maturity. Because of a narrow stock market definition of the market, these results might be viewed as inconclusive. Nevertheless, our model seems more akin to segmentation than to capital asset theories of term structure. An observer with a high degree of prior confidence in the appropriateness of the capital asset pricing model as a description of the government debt market would construct a term structure simulation technique that embodied this model in its assumptions. We made no attempt to construct such a simulation technique for this paper.

Fair insurance premiums for a twelve-month examination period are given in table 15.5. The examination period begins in June 1975. The insurance premiums are calculated:

$$\text{Fair premium}_t = E_t\,(\text{capital}_{t+\alpha} \mid \text{capital}_{t+\alpha} < 0) \cdot \text{Probability}\,(\text{capital}_{t+\alpha} < 0),$$

where the examination period runs from t to $t + \alpha$. The premiums are small. With 3 percent capital, the premium on the average portfolio is

Table 15.5 **Insurance Cost for Twelve-Month Examination Period of Government Par Bond Portfolio with Stated Maturity (700 ARIMA Simulations) (Cost per Year as a Percentage of Assets)**

Percentage Capital	Average Government Portfolio	Maturity in Years: 1	2	3	5	7	10	12
0	.49433	.15423	.40068	.60045	1.17292	1.41348	1.42786	1.48770
1	.15528	.00034	.09795	.26428	.70098	.92067	.96955	1.03696
3	.00489	.00000	.00088	.02724	.20236	.31950	.36348	.41962
5	.00000	.00000	.00000	.01483	.04300	.09502	.12003	.13470
7	.00000	.00000	.00000	.00109	.00501	.01989	.02798	.02997
10	.00000	.00000	.00000	.00000	.00000	.00000	.00120	.00275

.005, or one-half cent on one hundred dollars of securities. This small premum reflects directly the information contained in table 15.4 and the fact already noted that the assumptions underlying that table lead to fairly sizable increases in capital for most simulations. The probabilities of failure depend on both the initial capital and these assumed earnings.

The table shows, as one would expect, that fair premiums increase with increases in maturity and in examination period and decrease with increases in capital.

The insurance premiums in this section are derived on the assumption of an autoregressive integrated moving average (ARIMA) structure in past rates. In the following section premiums are derived on the assumption that past rates contain no useful information about changes in future rates.

15.2.5 Insurance Premiums When Adjacent Term Structures Differ Only by Noise

The term structure simulation technique described above adjusts for the estimated correlation of the ΔF across time. Under some notions of the term structure, this correlation may be viewed as stemming from changes in term premiums. This view seems consistent with market efficiency in Roll's sense (Roll 1970), in that simulated expected future spot rates are a martingale; however, some notions of market efficiency hold that there is no exploitable regularity in interest rate movements (cf. Phillips and Pippenger 1976). The results presented in this section assume that there is no correlation of the ΔF over time and thus are consistent with the no-exploitable-regularity view of market efficiency.

Successive term structures are simulated by "aging" the previous period's structure one period, adjusting for a constant term premium, and adding noise. As before, this is done in terms of principal components to account for the tendency of adjacent forward rates to fluctuate together over time.

For this section we calculated the principal components as in the previous section. However, no time-series analysis was done on these components. Rather, each component was assumed to come from independent standard normal distributions. Drawings on these independent distributions yielded simulated values for each component. Simulated values of ΔF were obtained by combining the simulated components utilizing the loadings, α_i, as weights. Each consecutive term structure in a sequence was then formed by "aging" the prior term structure, adjusting for a constant term premium and adding the simulated ΔF.

Each simulation drawing consists of a sequence of twenty-four consecutive monthly observations on the term structure; each observation consists of a one-period spot rate and 149 one-period forward rates. The program generates 700 such drawings. Simulated observations from two different sequences on the term structure corresponding to the same data may differ owing to differences in current and previous drawings on the principal components.

For ease of reference the insurance premiums of the previous section are referred to as ARIMA-based estimates, while the insurance premiums of the instant section are referred to as noise-based estimates.

As in the ARIMA case, the noise-based insurance premium analysis was done with seven principal components.

