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12 Multivariate Analysis of Interest Rate Risk

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12.1 Introduction

Much of the literature on the adequacy of bank capital is concerned with such factors as default risk and faulty management. These factors are important, but they neglect the role that purely stochastic elements can play in affecting the capital of a well-managed bank, even if it is free of default risk. Because banks raise funds by issuing liabilities with maturities different from those of the assets they acquire, changes in the interest rates paid on these liabilities relative to the interest rates on assets will affect earnings and, hence, bank capital.

As long as assets and liabilities have different maturities, there is no way to avoid the risk of unanticipated movements in the interest rate spread.[1] One role of bank capital is to provide a buffer that absorbs fluctuations in bank earnings caused by unexpected changes in the term structure of interest rates. Thus, banks are self-insuring against term structure risk through their capital account. Interest rate risk represents a claim on bank capital just as does default risk. It becomes important, therefore, to assess the size of this claim relative to the size of the capital position. This paper presents an empirical measure of the size of interest rate risk.

The efficient markets hypothesis requires that forward interest rates equal expected interest rates where these expectations incorporate all available information—that is, where the expectations are rational. This requirement, assuming linear optimal forecasts, implies that forecast

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1. In principle, it is possible to have insurance against term structure risk. Such insurance is beyond the scope of this paper.

revisions (expectations) are serially uncorrelated. The magnitude of the forecast errors, however, depends on the unspecified information set.

Most previous work applying efficient market models to interest rates uses a single equation, or a single time series, to forecast interest rates. These models imply restrictions on the dynamic structure or the information set or both. In this paper we show that by explicitly modeling the dynamic interaction between short and long rates and by including the inflation rate in the information set we can substantially reduce the variance of the forecasts, that is, interest rate risk. The information used in the more complicated model is readily available and should be incorporated in a rational expectation.

Section 12.2 presents the vector stochastic model of the determination of interest rates and shows its relation to a single-series model. Section 12.3 gives the empirical estimates, and section 12.4 presents the postsample forecasts and compares them with forecasts from a random walk specification.

12.2 The Model

12.2.1 Efficient Markets Definition

A form of the efficient markets hypothesis contends that the j^{th}period forward interest rate ${}_tF_{t+j}$ should equal the expectation of the spot rate ${}_tr^*{}_{t+j}$ made at time t for period $t+j$ where the expectation incorporates all the currently available information Ω_t; that is,

$$(1) \qquad {}_tF_{t+j} \equiv {}_tr^*{}_{t+j} \equiv E(r_{t+j}|\Omega_t).$$

If the forward rate deviates from the expected future spot rate, then expected profits exist, and, except for transaction costs, the forward rate will be forced to the expected spot rate. Under these conditions it has been shown (e.g., see Samuelson 1972; Sargent 1972; or Shiller 1973) that the sequence of forward rates,

$$(2) \qquad {}_{t+i}F_{t+j} \equiv E(r_{t+j}|\Omega_t) = {}_tF_{t+j}\,, \qquad i>0$$

satisfies the definition of a martingale; that is, the changes in the forward rate or the forecast revisions are serially uncorrelated. To give the efficient markets definition some empirical content, it is necessary, of course, to assume a probability distribution that describes the spot rate.

12.2.2 Single-Series Models

Recently, univariate time-series techniques have been used to model the determination of individual interest rates, (e.g., see Brick and Thompson 1978 and Nelson and Schwert 1977). In this part we discuss the assumptions behind single-series models and their relation to the efficient markets assumptions. We show that because single-series models empirically do not satisfy the efficient market assumption, a multivariate approach is called for.

