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14 Calculating the Present Value of an Asset's Uncertain Future Cash Flows

Stephen D. Nadauld

14.1 Introduction

This chapter explains the application of a multiperiod uncertainty model whose theory has been discussed in previous chapters. The model makes it possible to calculate the present value of uncertain future cash flows generated by any asset. The approach may be applied to a wide variety of asset classes. However, this paper uses as an example an actual computer program designed to calculate the present value of a mortgage portfolio and comments on how such a program could be modified and applied to other types of assets.

For illustration, the approach is applied to mortgage portfolio data obtained from the Federal Home Loan Bank Board for eighteen savings and loan associations in the San Francisco standard metropolitan statistical area (SMSA).

The usefulness of this approach to multiperiod valuation under uncertainty stems from its similarity to the common certainty formulation, which may be written as

$$PV = \frac{x_1}{(1+r)^1} + \frac{x_2}{(1+r)^2} + \ldots + \frac{x_T}{(1+r)^T} \tag{1}$$

$$= \sum_{t=1}^{T} \frac{x_t}{(1+r)^t},$$

where

PV = present value of the income stream
x_t = *certain* cash flow received in period t
t = the risk-free interest rate

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The equation (1) approach of discounting *certain* future cash flows by a rate that is the same for each period in the future should be familiar to the reader.

A similar simple closed form equation for discounting *uncertain* future cash flows is as follows:

$$(2) \qquad PV = \sum_{t=1}^{\infty} \frac{E(X_t) - \dfrac{COV(X_t, -R_{mt}^{-b})}{E(R_{mt}^{-b})}}{R_{Ft}},$$

where

PV = present value of the income stream
X_t = the uncertain cash flow in period t
R_{mt} = the future value of \$1 invested for t periods in the market portfolio
R_{Ft} = the future value of \$1 invested for t periods in risk-free bonds
b = level of proportional risk aversion which will be assumed equal to one.

Since the theoretical development behind this model has been discussed in previous chapters, attention in this chapter will focus on application of equation (2).

To understand the application, it is important to note both the similarities and the differences of equations (1) and (2). The two formulations are similar in that both have cash inflows in the numerator, and both discount the cash flows for time value in the denominator. However, there are three important differences between the two equations. First, equation (2) uses expected cash inflows in the numerator ($E(X_t)$) instead of the single-point certainty cash flows of equation (1). Second, equation (2) has a term in the numerator that adjusts the expected cash flows for uncertainty. While this term appears complex, it may be thought of as simply a dollar amount that, when subtracted from the expected cash flows, supplies a certainty equivalent in place of the expected value. Third, the denominator in equation (2) employs a full specification of the term structure in place of the single level rate assumption of equation (1).

As suggested by the differences between equations (1) and (2), equation (2) is the conceptual basis for a much richer approach to the valuation of financial instruments. In addition to the obvious allowance for uncertainty, equation (2) allows for the cash flow in each period to be discounted by the risk-free discount factor appropriate to that period and thus uses all the information available in the term structure. Also, the approach is extremely useful in dealing with the question of interest sensitivity, since equation (2) makes provision for specifying differences in the response of short and long rates and allows computation of the resulting effect on valuation.

Because the approach affords important insights into the impact of interest rate changes on valuation, it is important at the outset to specify how interest rate changes must be incorporated in the analysis. Interest rate changes have three separate effects. First, they may affect the expected future cash flows. In the mortgage application, for example, the effect of rate changes on future prepayments and defaults may considerably alter the expected cash flows. Second, a change in interest rates obviously changes the discount factors in the denominator. Third, and potentially the most troublesome, is the effect of interest rate change on the risk premium. The risk premium may be affected because of changes in aggregate risk tolerance or through changes in the systematic riskiness of the asset that reflect a change in the joint distribution of asset cash flows and market returns. To make maximum use of the model, specific equations linking each of these three areas to interest rate change need to be incorporated into the analysis.

The process of applying equation (2) consists of four steps for any type of asset:

1. Determination of the expected cash flows for the asset. This may be easy or more difficult depending on the nature of the instrument and the distribution of the cash flows. Mortgages, for example, require specification of future contract payments and future prepayments, and some estimate of defaults. In addition, time-series or cross-sectional data are necessary to give some insight into the distribution.

2. Specification of the risk-free term structure. There are several ways to determine the necessary term structures: (*a*) present or past term structures may be calculated from risk-free government securities; (*b*) future term structures may be obtained from macro models such as the Penn-MIT model, from systematic adjustments of forward rates obtained from present term structures, or from arbitrary specifications.

3. Calculation of the uncertainty adjustment term. To calculate the uncertainty term, it is necessary to make assumptions that allow the computations to be done in the form

$$U_t = COR(X_t\,, \frac{1}{r_m})\ Std(\frac{1}{r_m})\ Std(X_t)\ E(r_{mt})\ ,$$

where

U_t = uncertainty adjustment for period t
COR = coefficient of correlation
Std = standard deviation
E = expectation operator
r_m = the one-period return on the market.

The correlation, standard deviations, and expectation must be calculated from time-series or cross-sectional data obtained for asset cash flows and market returns.

4. Computation of the present value. The respective elements of equation (2) are combined by the computer program into the calculated present value.

The rest of the chapter will be divided into two sections. Section 14.2 will explain the basic calculations in steps 1 through 4 and apply them to the question of valuing a mortgage portfolio, and section 14.3 will demonstrate the computer program operations and give sample inputs and outputs.

14.2 Basic Calculations and Applications

14.2.1 Step 1: Determining the Expected Cash Flows

The cash flows from a mortgage portfolio are made up of regular mortgage payments (including principal plus interest) and of mortgage prepayments. For the typical mortgage, the contract rate and maturity are fixed at the time of origination. The combination of rate and maturity specifies a fixed-payment cash inflow that the mortgage promises to generate at each point in the future. The payment is made up of principal plus interest and follows a prescribed amortization schedule depending on the rate and maturity. Because in the typical situation the promised individual mortgage payments are fixed, the promised portfolio cash inflow is fixed. Therefore in general the value of the portfolio varies inversely with fluctuating market rates.

The difficulty that financial institutions have with fixed-rate instruments is well known, and some mortgage-granting institutions have recently attempted to solve the problem by issuing variable-rate mortgages. A variable-rate mortgage allows the institution to raise (or lower) the mortgage contract rate according to the rise or fall of an index of appropriately constructed market rates. This kind of mortgage has some advantages to the institution, especially in times of rising interest rates. Variable-rate mortgages have become popular with selected institutions and are increasingly being issued, especially by California institutions. However, the trend toward variable-rate mortgages is relatively recent. Since the data gathered for this application are from portfolios formed in 1975 or before, they are made up almost wholly of fixed-rate mortgages; for this reason, the analysis in this paper will concentrate on their behavior.