More formally, the simulated principal components:

$$\tilde{C}_{i\tau+1} \qquad i = 1, \ldots, 7 \tag{13}$$

were drawn from independent normal distributions. The single tilde indicates noise-based simulation. The ΔF were derived in a manner similar to equation (11) above by:

$$\widetilde{\Delta F_{\tau+1}(j)} = \sum_{i=1}^{F} \alpha_i(j)\, \tilde{C}_{i\tau+1}\,. \qquad j = 2, \ldots, M-1 \tag{14}$$

By analogy to equation (12):

$$\widetilde{F_{\tau+1}(j-1)} = F_\tau(j) + \widetilde{\Delta F_{\tau+1}(j-)}\,. \qquad j = 2, \ldots, M \tag{15}$$

Equation (15) provides one complete simulated observation on the term structure at time $\tau + 1$.

The balance of the noise-based procedure for the derivation of the 700 sequences of twenty-four consecutive monthly term structures is identical to that used for the ARIMA simulations.

Mean portfolio values and insurance premiums derived from the noise-based term structure sequences were calculated and are presented in tables 15.6 and 15.7. The formats of these tables are identical to those of the previous tables. All examination periods begin in June 1975.

Inspection of table 15.6 reveals that average end-of-period value peaks for maturities of about two or three years and then generally declines.

Table 15.6 **Mean Value at End of Twelve-Month Examination Period of Government Par Bond Portfolio with Stated Maturity (700 Noise Simulations)**

Percentage Capital	Maturity in Years 1	2	3	5	7	10	12
0	.18	.45	.52	.38	.23	.00	−.16
1	1.24	1.51	1.58	1.44	1.29	1.06	.90
3	3.37	3.63	3.71	3.56	3.41	3.19	3.03
5	5.49	5.76	5.83	5.68	5.53	5.31	5.15
7	7.61	7.88	7.95	7.81	7.66	7.43	7.27
10	10.80	11.07	11.14	10.99	10.77	10.62	10.46

Table 15.7 **Insurance Cost for Twelve-Month Examination Period of Government Par Bond Portfolio with Stated Maturity (700 Noise Simulations) (Cost per Year as a Percentage of Assets)**

Percentage Capital	Maturity in Years 1	2	3	5	7	10	12
0	.14520	.49827	.84887	1.39974	1.75911	2.23395	2.55016
1	.00250	.19410	.47361	.97340	1.30236	1.73904	2.02352
3	.00000	.01433	.12071	.40557	.64739	.9960	1.21309
5	.00000	.00056	.01798	.14631	.29668	.52976	.67879
7	.00000	.00000	.00054	.04379	.11788	.26544	.35584
10	.00000	.00000	.00000	.00231	.01560	.06854	.11190

This pattern primarily reflects the magnitude of term premiums. Casual comparison of sample period mean term premiums with differences across maturities of portfolio end-of-period values showed high correspondence. In other words, in the sample period ex post holding period returns were often lower for long-term securities than for short-term securities. The simulation results, based on the sample period, simply reflect this fact.

The noise-based insurance premiums are generally larger than those derived using the ARIMA techniques. Nevertheless, the premiums are still relatively small. With 3 percent capital, the premium on a three-year par bond is 12 basis points. This is about four times larger than the corresponding ARIMA-based premium.

For some maturity-capital combinations the noise-based insurance premiums are greater than the ARIMA-based premiums even though the mean portfolio values are greater for the former than for the latter. This is because the μ_{t+1}, the change between t and $t + 1$ in the expected spot rate corresponding to a fixed future date, is larger for the noise estimates than for the ARIMA. Thus, while in some cases the noise estimates have larger means than the ARIMA, they also generally have larger variance around these means. It is this larger variance that produces the larger insurance premiums.

Wealth effect risk is likely to be greater for municipal and mortgage elements of portfolios than for governments, since these elements typically have longer duration than governments.

The following sections address the problems of calculating insurance premiums on municipal and mortgage portfolios.

15.3 Insurance Premiums for Portfolios of Municipal Assets

This section presents a simulation model of the municipal term structure and uses this model to estimate insurance premiums for portfolios of municipal bonds. The simulated municipal term structure runs off the ARIMA-based simulated government structure; however, adjustments are made for certain systematic tendencies of municipal and government rates to behave differently.