Assume that the first difference of the one-period spot rate r_t is a stationary stochastic process with a finite variance (which by itself is a fairly weak assumption), so that it may be given the autoregressive moving average (ARMA) representation,

$$a(z)r_t = b(z)u_t \ , \tag{3}$$

where u is a mean zero, serially uncorrelated, constant variance (white noise) error. The coefficients in equation (3) are polynomials in the lag operator z,

$$a(z) \equiv \sum_{i=0} a(i)z^i$$
$$b(z) \equiv \sum_{i=0} b(i)z^i \ ,$$

and the lag operator z is defined as $z^i x_t \equiv x_{t-i}$. Wold has shown that the prediction ${}_t\hat{r}_{t+j}$ at time t of the spot rate r_{t+j} that has minimum expected variance and that is a linear function of the observable single series r_t, $r_{t-1}, \ldots$ is given by

$${}_t\hat{r}_{t+j} = \sum_{i=0}^{\infty} w(i+j)u_{t-i} \ , \tag{4}$$

where

$$w(z) \equiv \frac{b(z)}{a(z)} \equiv \sum_{i=0}^{\infty} w(i) \ .$$

Thus, if it is assumed (a) that the available information Ω_t consists of the single time series, r, and (b) that the forecasts are linear, then the efficient markets hypothesis implies that the expectations of future spot rates should equal the forecasts from single-series ARMA models.

Brick and Thompson estimated equations for (the first difference of) seven federal and municipal interest rates of the form[2]

$$a(z)r_{k_t} = b(z)u_{k_t} \ . \qquad k = 1, 2, \ldots 7. \tag{5}$$

Although the errors from the single-series models are serially uncorrelated and they contain all the information in the single-series, they may be correlated with other information that would be used in a rational expectation.

Brick and Thompson cross-correlated the residuals from the single-series models to determine if there was additional information about the lead-lag structure contained in the other interest rate series. For example, assume the errors for two series—the short-rate u_s and the long-rate u_L—have the following relationship:

2. They found that a random walk representation was adequate for most of the series (Brick and Thompson 1978, p. 96).

(6) $$u_{L_t} = a_{Ls}(z)u_{s_t} + c_L(z)e_{l_t}$$

$$u_{s_t} = a_{sL}(z)u_{L_t} + c_s(z)e_{2_t},$$

where e_1 and e_2 are independent white-noise errors.[3] The cross-covariances

(7) $$E(u_{L_t}, u_{s_{t-i}}) = \lambda_{L,s_i} - \infty < i < \infty$$

will have some nonzero values and will be two-sided (i.e., $i \gtrless 0$) unless $a_{Ls}(z)$ or $a_{sL}(z)$ is identically equal zero. Brick and Thompson found significant sample cross-correlations, leading them to conclude that, "there was apparently a complex feedforward-feedback relationship [between the rates] rather than a simple leading or lagging relationship." But they contend that the relationship is not stable over time.[4]

12.2.3 A Vector Model

Brick and Thompson's results indicate that there is significant information in the other interest rate series so that expectations in an efficient market should incorporate this information. In other words, the assumption in the second section is too restrictive. If the cross-series information is stable, it can be incorporated in a more general vector ARMA representation. To derive a bivariate form of this model, substitute the definition of the errors from the single-series model, equation (5),

(8) $$u_{k_t} = \frac{a_k(z)}{b_k(z)} r_{k_t},$$

into the equation (6), which defines the relationship between the single-series errors and gives the bivariate stochastic process

(9) $$\frac{a_L(z)\, r_{L_t}}{b_L(z)} - \frac{a_{Ls}(z)a_s(z)\, r_{s_t}}{b_s(z)} = c_L(z)e_{L_t}$$

$$-\frac{a_{sL}(z)a_L(z)\, r_{L_t}}{b_L(z)} + \frac{a_s(z)\, r_{s_t}}{b_s(z)} = c_s(z)e_{s_t}.$$

Notice that both current and lagged long and short rates determine the current long rate and the current short rate.[5]

The variance of the forecasts from the bivariate model conditional on the information set that includes both series ($\Omega_t \equiv r_{L_{t-1}}, r_{L_{t-2}} \cdots,$ $r_{s_{t-1}}, r_{s_{t-2}}, \ldots$) is necessarily less than or equal to the forecast variance

3. The polynomial coefficients are restricted, since each series u_k is serially uncorrelated. See Granger and Newbold (1975).

