Volume Title: Risk and Capital Adequacy in Commercial Banks

Volume Author/Editor: Sherman J. Maisel, editor

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-50281-3 (cloth); 0-226-50282-1 (paper)

Volume URL: http://www.nber.org/books/mais81-1

Conference Date:

Publication Date: 1981

Chapter Title: Interest Rate Risk and Capital Adequacy for Traditional Banks and Financial Intermediaries

Chapter Author(s): J. Huston McCulloch

Chapter URL: http://www.nber.org/chapters/c13528

Chapter pages in book: (p. 223 - 248)

10 Interest Rate Risk and Capital Adequacy for Traditional Banks and Financial Intermediaries

J. Huston McCulloch

10.1 Introduction

In this study we investigate the interest rate risk confronting banks and thrift institutions. We do not deny that other types of risk are important, often more important than pure interest rate risk. However, the interest risk is particularly interesting in view of the traditional practice followed by financial intermediaries of "transforming maturities" by borrowing short and lending long. We quantify the value of insurance against this risk empirically, using a Paretian stable option pricing model.

These results can be applied in either of two ways. Currently, banks all pay a given premium to the insuring agency and are subjected to a more or less arbitrary set of regulations regarding capital structure and activities, intended to make them fairly safe from failure. Mayer (1965) has proposed a graduated deposit insurance plan, under which banks would be allowed (within reason) to take whatever capital position and risks they choose and in exchange would be required to pay a variable premium that covered the fair value of insurance for the risk category chosen. Given the riskiness of the bank's activities, the fair value of such insurance will decline as the bank's capital/asset ratio increases, because the more capital the bank has, the larger the share of any losses on the assets that will be borne by the bank's stockholders, rather than by the insuring agency. Therefore, for any given premium and riskiness of operations there will be some amount of capital that will be adequate to make the premium in question cover the fair value of insurance for the bank. Sharpe (chap. 8) has proposed that this be the criterion for deciding whether a bank's capital is "adequate," given the premium it pays and the

J. Huston McCulloch is associate professor of economics and finance at Ohio State University.

risks it takes. There is no essential difference between the two proposals, since they are just two ways of interpreting the same underlying relationship between permissible risk, adequate capital, and fair premium, which we here attempt to quantify.

10.2 Capital and the Division of Losses

To simplify the analysis, we will concentrate on a "traditional bank," which has demand or virtually demand liabilities and longer-term assets. We will assume that these assets are perfectly marketable with no transactions costs and that there is no risk of default or possibility of being called before maturity. At first we will assume that the bank's entire portfolio is invested in one type of asset and in one maturity. Later we will relax many of these assumptions.

Let A_0 be the initial value of the bank's assets and L_0 be the initial value of its liabilities, so that

(1) $$C = A_0 - L_0$$

is the initial economic value of its capital. Define

(2) $$q = C/A_0$$

as its initial capital/asset ratio, so that

(3) $$L_0 = A_0(1 - q).$$

Let $\tilde{A}$ be the random value of the bank's assets when it is next examined. If $\tilde{A}$ is greater than A_0, so that the bank has excess capital, we assume the bank will be allowed to distribute this excess capital to shareholders in order to return its capital/asset ratio to q. If it is less than A_0, the examiners will require the shareholders to restore the deficient capital. If they do not, the bank will be liquidated. The shareholders could restore the capital by putting up more money of their own, by selling new stock on the open market so as to dilute their own stock, or by following a policy of retaining some of the anticipated return on the assets as a buffer against unanticipated capital losses. As long as $\tilde{A}$ is above L_0, so that their stock has positive net worth, they will choose to replenish the capital by one of these means. However, if $\tilde{A}$ is below L_0 (that is, if $\tilde{A}/A_0$ is below $1 - q$), they will (abstracting from the value of the bank's charter and customer relationships) prefer to abandon the bank and let the insuring agency pay off the depositors, taking a loss of $L_0 - \tilde{A}$. Since the shareholders reap the unanticipated capital gains if $\tilde{A}$ is unusually high but do not take all the losses if $\tilde{A}$ is unusually low, the insuring agency would have to charge some positive insurance premium I to compensate itself for the risk of having to take up part of the losses.

This premium should be retained by the party guaranteeing the deposits even in years when there are no failures, since the risk cannot be expected to average out over banks in any given year. Insurance against loan default or against embezzlement may behave to a large extent like casualty insurance, in which the premiums can be expected to pay off the losses in each year, but insurance against interest rate risk is undiversifiable. In most years, no banks will fail from this cause, but occasionally—once every ten or one hundred years, depending on the risks the banks take—all the banks will be in trouble.

10.3 The Value of Deposit Insurance

Formally, deposit insurance is equivalent to an option on the bank's assets. Deposit insurance essentially gives the banking firm (construed to include both depositors and shareholders) a "put" option, entitling it to sell the bank's assets to the insurer at a prearranged "execution price," determined by the nominal value of the bank's liabilities. The liability/asset ratio, $1 - q$, may therefore be thought of as the execution price/current price ratio in a put option contract.

Robert Merton (1977*a*) has thus applied the well-known Black-Scholes option pricing formula to the problem of evaluating bank deposit insurance. However, this formula relies on the strong assumption that $\tilde{A}$ is log *normal.* The distribution of most prices, including the prices of interest-bearing securities, seems to be much too fat tailed or leptokurtic to be consistent with a simple normal or log normal distribution.

We therefore assume that the logarithm of $\tilde{A}$ is instead distributed according to the symmetric Paretian stable class of distributions, whose use in financial applications has been pioneered by Mandelbrot (1963), Fama and Roll (1968, 1971), and Roll (1970). If $\tilde{A}$ itself had a symmetric stable distribution, there would be a small but positive probability that the value of the assets would actually go negative. By making its logarithm symmetric stable, we eliminate this possibility.

Symmetric stable distributions are characterized by three parameters: the *characteristic exponent* α, which governs how fast the tails taper off, the *standard scale* c, which roughly equals the semi-interquartile range, and the mean.

The characteristic exponent may range between 0 and 2, though in financial applications it is ordinarily assumed to be between 1 and 2. When it equals 1, the Cauchy distribution results, and when it equals 2, the normal (Gaussian) distribution is obtained. Except in the limiting normal case, the variance is infinite, which is why we must use the standard scale in place of the standard deviation to characterize its spread. If we were to restrict ourselves to the normal distribution, we would greatly underestimate the probability of a sudden large change in the value of the bank's assets and hence would greatly underestimate the fair value of insurance.

Figure 10.1 shows how the bell-shaped probability density function of the symmetric stable distribution changes with α. In all three cases, the standard scale is chosen to be 1.0, so that the probability is roughly 0.5 that $\tilde{X}$ will lie between + 1 and − 1 on the horizontal axis. When α is less than 2.0, the curve has a higher mode, lower shoulders, and higher tails

