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Pledgability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity

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Collateral is, after all, only good if a creditor can get his hands on it.
—Niall Ferguson, *The Ascent of Money*

I. Introduction

We develop a theory of the role of assets in the exchange process and use it to study several issues in macro, monetary, and financial economics, both analytically and quantitatively. Our approach begins with the premise that the intertemporal allocation of resources is hindered by limited commitment. Interacted with some notion of imperfect monitoring or record-keeping, as stressed by Kocherlakota (1998) and Wallace (2010), limited commitment implies that assets have a role in facilitating credit transactions. In our view, desiderata for a theory that takes this seriously are: (a) it must use a general equilibrium approach, in the sense of working within a *complete and internally consistent* description of an economic environment; and (b) it must go beyond classical equilibrium analysis by modeling agents as *trading with each other*, not merely against their budget constraints. When one has such a theory, one can start to ask *how* agents trade: Is it bilateral or multilateral? Are the terms of trade taken parametrically or set strategically? Do they use barter, money, or credit? If they use credit, how is repayment enforced? It is from this vantage that we study macro and financial activity.

By way of example, suppose that you want something, either a consumption or a production good, from someone now, but you have no good that they want at the moment, so you cannot barter directly. If you will have something at a later date that they want—maybe cash, maybe goods or claims to goods, or general purchasing power—you

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could promise that if they give you what you want now you will reciprocate by transferring something to them in the future. But they worry you may renege (that is what a lack-of-commitment friction means). What mechanism can provide incentives that encourage you to honor your obligations? Theories like Kehoe and Levine (1993, 2001) and Alvarez and Jermann (2000) punish those who default by taking away their access to future credit. That can be difficult, however, when there is imperfect monitoring or record-keeping. With limited ability to punish those who renege, unsecured credit does not work well.

In this situation there emerges a role for assets. There are two ways this can work. First, if you want something and have assets at hand, you can turn them over to a counterparty now, finalizing the transaction. In this case assets serve as a medium of exchange, as in Kiyotaki and Wright (1989, 1993). Second, you can assign to the seller the right to seize some of your assets in the event that you renege on the deferred payment. In this case the assets serve as collateral, as in Kiyotaki and Moore (1997, 2005). Collateral is useful in the presence of commitment issues because it helps ensure compliance: if you fail to honor an obligation you lose the collateral, and, to the extent that you value it, this helps deter opportunistic misbehavior (notice for this to work it is not necessary that the counterparty values the collateral; it is enough that you do). While these two ways in which assets may facilitate intertemporal exchange—serving as a medium of exchange or as collateral—look different on the surface, they are often equivalent.[1]

We proceed with the interpretation that assets serve as collateral. In this situation, what matters is the fraction of one's assets that can be seized in a default. As Holmstrom and Tirole (2011) put it, what matters is *pledgability*, which is related to *liquidity*. We formalize this in a framework that nests standard growth and asset-pricing theory, and can be viewed as an extension of the New Monetarist models recently surveyed by Williamson and Wright (2010) and Nosal and Rocheteau (2011). Since we can price currency as well as capital, equity, real estate, and so forth, we can analyze the effects of monetary policy on investment, stock markets, housing markets, and the like. Classic results by Fisher, Mundell, Tobin et. al. emerge as special cases, clarifying how inflation affects asset returns. It also affects output and employment, and the model can generate a stable Phillips curve. One can also analyze open-market operations, or other policies where the public and private sectors swap assets, as well as the impact of financial development.

While many results can be derived analytically, we also calibrate the model to study the effects of policy and innovation quantitatively. In the baseline calibration, higher inflation rates over some range increase output, employment, investment, the price and quantity of housing, and the value of the stock market. This is driven by a Mundell-Tobin effect that makes agents substitute out of real balances and into other assets when inflation rises. The nominal returns on illiquid assets go up one-for-one with inflation, but the nominal returns on partially-liquid assets go up by less. Hence, inflation reduces real returns on bonds, capital, and housing. In the baseline model welfare is *increasing* in inflation over a reasonable range, due to the Mundell-Tobin effect combined with the fact that capital accumulation is too low due to the taxation of asset income (without such taxes, the Friedman rule is optimal). For the baseline calibration the optimal inflation rate is close to the mean in the data, although this is sensitive to parameters. In terms of the financial variables, an increase in the pledgability of home equity, for example, initially leads first to a boom in house prices, construction, investment, and employment, but then a bust. Still, it can increase welfare, even if it might look bad for some macro variables.[2]

The rest of the paper is organized as follows. Section II lays out the basic assumptions. This includes a discussion of debt limits, since they are the heart of the model, and of pricing mechanisms, since the theory allows different approaches to determining the terms of trade. Section III defines equilibrium and describes three possible outcomes: liquidity may be plentiful and hence the economy achieves efficiency without using money; liquidity may be less plentiful but money does not help; and liquidity may be sufficiently scarce that money becomes essential. For each case we derive analytic predictions. Section IV discusses extensions. Section V presents the quantitative analysis. The model is calibrated and used to study the effects of inflation on allocations, prices, and welfare, and to study the effects of financial innovation. Section VI concludes.[3]

To close these introductory remarks, before we begin the formal presentation, we take up a suggestion from the editors to briefly discuss what we have in mind by a New Monetarist model. The general idea is discussed at length in Williamson and Wright (2010) and Nosal and Rocheteau (2011), but we can summarize it as follows. First, the approach is meant to be distinct from New Keynesian models because it downplays nominal rigidities. It is not that people cannot (or have not) put sticky prices into these models, but that it is not a critical component, the way it is in New Keynesian economics. This is partly because users of these models are often more interested in longer run issues. It is also because they want to focus on other frictions. As mentioned, a defining characteristic of the approach is that agents in the model trade with each other, which is not true in Walrasian models. To us, Walrasian models with cash-in-advance constraints are *not* a satisfactory compromise between general equilibrium theory and macroeconomics. The reason is not because those models are less "deep," but because agents still trade along their budget lines, subject to an ad hoc restriction.

Once we model agents trading with each other, explicitly, we can start to endogenize how they trade. Missing markets, incomplete contracts, and nominal rigidities are to be avoided, unless they can be microfounded, by which we mean they emerge as outcomes and are not forced on the model as assumptions. The reason is this: if the agents in the model can find better ways to trade than those assumed by the modeler, then we think the agents should be allowed to do so. Some people probably put too much faith in the (competitive) price mechanism. Sticky price or wage models certainly do. At the risk of generalizing, in these models, agents adhere strictly to competitive price taking, and then the modeler confronts them with the wrong relative prices. Of course the outcome is inefficient. A principle to which it may be hard to adhere in practice, but is nevertheless worth aiming for, is the following: if the agents in the model are being poorly treated by, say, a perverse Walrasian auctioneer in cahoots with the Calvo fairy, the agents in the model should be allowed to find better ways to trade.

No one ought to object to this in spirit, and it is of course a matter of degree. Microfoundations are not usually black or white, only shades of grey. But relative to some other approaches, we are more inclined to strive for better microfoundations, even if at times it means compromising on other dimensions, including those related to empirical or policy issues. It is not that we are uninterested in those issues—as this paper hopefully shows—but that at the margin we are relatively more concerned with logical consistency, or at least we try to be. Moreover, of course it may not be possible to achieve first-best outcomes when there are genuine frictions in the environment. Believing that agents in a model should be allowed to exploit all the possible gains from trade, or at least that they should not be forced to leave obvious gains from trade sitting on the table, does not mean there are not genuine impediments to trade that should be explicitly modeled.

Genuine frictions include spatial or temporal separation, although it would be very wrong to think, as some people do, that search is a critical component in New Monetarist economics. Still, a healthy outcome of modeling some of the issues in terms of search theory in early work was that some interesting new ingredients came to be incorporated in the theory, including arrival rates (or more generally, matching considerations) and bargaining powers (or more generally, different mechanisms for determining the terms of trade, e.g., price posting and auctions). The claim is *not* that New Monetarists invented these tools—that would be foolish—only that their models incorporate them. Textbook versions of New Keynesian models do not; nor do models with cash-in-advance constraints or money in the utility function. In studying money, banking, and credit arrangements we borrow from other literatures where frictions matter, like the literature on unemployment. However, in our models, the more relevant frictions turn out to be limited commitment or enforcement, and some notion of imperfect information.[4]

Again, these issues are discussed in depth elsewhere, and can only be touched on here. Still, it may be useful to keep them in mind when reading the paper. It may help, for instance, in understanding the way we construct and motivate certain elements of the theory. Limited commitment is what gives assets a role in exchange because the seizure of collateral is a punishment device. This sounds right. Yet we are not completely satisfied with pledgability as a modeling device, and think that it must be important to delve more deeply into the microfoundations at some point. A few hints about ways to proceed along these lines are provided later, but this paper is not about that. We provisionally take pledgability restrictions as given, to see what they imply in a full-blown macro model, with labor, capital, housing, money, taxation, and so forth. One always has to be wary of falling into the trap of getting too comfortable with more-or-less exogenous restrictions—like sticky prices—but for now, we make a compromise in terms of theory in order to confront data and policy issues. We learned a lot from the exercise, and hope readers will too.

II. Environment

This section describes preferences, technology, and so forth, then discusses credit frictions and mechanisms for determining the terms of trade.

A. *Fundamentals*

There is a [0, 1] continuum of infinitely-lived households. Each period in discrete time has two distinct markets that meet sequentially. One is a frictionless centralized market, called AD for Arrow-Debreu, where agents trade assets, labor, and some goods. The other is a market where they trade different goods subject to various frictions that impede credit, as detailed following, called KM for Kiyotaki-Moore. We assume KM convenes before AD, but little depends on this. All agents always participate in AD, while only a measure $2\sigma \leq 1$, chosen at random each period, participate in KM. By not participating, we mean some households neither derive utility from, nor have an endowment of, KM goods that period. Of the measure σ that participate, they all have an endowment $\bar{q}$, while $\sigma/2$ have utility function $u_b(Q)$ and $\sigma/2$ have utility function $u_s(Q)$, where $u_b'(Q) > u_s'(Q)$ for all Q, and the subscripts signify buyer and seller. Buyers and sellers meet in KM, and potentially trade because the former have higher marginal utility.[5]

If a seller in KM gives $q \leq \bar{q}$ to a buyer, the cost to the former is $c(q) \equiv u_s(\bar{q}) - u_s(\bar{q} - q)$and the gain for the latter is $u(q) \equiv u_b(\bar{q} + q) - u_b(\bar{q})$. Strictly speaking, q is a transfer, while $Q_b = \bar{q} + q$ and $Q_s = \bar{q} - q$ are net consumption for buyers and sellers, but we sometimes refer to q as KM consumption. Given u_b and u_s satisfy the usual monotonicity and curvature assumptions, so do u and c. This notation looks just like the setup in models where there is random matching and sellers are households that produce (e.g., Lagos and Wright 2005). The interpretation here, making KM a pure-exchange market, implies that all production occurs in AD, which is not at all crucial but is convenient for some purposes discussed later. In any case, for a seller to hand over q, he must get something in return. While many models of this type adopt the interpretation of assets as a medium of exchange, as we said earlier, they are used here as collateral to secure promises of payment in the next AD market.

The specification captures in an abstract way the notion that households sometimes want to make certain purchases—for example, household or automobile repairs, or medical treatment—for which they need loans. They need loans in the model because in between two AD markets they have no current income. These loans require collateral because limited commitment means agents are free to renege on promises. If no punishments are available beyond seizing collateral, sellers will only accept pledges of future payments up to some limit that depends on the value of one's assets. In reality, unsecured credit is also impossible, and some expenditure on home improvement, medical treatment, and so forth can be put on one's credit card. But as long as there are limits, some loans require collateral. We allow unsecured debt up to a limit; beyond this, assets must be used to secure loans.

Moving to the AD market, as in standard growth theory, there is a numeraire good x used for consumption or investment, produced by firms using capital and labor with technology $f(k, \ell)$ that is strictly increasing and concave. Usually, f displays CRS (constant returns to scale), but for some results that is not necessary, and it suffices to assume inputs are complements, $f_{k\ell} > 0$. Profit maximization implies

$$\omega = f_\ell(k, \ell) \text{ and } \rho = f_k(k, \ell), \tag{1}$$

where ω is the wage and ρ the rental rate on capital in terms of numeraire.[6] Firms are owned by households, and if there are profits they are dispersed as dividends. As always, CRS implies that profits are 0 in equilibrium.