4. Brick and Thompson (1978, p. 98 and pp. 101–2).

5. Sims (1972) has shown that distributed lag estimates of a single equation, for example, term structure models, from the bivariante system (2.3.2) cannot be interpreted as a causal or behavioral relation because the feedback has been ignored.

from the single-series models. The bivariate model includes the additional restriction that the cross-covariances

$$\lambda_{Ls_i} = E(e_{L_t}, e_{s_{t-i}}) = 0 \qquad -\infty < i < \infty$$

(as well as the autocovariances) are equal to zero. As a result the vector of forecast revisions,

$$[\,{}_t\hat{\underline{r}}_{t+j} - {}_{t-1}\hat{\underline{r}}_{t+j}\,], \qquad j \geq 0$$

where

$$\underline{r}_t \equiv \begin{bmatrix} r_{L_t} \\ r_{s_t} \end{bmatrix}$$

is serially uncorrelated.

The basic model used in this paper is a slight generalization of the bivariate model. We also included the first difference of the inflation rate, p, as an exogenous driving variable. The inflation rate was included in the information set because theory and previous empirical work suggest that it should be (see Modigliani and Shiller 1973) and because of its easy observability. The vector ARMA model is

$$A(z)\underline{r}_t = B(z)p_t + C(z)\underline{e}_t, \tag{10}$$

where $\underline{r}$ is the two-element column vector containing the first difference of the long rate r_L and the short rate r_s and e is a corresponding two-element white-noise error vector. The coefficients are matrix polynomials in the lag operator z. The first term in the autoregressive power series is normalized to an identity,

$$A(z) = I + A(1)z + \ldots,$$

so that (10) is a reduced form, and the moving average matrix polynomial $C(z)$ is diagonal so that each equation contains a single moving average error.[6] The first difference of the inflation rate is assumed to follow the independent ARMA stationary-stochastic process,

$$g(z)p_t = h(z)d_t, \tag{11}$$

where d is a white-noise error.

12.3 Empirical Estimates

12.3.1 Data and Preliminary Specification Tests

The data for the model estimation consist of monthly time-series observations on three variables—the long interest rate, which is Moody's BAA corporate bond rate, the short rate, which is the four- to six-month

6. The normalization involves no loss in generality if we allow the reduced-form error vector to be contemporaneously correlated, since it is still serially uncorrelated.

prime commercial paper rate, and the inflation rate, which is the seasonally adjusted annual growth rate of the consumer price index. The data come from the NBER data bank, and the period of observations is from 1953–3 (post-Korean War) to 1971–7 (just before the wage-price freeze).

The series were first-differenced and the first twenty-four sample autocorrelations were calculated for the entire period and for the sample split into pre- and post-1965 data. The autocorrelations tended to die out, indicating that the series were stationary. Brick and Thompson, however, found a significant (at the 95 percent confidence level) increase in the sample variance of their post-1965 data. Our series displayed a similar increase in the sample variance for the post-1965 data. In contrast to Brick and Thompson's results, however, the point estimates of all but one of the sample autocorrelations from the pre-1965 data fell within the confidence ban (two standard errors) of post-1965 estimates, and the majority were within one standard error. From this we concluded that the time structure was stationary but that the white-noise errors, u_k in equation (5), were heteroskedastic. If there was a one-time shift in the error variance, or if the model variance is bounded, and if the model can be correctly identified, then the final model parameter estimates are consistent but not asymptotically efficient.[7]

We also did a preliminary test of the causal structure specified in the vector model (10). Sims (1977)[8] suggested an exogeneity test based on the standard regression model

$$y = Xb + u \ , \tag{12}$$

where the hypothesis that X is strictly exogenous is the hypothesis that $E(u|X) = 0$. If exogeneity holds for this model with sample size up to $T + s$, then we can add to the right-hand side of (12) the variable Z, whose t th component is the $t + s$ th component of X, to get

$$y = Xb + Zc + u . \tag{13}$$

On the null hypothesis that (12) satisfies the assumptions of the Gauss-Markov theorem, (13) does also, with $c = 0$. Testing $c = 0$ by standard methods thus tests the null hypothesis of strict exogeneity of X in (12).[9]

To test the hypothesis that the inflation rate was exogenous in the long interest rate equation, we ran the autoregressive model

$$r_{L_t} = \sum_{i=1}^{12} b_{o_i} r_{L_{t-i}} + \sum_{i=0}^{12} b_{1_i} r_{s_{t-1}} + \sum_{i=0}^{12} b_{2_i} p_{t-i} \tag{14}$$

7. The vector model was estimated using a FIML technique (see Wall 1976) so that if the errors were homoskedastic the estimates would be asymptotically efficient.

