Volume Title: NBER Macroeconomics Annual 2014, Volume 29

Volume Author/Editor: Jonathan A. Parker and Michael Woodford, editors

Volume Publisher: University of Chicago Press

Volume ISBNs: 978-0-226-26873-6 (paper); 978-0-226-26887-3 (e-ISBN)

Volume URL: http://www.nber.org/books/park14-1

Conference Date: April 11–12, 2014

Publication Date: July 2015

Chapter Title: Comment on "Information Aggregation in a Dynamic Stochastic General Equilibrium Model"

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Chapter URL: http://www.nber.org/chapters/c13416

Chapter pages in book: (p. 208 – 214)

Comment

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In general equilibrium models with dispersed information, consumers and firms do not observe directly the aggregate shocks causing fluctuations, but only learn gradually about them from looking at prices and quantities. Models with these features have received increased attention from macroeconomists in recent years. The hope is they can help explain the sluggish responses of some variables to some shocks (e.g., the response of the price level to a monetary policy shock) and, at the same time, introduce new expectational shocks that can capture the role of "sentiments" or "animal spirits" in business cycles. A recent paper that develops a fairly rich dynamic stochastic general equilibrium model with these features is Maćkowiak and Wiederholt (2010). Macro models in this vein typically focus on real activity and inflation, so they feature a rather rudimentary financial sector and little or no analysis of asset pricing. One can safely conjecture that a richer asset-pricing structure matters for these models, as asset prices can reveal information about aggregate shocks and also add sources of aggregate noise. But, at the moment, we do not have tractable models that can be used to explore this conjecture.

Models with dispersed information have long been used in finance to understand how the prices of financial assets help to aggregate information. However, the finance literature on dispersed information has mostly worked with two- or three-period models, focusing on high-frequency phenomena and on the role of the market microstructure on information diffusion.

Hassan and Mertens take the challenge of building a dynamic stochastic general equilibrium model with a fully fledged asset pricing side. Such a model aims to bridge the gap between the macro and fi-

978-0-226-26873-6/2015/2014-0302$10.00

nance literature and could be used to ask many interesting questions: How does information aggregation affect consumption and investment decisions? How does it affect asset prices? However, the first difficulty they have to address is a methodological one. Macro-oriented DSGE exercises usually consider a linearized version of the model, in which standard signal-extraction tools can be applied. In applications more geared toward finance this approach is not satisfactory, as it does not allow to capture movements in correlations and risk premia that are crucial for asset pricing. Therefore, the main hurdle is to build models that preserve the relevant nonlinearities, while, at the same time, keeping the agent's signal-extraction problem tractable. Hassan and Mertens propose a way of modeling noise traders that overcomes this difficulty and derive some illustrative results.

In this comment, I will focus on the methodological contribution, which is the main contribution of the present paper. I will first illustrate the logic of their approach in the context of a simple two-period model. I will then discuss some of the advantages and limitations of this approach and discuss some potential alternatives.

Consider an economy that lasts two periods, $t = 0, 1$, populated by two groups of agents, old and young. Old agents are all identical and are endowed with a fixed, random supply of capital K_s, which they sell inelastically at time 0. Young agents are also identical, but receive different pieces of information. There is a continuum of young agents indexed by $i \in [0, 1]$. Agent i works in period 0 and uses labor income to purchase capital k_i from old agents. In period 1, agent i consumes the returns of the capital invested Ak_i. There is uncertainty on the rate of return A and young agents receive private information about it. Namely,

$$A = e^{\eta}$$

with $\eta \sim N(0, \sigma_\eta^2)$ and agent i observes the signal

$$s_i = \eta + \nu_i$$

with $\nu_i \sim N(0, \sigma_\nu^2)$.

The preferences of the young consumer are represented by the utility function $E[u(c_i) - v(n_i)]$, where c_i is consumption in period 1, n_i is labor supply in period 0 and we assume that labor produces output 1:1 in period 0. Therefore, the demand for capital by consumer i comes from maximizing

$$E\left[u\left(Ak_i\right) - v\left(Qk_i\right) \mid s_i, Q\right],$$

where Q is the price of capital. Notice that the price Q is a public signal which, in general, will contain useful information about η. Therefore, to solve this maximization problem we need to derive the distribution of A conditional on (s_i, Q). This is the inference problem which is, in general, hard to solve outside the linear-Gaussian case usually adopted in the literature. Usually, to ensure that Q is a linear function of the Gaussian shocks in the model, one needs to restrict attention to specific preferences, namely constant absolute risk aversion (CARA) preferences. Hassan and Mertens's advance is to find a way to use any utility function $u(\cdot)$, while keeping the inference problem a linear-Gaussian problem.