In addition to k, households own housing, h. Let δ_k and δ_h be the depreciation rates on k and h. Housing can be in fixed supply H, in which case $\delta_h = 0$, or produced endogenously, as discussed later. There is also equity e in a Lucas (1978) tree paying dividend γ in each AD market. This is always in fixed supply, normalized to 1. There is also fiat money m. Its supply M evolves over time depending on policy: $M' = (1 + \pi)M$. There is also a real bond b that can be purchased in one AD market and redeemed in the next. Its supply B also depends on policy. A portfolio $\mathbf{a} = (b, e, h, k, m)$ lists these assets (as a mnemonic device, in alphabetical order). Let $\boldsymbol{\phi} = (\phi_b, \phi_e, \phi_h, \phi_k, \phi_m)$ be the asset price vector, with $\phi_k = 1$ because k and x are the same object. Notice a has some reproducible assets, like k, some in fixed supply, like e, and some that can be either, like h. It includes m and b so one can discuss traditional monetary policy. It is also worth mentioning that m does not need to be cash, and can include bank deposits, avoiding a common disconnect between theory and measurement (see, e.g., Lucas 2000).

Households have period utility over AD consumption, housing, and labor $\mathcal{U}(x, h, \ell) = U(x, h) - \zeta\ell$, where for now $\zeta = 1$. Quasi-linear utility simplifies analytic results, but there should be no presumption that the key insights hinge on it. Chiu and Molico (2010, 2011) numerically study models with more general preferences, and derive results quantitatively similar to those in quasi-linear versions. Also, Rocheteau et al. (2008) and Wong (2012) show different ways to relax this without affecting tractability, but for now we stick with quasi-linear utility. In addition to usual monotonicity and concavity restrictions, for a few results x and h are assumed to be normal goods. Finally, in terms of preferences, households have discount factor $\beta \in (0, 1)$ between the AD market and the next KM market, but without loss of generality there is no discounting between KM and AD.

A household in AD with portfolio a has net worth in terms of numeraire

$$y = y(\mathbf{a}) = b + (\gamma + \phi_e)e + (1 - \delta_h)\phi_h h + (\rho + 1 - \delta_k)k + \phi_m m - d + I, \quad (2)$$

where d is debt (which could be negative) from the previous KM market and I denotes other income, including dividends and lump-sum transfers minus taxes. All KM debt is settled each period in AD; given the preference structure, this is without loss in generality as long as $|d|$ is not so big that we get a corner solution. Also, (2) presumes there is no default, which is true in equilibrium; if one were to default, d would vanish from the RHS, and any assets that were pledged would be subtracted. The individual state variable is (y, h), since by assumption one has to own housing at the start of a period to enjoy its service flow.

If $W(y, h)$ denotes the value function of a household in AD then

$$W(y, h) = \max_{x,\ell,\hat{\mathbf{a}}}\{U(x, h) - \ell + \beta V(\hat{\mathbf{a}})\} \text{ s.t. } x = y + \omega\ell - \phi\hat{\mathbf{a}},$$

where $V(\hat{\mathbf{a}})$ is the continuation value at market closing, generally depending on the composition of the portfolio $\hat{\mathbf{a}}$, not just its value. Eliminating ℓ using the budget equation, we reduce this to

$$W(y, h) = \frac{y}{\omega} + \max_{x}\left\{U(x, h) - \frac{x}{\omega}\right\} + \max_{\hat{\mathbf{a}}}\left\{-\frac{\phi\hat{\mathbf{a}}}{\omega} + \beta V(\hat{\mathbf{a}})\right\}. \quad (3)$$

This implies W is linear in y with slope $1/\omega$. Also, the choice of $\hat{\mathbf{a}}$ is independent of y, so all households exit the AD market with the same portfolio. Hence we do not have to track a distribution of $\hat{\mathbf{a}}$ in KM as a state variable, which is the simplification that follows from quasi-linearity.

The value function for a household entering the KM market is

$$V(\mathbf{a}) = W[y(\mathbf{a}), h] + \sigma[u(q) - d/\omega] + \sigma\tilde{d}/\omega - c(\tilde{q})], \quad (4)$$

where (q, d) denotes the terms of trade when the agent is a buyer, comprised of a quantity q and a debt obligation d coming due in the following AD market, and $(\tilde{q}, \tilde{d})$ denotes the terms of trade when the agent is a seller. The first term on the RHS is one's payoff if one does not participate in KM. The second term is the expected surplus from being a KM buyer, since

$$u_b(\bar{q} + q) + W[y(\mathbf{a}) - d, h] - u_b(\bar{q}) - W[y(\mathbf{a}), h] = u(q) - d/\omega,$$

since $u(q) = u_b(\bar{q} + q) - u_b(\bar{q})$, and W is linear in wealth. The final term is the expected surplus from being a KM seller.

B. *Debt Limits*

A household's debt position is

$$d = d(\mathbf{d}) = d_b + (\gamma + \phi_e)d_e + (1 - \delta_h)\phi_h d_h + (\rho + 1 - \delta_k)d_k + \phi_m d_m + d_u, \quad (5)$$

where $\mathbf{d} = (d_b, d_e, d_h, d_k, d_m, d_u)$ is a vector of asset pledges, plus unsecured debt d_u. In (5), bond pledges are evaluated at face value, as are money and unsecured debt; pledges of equity are evaluated cum dividend; pledges of capital are evaluated before factor markets convene; and home equity pledges are evaluated at market prices after depreciation. As regards housing, this reflects a timing assumption, that a creditor can seize h if a debtor defaults, but foreclosure occurs at the end of the period.

The more important point is that we do *not* have in mind borrowers making promises that oblige them to deliver particular quantities of individual assets—they only pledge to deliver general purchasing power (numeraire). Since AD is a centralized market, neither borrowers nor lenders care about the instrument of settlement, and d_j is only interesting off the equilibrium path in the event of default. If you owe d and renege, the creditor—or maybe the court, or some other institution—seizes $D_j(a_j) \leq a_j$ of your holdings of a_j. Note that $D_j(a_j)$ is not the gain realized by the creditor from asset seizure, but the loss to the debtor. Sellers give up goods in KM because they want general purchasing power, not specific assets, and they believe you will deliver it up to a point to avoid punishment.

It is a best response to renege when the loss from forfeiture is less than the value of one's obligations. Of course, there can be additional punishments, as it used to be standard in practice to incarcerate defaulters, and it is now standard in theory to take away their future credit, although this punishment is not without potential problems. One such problem is that after defaulting on a particular creditor, even leaving renegotiation issues aside, it is not always clear why one cannot go to a different creditor. In any case, we impose the following *pledgability restrictions*

$$d_j \leq D_j(a_j) \text{ for } j = b, e, h, k, m, \text{ and } d_u \leq D_u, \tag{6}$$

where $D_j(0) = 0$, $D_j(a_j) \leq a_j$, and $\partial D_j / \partial a_j \geq 0$, in general, with $D_m(m) = m$ for money. The upper bound on debt comes from pledging oneself to the hilt,[7]

$$\begin{aligned} \bar{D}(\mathbf{a}) \equiv\ & D_b(b) + (\gamma + \phi_e)D_e(e) + (1 - \delta_h)\phi_h D_h(h) + (\rho + 1 - \delta_k)D_k(k) \\ & + \phi_m m + D_u. \end{aligned} \tag{7}$$

Although debtors honor obligations in terms of general purchasing power here, one might imagine situations where they pledge specific assets. In this case, however, buyers may as well hand the assets over at the time of sale and finalize the transaction. One might say this sounds more like Kiyotaki-Wright than Kiyotaki-Moore, and that would be right, but it does not change the equations. It may be reasonable to think some assets, like cash, are more naturally used as media of exchange, while others, like home equity, are more naturally used as collateral. To be precise about this, one ought to explicitly incorporate assumptions about asset attributes, including portability, divisibility, recognizability, and so forth (see Nosal and Rocheteau [2011] for a modern take). While this is interesting for some purposes, the distinction between a medium of exchange and collateral does not matter much here, so we do not dwell on it a lot more.

C. *Mechanisms*

Here we determine the KM terms of trade using an abstract mechanism. To some readers this may seem strange, but one of the key innovations in modern monetary economics involves exploring various options, and we see no need to be wed to Walrasian pricing, sticky or otherwise, although it is a special case. Starting with an example using bargaining, suppose KM trade is bilateral and buyers make take-it-or-leave-it offers. If one asks for q, one must compensate a seller for his cost, $d = \omega c(q)/\zeta$, subject to $d \leq \bar{D} = \bar{D}(\mathbf{a})$. Notice $c(q)$ is measured in utils, so dividing by ζ converts it to time, and multiplying by ω converts it to numeraire. Given $\zeta = 1$, the best take-it-or-leave-it offer is described as follows: let $u'(q^*) = c'(q^*)$, so $d^* = \omega c(q^*)$ is the promise one has to make to get q^*. If $d^* \leq \bar{D}$ a buyer asks for q^* and promises d^*; but if $d^* > \bar{D}$ he cannot credibly promise d^*, so he offers $d = \bar{D}$ and gets $q = c^{-1}(\bar{D}/\omega) < q^*$.

A generalization of this is the Kalai (1977) proportional bargaining solution, described as follows. Let

$$z(q) = \theta c(q) + (1 - \theta)u(q), \tag{8}$$

where θ is the buyer's bargaining power, and let $d^* = \omega z(q^*)$. Then if $d^* \le \bar{D}$ the buyer gets $q = q^*$ and promises $d = d^*$; but if $d^* > \bar{D}$ he promises $d = \bar{D}$ and gets $q = z^{-1}(\bar{D}/\omega) < q^*$. Some may be more familiar with generalized Nash bargaining, but Kalai has several advantages.[8] For now, however, all we need is:

Assumption 1 *There is some function z, continuously differentiable on $(0, q^*)$, with $z'(q) > 0$, $z(0) = 0$, and $\omega z(q^*) = d^*$, such that: if $d^* \le \bar{D}(\mathbf{a})$ then $q = q^*$ and $d = d^*$; and if $\omega z(q^*) > \bar{D}(\mathbf{a})$ then $d = \bar{D}(\mathbf{a})$ and q solves $\bar{D}(\mathbf{a}) = \omega z(q)$.*

Approaches used in related models that are consistent with assumption 1 include price posting with either directed or undirected search as in Lagos and Rocheteau (2005) or Head et al. (2012); abstract mechanism design as in Hu, Kennan, and Wallace (2009) or Gu, Mattesini, and Wright (2012); and auctions as in Galenianos and Kircher (2008) or Dutu, Julien, and King (2009). Some of these, like auctions, or price posting along the lines of Burdett and Judd (1983), are more interesting and easier to motivate once one departs from bilateral trade. If we have multilateral trade, it also makes sense to consider Walrasian pricing, as discussed by Rocheteau and Wright (2005) (in the context of labor-search models, think of switching from Mortensen and Pissarides 1994 to Lucas and Prescott 1974). To use Walrasian pricing, simply set $z(q) = Pq$, where individuals take P as given, then set $P = c'(q)$ in equilibrium. If $c(q) = q$ this is the same as bargaining with $\theta = 1$, although for the quantitative work below we prefer $\theta < 1$ calibrated to match markups. In any case, all we need for now is some $z(q)$ describing the terms of trade when $d \le \bar{D}$ binds.[9]

III. Equilibrium

To solve for the choice of $\hat{\mathbf{a}}$ in (3), form the Lagrangian:

$$\mathcal{L} = -\phi\hat{\mathbf{a}}/\omega + \beta W[y(\hat{\mathbf{a}}), \hat{h}] + \beta\sigma[u(q) - z(q)] + \Sigma_j \lambda_j [D_j(\hat{a}_j) - d_j] + \lambda_u (D_u - d_u)$$
$$+ \lambda_q [d_b + (\gamma + \phi'_e) d_e + (1 - \delta_h)\phi'_h d_h + (\rho' + 1 - \delta_k) d_k + \phi'_m d_m + p_d - \omega' z(q)].$$

The constraints with multipliers λ_u and λ_j, $j = b, e, h, k, m$, say that unsecured pledges are limited by D_u and pledges secured by $\hat{a}_j$ are limited by $D_j(\hat{a}_j)$. The constraint with multiplier λ_q says KM trade must respect the mechanism, $d = \omega' z(q)$ if the debt limit is binding with $d = d(\mathbf{d})$ given by (5). Note that ω' and ϕ' are prices next period, since that is when the relevant KM trades occurs.

The first-order conditions (FOCs) are

$$\hat{a}_j : -\frac{\phi_j}{\omega} + \beta \frac{\partial W[y(\hat{\mathbf{a}}), \hat{h}]}{\partial \hat{a}_j} + \lambda_j \frac{\partial D_j(\hat{a}_j)}{\partial \hat{a}_j} \leq 0, = \text{if } \hat{a}_j > 0 \tag{9}$$

$$d_j : -\lambda_j + \lambda_q \frac{\partial d(\mathbf{d})}{\partial d_j} \leq 0, = \text{if } d_j > 0 \tag{10}$$

$$q : \beta\sigma \left[\frac{\partial u(q)}{\partial q} - \frac{\partial z(q)}{\partial q} \right] - \lambda_q \omega' \frac{\partial z(q)}{\partial q} \leq 0, = \text{if } q > 0. \tag{11}$$

In (10), $\partial d(\mathbf{d}) / \partial d_j$ is the marginal value of a d_j pledge—for example, $\partial d(\mathbf{d}) / \partial d_e = \gamma + \phi_e$ is how much being able to pledge more e buys you, since each unit is worth $\gamma + \phi_e$ in the AD market. A solution to the household's problem is given by equations (9) through (11), plus $\omega U_x(x, h) = 1$, which determines x, and the budget equation, which determines ℓ.

