Chapter Title: Dormant Shocks and Fiscal Virtue

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1

Dormant Shocks and Fiscal Virtue

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I.   Introduction

The importance of modeling the interaction between fiscal and monetary policies goes back to the seminal contribution of Sargent and Wallace (1981). However, in many of the models that are routinely used to investigate the sources of macroeconomic fluctuations, fiscal policy plays only a marginal role. The vast majority of papers resolve the problem of monetary/fiscal policy coordination assuming that the fiscal authority stands ready to accommodate the behavior of the monetary authority, keeping the process for debt on a stable path. This is a strong assumption as a casual observation of the data shows that countries often experience prolonged periods of severe fiscal imbalance. Quite interestingly, these episodes are frequently followed by significant increases in inflation. In some cases, such increases are short lasting and remarkably violent. In other cases, they unfold over many years, generally starting small and then gaining momentum. In this paper, we develop a theoretical framework that can quantitatively account for persistent and accelerating increases in inflation and for the heterogeneity, across countries and over time, of the link between inflation and fiscal discipline.

We model an economy populated by a continuum of agents that are fully rational and understand that debt can be stabilized through movements in taxes or movements in inflation. When the fiscal authority is virtuous and moves primary surpluses in response to fluctuations in the ratio of debt to gross domestic product (GDP), the Central Bank has full control over inflation. Under the assumption of nondistortionary taxation, fiscal shocks do not have any effect on the real economy

as they only redistribute the timing of taxation. When policymakers deviate from the virtuous regime, with the fiscal authority not reacting to debt fluctuations and the Central Bank disregarding the Taylor principle, two situations can arise. If agents expect the return to the virtuous regime to be close enough in time, inflation stability is preserved. On the other hand, if the deviation is expected to last for a long period of time, high levels of debt require an increase in inflation.

We build on this basic intuition and assume that when facing a deviation from the virtuous rule, agents do not know how long it will take to move back. Instead, they have to conduct Bayesian learning to infer the nature of the deviation. As they observe more and more deviations, they get increasingly convinced that a prompt return to the virtuous regime is very unlikely. Given that agents are fully rational and understand that debt has to be financed in one way or the other, the drift in agents' beliefs determines a progressive increase in inflation. The initial movement can be almost undetectable, but as initially optimistic agents become relatively pessimistic, inflation accelerates, gaining momentum and getting out of control. At the same time, expected and realized volatilities go up as shocks that are *dormant* under the virtuous regime slowly start manifesting themselves. Therefore, if an external observer were monitoring the economy focusing exclusively on output and inflation, he would detect a run-up in inflation and an increase in volatility without any apparent explanation. The observer might then conclude that the volatility of the exogenous shocks and the target for inflation have both increased.

Dormant shocks are undetectable when policymakers are virtuous or agents are optimistic that they will be virtuous in the future because agents understand that any imbalance in the debt-to-GDP ratio will be followed by a fiscal adjustment. As agents become discouraged about policymakers' future behavior, the effects of dormant shocks arise. Therefore, dormant shocks can have effects many years after they occurred, as long as the fiscal imbalance that they generated is not totally reabsorbed by the time the deviation from the virtuous regime takes place. Furthermore, even after a regime change, their effects can barely be detected if agents find it extremely unlikely that policymakers will engage in a long-lasting deviation from the virtuous regime. In other words, depending on policymakers' *fiscal virtue*, inflation can stay low for many periods, as it takes time for agents to become convinced that the economy has entered a long-lasting deviation. According to the same logic, if on average policymakers spend a lot of time in the virtu-

ous regime, agents might remain confident about ultimately responsible fiscal behavior even when observing a long sequence of deviations. However, no matter how optimistic agents are or how virtuous policymakers have been in the past, if a deviation lasts for an extended period of time, agents will eventually become convinced that a quick return to the virtuous regime is unlikely. In other words, following a deviation, fiscal virtue can delay the effects of dormant shocks, but it cannot eliminate them.

The interaction between dormant shocks and fiscal virtue also provides an appealing explanation for why countries with different levels of debt might have similar levels of inflation for prolonged periods of time, but then experience very different outcomes during *hard times*. When a virtuous regime prevails or agents are confident that it will prevail in the future, the level of debt is substantially irrelevant. However, if agents become convinced that the economy has entered a long-lasting deviation, then the differentials between the interest rate and inflation open up. The larger the difference in fiscal virtue, the larger the difference in the speed of learning, the faster the opening of the differentials between the interest rate and inflation.

Therefore, our theoretical framework is capable of accounting for the instability of the link between fiscal discipline and inflation. In our model, agents are fully rational, but uncertain about the way the trade-off between inflation and taxation will be resolved. This creates a continuum of regimes indexed according to agents' beliefs and a smooth transition from the law of motion that prevails under the virtuous regime to the one that characterizes a long-lasting deviation. Therefore, the strict distinction between Ricardian and non-Ricardian regimes typical of the fiscal theory of price level literature (Leeper 1991; Sims 1994; Woodford 1994, 1995, 2001; Schmitt-Grohe and Uribe 2000; Bassetto 2002; and Cochrane 1998, 2001, among others) breaks down and is replaced by a series of intermediate regimes that reflect the evolution of agents' expectations about the future conduct of fiscal and monetary policies.[1]

Furthermore, agents *know that they do not know*. Therefore, when forming expectations, they take into account that their beliefs will evolve according to what they observe. Given this feature, our approach is clearly different from the one used in the traditional literature about learning, which assumes anticipated utility; that is, that agents form expectations conditional on their beliefs without taking into account that these are likely to change in the future. In our context, it is possible

to go beyond the anticipated utility assumption because there is only a finite number of relevant beliefs and they are strictly linked to policy-makers' behavior through the learning mechanism, in a way that we can keep track of their evolution.

In this respect, our paper is related to Eusepi and Preston (2012), who study the problem of macroeconomic stability in a model in which agents use adaptive learning to make forecasts about the future evolution of fiscal and monetary variables. In their model, there are not regime changes. If agents were fully rational, fiscal policy and the maturity structure of debt would be irrelevant because the Taylor principle always holds and fiscal policy is always Ricardian. However, agents do not know the parameters of the model and they erroneously believe that the economy is subject to regime changes. For this reason they use a constant gain learning algorithm in which recent observations receive more weight than observations that are far into the past. In this context, non-Ricardian effects arise because agents might erroneously regard bonds as net-wealth as in Barro (1974). Instead, in this paper non-Ricardian effects arise in the moment fully rational agents, in response to changes in policymakers' behavior, become discouraged about debt sustainability being guaranteed by movements in primary surpluses.

Given that the underlying mechanism relies on uncertainty around the source of financing for the debt-to-GDP ratio, all shocks that move this variable are potentially candidates for dormant shocks. In an environment with nondistortionary taxation, shocks to transfers and taxes are particularly interesting, given that they do not have *any* effect on the macroeconomic variables when the virtuous regime is in place but can generate large fluctuations in inflation once policymakers start deviating. Furthermore, given that agents are forward-looking, even announced changes in expenditure or taxation would trigger the inflationary mechanism.

We illustrate the key properties of the model using the basic three-equations new-Keynesian model used by Clarida, Gali, and Gertler (2000), Woodford (2003), and Lubik and Schorfheide (2004), augmented with a fiscal block. We then conduct a quantitative analysis building on the empirical results obtained by Bianchi and Ilut (2012). The estimates from that paper are used to calibrate a richer model and to provide guidance in characterizing the evolution of policymakers' behavior: A prolonged deviation from the virtuous regime started in the late 1950s and ended with the appointment of Paul Volcker as Federal Reserve Chairman. The transition matrix controlling the evolution of policy-

makers' behavior is then chosen in a way to match the stylized facts regarding the US Great Inflation: inflation started increasing in the mid-1960s, gained momentum in the early 1970s, got out of control toward the end of that same decade, and experienced a sudden drop in the early 1980s. The entire run-up of inflation of the 1970s can be obtained by considering only two shocks. The first rise of inflation would be the result of the *announcement* of the Great Society initiatives of President Lyndon Johnson around 1964, while the second acceleration would be caused by the tax cuts enacted by President Gerald Ford's administration. The progressive deterioration of agents' beliefs explains why inflation seemed to gain momentum over time. The appointment of Volcker at the end of the 1970s marks the return to the virtuous regime and determines the sudden drop in inflation of the early 1980s.

We then use the model to analyze the current situation. Given that dormant shocks might take a long time to unfold, we should not interpret the current low levels of inflation expectations and long-term interest rates as reflecting a low risk of high inflation for the US economy. We show that if US policymakers were to follow the current policy mix for a prolonged period of time, inflation might quickly accelerate and get out of control. In other words, the low inflation expectations and long-term interest rates reflect the reputation that US policymakers have built in the past twenty to thirty years since the Volcker disinflation. This stock of reputation is not unlimited, and it slowly deteriorates as policymakers keep deviating. This also suggests that if inflation is the result of a lack of fiscal discipline, central bankers cannot simply wait to see inflation in order to decide to worry about that. At that point, only an immediate change in *both* fiscal and monetary policies would be able to cut the inflation spiral.

This paper is part of a broader research agenda that aims at understanding the role of fiscal policy in explaining changes in the reduced form properties of the macroeconomy. In this regard, the current paper is related to Bianchi and Ilut (2012), who estimate a dynamic stochastic general equilibrium (DSGE) model for the US economy that allows for a structural break from a non-Ricardian regime to a Ricardian one. The main contribution of that paper is to identify the timing of the structural break, occurring a few quarters after the appointment of Volcker, and to show that the policy change can account for the rise and fall of inflation and the changes in the reduced form properties of the macroeconomy. The current paper does not present a fully specified estimation exercise. Instead, we use the results obtained by Bianchi and Ilut

(2012) to fix parameter values and the timing of regime changes. That said, the current paper contributes to this research agenda in several ways. First, it introduces the notion of dormant shocks. This resolves an apparent puzzle of the fiscal theory of price level; that is, the fact that in the data fiscal shocks do not seem to cause an immediate increase in inflation. Instead, thanks to the learning mechanism, dormant shocks can cause accelerating and persistent increases in inflation that unfold over *decades*. Second, it illustrates how the interaction between dormant shocks and fiscal virtue can account for the heterogeneity across countries in the link between fiscal discipline and inflation. Finally, it puts the theory to work to discuss its implications for the future behavior of inflation.

Our paper is related to the extensive literature that explores the evolution of output and inflation over the past sixty years using micro-founded models. Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010) consider models with time-varying structural parameters and find substantial evidence of parameter instability. Using a large-scale DSGE model augmented with stochastic volatilities, Justiniano and Primiceri (2008) find that changes in the volatility of investment shocks play a key role in explaining the evolution of the reduced form properties of the economy. Davig and Leeper (2007), Bianchi (2013), and Davig and Doh (2008) allow for heteroskedasticity and changes in monetary policy. Bianchi and Melosi (2012a) develop a theoretical framework to quantitatively assess the general equilibrium effects and welfare implications of central bank reputation and transparency. Finally, Ireland (2007), Liu, Waggoner, and Zha (2011), and Schorfheide (2005) consider models in which the target for inflation is moving over time. Our model is able to account for changes in the low-frequency component of inflation and in the volatility of the endogenous variables.

Our work is also related to Benati (2008); Cogley, Primiceri, and Sargent (2010); Cogley, Sargent, and Surico (2011); and Coibion and Gorodichenko (2011). Benati (2008) documents that inflation persistence is not stable across time and across countries. Cogley et al. (2010) study changes in the persistence of the inflation gap measured in terms of short- to medium-term predictability. Cogley et al. (2011) show that the Gibson's paradox (i.e., low correlation between inflation and nominal interest rates) vanished during the Great Inflation and reappeared after 1995. Coibion and Gorodichenko (2011) point out that the determinacy region in a model with positive trend inflation could be smaller

than what is implied by the Taylor principle. They conclude that the US economy was still at risk of indeterminacy in the 1970s, even if the Taylor principle was likely to be satisfied, because of the high level of trend inflation. Our model is able to generate variability in the persistence and low frequency component of inflation as a result of the evolution of agents' beliefs about policymakers' future behavior. Finally, our work is also linked to papers that study the impact of monetary policy decisions on inflation and inflation expectations, such as Mankiw, Reis, and Wolfers (2004); Nimark (2008); Del Negro and Eusepi (2011); and Melosi (2012, 2013).

This paper can be summarized as follows. In section II, we describe the model, outlining its properties under fixed coefficients. In section III, we introduce regime changes and learning. In section IV, we introduce the notion of dormant shocks and explain how they are related to fiscal virtue. We put the theory to work in section V: first, we look at the past; and then we look at the current situation and beyond under the same theoretical framework. We present our conclusions in section VI.