Two factors substantially complicate the cash inflows generated by a mortgage portfolio. The first of these is the prepayment option. Although the contract payments are spoken of as fixed, the mortgagee has the option to prepay part or all of the principal in conjunction with the refinancing or sale of the mortgaged property. Because of the mobility of many homeowners, the option to prepay is often exercised, and the resulting prepayment cash flows are a substantial factor in analyzing the total cash flows from the mortgage portfolio. The second factor concerns mortgage defaults. The payments spoken of as fixed are fixed only in the sense that they are promises to pay. In the case of default, these promised payments are not realized by the mortgage-granting institution.

Fixed Payments

The basic element of mortgage portfolio cash flows is derived from the fixed mortgage payment. The standard parameters of the fixed-payment, self-amortizing mortgage, are straightforward. The regular monthly payment is determined as a function of the amount of the principal, contract rate, and term to maturity and may be computed by applying the following simple payment algorithm:

$$\text{payment} = \frac{\text{Principal}}{1 - \dfrac{1}{(1+r)^t}},$$

where

r = contract rate on monthly basis
t = term to maturity in months.

These fixed payment amounts must be adjusted in each period to take into account foreclosures or prepayments that occurred in prior periods. The effect of the foreclosure or prepayment is to eliminate loans from the portfolio and therefore reduce the fixed payment portion of the cash flow.

Prepayments

Since mortgage holders have the option to pay down their mortgage by amounts in excess of the scheduled amortization, mortgage portfolios have cash flows that may in any period substantially exceed the prescribed fixed payments for the period. It is important to consider these prepayments, since their timing may greatly affect the value of the mortgage portfolio. There are several ways to include prepayments in the analysis. It is possible, for example, to use an arbitrary set of prepayment data in the form of annual mortgage termination rates. Termination rates are defined as a fraction of the portfolio balance that would be paid off in any year. The rates may be stated as a fraction of the original portfolio amount (fixed basis) or as a fraction of the remaining portfolio amount (current basis). Typical termination patterns are available from Federal Housing Administration (FHA) data.

Another more interesting alternative is available for dealing with prepayments and illustrates the concept of linking cash flows to interest rate changes. The approach is based on a model developed by Curley and Guttentag (1974). The model explains mortgage termination rates as a function of policy year, maturity, the relationship between the current mortgage contract rate and original contract rate, and the discount points charged in the specified year. Their results are contained in a single equation of the following form:[1]

$$\begin{aligned} \log TR_t = &- .56178 + .90249 \log(P/M) - .10580 \\ &(-32.67) \quad (56.29) \qquad\qquad (15.61) \\ &(C_t - C) - .02179 D_t , \\ &\qquad\qquad\quad (5.97) \end{aligned}$$

where

$$\begin{aligned} R^2 &= .867 \\ TR_t &= \text{annual termination rate in year } t \text{ (current basis)} \\ P &= \text{policy year} \\ M &= \text{maturity} \\ C_t &= \text{mortgage rate in year } t \\ D_t &= \text{discount in year } t \\ C &= \text{original contract rate.} \end{aligned}$$

The termination rate in year t (TR_t) is defined as a function of the portfolio balance that will be paid off in that year. The prepayment equation estimates current basis termination rates, but these may easily be converted to fixed basis rates if desired. The prepayment equation was estimated using FHA annual data covering the period 1951–67 for twenty- , twenty-five- , and thirty-year mortgages. The absolute level of both long- and short-term interest rates was generally lower during this period than during subsequent periods to which the equation may be applied. However, the variables in the prepayment equation that would be affected, namely the original contract rate and the mortgage rate in year t, enter the equation in the form of a yield differential. Since relative

1. The numerator of the valuation equation requires expected cash flows. The prepayment cash flows are determined by finding the prepayment function (TR) which is the analog of the log TR_t equation. It is clear that $E(x) \neq \exp[E(ln(x))]$; but it can be shown that the difference is small. One approach is to suppose the generalized function $g(x) = a + bx + cx^2 + dx^3 + \ldots$. Both $ln(x)$ and e^x meet this criterion. Expanding $g(x_o + \varepsilon)$ and taking the expectation ($E[g(x_o + \varepsilon)]$) gives $x_o + E(\varepsilon)$ plus the expectation of higher-order term of ε^2, i.e., $E[0(\varepsilon)^2]$. In the same fashion, $g[E(x)] = g[x_o + E(\varepsilon)] + x_o + E(\varepsilon)$ plus higher-order terms of the square of the expectation, i.e., $0[E(\varepsilon)]^2$. If ε is small, ε^2 is very small, so $E[0(\varepsilon)^2] \cong 0$; and $E[\varepsilon]$ is small, so $0(E[\varepsilon])^2$ is also $\cong 0$. This leaves the constant and first-order terms the same; and with only small differences in the second-order terms, it appears safe to use the analog of the log TR_t equation in place of the expectation.

differentials or spreads have approximately the same range in subsequent periods as in the estimation period, the prepayment equation should retain its usefulness when applied to more recent data.

The prepayment equation makes it possible to calculate prepayments and add them to the fixed mortgage payments for each period in the future. However, because the equation uses projected future mortgage rates, it is necessary to supply these rates in some fashion. This may be done arbitrarily or, since other numerator and denominator changes are driven by changes in short-term rates, it may be done by attempting to link future mortgage rates to future short-term interest rates.

Mortgage Rates

As an illustration of an attempt to determine the nature of the relationship between mortgage rates and short-term rates, a monthly mortgage rate series from December of 1969 through December of 1975 was analyzed in conjunction with three-month Treasury bill rates over the same period. The most notable feature of these two series is the extremely high level of serial correlation that exists in each. For example, a regression of the spread between the mortgage rate and the Treasury bill rate against the Treasury bill rate yields the following equation:

$$MRT - TBRT = 6.11 - 0.579\ TBRT + e$$
$$\bar{R}^2 = 0.60 \qquad (.34) \qquad (.056) \qquad D\text{-}W = 0.1398$$
$$N = 73,$$

where

MRT = mortgage rate,
TBRT = Treasury bill rate,
$D\text{-}W$ = Durbin-Watson statistic,
and the figures in parentheses are standard errors. The resulting t-statistics are significant at the 99 percent level.