There are three possible situations: (1) liquidity is not scarce, in which case m cannot be valued; (2) liquidity is somewhat scarce, but m cannot help; and (3) liquidity is more scarce, and m is essential. We study these in turn.

A. Liquid Nonmonetary Equilibrium

If households have sufficient pledgability to acquire q^*, liquidity is not scarce and money cannot be valued: $\phi_m = 0$. In this case, with $q = q^*$, the pledgability constraints are slack and $\lambda_j = 0$. Then the FOC (9), which holds at equality in equilibrium, becomes $\phi_j = \omega\beta\, \partial W / \partial \hat{a}_j$. Deriving $\partial W / \partial \hat{a}_j$ and simplifying, we get the asset-pricing conditions:

$$\phi_b = \frac{\beta\omega}{\omega'} \tag{12}$$

$$\phi_e = \frac{\beta\omega}{\omega'} (\gamma + \phi_e') \tag{13}$$

$$\phi_h = \frac{\beta\omega}{\omega'} [(1 - \delta_h)\phi_h' + \omega' U_h(x', h')] \tag{14}$$

$$1 = \frac{\beta\omega}{\omega'} (\rho' + 1 - \delta_k). \tag{15}$$

Because $\omega = 1/U_x(x, h)$, (12) says the bond price equals the MRS, $\beta U_x(x', h')/U_x(x, h)$. Similarly, (13) through (14) set the prices of e and h to the MRS times their payoffs. And (15) is the usual capital Euler equation, which in steady state is $\rho = r + \delta_k$ with $r = (1-\beta)/\beta$. The *accounting return* r_j on asset j is next period's payoff over the current price,

$$1 + r_b = 1/\phi_b \tag{16}$$

$$1 + r_e = (\gamma + \phi'_e)/\phi_e \tag{17}$$

$$1 + r_h = (1 - \delta_h)\phi'_h/\phi_h \tag{18}$$

$$1 + r_k = \rho' + 1 - \delta_k, \tag{19}$$

although for housing the *true return* is $[(1 - \delta_h)\phi'_h + \omega U_h(x', h')]/\phi_h$. From (12) through (15), the true return on all assets is $(1 + r)\omega'/\omega$ when liquid is plentiful.

The previous results follow directly from the household problem. The next step is to discuss macroeconomic equilibrium in two versions of the model, one with a fixed housing supply H, and one with endogenous supply. In the first version, given the initial stocks of k and h, equilibrium consists of time paths for: (1) AD consumption, capital investment, housing investment, and employment (x, k', h', ℓ), satisfying

$$1 = f_\ell(k, \ell)U_x(x, H) \tag{20}$$

$$U_x(x, H) = \beta U_x(x', H)[f_k(k', \ell') + 1 - \delta_k] \tag{21}$$

$$h' = H \tag{22}$$

$$x = \gamma + f(k, \ell) - [k' - (1 - \delta_k)k]; \tag{23}$$

(2) KM consumption and debt $q = q^*$ and $d = f_\ell(k, \ell)z(q^*)$; and (3) asset prices as described above.[10] A steady state satisfies stationary versions of these conditions.

We now impose stationarity in equations (20) to (23) and derive

$$\begin{bmatrix} f_{kk} & f_{k\ell} & 0 \\ f_k - \delta_k & f_\ell & -1 \\ U_x f_{\ell k} & U_x f_{\ell\ell} & f_\ell U_{xx} \end{bmatrix} \begin{bmatrix} dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} dr + d\delta_k \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh} dh \end{bmatrix}.$$

Let the square matrix be $\mathbf{C}_1$. Then $\Delta_1 = \det(\mathbf{C}_1) = f_\ell[f_\ell f_{kk} - (f_k - \delta_k)f_{k\ell}]U_{xx} + |f|\,U_x > 0$, where $|f| = f_{kk}f_{\ell\ell} - f_{k\ell}^2 \geq 0$, with equality if f displays CRS. It is now routine to compute the effects of parameters on the allocation and prices, including factor prices and the rental rate on housing, $R_h = (r + \delta_h)\phi_h$, as given in table 1.

Table 1
Results in case 1 with k endogenous, h exogenous

	k	ℓ	x	ω	ρ	R_h
γ	–	–	+/0	+/0	0	$+N_h/0$
r	–	?	–	–	+	$-N_h$
δ_k	?	?	–	–	+	$-N_h$
H	$+U_{xh}$	$+U_{xh}$	$+U_{xh}$	$-U_{xh}/0$	0	$-N_x$

Note: $+U_{xh}$ means the same sign as U_{xh} and similarly for $-U_{xh}$; +/0 means = 0 for concave f and =0 for CRS; and $-N_j$ means =0 if good j is normal.

Consider those related to housing, for example,

$$\Delta_1\, \partial k/\partial H = f_\ell f_{k\ell} U_{xh} \approx U_{xh}$$

$$\Delta_1\, \partial \ell/\partial H = -f_\ell f_{kk} U_{xh} \approx U_{xh}$$

$$\Delta_1 \partial x/\partial H = f_\ell[(f_k - \delta_k) f_{k\ell} - f_\ell f_{kk}] U_{xh} \approx U_{xh},$$

where $A \approx B$ indicate that A and B take the same sign. Naturally, these depend on whether x and h are complements or substitutes in the sense $U_{xh} \gtreqless 0$. However, $\partial\phi_h/\partial H < 0$ is unambiguous, at least if x is normal, which we interpret as a downward-sloping long-run demand for housing.

For the model with endogenous h, introduce in the AD market competitive home builders with convex cost $g(\cdot)$, so in equilibrium $\phi_h = g'[h' - (1 - \delta_h)h]$. Combining this with the FOC for h', we get the housing Euler equation

$$U_x(x, h) g'[h' - (1 - \delta_h)h] = \beta U_x(x', H)\{(1 - \delta_h) g'[h'' - (1 - \delta_h)h'] + U_h(x', h')\}.$$

Equilibrium with endogenous h uses this instead of (22), and uses

$$x = \gamma + f(k, \ell) - [k' - (1 - \delta_k)k] - g[h' - (1 - \delta_h)h]$$

instead of (23). In steady state $g'(\delta_h h) = \phi_h$, implying an upward-sloping long-run supply curve. So there is a unique (h, ϕ_h) clearing the housing market. Symmetric with table 1, table 2 gives parameter effects with h endogenous but k fixed.[11]

We close this case with three other observations. First, we still have to ask, under what conditions do we get equilibrium where liquidity is plentiful? Focusing on steady state, this obtains if and only if $\bar{D}(\mathbf{a}) > f_\ell(k, \ell) z(q^*)$, where $\bar{D}(\mathbf{a})$ is given by (7). Second, in this equilibrium the form of payment is indeterminate, and one can use b, e, or any combination. Third, when liquidity is plentiful, the model *dichotomizes*: the

Table 2
Results in case 1 with h endogenous, k exogenous.

	h	ℓ	x	ω	ρ	R_h
γ	$+N_h$	$-$	$+N_x$	$+$	$-$	$+N_h$
r	$-$	$-N_h$	?	$+N_h$	$-N_h$	$+N_{h,x}$
δ_h	$-N_h$	?	?	?	?	$+N_{h,x}$
K	$+N_h$	?	$+N_x$	?	?	$+N_h$

Note: $+U_{xh}$ means the same sign as U_{xh} and similarly for $-U_{xh}$; $+/0$ means $=0$ for concave f and $=0$ for CRS; and $-N_j$ means $=0$ if good j is normal.

AD allocation (x, k', h', ℓ) is independent of q^*. It is known that one can break this dichotomy by interacting q with (x, k', h', ℓ) in preferences or technology; in what follows we break it by assuming liquidity is scarce, so AD and KM interact via financial considerations.

B. *Illiquid Nonmonetary Equilibrium*

Consider next a nonmonetary equilibrium where $\bar{D}(\mathbf{a})$ is such that buyers cannot get q^*. The FOC for q implies $\lambda_q = \beta\sigma L(q)/\omega' > 0$, where

$$L(q) = \frac{u'(q) - z'(q)}{z'(q)}. \tag{24}$$

To ease the presentation, let us assume $L'(q) < 0 \ \forall q$. This holds automatically for many mechanisms, including Kalai bargaining and Walrasian pricing. Also, one can prove the same results without $L'(q) < 0 \ \forall q$, as in Wright (2010), but we prefer to avoid these technicalities.

Moving on to substantive results, for $j \neq m$ the FOC for d_j now implies $\lambda_j = \lambda_q \, \partial d/\partial d_j > 0$. Hence, KM buyers borrow to the limit $\bar{D}(\hat{\mathbf{a}})$, and q solves $z(q)\omega' = \bar{D}(\hat{\mathbf{a}})$. In terms of asset prices, we have

$$\phi_b = \frac{\beta\omega}{\omega'}[1 + D_b'(B)\sigma L(q)] \tag{25}$$

$$\phi_e = \frac{\beta\omega}{\omega'}(\gamma + \phi_e')[1 + D_e'(1)\sigma L(q)] \tag{26}$$

$$\phi_h = \beta\omega U_h(x', h') + \frac{\beta\omega}{\omega'}(1 - \delta_h)\phi_h'[1 + D_h'(h')\sigma L(q)] \tag{27}$$

$$1 = \frac{\beta\omega}{\omega'}(\rho' + 1 - \delta_k)[1 + D_k'(k')\sigma L(q)]. \tag{28}$$

Compared to (12) through (15), the *liquidity premium* $D_j'(\hat{a}_j)\sigma L(q)$ now appears on the RHS, because as long as $D_j'(a_j) > 0$, having more a_j relaxes debt limits.

Suppose $h = H$ is fixed (endogenous h can be handled as above). An illiquid nonmonetary equilibrium consists of paths for: (1) (x, k', h', ℓ) satisfying

$$1 = f_\ell(k, \ell)U_x(x, H) \tag{29}$$

$$U_x(x, H) = \beta U_x(x', H)[f_k(k', \ell') + 1 - \delta_k][1 + D_j'(k)\sigma L(q)] \tag{30}$$

$$h' = H \tag{31}$$

$$x = \gamma + f(k, \ell) + (1 - \delta_k)k - k'; \tag{32}$$

(2) (q, d) satisfying $d = \bar{D}(\hat{\mathbf{a}})$ and $z(q) = d/\omega'$; and (3) asset prices as described above. Compared to the previous case, (30) has $1 + D_j'(k)\sigma L(q)$ multiplying the RHS because liquidity considerations now affect investment in productive capital. Steady state satisfies stationary versions of these conditions, including

$$\bar{D}(\hat{\mathbf{a}}) = D_b(B) + D_e(1)\phi_e + D_h(H)\phi_h + [f_k(k, \ell) + 1 - \delta_k]D_k(k) + D_u, \tag{33}$$

which makes q depend on asset prices (for b, e, and h) and quantities (for k and for h when it is endogenous).

For illustration let $D_j(\hat{a}_j)/\hat{a}_j = \mu_j$ and assume $\mu_b = \mu_e = \mu_h = 0 < \mu_k$, so that for now k and only k serves as collateral. Then steady state is summarized by an allocation (x, k, ℓ, q) satisfying:

$$1 = f_\ell(k, \ell)U_x(x) \tag{34}$$

$$r + \delta_k = f_k(k, \ell) + [f_k(k, \ell) + 1 - \delta_k]\sigma\mu_k L(q) \tag{35}$$

$$x = \gamma + f(k, \ell) - \delta_k k \tag{36}$$

$$f_\ell(k, \ell)z(q) = [f_k(k, \ell) + 1 - \delta_k]\mu_k k + D_u. \tag{37}$$

From these, one derives

$$\mathbf{C}_2 \begin{bmatrix} dq \\ dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} dD_u + (f_k + 1 - \delta_k)kd\mu_k - \mu_k kd\delta_k \\ dr + (1 + \sigma\mu_k L)d\delta_k - (f_k + 1 - \delta_k)L(\sigma d\mu_k + \mu_k d\sigma) \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh} dH \end{bmatrix}$$

where

Table 3
Results in case 2 with k endogenous, h exogenous.

	q	k	ℓ	x	ω	ρ
γ	–	–*	–*/0	+*	+*/0	–
r	+*	–	?	–	–	+
H	$+U_{xh}$	$+U^*_{xh}$	$+U^*_{xh}$	$+U^*_{xh}$	$-U^*_{xh}$	$+U_{xh}$
D_u	+	–	?	+	–	+
μ_k	?	+*	–*	+*	–*	–*
σ	–*	+	?	+	+	–

Notes: $+U_{xh}$ means the same sign as U_{xh} and similarly for $-U_{xh}$; +* means the result is ambiguous in general, but >0 if k is small, and similarly for –*; and +/0 means =0 for concave f and =0 for CRS, and similarly for –/0.

$$\mathbf{C}_2 = \begin{bmatrix} f_\ell z' & zf_{\ell k} - \mu_k(f_k + 1 - \delta_k) - \mu_k f_{kk} & zf_{\ell\ell} - \mu_k kf_{k\ell} & 0 \\ (f_k + 1 - \delta_k)\mu_k \sigma L' & (1 + \mu_k \sigma L) f_{kk} & (1 + \mu_k \sigma L) f_{k\ell} & 0 \\ 0 & f_k - \delta & f_\ell & -1 \\ 0 & f_{k\ell} U_x & f_{\ell\ell} U_x & f_\ell U_{xx} \end{bmatrix}.$$

One can show $\Delta_2 > 0$, at least if $\sigma\mu_k$ is not too big. Then the effects of parameter changes shown are in table 3.