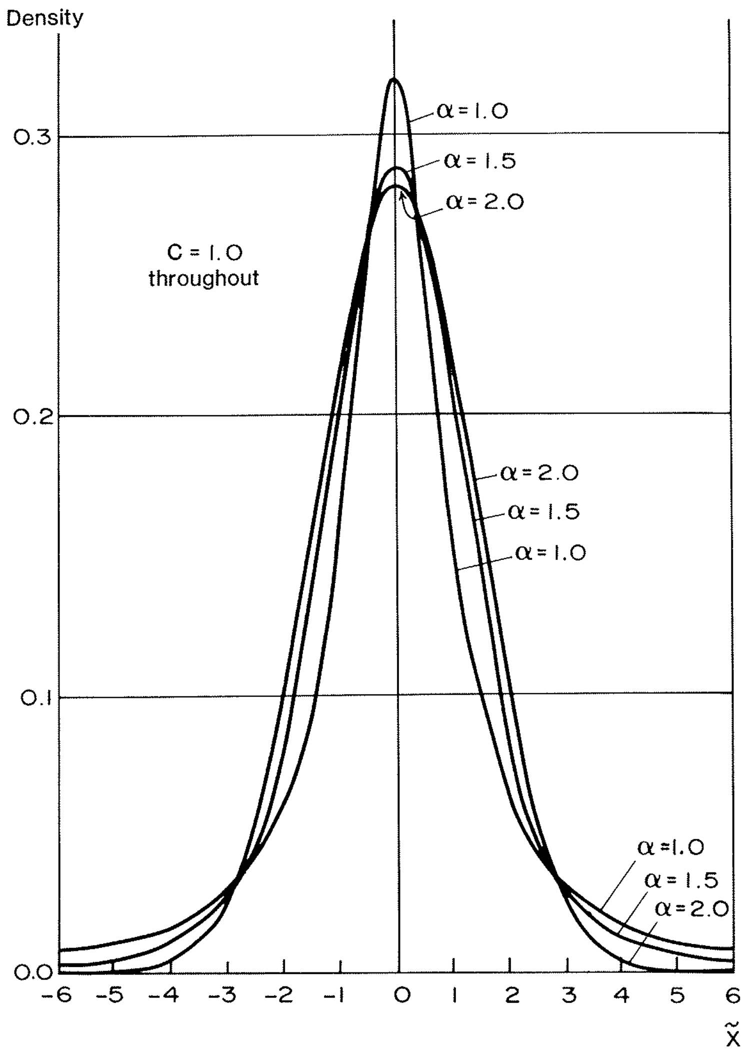

Fig. 10.1 The effect of the characteristic exponent.

than the familiar normal distribution. Note that the normal density virtually disappears by $\tilde{X} = 5.0$.

Figure 10.2 shows how the stable density function is affected by the scale parameter c. In each case, the characteristic exponent equals 1.5, the intermediate case of figure 10.1. If c is 2.0 instead of 1.0, the distribution has the same shape but is twice as spread out (and has only half as high a mode, in order to continue to integrate to unity). When c is 0.5 instead of 1.0, it is more squeezed together. The mode is now at 0.576, which is off scale in figure 10.2. Note that the tails are still perceptibly

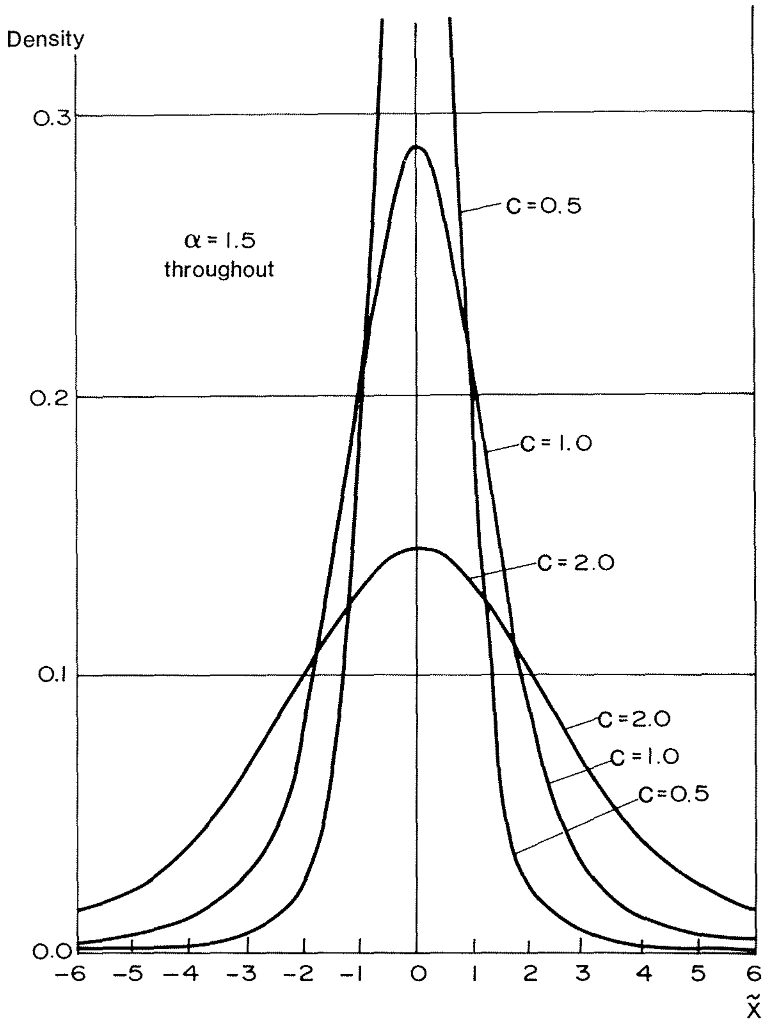

Fig. 10.2 The effect of the scale parameter.

above zero at $\tilde{X} = 6.0$, which corresponds to 12 standard scales. For the normal distribution ($\alpha = 2.0$), the standard scale exactly equals the standard deviation divided by $\sqrt{2}$, so that 54.7 percent of the probability density lies between $+c$ and $-c$.

In two papers, I have developed some of the properties of stable distributions in a continuous time context and have developed an option pricing formula based on the log symmetric stable distribution. In continuous time the sample path of a process governed by these distributions is full of discontinuities. Therefore, even if the regulators are constantly vigilant—essentially examining the bank continuously—there is some possibility that the value of the bank's assets may change so suddenly that net worth will be deeply negative before they have a chance to close it. If the bank's assets fell in value, but not by enough to wipe out its capital, the regulators could either insist on a capital injection or else raise the premium for insurance in keeping with the deteriorated capital ratio of the bank. Therefore, with continuous examination, the only way the bank could fail is through a single discontinuity large enough to wipe out the bank's capital; that is, a change in log $\tilde{A}$ greater than log $(A_0/L_0) = -\log(1-q)$. In McCulloch (1978*a*), it is shown that the annual rate of occurrence of such discontinuities is

$$\lambda = \frac{k_\alpha}{2}\left(\frac{c_0}{-\log(1-q)}\right)^\alpha, \tag{4}$$

where c_0 is the standard scale of log $\tilde{A}$ that accumulates in one year, and k_α is a constant (depending on α) that is tabulated in that article. For the sake of illustration, $k_1 = .6366$, $k_{1.5} = .3989$, and $k_2 = 0$. (Since $k_2 = 0$, a normal diffusion process never, with probability 1, has discontinuities.)

When a change in the value of the bank's assets sufficiently large to close the bank suddenly occurs (or is suddenly perceived to have already occurred), the change is likely to have been more than large enough to have wiped out the bank's capital, imposing some losses on the insuring agency. McCulloch (1978*b*) shows that the fair value of these losses is given by

$$\frac{I}{A_0} = \lambda H(1-q, \alpha)dt, \tag{5}$$

where $H(\,)$ is a function that is tabulated in that article, I is insurance, and dt is the life of the "option" (or examination period), which approaches 0. Thus the value per year of insurance, computed (as is conventional) as a fraction of liabilities, is

$$\pi = \frac{I}{L_0 dt} = \frac{\lambda H(1-q, \alpha)}{1-q}. \tag{6}$$

This equation, in conjunction with (4), tells us how to compute the fair value of insurance from q, α, and c_0.

10.4 The Data

To evaluate formulas (4) and (6), we need empirical values for c_0 and α for idealized default-free, perfectly marketable assets, of various maturities. United States Treasury securities are default-free, and very highly marketable, so we will use their empirical behavior as a proxy for the idealized assets we desire.

For each of several key maturities, we would like estimates of c_0 and α for three types of assets a bank or thrift institution might hold: single payment or "discount" instruments, "par bonds," by which we mean coupon bonds that happen to be selling exactly at par, and "amortized" loans that pay a constant amount each month with no balloon at final maturity. We cannot use raw Treasury securities price behavior to estimate these parameters directly, since the Treasury issues no marketable amortized securities, since its discount instruments (Treasury bills) have maturities only out to one year, and since most outstanding Treasury bonds are ordinarily selling substantially above or below par, depending on their coupon rates, and in any event do not coincide with the key maturities we would like to investigate.