The Durbin-Watson statistic of 0.1398 shows the high level of serial correlation in the residuals. This could be anticipated, since the mechanism described by several practitioners for mortgage rate determination is to set the current period mortgage rate equal to the previous period's rate and adjust slightly upward or downward, depending on short-term rate movements. The autocorrelation is strong enough to require second-order autocorrelation correction, which yields the following results:

$$\begin{aligned} MRT - TBRT = {} & \underset{(.45)}{8.77} - \underset{(.03)}{1.03}\,TBRT + \underset{(.085)}{1.65}\,RHO1 \\ & - \underset{(0.087)}{0.69}\,RHO2 \end{aligned}$$

$$D = W = 1.83$$
$$\bar{R}^2 = 0.98$$
$$N = 73.$$

The resulting t-statistics are again significant at the 99 percent level. However, there is a difficulty involved in using the equation above to predict future mortgage rates in the current setting. The prepayment algorithm requires a pattern of future mortgage rates over a thirty-year period in order to compute a thirty-year pattern of prepayments and the resulting pattern of cash flows. When the equation is used to predict beyond five to ten periods, the predicted mortgage rates diverge rapidly from the average relationship that has existed in the past between mortgage rates and short-term Treasury bill rates. This average spread was computed for 1961 through 1975 and was found to be approximately 210 basis points. Using the 210 basis point average spread to predict future mortgage rates was found to produce more reasonable prepayment patterns than those developed from the autocorrelation-corrected equation.

Defaults

While it is possible to relate the question of prepayments and interest rate changes in a fashion sympathetic to the overall approach, the problem of defaults is not so easily handled. The numerator-related concern is essentially a question of how the expected value of cash flows are affected by defaults. Since it is reasonable to assume that short-term interest rates, either directly or as a proxy, may have some influence on default, it should be possible in principle to link rate changes through defaults to the expected cash flows. However, because default data were not available for the portfolios in the data sample, it was not possible to examine the linkage between rate changes and defaults. Nevertheless, the computer program written to do the calculation allows for the possibility of arbitrarily varying the default rates by year for each association. For example, the first year foreclosure rate could be determined as 2 percent, meaning that 2 percent of the book value of the portfolio would be eliminated for all subsequent periods. However, the program allows the user to specify an average percentage of the foreclosed amount to be recovered, and the recovered amount is added to the normal payment and prepayment cash flows for the period. The foreclosed mortgages are subtracted from the portfolio, and future cash flows from both regular payments and prepayments are appropriately reduced.

14.2.2 Step 2: Specification of the Risk-Free Term Structure

As suggested previously, there are several ways to obtain the necessary risk-free interest rates that make up the denominator of equation (2). To examine the question of how changes in rates affect the resulting present values, it is useful to have both present and projected future term structures that can be used to discount the projected cash flows.

Current estimates of the term structure may be obtained from an estimation of the discount function accomplished by a computer program developed by J. Huston McCulloch (1975*a*). The basic concept is to fit a discount function to the observed prices of United States government Treasury obligations. The program uses as inputs the bid and ask prices of bill, note, and bond quotes for a given date and applies an instrumental variables technique to fit the discount function to a cubic spline (a flexible curve much like a polynomial). An example of the program's output is the point yield for 2 January 1976, which appears in table 14.1. These data are used directly as inputs to the present value calculation. The calculations are done in continuous time so that the denominator at any quarter t

Table 14.1 **"2 January 1976," "Shifted," and "Arbitrary" Term Structures, Quarterly for Five Years**
(Shift Parameter Values: RR = .015 RR-HAT = .015
II = .0377 II-HAT = .1477)

Month/Year	2 January 1976 Term Structure	Shifted Term Structure	Arbitrary Term Structure
3/76	5.27	10.92	11.40
6/76	5.62	9.95	10.70
9/76	5.89	9.20	10.00
12/76	6.13	8.67	9.80
3/77	6.35	8.29	9.50
6/77	6.54	8.02	9.25
9/77	6.69	7.82	9.10
12/77	6.81	7.68	9.00
3/78	6.92	7.58	8.91
6/78	7.02	7.53	8.84
9/78	7.10	7.49	8.78
12/78	7.17	7.47	8.73
3/79	7.24	7.47	8.68
6/79	7.29	7.46	8.63
9/79	7.35	7.48	8.59
12/79	7.39	7.49	8.55
3/80	7.44	7.52	8.52
6/80	7.48	7.54	8.50
9/80	7.52	7.56	8.50
12/80	7.55	7.58	8.50

is $DNOM_t = EXP(K_t * t/4)$, where K_t is the point yield for period t determined as in table 14.1.

The current term structure may be used as a basis for obtaining future term structures through a technique developed by Morrison and described in chapter 13. The function used to shift the term structure takes the following form:

$$\frac{\Delta r_k}{R_{Fk}} = \frac{\Delta rr_1 \, e^{-.41 - .14k}}{R_{F1}} + \frac{\Delta i_1 \, e^{-.14 - .09k}}{I_1}, \tag{3}$$

where

rr_1 = real one-period rate
R_{Fk} = 1 + the k-period risk-free nominal compound rate
i_1 = one-period inflationary expectation
$I_1 = 1 + i_1$
$r_1 = rr_1 + i_1$
Δ = difference operator
k = time period in months.

As an example of how the function works, suppose:

$r_1 = .06$
$rr_1 = .03$
$i_1 = .03$
$\Delta rr_1 = .01$
$\Delta i_1 = .03$.

Since $\Delta r_k = r_k{}^* - r_k$, it is possible to find a new value of the k-period rate from $r_k{}^* = (1 + r_k)\, \frac{.01}{1.06} e^{-.41 - .14k} + \frac{.03}{1.03} e^{-.41 - .09k} - r_k$.
For $k + 4$ and $r_k = .07$,

$$r_k{}^* = (1.07)\, \frac{.01}{1.06} e^{-.97} + \frac{.03}{1.03} e^{-.77} + .07$$

$$= .0883.$$

The rate of each period making up the term structure may be shifted in the same manner.

Table 14.1 contains an example of a shifted term structure that has been computed using equation (3) and the shift parameters shown in the table. As an alternative, it is possible to choose arbitrary term structures and employ them as the denominator of equation (2). An example of such an arbitrary designated term structure is also shown in the table. Figure 14.1 shows the shapes of the 2 January 1976, revised, and arbitrary term structures. Each of these three-term structures will subsequently be used to discount mortgage portfolio cash flows. Of course, the method

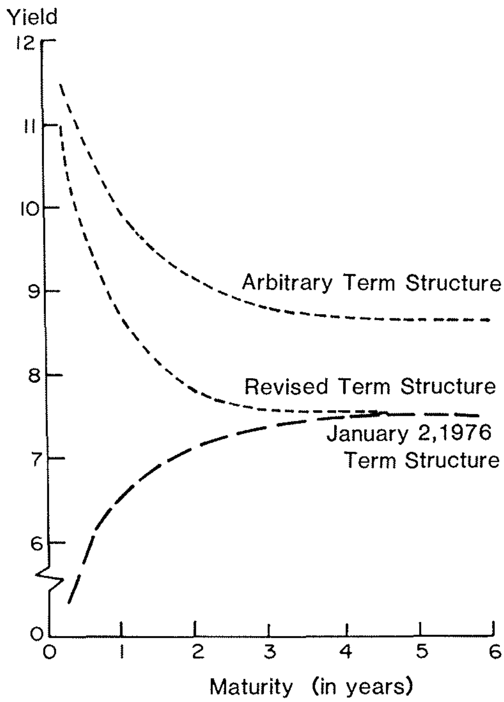

Fig. 14.1 Shapes of the "2 January 1976," "revised," and "arbitrary" term structures.

used for determining the term structure will restrict what may or may not be said concerning the resulting present values.