There are some twists here, compared to tables 1 and 2, because now financial conditions matter. First, an increase in r lowers k, but at least when μ_k is not too big this increases q. This is because $\rho = f_k$ is higher when there is less k, and on net credit constraints can be relaxed. Second, supposing h and x are complements, if H increases then x and k do, too, so credit constraints ease with higher H even though $\mu_h = 0$. The loan-to-value ratio μ_k has an ambiguous effect on q, because k rises but ρ falls, and on net debt limits can fall. Similarly, an increases in σ makes agents put more weight on liquidity, which increases k and hence x, but has an ambiguous effect on q. Indeed, $\partial q / \partial\sigma < 0$ for sure when μ_k is small.[12] Finally, we have to ask when the illiquid nonmonetary equilibrium exists. The answer is $f_\ell(k, \ell)z(q^*) > \bar{D}(\hat{\mathbf{a}}) \geq f_\ell(k, \ell)z(q^i)$. The first inequality says agents cannot borrow enough to get q^*, while the second says they can borrow enough to get at least what they would get in monetary equilibrium, as described next.

C. Monetary Equilibrium

In monetary equilibrium, the constraints bind, so $\lambda_q = \beta\sigma L(q) > 0$ and $\lambda_j = D_j'(p_j)\lambda_q > 0$. Again, buyers go to the limit $\bar{D}(\hat{\mathbf{a}})$, but now this includes real balances. The equations for $(\phi_b, \phi_e, \phi_h, \phi_k)$ are the same as above, (25) through (28), and now there is a new condition for pricing currency,

$$\phi_m = \frac{\beta\omega}{\omega'}\phi'_m[1 + \sigma L(q)]. \tag{38}$$

Equilibrium satisfies the relevant conditions for AD, KM, and asset prices, as above, but in a monetary economy the determination of q is very different.

To see this, rearrange (38) as $\omega'\phi_m/\beta\omega\phi'_m = 1 + \sigma L(q)$. The LHS is the inflation rate $1 + \pi = \phi_m/\phi'_m$ times the real interest rate $1 + r = \omega'/\beta\omega$ on an illiquid bond (i.e., one that is not pledgable at all) which we can always price even if it does not trade in equilibrium. Therefore, by the Fisher equation, the LHS is the $1 + i$, where i is the return on an illiquid nominal bond. Hence, (38) can be rewritten succinctly as

$$i = \sigma L(q). \tag{39}$$

Monetary policy can set inflation or the growth rate of M (both equal π in steady state). Or, it can peg the interest rate on illiquid nominal bonds i, then let π and M evolve endogenously. For concreteness let's say policy pegs i. Then (39) pins down $q = q^i$, with $\partial q/\partial i = 1/\sigma L'(q) < 0$. Thus, increasing i raises the cost of carrying real balances, and this lowers purchases of q.

A monetary steady state can be summarized by an allocation satisfying similar conditions to the previous case, except we replace $\omega z(q) = \bar{D}(\hat{\mathbf{a}})$ with $i = \sigma L(q)$. For monetary equilibrium to exist we need $\bar{D}(\hat{\mathbf{a}}) < f_\ell(k, \ell)z(q^i)$, which says the q^i that solves (39) exceeds what one could get using only credit. Obviously, this is more likely to be true when i is lower. In steady state, (x, k, ℓ, q) satisfies

$$1 = f_\ell(k, \ell)U_x(x) \tag{40}$$

$$r + \delta_k = f_k(k, \ell) + [f_k(k, \ell) + 1 - \delta_k]\sigma D'_k(k)L(q) \tag{41}$$

$$x = \gamma + f(k, \ell) - \delta_k k \tag{42}$$

$$i = \sigma L(q). \tag{43}$$

At the *Friedman rule* $i = 0$, $L(q) = 0$, and (40) through (42) reduce to the conditions for (x, k, ℓ) in section A of section III. Thus, $i = 0$ delivers the efficient AD allocation—but this will not necessarily be true when taxes are included in section V. Even if it delivers AD efficiency, $i = 0$ may not deliver KM efficiency $q = q^*$—that depends on the mechanism.[13]

In any case, setting $D(a_j) = \mu_j a_j$ and combining (39) and (41), we get

$$f_k(k, \ell) = \frac{r + \delta_k - (1 - \delta_k)\mu_k i}{1 + \mu_k i}.$$

While μ_k affects this condition, it cannot affect q, which is determined by (39). How can pledgability not affect KM trade? The answer is that in monetary equilibrium i pins down q^i and then q^i pins down the debt limit $\bar{D}(\hat{\mathbf{a}}) = f_\ell(k, \ell) z(q^i)$, not vice versa. Heuristically, when the pledgability of k increases, other forms of liquidity are crowded out, leaving

$$\bar{D} = \mu_b B + \frac{(1 + r)\gamma\mu_e}{r - \mu_e i} + \frac{(1 - \delta_h)(U_h / U_x)\mu_h H}{r + \delta_h - (1 - \delta_h)\mu_h i} + (f_k + 1 - \delta_k)\mu_k K + D_u + \phi_m M$$

the same. Relatedly, increasing B has no effect on the allocation, as real balances get completely crowded out. This suggests that open market operations, or more generally, *quantitative easing*, might not have the impact one expects.

To characterize these and other effects in more detail, one can derive

$$\mathbf{C}_3 \begin{bmatrix} dq \\ dk \\ d\ell \\ dx \end{bmatrix} = \begin{bmatrix} Ld\sigma - di \\ dr + (1 + \mu_k \sigma L) d\delta_k - (f_k + 1 - \delta_k) L(\sigma d\mu_k + \mu_k d\sigma) \\ kd\delta_k - d\gamma \\ -f_\ell U_{xh} dH \end{bmatrix}$$

where

$$\mathbf{C}_3 = \begin{bmatrix} -\sigma L' & 0 & 0 & 0 \\ (f_k + 1 - \delta_k)\mu_k \sigma L' & (1 + \mu_k \sigma L) f_{kk} & (1 + \mu_k \sigma L) f_{k\ell} & 0 \\ 0 & f_k - \delta_k & f_\ell & -1 \\ 0 & f_{\ell k} U_x & f_{\ell\ell} U_x & f_\ell U_{xx} \end{bmatrix}$$

and $\Delta_3 = \det(\mathbf{C}_3) = -\sigma L'(1 + \mu_k \sigma L)\{U_x |f| + f_\ell [f_\ell f_{kk} - f_{k\ell}(f_k - \delta_k)] U_{xx}\} > 0$. The (extremely sharp) effects of parameters are shown in table 4, including what is usually called the *Tobin effect*,

$$\Delta_3 \, \partial k / \partial i = (f_k + 1 - \delta_k)\mu_k \sigma L' (f_\ell^2 U_{xx} + f_{\ell\ell} U_x) > 0 \text{ if } \mu_k > 0.$$

Intuitively, higher inflation gives households the incentive to substitute k for m in their portfolios, which is relevant for several of the findings to follow.

Table 4
Results in case 3 with k endogenous, h exogenous

	q	k	ℓ	x	ω	ρ	ϕ_e
i	–	+	?	+	+	–	+
σ	+	0	0	0	0	0	0
γ	0	–	–	+	0	0	0
μ_k	0	+	?	+	+	–	0
μ_e	0	0	0	0	0	0	+
R	0	–	?	–	–	+	0
δ_k	0	?	?	–	–	+	0

In particular, since monetary policy affects investment in capital it also affects the labor market. In general, one cannot sign

$$\Delta_3\, \partial \ell / \partial i = -(f_k + 1 - \delta_k)\mu_k \sigma L'[f_{k\ell} U_x + (f_k - \delta_k) f_\ell U_{xx}],$$

and so the *Phillips curve* can go either way, but if U_{xx} is small then we know for sure that $\partial \ell / \partial i > 0$.[14] Here movements in ℓ come along the intensive margin, as changes in hours worked by the representative individual, but as mentioned above, one can assume indivisible labor and use lotteries to generate unemployment. The same equations hold in that model, but now movements in ℓ come along the extensive margin, as changes in the number of agents employed. Thus, the model can generate a long-run exploitable trade-off between inflation and unemployment. Of course, just because it is feasible to reduce unemployment by increasing inflation, that does not mean it is a good idea. As we said, $i = 0$ is optimal here, although it may not be once one introduces other distortions.

In terms of financial parameters, higher μ_k makes k a better payment instrument, and so it increases investment, but again q does not change—restricting attention, obviously, to parameter changes that do not take us out of monetary equilibrium. Similarly, increasing μ_e raises the price and lowers the return on e but has no effect on q. In terms of policy's impact on returns, increasing i lowers the demand for money and raises the demand for other assets, which we call a *Mundell effect*. This helps clarify the nature of *Fisher's theory* that nominal interest rates increase one-for-one with inflation, leaving real returns the same. One version of this theory, the Fisher equation, says the real return on an illiquid (nonpledgable) bond is pinned down in steady state by $1 + r = 1/\beta$, independent of policy. But the critical qualification there is that the asset is illiquid.

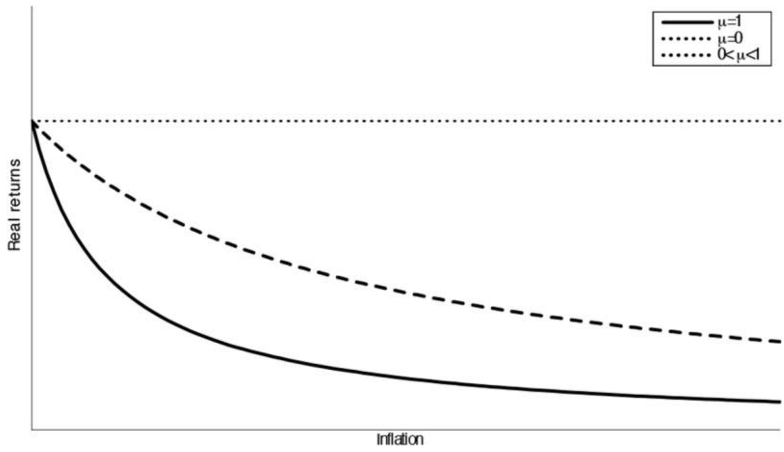

Fig. 1. Inflation and asset returns

Fisher's theory cannot hold for all assets, since m is an asset, with a 0 nominal return, and so its real return must fall with inflation. How about partially-liquid assets? The real return on equity is

$$r_e = \frac{r - \mu_e i}{1 + \mu_e i} > 0,$$

assuming i is sufficiently low that monetary equilibrium exists. If $\mu_e > 0$ then $\partial r_e / \partial i < 0$. More generally, for any asset j, $\partial r_j / \partial i \leq 0$ with equality if and only if $\mu_j = 0$ (i.e., if and only if the asset is not pledgable at all). In figure 1, the dotted line is the steady state return on an illiquid asset, $1 + r = 1/\beta$, the solid curve is the real return on cash, $1 + r_m = \phi'/\phi$, and the dashed curve is the real return on an asset with $0 < \mu_j < 1$. We conclude that real returns are independent of inflation for illiquid but not liquid assets.

IV. Extensions

Before getting into the quantitative work, we briefly mention how the framework can be extended in various directions (one can skip this without loss of continuity). First, in an stochastic economy liquidity risk is also a source of variation in asset returns. Intuitively, assets that provide liquidity in states where it is most needed command a higher premium. Suppose the dividend on the Lucas tree γ is an i.i.d. random variable with mean $\bar{\gamma}$, and a realization that is known at the point of KM trade. In monetary equilibrium, the average return on equity is

$$1 + r_e = \frac{\phi_e + \bar{\gamma}}{\phi_e} = \frac{1+r}{1+\mu_e i} - \frac{\mu_e \mathrm{cov}[L(q), \gamma/\phi_e]}{1+\mu_e i}.$$

The first term on the RHS is the same as before; the second is an adjustment for risk, making r_e lower if the dividend γ is big in states when λ_q is high.