However, we can bypass this problem by curve-fitting a "discount function" to empirical Treasury quotations and using this smooth function to construct a synthetic price for any type or maturity of security we care to define. For this purpose we employ my cubic-spline term structure curve-fitting program, developed while at NBER-West. This version of the progam is a modification of the tax-adjusted term structure program developed in 1973 for the United States Treasury and described in McCulloch (1975*a*). The new modifications account precisely for the fact that the coupons on Treasury bonds arrive semiannually. For the sake of convenience and computational speed, the version developed for the Treasury assumed a continuous stream of coupons, which slightly distorts the shape of the term structure in the maturities where bonds and bills interface.

The data base we have available consists of bid and asked quotations for United States Treasury securities for the last business day of each month, from the end of December 1946 to the end of May 1977, a total of 366 months. Since these dates represent the dividing line between two months, they could equally well be associated with either month. We will refer to them as representing the "beginning" of the subsequent month, that is, January 1947 to June 1977. In fact, the quotations are for actual delivery and payment early in these months, about two business days after the quotation date. The data for January 1947 to April 1966 were collected by Reuben A. Kessel from the quotation sheets of Salomon Brothers and were processed by Myron Scholes under the supervision of Merton H. Miller. The data for May 1966 to June 1975 were collected from Salomon Brothers quotation sheets by Joel Messina and obtained with the assistance of Jay Morrisson. The data for July 1975 to June 1977 were collected from *Wall Street Journal* composite dealer quotations by Krista Chinn under my direction.

All tax-exempt securities were rejected as being nonrepresentative of the market as a whole. (By the mid-1950s all but a handful of these had disappeared.) "Flower bonds" often sell at a price premium because they can be surrendered at par value in payment of estate taxes if they are owned by the decedent at the time of his death. It was not practical to omit all of them, because for many years they constituted most if not all of the long-term securities. The following compromise was adopted for flower bonds: Those that were selling below par; matured after 1982; and were selling within $4 per $100 of face value of the lowest-priced flower bond were excluded. Any that did not meet all three of these criteria were included.[1] No attempt was made to compensate for the price discount that existed on many bonds in the earlier part of the period because of their ineligibility for commercial bank purchase. This discount was greatly reduced after the Accord of March 1951, and most of these bonds became eligible by the mid-1950s. Except for the tax-exempt and selected flower bonds, almost all United States Treasury bills, notes, and bonds were included. It would have been desirable in principle to exclude callable bonds, but this was not practical, since they constitute almost all of the longer-term securities for many years. Therefore they were treated as maturing on the final maturity date if selling below par, and as maturing on the call date if selling above par.

For each time t, a discount function $\delta(t, m)$ was fit to the bid/asked mean prices of the securities available. This function gives the present value, as a fraction of unity, of a dollar to be repaid after maturity m, that is, at future time $t + m$. The price $a(t, m)$ of an amortized loan paying $1 per year in continuous installments for m years can be derived from the discount function as follows:

$$a(t, m) = \int_0^m \delta(t, \mu)\, d\mu. \tag{7}$$[2]

Finally, the par bond yield $y(t, m)$ that gives the coupon rate (as a fraction of unity) that would be necessary to make a continuous-coupon bond sell just at par can be derived as follows:

$$y(t, m) = \frac{1 - \delta(t, m)}{a(t, m)} . \tag{8}$$

1. See McCulloch (1975*a*, pp. 817–22) for further discussion of these estate tax bonds.
2. For technical reasons, $a(t,m)$ was not calculable to adequate precision for the three-month and six-month maturities. Therefore no results are reported for amortized loans of these maturities.

10.5 Parameter Estimates

At time t, a pure discount security with maturity Δt can be purchased for $\delta(t, \Delta t)$ dollars. At time $t + \Delta t$, it can be sold (i.e., cashed, since it has just matured) for \$1. The log price relative on this investment is

$$\log \frac{1}{\delta(t, \Delta t)} = -\log \delta(t, \Delta t). \tag{9}$$

The log price relative over the same holding period of duration Δt on an investment in a security with any longer maturity is random, since it depends on an unknown future price. However, its expected value will approximately equal (9), since investors have the option of investing in any maturity and will compare expected returns on all of them. In practice, the expected log price relative will be slightly higher the longer the maturity, because of a reliable, but small, liquidity premium (see McCulloch 1975*b*). Therefore the realized log price relative on a longer maturity security, minus expression (9) (i.e., plus $\log \delta(t, \Delta t)$), will equal the unanticipated return plus a small, relatively constant liquidity premium. For a pure discount security with initial maturity m, this difference is given by

$$\log \frac{\delta(t + \Delta t, m - \Delta t)}{\delta(t, m)} + \log \delta(t, \Delta t), \tag{10}$$

since, after Δt has elapsed, it can be sold for $\delta(t + \Delta t, m - \Delta t)$.

For coupon bonds and amortized loans, our problem is somewhat complicated by the payments that arrive during the holding period, that is, between t and $t + \Delta t$. We will therefore consider the log price relatives of modified bonds and amortized loans that have had these first few payments removed. Such a modified par bond can be purchased at time t for $1 - y(t, m)\, a(t, \Delta t)$. When it is sold at $t + \Delta t$, its principal is worth $\delta(t + \Delta t, m - \Delta t)$, and its coupons are worth $y(t, m)\, a(t + \Delta t, m - \Delta t)$, so the amount by which its log price relative exceeds that on a discount security with maturity Δt is given by

$$\log \frac{\delta(t + \Delta t, m - \Delta t) + y(t, m)\, a(t + \Delta t, m - \Delta t)}{1 - y(t, m)\, a(t, \Delta t)} \tag{11}$$
$$+ \log \delta(t, \Delta t).$$

Our modified amortized loan can be purchased at time t for $a(t, m) - a(t, \Delta t)$ and sold at $t + \Delta t$ for $a(t + \Delta t, m - \Delta t)$, so the relevant difference is given by

$$\log \frac{a(t + \Delta t, m - \Delta t)}{a(t, m) - a(t, \Delta t)} + \log \delta(t, \Delta t). \tag{12}$$

The monthly standard scale c_{1mo} and the characteristic exponent of the unanticipated returns were estimated for each of the three types of asset using the methods suggested by Fama and Roll (1971), for both the entire post-Accord period (roughly the past twenty-six years), and for the past ten years. Before the Accord of March 1951, the Federal Reserve System artificially stabilized interest rates on United States Treasury securities. Since that period is not representative of what may be expected in years to come, we did not make any use of that part of our data set. Table 10.1 shows our standard scale estimates, based on the .28 and .72 fractiles of the distributions of expressions (10)–(12). In table 10.1, these values are multiplied by 100 in order to express them as percentage unanticipated changes. These figures mean that in roughly half the months observed, the unanticipated capital gain or loss on the asset indicated was within the value indicated, as a percentage of the initial value of the asset. The standard scale for unanticipated changes accumulated over one year, c_0, can be computed from these values by equation (13), to be introduced presently.

In Figures 10.3 and 10.4 these standard scale estimates are plotted versus maturity on double logarithmic graph paper. We see that for

Table 10.1 Standard Scale of Unanticipated Logarithmic Returns (100 c_{1mo})

Maturity	Months	Discount Instruments	Par Bonds	Amortized Loans
		Past 26 Years (Post-Accord)		
3 mo	314	.0282	.0281	**
6 mo	314	.0698	.0695	**
1 yr	314	.141	.1378	.0686
2 yr	314	.280	.268	.1435
5 yr	314	.588	.523	.277
10 yr	314	.861	.698	.481
20 yr	274	1.29	.878	.648
30 yr	64	1.28	.849	.605
		Past 10 Years		
3 mo	120	.0462	.0459	**
6 mo	120	.1159	.1137	**
1 yr	120	.253	.247	.1241
2 yr	120	.492	.465	.264
5 yr	120	.866	.730	.492
10 yr	120	1.280	.945	.695
20 yr	85	2.91	1.437	1.132
30 yr	0	—	—	—
20 yr		1.887*	1.223*	.980*
30 yr		2.45*	1.422*	1.199*

*Extrapolated.
**See note 2.

maturities under one year, discount instruments and par bonds are virtually indistinguishable, but that the difference becomes important after five years or so. Amortized loans behave very much like par bonds with half the terminal maturity.