II. The Model

In order to illustrate the key properties of the model, we consider the basic new-Keynesian model employed by Clarida, Gali, and Gertler (2000), Woodford (2003), and Lubik and Schorfheide (2004), augmented with a fiscal rule. This model has very little built-in persistence, given that it features a purely forward-looking Phillips curve. This will allow us to isolate the effects of the learning mechanism.

A.   A New-Keynesian Model

The economy consists of a continuum of monopolistic firms, a representative household, and a monetary policy authority. The household derives utility from consumption C_t and disutility from labor h_t:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \exp(d_t) \left[\log(C_t) - h_t \right] \right], \tag{1}$$

where β is the household's discount factor and the preference shock d_t follows an autoregressive process: $d_t = \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}$, $\epsilon_{d,t} \sim N(0,1)$. The household budget constraint is given by

$$P_t C_t + B_t + P_t T_t = P_t W_t h_t + R_{t-1} B_{t-1} + P_t D_t, \tag{2}$$

where B_t represents nominal bond holdings, D_t captures dividends paid by firms, W_t is the real wage, T_t is a net lump sum tax, P_t is the price level, and R_t is the one-period gross nominal interest rate.

Each of the monopolistically competitive firms faces a downward-sloping demand curve:

$$Y_t(j) = \frac{P_t(j)}{P_t}^{-1/\upsilon} Y_t,$$ (3)

where $P_t(j)$ is the price chosen by firm j and the parameter $1/\upsilon$ is the elasticity of substitution between two differentiated goods. The firms take as given the general price level, P_t, and level of real activity Y_t. Whenever a firm wants to change its price, it faces quadratic adjustment costs represented by an output loss:

$$AC_t(j) = \frac{\varphi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi \right)^2 \frac{Y_t(j)P_t(j)}{P_t}.$$ (4)

The firm's problem consists in choosing the price $P_t(j)$ to maximize the present value of future profits:

$$E_0 \left[\sum_{t=0}^{\infty} Q_t \left(\frac{P_t(j)Y_t(j)}{P_t} - W_t h_t(j) - AC_t(j) \right) \right],$$

where Q_s is the household's stochastic discount factor. Labor is the only input in a linear production function, $Y_t(j) = A_t h_t(j)$, where total factor productivity $z_t = \ln(A_t/A)$ follows an autoregressive process: $z_t = \rho_z z_{t-1} + \epsilon_{z,t}, \epsilon_{z,t} \sim N(0, 1)$.

The government budget constraint is given by

$$b_t = b_{t-1} \left(\frac{Y_t \Pi_t}{Y_{t-1}} \right)^{-1} R_{t-1} - s_t,$$

where $b_t = B_t/(P_t Y_t)$ and $s_t = S_t/(P_t Y_t)$ are the debt-to-GDP ratio and the primary-surplus-to-GDP ratio, respectively. We assume that the government only moves lump-sum taxes and provides a subsidy. In other words, we exclude government purchases and we assume that the primary surplus coincides with net lump sum taxes. This will allow us to completely isolate the effects of fiscal shocks deriving from the lack of fiscal discipline. Introducing government purchases (G_t) would not modify the mechanism outlined here, but would make the interpretation of the results less immediate. The fiscal authority moves the primary surplus in response to deviations of debt from its own steady state (b):

$$(s_t - s) = \rho_s(s_{t-1} - s) + (1 - \rho_s)\delta_{b,\xi_t}(b_{t-1} - b) + \sigma_s \epsilon_{s,t}, \epsilon_{s,t} \sim N(0, 1),$$ (5)

while the Central Bank follows the rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_{R,\xi_t}}\left[\left(\frac{\Pi_t}{\Pi}\right)^{\psi_{\pi,\xi_t}}\left(\frac{Y_t}{Y_t^n}\right)^{\psi_{y,\xi_t}}\right]^{(1-\rho_{R,\xi_t})} e^{\sigma_R \epsilon_{R,t}}, \quad \epsilon_{R,t} \sim N(0,1), \quad (6)$$

where R is the steady-state gross nominal interest rate, Y_t^n is natural output, and Π is the target/steady-state level for gross inflation. The unobserved state ξ_t captures the monetary/fiscal policy combination that is in place at time t. The unobserved state takes on a finite number of values $j = 1, \ldots, m$ and follows a Markov chain that evolves according to a transition matrix H.

The model is solved and linearized around the steady state. From now on, all variables should be interpreted as deviations from steady state.[2] The private sector can be described by the following system of equations:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - z_t), \quad (7)$$

$$y_t = E_t(y_{t+1}) - (R_t - E_t(\pi_{t+1})) + (1 - \rho_d)d_t, \quad (8)$$

$$d_t = \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}, \epsilon_{d,t} \sim N(0,1), \quad (9)$$

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}, \epsilon_{z,t} \sim N(0,1), \quad (10)$$

where $\kappa \equiv (1 - \upsilon)/(\upsilon\phi\Pi^2)$ is the slope of the expectational Phillips curve (7). Equation (8) is the linearized intertemporal Euler equation describing the households' optimal choice of consumption and bond holdings.

The linearized government budget constraint is given by

$$b_t = \beta^{-1}b_{t-1} + b\beta^{-1}(R_{t-1} - \pi_t - \Delta y_t) - s_t, \quad (11)$$

where b_t and s_t represent now debt and surplus in terms of GDP in linear deviations from the steady state. The fiscal rule is given by

$$s_t = \rho_s s_{t-1} + (1 - \rho_s)\delta_{b,\xi_t}b_{t-1} + \sigma_s \epsilon_{s,t}, \epsilon_{s,t} \sim N(0,1), \quad (12)$$

while the linearized monetary policy rule is

$$R_t = \rho_{R,\xi_t}R_{t-1} + (1 - \rho_{R,\xi_t})(\psi_{\pi,\xi_t}\pi_t + \psi_{y,\xi_t}[y_t - z_t])$$
$$+ \sigma_R \epsilon_{R,t}, \epsilon_{R,t} \sim N(0,1). \quad (13)$$

B.   Fixed Coefficients and Determinacy Regions

Before describing the features of the model with regime changes, it is useful to review the properties of its *fixed coefficient* counterpart. Follow-

Table 1
Partition of the parameter space according to existence and uniqueness of a solution (Leeper 1991)

	Active Fiscal (AF)	Passive Fiscal (PF)
Active Monetary (AM)	No Solution	**Determinacy**
Passive Monetary (PM)	**Determinacy**	Indeterminacy

ing Leeper (1991), we can distinguish four regions of the parameter space according to existence and uniqueness of a solution to the model. These regions are summarized in table 1 and in general they are a function of all parameters of the model. However, in practice, the two policy rules are key in determining the existence and uniqueness of a solution. There are two determinacy regions. The first one, Active Monetary/ Passive Fiscal (AM/PF), is the most familiar one: the Taylor principle is satisfied and the fiscal authority moves taxes in order to keep debt on a stable path: $\psi_\pi > 1$ and $\delta_b > \beta^{-1} - 1$.[3] To grasp the intuition behind this result, substitute the tax rule in the law of motion for government debt (assuming for simplicity $\rho_s = 0$) and isolate the resulting coefficient for lagged government debt:

$$b_t = (\beta^{-1} - \delta_b)b_{t-1} + b\beta^{-1}(R_{t-1} - \pi_t - \Delta y_t) - \sigma_s \epsilon_{s,t}.$$

Intuitively, in order to guarantee stability of government debt, we need this coefficient to be smaller than one ($\beta^{-1} - \delta_b < 1$), so that debt is mean reverting. To get this, in turn, requires the coefficient on debt in the tax rule to satisfy the condition $\delta_b > \beta^{-1} - 1$. Therefore, we can think of fiscal policy as passive to the extent that it *passively* accommodates the behavior of the monetary authority ensuring debt stability. Woodford (1995) refers to this regime combination as Ricardian in the sense that the fiscal authority is committed to making the necessary adjustments to fiscal tools in order to neutralize any disturbance affecting the government budget constraint. Under this parameter combination, the macroeconomy is *insulated* from fluctuations of the debt-to-GDP ratio as agents understand that any imbalance will eventually be reabsorbed by means of adjustments to primary surpluses.

The second determinacy region, Passive Monetary/Active Fiscal (PM/AF), is less familiar and corresponds to the case in which the fiscal authority is not committed to stabilizing the process for debt: $\delta_b < \beta^{-1} - 1$. Now it is the monetary authority that *passively* accommodates the behavior of the fiscal authority, disregarding the Taylor principle and allowing inflation to move in order to stabilize the process for

debt: $\psi_\pi < 1$. Woodford (1995) refers to this regime combination as non-Ricardian. Under this regime, the macroeconomy is *not insulated* from the fiscal block. In other words, even in the absence of distortionary taxation, shocks to net taxes can have an impact on the macroeconomy as agents understand that they will not be followed by future offsetting changes in the fiscal variables. Finally, when both authorities are active (AM/AF) no stable equilibrium exists, whereas when both of them are passive (PM/PF) the economy is subject to multiple equilibria.

As it will be highlighted in the next section, this one-to-one mapping between the regions of the parameter space identified by Leeper (1991) and the terminology introduced by Woodford (1995) applies only in the context of a model with fixed coefficients. When regime changes are introduced, the distinction between Ricardian and non-Ricardian regimes becomes more subtle and crucially depends on agents' beliefs about the future evolution of the policy mix.

Much attention has been given by scholars to the behavior of the economy under the AM/PF regime and the problem of indeterminacy, whereas less attention has been devoted to the PM/AF determinacy region. A popular argument is based on the idea that even if the government does not constantly move taxes in order to stabilize debt, this does not imply that it will never do it. In other words, even if agents observe the PM/AF regime for a while, this does not mean that they are going to expect such a situation to prevail forever. We start from this argument to construct a model in which the presence of a period of PM/AF policy mix does not necessarily imply that the economy is subject to inflationary pressure. At the same time, we want to allow for the possibility that if a deviation lasts for a prolonged period of time, agents can rightfully lose confidence about the commitment of the government to stabilize debt. In section A of section III, we start building the intuition using a model in which agents can exactly infer the likely duration of a deviation from the virtuous regime. This model will serve as the benchmark for the full model with learning that will be introduced in section B of section III.

III.   Regime Changes and Agents' Beliefs

Consider the model described by the system of equations (7) through (13) and assume that the evolution of the monetary/fiscal policy mix can be described by a three-regime Markov-switching process whose transition matrix H is:

Table 2
Parameter choices of the DSGE parameters and of the transition matrix elements

Parameter	Value	Parameter	Value	Parameter	Value
ψ_π^A	0.80	ρ_s	0.90	$100\sigma_R$	0.20
ψ_π^P	2.00	ρ_z	0.90	$100\sigma_s$	0.50
ψ_y^A	0.10	ρ_d	0.90	$100\sigma_z$	0.70
ψ_y^P	0.10	b	1.00	$100\sigma_d$	0.40
ρ_R^A	0.75	κ	0.05	p_{11}	95%
ρ_R^P	0.75	β	0.99	p_{12}	4.95%
δ_b^A	0			p_{22}	70%
δ_b^P	0.03			p_{33}	99%

$$H = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 1 - p_{22} & p_{22} \\ 1 - p_{33} & & p_{33} \end{bmatrix}.$$

We calibrate the model using the values reported in table 2. We made use of the following two guidelines to characterize the matrix H and the three regimes. First, fiscal and monetary authorities do not have to pursue their goals on a daily basis. Deviations from the Taylor principle are possible and the fiscal authority does not have to constantly move taxes in order to keep debt on target. What is truly necessary is that over a medium to long horizon policymakers act responsibly and that agents understand this. In fact, it is quite reasonable that policymakers want to retain some flexibility in order to respond effectively to extraordinary events. To model this feature, we introduce two regimes. Regime 1 is the virtuous regime: the Taylor principle is satisfied, and fiscal policy accommodates the behavior of the monetary authority (i.e., monetary policy is active and fiscal policy is passive, $\psi_{\pi,1} = \psi_\pi^A > 1$ and $\delta_{b,1} = \delta_b^P > \beta^{-1} - 1$). Under regime 2, the central bank reacts less than one-for-one to inflation, and the fiscal authority does not move surpluses in response to movements in government debt (i.e., monetary policy is passive and fiscal policy is active, $\psi_{\pi,2} = \psi_\pi^P < 1$ and $\delta_{b,2} = \delta_b^A = 0 < \beta^{-1} - 1$). To capture the idea that these deviations are short lasting, we set the persistence of regime 2 to a relatively low value: $p_{22} << 1$.