Equations determining municipal rates for specified maturities are estimated by regressing municipal rate relatives on roughly corresponding government rate relatives. Values for municipal rates between those specified are interpolated. This procedure yields the municipal term structure as a function of the current level and recent history of the government term structure. Drawings from the government term structure simulator are then used to provide a simulated history necessary to generate a simulated observation on the municipal term structure. Repeated drawings on the government term structure provide repeated simulated observations on the municipal term structure.

Using the simulated municipal term structures with procedures similar to those described above for government term structures, frequency distributions of portfolio end-of-period value are generated, and fair insurance premiums against pure interest rate risk are calculated for intermediaries with specified municipal portfolios and initial capital positions.

Finally, through a simple extension of the techniques of the previous section, mixed portfolios of government and municipal securities are evaluated and fair insurance premiums calculated.

15.3.1 Some General Characteristics of the Municipal Securities Market

Before we consider the details of the municipal simulation model, some general comments about the characteristics of the municipal securities market are in order. Any adequate simulation model of this market must attempt to deal with these characteristics.

Differences in behavior of municipal interest rates and other rates primarily reflect differences in tax treatment. Coupon payments on most securities are subject to federal income taxes, while those on municipals are not. The differential tax treatment means that yields on municipals are generally substantially lower than those on corresponding taxable securities. However, the term structure for municipals tends to be steeper than that for taxable securities, and in high interest rate periods, yield differentials between these two classes of securities tend to close.

Although this converging effect takes place throughout most of the maturity structure, it is most pronounced for the longer maturities. Thus long municipal rates tend to display more variability than corresponding government rates. Finally, the shorter municipal rates tend to be less responsive to changes in the one-month government rate than do corresponding government rates.

The factors determining the municipal rate behavior described are not clearly understood, but explanations resting on institutional factors are common. It is argued that, because of the short maturities of their liabilities, banks wish to hold short-maturity municipals. Owing to their size, banks dominate this sector of the market, and yields on short-term municipal issues tend to average about 52 percent of those on corresponding government issues. This ratio results in rough equality in bank after-tax earnings on municipal and government securities. However, banks resist getting heavily into longer-term municipal instruments; hence this sector is left to lower-bracket taxpayers who drive up long-term rates, since they require a higher ratio of municipal risk-free yields to equate after-tax earnings. The tendency of municipal rates to rise more than other rates in high interest rate periods probably reflects the banking community's attitude that maintaining customer relationships by assuring availability of credit is of primary importance in maximizing long-run earnings. Thus, in high rate periods banks tend to unload municipals to finance commercial lending. This scenario seems to be an accurate description of events in the high rate days of 1966 and 1969, when banks sold substantial volumes of municipals and rates on tax-exempts soared.

15.3.2 Municipal Term Structure Simulation and Insurance Premiums

We simulated the municipal term structure by relating three municipal rates to roughly corresponding government rates and interpolating to obtain the entire term structure for the remaining municipal rates.

We ran regressions explaining the 12- , 60- , and 360-month municipal rate by the 12- , 60- , and 120-month government rates, respectively. We used the 120-month government rate because 360-month government data were not available. The municipal rate data were taken from Salomon Brothers (1974) quarterly "good" grade municipal yield series. The government data were the McCulloch data described above. The sample period was mid-1965 to mid-1975.

The regression equations were:

(16)
$$\left[\frac{MN_t(m)}{MN_{t-1}(m)} - 1\right] = \alpha_m + \beta_m\left[\frac{G_t(m)}{G_{t-1}(m)} - 1\right] \qquad m = 12, 60$$

and

$$\left[\frac{MN_t(360)}{MN_{t-1}(360)} - 1\right] = \alpha_{360} + \beta_{360}\left[\frac{G_t(120)}{G_{t-1}(120)}\right] - 1 \quad ,$$

where

$MN_t(m)$ is the m month municipal yield at time t, and $G_t(m)$ is the m month government yield at time t.

The regression results are presented in table 15.8. All the estimated parameters are significant at the 5 percent level. About half of the variation in the municipal relatives is explained by variation in the government relatives. The presence of a significant constant indicates that during the sample period there was a slight tendency for municipal rates to increase relative to government rates. The magnitude of the coefficients of determination indicates that factors other than changes in government rates play an important role in explaining municipal rates. We attempted to increase the explanatory power of the equations by adding variables representing government rate levels. The coefficients these attempts yielded were generally not significant, and hence we used the simple form of equation (16).