8. Also see Sims (1972).

9. Sims (1977, p. 24).

$$+ \sum_{i=1}^{12} c_i \, p_{t+i} = u_t$$

and tested the null hypothesis that the coefficient vector $c = 0$. We then tested for exogeneity of the short rate in the long rate equation by replacing the led values of the (first difference of the) inflation rate with led values of the (first difference of the) short rate. Table 12.1 reports the F values for all combinations of the three variables. The critical $F_{(12,294)}$ value at the 5 percent level is approximately 1.79; the starred values are significant. The first row in table 12.1 indicates that the null hypothesis that short rates are exogenous in the long rate equation can be rejected, element (1,2); but the null hypothesis that the inflation rate is exogenous cannot be rejected, element (1,3). Row 2 presents a similar picture for the short rate. Row 3 indicates that we cannot reject the null hypothesis that the short interest rate is exogenous in the inflation equation, but we reject the null hypothesis that the long rate is exogenous. In short, table 12.1 supports the specification of the vector model (10) and (11). There is feedback between the two interest rates, but a unidirectional flow from the inflation rate to the long rate.

12.3.2 Estimation

Estimation of the vector model (10) is an iterative multistage procedure that is described in Granger and Newbold (1975), chapter 7. Briefly, the technique is to:

1. Fit single-series models to each endogenous variable using univariate techniques. Differencing may be necessary to obtain stationarity—for example, equation (5).
2. Calculate the cross-correlations between the single-series residuals and use them to identify the transfer functions between the residuals—for example, equation (6).
3. Identify the error structures—that is, the transfer function for the errors in the bivariate model.
4. Estimate the bivariate model—for example, equation (9).
5. Calculate the cross-covariances between the residuals from the bivariate model and the residuals from the single-series model for the

Table 12.1 Pseudo-Sims Test F Values

Dependent Variable	Coefficients on Future Values r_L	r_s	p
r_L	—	1.92*	0.45
r_s	2.62*	—	0.99
p	6.41*	0.49	—

*Value is significant.

exogenous variable and use these to identify the transfer function on the exogenous variable.

6. Estimate the complete model.

7. Check the adequacy of the representation and, if necessary, modify and reestimate it.

Our estimates of the single-series models for the interest rates show a fairly simple time structure, but they are definitely more complicated than the random walk, accepted by Brick and Thompson:

$$\underset{(.084)}{(1 - .31z} - \underset{(.067)}{.20z^6} + \underset{(.075)}{.20z^7})r_{L_t} = (1 + \underset{(.084)}{.37z})u_{l_t} \tag{15}$$

$$\hat{\sigma}^2_{u_1} = .0089$$

$$(1 - \underset{(.082)}{.30z^3} - \underset{(.070)}{.14z^{12}})r_{s_t} = (1 + \underset{(.067)}{.56z} + \underset{(.082)}{.29z^2})u_{2_t} \tag{16}$$

$$\hat{\sigma}^2_{u_2} = .0514$$

The autoregressive structure reflects complicated seasonal movements, and there are moving average errors whose effects persist for up to three months.

The cross-correlations of the single-series residuals given in table 12.2 indicate a significant relationship between the residuals of the short rate and the residuals of the long rate (column 1) at lags 1, 4, 8, and possibly 14; the asymptotic standard error of the cross-correlations is approximately .06. Somewhat to our surprise, however, the cross-correlations between the residuals of the long rate and the lagged residuals from the short rate showed no significant relationship. The cross-correlations in table 12.2 seem to indicate a recursive bivariate relationship in which lagged short rates and long rates plus an error process cause the short rate, but only lagged long rates plus an error process cause the long rate. Based on the Sims test and economic theory (intuition?) we decided to contradict the rule of parsimonious parameterization and allowed for feedback.