For the post-Accord period, we have a full 314 observations for maturities ten years or less, 274 observations for the twenty-year maturity, and only 64 observations for the thirty-year maturity. In figure 10.3, which is based on this period, we see that the twenty-year maturity standard scales lie almost on a straight line extrapolated from the five- and ten-year points, at least for discounts and par bonds. The thirty-year standard scales seem not to lie on the curve derived from the shorter maturities. I attribute this to the fact that we have only a highly curtailed sample for the thirty-year maturity, which apparently is not representative of the

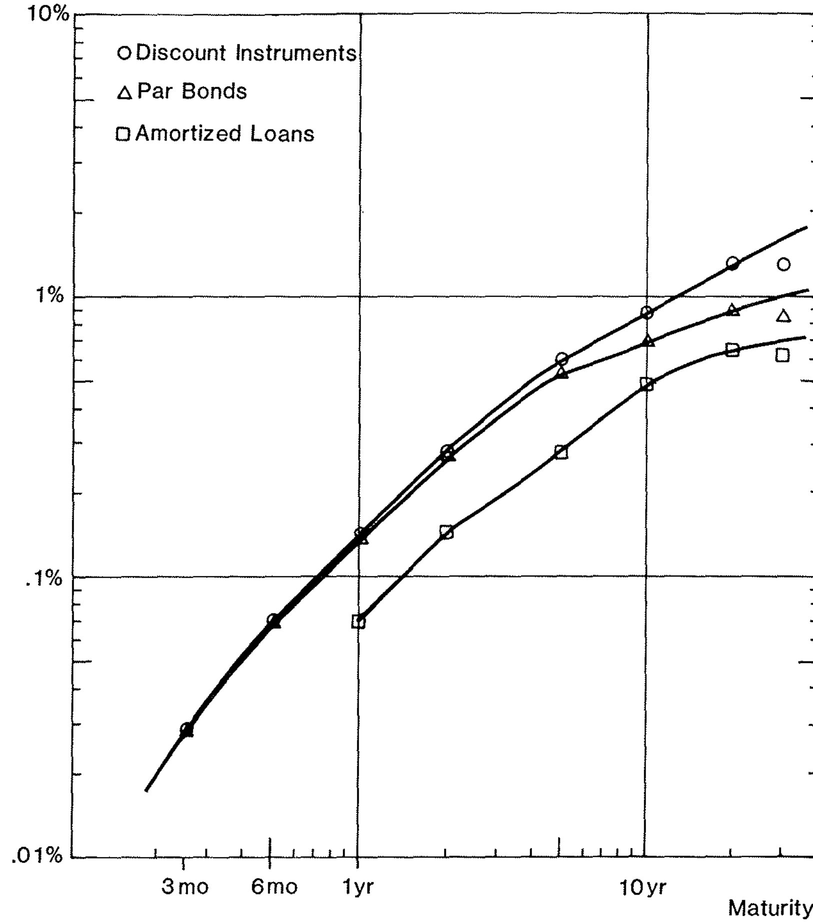

Fig. 10.3 Standard scale of unanticipated returns (post-Accord).

period as a whole. We therefore believe that if we had a full 314 observations for the thirty-year maturity, its standard scale would lie on a relatively straight line, or at least on a smooth curve, with the shorter maturities, when plotted on double logarithmic graph paper.

For the past ten years, the sample falls off even faster. We have a full 120 observations for maturities ten years or less, 85 observations (some 30 percent fewer) for the twenty-year maturity, and no observations at all for the thirty-year maturity. We see from figure 10.4 that here even the twenty-year maturity standard scales do not seem to line up with the shorter maturity standard scales. We attribute this to the fact that the twenty-year maturity sample size is more curtailed, in percentage terms, and therefore less representative, here than for the post-Accord period. If we had a full 120 observations for the twenty- and thirty-year matur-

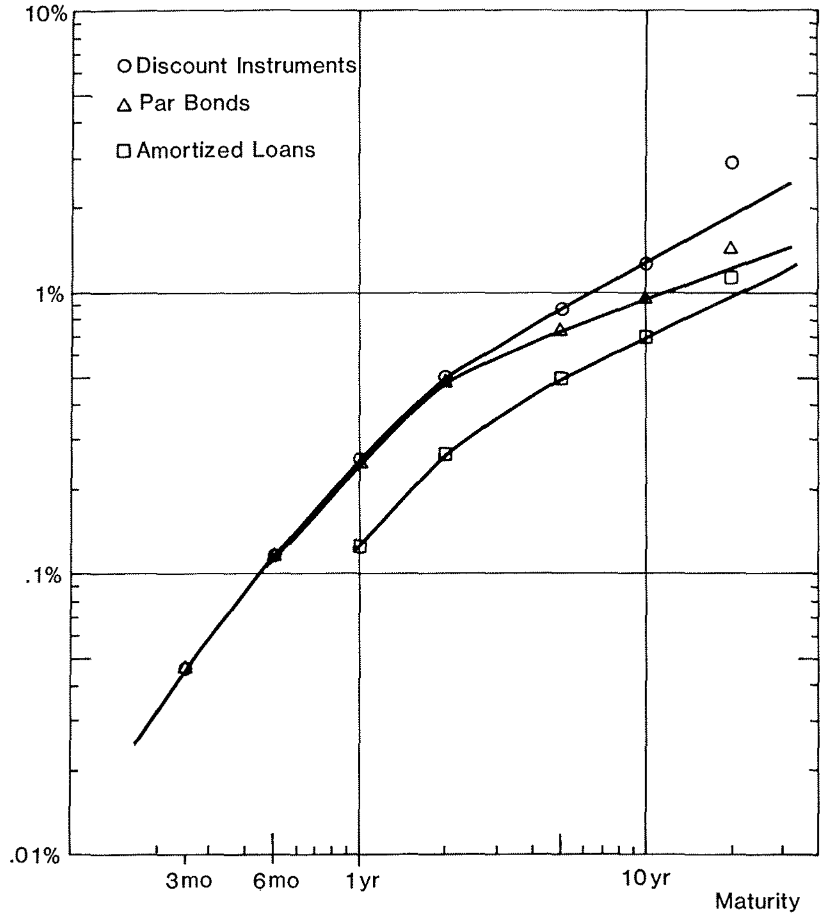

Fig. 10.4 Standard scale of unanticipated returns (past ten years).

ities, it seems reasonable to believe that they would lie on a straight line extrapolated from the five- and ten-year points. These maturities are of potential interest, especially in the case of thrift institutions, so it is important that we have reasonable estimates for these values. The straight-line double logarithmic extrapolations seems more reasonable than the curtailed-sample actual estimates, so we will use the extrapolated values to evaluate our formulas for these maturities. These extrapolated values are indicated with asterisks in table 10.1.

Table 10.2 shows estimates of the characteristic exponent, using the estimator $\hat{\alpha}_{.95}$ suggested by Fama and Roll. This estimator of α is computed from the .05 and .95 fractiles of the observed distribution. Except for the curtailed-sample thirty-year maturity, most of the estimates for the past twenty-six years lie in the range 1.24 to 1.39. The values for the past ten years are somewhat higher. Except for the curtailed-sample twenty-year maturity, most of the estimates lie in the range 1.35 to 1.52.