Lagos (2010, 2011) assumes b is riskless while e has a random dividend. Then the excess return on equity has two parts, one coming from the liquidity differential and one from risk:

$$r_e - r_b = \frac{(1+r)(\mu_b - \mu_e)i}{(1+\mu_e i)(1+\mu_b i)} - \frac{\mu_e \mathrm{cov}[L(q), \gamma/\phi_e]}{1+\mu_e i}.$$

Notice this premium depends on the policy variable i. Although this can be explored in more detail, we focus below on the pure liquidity premium. But to be clear, there are two parts to Lagos' approach. One is that differences in μ_j help explain $r_e - r_b$. The other is to notice that even if $\mu_e = \mu_b$, liquidity affects how one interprets the data. Heuristically, suppose $r_e = 6\%$ and $r_b = 1\%$. A factor of 6 looks like a lot to explain based on risk. But suppose the liquidity value of both assets is worth 4 percent. Then the "corrected" return on e is 10 percent and on b is 5 percent, only a factor of 2.

Next, we mention that it is easy to allow match-specific pledgability limits: the loan-to-value ratio μ_j for asset j can depend on who one trades with in the KM market. It is worth pursuing this, in general, but here we only use the idea to demonstrate one way of making pledgability endogenous, following Lester, Postlewaite, and Wright (2012). Suppose there are two assets, e and m. Every seller takes m at face value, while there are two technologies for enforcing debt secured by e. The first is free, and allows creditors to seize $\mu_1 e$ in the event of a default. The second has a fixed cost κ, and enables sellers to seize $\mu_2 e$, with $\mu_2 > \mu_1$. Let us assume κ_s has to be paid each period in the AD market, although it is also interesting to consider once-and-for-all investments. The cost is specific to the individual: agent $s \in 0, 1]$ must invest $\kappa_s \in \underline{\kappa}, \bar{\kappa}]$ to access the superior technology.

Label agents so κ_s is increasing in s. Then let $H(\kappa)$ be the corresponding CDF with support $[\underline{\kappa}, \bar{\kappa}]$, and χ the endogenous fraction of agents investing in the better technology. Conditional on being a KM seller, one's state is whether one has made the investment. Conditional on being a buyer, χ is the fraction of meetings where $\mu_e = \mu_2$. The benefit for a seller from the better technology is

$$\Omega = \Omega(\chi) \equiv z(q_2) - c(q_2) - z(q_1) + c(q_1),$$

where q_1 and q_2 are KM output in the two types of meetings, which depend on χ. Agent s invests if $\Omega(\chi)$ exceeds κ_s. Defining $T(\chi) = H[\max\{s : \kappa_s \leq \beta\sigma\Omega(\chi)\}]$, an equilibrium with endogenous pledgability is a fixed point $\chi = T(\chi)$. While $T(\chi)$ is not necessarily continuous, Lester et al. (2012) show, using Kalai bargaining, that it is increasing.[15] So by Tarski's fixed point theorem equilibrium exists. There can easily be multiple equilibria. It is possible that no one invests $\chi = 0$, that everyone does $\chi = 1$, or that a fraction does $\chi \in (0, 1)$. Importantly, pledgability is not invariant to changes in policy.

One can also make pledgability endogenous with a simple moral-hazard model. Again consider two assets, e and m, and suppose the former requires maintenance to yield the full dividend γ. Agents with e units of equity choose a maintenance level $n \in [0, 1]$, after trading, at a utility cost εne. Given n, the dividend per unit of equity is $n\gamma e$. In principle, a debtor can choose to not maintain an asset that has been pledged to a creditor, renege on his obligation, and forfeit e. One can show (details available on request) the inability of debtors to commit to n leads to an endogenous pledgability limit $D_e(e) = \mu_e e$ with $\mu = 1 - \varepsilon\omega'/(\gamma + \phi_e)$. That arises because he will maintain the asset to keep the return to his share (after forfeiture) high. Also, when all agents face the same maintenance cost ε, then it is equivalent to use the asset as a medium of exchange or as collateral. But this is no longer true when ε differs across agents.

Consider two types, with costs ε_1 and $\varepsilon_2 > \varepsilon_1$, and the fraction of type-1 is common knowledge. We consider two cases: ex ante heterogeneity, where type is observed in the KM market; and ex post heterogeneity, where it is realized after KM trade and privately observed. In the first case, collateralized borrowing can be strictly preferred in some meetings, and transfers in others. Thus, if one agent is known to be better at maintaining the value of e, he should hold it. But if differences emerge after KM trade and are privately observed, immediate transfers can be better. Private information limits the extent to which ex post liquidity can be pledged. Thus, assets characterized by observable differences in the ability to maintain value are more likely to be used as collateral. See Holmstrom and Tirole (2011) for more discussion. The point here is simply that it is possible to endogenize pledgability, in various ways.

V. Quantitative Analysis

We now ask what the theory has to say quantitatively. This is *not* a business cycle analysis; the interest here is on longer-run phenomena. This means looking at differences across steady states, or deterministic transitions between steady states, although one obviously *could* simulate the model with high-frequency shocks to technology, monetary, financial, and other variables. In terms of the data, we are mainly interested in observations after filtering. This is partly because we want the empirical analysis to correspond to the results in section III, and partly because we think longer-run changes in monetary and financial variables are more interesting.[16]

By way of preview here are some findings:

- Liquidity effects generate a large return premium for less-pledgable assets.
- The model accounts well for the effects of inflation on standard macro aggregates.
- The calibrated parameters imply that reducing inflation to the Friedman rule leads to a *decrease* in welfare, and optimal inflation is close to the average in the data.
- Financial innovation can generate significant movements in asset prices and economic activity, including expansions and recessions fueled by housing markets.

Before proceeding, we make three changes to the model. First, we set $D_u = 0$, and ignore Lucas trees, leaving four assets to facilitate intertemporal exchange: $\mathbf{a} = (b, h, k, m)$. Second, we add taxes on labor income and asset income (the returns to b and k), denoted τ_ℓ and τ_a, and assume pledgability limits are post-tax: $d_b \le [1 - (1 - \phi_b)\tau_a]\mu_b b$ and $d_k \le [1 + (\rho - \delta_k)(1 - \tau_a)]\mu_k k$. While taxes would have been a distraction for the analytic results, they are critical for calibration. Third, the probability that a household needs a KM loan each period is now $\sigma \in 0, 1]$. When households were interpreted as trading bilaterally in the KM market, previously we have $\sigma \in 0, 1/2]$ since not more than a fraction $\sigma = 1/2$ can be buyers if each one needs a seller. However, to match some observations, it helps to allow $\sigma > 1/2$ (this is no problem for the theory, as we can simply assume some sellers serve multiple buyers in the KM market).

Also, we address a measurement issue. With the KM sector modeled as a pure-exchange market, total output in the model economy is well defined according to standard accounting practice—AD output plus KM output all measured in numeraire—and total employment comes only from AD hours. But there are alternative approaches. What if not all KM activity is recorded in the data? At least part of this activity involves cash transactions, some of which may not show up in the official accounts (Wallace [2010] and Aruoba [2010], for example, interpret the analog of our KM market as the underground economy). Moreover, instead of making KM a pure exchange market, one can assume that q is produced and add KM labor to total employment. Usually we stick to the benchmark interpretation, where q is counted in GDP while only ℓ is counted as employment, but we also discuss some implications of changing this interpretation.

A. Calibration

The AD utility and production functions are $\mathcal{U}(x, h, \ell) = \log x + \psi \log h - \zeta\ell$ and $f(k, l) = k^{\alpha}l^{1-\alpha}$. The KM utility and cost functions are $u(q) = \upsilon q^{1-\eta}/(1-\eta)$ and $c(q) = q$. The housing cost function is $g(I) = I^{1+\xi}/(1+\xi)$, where $I = H' - (1-\delta_h)H$ is net residential investment. The KM terms of trade are determined by Kalai bargaining, $z(q) = (1-\theta)u(q) + \theta c(q)$, although Walrasian pricing emerges as a special case when $\theta = 1$. As in much of the literature, α in the AD production function is set to 0.33 to match labor's and capital's shares in the income accounts. We let the time period be a year, and set $\beta = 0.95$ so the annual real interest rate on an illiquid bond is 5.26 percent. However, interest rates on liquid assets can be considerably lower, and 5.26 percent should be interpreted as the return one would require if an asset could not be used as collateral at all. Estimates of tax rates vary widely; we use $\tau_a = 0.4$ and $\tau_\ell = 0.3$ in the range of previous studies. The remaining parameters are calibrated to US data, from 1954 to 2000, unless otherwise noted. There are two reasons for stopping at 2000. First, we do not claim to have a great theory of the great contraction, although models that take financial considerations seriously may be on the right track. Second, even before the crisis, developments in financial and housing markets make the 2000s different.[17]

The appendix contains details of the data sources, although only housing merits much discussion. For this, the value of structures plus land is taken from Davis and Heathcote (2007), which has the disadvantage of starting only in 1975, but has the advantage of reliability. To this we add consumer durables since many of these (e.g., home appliances) are part of household capital. To be consistent, the value of housing services is removed from the GDP data, and durable consumption is added to residential investment. Annual depreciation rates are set to

$\delta_h = \delta_k = 0.10$ to approximately match investment rates in h and k. These numbers are slightly higher than those seen in some other studies, at least in part due to the way we adjust our housing and output measures. Inflation averages 3.7 percent in our sample. Given this, and an average pretax real return on three-month T-bills of 1.8 percent, we can pin down $\mu_b = 0.45$ from the optimality condition for b, which with taxes is $1 + r_b = [1 + r - (1 + \mu_b i)\tau_a]/(1 + \mu_b i)(1 - \tau_a)$.

This leaves ten parameters, $(\mu, \upsilon, \sigma, \theta, \psi, \xi, \mu_k, \mu_h, \zeta)$, which are calibrated as follows. The KM parameters—the curvature and level of utility η and υ, plus the fraction of agents needing a loan σ—are set to fit: (a) the empirical relationship between inflation and M/PY (money demand or inverse velocity), defined using sweep-adjusted $M1$ data, consistent with the idea that in this model m can well represent money in the bank; and (b) the empirical relationship between inflation and real output. While (a) is standard procedure, (b) is less so and deserves discussion, but we defer that until we see some results. The remaining seven parameters are set to match the targets indicated in table 5.[18]

Although the parameters are set jointly to match the targets, heuristically one can think of subsets of parameters being identified by subsets of the targets, as described in table 5. Already from these numbers one can ask how the model does in terms of some issues, including the equity premium. From the calibrated μ_b and μ_k, k earns a higher return than b, with the difference $r_b - r_k$ approximately $i(\mu_k - \mu_b)/(1 - \tau) = 0.04$. This is remarkably close to the empirical equity premium, as measured by the difference in average returns to stocks and bonds over the last 100 years. Mehra and Prescott (1985) highlight the inability of standard models to explain this difference based on risk aversion, and a large body of work has attempted to resolve the issue by modifying standard models in various ways. It turns out that liquidity goes a long way.

As we said earlier, Lagos (2010, 2011) makes a similar point when he demonstrates that models where liquidity is incorporated explicitly can explain a substantial part of the equity premium. Our approach is somewhat different, in that he only considers differences in liquidity on the extensive margin—assets either are or are not accepted in decentralized trade—while our model captures liquidity differences along the intensive margin—assets are accepted only up to certain limits. The point is not that these two approaches are so different; rather, we want to emphasize how models of this class generally have considerable potential for studying macro-finance issues, compared to models that ignore liquidity considerations.

Table 5
Calibration

Parameter	Description	Baseline	Alternate	Identified by
β	Discount factor	0.95	0.95	Set directly
α	Capital's share	0.33	0.33	"
π	Inflation	0.037	0.037	"
δ_h	Depreciation of housing	0.10	0.10	"
δ_k	Depreciation of capital	0.10	0.10	"
τ_ℓ	Labor-income tax	0.30	0.30	"
τ_a	Asset-income tax	0.40	0.40	"
ψ	Coefficient on h in U	0.86	0.73	Housing/GDP
ζ	Coefficient on ℓ in U	0.38	0.41	Hours worked
ξ	Curvature of h cost function	0.30	0.30	Housing investment/GDP
μ_b	Pledgability of bonds	0.45	0.45	Real return on bonds
μ_k	Pledgability of capital	0.17	0.06	Capital/GDP
μ_h	Pledgability of housing	0.06	0.04	Home equity loans/housing
B	Bond supply	0.08	0.07	T-bills/GDP
θ	Bargaining power	0.86	0.68	Retail markup
η	Curvature of $u(q)$	0.63	0.39	M/PY and Y vs. inflation
υ	Constant in $u(q)$	1.39	1.11	"
σ	Fraction of KM buyers	0.67	0.75	"

B. Main Results

Figure 2 shows steady-state values for the standard macro aggregates as a function of inflation π, from a high of 10 percent to a low of –0.05, the value associated with the zero lower bound on i, or the Friedman rule. Inflation is bad for KM consumption q, as we knew from the analytic results, but now we see the effect is sizable: reducing π from 10 percent to the Friedman rule increases q from around 1.1 to 1.7, and even reducing π from 10 percent to 0 increases q to almost 1.5. Offsetting this is the higher AD consumption x, which results mainly from k increasing. The increase in k also generates an increase in employment, although the scale on the vertical axis indicates that this effect is not very big (we return to this later). Although this is not true for all parameter values, for the calibrated values, total output increases with π, as the increase in AD production more than offsets the fall in KM trade.