Using the results in the table, we obtained 12- , 60- , and 360-month municipal yields by plugging simulated government rates into the equations and solving. In this way it was possible, for each term structure in a simulated sequence of government term structures, to obtain three simulated municipal rates that were dependent on the associated government structure. However, this technique did not provide a complete municipal term structure. To obtain such a term structure, we needed some interpolation technique. We selected a technique used by Bradley and Crane (1975) for interpolating municipal yield curves. The yield curve is assumed to follow a function of the form:

(17)
$$MN(m) = am^b e^{cm} \quad ,$$

where $MN(m)$ is the yield to maturity of an m period municipal bond and a, b, and c are parameters. Given three yields on a particular yield curve, it is possible to treat (17) as three equations in these unknowns and solve for these parameters; the remaining yields along the given curve can then

Table 15.8 **Regressions of Municipal Rates on Government Rates**

Dependent Variable	Independent Variable	α (Standard Error)	ß (Standard Error)	R^2
MN(12)	G(12)	.00167 (.00036)	.69381 (.05557)	.63
MN(60)	G(60)	.00138 (.00038)	.68436 (.07315)	.49
MN(360)	G(120)	.00148 (.00027)	.57761 (.05917)	.15

be solved for using these parameters. New parameter values must, of course, be obtained for each set of three municipal yields belonging to a single given yield curve.

Using equation (16) and a given simulated ARIMA government term structure, we obtained simulated 12- , 60- , and 360-month municipal rates along a single municipal yield curve. We then used these three simulated municipal yields with equation (17) to obtain the complete yield curve corresponding to the three simulated yields. For each successive set of three simulated municipal yields (generated from equation 16 and successive simulated government term structures), we repeated this procedure to obtain a complete simulated municipal yield curve.

The final result is similar to that obtained for governments. Seven hundred simulated municipal term structure sequences were generated. Each sequence consists of eight consecutive simulated quarterly observations on a municipal term structure composed of the one-month municipal spot rate and 359 one-month forward rates.

The simulated sequences of municipal rates were used in the same fashion as the simulated risk-free rates to generate estimates of mean portfolio values and of fair insurance premiums. The results are presented in tables 15.10 and 15.11. The weights used for the average municipal portfolio, obtained from the *1976 Annual Report of the Federal Deposit Insurance Corporation*, are displayed in table 15.9. The insurance premiums for municipals are generally from 35 to 60 percent of the premiums on corresponding government bonds. Often they are far smaller than this. These small premiums reflect the relatively low coefficients of determination of table 15.8. Government rates explain only about half the variation in municipal rates. Thus the simulated municipal rates, dependent as they are only on government rates, display less variability than do actual municipal rates. That the simulated municipal term structures have low variability does not render the insurance premiums derived from them useless. However, care must be taken in interpreting these premiums. The premiums in table 15.11 must be interpreted as fair insurance premiums against pure government interest rate variability risk only. These premiums do not account for all sources of variability in municipal rates; they are appropriate insurance premiums on municipals owing to variations in government rates alone.

Table 15.9 **Weights in Average Municipal Portfolio**

Par Bond Maturity in Years	Weight
1	.17
3	.15
5	.14
7	.14
10	.14
15	.10
20	.08
25	.05
30	.03

Source: Annual Report of the Federal Deposit Insurance Corporation, 1975.
Note: Weights are calculated from reported par values, not market values.

Table 15.10 **Mean Value at End of Twelve-Month Examination Period of Municipal Bond Portfolio with Stated Maturity (700 Simulations)**

Percentage Capital	Average Municipal Portfolio	Maturity in Years: 1	3	5	7	10	15	20	25	30
0	.27	.04	.07	.07	.16	.34	.60	.72	.69	.59
1	1.31	1.08	1.11	1.11	1.20	1.38	1.65	1.76	1.74	1.63
3	3.39	3.16	3.19	3.20	3.28	3.47	3.73	3.84	3.82	3.71
5	5.47	5.25	5.28	5.28	5.37	5.55	5.81	5.92	5.90	5.80
7	7.56	7.33	7.36	7.36	7.45	7.63	7.89	8.01	7.98	7.88
10	10.68	10.45	10.48	10.49	10.57	10.76	11.02	11.13	11.11	11.00