Examination of the raw data suggests that the volatility of interest rates has been relatively constant over the past ten years, whereas it has undergone significant changes over the past twenty-six years. The early 1950s and 1960s were periods of relatively low interest rate volatility, whereas the late 1950s and the most recent ten years were periods of relatively high interest rate volatility. When stable variables having the same characteristic exponent but different standard scales are mixed

Table 10.2 **Characteristic Exponent (α)**

Maturity	Months	Discount Instruments	Par Bonds	Amortized Bonds
		Past 26 Years (Post-Accord)		
3 mo	314	1.36	1.36	*
6 mo	314	1.43	1.44	*
1 yr	314	1.25	1.24	1.24
2 yr	314	1.27	1.29	1.31
5 yr	314	1.35	1.35	1.24
10 yr	314	1.28	1.28	1.35
20 yr	274	1.33	1.38	1.36
30 yr	64	1.49	1.62	1.59
		Past 10 Years		
3 mo	120	1.54	1.54	*
6 mo	120	1.52	1.51	*
1 yr	120	1.41	1.41	1.45
2 yr	120	1.47	1.47	1.52
5 yr	120	1.35	1.35	1.48
10 yr	120	1.39	1.32	1.36
20 yr	85	1.70	1.60	1.69
30 yr	0	—	—	—

*See note 2.

together, the resulting sample distribution tends to have fatter tails than do the distributions of the variables thus mixed. The estimator $\hat{\alpha}_{.95}$ would therefore tend to come out lower than the true characteristic exponent of the distributions generating the variables mixed. This may explain why the estimates for the past twenty-six years come out lower than for the past ten years: Unanticipated returns may have had a relatively constant characteristic exponent in the range 1.35 to 1.52, but a standard scale that has changed gradually over time, giving biased estimates of α when the whole period is pooled. It is therefore reasonable to assume that, in the immediate future, unanticipated returns will have roughly the standard scale of the past ten years, with α in the range 1.35 to 1.52. The probability of default and fair value of insurance are sharply declining functions of the characteristic exponent (except for extremely risky banks), so, in order not to overstate these values, we will use a relatively high value of α in this range, namely 1.5, in all our calculations below.

In the long-run future, we may expect the standard scale of unanticipated changes to either rise or fall from its level of the past ten years. We could probably approximate this compound distribution with a stable distribution using the standard scale and (apparently biased) characteristic exponent values estimated for the entire post-Accord period. However, this could actually give higher estimates of bank riskiness than would obtain using the past ten years' estimates, in spite of the lower standard scales, because of the powerful effect of the lower α we would be forced to use in order to capture the uncertainty in the standard scale. In the interest of keeping our estimates of the risk on the downward-biased side, we will therefore use the standard scale estimates for the past ten years (as extrapolated in the case of the twenty- and thirty-year maturities), in conjunction with an α value of 1.5 for all maturities and types of asset.[3]

10.6 Capital Adequacy: The Probability Criterion

Formula (4) above is based on the one-year standard scale c_0, while our monthly data have given us the one-month standard scale c_{1mo}. For stable distributions the standard scale accumulates according to the rule

$$c_0^\alpha = 12\, c_{1mo}{}^\alpha. \tag{13}$$

Therefore, in place of (4) we may use

$$\lambda = 6k_\alpha \left(\frac{c_{1mo}}{-\log(1-q)} \right)^\alpha \tag{14}$$

to evaluate the annual rate of failure. Recall that $k_{1.5} = .3989$.

Table 10.3 shows the probability per year of failure for various capital/asset ratios and eight key maturities, using formula (14). At the high

3. In footnotes 4 and 5 below, the effect of alternate values of c_0 and α will be illustrated.

Table 10.3 **Average Annual Rate of Failure as a Percentage (100λ)**

Capital/ Asset Ratio	Maturity Asset: 3 Months	6 Months	1 Years	2 Years	5 Years	10 Years	20 Years	30 Years
			Discount Instruments					
1%	2.35	9.38	30.2	82.0	191.4	344.	617.	911.
2	.827	3.29	10.60	28.7	67.1	120.7	217.	319.
4	.288	1.145	3.69	10.02	23.4	41.8	75.4	111.3
7	.1216	.483	1.558	4.23	9.87	17.73	31.8	46.7
10	.0695	.276	.890	2.42	5.64	10.14	18.14	26.9
15	.0363	.1443	.465	1.260	2.94	5.29	9.49	14.01
			Par Bonds					
1%	2.34	9.10	29.3	75.3	148.3	218.	321.	403.
2	.821	3.19	10.25	26.5	52.0	76.6	112.5	141.4
4	.285	1.112	3.57	9.20	18.11	26.7	39.3	49.2
7	.1206	.469	1.503	3.89	7.64	11.25	16.56	20.8
10	.0689	.269	.860	2.22	4.37	6.44	9.47	11.87
15	.0360	.1400	.450	1.158	2.28	3.36	4.94	6.18
30	.01108	.0431	.1382	.357	.702	1.031	1.520	1.904
60	.00269	.01046	.0335	.0866	.1704	.251	.369	.463
90	.000675	.00263	.00843	.0218	.0428	.0629	.0926	.1161
			Amortized Loans					
1%	*	*	10.38	32.2	82.1	137.6	231.	312.
2	*	*	3.64	11.31	28.8	48.2	80.9	109.4
4	*	*	1.268	3.94	10.03	16.80	28.2	38.1
7	*	*	.535	1.660	4.23	7.09	11.89	16.06
10	*	*	.306	.949	2.42	4.05	6.79	9.19
15	*	*	.1598	.495	1.262	2.12	3.54	4.79

*See note 2.

extreme, we see that a bank with 1 percent capital and a portfolio consisting of nothing but hypothetical thirty-year "Treasury bills" would have a failure rate of 911 percent per year. By this we mean that we would expect it to fail about 9.11 times per year, assuming it were reorganized immediately after each failure. At the low extreme, we see that a bank with 90 percent capital that rolls over from month to month in three-month par bonds has a failure rate of 0.000675 percent per year. That is, the expected time until its next failure is 1/0.00000675 = 148,000 years, or something like twenty-five times the length of recorded history.

Between these extremes, we find more down-to-earth values. The average capital/asset ratio for the domestic operations of United States commercial banks is about 7 percent. If such a bank held a portfolio of one-year maturity par bonds (which is actually somewhat less than the present average maturity of United States commercial bank assets), its mean rate of failure would be 1.503 percent per year, and its expected life to next failure would be about 66 years, so that it would probably outlive its present management and depositors.[4] If it were to reduce the maturity of its assets to six months, its failure rate would fall to 0.469 percent per year, giving it an expected life of some 213 years.

On the other hand, a thrift institution with 7 percent capital and a portfolio of twenty-year amortized mortgages has an annual rate of occurrence of insolvency of 11.89 percent. For such an institution, a year sufficiently bad to actually make its net worth (based on market value) negative would come about once every eight years. The years 1966 and 1969 may have been recent examples of this. The authorities did not actually close the savings and loans en masse in those years, but rather imposed ceilings on the rates they were allowed to pay when competing for deposits. The monopolylike profits these ceilings gave to the thrift institutions helped to restore their battered capital positions.

Throughout table 10.3 we see that the probability of failure increases sharply with asset maturity and declines dramatically with capital/asset ratio. There is therefore a considerable trade-off between capital and this measure of risk.

The data from table 10.3 for par bonds are plotted on double logarithmic graph paper in figure 10.5. A smooth curve has been fitted to the points for each capital/asset ratio. This diagram essentially depicts a relationship between three variables: maturity, capital/asset ratio, and probability of default. We can use this diagram to derive another way of looking at this relationship, as follows: for any probability (or, equivalently, expected longevity) that we are particularly interested in, say 1 percent (one hundred years) or 4 percent (twenty-five years), we can pick off a set of maturity/capital asset ratio points that give just the probability selected. Figure 10.6 shows the results of this procedure. This diagram essentially tells us how much capital is adequate to keep the probability of default below any given level chosen, as a function of maturity. For example, 9.0 percent capital would be necessary to reduce the probability of default to 1 percent for a bank with a portfolio of one-year par bonds. A mere 2.0 percent capital would be adequate for this bank if we were content to let it fail once every ten years, but 34 percent capital would be necessary if we insisted that it fail only once in a millennium. Similar diagrams could be constructed for discount instruments or amortized loans.

10.7 Capital Adequacy: Fair Insurance Criterion

Table 10.4 shows the annual value of deposit insurance, expressed as a percentage of liabilities. At the upper extreme we see that our bank with

4. Using the same ten-year c_{1mo} of .00247 and the actual point estimate of α of 1.41, this failure rate would instead be 2.30 percent. With the twenty-six-year c_{1mo} (.001378) and α (1.25) it is 2.25 percent.

1 percent capital and thirty-year Treasury bills would have to pay the FDIC at the rate of 15.4 percent of its liabilities per year to compensate the FDIC for the risk it would run by insuring it. Again, the lower extreme is given by a bank with ninety percent capital and a portfolio of three-month par bonds. This bank would have to pay only a sum equal to 0.000444 percent of its liabilities per year (i.e., 0.0444 basis points) to fully compensate the FDIC.