Second, we want to allow for the possibility of long-lasting deviations from the standard policy practice. Therefore, we characterize regime 3 as having the same parameters of regime 2, but with a different persistence: $\psi_{\pi,3} = \psi_{\pi,2} = \psi_\pi^P < 1$, $\delta_{b,3} = \delta_{b,2} = \delta_b^A = 0 < \beta^{-1} - 1$, and

$p_{33} \gg p_{22}$. It is important to stress that even if the parameters entering the Taylor and fiscal rules are the same, the two regimes are in fact different. This is because the different persistence has deep implications for what agents expect about the future, as it will be illustrated in subsection A of section III. Given that a long-lasting deviation represents a substantive shift in the conduct of monetary and fiscal policies, we assume that when in the virtuous regime policymakers are more likely to engage in a short-lasting deviation—that is, they are more likely to move to regime 2: $p_{12} > p_{13}$.

A. Perfect Information

Before moving to the full model with learning, we will analyze the properties of the companion model in which agents can observe all aspects of the economy, including the regime that is in place at each point in time. When agents are aware of regime changes, standard solution methods do not apply. Instead, we need to use one of the solution methods developed to handle Markov-switching general equilibrium models. The solution algorithm employed in this paper is based on the work of Farmer, Waggoner, and Zha (2010). The authors show that it is possible to reduce the task of finding a Minimal State Variable solution to that of computing the roots of a quadratic polynomial in several variables. When a solution exists, it can be characterized as a regime-switching vector autoregression, of the kind studied by Hamilton (1989), Chib (1996), and Sims and Zha (2006):

$$S_t = T(\xi_t, \theta, H)S_{t-1} + R(\xi_t, \theta, H)\varepsilon_t. \tag{14}$$

It is worth emphasizing that the law of motion of the DSGE states depends on the model parameters (θ), the regime in place (ξ_t), and the probability of moving across regimes (H). This means that what happens under regime i depends not only on the structural parameters describing that particular regime, but also on what agents expect is going to happen under alternative regimes and on how likely it is that a regime change will occur in the future.

The simplest way to understand the properties of the different regimes is to analyze the impulse responses. We shall focus on a primary surplus shock to isolate the effects of fiscal discipline. Under the assumption of nondistortionary taxation such a shock does not have any impact on the macroeconomy if agents anticipate that policymakers will make the necessary fiscal adjustments in order to repay the resulting in-
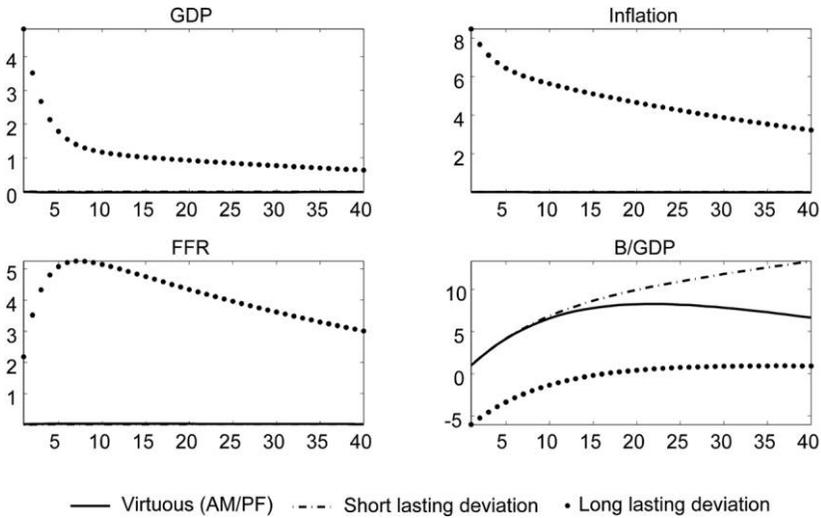
Fig. 1. Impulse responses to a two standard deviation primary deficit shock under perfect information

Note: FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

crease in debt. Therefore, any detected effect on the macroeconomy will be due to the lack of confidence that such adjustments will be made. On the other hand, it is important to keep in mind that fiscal discipline affects the propagation of all shocks. However, for the other shocks, we would need to separate the direct impact of the shock from the effect due to the lack of fiscal discipline (see appendix).

Figure 1 reports the impulse responses to a large primary deficit shock. The difference between the long-lasting deviation and the other two regimes is particularly striking. Under the virtuous regime and the short-lasting deviation, we do not observe any effect on inflation and output from this shock, whereas under the long-lasting deviation we observe a large and persistent increase in inflation and an expansion in output. Under the virtuous regime and the short-lasting deviation, the debt-to-GDP ratio starts increasing slowly and steadily in response to the increase in expenditure. The two paths differ only to the extent that under the virtuous regime, the government is adjusting primary surpluses in response to the increase in debt. Under the long-lasting deviation, the debt-to-GDP ratio experiences a sudden drop, because of the increases in inflation and GDP, followed by a smooth return to the steady state from below, because of the increased level of expenditure and the slowdown in growth.
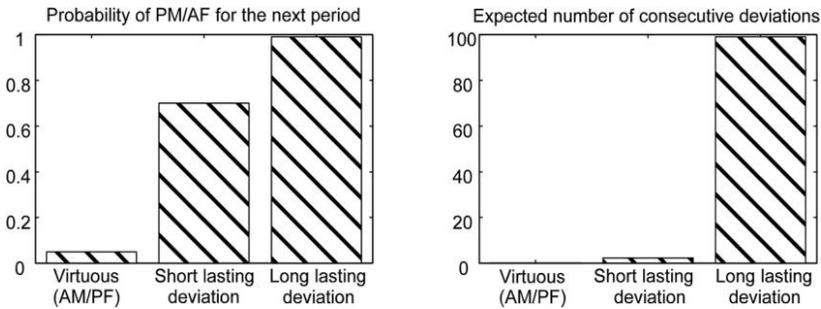
Fig. 2. The two panels report, respectively, the one-step-ahead probability of a deviation from the virtuous regime and the expected number of consecutive deviations under the different regimes under the assumption that agents have perfect information

From these impulse responses, it should be clear that compared with a long-lasting deviation from the virtuous regime, a short-lasting deviation has very different implications. The interesting fact here is that the behavior of the two authorities is *identical* across the two regimes. However, the two regimes differ to the extent that they induce different expectations about the policymakers' *future* behavior. To illustrate this point, for each regime, figure 2 reports the one-step-ahead probability of observing a deviation from the virtuous regime and the expected number of consecutive deviations. When the economy is under the virtuous regime, agents are confident of staying there for a while, and the expected number of consecutive deviations is very low. When under the short-lasting deviation, the one-step-ahead probability increases substantially, but the expected number of consecutive deviations is still very low (2.33). Finally and most importantly, when policymakers engage in a long-lasting deviation, the number of expected consecutive periods of PM/AF policy mix increases substantially. Therefore, under the short-lasting deviation, agents are confident that in the near future the government will run a sequence of primary surpluses to balance the debt-to-GDP ratio. Instead, when a long-lasting deviation occurs, agents are discouraged about a prompt return to the virtuous regime. In this case, the fiscal imbalance generated by the increase in primary deficits calls for an increase in inflation. Given that the Taylor principle does not hold, the monetary authority accommodates such a rise in inflation. This determines a drop in real interest rates, and output increases. High inflation and the jump in output cause the drop in the debt-to-GDP ratio.

These results allow us to make a first important point: in a model

with recurrent regime changes, the policy mix is not enough to establish if a regime is Ricardian or not. Instead, the persistence of the regime becomes a key ingredient given that it affects agents' expectations about the conduct of fiscal and monetary policies in the medium-long run. When agents are confident about a prompt return to the virtuous policy mix, a fiscal imbalance is not inflationary even if policymakers do not immediately take care of it. Agents understand that with high probability the necessary adjustments will occur in the future. Instead, when the regime change is perceived to be too far into the future, inflation starts moving in response to the fiscal imbalance.

B.   Bayesian Learning

We are now ready to analyze the case in which agents cannot observe the regime they are in. Let $\mathcal{F}_t$ be the agents' information set. It is assumed that agents observe the history of the endogenous variables as well as the history of all shocks. However, agents do not observe the history of regimes. Instead, they need to conduct Bayesian learning in order to infer the regime that they are in. Regime changes are modeled as the three-regime Markov-switching process described earlier. In this context, the transition matrix also reflects agents' priors about the evolution of the monetary/fiscal policy mix.

Since agents know the history of endogenous variables and shocks, they can exactly infer the policy mix that is in place at each point in time. However, while the virtuous regime is fully revealing, when the PM/AF mix is prevailing, agents do not know whether policymakers are engaging in a short-lasting deviation or a long-lasting one. Agents have to learn the nature of the deviation in order to form expectations over the endogenous variables of the economy. An important result is then the following: *Agents will grow more and more pessimistic about moving back to the virtuous regime, the longer the time spent under the alternative policy mix*. To see this, note that after having observed $\tau \geq 1$ consecutive deviations from the virtuous regime, agents believe that policymakers will keep deviating in the next period with the following probability:[4]

$$prob\{s_{t+1} \neq 1|\mathcal{F}_t\} = \frac{p_{22}(p_{12}/p_{13})(p_{22}/p_{33})^{\tau-1} + p_{33}}{(p_{12}/p_{13})(p_{22}/p_{33})^{\tau-1} + 1}. \tag{15}$$

The probability $prob\{s_{t+1} \neq 1|\mathcal{F}_t\}$ has a number of properties that shed light on the key features of the learning mechanism. Since $p_{22} < p_{33}$, this probability is monotonically increasing with respect to the number of

last consecutive deviations τ. As the number of periods τ in which the PM/AF policy mix has prevailed increases, agents become more and more pessimistic about the odds of switching to the virtuous regime in the next period. The reason is that as policymakers keep deviating from the virtuous regime, agents get increasingly convinced that the two authorities are engaging in a long-lasting deviation (regime 3) and are, therefore, more unlikely to switch to the virtuous regime ($p_{22} < p_{33}$).

Furthermore, agents' pessimism admits an upper and lower bound. The intuition goes as follows. For a given number of consecutive deviations τ, equation (15) shows that the probability of observing another deviation next period will be a weighted average of p_{22} and p_{33}. When agents observe policymakers deviating from the virtuous regime for the first time ($\tau = 1$), the weights are $p_{12}/(p_{12} + p_{13})$ and $p_{13}/(p_{12} + p_{13})$. This weighted average is the lower bound for agents' pessimism. The smaller the conditional probability of a long-lasting deviation, $p_{13}/(p_{12} + p_{13})$, the closer the lower bound will be to p_{22}. As policymakers keep deviating, agents get increasingly convinced that the economy has entered a long-lasting deviation, given that under this regime long deviations are more likely. As the weight assigned to the long-lasting deviation increases, the probability of observing an additional deviation increases, given that $p_{33} > p_{22}$. If policymakers deviate from the virtuous regime for a very long time, agents will eventually become convinced of being in the long-lasting PM/AF regime, and the probability of observing an additional deviation in the next period degenerates to the persistence of such a regime:

$$\lim_{\tau \to \infty} prob\{s_{t+1} \neq 1 | \mathcal{F}_t\} = p_{33}. \tag{16}$$

Hence, p_{33} is the upper bound for agents' pessimism. This implies that for each $\varepsilon > 0$, there exists an integer τ^* such that:

$$p_{33} - prob\{s_{t+1} \neq 1 | \tau = \tau^*\} < \varepsilon, \tag{17}$$

with the important result that for any $\tau > \tau^*$, agents' beliefs can be effectively approximated using the properties of the long-lasting PM/AF regime. In other words, agents cannot be more pessimistic than when they are convinced of being in the long-lasting PM/AF regime.