The value of the housing stock $\phi_h h$ also increases with π, consistent with the facts documented in Aruoba, Davis, and Wright (2012), but the explanation is different. The suggestion there is that agents substitute

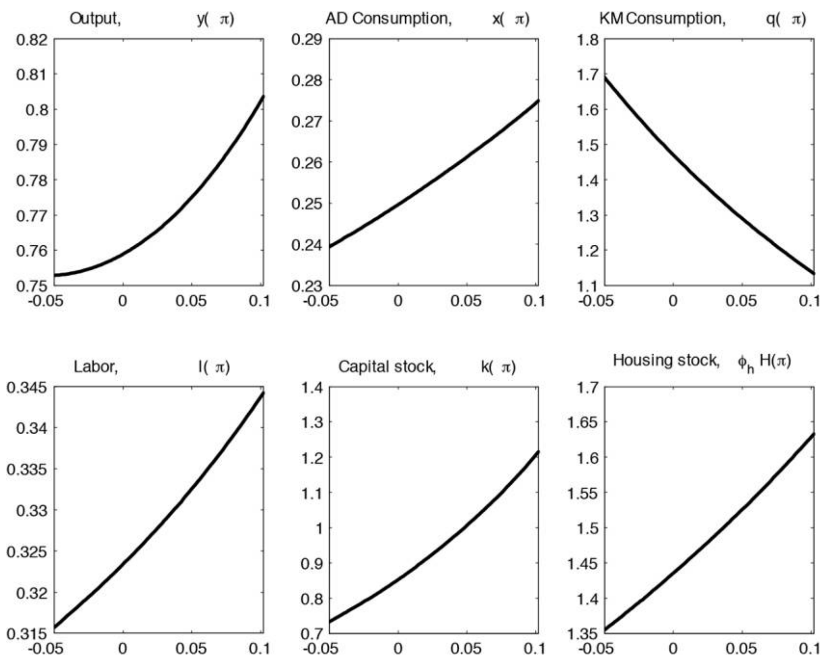

Fig. 2. Inflation and aggregate variables (baseline calibration).

out of market and into household production as π increases, leading to higher demand for inputs to household production, including h. Here the demand for h increases with π as agents try to substitute other pledgable assets for cash, similar to the effect on k. An advantage of this explanation is that it has h and k both rising with π, as in the data (see below); the other story tends to make them move in opposite directions. Something similar applies to bonds, which we emphasize because T-bills are the subject of recent empirical work by Krishnamurthy and Vissing-Jorgensen (2012). Figure 3 shows how π reduces the returns to b, k, and h, which we knew from the analytic results, but here we quantify the effects. The fall in r_b is larger than r_h and r_k, since the calibration implies b is more pledgable. The lower left panel of figure 3 shows the fall in real balances with π, while the next panel shows the increase in liquidity from other assets, $\bar{D} - \phi_m M$. On net, total liquidity falls—or credit tightens—with π since q falls. Part of this comes from an increase in ω, since $z(q)\omega/\zeta = \bar{D}$ means that at higher wages KM buyers get less for their money.

The last panel of figure 3 shows the cost of inflation, computed as

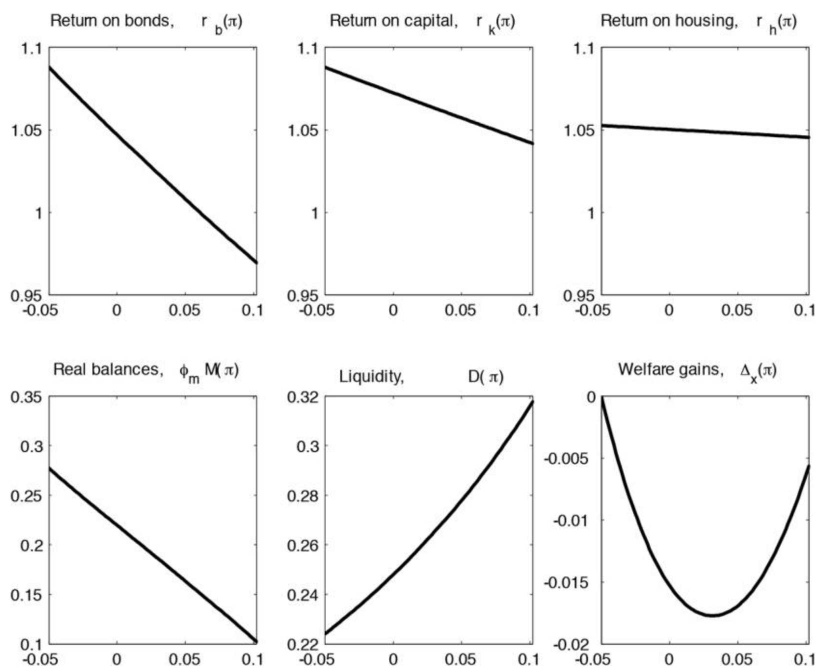

Fig. 3. Inflation and aggregate variables (baseline calibration).

the change in AD consumption that gives the same increase in utility as switching to the Friedman rule, not across steady states but taking into account the transition as k and h adjust. In the calibrated model this cost is *negative*—that is, switching to the Friedman rule is like a *reduction* in x. Going from the average π in the sample to the Friedman rule is like reducing x about 1.5 percent. This is due to taxes, which tend to make k too low. Inflation is beneficial because the Tobin effect partially offsets the tax effect on k, more than compensating for the effect on q. If π did not affect k then going from $\pi = 0.1$ to the Friedman rule would be like an increase in x of nearly 5 percent, purely due to the impact on q. This is much higher than what one finds in reduced-form models like Lucas (2000), but similar to Lagos and Wright (2005). Even with this big impact on q, the beneficial impact on k, absent in Lucas (2000) or Lagos and Wright (2005), dominates effect. Without taxes the Friedman rule is optimal; with taxes, the calibration implies the optimal inflation rate is 3.5 percent, which is surprisingly close to the average inflation rate in the data.

While these results seem interesting, especially the welfare effects, one might wonder if the model does well enough at capturing the data to take them seriously. The answer seems to be *yes*. Figure 4 overlays some model relationships with the facts, normalizing nonstationary variables by GDP (e.g., reporting k/y rather than k). The stars indicate raw data, and the circles indicate HP (Hodrick-Prescott) trends with filtering parameter 100. These pictures reveal that changes in inflation over the period account for a significant fraction of movements in the data, especially the lower-frequency components captured by the filtered series. The only relationship shown in figure 4 that fits in the calibration routine is M/PY versus inflation, yet the model is in line with data in terms of c/y, k/y, h/y, and r_b.

The one dimension along which the model does less well is the labor market: it generates a fairly flat but slightly upward-sloping relationship, somewhat at odds with the data. There are two points to make. First, one can argue that the scatter plot of π versus ℓ is basically a "cloud," as Christiano and Eichenbaum (1992) argue for the relationship between productivity and ℓ, and hence, one is not going to match the data with any one forcing variable. Second, as we said above, one can interpret employment differently in the model by having production in the KM market. Figure 5 compares the model-implied Phillips curve to the data under the two interpretations of labor. The left panel is reproduced from the baseline results, which only count AD labor. The

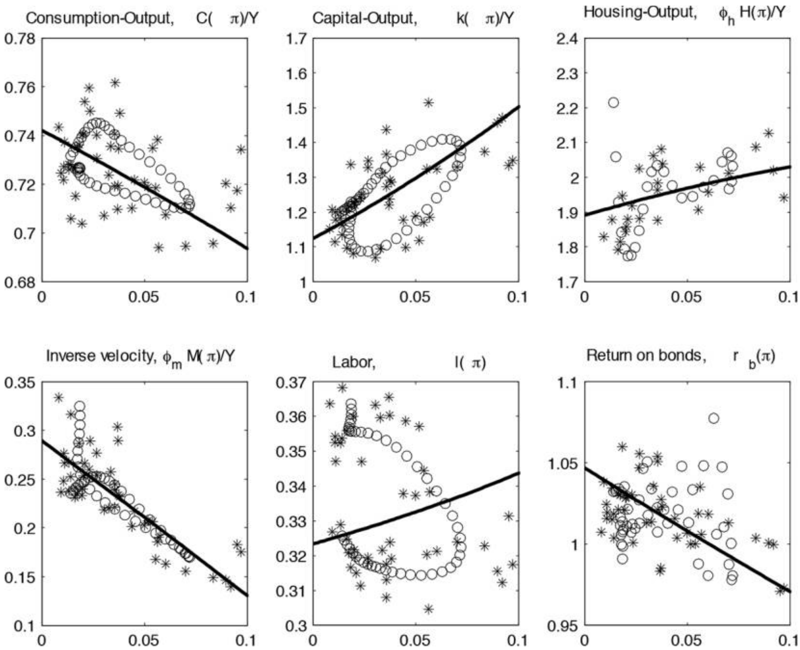

Fig. 4. Inflation and ratios (baseline calibration): * = data; ○ = HP trends.

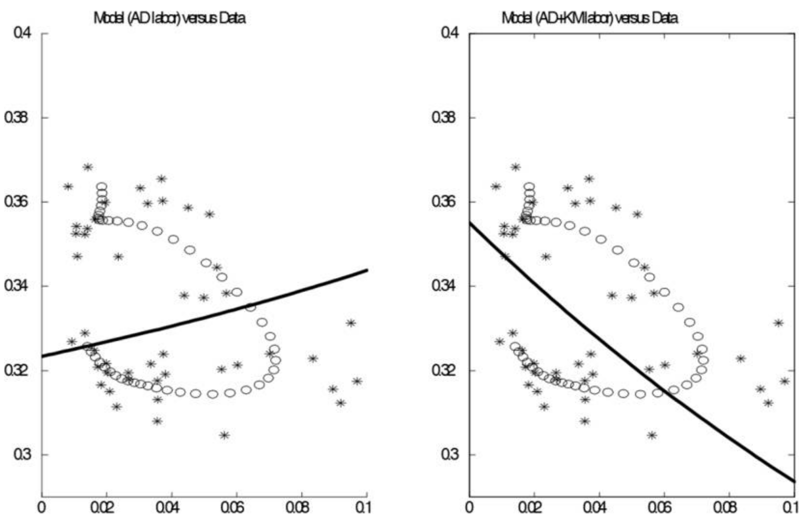

Fig. 5. Phillips curves: * = data; ○ = HP trends.

right panel adds ℓ and q, assuming the KM technology produces output one-for-one with hours, and rescaling the sum so that the model is still consistent with the calibration target 0.33. Under this interpretation, the model lines up well with the facts.

While we do not want to oversell any match between model and data, we feel obliged to defend its relevance against the obvious critique that it is generated assuming no changes in other driving forces, including taxes, technology shocks, and so forth. It is standard in macro to ask, counterfactually, what might have happened in the economy if the only driving process were changes in some variable. Here the variable is given by medium-run changes in inflation, and the answer is that the main variables of interest would have looked a lot like what was observed in the data. This should shift one's prior toward thinking that maybe inflation is interesting and that the model is reasonable. One still ought to think about robustness, however, particularly with respect to the choice of the targets generating the parameter values.

Much of our calibration strategy is standard, except perhaps matching the relationship between filtered output and inflation. One can try to set the KM parameters (η, υ, σ) to match only the empirical money demand relationship, although identification is fairly weak—there are different combinations of these parameters that fit the M/PY versus π data about as well. Table 5 shows the alternative calibration where only M/PY versus π data are used to set (η, υ, σ). The main change is in the value for μ_k. Figures 6 and 7 show the results for this calibration, while Figure 8 overlays the relevant data. Comparing these to figures 2 through 4, for most variables the results are very similar, and the predictions for the ratios k/y, c/y, and so forth, are still in line with the data. This implies that many predictions are robust along these dimensions, but there are two exceptions.