14.2.3 Step 3: Calculation of the Uncertainty Adjustment Term

As will be recalled from equation (2), the uncertainty adjustment term takes the form:

$$U = -\frac{COV\,(X_t\,,\,-R_{mt}^{-b})}{E(R_{mt}^{-b})},$$

where

R_{mt} = the one plus compound return on the market portfolio in period t
X_t = cash flow in period t
b = the level of proportional risk aversion.

To calculate a value for the term, it is necessary to make the following assumptions:

1. The level of proportional risk aversion (b) is set equal to one.

2. The returns on the market portfolio are serially uncorrelated, and the variance of market returns is constant.

3. The cash flows are not correlated with previous period's market returns.

4. The correlation between current period cash flows and market returns is not a function of time or interest rates.

These assumptions allow the covariance between the cash flow (X_t) and the product of market returns (R_{mt}) to be factored into the expectation of a product term and the covariance of a simpler term that can be calculated. Thus

$$- COV\,(X_t\,,\ - R_{mt}^{\ -1}) = E\left(\frac{1}{R_{mt-1}}\right) COV\left(X_t\,,\ \frac{1}{r_{mt}}\right) .$$

The whole adjustment term becomes

$$\frac{-COV(X_t,\ - R_{mt}^{\ -1})}{E(R_{mt}^{-1})} = \frac{E\left(\dfrac{1}{R_{mt-1}}\right) \ COV\left(X_t,\ \dfrac{1}{r_{mt}}\right)}{E\left(\dfrac{1}{R_{mt}}\right)}$$

$$= \frac{COV\left(X_t,\ \dfrac{1}{r_{mt}}\right)}{E\left(\dfrac{1}{r_{mt}}\right)}$$

$$(4) \qquad = E(r_{mt})\ COR\left(X_t,\ \frac{1}{r_{mt}}\right) \ Std\,(X_t)\ Std\left(\frac{1}{r_{mt}}\right) .$$

As suggested previously, the risk premium may be affected either through changes in aggregate risk tolerance or through changes in the joint distribution of mortgage cash flows and market returns. Setting $b = -1$ is theoretically justified (see Hakansson 1971*c*, Rubinstein 1973*b*), and in this context it essentially means that interest rate changes are assumed not to affect the risk premium through any influence on aggregate risk tolerance.

The second effect, namely the impact of interest rate change on the joint distribution of portfolio cash flows and market returns, can be seen quite clearly from equation (4). Equation (4) contains terms for the mean and standard deviation of the reciprocal of market returns, the standard deviation of cash flows, and the correlation between the cash flows and the reciprocal of market returns. Fortunately, it is possible to say something about how expected market returns vary with short-term interest rates, as will be shown subsequently.

It is also possible that interest rate changes affect the correlation of cash flows with market returns or the standard deviation of cash flows. For example, in the case where investor expectations concerning interest rates influence the distribution of cash flows through defaults, both the correlation and the standard deviation of cash flows would be affected. To measure the effect requires cross-sectional data, including a number of observations of portfolio cash flows at each of several interest rates. Because these data are not available, and since it is still useful to calculate the correlation coefficient, it is necessary to assume that the correlation is not a function of interest rates, and it can then be calculated from the available time-series observations. For the same reason, the standard deviations of both the cash flows ($Std[X_t]$) and the reciprocal of market returns ($Std\left[\frac{1}{r_{mt}}\right]$) are assumed to be invariant with respect to time and interest rates. It is useful to note that the data used to calculate both the correlations and standard deviations of cash flows are actual savings and loan mortgage portfolio data, which include the effect of defaults, and therefore the correlations and standard deviations may be thought of as including average default characteristics.

Thus, under the stated assumptions, interest rate changes do influence the risk premium, but only through an effect on the expected market return. It is difficult to estimate the size or nature of the bias that may be introduced by the assumptions above. When risk premiums are stated in terms of yield differentials, the suspicion is that they exhibit sizable changes. For example, the yield spread between long-term government bonds and mortgage rates ranged from 145 to 375 basis points over the time period studied. However, it is important to note that a "yield" calculation assumes that interest rates are level through time and that contract rates or coupon rates represent certain cash flows. Therefore, any adjustments in expected cash flows, in the term structure, or in strict risk premium changes are thrown together into the reported "yield." Since the analysis above does incorporate interest rate changes into the term structure, the expected cash flows, and the risk premium, it is difficult to estimate how much of the difference in observed yield spreads may be unaccounted for.

To incorporate the uncertainty adjustment term in the analysis, we must estimate values for each of the four terms of equation (4). Consider first the term $E(r_{mt})$, which is the expected one plus return on the market portfolio. In this application the return is assumed to be a function of the level of risk-free interest rates. Since the uncertainty adjustment is done quarterly, the appropriate relationship is between the quarterly return on the market and the three-month Treasury bill yield. This relationship was estimated for the twenty-five quarters from 1969-IV to 1975-IV. Table 14.2 gives the results:

Table 14.2 **Excess Market Return (r_x) versus the Risk-Free Rate (r_t)**

$r_x = .140 - 3.19\, r_t$	Durbin-Watson = 1.83
(1.99) (2.79)	Standard error = .084
	N = 25

Figures in parentheses are t-statistics.

Since $r_x = r_m - r_t$, the return on the market portfolio in any period can be determined from the risk-free rate for the period in the form:

$$\hat{r}_{mt} = .14 - 2.19\hat{r}_{ft} .$$

One way of obtaining a series of future three-month spot rates needed as the independent variable is to obtain from the term structure the liquidity-adjusted three-month-forward rates. (Assuming that the liquidity-adjusted three-month-forward rates are unbiased projection of future three-month spot rates.) These forward rates can be computed from the term structure by using the relationship

$$e^{(r_t)(1/4)}\, e^{(k_t)(1/4)} = e^{\frac{(k_{t+1})(t+1)}{4}},$$

which, when solved for r_t, gives

$$r_t = 4 \ln^{e} \frac{(k_t + 1)\left(\frac{t+1}{4}\right)}{e^{(k_t)(t/4)}},$$

where

r_t = rate on three-month loan to begin at time t
k_t = point yield on a risk-free security of maturity t.