These ideas are summarized in figure 3. The first panel describes the evolution of agents' beliefs as a function of the number of observed deviations from the virtuous regime. The virtuous regime is fully revealing, therefore agents do not face any uncertainty.[5] When agents observe the first deviation, they are relatively confident that the prevailing
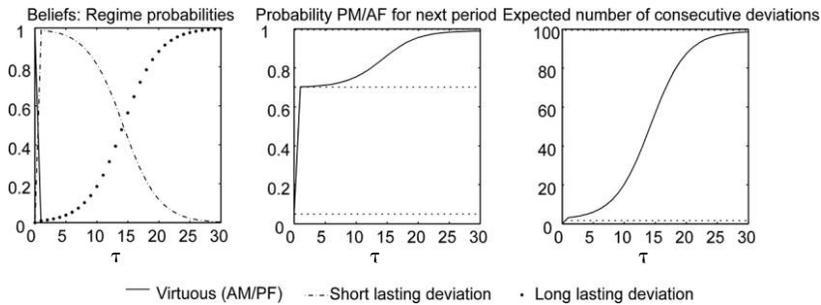
Fig. 3. From left to right, the three panels describe, as a function of the number of con-secutive deviations τ, the evolution of agents' beliefs, the one-step-ahead probability of observing the PM/AF policy mix, and the expected number of consecutive deviations from the virtuous regime

regime is the short-lasting PM/AF regime, given the assumption $p_{12} > p_{13}$. However, as the number of deviations grows, agents become more and more convinced that the economy has entered a long-lasting deviation. Initially the probability attached to the long-lasting PM/AF regime increases slowly, but it eventually accelerates as agents become less and less optimistic about a return to the virtuous regime. This has important implications on agents' expectations regarding the policy-makers' future behavior. The second and third panels of figure 3 report the one-step-ahead probability of observing the PM/AF policy mix and the expected number of consecutive deviations implied by the drift in beliefs. As agents observe more and more deviations, they become more and more pessimistic and the expected number of consecutive devia-tions increases. As agents go from being relatively optimistic to being relatively pessimistic, the expected number of consecutive deviations accelerates. Finally, as agents become convinced of being in a long-last-ing deviation, the expected number of consecutive deviations converges to the one prevailing under the long-lasting PM/AF regime. This drift in beliefs about the future conduct of fiscal and monetary policies has deep implications for the way shocks propagate through the economy. However, we will first describe the solution method for the model with learning.

Solving the Model with Learning

It is very important to emphasize that the evolution of agents' beliefs about the future conduct of fiscal and monetary policies plays a critical

role in the Markov-switching model with learning. In fact, unlike the perfect-information model described in section A of section III, the dynamics of the endogenous variables in the model with learning cannot be fully captured by the three policy regimes. Instead, agents expect different dynamics for next period's endogenous variables depending on their beliefs about a return to the virtuous regime.

Therefore, accounting for agents' learning requires expanding the number of regimes and redefining them as a combination between policymakers' behavior and agents' beliefs. Such new regimes reflect different degrees of pessimism while agents are learning about the persistence of the deviation from the virtuous regime. Bianchi and Melosi (2012b) show that the Markov-switching model with learning described previously can be recast in terms of an expanded set of $(\tau^* + 1) > 3$ new regimes, where $\tau^* > 0$ is defined by the condition (17). The increased number of regimes captures the degree of pessimism associated with observing deviations from the virtuous regime for $\tau = 1 \ldots \tau^*$ periods. The $\tau^* + 1$ regimes are given by

$$[(\xi_t = 1, \tau_t = 0), (\xi_t \neq 1, \tau_t = 1), (\xi_t \neq 1, \tau_t = 2), \ldots, (\xi_t \neq 1, \tau_t = \tau^*)],$$

and the transition matrix $\tilde{H}$ is defined as

$$\tilde{H} = \begin{bmatrix} p_{11} & p_{12} + p_{13} & 0 & \ldots 0 & 0 \\ 1 - \dfrac{p_{12}p_{22} + p_{13}p_{33}}{p_{12} + p_{13}} & 0 & \dfrac{p_{12}p_{22} + p_{13}p_{33}}{p_{12} + p_{13}} & \ldots 0 & 0 \\ 1 - \dfrac{p_{12}p_{22}^2 + p_{13}p_{33}^2}{p_{12}p_{22} + p_{13}p_{33}} & 0 & 0 & \ldots 0 & 0 \\ \vdots & \vdots & \vdots & \ddots \vdots & \vdots \\ 1 - \dfrac{p_{12}p_{22}^{(\tau^*-1)} + p_{13}p_{33}^{(\tau^*-1)}}{p_{12}p_{22}^{(\tau^*-2)} + p_{13}p_{33}^{(\tau^*-2)}} & 0 & 0 & 0 \ 0 & \dfrac{p_{12}p_{22}^{(\tau^*-1)} + p_{13}p_{33}^{(\tau^*-1)}}{p_{12}p_{22}^{(\tau^*-2)} + p_{13}p_{33}^{(\tau^*-2)}} \\ 1 - \dfrac{p_{12}p_{22}^{\tau^*} + p_{13}p_{33}^{\tau^*}}{p_{12}p_{22}^{(\tau^*-1)} + p_{13}p_{33}^{(\tau^*-1)}} & 0 & 0 & 0 \ 0 & \dfrac{p_{12}p_{22}^{\tau^*} + p_{13}p_{33}^{\tau^*}}{p_{12}p_{22}^{(\tau^*-1)} + p_{13}p_{33}^{(\tau^*-1)}} \end{bmatrix}.$$

Hence, one can recast the Markov-switching DSGE model with learning as a Markov-switching Rational Expectations system, in which the regimes are redefined in terms of the realized duration of the passive regimes, τ_t. This result allows us to solve the model with regime switches and learning by applying any of the methods developed to solve Markov-switching rational expectations models.

It is worth emphasizing that this way of recasting the learning process allows us to easily model the behavior of agents that *know that*

they do not know. In other words, agents are aware of the fact that their beliefs will change in the future according to what they observe in the economy. This represents a substantial difference with the anticipated utility approach, in which agents form expectations without taking into account that their beliefs about the economy will change over time. Furthermore, our approach differs from the one traditionally used in the literature about learning in which agents form expectations according to a reduced-form law of motion that is updated recursively (for example, using discounted least squares regressions). The advantage of adaptive learning is the extreme flexibility given that, at least in principle, no restrictions need to be imposed on the type of parameter instability characterizing the model. However, such flexibility does not come without a cost, given that agents are not really aware of the model they live in, but only of the implied law of motion. Instead, in this paper, agents fully understand the model and they are aware of the trade-offs that characterize it. However, they are uncertain about the policymakers' future behavior, and this uncertainty has important consequences for the law of motion of the economy.

Impulse Responses

In order to understand the properties of the model, we will start illustrating how learning affects the propagation of the shocks. Figure 4 contains impulse responses to a negative primary surplus shock conditional on different starting values of the number of consecutive deviations, τ, under the assumption that the PM/AF policy mix is in place over the entire horizon. The dark areas correspond to large *starting* values for τ, while as the colors become lighter and lighter, the starting number of deviations falls to 1 (i.e., at the time of the shock, agents observe the first deviation from the virtuous regime). We do not report here the response under the virtuous regime because this would look very similar to the one prevailing under perfect information, as the regime is fully revealing.[6]

   Before proceeding, it is worth stressing an important point. The assumption that the PM/AF policy mix prevails over the entire horizon implies that the number of observed deviations grows over time. This in turn determines a progressive change in the law of motion as agents become more and more convinced that the economy has entered a long-lasting deviation. The law of motion then stabilizes in the moment that agents become certain that they entered a long-lasting deviation ($\tau = \tau^*$). The law
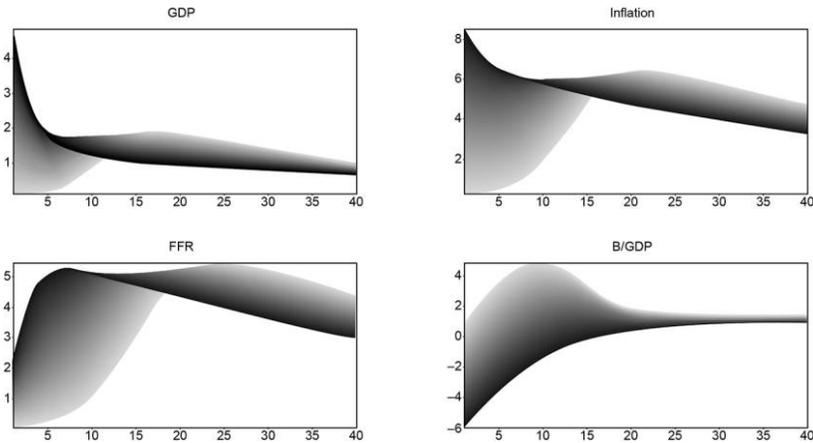
Fig. 4. Impulse responses to a two standard deviation primary deficit shock under learning, assuming that the PM/AF rule prevails over the relevant horizon

Notes: The dark areas correspond to large starting values for tau, while as the colors become lighter and lighter, the starting number of deviations falls to 1 (i.e., at the time of the shock, agents observe the very first deviation from the virtuous regime). The impulse responses under the virtuous regime are very similar, although not identical, to the ones that prevail under perfect information, and are therefore omitted. FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

of motion would change again if agents were to observe a return to the virtuous regime. This case will be analyzed in section A of subsection V.

Consider the case in which agents have observed a large number of consecutive deviations, implying that agents are already convinced of being in the long-lasting PM/AF regime. As a result, the impulse responses resemble very closely the ones implied by the same regime under perfect information.[7] Following the shock, inflation increases immediately, and then it slowly declines. At the same time, the debt-to-GDP ratio experiences a large drop as a result of higher growth and larger inflation, despite the increase in the primary deficit. As it was for the perfect information case, these dynamics reflect the expectation that the government will not increase future surpluses in order to cover the current deficits.

When the starting number of deviations is low, these effects are initially mitigated because agents are confident about a prompt return to the virtuous regime. During the first few periods, inflation and GDP barely move, even if the current policymakers' behavior is still characterized by the PM/AF policy mix. Consequently, we do not observe the drop in the debt-to-GDP ratio, which instead starts increasing because

of the primary deficits. However, as agents observe more and more deviations, their expectations start drifting, the law of motion evolves, and the non-Ricardian dynamics start arising. Inflation starts to increase smoothly, then it accelerates in the moment that agents become relatively pessimistic, and it finally reaches its peak after approximately seven years. Symmetrically, real interest rates decline because the Taylor principle does not hold, output starts growing, and the debt-to-GDP ratio falls.

IV. Dormant Shocks and Fiscal Virtue

As shown in section III, when learning is introduced in the model with regime changes, the economy responds gradually to the shocks as agents update their beliefs about policymakers' future behavior. The response to a fiscal shock is particularly interesting given that the learning mechanism can substantially prolong its effects and move the peak of the responses further into the future. In this section, we will analyze these features more in detail introducing the notion of *dormant shocks* and characterizing their effects as a function of *fiscal virtue*.

A.   Dormant Shocks

Figure 5 considers a large negative shock to primary surpluses occurring at time 0 under the Virtuous regime. After five years policymakers start deviating from the virtuous regime and they keep deviating for twenty years. The panels on the left report the impulse responses, while the panels on the right contain the evolution of expected volatility at different horizons, from one year (light areas) to five years (dark areas). This measure of uncertainty is computed taking into account the possibility of regime changes and the evolution of agents' beliefs, using the methods described in Bianchi (2012). For a variable X_t and an horizon T, it corresponds to $\sqrt{V(X_{t+T}|\mathcal{F}_t)}$.

Notice that as long as the virtuous regime prevails, the effects on inflation, output, and federal funds rate (FFR) are basically undetectable as agents expect taxes to be raised in order to repay the growing debt. However, as soon as the policy mix changes, the learning process begins. At this point an external observer focusing exclusively on the three standard macroeconomic variables would be observing a slow moving increase in inflation and an acceleration in output growth (and a positive output gap), along with a weak response of the FFR. All of
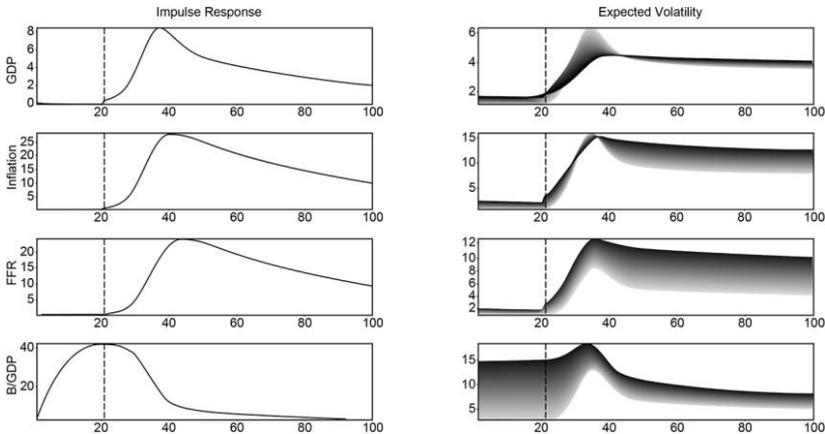
Fig. 5.   The two sets of panels report the evolution of the endogenous variables and the correspondent expected volatilities assuming a large primary surplus shock occurring at time 0 under the virtuous regime

Notes: After five years, policymakers start deviating from the virtuous regime (dashed vertical line). In the panels on the right, when moving from the light lines to the dark lines, the time horizon increases from one year to five years. Expected volatilities are computed taking into account the possibility of regime changes. FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

these changes would be without any apparent explanation, as no new shocks have occurred. Fiscal shocks have therefore an interesting property in this environment: they can manifest themselves many years after they occurred. In the meantime, they are just dormant shocks, given that they do not have any apparent effect on the three standard macro-economic variables. Notice that this aspect makes them very hard to identify. If an econometrician were trying to understand the causes of the slow-moving increase in inflation, he might be tempted to conclude that a change in the target for inflation occurred. Alternatively, he might want to include fiscal variables in order to estimate the model under the assumption that a non-Ricardian regime is in place. However, if he happens to exclude the early years of the sample, he might have a very hard time trying to recover any movement in the fiscal variables that could in fact explain the slow-moving increase in inflation.