The parameter estimates for the complete model in rational distributed lag format are given in equations (17) and (18), with the summary statistics—parameter estimates and standard errors, and the covariance matrix of the estimated residuals—given in table 12.3.

$$a_{11}(z)r_{L_t} = \frac{a_{12}(z)_{r_{s_t}}}{a^*{}_{11}(z)} + b_1(z)p_t + c_1(z)e_{1_t} \tag{17}$$

$$(1 + .123z^5 + .129z^7 + .208z^8)r_{L_t} = \frac{.07z}{1 - .454z - .537z^2}\,r_{s_t}$$

Table 12.2 Cross-Correlation of the Single-Series Residuals

Lag	Correlations $\hat{u}_{r_S}, \hat{u}_{r_L}(z)$	Correlations $\hat{u}_{r_L}, \hat{u}_{r_S}(z)$
1	.1424	.0827
2	−.0590	.0005
3	−.0941	.0288
4	−.1707	.0201
5	−.0957	.0187
6	−.0045	−.0123
7	−.0642	−.0099
8	−.1743	.0467
9	−.0713	−.0636
10	−.0164	.0365
11	.0282	.0608
12	.0570	.0991
13	−.0108	−.0641
14	−.1278	.0928
15	−.0888	.0154
16	.0077	.0065
17	−.0773	.0911
18	.0377	−.0228
19	.0758	−.0535
20	.0404	.0144
21	−.0100	.0066
22	−.0147	−.0958
23	−.0626	−.0347
24	.0408	.0703

$$+ (.437 + .46z)p_t + (1 + .495z)e_{1_t}$$

(18) $$a_{22}(z)r_{s_t} = \frac{a_{21}(z)}{a^*_{22}(z)} r_{L_t} + b_2(z)p_t + c_2(z)e_{2_t}$$

$$(1 + .352^3 + .143z^7 + .149z^8)r_{st}$$

$$= \frac{.326z}{(1 - .071z + .214z^2 + .725z^3)} r_{L_t}$$

$$+ (.863 + .363z)p_t + (1 + .445z + .207z^2)e_{2_t} .$$

The final model exhibits a strong feedback relation between the interest rates, with the inflation rate exerting a driving influence on both. The more complicated vector model yields a substantial reduction in the residual variance (recall that all the series are first differences) of 12

Table 12.3 Final Estimates

β_{ij} (Lag)	Estimated Parameters	Standard Errors
Equation (17)		
$a_{11}(5)$	−.123172	.094460
$a_{11}(7)$	−.128601	.084685
$a_{11}(8)$	−.207969*	.094560
$a^*_{11}(1)$	−.453966*	.069506
$a^*_{11}(2)$	−.536739*	.068979
$a_{12}(1)$	.069834*	.013565
$b_1(0)$	.436824*	.092618
$b_1(1)$	.459561*	.097379
$c_1(1)$	.495315*	.093891
Equation (18)		
$a_{22}(3)$	.351979*	.086579
$a_{22}(7)$	−.143352	.087456
$a_{22}(8)$	−.148512	.094050
$a^*_{22}(1)$	−.070607	.089722
$a^*_{22}(2)$	.214318*	.086498
$a^*_{22}(3)$	.724717*	.094354
$a_{21}(1)$	.326328*	.093421
$b_2(0)$	.863319*	.099563
$b_2(1)$	.352870*	.106983
$c_2(1)$	.445171*	.089041
$c_2(2)$	.207059*	.075655

Residual Covariance Matrix

	r_L	r_s
r_L	$.5353E-02$	$.6856E-02$
r_s	$.6856E-02$	$.3065E-01$

percent for the long rate and 15 percent for the short rate from the single-series specification. It is also interesting from a theoretical point of view that the inclusion of the other rate and the inflation term makes most of the seasonal autoregressive parameters insignificant; the seasonal patterns seem to be explained by the complicated interaction of the interest rates plus the inflation term. We have not reestimated the model with the insignificant parameters deleted.

12.4 Predictions with the Model

We conducted several experiments to assess the predictive performance of the model. It was tested outside the sample period for the period August 1971 through October 1977—the last date for which we had collected data.