Before the calculated forward rate (r_t) can be used as a spot rate projection, it must be adjusted by subtracting off a liquidity premium. The three-month liquidity premiums estimated by McCulloch (1975*b*) are shown in table 14.3.

Calculation of the correlation coefficient, $COR\left(x, \frac{1}{r_m}\right)$, requires cash flow data from the mortgage portfolio and a corresponding series of market returns. For this application, the data employed were six years of quarterly cash flows from a savings and loan association mortgage portfolio and market return data for the same period from the Center for Research in Security Prices (CRSP) tapes. Since the mortgage portfolio cash flows were separated into interest and principal payments, three separate correlation coefficients were calculated: ρ_1 is the correlation between the interest payments and the reciprocal of market returns, ρ_2 is

Table 14.3 **Estimates of Mean Liquidity Premium $p(m_1,m_2)$ (Standard Errors in Parentheses)**

m_1	m_2 = 3 Months	m_1	m_2 = 3 Months
1 month	0.09 (0.01)	2 years	0.22 (0.06)
2 months	0.14 (0.02)	3 years	0.22 (0.06)
3 months	0.17 (0.03)	5 years	0.22 (0.06)
6 months	0.21 (0.05)	10 years	0.22 (0.06)
9 months	0.22 (0.06)	20 years	0.22 (0.06)
1 year	0.22 (0.06)	30 years	0.22 (0.06)

Source: J. Huston McCulloch, "An Estimate of the Liquidity Premium" (dissertation submitted to Department of Economics, University of Chicago, June 1973), p. 57.
Note: Table 14.3 gives the premium on an m_2 maturity loan entered into m_1 years into the future.

the correlation between principal payments and the reciprocal of market returns, and ρ_3 is the correlation between the combined cash flows and the reciprocal of market returns. Table 14.4 shows the results of the calculations for a typical savings association's mortgage portfolio.

When examining the results, one should note that the sign of the uncertainty adjustment term in equation (4) is positive. This may seem counterintuitive, since the usual notion is to subtract from the expected cash flow a term that reflects the undesirability of highly correlated cash flows. However, because the correlation calculated here is between a cash flow and the reciprocal of the market return, the signs will be opposite those normally expected. Since the sign of the correlation may be either positive or negative, the sign of the whole adjustment term may be either positive or negative. Thus the sign of the correlation coefficient determines whether the mortgage portfolio's value is increased or decreased when held in a portfolio with other assets. For the example above, the combined cash flows have a positive correlation with the reciprocal of market returns (meaning the correlation between cash flows and regular market returns would be negative), and therefore the value of the mortgages is increased when held in conjunction with the market portfolio.

Table 14.4 **Correlations between the Reciprocal of Market Returns and Mortgage Portfolio Cash Flows**

ρ_1	ρ_2	ρ_3
−.22	.52	.45

Correlation coefficients greater than 0.36 are significant at the 90 percent level for $\frac{\rho^2}{k-1} \cdot \frac{n-k}{1-\rho^2} \sim F(1,20)$.

To complete the risk adjustment term, it is necessary to calculate the standard deviation of both the reciprocal of market returns and the portfolio cash flows. From the CRSP data, the standard deviation of the reciprocal of the one-plus market return was computed to be 0.0882 for the sample time period. For the portfolio cash flows, the quarterly standard deviation was calculated to be \$1,708.

It should be noted that for cash flows that are serially uncorrelated and have a growth rate with a stationary mean, the standard deviation can be computed as:

$$Std(X_t) = X_0\,[(\sigma_g^2 + \mu_y^2)^t]^{1/2},$$

where

X_t = cash flow at time t
g = 1 + rate of growth of cash flows
σ = standard deviation of the growth rate
μ = mean of the growth rate.

However, the cash flows from a mortgage portfolio do not fit this pattern because they do not grow over time. Since regular payments are fixed, any variation in the cash flows results from variation in the prepayment or foreclosure rates. It is generally assumed that, as a portfolio ages, the absolute number of foreclosures and prepayments decreases. Consequently, the standard deviation of cash flows also ought to decrease with time. However, because analysis of how the standard deviation of cash flows behaves as a function of foreclosure rates, prepayments, and time was not possible, the program assumes for this application that the standard deviation is constant over time.

14.2.4 Step 4: Calculation of the Present Value

Calculation of the present value requires that each of the previously described steps can be combined in the appropriate relationship according to equation (2). The computer program developed to do the calculations performs essentially five operations; four operations are related to the numerator, and one is related to the denominator. These operations are discussed relative to the mortgage portfolio application and may need to be altered slightly for use in other contexts. They are essentially a review of steps 1 through 3.

Certainty Equivalences

With respect to the numerator, the program first takes the individual mortgage data for each mortgage in the portfolio and calculates the future fixed monthly payments. Second, it calculates a series of future mortgage rates based on a specified term structure and uses them in conjunction with the prepayment algorithm to calculate the dollar amount of prepayments in each period. The program then applies an arbitrarily determined set of foreclosure rates to the portfolio. The foreclosure rates can be varied at will but are not included as a function of any other endogenous or exogenous parameters. At this point the program combines the fixed payments, prepayments, and foreclosures for each loan into a specified quarterly cash flow for the entire portfolio. As a fourth step, the parameters of the uncertainty adjustment term are computed and added to or subtracted from the expected cash flow for each period.

Discounts

The denominator calculation consists of taking the series of forward rates describing the term structure and forming the appropriate discount factor for each period. The discounted, uncertainty-adjusted cash flows are summed over the total number of periods, and the sum is the resulting present value.