Table 15.11 **Insurance Cost for Twelve-Month Examination Period for Municipal Bond Portfolio with Stated Maturity (700 Simulations) (Cost per Year as a Percentage of Assets)**

Percentage Capital	Average Municipal Portfolio	Maturity in Years: 1	3	5	7	10	15	20	25	30
0	.50813	.06882	.30736	.47354	.59920	.68616	.80772	.90854	1.01365	1.12029
1	.17575	.00000	.03785	.13366	.22164	.32263	.44340	.53175	.61233	.70008
3	.00975	.00000	.00000	.00337	.01671	.05185	.10644	.14648	.18477	.22205
5	.00000	.00000	.00000	.00000	.00000	.00239	.01349	.02497	.03624	.04756
7	.00000	.00000	.00000	.00000	.00000	.00000	.00042	.00188	.00282	.00516
10	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

While the long maturity of municipal securities apparently does not require large premiums to cover risk of variation in default-free rates, the same need not be true of other long-maturity instruments. The following section provides rough estimates of premiums on commercial bank mortgage portfolios.

15.4 Insurance Premiums on a Commercial Bank Mortgage Portfolio

Because of their relatively long nominal maturities, real estate mortgages might be expected to be a source of considerable interest rate risk. The interest rate sensitivity of the commercial bank mortgage portfolio is carefully dealt with in the chapter by Nadauld (chap. 14). However, some rough calculations are done in this section to get a notion of the magnitude of fair insurance premiums on commercial bank mortgage portfolios.

We calculate the insurance premiums using the technique developed above for government securities. Rather than simulating appropriate discount rates for mortgages, we discount the flows off the mortgage portfolio by the previously derived ARIMA-based simulated government term structure sequences. While mortgage portfolios generally include little default loss, the appropriateness of discounting by government rates is open to question.

Since the simulated government term structures extend only to maturities of twelve years, it was necessary to specify values for rates from twelve to thirty years. We did this by assuming that the thirty-year yield was 50 basis points greater than the twelve-year yield and linearly interpolating to obtain rates between these two rates. The 50 basis point spread is one that has prevailed at times in the past. While this assumption too is open to question, it should not cause substantial error in the premium estimates, since most of the value of a mortgage portfolio derives from flows within the first twelve years.

A commercial bank mortgage portfolio that appeared to be representative was selected for analysis. The portfolio was composed of new and seasoned mortgages. We calculated expected flows off the portfolio by correcting the scheduled nominal flows for average experiences with prepayments and defaults. The method is described in detail in Nadauld's chapter.

Mean end-of-period mortgage portfolio values and corresponding insurance premiums are reported in table 15.12. Examination periods begin in June 1975. Once again the premiums are small. With 5 percent capital, the fair premium on an average mortgage portfolio is only 5 basis points. The small premiums reflect the short actual duration of mortgages. Prepayments and amortization bring the duration of most residential mortgages to below ten years.

Table 15.12 **Mean Value at End of Twelve-Month Examination Period and Insurance Cost for an Average Mortgage Portfolio (700 ARIMA Simulations)**

Percentage Capital	Mean Value	Insurance Cost per Year as Percentage of Assets
0	.28	1.16002
1	1.35	.71988
3	3.47	.20903
5	5.60	.04344
7	7.73	.00459
10	10.92	.00166

15.5 Insurance Premiums on a Mixed Portfolio of Governments, Municipals, and Mortgages

The previous sections have presented fair insurance premiums for portfolios composed of single classes of assets—government securities, municipal securities, or mortgages. Commercial banks often hold all these assets simultaneously. This section presents estimates of fair insurance premiums on what might be considered an average commercial bank portfolio composed of all three of these assets.

The average portfolio was formed by using the average government, municipal, and mortgage portfolios considered above and weighting these portfolios by the weights displayed in table 15.13.

Insurance premiums were derived by combining techniques used in earlier sections. First, we obtained a simulation drawing on a sequence of consecutive ARIMA-based government term structures, and we applied the procedure outlined in the section on governments to the government element of the average portfolio to obtain a single observation on this element's end-of-period value. Second, we determined the municipal term structure sequence associated with the given government sequence and applied the procedure outlined in the section on municipals to the municipal element of the average portfolio to obtain an observation on this element's end-of-period value. Finally, we extrapolated the term structures in the given government sequence to thirty years and applied the procedure outlined in the section on mortgages to the mortgage element of the average portfolio to obtain an observation on its end-of-period value.