The test of the model is particularly severe because the postsample period contained the effects of an incredible number of large shocks to interest rates. Among these were price controls in their various phases, devaluation, and OPEC. To these shocks one must add the effects of monetary policy in 1974—probably the most restrictive monetary policy ever experienced in the United States—the deepest recession since the 1930s, and an unusually high and variable rate of inflation. It is asking a great deal of any model to predict the movements of interest rates during this six-year period.

Although the accuracy of the predictions deteriorated outside the sample period, the model performed very well over this difficult period. The mean squared error of the forecasts were calculated for one-period-ahead forecasts. The errors of the one-period forecast are plotted in figures 12.1 and 12.2 and are compared with the errors from a random

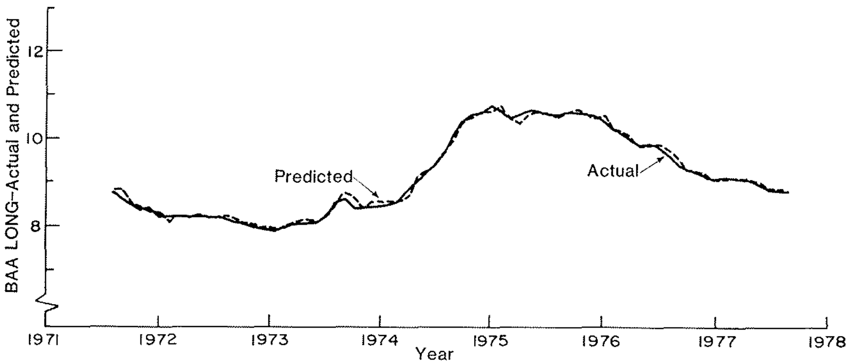

Fig. 12.1 Actual and predicted values for commercial paper rate.

Fig. 12.2 Actual and predicted values for the BAA rate.

walk specification in table 12.4. Table 12.4 contains the variance of the residuals in the sample period and the mean squared errors from the one-step-ahead model predictions and a random walk specification. Although the model errors, especially in the short rate, increase substantially for the postsample forecasts, they are still considerably better than the random walk specification.

The lower mean squared forecast errors for the vector model suggest that there is a stable time relationship among long- and short-term interest rates and the inflation rate. Including the inflation rate does not alter this conclusion, because if a random walk properly characterized the interest rate processes (the model accepted by Brick and Thompson) then the lagged inflation rates would contain no information.

Figure 12.1 plots actual and predicted values for the commercial paper rate. This figure suggests why the accuracy of prediction deteriorated outside the sample period. The period 1973–75 experienced unprecedented swings in short-term interest rates. During 1973, the commercial paper rate soared from 5.8 percent to 10.2 percent. The rate fell temporarily in 1974 before taking off for a high of 11.6 percent in August. By May of 1975 the rate was back down to 5.8 percent.

Figure 12.2 plots actual and predicted values for the BAA rate. As one would expect, the fluctuations in the long rate were much less than for short rates. The fluctuations were large by historical standards, however, and the model predicts them well.

As a final test of the model, a dynamic simulation was run over the period 1959–7 through 1977–10. For this entire period, r_s and r_L were generated endogenously, but the inflation rate was taken as its actual value in each month. The model proved to be remarkably stable over this eighteen-year period. Given the actual behavior of inflation, there was no tendency for the predicted levels of either interest rate to drift very far from their actual values. These results confirm the plausible assertion that the major problem in forecasting interest rates far into the future lies in forecasting the inflation rate. Given the inflation rate, the autoregressive processes generating interest rates appear to be highly stable.

Table 12.4 **Mean Squared Errors**

r_L	r_s	
.0053	.0307	Model, sample period
.0077	.2466	Model, postsample
.0164	.2993	Random walk, postsample

12.5 Conclusions

The interest rate risk faced by a bank depends on the distribution of the interest rate forecasts. The efficient markets criterion requires that all available information be used when forming expectations. We have shown that single-series models omit significant information that is readily available, implying that the forecast variance of these models overstates the true interest rate risk.