Our bank with 7 percent capital and one-year par bonds should be paying a premium of 13.9 basis points.[5] The fair value of insurance

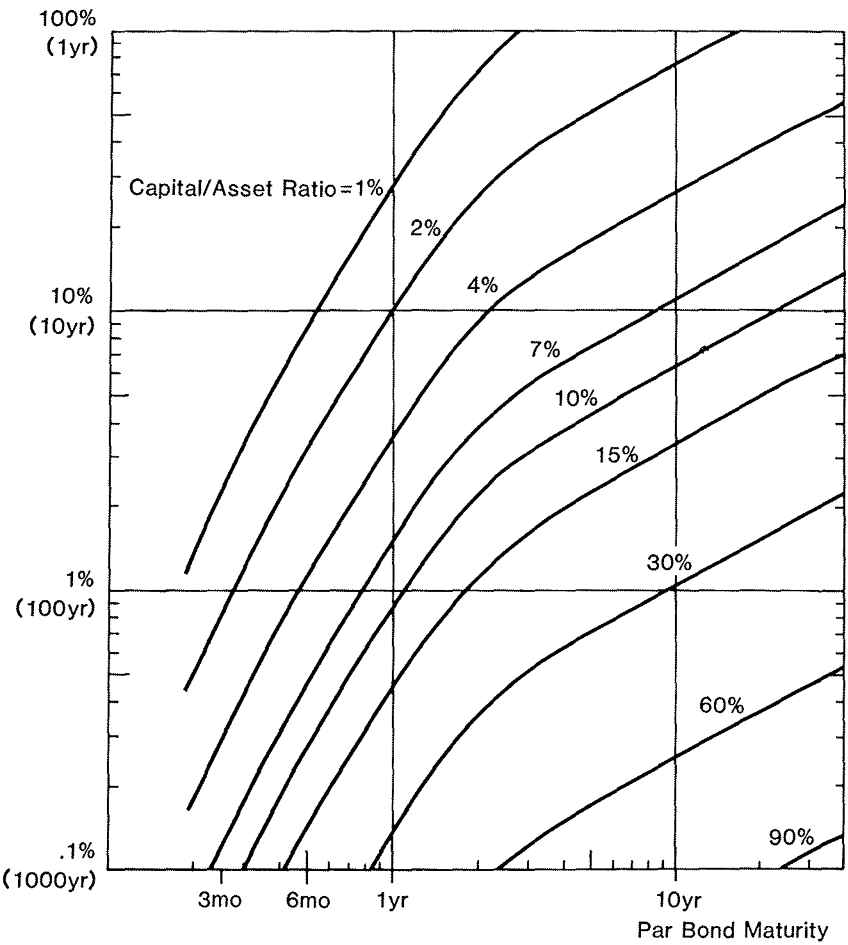

Fig. 10.5 Average rate of failure (par bonds).

5. With the same c_{1mo} but $\alpha = 1.41$, this value becomes 23.4 basis points. With $c_{1mo} = .001378$ and $\alpha = 1.25$, it becomes 27.3 basis points. Thus our estimates are, if anything, definitely on the low side. Compare footnote 4.

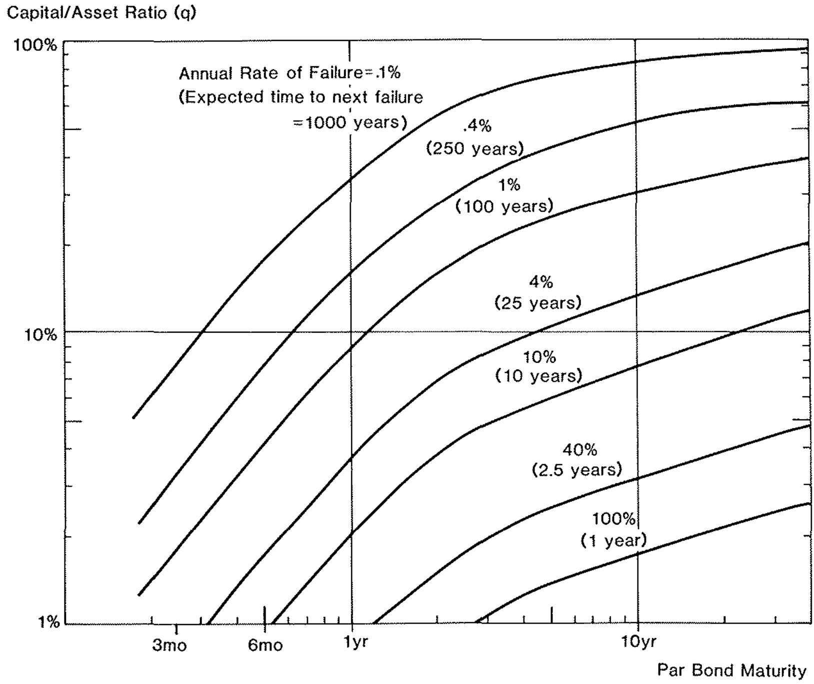

Fig. 10.6 Capital adequacy: probability criterion.

increases sharply with asset maturity in table 10.4. The hypothetical thrift institution discussed in the previous section, with 7 percent capital and a portfolio of twenty-year mortgages, imposes a liability worth 110 basis points on its insuring agency. This high a premium would make a substantial dent in the gross return it makes on its assets. If it were actually charged this premium, it would quickly try to change the structure of its balance sheet. (In practice, United States thrift institutions have an averge capital ratio more like 6 percent which would make the fair premium even higher. On the other hand, many mortgages are paid off early, making their effective maturity considerably less than their nominal maturity.)

The data for par bonds from table 10.4 have been plotted in figure 10.7. This graph can be used to find combinations of maturity and capital that generate any particular insurance value we might be interested in. This derived relationship is shown in figure 10.8.

From Figure 10.8 we see that 14 percent capital would be necessary to make the actual premium of 1/12 percent adequate for a bank whose assets consist of one-year bonds. In order for 7 percent capital to be adequate, its assets could have no more than 0.73 year (8.8 month) maturities.

Table 10.4 **Annual Value of Insurance as a Percentage of Liabilities (100 π)**

Capital Asset Ratio	Asset Maturity							
	3 Months	6 Months	1 Year	2 Years	5 Years	10 Years	20 Years	30 Years
				Discount Instruments				
1%	.0397	.158	.510	1.39	3.23	5.81	10.4	15.4
2%	.0262	.104	.336	.909	2.13	3.82	6.87	10.1
4%	.0167	.0665	.214	.582	1.36	2.43	4.38	6.46
7%	.0113	.0448	.144	.392	.915	1.64	2.95	4.33
10%	.00860	.0341	.110	.299	.698	1.25	2.24	3.33
15%	.00616	.0245	.0789	.214	.499	.898	1.61	2.38
				Par Bonds				
1%	.0395	.154	.495	1.27	2.51	3.68	5.42	6.81
2%	.0260	.101	.325	.839	1.65	2.43	3.56	4.48
4%	.0165	.0645	.207	.534	1.05	1.55	2.28	2.86
7%	.0112	.0435	.139	.361	.709	1.04	1.54	1.93
10%	.00852	.0333	.106	.275	.540	.796	1.17	1.47
15%	.00611	.0238	.0764	.196	.387	.570	.838	1.05
30%	.00314	.0122	.0392	.101	.199	.292	.431	.540
60%	.00125	.00487	.0156	.0403	.0794	.117	.172	.216
90%	.000444	.00173	.00554	.0143	.0281	.0413	.0608	.0763
				Amortized Loans				
1%	*	*	.175	.544	1.39	2.33	3.90	5.27
2%	*	*	.115	.358	.912	1.53	2.56	3.47
4%	*	*	.0736	.229	.582	.975	1.64	2.21
7%	*	*	.0496	.154	.392	.658	1.10	1.49
10%	*	*	.0378	.117	.299	.501	.840	1.14
15%	*	*	.0271	.0840	.214	.360	.601	.813

*See note 2.