Tables 14.5 and 14.6 show the results of applying the approach to eighteen San Francisco SMSA savings and loan association portfolios. The results in table 14.5 have been calculated assuming uncertainty, no

Table 14.5 **Value of Mortgage Portfolios under Uncertainty: No Foreclosures–Revised Term Structure**

Association	Book Value	Present Value 2 January 1976 Term Structure	% Present Value Exceeds Book Value	Present Value Revised Term Structure	% Change in Present Value
1	$ 759,921	$ 800,833	5.4	$ 795,506	−0.67
2	129,936	135,400	4.2	134,493	−0.67
3	685,939	720,440	5.0	715,673	−0.66
4	155,617	164,576	5.8	163,483	−0.66
5	424,979	443,282	4.3	440,346	−0.66
6	1,213,133	1,282,471	5.7	1,274,043	−0.66
7	333,094	354,720	6.5	352,385	−0.66
8	109,255	115,396	5.6	114,635	−0.66
9	457,858	483,613	5.6	480,400	−0.66
10	80,478	85,171	5.8	84,602	−0.67
11	155,200	163,823	5.6	162,731	−0.67
12	133,800	140,774	5.2	139,841	−0.66
13	50,124	53,656	7.0	53,302	−0.66
14	1,237,126	1,296,441	4.8	1,287,847	−0.66
15	26,851	27,674	3.1	27,488	−0.67
16	26,215	27,768	5.9	27,583	−0.66
17	38,210	40,253	5.3	39,985	−0.67
18	24,139	25,698	6.5	25,526	−0.67

foreclosures, and the term structure and revision discussed in step 2. Note that the portfolio values as of 2 January 1976 exceeded the book values in every case. This is because the rates represented by the 2 January 1976 term structure were lower than the average mortgage portfolio contract rates. Note also that, even when the portfolio cash flows were valued using the revised downward sloping term structure, the present values remained above book value. This occurred because the portfolio cash flows are received over a very long period of time; and, even though the term structure was shifted upward, the nature of the shift was such as to leave long-term rates relatively unchanged.

Table 14.6 shows the effect of discounting the expected mortgage cash flows by the arbitrarily designated term structure described previously. As seen most clearly from figure 14.1, the main difference between the revised and arbitrary term structures is that the arbitrary term structure exhibits higher long-term rates. Discounting by this term structure decreases the mortgage portfolio values by more than 6 percent in every case.

14.3 Program Operation and Sample Inputs and Outputs

This section contains more specific information and description concerning the computer program discussed previously. The first part of the section describes the real time program requests and the nature of the inputs requested. The second part contains a sample of the program output.

The program is currently set to run on a Dec 10 computer system; however, only slight modifications of certain control statements would be required to make it compatible with similar time-shared systems. The input steps correspond with the previous discussion on projecting cash flows, adjusting for uncertainty, and so forth. Table 14.7 contains a sample of the program statements and inputs and should be referred to in conjunction with the description of program steps.

After logging in and receiving the prompt character, the user types RUN FLO, and the computer responds with the first input requested.

14.3.1 Projecting Cash Flows

Items 1–12 allow for the calculation of cash flows for a savings and loan mortgage portfolio. These items could be modified to accept coupon bond data or any other data representing cash flows from financial instruments.

1. ENTER THE SAVINGS & LOAN DOCKET NO. AND THE NUMBER OF LOANS
The user must input two numbers separated by a comma. The first number is simply a number that identifies the run. This number will appear in the heading of the output. The second number indicates how

Table 14.6 **Value of Mortgage Portfolios under Uncertainty: No Foreclosures–Arbitrary Term Structure**

Association	Book Value	Present Value 2 January 1976 Term Structure	Present Value Arbitrary Term Structure	% Change in Present Value
1	$ 759,921	$ 800,833	$ 749,391	−6.42
2	129,936	135,400	126,777	−6.37
3	685,939	720,440	673,916	−6.46
4	155,617	164,576	154,015	−6.42
5	424,979	443,282	414,687	−6.45
6	1,213,133	1,282,471	1,199,008	−6.51
7	333,094	354,700	331,642	−6.51
8	109,255	115,396	107,923	−6.48
9	457,858	483,613	452,558	−6.42
10	80,478	85,171	79,738	−6.38
11	155,200	163,823	153,388	−6.37
12	133,800	140,774	131,672	−6.47
13	50,124	53,656	50,178	−6.48
14	1,237,136	1,296,441	1,213,063	−6.43
15	26,851	27,674	25,917	−6.35
16	26,215	27,768	25,986	−6.42
17	38,209	40,253	37,685	−6.38
18	24,139	25,698	24,060	−6.37

many separate loans will be entered for analysis. After the second number, the user enters a carriage return that tells the computer to go to the next step.

2. WHEN DO YOU WANT THE CASH FLOWS TO START PRINTING?

User enters a two-digit number (e.g., 76) that specifies the first year the output will begin to appear.

3. ARE THE LOAN DATA ON DISK?

If the user inputs "Y," meaning yes, the computer responds with statement 4. If the user inputs "N," the computer responds with statement 3A.

3A. WHAT WOULD YOU LIKE TO CALL THE FILE?

The user inputs a file name to which the loan data will be assigned.

3B. ENTER 1—THE LOAN AMOUNT

2—THE YEAR LOAN HAD THAT AMOUNT

3—THE CONTRACT RATE

4—THE NUMBER OF MONTHS LEFT TO MATURITY

5—THE AGE (IN MONTHS) OF LOAN

4. WHAT FILE ARE THEY IN?

The response to this statement must be the name of a data file in which the loan data have previously been put. Loan data must include for each loan the four items described in 3B.

Table 14.7 Sample Program Statements and Inputs

```
.RUN FLO

ENTER THE SAVINGS & LOAN DOCKET NO. AND THE # OF LOANS
2.6

WHEN DO YOU WANT THE CASH FLOWS TO START PRINTING?
76

ARE THE LOAN DATA ON DISK?
Y

WHAT FILE ARE THEY IN?
LOAN.A2

ARE THE PREPAYMENT DATA ON "DISK", TO BE "HAND" ENTERED, OR TO BE "CALCULATED"?
C

DO YOU WANT THEM CALCULATED FROM "ARBITRARY" MORTGAGE RATES, OR THE "TERM"
  STRUCTURE DETERMINED MORTGAGE RATES?
T

YOU NEED 5. PAST MORTGAGE RATES AND DISCOUNT POINTS. WHAT FILE ARE THEY IN?
MORT.PST

WHAT FILE ARE THE LIQUIDITY PREMIUMS IN?
L.DT

WHAT SPREAD DO YOU PROJECT FOR THE MORTGAGE RATE OVER THE TREASURY BILL RATE? (I.E.
  0.021)
.021

WHERE ARE THE EXPECTED FORECLOSURE RATES FOUND?
FORC.RAT

WHAT PERCENTAGE OF FORECLOSURES DO YOU EXPECT TO RECOVER?
100

HOW MANY MONTHS INTEREST WILL BE CHARGED FOR THE PREPAYMENT PENALTY (ENTER 100 FOR
  THE FHLMC METHOD)?
0

WHERE ARE THE K'S FOUND?
K.DT

DO YOU WANT TO ARBITRARILY SHIFT THE TERM STRUCTURE?
N

ENTER RR, RR-HAT, II, AND II-HAT
.015,.015,.0377,.1477

DO YOU WISH TO ADJUST FOR UNCERTAINTY?
Y

ENTER 1—THE CORRELATION BETWEEN CASH FLOWS AND RETURN ON THE MARKET,
      2—THE MEAN OF (1+ THE GROWTH RATE OF THE CASH FLOWS),
      3—THE STANDARD DEVIATION
.5,0,2000

DO YOU WANT THE MONTHLY CASH FLOWS PRINTED OUT?
N

DO YOU WANT DETAILED PREPAYMENTS, ETC.?
N

STOP

END OF EXECUTION
CPU TIME: 7.86 ELAPSED TIME: 1:28.45
EXIT
```

5. ARE THE PREPAYMENT DATA ON "DISK" TO BE "HAND" ENTERED OR TO BE "CALCULATED"?

The user may respond with "D," "H," or "C."