The end-of-period values of each of the three elements of the average portfolio were summed to obtain a single end-of-period observation on the value of the entire portfolio. This observation is associated with a single given government term structure sequence. For each examination period we repeated this entire procedure 700 times to obtain a frequency

Table 15.13 Weights of Component Portfolios in Average Bank Portfolio

Portfolio	Weight
Average government portfolio	.26
Average municipal portfolio	.31
Representative mortgage portfolio	.43

Source: Annual Report of the Federal Deposit Insurance Corporation, 1976.

distribution of end-of-period values of the average portfolio. From this distribution we calculated insurance premiums.

The mean end-of-period values and the associated insurance premiums are reported in table 15.14. All simulated examination periods begin in June 1975. Once again the premiums are small.

15.6 Conclusion

This chapter demonstrates a method for simulating sequences of government term structures and associated sequences of municipal term structures. We used the simulated term structure sequences to obtain frequency distributions of net end-of-period values of relatively simple hypothetical portfolios. From these distributions we calculated fair insurance premiums as the average loss (per dollar of assets) paid out by a deposit insurer. Insurance premiums were calculated for individual government, municipal, and mortgage securities as well as for combinations of these assets roughly corresponding to average commercial bank portfolios.

The insurance premiums depend upon the expected end-of-period net worth and its probability distribution. In these simulations expected income from assets depends upon initial capital, upon the distribution of the asset portfolio by type of asset and maturity, by past relationship of returns to the risk-free rate, and by movements in the risk-free interest

Table 15.14 Mean Value at End of Twelve-Month Examination Period and Insurance Cost for an "Average" Bank Portfolio (700 Simulations)

Percentage Capital	Mean Value	Insurance Cost per Year as Percentage of Assets
0	.23	.76407
1	1.29	.36742
3	3.40	.05262
5	5.52	.00236
7	7.63	.00000
10	10.80	.00000

rates across the term structure. The cost of liabilities is assumed to be equal to the six-month Treasury bill rate.

We use two methods of simulating movements in the term structure. In the first (ARIMA) method, the implicit forecasts of future rates contained in the previous period's term structure are "aged" by adjusting for expected movements. The movements are projected from a time-series analysis of the principal components of past interest rate changes and by an added error term. An entire 150-month term structure is aged, with a separate estimate for each interest rate in the structures for each of the following twelve months. (Six and twenty-four month simulations were also available.) Each of these processes or simulation drawings is repeated 700 times to obtain a distribution of term structures for the end of the period as shown in figure 15.1. In the second technique, successive term structures are aged by adjusting for the constant term premiums (cf. chap. 9 and table 14.3) and by drawing from a normal distribution fitted to month-to-month interest movements in the period 1 January 1960 to 1 June 1975.

The expected value of each specific portfolio at the end of the period is calculated for each of 700 term structure simulations. The value for a portfolio under a drawing depends upon: (*a*) the initial capital; (*b*) the June 1975 level and term structure of interest rates; (*c*) the content of the portfolio by type of asset and maturity; (*d*) the assumption that all liabilities carry the six-month simulated bill rate; (*e*) that all receipts from principal are used to reduce the size of the portfolio, while all receipts from income are reinvested at the six-month bill rate; (*f*) that no earnings are paid out; and, most important (*g*), that remaining assets are revalued at the term structure projected for the end of the period.

Fair insurance premiums are calculated by examining all the drawings with negative values among the 700 simulations for a specific portfolio, estimating the expected loss for each of these, and summing costs by applying to each negative value its proportion of the total simulations. Required insurance premiums increase with the maturity of a portfolio and fall as the capital/asset ratio rises.

The insurance premiums under these particular assumptions are low. For example, existing FDIC rates, according to table 15.14, would cover the simulated interest rate risk for this portfolio in a bank with 3 percent capital. This is a far lower risk than estimated in chapter 9, and somewhat less than estimated in chapter 4. Major differences arise from the assumptions concerning the shape of the variance distribution and from the expected net worths around which the variances are measured. Clearly these tables cannot be applied directly in their present form. They must be adjusted to the time of evaluation, to the specific portfolio, and for other risks.