The increase in inflation is not the only effect of the changed economic environment. As outlined in panels on the right of figure 5, agents also face an increase in uncertainty. The expected volatilities of the macro-economic variables start increasing smoothly and reach a peak in the moment that agents change from being relatively optimistic about a

prompt return to the virtuous regime to being relatively pessimistic about this happening. These beliefs coincide with the peak of uncertainty because agents attach similar probabilities to two very different scenarios: (1) a large spur of inflation to cover the large debt and (2) a return to the virtuous regime with a subsequent drop in inflation. This also explains why uncertainty is *larger* at short horizons than at long horizons: In the long run, debt is expected to be closer to the steady state, regardless of the path taken by policymakers, while in the short run, the large stock of debt can have pervasive effects on the macroeconomy. Dormant shocks have therefore another interesting effect: After many years they can cause an increase in the volatility of the endogenous variables and consequently in agents' uncertainty. Once again, these effects are dormant as long as the economy is under the virtuous regime.

As agents become convinced that debt will be inflated away, the volatilities approach the values associated with the long-lasting PM/ AF regime. Under this regime, macroeconomic uncertainty is larger than under the virtuous regime for three reasons. First, given that the Federal Reserve reacts less strongly to deviations of inflation from the target, any shock has a larger *direct* impact on the dynamics of inflation. Second, any shock that moves the debt-to-GDP ratio is also going to have an *indirect* impact on all the macroeconomic variables. Third, the fiscal shocks that are dormant under the virtuous regime affect the macroeconomic variables under the long-lasting PM/AF regime.

Based on these results, it should be clear that the virtuous regime leads to a more stable macroeconomic environment. Both inflation and output volatilities are smaller when policymakers refrain from long-lasting deviations from the virtuous regime.[8] However, it is worth pointing out that this result might change in the moment more shocks are introduced in the analysis. Bianchi and Melosi (2013) consider a particularly topical case: a large contraction in aggregate demand that forces monetary policy to the zero lower bound. In this case, policymakers understand that a large recession and deflation can be avoided by abandoning fiscal discipline, but this decision would also result in a sharp increase in macroeconomic instability once out of the recession. On the other hand, announcing a period of fiscal austerity is detrimental in the short run, but it preserves macroeconomic stability in the long run. Therefore, the moment that monetary policy is constrained, it is not clear anymore that sticking to fiscal discipline is always preferable. We show that the policy trade-off can be resolved by committing to inflate away only the portion of debt resulting from the unusually large

recession. This determines an increase in short-run inflation expectations that mitigates the recession. At the same time, long-run inflation expectations and uncertainty remain anchored as policymakers' rules in response to other shocks are unchanged and the precrisis debt is still backed by future fiscal adjustments.

Summarizing, an external observer monitoring the evolution of our hypothetical economy would detect a progressive increase in volatility and uncertainty, measured by *expected* volatility. At the same time, she would observe an increase in inflation that seems to gain momentum over time. Our external observer would probably conclude that the target for inflation has changed and that the volatility of the exogenous shocks has increased.

B. Fiscal Virtue

The previous section emphasized that dormant shocks can have effects many years after they occurred. In this section, we will elaborate more on this point, trying to understand what determines the lag between the time of the regime change and the peak of the inflation increase that such a regime change triggers. We will show that two margins are important to understand how quickly these events unfold. The first margin is represented by the relative probability of entering a short-lasting deviation versus a long-lasting deviation and controls agents' *a priori beliefs* following a first deviation from the virtuous regime. The second margin is given by the relative persistence of the short- and long-lasting deviations and reflects agents' *confidence* in ultimately responsible fiscal behavior.

Figure 6 considers the same exercise of the previous subsection for different values of the ratio $p_{13}/(p_{12} + p_{13})$. Recall that this ratio controls agents' a priori beliefs of entering a long-lasting versus a short-lasting deviation. In other words, it determines the probability attached to the long-lasting deviation once agents observe the first realization of the PM/AF policy mix. As this ratio declines, agents are a priori more and more optimistic about the possibility of facing only a temporary deviation from the virtuous regime. Consequently, it takes longer for agents to become convinced that they entered a long-lasting deviation. To capture this idea, the different curves are labeled according to the implied τ^*, that is, the number of deviations required for agents' beliefs to approximately coincide with the ones implied by the long-lasting PM/AF regime. As explained in subsection B of section III, this value represents
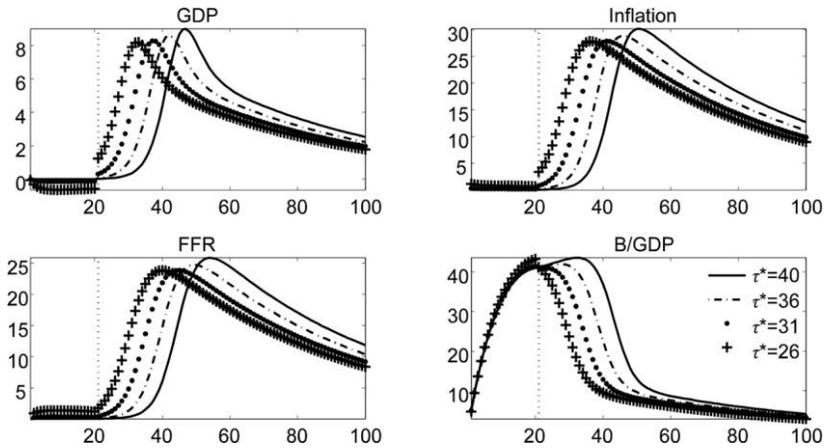
Fig. 6. The figure considers the effects of a large "dormant shock" to primary surplus for transition matrices that differ according to the a priori relative probability of a short-lasting deviation.

Notes: The shock occurs at time 0, while the regime change occurs after 20 periods (dotted vertical line). The different a priori beliefs are captured by the different periods necessary to convince agents that they entered a long-lasting deviation (τ^*). FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

Table 3
Parameter values used in the study of dormant shocks

| $100p_{13}/(p_{12}+p_{13})$	p_{22}	τ^*	$100p_{13}/(p_{12}+p_{13})$	p_{22}	τ^*
.04	0.70	40	1.00	.90	97
.20	.070	36	1.00	.80	48
1.00	**0.70**	**31**	**1.00**	**.70**	**31**
5.00	0.70	26	1.00	.60	23

Note: The persistences of the long-lasting PM/AF regime and of the AM/PF regime are fixed at .99 and .95, respectively.

an upper bound on agents' pessimism about a prompt return to the virtuous regime. The dotted line corresponds to the benchmark case in which $p_{13}/(p_{12}+p_{13}) = 1\%$ and $\tau^* = 31$. Table 3 summarizes the mapping for the other three curves.

As the ratio $p_{13}/(p_{12}+p_{13})$ declines and τ^* increases, the peak of the inflation spur moves to the right. When agents attach a conditional probability of .04 percent to the long-lasting regime, the peak in inflation occurs thirteen years after the shock took place and eight years after the regime change occurred. When instead agents are relatively pessimistic and $p_{13}/(p_{12}+p_{13}) = 5\%$, the learning process is faster and the

peak of inflation occurs only six years after the regime change. Given that the ratio $p_{13}/(p_{12} + p_{13})$ controls a priori agents' beliefs, it is capturing the credibility of policymakers in agents' eyes. Agents' beliefs are likely to reflect some historical evidence, in which case the matrix H would be pinned down by the relative frequencies of both short-lasting and long-lasting deviations. Alternatively, we could imagine that agents form subjective conjectures about how likely it is that the government will engage in a persistent deviation from the virtuous regime. In other words, agents could believe the occurrence of a long-lasting deviation is more or less likely according to the policymakers' reputation. However, no matter what agents' a priori beliefs are, *as long as agents update them according to what they observe, the government cannot indefinitely avoid increasing taxes*. In other words, no matter how optimistic agents are, if the government deviates from the virtuous regime for a long period of time, eventually it will induce a change in expectations leading to an increase in inflation and uncertainty. At the same time, if the government has been virtuous in the past, it has probably built some reputation for avoiding long-lasting deviations. This translates into a low value for $p_{13}/(p_{12} + p_{13})$ and implies that policymakers can deviate for a longer period of time without losing control of inflation expectations.

However, the a priori relative probability of the two deviations is not the only margin that affects the timing of the peak of inflation. The relative persistence of both the short- and long-lasting deviations, p_{22}/p_{33}, is also important. When p_{22}/p_{33} is low, the learning process is faster as agents need only a few consecutive observations to conclude that they entered a long-lasting deviation. In the limiting case in which the ratio is zero, agents just need two consecutive deviations to conclude that they entered a long-lasting regime and that with very high probability they will face a long series of PM/AF realizations. In contrast, when p_{22}/p_{33} is large, the learning process unfolds very slowly, and consequently the peak of inflation occurs later (i.e., moves further to the right when plotted). Figure 7 illustrates this point. The benchmark case is once again represented by the dotted line and the parameter choices are summarized in Table 3. Notice that when $p_{22}/p_{33} = .9/.99 = 0.4762$, the peak of inflation occurs more than twenty years after the shock occurred and more than fifteen years after the regime took place. Even in this case, the ratio can be interpreted as characterizing policymakers' credibility, but for a different aspect. Specifically, agents might have different views about what it means for a deviation to be short lasting. When, for a given persistence of the long-lasting regime, this ratio in-
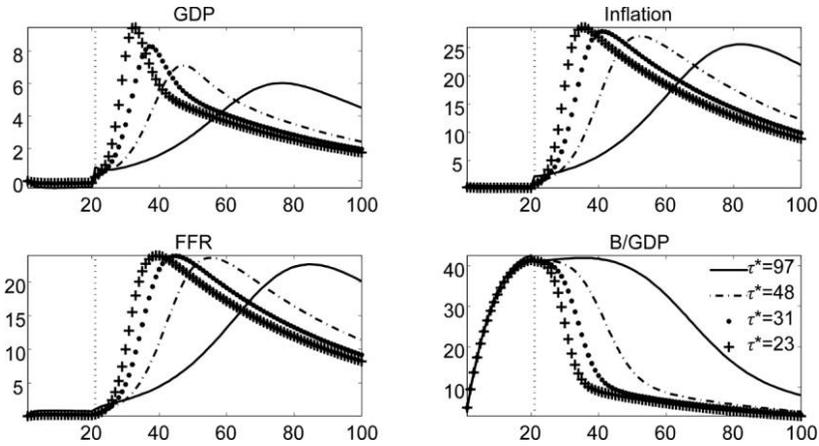
Fig. 7. The figure considers the effects of a large "dormant shock" to primary surplus for transition matrices that differ according to the relative persistence of the two PM/AF regimes

Notes: The shock occurs at time 0, while the regime change occurs after 20 periods (dotted vertical line). The different values for the relative persistence of the deviations are captured by the number of periods necessary to convince agents that they entered a long-lasting deviation (τ^*). FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

creases, it means that agents remain confident about a return to the virtuous regime even after a relatively long sequence of deviations. Even in this case, the relative persistence of the two regimes can be the result of some past evidence or determined by an arbitrary conjecture about what it means for a regime to be short lasting. What matters is that the persistences of the two regimes differ largely enough to make the distinction meaningful. Furthermore, it is important to keep in mind that the characterization of one regime affects the laws of motion of all the others. This implies that for a given set of parameters, there is a limit to how large the ratio p_{22}/p_{33} can be made without causing the short-lasting regime—and possibly the virtuous regime—to be inflationary.

Fiscal virtue can make the unfolding of the effects of dormant shocks smooth—a property that seems appropriate to characterize inflation dynamics in economies in which policymakers have a strong reputation, such as the US economy. Countries for which the commitment to fiscal responsibility is less clear are more likely to be subject to sudden shifts in agents' expectations as agents need only a few deviations in order to decide that the economy has entered a long-lasting deviation. This suggests an interesting interpretation of the different impulse responses considered in this subsection: following a shock of the same

magnitude, countries with different levels of fiscal virtue might experience similar levels of inflation as long as a virtuous regime is in place. However, once they experience a deviation from the virtuous regime, substantial inflation differentials would arise as the speed of learning greatly differs across countries.