First, the baseline model predicts output increases with π, while the alternative predicts it falls. This is driven by the difference in the pledgeability of capital. The baseline calibration selects values for (μ, σ, υ) that require a higher μ_k to match the target capital-ouput ratio, and this makes the Tobin effect stronger. Second, there is a big difference in the implications for welfare. Welfare rises with π over a broad range in the baseline model, since ameliorating the tax-related investment distortion more than compensates for the decline in KM activity. This is not true for the alternative calibration, where the KM effect dominates, and there are substantial welfare losses from positive inflation: reducing π from 10 percent to the Friedman rule is worth almost 5 percent of x. The

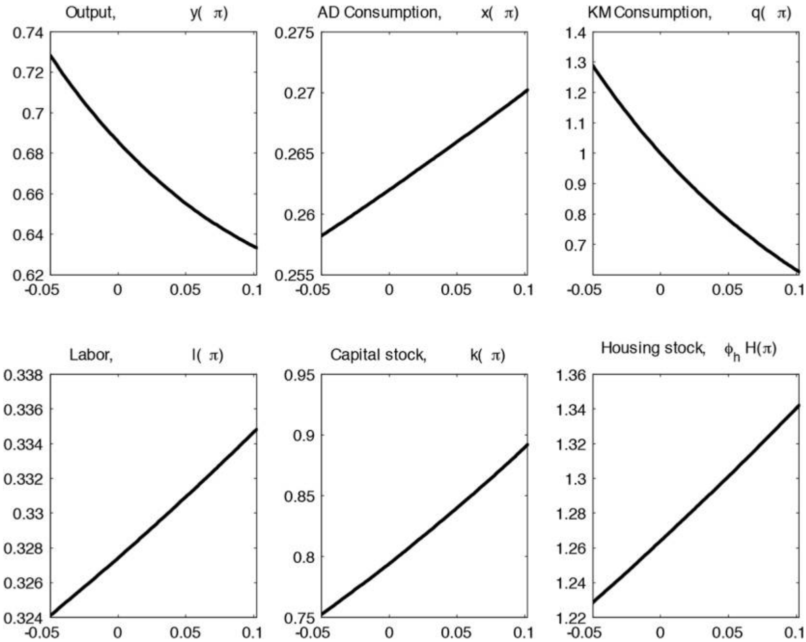

Fig. 6. Inflation and aggregate variables (alternative calibration).

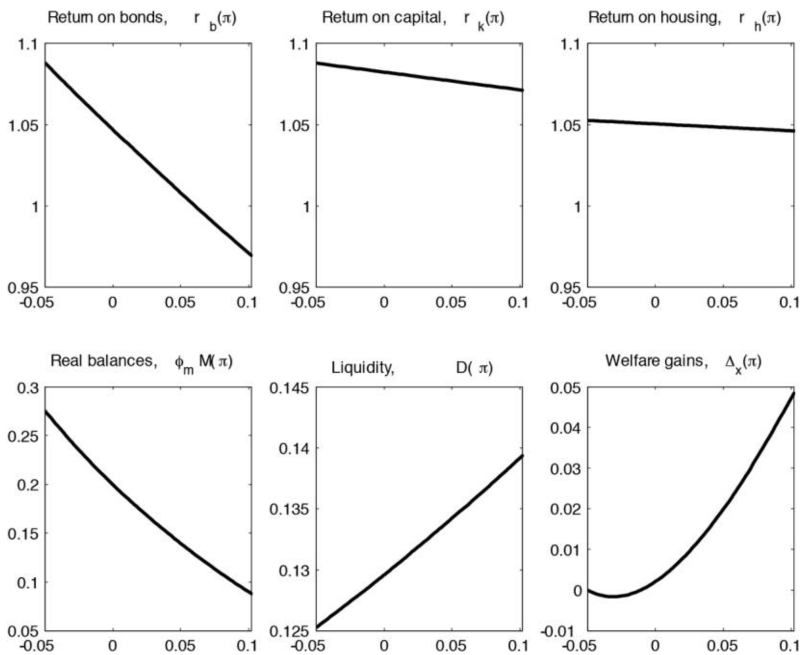

Fig. 7. Inflation and aggregate variables (alternative calibration).

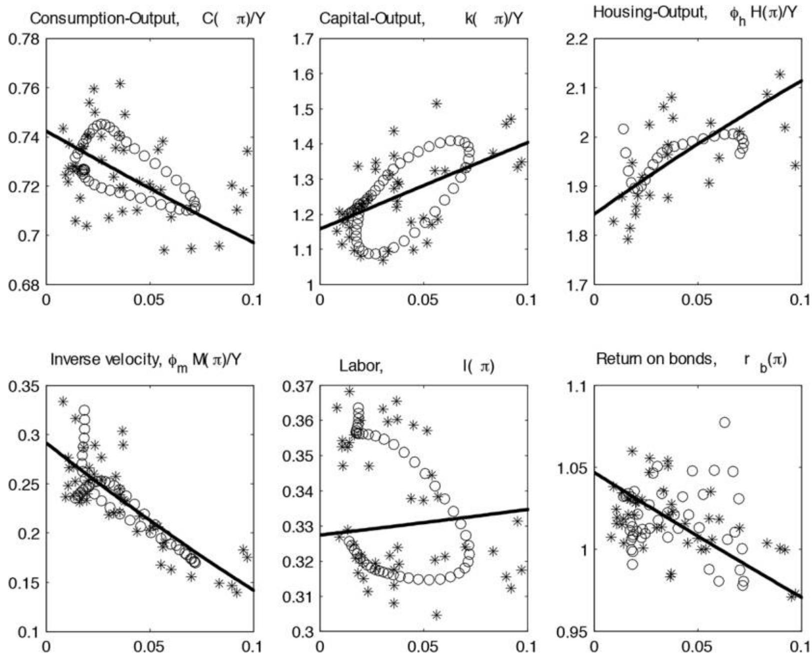

Fig. 8. Inflation and ratios (alternative calibration): * = data; ○ = HP trends.

optimal inflation rate is now negative, although still above the Friedman rule. Thus, while many of the positive implications of the theory are quite robust, some normative implications are less so.[19]

In terms of other robustness issues, we recalibrated the model after setting $\theta = 1$, which can be interpreted as Walrasian pricing. The results (not shown) are similar: although q is higher, the effect of going to the Friedman rule is slightly lower, and there are minor differences in the impact of π on k and h. Where $\theta = 1$ misses, of course, is that it gives a KM markup of 0. We also checked robustness by shutting down taxes. The results (not shown) differ from the benchmark because taxes have big effects on capital. Consider the cost of inflation. With $\tau_a = \tau_\ell = 0$, going from $\pi = 0.1$ to the Friedman rule, which is now optimal, is worth nearly 18 percent of x, with about half coming from KM and half from AD. Even going from $\pi = 0.04$ to the Friedman rule is worth nearly 5 percent of x. We do not think these numbers are too meaningful, however, since $\tau_a = \tau_\ell = 0$ is not a compelling calibration.

One can also use the model to quantify the effects of financial innovation, interpreted as changes in the pledgability parameters. Figure 9

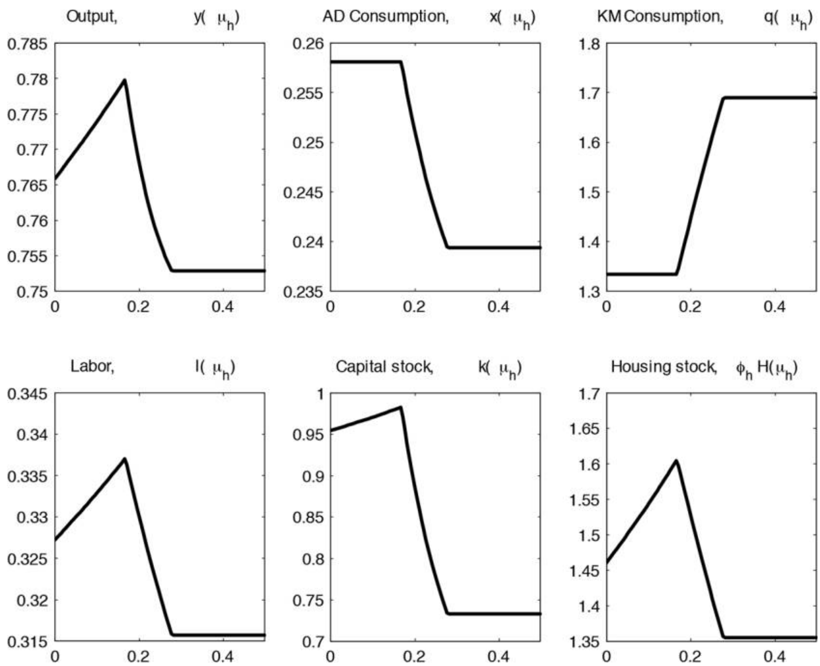

Fig. 9. Housing pledgability and aggregate variables.

shows the effect of raising μ_h. When μ_h is small, an increase leads to a housing boom, combined with higher output, investment, and employment; as the innovation continues, however, it ultimately leads to a bust, with declines in output, investment, employment, and the housing market. The reason is that as liquidity becomes more abundant, its marginal value decreases, reducing liquidity premia and asset prices. In particular, once liquidity is sufficiently abundant that buyers can get at least q^i on credit, money is no longer valued. The kinks in the graphs indicate points where we switch across the different types of equilibria described in section III.

He, Wright, and Zhu (2012) use a related but simpler model to evaluate the possibility that financial innovation spurred the US housing experience since 2000, along with alternatives, like self-fulfilling prophecies. The findings are similar: while a housing-fueled expansion and contraction can be explained qualitatively, it is difficult to match the facts quantitatively. The ratio of the value of the housing stock (the last panel of figure 9) to GDP (the first panel) peaks at just over 2, while in the data it reached approximately 2.76 at the height of the boom. Hence,

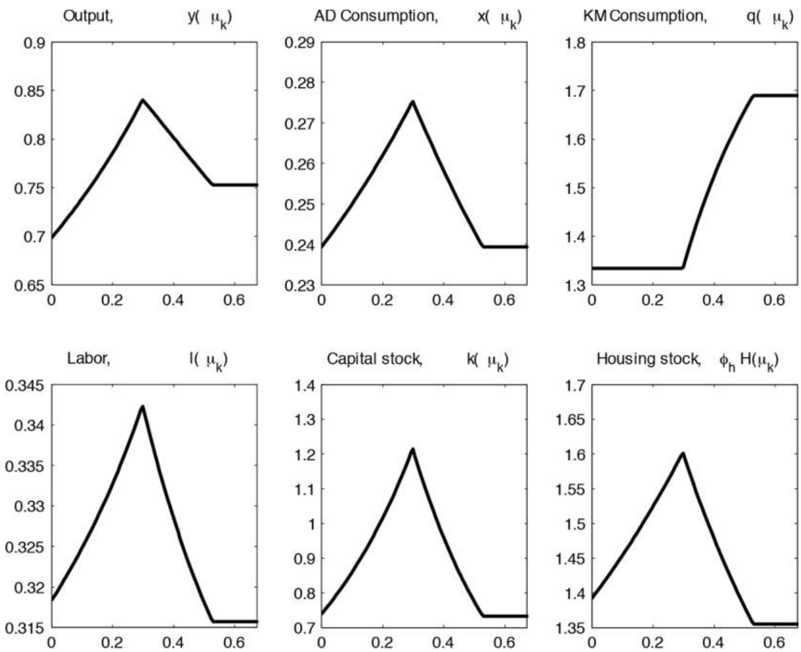

Fig. 10. Capital pledgability and aggregate variables.

while financial effects may have been significant, there is room for other forces. For comparison, Figure 10 repeats the analysis for an increase in μ_k. As with μ_h, the impact on output, employment, capital, and housing is first positive and then negative, for the same reason. What is not reported in these figures is that increases in pledgability typically improve welfare. The general rule is that improvements in credit conditions can be good, even if it looks bad for some macro aggregates.

VI. Conclusion

In modeling the role of assets in facilitating intertemporal exchange, a key ingredient is the asynchronization of receipts and expenditure. Credit was modeled as in Kiyotaki and Moore (1997, 2005), but focusing on consumers rather than producers, in the spirit of Kehoe and Levine (1993, 2001), and models generally going back to Bewley (1977). Debt limits were endogenized by limits to punish defaulters by seizing assets. This was guided by work on the microfoundations of money going back to Kiyotaki and Wright (1989, 1993), except here agents do not use assets as media of exchange, they have a portfolio of each element, which can be used to a greater or lesser extent as collateral. As in Holmstrom and Tirole (2011), we focus on pledgability. The setup is tractable enough to yield many analytic results, although to some readers a few ingredients might be unfamiliar, like abstract pricing mechanisms. In terms of substantive economics, we used the model to study a variety of issues in macro and finance. We analyzed how inflation affects returns as a function of asset pledgability, and how it affects output and employment. Also, we showed there is crowding out of liquidity: inflation reduces real balances, but given alternative pledgable assets, total liquidity changes by less.

The quantitative results imply that over a relevant range inflation can increase output, employment, investment, and consumption. While the nominal returns on illiquid assets go up one-for-one with inflation, nominal returns on liquid assets go up significantly less. This is especially true for bonds, which are very liquid and hence earn a lower return than capital, consistent with the empirical equity premium. The price and quantity of housing increase with inflation, consistent with the data. The benchmark model also implies welfare is increasing in inflation over a relevant range, due to a Mundell-Tobin effect combined with investment being too low due to taxation. For the baseline calibration, the optimal inflation rate was just about the mean in the data, although this is sensitive to the parameterization. In an alternative calibration, the predictions for most variables are similar, except inflation decreases output and the optimal inflation rate is negative.