A "D" response means that the user has previously placed in a disk file the fraction of mortgages in the specified year that will be terminated. A typical data file would contain thirty decimal fractions similar to those that appear on the output for each loan under "has the following prepayment experience."

An "H" response means the user is prepared to input the decimal fractions at the terminal one at a time separated by commas.

A "C" response means the user wishes the computer to calculate the prepayment fractions based on the algorithm described in the text. The calculated fractions are printed out for each loan on the output. A "C" response involves statement 6.

6. DO YOU WANT THEM CALCULATED FROM "ARBITRARY MORTGAGE RATES" OR THE TERM STRUCTURE DETERMINED MORTGAGE RATES?

The user may respond with an "A" or a "T." The "A" response means the user wants to specify a set of future annual mortgage rates that will be used in the prepayment algorithm to calculate the prepayment fractions. This allows the user to uncouple the prepayment fractions from the term structure and have them calculated from arbitrarily determined future mortgage rates. After an "A" response, the computer will come back and ask for a file name in which the future mortgage rates have been stored. The "T" response instructs the computer to calculate the forward rates from the term structure and use the forward rates to calculate the future mortgage rates that are to be used to determine the prepayment fractions.

7. YOU NEED X PAST MORTGAGE RATES AND DISCOUNT POINTS. WHAT FILE ARE THEY IN?

The program has been written so that prepayment fractions would be applied to loans that were closed in previous years. To calculate the prepayment experience, the computer uses the prepayment algorithm and requires as inputs the past mortgage rates and discount points. These need to be placed in a data file.

8. WHAT FILE ARE THE LIQUIDITY PREMIUMS IN?

The user responds with the file name. The file may specify the McColloch liquidity premium or any arbitrarily chosen set.

9. WHAT SPREAD DO YOU PROJECT FOR THE MORTGAGE RATE OVER THE TREASURY BILL RATE? (I.E. 0.021)

The user enters a decimal fraction. For example, the 0.021 adds 2.1 percent to an 8.0 percent forward rate to give a 10.1 percent mortgage rate.

10. WHERE ARE THE EXPECTED FORECLOSURE RATES FOUND?

The user enters a file name. See, for example, FORC.RAT.

11. WHAT PERCENTAGE OF FORECLOSURES DO YOU EXPECT TO RECOVER?
The user enters a number such as 50 (50 percent) or 100 (100 percent), that specifies the percentage of the value of foreclosed property that will be realized.
12. HOW MANY MONTHS INTEREST WILL BE CHARGED FOR THE PREPAYMENT PENALTY (ENTER 100 FOR THE FHLMC METHOD)?
User enters a number like 6 (6 months) or 0 (no months interest penalty).

14.3.2 Specification of the Risk-Free Term Structure

Items 13–15 relate to the development of discount factors that make up the denominator of equation (2).
13. WHERE ARE THE K'S FOUND?
User enters a file name. See for example K.DT. The Ks are the quarterly rates (on annual basis) obtained from the McColloch program that specify the term structure.
14. DO YOU WANT TO ARBITRARILY SHIFT THE TERM STRUCTURE?
User responds with "Y" or "N." A "Y" response means the user does not wish to have the term structure shifted by using the computational algorithm but prefers to arbitrarily designate a new shifted term structure. The computer requests the file name of the shifted term structure.

An "N" response means the user wishes to use the algorithm for shifting the term structure, and the computer responds with statement 15.
15. ENTER RR, RR-HAT, II, AND II-HAT
The user enters four decimals separated by commas. RR and II are respectively the real rate and rate of inflation components that make up the nominal short term (three-month)rate. RR-HAT and II-HAT are the real and inflationary rate components that are projected to make up the short-term nominal rate in the shifted term structure.

14.3.3 The Uncertainty-Adjustment Term

Items 16 and 17 allow for the computation of the uncertainty-adjustment term as previously described.
16. DO YOU WISH TO ADJUST FOR UNCERTAINTY?
User enters "Y" or "N." An "N" response sets the uncertainty-adjustment term at zero. If the user responds in the affirmative, the computer proceeds to statement 17.
17. ENTER 1—THE CORRELATION BETWEEN CASH FLOWS AND RETURN ON THE MARKET
2—THE MEAN OF (1 + THE GROWTH RATE OF THE CASH FLOWS)
3—THE STANDARD DEVIATION
Two separate versions of the uncertainty-adjustment term are available in the program. Both use the correlation coefficient, which is entered as the first input. The first version for uncertainty adjustment uses the mean and standard deviation of the 1 + growth rates of cash flows. These data

would be entered, for example, as $\mu = 1.05$, $\sigma = .06$. The second version of uncertainty adjustment uses only the standard deviation of the cash flows themselves, in which case the mean is entered as zero and the standard deviation is entered as a whole number.

14.3.4 Calculation of the Present Value

In addition to the present value, which is automatically provided by the program, it is possible to specify more detailed output such as that described by items 18 and 19.

18. DO YOU WANT THE MONTHLY CASH FLOWS PRINTED OUT?

This option allows the user to obtain cash flow figures by month for the whole portfolio for interest, principal, principal repayments, and total cash flow.

19. DO YOU WANT DETAILED PREPAYMENTS, ETC?

An affirmative response for this option gives the user a detailed printout of cash flows by month for each individual loan in the portfolio.

14.3.5 Sample Output

An example of the abbreviated program output appears as table 14.8. The more detailed outputs are much longer, and therefore examples have not been included. The output first lists the assumptions that have been used in the particular run called for. Since many of the data files are extensive, only the file names are referred to in the assumptions. The output then details each of the loans in the portfolio and prints the termination fractions that are assumed for the loan prepayments. The program then prints a present value and book value based on the initially specified term structure. The output then presents a second present value calculation, both of which are based on the revised or arbitrary term structure as specified. The percentage change in present value is calculated as the concluding data item.