V. The History and Future of US Inflation

In this last part of the paper, we conduct a quantitative analysis based on a richer model augmented with external habits, government purchases, inflation indexation to past inflation, and a maturity structure for government debt, as well as a richer fiscal block in which government expenditure and taxation are modeled separately. These features make the model more realistic and allow us to calibrate the model using the estimates obtained by Bianchi and Ilut (2012). Bianchi and Ilut (2012) estimate a DSGE model subject to a structural break from a PM/ AF regime to an AM/PF regime, and show that such a break can account for the rise and fall of US inflation and the change in the reduced-form properties of the macroeconomy. Here we expand the number of regimes to allow for a short-lasting and a long-lasting PM/AF regime and introduce the learning mechanism while relying on the estimates to pin down the timing of the switch to the virtuous regime. A fully specified estimation exercise is beyond the scope of this paper and, although feasible, poses some significant computational challenges that we plan to take up in future research (Bianchi, Melosi, and Ohanian 2012). The appendix contains an accurate description of the model, the parameter values, and impulse responses under perfect information and learning.

A.   A Look at the Past

In the previous section, we introduced the concept of dormant shocks and showed how they can propagate slowly over time and have the largest impact many years after they took place. We will now put the theory to work showing how a few shocks, combined with the learning mechanism, can go a long way in explaining the historical dynamics of inflation. Specifically, we will show that three events could be central to understanding the run-up of inflation in the 1970s. The first event is a prolonged deviation from the virtuous regime that lasted from the 1960s until the appointment of Paul Volcker as Federal Reserve Chairman in 1979. The second event is a large shock to the long-term component of government expenditure coinciding with the first reference to

the "Great Society" made by President Lyndon Johnson in May 1964. Following Bianchi and Ilut (2012), we model this as a large and persistent increase in government transfers. The third event is the tax cuts made by President Gerald Ford's administration in the mid-1970s.

We choose a transition matrix in order to satisfy two criteria. First, we make the learning process substantially slower, making the persistences of the short-lasting and long-lasting deviations more similar. Second, we make the ex ante conditional probability of a long-lasting deviation very low. We then consider the following parameterization for the transition matrix:

$$H = \begin{bmatrix} .9750 & .0249 & .0001 \\ .0700 & .9300 & \\ .0100 & & .9900 \end{bmatrix}$$

The a priori beliefs that agents assign to the policymakers having engaged in a long-lasting deviation are very low (0.4 percent), and agents tolerate relatively long deviations as the ratio p_{22}/p_{33} is relatively high. Therefore it takes around thirty-seven years for agents to become convinced that they entered a long-lasting deviation from the virtuous regime ($\tau^* = 148$).

We initialize the economy using the level of debt that was in place in 1955 and set all the other variables to their respective steady states. We then assume a path for tax revenues and total government expenditure that approximates the actual ones. These are reported in the second and third panels of figure 8. Given these paths and a sequence for policymakers' behavior to be described later, we reconstruct a path for inflation. This is reported in the first panel of figure 8. We assume that policymakers started deviating during the late 1950s, and we use the estimates of Bianchi and Ilut (2012) to pin down the return to the virtuous regime, occurring in mid-1980, a few quarters after the appointment of Volcker as Federal Reserve Chairman (August 1979). These two events are marked by two vertical dashed lines, and the gray area represents the period during which policymakers deviated from the virtuous regime. The two solid vertical lines mark the key shocks that are used to explain the increase in trend inflation of the 1970s: the first reference made to the Great Society plans by President Johnson and the Ford administration's tax cuts. Finally, the dotted line in the first panel represents the path of inflation under the assumption that policymakers always follow the virtuous regime.

Several aspects of this simulation are worth mentioning. Following
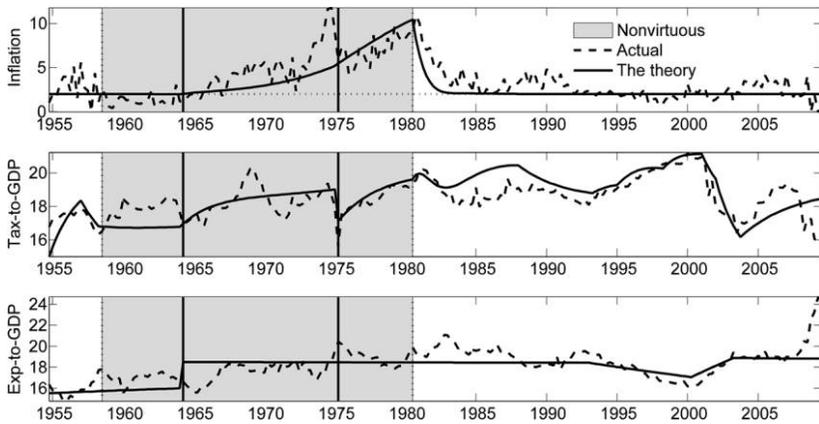
Fig. 8. The figure shows the results for a simulation meant to illustrate the properties of the model with learning

Notes: Four events are key: (1) policymakers start deviating from the virtuous regime in the late 1950s (first dashed vertical line); (2) a large shock to the long-term component of expenditure assumed to coincide with the first reference to the Great Society made by President Johnson (first solid vertical line); (3) the Ford administration's tax cut (second solid vertical line); and (4) the return to the virtuous regime a few quarters after the appointment of Volcker as Fed Chairman (second dashed vertical line). The solid lines correspond to the simulated data, the dashed line corresponds to the actual data, and the dotted line in the first panel shows the behavior of inflation under the assumption that policymakers always follow the virtuous regime.

the *Great Society shock*, inflation starts to rise, but very smoothly. In the short run, such low-frequency movements are probably hard to detect. However, inflation keeps increasing, slowly but steadily, even if in the simulated fiscal series there are no other shocks until the mid-1970s. By the time the Ford tax cuts hit, inflation has already gained some momentum because of the acceleration in agents' pessimism. Agents are now more pessimistic about the possibility of a quick return to the virtuous regime. Consequently, inflation rises faster following this second fiscal shock and accelerates. It is worth pointing out that none of these events would have caused movements in inflation if policymakers had always followed the virtuous regime, as exemplified by the dotted horizontal line in the first panel: All fiscal shocks are dormant when policymakers follow the virtuous regime. This inflationary spiral comes to an end with the appointment of Volcker as Federal Reserve Chairman that can be regarded as a strong signal about the future conduct of monetary and fiscal policies. The switch to the virtuous regime determines a sudden drop in inflation as agents' beliefs about policymakers' future

behavior are subject to a drastic swing. Fiscal shocks are now neutralized as agents revise their expectations about the way the debt will be financed. As a result, all the movements in taxation and expenditure that follow the return to the virtuous regime have a negligible effect on inflation.

In summary, we have taken a historical perspective in this subsection to show how dormant shocks can be used to explain low-frequency movements in the process for inflation. An important lesson is that it might take a long time to see significant effects of these shocks on inflation, which may give a false sense of security to policymakers. Finally, it is worth emphasizing that the shocks by themselves would not have any impact on the macroeconomy. It is the interaction between policymakers' behavior and the shocks that determines inflation dynamics.

B. To Understand Today and the Future

Having shown that the model can account for the events of the past, we now use the same calibration to analyze the current situation. Is the US economy heading toward a prolonged period of high inflation? Should policymakers feel assured that long-term interest rates and inflation expectations seem to be under control? In order to answer these questions, we will compute forecasts conditional on two different scenarios about policymakers' future behavior. For each forecast, we will use the state of the economy during the third quarter of 2009. For each simulation, we compute the path for five-year-ahead inflation expectations and the five-year bond yield. Both of them are computed taking into account the possibility of regime changes using the methods developed in Bianchi (2012).

Policymakers announce that the economy will be at the zero lower bound for one year. We approximate such an announcement using a third regime that has the same characteristics of the PM/AF regime, but for which the response to the output gap is set to zero. We believe this is a convenient and potentially appropriate way to model the zero lower bound, given that the Central Bank would probably react if inflation started rising substantially.

In the first scenario, the zero-lower-bound period is followed by the PM/AF policy mix, while in the second case, the virtuous policy mix prevails. Furthermore, we assume that policymakers do not make any announcement regarding the exit strategy, so agents do not know how the economy will evolve after the zero-lower-bound period. Therefore,
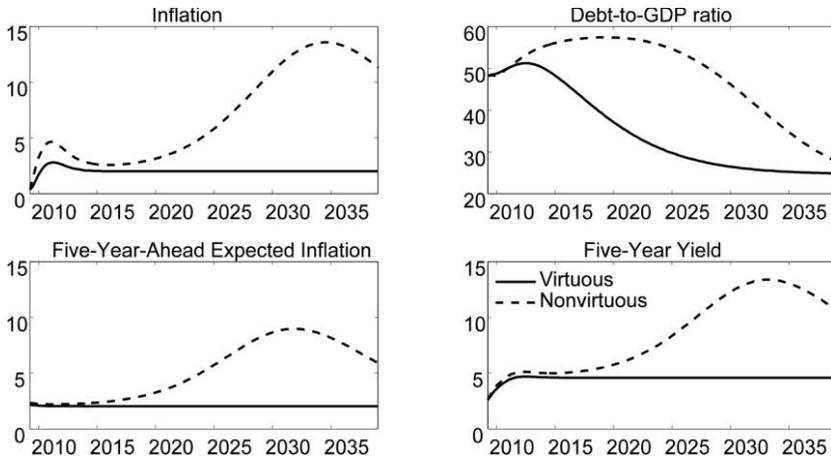
Fig. 9. Conditional forecasts based on a transition matrix that implies high reputation of policymakers in preventing large fluctuations in inflation

Notes: The zero lower bound is announced for one year. After that, two scenarios are considered: always PM/AF or always AM/PF. In both cases, no announcement is made about the exit strategy from the zero lower bound. Five-year-ahead inflation expectations and the five-year bond yield are computed, taking into account the possibility of regime changes.

it is very important to specify what agents expect is going to happen once the deviation is over. We assume that in the last period of the announced deviation, the probability that agents attach to going back to the virtuous regime is equal to the one that would prevail if they had observed one year of PM/AF policy. Notice that agents anticipate that this is the way they will think one year from the time of the announcement.

Figure 9 reports the results. The first aspect that is worth noting is that long-term inflation expectations are initially very well anchored regardless of policymakers' behavior. In a similar fashion, the five-year bond yield is very low, in line with what is observed in the data. This is because even if the PM/AF policy mix prevails, agents are initially optimistic about the probability of moving back to the virtuous combination. However, as time goes by, policymakers' behavior starts making a difference. If policymakers insist in following the PM/AF combination, agents get increasingly convinced that the economy has entered a long-lasting deviation from the virtuous regime. Accordingly, inflation and inflation expectations start drifting. Eventually inflation gets to levels that are comparable to the ones observed in the 1970s. If instead the

virtuous policy mix is in place, this pattern is absent, as agents observe taxes being raised in order to stabilize debt. In both cases, GDP growth experiences an acceleration in the short run that reflects the fact that the economy is going back to the steady state.

Interestingly, the first spur of inflation that is associated with the recovery dies out very quickly, indicating that an external observer could be induced to think that in both cases the central bank still has full control of inflation dynamics. But this is just an illusion, as the run-up of inflation that follows makes clear.

These conditional forecasts highlight the perils associated with trying to infer the risk of high inflation looking at current inflation expectations or long-term interest rates. If it takes time for agents to learn about the nature of the deviation that they are currently experiencing, then their expectations are likely to be initially very well anchored. This is likely to be especially true for an economy like that of the United States, which has experienced a prolonged period of monetary dominated policy mix. In such an economy, the a priori beliefs about the possibility of entering a long-lasting deviation are arguably very low. Furthermore, it is worth pointing out that short-lasting interventions that do not resolve the long-run problems of debt sustainability cannot be interpreted as changes in policy mix. These are only shocks that do not move agents' beliefs about the resolution of the long-term sustainability of government debt.

VI.   Conclusion

When agents are uncertain about the way debt will be stabilized, the strict distinction between Ricardian and non-Ricardian regimes typical of the literature on the fiscal theory of price level breaks down. In its stead, a continuum of regimes reflecting agents' beliefs about the future conduct of fiscal and monetary policies arises. As agents observe more and more deviations from a virtuous regime in which the central bank has full control of inflation, they become increasingly convinced that debt will be inflated away. This implies that the law of motion characterizing the economy evolves over time in response to what agents observe.

We introduced the notion of dormant shocks. These are shocks that move the debt-to-GDP ratio and that have no effect on the macroeconomic variables when policymakers behave according to a virtuous regime. However, as policymakers start deviating from such a regime

and agents become more and more discouraged about the possibility of moving back to the virtuous regime, the effects of the dormant shocks arise, with a progressive increase in inflation and uncertainty. Therefore, the model is able to generate a persistent and accelerating run-up in inflation as relatively optimistic agents become more and more pessimistic. This resolves an apparent puzzle of the fiscal theory of price level, that is, the fact that in the data fiscal shocks do not seem to cause an immediate increase in inflation.