It would therefore be good to know whether inflation increases output in reality. There is a big literature on this, which can only be discussed briefly here. Bullard and Keating (1995) find that permanent changes in inflation do not increase in real output for most countries, but do for some low-inflation countries. Ahmed and Rogers (2000, p. 3) conclude from a century of US data that "the long-run effects of inflation on consumption, investment, and output are positive . . . [and] ratios like the consumption and investment rates are not independent of inflation, which we interpret in terms of the Fisher effect." Therefore, they say "models generating long-term negative effects of inflation on output and consumption seem to be at odds with data." On the other hand, for OECD (Organization for Economic Cooperation and Development) economies Madsen (2003) finds investment is negatively related to inflation. In terms of returns, King and Watson (1997) find that nominal interest rates do not adjust one-for-one with permanent inflation, for many plausible identification schemes.[20]

After surveying much literature, Bullard (1999, p. 74) concludes: "While the overall evidence on these questions is mixed, considering only lower inflation countries leads to the conclusion that permanently higher money growth or inflation is associated with permanently higher output and permanently lower real interest rates." He also says "this result is inconsistent with many—almost all?—current quantitative business cycle models, which generally predict that permanently higher inflation permanently lowers consumption and output. There is little support for such a prediction in the studies surveyed here. This is an important empirical puzzle that stands as a challenge for future research." Our model *is* consistent with these findings. Future work might try to better understand the facts, to look at higher-frequency implications of the model, and to apply it to additional policy issues. That is left to future work.

Appendix
Data Sources

1. Nominal GDP: Table 1.1.5 from NIPA, Bureau of Economic Analysis. Housing services and utilities are deducted.

2. Consumption: Table 1.1.5 from NIPA, Bureau of Economic Analysis. Housing services, utilities, and durables are deducted.

3. Nonresidential capital stock: Table 4.1 of the Fixed Asset Account tables.

4. Housing stock: From Davis and Heathcote (2007).

5. Investment (residential and nonresidential): Table 1.1.5 from NIPA, Bureau of Economic Analysis.

6. Sweep-adjusted M1: Series M1S in Cynamon, Dutkowsky, and Jones (2006).

7. Inflation: Tables 1.1.4 and 2.3.4 from NIPA, Bureau of Economic Analysis.

8. Return on bonds: Nominal returns from the H15 release of the Board of Governors of the Federal Reserve.

9. Treasury bills outstanding: Table L-105 in the Flow of Funds tables of the Board of Governors of the Federal Reserve.

10. Residential mortgages and home equity loans: Table L-218 in the Flow of Funds tables published by the Board of Governors of the Federal Reserve.

11. Labor: Aggregate hours and labor force data published by the Bureau of Labor Statistics.

Endnotes

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1. Suppose at date t you have assets that will be worth ϕ_s at date $s > t$, and you use them to secure a loan between t and s. If no punishment is available except forfeiture of collateral, clearly your debt limit is ϕ_s because you will honor an obligation if and only if it is less than the value of the collateral. It is equally clear that, instead of using the assets as collateral, you can turn them over and finalize the transaction at t. At least, this is the case without some reason to prefer either immediate or deferred settlement. We talk more later about why one may have such a preference.

2. We think these kinds of computational exercises constitute a step in the right direction for research that tries to model the microfoundations of the exchange process. Early work in this vein was not meant to be quantitative; the goal was to elucidate the roles of various frictions on transactions patterns. As methods and models advance, it becomes increasingly possible to incorporate elements of the theory into more standard macro models. Ours is obviously not the first attempt, but rather than listing individual contributions, we refer readers to Aruoba, Waller, and Wright (2012).

3. A related paper is Lester, Postlewaite, and Wright (2012), where differential liquidity is modeled using information frictions: some traders are unable to recognize asset quality. In that paper, agents who do not recognize quality reject assets outright, which avoids bargaining under asymmetric information, but then liquidity differs only on the extensive margin (acceptance by more or fewer counterparties). One can tackle bargaining under asymmetric information in the model, as in Rocheteau (2011), Li and Rocheteau (2011), and Li, Rocheteau, and Weill (2012), and also get liquidity differentials on the intensive margin (acceptance of assets up to endogenous limits), but that is complicated, and often relies on special protocols like take-it-or-leave-it offers. Our approach is based on commitment rather than information frictions—that is, on pledgability rather than recognizability—which is much easier. This allows us to go well beyond those papers in terms of applications and quantitative analysis. There is much more work on the microfoundations of monetary economics that is related, some of which is discussed later, but there are too many papers to list individually. Therefore we refer readers to the abovementioned surveys on New Monetarist economics, and to Gertler and Kiyotaki (2010) for a survey of related work from a somewhat different perspective.

4. Frictions like spatial or temporal separation, and commitment or information issues, are problems in the *environment*, while sticky prices are problems with the *mechanism* for determining the outcome. We think it is important to make the distinction, and to assert that the former are better. Of course, problems with pricing are also worth considering, including holdup problems in bargaining models. When holdup problems arise, it is usually possible to ground them in terms of the timing of whom meets whom and when, as well as explicit commitment problems.

5. In terms of modeling strategy, the alternating-market structure is meant to capture in a tractable way the obviously correct notion that in reality not all economic activity

takes place in frictionless settings, nor does it all take place in settings with search, limited commitment, and so forth. Instead of alternating the markets, one can have them both always open, with agents transiting randomly between (e.g., Williamson 2006). The theory is robust to changing many other details. One can assume households in the KM have the same utility but different endowments. Or that q is a factor of production and they realize different investment-opportunity shocks. Or that gains from trade arise from random matching, with σ interpreted as the probability of meeting someone who can produce something you want. Or, instead of trading with each other in KM, households can trade with producers or retailers.

6. Recognizing that the structure described in the following is complicated, we impose method as much as we can on notation. Thus, all quantities are represented by Roman and all prices by Greek letters.

7. The loan-to-value ratio is sometimes assumed to be constant below, $D_j(a_j) = \mu_j a_j$. ~~There are various ways in the literature to rationalize debt limits, and in particular $\mu_j < 1$.~~ Sometimes there is appeal to diversion: creditors can seize only a fraction of your assets, while you abscond with the rest. Sometimes there is appeal to resources getting used up by seizure, including litigation costs (e.g., Iacoviello 2005), but for us that is irrelevant—compliance can be encouraged here by the threat of *burning* your assets. Holmstrom and Tirole (2011) take pledgability as a primitive, but provide several ways to motivate it. There are also formalizations based on private information, as discussed in note 3.

8. Kalai is certainly more tractable: as Lagos and Wright (2005), generalized Nash leads here to a similar outcome, except

$$z(q) = \frac{\theta u'(q)c(q)}{\theta u'(q) + (1-\theta)c'(q)} + \frac{(1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)}.$$

This is the same as (8) when $u(q) = c(q) = q$, or when $\theta = 1$. Otherwise, they give the same outcome i.f.f. $d \le \bar{D}$ does not bind. Aruoba, Rochteau, and Waller (2007) show that Kalai guarantees the trading surpluses are increasing in $\bar{D}$, that V is concave, and that buyers have no incentive to conceal assets. Generalized Nash does not guarantee these results, but still can be and has been used in many New Monetarist models.

9. This can also be derived using an axiomatic approach: Gu, Mattesini, and Wright (2012) show that any mechanism satisfying feasibility, individual rationality, bilateral efficiency, and monotonicity must satisfy assumption 1. But the important point here is simply that none of our theoretical results depend on a particular way of splitting gains from trade.

10. In case it is not obvious, (20) and (21) are the FOCs for x and $\hat{k}$, after inserting factor prices ω and ρ from (1); (22) clears the housing market, with ϕ_h adjusting to make that happen; and (23) clears the AD goods market. Note that ℓ denotes aggregate labor in these equations. Individual household labor depends on net wealth in AD, which generally differs across households depending on debt from the previous KM market, but we only need aggregate ℓ to define macro equilibrium.

11. A few of these results impose that r is not too big. More substantively, in terms of modeling, having housing allows us to make a few points. First, when h is endogenous, supply and demand jointly determine (h, ϕ_h), and when $h = H$ is fixed, demand simply pins down price. One could make a similar point by comparing k and e, but that is less interesting because $\phi_k = 1$, which is not true for ϕ_h. Second, different from other assets, h affects utility directly and not only via the budget equation. Suppose, for example, that $U(x, h) = \tilde{U}(x) + h^{1-\xi}/(1-\sigma)$. Then when H increases, home equity $\phi_h H$ can go up or down, so liquidity can become less or more scarce, depending on $\xi \gtrless 1$. Because of this, welfare can also go up or down as H increases. By contrast, increases in the supply of e always raises liquidity and welfare. Third, h provides a plausible situation where it is not equivalent to use an asset as collateral or as a medium of exchange: even if it were possible to hand over part of your house in a KM transaction, our assumption is that then it cannot be used that period, so you prefer deferred settlement secured by h.

12. At a risk of introducing jargon, one might call this a *paradox of liquidity*: individuals

can try to relax borrowing constraints by investing in k, but if everyone does so ρ can fall by enough to tighten credit conditions.

13. It is not hard to show $i = 0$ implies $q = q^*$ with Walrasian pricing or Kalai bargaining, but not with generalized Nash bargaining unless $\theta = 1$. For Nash with $\theta < 1$, $i = 0$ is optimal but it does not achieve q^*. Hence, it would be desirable to set $i < 0$, but there is no equilibrium with $i < 0$. This is the New Monetarist version of the New Keynesian *zero lower bound* problem. Here, $i < 0$ is desirable if only it were feasible because it corrects a problem with bargaining.

14. If $U(x) = x^{1-\eta}/(1-\eta)$ and $f(k, \ell) = k^{\alpha}\ell^{1-\alpha}$ then one can show $\partial\ell/\partial i > 0$ when $\eta < \hat{\eta}$, where $\hat{\eta} > 1$. Also, by employment we mean AD labor, since q is not produced. We set it up this way precisely to identify employment by ℓ, but alternatively, if q is produced there is an effect going the other way (see section V). And we abstract from the effect in Rocheteau, Rupert, and Wright (2007) and Dong (2011), where x and q interact in utility, making $\partial\ell/\partial i \gtreqless 0$ depend on whether they are substitutes or complements. Some people seem to think employment increases with inflation in the data, although our reading is the opposite (see Berentsen, Menzio, and Wright [2011] or figure 5). So perhaps it is good the framework is flexible in its prediction for $\partial\ell/\partial i$.

15. If more sellers have the μ_2 technology, it is easier to use e as collateral, which increases the demand for e. This increases the price ϕ_e and makes agents more keen on being able to trade using e as collateral, so χ goes up.

16. The exercises are in the nature of controlled experiments, or numerical partial derivatives—for example, we ask, counterfactually, what would the model predict if inflation were the sole driving force? While this is not the only possible approach, it has much precedent, including the practice in macro following Kydland and Prescott (1982) of asking what a model would predict if technology shocks were the sole impulse.

17. As Holmstrom and Tirole (2011) say, " In the runup to the subprime crisis, securitization of mortgages played a major role . . . by making nontradable mortgages tradable, [and this] led to a dramatic growth in the US volume of mortgages, home equity loans, and mortgage-backed securities in 2000 to 2008." As Ferguson (2008) or Reinhart and Rogoff (2009, p. 87) put it, this allowed consumers to start treating their houses as "ATM machines." To be clear, this is not a problem for the theory, which in fact puts front and center issues relating to credit and pledgeability. But our strategy is to calibrate to more "normal" times, then see what the model says about events since the turn of the millennium after adjusting financial parameters.

18. See Aruoba, Waller, and Wright (2012) for a discussion of markup data taken from the Annual Retail Trade Survey. Also, for μ_h, we need to adjust for two factors. First, around 30 percent of home capital is used to collateralize mortgages, and is therefore not available to secure consumption loans. Thus, home equity is given by $0.7(1 - \delta_h)\phi_h h = 0.63\phi_h h$. Second, in the model, only a fraction σ of households need loans in any period. So home equity loans over housing wealth is $0.63\sigma\mu_h$. A target for this ratio and a choice for σ pins down μ_h.

19. We discuss this more, in the context of the empirical literature on the effects of inflation, in the conclusion.

20. Other papers focus more on growth rates, not levels. Gillman and Kejak (2011) find that growth, investment, and real interest rates tend to be negatively affected by inflation. Gillman and Nakov (2003) find evidence for the United States and United Kingdom consistent with a Tobin effect, while the growth rate of output is reduced by inflation. Ericsson, Irons, and Tryon (2001) find in cross-country regressions that output growth is often negatively related to inflation, but that result is not robust, and correcting for several problems leads to a positive effect for most countries. Benhabib and Spiegel (2009) find nonmonotone effects: when inflation is high it is negatively correlated with growth, while a positive relationship exists for negative to moderate inflation. López-Villavicencio and Mignon (2011) also find nonlinear effects: above a threshold inflation decreases growth, and below it inflation increases growth.

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