In figure 10.8 we also include a line for a premium of 1/48 percent. This value is of interest, since in recent years the FDIC has rebated approximately half of the official premium to the banks, making the total premium more nearly 1/24 percent, and this reduced premium has to cover many other types of risk beside pure interest rate risk. If these other types of risk use up half of the reduced premium, that leaves only 1/48 percent (2.08 basis points) to cover interest rate risk. We see that asset maturities would have to be reduced to about 0.34 year (4.1 months) to make 7 percent capital adequate with this low an effective premium. One-year assets would require 51 percent capital.

Using the rival normal assumption, Merton (1977*a*, pp. 10–11) estimates the fair value of insurance for a bank with 10 percent capital and a portfolio of long-term United States government bonds as being in the neighborhood of 6 basis points (setting Merton's τ equal to 0.003), if the bank is annually inspected. We see from table 10.4 that with 10 percent capital and twenty-year par bonds, the value of the risk is actually more like 117 basis points per year. As the bank becomes safer, so that we are concerned with events even farther out on the tail of the distribution, the difference becomes still more striking. With 15 percent capital and twenty-year par bonds, we estimate 83.8 basis points, while Merton's estimate drops below 1 basis point. Thus, the normal assumption leads to substantial underestimation of the value of bank insurance.

10.8 Mixed-Maturity Portfolios

The fair premium formula for a bank with a given capital/asset ratio is proportional to c_0^{α}, where c_0 is the annualized standard scale of the

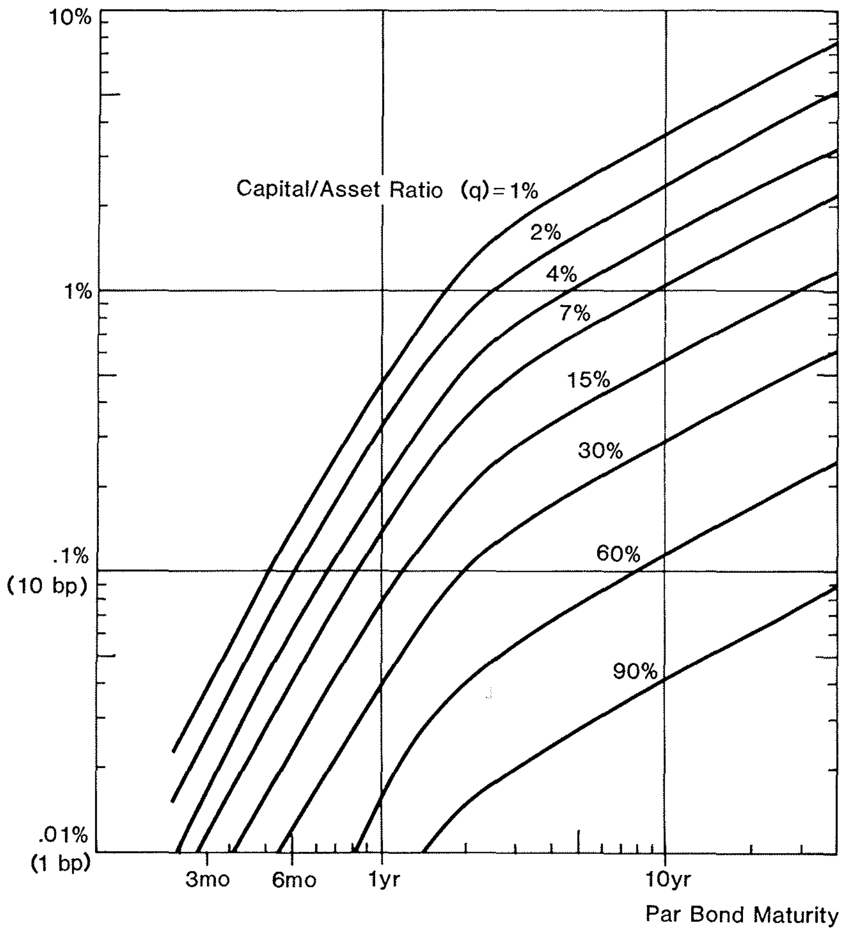

Fig. 10.7 Annual value of insurance as a percentage of liabilities (par bonds).

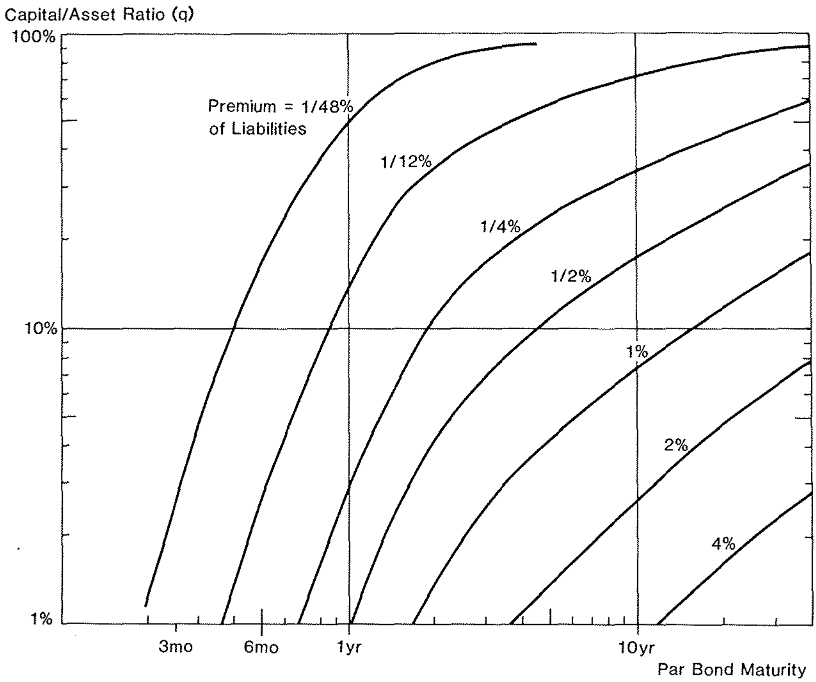

Fig. 10.8 Capital adequacy: fair insurance criterion.

unanticipated change in the logarithm of the value of the bank's assets, assumed thus far to be of one type. If the bank has a mixed portfolio of assets, we must instead base our calculation in the variability of the mixed portfolio. While the *product* of n log stable variables with the same α is log stable, their *sum* is not precisely log stable. Nevertheless, the sum is *approximately* log stable. I will therefore treat the mixed portfolio as if it really were log stable, with standard scale c_p.

Consider a bank with n types of asset, each of whose returns are log stable, and demand liabilities with fixed value. Asset i, which by itself has annualized standard scale c_i, forms fraction θ_i of the bank's assets, where

$$(15) \qquad \sum_{i=1}^{n} \theta_i = 1.$$

The effective standard scale of the approximately log stable mixed portfolio depends in a complicated way on the correlation of unanticipated returns for the different assets. Two cases are mathematically tractable: that of zero correlation, in which case we would have

$$(16) \qquad c_p^\alpha = \sum_{i=1}^{n} \theta_i^\alpha c_i^\alpha ,$$

and that of perfect positive correlation, in which case we would have

$$(17) \qquad c_p = \sum_{i=1}^{n} \theta_i c_i .$$

In actual fact, interest rate movements are highly (though by no means perfectly) correlated across maturities, which means that the unanticipated returns for different maturities will also be highly and positively correlated. We therefore regard (17) as a much better approximation to the truth than (16).

Therefore, either equation (17) can be used directly for a bank with mixed assets, or else a "pure" fair premium π_i can be found from table 10.4 or figure 10.7, and these pure premiums mixed to obtain a premium π_p for the entire portfolio, by using the formula

$$\pi_p = \left(\sum_{i=1}^{n} \theta_i \, \pi_i^{1/\alpha}\right)^{\alpha} . \tag{18}$$

For example, suppose a bank with 7 percent capital held 90 percent of its assets in three-month Treasury bills (which would require a premium of 1.13 basis points by themselves) and 10 percent of its assets in ten-year bonds selling near par (requiring 104 basis points by themselves). The composite premium required for pure interest rate risk would then be

$$\pi_p = (.9(1.13)^{1/1.5} + .1(104)^{1/1.5})^{1.5} = 5.69 \text{ basis points.} \tag{19}$$

Interestingly, this is less than 10 percent of the 104 basis point premium required for a pure ten-year bond portfolio, because of the nonlinear form of (18).