   We used the model to point out that currently low inflation expectations and low long-term interest rates are likely to reflect the reputation US policymakers have built over the years. This means that the true risk of inflation might be higher than it appears and crucially related to the way policymakers will behave in the future. If policymakers keep deviating from the policies that have characterized their behavior in the three decades before the most recent financial crisis, agents' pessimism about policymakers' future behavior will eventually accelerate, and so will inflation. At that point, only an immediate change in *both* fiscal and monetary policies will be able to cut the inflationary spiral.

Appendix
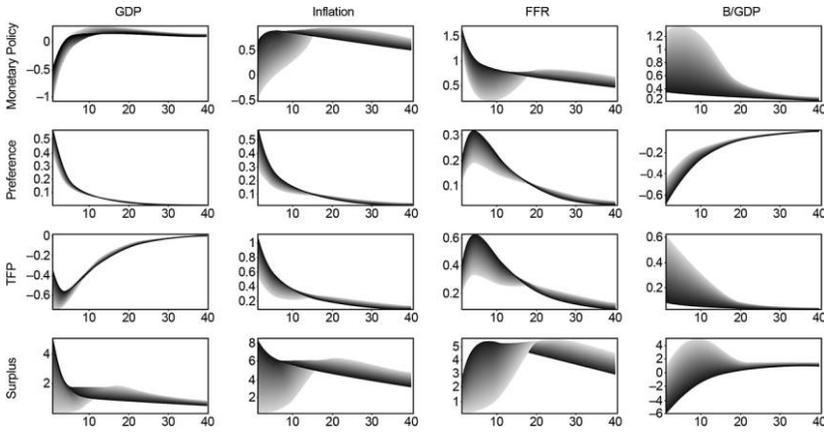**Impulse Responses for Benchmark Model**

Figure A1 reports the impulse responses for the benchmark model under perfect information. In the main text, we have already described how agents' expectations affect the propagation of a fiscal shocks. Agents' expectations also affect the response of the economy to the other shocks. The first row reports the responses to a monetary policy shock. Under all regimes, the Federal Reserve retains the ability to generate a recession. However, under the long-lasting PM/AF regime, inflation increases instead of declining. This "stepping on a rake" effect (Sims 2011) implies that the Central Bank loses its ability of controlling inflation the moment that its actions are not adequately supported by the fiscal authority. Notice that the inflation dynamics under the short-lasting deviation are very similar to the ones implied by the virtuous regime. This occurs despite the fact that the fiscal rule in place at the time of the shock determines an increase in the debt-to-GDP ratio that could appear to be permanent to an external observer. In the short run, given the presence of nominal rigidities, the increase in the FFR determines an increase in the real interest that makes the cost of debt larger. Under the short-lasting deviation, agents are confident that a return to
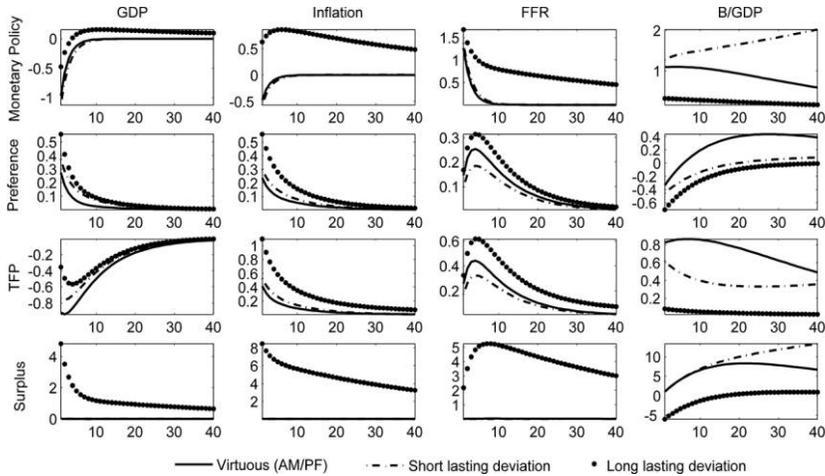
Fig. A1. Impulse responses for the benchmark model under perfect information
Note: FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

the virtuous regime will soon occur. This is why the increase in debt is not inflationary. Under the long-lasting deviation agents anticipate that the government will not increase primary surpluses in the near future and inflation increases in order to stabilize debt. The Central Bank accommodates the increase of inflation, and debt slowly declines because of negative real interest rates.[9]

The first row of figure A2 illustrates that the stepping on a rake effect also arises when introducing learning. When the initial number of deviations is small, in response to a monetary policy shock, inflation declines on impact and stays below the steady state for several periods. However, as agents become aware of the possibility that the economy entered a long-lasting deviation, inflation starts increasing and the stepping on a rake effect appears. If agents have already observed a large number of deviations, the entire sequence of events occurs more quickly and inflation immediately increases. These dynamics have two important implications. First, when the initial number of deviations is small, a central bank might be initially induced to believe that it is still able to control inflation, given that the stepping on a rake effect does not immediately manifest itself. Second, as policymakers keep deviating and the ability of the central bank to control inflation deteriorates, central bankers might erroneously be induced to think that structural

Fig. A2. Impulse responses for the benchmark model under learning assuming that the PM/AF regime prevails over the relevant horizon

Notes: Moving from the light to the dark, the initial number of observed deviations increases. The impulse responses under the virtuous regime are very similar, although not identical, to the ones that prevail under perfect information. FFR stands for federal funds rate and B/GDP stands for public-debt-to-GDP ratio.

changes are occurring in the economy that are making the sacrifice ratio less favorable.

Extended Model

In this appendix, we describe the model used in the quantitative analysis of section V. The model is a simplified version of the one used in Bianchi and Ilut (2012).

A. Model Description

Households. The representative household maximizes the following utility function:

$$E_0 \left[\sum_{s=0}^{\infty} \beta^s e^{d_s} [\log(C_s - \Phi C_{s-1}^A) - h_s] \right], \tag{18}$$

subject to the budget constraint

$$P_t C_t + P_t^m B_t^m + P_t^s B_t^s = P_t W_t h_t + B_{t-1}^s + (1 + \rho P_t^m) B_{t-1}^m + P_t D_t - NT_t,$$

where D_t stands for real dividends paid by the firms, C_t is consumption, P_t is the aggregate price level, h_t is hours, W_t is the real wage, NT_t stands for net taxes, and C_t^A represents the average level of consumption in the

economy. The parameter Φ captures the degree of external habit. The preference shock d_s has a mean of zero and the following time series representation: $d_t = \rho_d d_{t-1} + \sigma_d \varepsilon_{d,t}$. In line with Cochrane (2001), we recognize the importance of allowing for a maturity structure of government debt. Longer maturities imply important fluctuations in the return of bonds and consequently in the present value of debt. News about future surpluses can then translate into smooth changes in inflation, as opposed to a discrete jump in the current price level, even in absence of any additional friction. Hall and Sargent (2011) show that revaluation effects explain a significant fraction of the fluctuations of the debt-to-GDP ratio. Following Eusepi and Preston (2012) and Woodford (2001), we assume that there are two types of government bonds: one-period government debt, B_t^s, in zero net supply with price P_t^s and a more general portfolio of government debt, B_t^m, in nonzero net supply with price P_t^m. The former debt instrument satisfies $P_t^s = R_t^{-1}$. The latter debt instrument has the payment structure $\rho^{T-(t+1)}$ for $T > t$ and $0 < \rho < 1$. The value of such an instrument issued in period t in any future period $t+j$ is $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$. The asset can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$. Varying the parameter ρ varies the average maturity of debt.

Firms. Each of the monopolistically competitive firms faces a downward-sloping demand curve:

$$Y_t(j) = (P_t(j)/P_t)^{-1/\upsilon}Y_t, \tag{19}$$

where the parameter $1/\upsilon$ is the elasticity of substitution between two differentiated goods. The firms take as given the general price level, P_t, and the level of real activity, Y_t. Whenever a firm changes its price, it faces quadratic adjustment costs represented by an output loss:

$$AC_t(j) = .5\varphi\left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi_{t-1}\right)^2 \frac{Y_t(j)P_t(j)}{P_t}, \tag{20}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation at time t.

The firm chooses the price $P_t(j)$ to maximize the present value of future profits:

$$E_t\left[\sum_{s=t}^{\infty} Q_s\left(\frac{P_s(j)}{P_s}Y_s(j) - W_s h_s(j) - AC_s(j)\right)\right],$$

where Q_s is the marginal value of a unit of the consumption good. Labor is the only input in a linear production function, $Y_t(j) = A_t h_t(j)$,

where total factor productivity A_t evolves according to an exogenous process: $\ln(A_t/A_{t-1}) = \gamma + a_t$, $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$, $\varepsilon_{a,t} \sim N(0,1)$.

Government. Imposing the restriction that one-period debt is in zero net supply, the flow budget constraint of the *federal* government is given by

$$P_t^m B_t^m = B_{t-1}^m(1 + \rho P_t^m) - T_t + E_t + TP_t,$$

where $P_t^m B_t^m$ is the market value of debt and T_t and E_t represent federal tax revenues and federal expenditures, respectively. The term TP_t is a shock that is meant to capture a series of features that are not explicitly modeled here, such as changes in the maturity structure and the term premium. This shock is necessary to avoid stochastic singularity when estimating the model given that we treat debt, taxes, and expenditures as observables.[10] We rewrite the federal government budget constraint in terms of debt-to-GDP ratio $b_t^m = (P_t^m B_t^m)/(P_t Y_t)$:

$$b_t^m = \frac{b_{t-1}^m R_{t-1,t}^m}{\Pi_t Y_t/Y_{t-1}} - \tau_t + e_t + tp_t,$$

where all the variables are now expressed as a fraction of GDP and $R_{t-1,t}^m = (1 + \rho P_t^m)/P_{t-1}^m$ is the realized return of the maturity bond. We assume $tp_t = \rho_{tp} tp_{t-1} + \sigma_{tp} \varepsilon_{tp,t}$, $\epsilon_{tp,t} \sim N(0,1)$.

The (linearized) federal government expenditure as a fraction of GDP, $\tilde{e}_t$, is the sum of a short-term component $\tilde{e}_t^S$ and a long-term component $\tilde{e}_t^L(\tilde{e}_t = \tilde{e}_t^L + \tilde{e}_t^S)$:[11]

$$\tilde{e}_t^L = \rho_{eL}\tilde{e}_{t-1}^L + \sigma_{eL}\epsilon_{eL,t}, \quad \epsilon_{eL,t} \sim N(0,1),$$

$$\tilde{e}_t^S = \rho_{eS}\tilde{e}_{t-1}^S + (1 - \rho_{eS})\phi_y(\hat{y}_t - \hat{y}_t^n) + \sigma_{eS}\epsilon_{eS,t}, \quad \epsilon_{eS,t} \sim N(0,1).$$

The long-term component is assumed to be completely exogenous, and it is meant to capture the large programs that arise as the result of a political process that is not modeled here. Consistently with this interpretation, we assume that this component of government expenditure is known one year ahead. Instead, the short-term component is meant to capture the response of government expenditure to the business cycle and responds to the (log-linearized) output gap $(\hat{y}_t - \hat{y}_t^n)$, where $\hat{y}_t^n$ is the natural output, that is, the level of output that would prevail under flexible prices. Notice that government expenditure is the sum of *federal* transfers and good purchases.

The federal and local/state governments buy a fraction ζ_t of total out-

put, equally divided among the J different goods. We define $g_t = 1/(1 - \zeta_t)$, and we assume that $\tilde{g}_t = \ln(g_t/g^*)$ follows the process

$$\tilde{g}_t = \rho_g \tilde{g}_{t-1} + (1 - \rho_g)\phi_{e^S}\tilde{e}_{t-1}^S + \sigma_g \epsilon_{g,t}, \, \epsilon_{g,t} \sim N(0, 1). \tag{21}$$

Before proceeding it is important to point out that we assume that local and state governments participate in purchasing goods and that they are supposed to run a balanced budget. Therefore, changes in net taxes at the state level are neutral as agents understand that they will be offset by future changes in the opposite direction. We believe this is a reasonable assumption as it is quite unlikely that in the United States local governments can exercise influence on the conduct of monetary policy.