Let's conjecture that Q is a strictly monotone function of a variable q, which we call the "price signal." Namely, let

$$Q = H(q),$$

and assume that the price signal takes the linear form

$$q = \eta + \psi\tau,$$

where $\tau \sim N(0, \sigma_\tau^2)$ is a shock to the supply of capital K_s. In particular, we assume that the supply is given by $K_s = K(q, \tau)$, so we allow for the possibility that the supply be sensitive to the price. Given the conjecture above on Q, the inference problem is still tractable, and we can easily compute the distribution of A conditional on (s_i, Q), since, by monotonicity, observing (s_i, Q) is equivalent to observing (s_i, q). In particular, η is normally distributed with mean

$$\beta_1 s_i + \beta_2 q$$

and variance V (which does not depend on the realizations of s_i or q). The coefficients β_1, β_2 and the residual variance V depend on σ_η, σ_τ, ψ. We then get the individual capital demand function from the optimization problem

$$K_d(\beta_1 s + \beta_2 q, Q) = \arg\max_k E[u(Ak) - v(Qk) \mid s, q].$$

This problem can be solved numerically for any utility functions u and v, using standard methods. Aggregating, we then obtain the capital demand function

$$\bar{K}_d(\beta_1\eta + \beta_2 q, Q) = \int_{-\infty}^{\infty} K_d(\beta_1\eta + \beta_2 q + \beta_1\nu, Q) d\Phi(\nu),$$

where Φ is the cumulative distribution function (CDF) of the private noise ν.

We can now impose market clearing in the capital market and, by reverse engineering, find properties of the random supply $K(q, \tau)$, which ensure we have an equilibrium. In particular, we need to check that

$$\bar{K}_d(\beta_1\eta + \beta_2 q, H(q)) = K(q, \tau),$$

is satisfied for some monotone function H and some nonzero parameter ψ.

To give a specific example, suppose we have constant relative risk aversion in consumption, $u(c) = c^{1-\gamma}/(1 - \gamma)$, and linear utility in labor effort, $v(n) = n$. Then the individual demand for capital k_i is

$$k_i = \{E[A^{1-\gamma} \mid s_i, q]/Q\}^{1/\gamma}$$

where the expectation can be computed as

$$E[A^{1-\gamma} \mid s_i, q] = e^{(1-\gamma)(\beta_1 s_i + \beta_2 q) + (1-\gamma)^2 V/2}.$$

Integrating across agents we then get the market clearing condition

$$[e^{(1-\gamma/\gamma)(\beta_1\eta + \beta_2 q) + [(1-\gamma)^2/\gamma](V/2) + \{[(1-\gamma)/\gamma]\beta_1\}^2(\sigma_v^2/2)}]Q^{-1/\gamma} = K_s.$$

In this specific example, we have a natural conjecture for functional forms for H and K that give a tractable solution:

$$H(q) = \bar{Q}e^{\lambda q}, \qquad K(q, \tau) = e^{\tau}.$$

We then get the requirement

$$\lambda(\eta + \psi\tau) = \frac{1-\gamma}{\gamma}(\beta_1\eta + \beta_2(\eta + \psi\tau)) - \tau$$

for all η, τ. After matching coefficients and rearranging, we get

$$\lambda = \frac{1-\gamma}{\gamma}(\beta_1 + \beta_2), \qquad \psi\beta_1 = \frac{\gamma}{1-\gamma}.$$

Substituting for the inference coefficient β_1, the last equation gives us a nonlinear equation in ψ,

$$\frac{\pi_s\psi}{\pi_0 + \pi_s + \pi_\tau/\psi^2} = \frac{\gamma}{1-\gamma},$$

and all remaining parameters can then be easily computed. The nice idea here is that by making the right assumption on the functional form of the supply shocks one can preserve the linear-Gaussian inference structure with a much broader class of utility functions.

The question is how can we generalize this approach? Notice that, if we do not impose any restriction on the function $K(q, \tau)$, the theory gives us too much freedom, as shown by the following proposition.