Table 14.8 Sample Output: Cash Flows for Savings and Loan Docket Number 2

THESE CASH FLOWS ASSUME:

1— 0.00% LATE PAYMENTS
2—100.0% RECOVERY OF FORECLOSURES
3— 0 MONTHS PREPAYMENT PENALTY
4—THE FOLLOWING FORECLOSURE EXPERIENCE

0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200

5—PREPAYMENTS WERE TERM STRUCTURE DETERMINED USING
FILE MORT.PST FOR PAST MORTGAGE RATES,
FILE L.DT FOR LIQUIDITY PREMIUMS,
AND A SPREAD OF 0.0210

6—TERM STRUCTURE SPECIFIED FROM FILE K.DT

7—SHIFTED TERM STRUCTURE USING PARAMETERS
RR = 0.0150 RR-HAT = 0.0150 II = 0.0377 II-HAT = 0.1477

8—UNCERTAINTY ADJUSTMENT ASSUMING
RHO = 0.500 MU = 0.000 SIGMA = 2000.000

LOAN # 1, AS OF '71 HAS A BALANCE OF 12898. AT A RATE OF .088
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0128	0.0239	0.0344	0.0444	0.0544	0.0643	0.0737	0.0830	0.0923	0.1015
0.1107	0.1198	0.1288	0.1377	0.1465	0.1553	0.1640	0.1727	0.1814	0.1899
0.1985	0.2070	0.2155	0.2239	0.2323	0.2407	0.2490	0.2573	0.2656	0.2739

LOAN # 2, AS OF '72 HAS A BALANCE OF 28722. AT A RATE OF .077
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0238	0.0342	0.0443	0.0544	0.0639	0.0734	0.0828	0.0920	0.1013
0.1104	0.1194	0.1284	0.1373	0.1461	0.1549	0.1636	0.1722	0.1808	0.1894
0.1979	0.2064	0.2149	0.2233	0.2317	0.2400	0.2483	0.2566	0.2649	0.2731

LOAN # 3, AS OF '73 HAS A BALANCE OF 26850. AT A RATE OF .074
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0237	0.0342	0.0444	0.0542	0.0638	0.0733	0.0827	0.0920	0.1013
0.1104	0.1194	0.1283	0.1372	0.1460	0.1548	0.1635	0.1721	0.1807	0.1893
0.1978	0.2063	0.2147	0.2231	0.2315	0.2399	0.2482	0.2564	0.2647	0.2729

LOAN # 4, AS OF '74 HAS A BALANCE OF 18275. AT A RATE OF .081
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0237	0.0343	0.0444	0.0542	0.0639	0.0734	0.0829	0.0922	0.1014
0.1105	0.1196	0.1285	0.1374	0.1462	0.1550	0.1637	0.1724	0.1810	0.1896
0.1981	0.2066	0.2151	0.2235	0.2319	0.2402	0.2486	0.2569	0.2651	0.2734

LOAN # 5, AS OF '75 HAS A BALANCE OF 20806. AT A RATE OF .094
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0239	0.0343	0.0445	0.0544	0.0641	0.0737	0.0832	0.0925	0.1018
0.1109	0.1200	0.1290	0.1379	0.1467	0.1555	0.1643	0.1730	0.1816	0.1902
0.1988	0.2073	0.2158	0.2242	0.2327	0.2410	0.2494	0.2577	0.2660	0.2743

Table 14.8 (continued)

LOAN # 6, AS OF '76 HAS A BALANCE OF 31368. AT A RATE OF .095
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0128	0.0238	0.0343	0.0445	0.0544	0.0641	0.0738	0.0832	0.0925	0.1018
0.1109	0.1200	0.1290	0.1379	0.1467	0.1556	0.1643	0.1730	0.1816	0.1903
0.1988	0.2073	0.2158	0.2243	0.2327	0.2411	0.2494	0.2578	0.2661	0.2743

PRESENT VALUE AS OF '76 = 133730.570
BOOK VALUE AS OF '76 = 122949.230

LOAN # 1, AS OF '71 HAS A BALANCE OF 12898. AT A RATE OF .088
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0128	0.0239	0.0344	0.0444	0.0544	0.0639	0.0738	0.0832	0.0924	0.1015
0.1107	0.1198	0.1288	0.1377	0.1465	0.1553	0.1640	0.1727	0.1814	0.1899
0.1985	0.2070	0.2155	0.2239	0.2323	0.2407	0.2490	0.2573	0.2656	0.2739

LOAN # 2, AS OF '72 HAS A BALANCE OF 28722. AT A RATE OF .077
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0238	0.0342	0.0443	0.0541	0.0640	0.0735	0.0828	0.0921	0.1013
0.1104	0.1194	0.1284	0.1373	0.1461	0.1549	0.1636	0.1722	0.1808	0.1894
0.1979	0.2064	0.2149	0.2233	0.2317	0.2400	0.2483	0.2566	0.2649	0.2731

LOAN # 3, AS OF '73 HAS A BALANCE OF 26850. AT A RATE OF .074
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0237	0.0342	0.0442	0.0543	0.0639	0.0734	0.0827	0.0920	0.1013
0.1104	0.1194	0.1283	0.1372	0.1460	0.1548	0.1635	0.1721	0.1807	0.1893
0.1978	0.2063	0.2147	0.2231	0.2315	0.2399	0.2482	0.2564	0.2647	0.2729

LOAN # 4, AS OF '74 HAS A BALANCE OF 18275. AT A RATE OF .081
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0237	0.0541	0.0445	0.0543	0.0640	0.0734	0.0829	0.0922	0.1014
0.1105	0.1196	0.1285	0.1374	0.1462	0.1550	0.1637	0.1724	0.1810	0.1896
0.1901	0.2066	0.2151	0.2235	0.2319	0.2402	0.2486	0.2569	0.2651	0.2734

LOAN # 5, AS OF '75 HAS A BALANCE OF 20806. AT A RATE OF .094
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0237	0.0344	0.0446	0.0544	0.0641	0.0737	0.0832	0.0925	0.1018
0.1109	0.1200	0.1290	0.1379	0.1467	0.1555	0.1643	0.1730	0.1816	0.1902
0.1988	0.2073	0.2158	0.2242	0.2327	0.2410	0.2494	0.2577	0.2660	0.2743

LOAN # 6, AS OF '76 HAS A BALANCE OF 31368. AT A RATE OF .095
IT HAS 360 MONTHS LEFT, AND HAS BEEN ON THE BOOKS 0 MONTHS
AND HAS THE FOLLOWING PREPAYMENT EXPERIENCE

0.0127	0.0239	0.0344	0.0445	0.0544	0.0642	0.0738	0.0832	0.0925	0.1018
0.1109	0.1200	0.1290	0.1379	0.1467	0.1556	0.1643	0.1730	0.1816	0.1903
0.1988	0.2073	0.2158	0.2243	0.2327	0.2411	0.2494	0.2578	0.2661	0.2743

PRESENT VALUE AS OF '76 = 132722.710
BOOK VALUE AS OF '76 = 122949.230
PERCENT CHANGE IN PV = −0.754