Monetary and Fiscal Rules. The Central Bank moves the FFR according to the rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_{R(\xi_t)}} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\psi_{\pi(\xi_t)}} \left(\frac{Y_t}{Y_t^n}\right)^{\psi_{y(\xi_t)}}\right]^{(1-\rho_{R(\xi_t)})} \sigma_R e^{\epsilon_{R,t}}, \, \epsilon_{R,t} \sim N(0, 1), \tag{22}$$

where R is the steady-state (gross) nominal interest rate, Y_t^n is natural output, and Π is gross steady-state inflation. The federal fiscal authority moves taxes according to the rule

$$\tilde{\tau}_t = \rho_{\tau(\xi_t)}\tilde{\tau}_{t-1} + (1 - \rho_{\tau(\xi_t)})[\delta_{b(\xi_t)}\tilde{b}_{t-1}^m + \delta_e\tilde{e}_t] + \delta_y(\hat{y}_{t-1} - \hat{y}_{t-1}^n)$$
$$+ \sigma_\tau \epsilon_{\tau,t}, \, \epsilon_{\tau,t} \sim N(0, 1), \tag{23}$$

where $\tilde{\tau}_t$ denotes (linearized) tax revenues with respect to GDP.

In equations (6) and (5), ξ_t is an unobserved state variable capturing the monetary/fiscal policy combination that is in place at time t. The unobserved state takes on a finite number of values $j = 1, \ldots, m$ and follows a Markov chain that evolves according to a transition matrix H. The targets for inflation and debt are assumed to be constant over time.[12] What changes is the strength with which the government tries to pursue its goals, not the goals themselves. This is in line with the idea that policymakers might find high inflation or high debt acceptable under some circumstances (perhaps in order to preserve output stability) but not desirable in itself. Once the model is solved, the variables can be rescaled in order to induce stationarity. The model is then linearized with respect to taxes, government expenditure, and debt, whereas it is log-linearized with respect to all the other variables. We obtain the following system of equations:

1. IS curve:

$$(1 + \Phi\gamma^{-1})\hat{y}_t = \tilde{g}_t(1 + \Phi\gamma^{-1} - \rho_g) + \Phi\gamma^{-1}(\hat{y}_{t-1} - \tilde{g}_{t-1})$$
$$- (1 - \Phi\gamma^{-1})[\tilde{R}_t - E_t[\tilde{\pi}_{t+1}] + (\rho_d - 1)d_t] + E_t[\hat{y}_{t+1}]$$
$$+ (\rho_a - \Phi\gamma^{-1})a_t;$$

2. Phillips curve:

$$(1 + \beta)\tilde{\pi}_t = \kappa(1 - \Phi\gamma^{-1})^{-1}[\hat{y}_t - \tilde{g}_t - \Phi\gamma^{-1}[\hat{y}_{t-1} - \tilde{g}_{t-1} - a_t]] + \kappa\mu_t$$
$$+ \tilde{\pi}_{t-1} + \beta E_t[\tilde{\pi}_{t+1}],$$

where $\kappa \equiv (1 - \upsilon)/\upsilon\phi\Pi^2$;

3. Monetary policy rule:

$$\tilde{R}_t = \rho_{R(\xi_t)}\tilde{R}_{t-1} + (1 - \rho_{R(\xi_t)})[\psi_{\pi(\xi_t)}\tilde{\pi}_t + \psi_{y(\xi_t)}(\hat{y}_t - \hat{y}_t^n)] + \sigma_R\epsilon_{R,t};$$

4. Total government purchases:

$$\tilde{g}_t = \rho_g\tilde{g}_{t-1} + (1 - \rho_g)\phi_{eS}\tilde{e}_{t-1}^S + \sigma_g\epsilon_{g,t};$$

5. Fiscal rule:

$$\tilde{\tau}_t = \rho_{\tau(\xi_t)}\tilde{\tau}_{t-1} + (1 - \rho_{\tau(\xi_t)})[\delta_{b(\xi_t)}\tilde{b}_{t-1}^m + \delta_e\tilde{e}_t] + \delta_y(\hat{y}_{t-1} - \hat{y}_{t-1}^n) + \sigma_\tau\epsilon_{\tau,t};$$

6. Debt:

$$\tilde{b}_t^m = \beta^{-1}\tilde{b}_{t-1}^m + b^m\beta^{-1}(\tilde{R}_{t-1,t}^m - \hat{y}_t + \hat{y}_{t-1} - a_t - \tilde{\pi}_t) - \tilde{\tau}_t + \tilde{e}_t^S + \tilde{e}_t^L + tp_t;$$

7. Return long-term bond:

$$\tilde{R}_{t,t+1}^m = R^{-1}\rho\tilde{P}_{t+1}^m - \tilde{P}_t^m;$$

8. No arbitrage:

$$\tilde{R}_t = E_t[\tilde{R}_{t,t+1}^m];$$

9. Expenditure, short-term component:

$$\tilde{e}_t^S = \rho_{eS}\tilde{e}_{t-1}^S + (1 - \rho_{eS})\phi_y(\hat{y}_t - \hat{y}_t^n) + \sigma_{eS}\epsilon_{eS,t};$$

10. Expenditure, long-term component (assumed to be known four periods in advance):

$$\tilde{e}_t^L = \rho_{eL}\tilde{e}_{t-1}^L + \sigma_{eL}\epsilon_{eL,t};$$

Table A1
Parameter choices of the DSGE parameters and of the transition matrix elements

Parameter	Value	Parameter	Value	Parameter	Value
$\psi_\pi(\xi_t = 1)$	0.6244	δ_e	0.7045	$100\sigma_R$	0.1972
$\psi_\pi(\xi_t = 2)$	2.3522	δ_y	0.0869	$100\sigma_\tau$	0.4564
$\psi_y(\xi_t = 1)$	0.3716	ρ_e	0.9950	$100\sigma_a$	0.6518
$\psi_y(\xi_t = 2)$	0.1527	Φ	0.7779	$100\sigma_e$	0.3653
$\rho_R(\xi_t = 1)$	0.8480	ρ_a	0.4540	$100\sigma_d$	6.9498
$\rho_R(\xi_t = 2)$	0.8132	ρ_d	0.6125		
$\delta_{\tau,b}(\xi_t = 1)$	0	κ	0.0128	p_{11}	0.9000
$\delta_{\tau,b}(\xi_t = 2)$	0.0327	$100\ln(\gamma)$	0.4896	p_{12}	0.0990
$\rho_\tau(\xi_t = 1)$	0.7306	b^*	0.9644	p_{22}	0.7000
$\rho_\tau(\xi_t = 2)$	0.8921	τ^*	0.1846	p_{33}	0.9900

Note: Based on Bianchi and Ilut (2012).
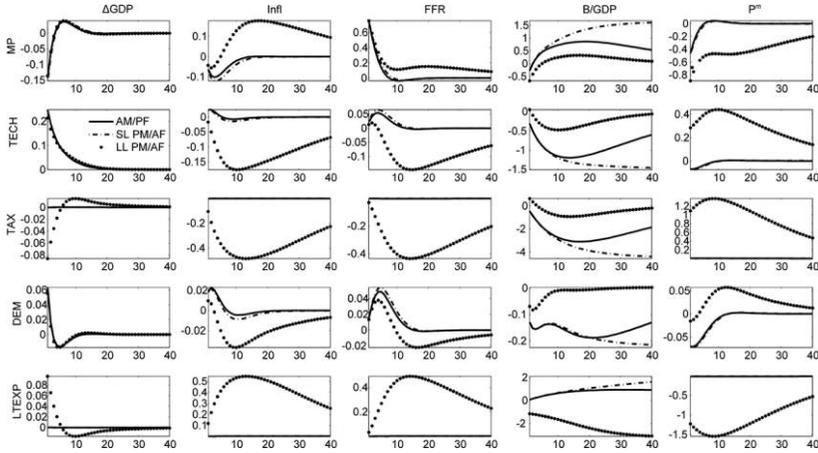
Fig. A3.   Impulse responses for the extended model under perfect information
Notes: MP, TECH, TAX, DEM, and LTEXP stand for monetary policy, technology, tax, demand, and long-term expenditure shock, respectively. FFR and B/GDP stand for Federal Funds rate and public-debt-to-GDP ratio.
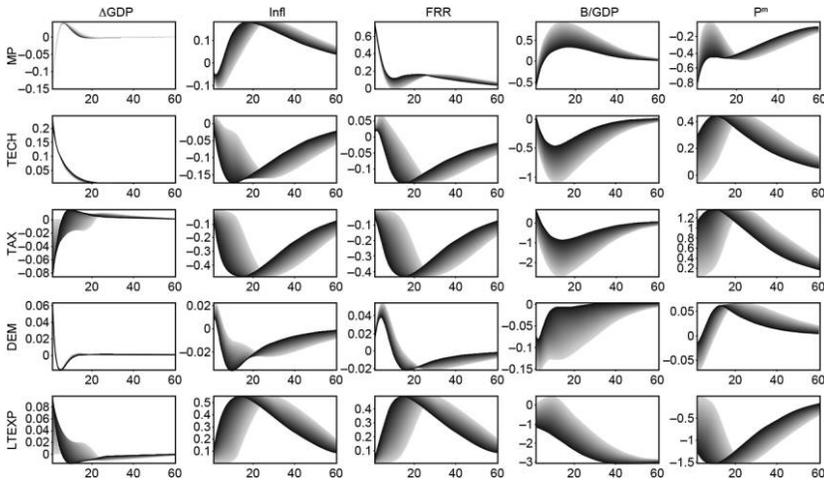
11. Term premium/maturity shock:

$$tp_t = \rho_{tp}tp_{t-1} + \sigma_{tp}\epsilon_{tp,t};$$

12. Technology:

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t};$$

13. Demand shock:

$$d_t = \rho_d d_{t-1} + \sigma_d \epsilon_{d,t}.$$

Fig. A4. Impulse responses for the extended model under learning assuming that the PM/AF regime prevails over the relevant horizon

Notes: When moving from the light lines to the dark lines, the initial number of observed deviations increases. The impulse responses under the AM/PF are very similar, although not identical, to the ones that prevail under perfect information. MP, TECH, TAX, DEM, and LTEXP stand for monetary policy, technology, tax, demand, and long-term expenditure shock, respectively. FFR and B/GDP stand for Federal Funds rate and public-debt-to-GDP ratio.

C. Impulse Responses

Figures A3 and A4 report the impulse responses for the extended model under perfect information and under learning, respectively.

Endnotes

1. See Cochrane (2011) for an effective discussion of the difference between the early approach of Sargent and Wallace (1981) and the subsequent analysis based on the fiscal theory of price level. See Atkeson, Chari, and Kehoe (2009) for an alternative approach to price determination in monetary general equilibrium models.

2. We linearize with respect to debt and primary surpluses, given that these variables can change sign, while we log-linearize with respect to all the others.

3. In the presence of nominal rigidities, the threshold for monetary policy is not exactly 1, given that the Central Bank's response to the output gap contributes to stabilize inflation. For simplicity, here we assume $\Psi_y = 0$.

4. This result can be derived by applying Bayes' theorem and then combining the resulting probabilities with the transition matrix H. See Bianchi and Melosi (2012b) for a detailed derivation.

5. It is not hard to extend the model to allow for a short-lasting and a long-lasting AM/PF regime. Bianchi and Melosi (2012b) consider this case.

6. Nevertheless, the impulse responses would not be *identical* because the uncertainty that prevails under the PM/AF regime also affects the law of motion under the virtuous regime.

7. However, even when agents think that they are in the long-lasting PM/AF regime with probability one, the law of motion still slightly differs from the one implied by the model with perfect information because agents' expectations about the future reflect the additional uncertainty that derived from the learning mechanism.

8. This also explains why we labeled the AM/PF regime "virtuous." In new-Keynesian models output gap and inflation volatilities are key to determine agents' welfare (see Bianchi and Melosi 2012a).

9. The stepping on a rake effect is a robust finding, and it holds also in the richer model considered by Bianchi and Ilut (2012). However, the short-run inflation dynamics can differ depending on the exact features of the model.

10. An alternative approach consists of excluding one of the fiscal components, for example, the series for debt. Our results are robust to this alternative specification. We also considered an alternative specification in which an observation error for the series of debt is included and the term premium shock is eliminated. The results are virtually identical.

11. In what follows, for a given variable X_t, $\hat{x}_t \equiv \log((X_t/A_t)/(X/A))$ represents the percentage deviation of a detrended variable from its own steady state. For all the variables normalized with respect to GDP (debt, expenditure, and taxes), $\tilde{x}_t$ denotes a linear deviation ($x_t = X_t - X$), while for all the other variables, $\tilde{x}_t$ denotes a percentage deviation ($\tilde{x}_t = \log(X_t/X)$). This distinction avoids having the percentage change of a percentage.

12. See Ireland (2007), Liu et al. (2011), and Schorfheide (2005) for models that allow for a time-varying target.

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