Proposition 1. *Given any invertible function H and any nonzero scalar* ψ, *define the function*

$$K(q, \tau) \equiv \bar{K}_d(\beta_1(q - \psi\tau) + \beta_2 q, H(q)).$$

If the supply of capital is given by $K(q, \tau)$, the price $Q = H(q)$ with $q = \eta + \psi\tau$ is a rational expectations equilibrium price in the economy above.

So with the right choice of K, we can support any price function, with any sensitivity to the two shocks η and τ. All the testable implications of the theory come from imposing restrictions on the choice of the function $K(q, \tau)$. To be clear, the paper's interpretation is that $K(q, \tau)$ represents disturbances coming from noise traders. Therefore, we need to introduce some restriction in choosing the distribution of noise traders shocks. The approach of the paper is to represent the function K as a Taylor series:

$$K(q, \tau) = K_0 + K_q q + K_\tau \tau + \frac{1}{2}(K_{qq}q^2 + 2K_{q\tau}q\tau + K_{\tau\tau}\tau^2) + \ldots,$$

and to impose restrictions by fixing the value of some of the coefficients $K_0, K_q, \ldots$.

To understand how many degrees of freedom are available in restricting the coefficients $K_0, K_q, \ldots$, let us also represent H as a Taylor series. The equation for q is linear by assumption, so this leaves us only with one additional first-order parameter to determine: ψ. A useful property of the solution approach proposed is that to pin down the coefficients of the Taylor expansion of H up to the j-th order we only need to impose restrictions on the coefficients of K up to the same order. This allows us to figure out how many degrees of freedom are available at each order. At zero order, we can choose K_0 to determine H_0. At order 1, we can choose both K_q and K_τ to determine the values of H_1 and ψ. At each order $j \geq 2$ we only have one parameter H_{q^j}, so we have one degree of freedom in choosing the j-th order coefficients of K.

The discussion above implies that if we are only interested in a linear approximation of the equilibrium prices, it is enough to choose K_0—say to match average capital—and to choose K_q and K_τ. Here a natural choice would be $K_q = 0$ and $K_\tau = 1$, which is analogous to the choice I did in the simple constant relative risk aversion (CRRA) example. If we want a second-order approximation, we have to impose some restriction on one of the three coefficients K_{qq}, $K_{q\tau}$, $K_{\tau\tau}$ and things get a bit more arbitrary. Which of the three should we restrict? Why? Here it is probably useful to experiment with different restrictions to evaluate the robustness of the model predictions.

Overall, the approch in the paper provides a useful addition to the toolbox. My impression is that, at the end of the day, the method should be thought of as an approximation. The open question is what are the specific advantages of introducing an approximation that twists the functional form of the noise traders' demand. An alternative approach would be to solve for the market clearing price exactly—for a given functional form for $K(q, \tau)$—and introduce an approximation at the individual inference stage. For example, assume that the agents in the model use a (misspecified) linear model and look for the linear model that better fits the price process generated by the actual, nonlinear model. To some extent, I find this bounded-rationality approach more transparent. The same authors have also pursued a modeling approach that emphasizes the imperfect rationality of the investors in Hassan and Mertens (2011). However, we are at a stage in which it may be useful to attack the same problem with different solution methods, hoping to better understand the properties of these different approximations.

From a broader perspective, an important limit of the computational approach proposed in this paper is in the dynamics. The model follows a long tradition by assuming that agents receive full information at the end of each period.[1] Of course, this greatly helps to achieve tractability, by limiting the signal extraction problem to a static problem. However, it also reduces the scope of the analysis by eliminating the possibility of informational inertia and by eliminating the possibility of confusion of different shocks that arises from the time-series structure, as, for example, in recent work of Rondina and Walker (2013). Allowing for persistent dispersed information is hard. Computing models with persistent dispersed information is challenging even in linear setups. Full revelation at the end of the period also helps in keeping heterogeneity tractable, by allowing full insurance from one period to the next. Nonetheless, finding ways to overcome these difficulties seems an important task for the quantitative development of this class of models.

Endnotes

For acknowledgments, sources of research support, and disclosure of the author's material financial relationships, if any, please see http://nber.org/chapters/c13416.ack.

1. I have followed this tradition myself in my work on optimal monetary policy.

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