Formula (18) has an important implication for reserve policy. Consider a bank that holds fraction r of its assets in the form of cash reserves and the remainder in a portfolio that by itself would require premium π. Formula (18) then implies that its composite premium should be

$$\begin{aligned} \pi_p &= (r \cdot 0 + (1 - r)\pi^{1/\alpha})^{\alpha} \\ &= (1 - r)^{\alpha}\pi. \end{aligned} \tag{20}$$

Differentiating this formula with respect to r yields

$$\frac{\partial \pi_p}{\partial r} = -\alpha(1 - r)^{\alpha - 1}\pi , \tag{21}$$

so that cash reserves reduce the bank's fair premium, but by an amount that diminishes as r increases. The maximum reduction therefore occurs at $r = 0$:

$$\left.\frac{\partial \pi_p}{\partial r}\right|_{r=0} = -\alpha\pi. \tag{22}$$

Let i be the expected return on the bank's risky assets. If cash reserves pay no interest, this is essentially the opportunity cost to the bank of holding reserves. As long as i is greater than $\alpha\pi$, as it almost surely would be, the bank only stands to lose by holding reserves, even the first few dollars of reserves that have the greatest impact on its fair premium. With a fair premium, the only reason the bank would voluntarily hold zero-interest cash would be to minimize transaction costs (which we have assumed away in this paper). The size of the inventory it would hold for this purpose could be modeled along the lines of the Baumol or Miller and Orr cash balance models. Zero-interest cash reserves therefore seem to be a relatively expensive way of providing bank safety. As long as banks pay a fair premium, coaxing them into short maturity assets, there seems to be no safety-related reason to require them to hold any cash reserves at all.

10.9 Liability Management

In this paper we have dealt only with a "traditional bank" that has only demand or virtually demand liabilities. In recent years, banks have relied increasingly on longer-term liabilities or certificates of deposit to partially hedge their long-term assets. This is a desirable development that would greatly reduce the riskiness of banks. If a bank perfectly matched its asset and liability maturities, it would have no interest rate risk at all. With a little capital, it would have the flexibility of not having to match perfectly and could still maintain zero interest rate risk, provided each contracted disbursement was preceded by contracted receipts. However, bank capital is limited, so the match would have to be relatively close.

If all intermediaries tried to match maturities in this manner, they would have to juggle the term structure of interest rates until savers and borrowers were coaxed into the same maturities. Elsewhere I have argued that this matching would improve the efficiency of the intertemporal economy. I argue that the traditional practice of mismatching maturities (which I call "misintermediation") actually disequilibrates macroeconomic activity.

We have made no attempt in this paper to deal with the difficult problem of how the fair premium should be calculated for a bank with mixed asset and liability maturities. Average duration is of only a little help, since the asset and liability payment profiles could have very different shapes and still represent the same average duration. If all the assets have one maturity and the liabilities have another maturity, the bank is roughly equivalent to one with demand liabilities and assets with maturity equal to the difference. Thus a thrift institution with twenty-year assets and one-year liabilities is roughly equivalent to one with nineteen-year assets and passbook liabilities.

10.10 Indexed and Variable-Rate Loans

Much of the volatility in interest rates in recent years has been due to the uncertainty of inflation. Long-term loan assets that are indexed to the cost of living would therefore probably have a substantially more certain return over a short holding period and therefore might justify lower insurance premiums, provided they were offset on the balance sheet by an equal value of indexed liabilities.

Indexed loans would have the additional advantage of reducing default risk, since a sudden unanticipated end to inflation today when nominal interest rates contain a substantial inflation premium would make it just as difficult for borrowers to repay as it was for them in 1929–33, when there was a sudden unanticipated *de*flation, while homeowners and farmers were committed to interest rates that would have been appropriate for constant prices. Defaults on such loans were a major cause of the wave of bank failures during those years. A sudden end to inflation could cause comparable problems today, unless bank balance sheets are indexed to the cost of living.

In spite of their desirable properties, indexed loans have apparently been, until very recently, illegal and unenforceable in the courts, under the second sentence of the 1933 Gold Clause Joint Resolution. It was not until October 1977 that this law was repealed.[6]

Denied indexed loans as a means of avoiding inflation uncertainty in long-term nominal interest rates, many banks have turned to "variable rate loans," whose nominal interest rates are tied to some index of short-term nominal interest rates. These loans substitute a series of relatively accurate short-run inflation forecasts for a relatively inaccurate long-run inflation forecast, so they do reduce inflation uncertainty to a degree, though not entirely. The effective maturity of these variable-rate loans is ambiguous for our purposes. If they had the same default risk as ordinary loans, they could be taken as having maturity equal to their interest computation interval, rather than the longer actual maturity of the loan. However, since they leave the real interest rate on long-term loans uncertain, the real portion of the interest rate risk is borne by the borrower rather than by the bank. On paper the bank does not face real interest rate risk from these loans, but in practice the real interest rate risk may simply be disguised as default risk. All in all, these variable-rate loans are inferior substitutes for fixed-rate indexed loans. Unless there are other hidden legal barriers to indexation, variable rate loans will probably soon disappear.

10.11 The "Going Business" Value of the Bank

For the calculations above, I have assumed that as soon as a bank has negative net worth (based on current market value of assets), the stockholders will take the option of allowing the bank to be liquidated, so that their stock becomes worthless and the insuring agency pays off the depositors in full. In practice, a bank with negative net worth may

6. See McCulloch (1980).

actually be financially viable, in the sense that eventually it may be able to pay off its liabilities with interest out of the income from its assets.

There are two reasons it may be able to do this. The first is the value of its customer relations. In fact, financial markets are not perfect, so that the bank receives a sort of rent from the fact that it has evaluated the credit worthiness of certain customers who are unknown to other banks. To the extent that this is true, the shareholders will be willing to put up additional capital or to dilute their own stock with outside capital in order to prevent the bank from being liquidated. Market imperfections are the stuff day-to-day business is made of, so it is important that a bank with negative net worth based on tangible assets be given a fair chance to raise more capital before liquidating it.

The second reason is the monopoly value of the bank's charter. Since 1935, entry into banking has been severely restricted. The value of the existing charters has been further enhanced by the 1933 ban on checking accounts interest, and the lingering interest ceilings on time and savings accounts and on the newly authorized NOW accounts.

These restrictions on competition cannot be considered in isolation from the issue of bank safety. They were originally introduced shortly after the massive bank failures of 1929–33 in hope that greater profitability would make banks safer. This hope was illusory. Pouring profits into a bank to make it safer is like trying to carry water across a room in a sieve. The profits may just flow out of the bank in the form of dividends to shareholders. The monopoly profits might instead be retained and added to economic capital, and to that extent they would indirectly make the bank safer. But the bank would be just as safe if the capital were raised by any other means.

If banks paid competitive interest on deposits but were charged a fair premium for deposit insurance, they would be forced to pass the premium on to depositors in the form of reduced interest. Therefore fair insurance would cost the demand deposit owner several basis points, depending on the maturity of the bank's assets. The above restrictions on competition, on the other hand, cost the depositor several *hundred* basis points. They are therefore an extremely costly means of providing bank safety. In a study of bank risk and capital adequacy, they cannot be taken as immutable institutional background, since they exist in the name of bank safety. If they were abolished, a large part of the "going business" value of the bank would be eliminated. The simple model we have used above would then be a more realistic one.

10.12 Conclusion

It should be remembered that the estimates in this paper are only point estimates derived from a few quartiles of the data. By using as high a value for the characteristic exponent as seemed justified by the data, we have tried to make our estimates of the fair value of insurance err, if anything, on the low side. I plan at some time in the future to improve upon these estimates by means of maximum likelihood estimates of the stable parameters. This procedure will give confidence intervals for the risk estimates in addition to improved point estimates. In the meantime the burden of proof should be on the banks to show at a high level of significance that the premium they are paying to government insuring agencies is at least sufficient to cover the fair value of insurance for the risks they are incurring.