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Comment

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Introduction

Adam, Kuang, and Marcet propose and analyze an interesting model of house price dynamics. It is pitched as an open economy model and used to study the relationship between aggregate debt and house prices. The model suggests a linkage between the current account and the value of the housing stock. The authors present some figures that motivate their analysis, with the financial crisis an important component of their data summaries. Their modeling of asset valuation follows Kiyotaki and Moore (1997) by exploring the role of collateral and follows Adam and Marcet (2011) (and related unpublished work) by exploring a particular way to relax an assumption of rational expectations. My comments focus primarily on a simplified version of their model, which I use to suggest ways to make this line of research more empirically ambitious. I also remark on the potential role of nonlinearities in the model specification for altering the implications.

Housing As an Asset

The paper adopts a simple valuation model as a device to model housing values. Consumers have preferences of the form:

$$E\left[\sum_{t=0}^{\infty}\delta^{t}(\xi_t h_t + c_t)\right],$$

where δ is a discount factor, ζ_t is a preference shock process and evolves as:

$$\log \xi_t = d + \log \xi_{t-1} + \log \varepsilon_t, \tag{1}$$

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and c_t is consumption. Preferences are linear in consumption. Strictly speaking their specification has $d = 0$, and thus the marginal rate of substitution between housing services and consumption is expected to increase over time at a rate given by one half the variance of log ε_t. It seems likely that adding a drift to this specification would not alter the analysis in an important way, and in what follows I include it.

To complete the preference specification, Adam et al. (2011) introduce the possibly nonlinear transformation:

$$h_t = G(H_t),$$

where H_t is the stock of housing. There are two possible interpretations of resulting preference specification. One interpretation is that h_t is the service flow from housing, in which case ζ_t is the implied rental rate as the authors write in their paper. This is not the only possible interpretation, however. Since consumers rent H_t not h_t, we may take $\xi_t G'(H_t)$ as the implied rental rate for housing where the service flow is assumed to be equal to the stock. This measure of the rental rate is distinct from ζ_t. While the authors think of G as a nonlinear transformation from housing stock to services, G might just as well represent curvature in preferences. This discussion is only interesting when G is nonlinear, and in this paper the nonlinearity of G seems not to play much of a role in the analysis. With the exception of the reported outcome of a Dickey-Fuller test for a unit root, time series on rental rates are used rather informally. Formally incorporating time series data on rental rates simultaneously with housing prices would direct the analysis toward a more serious discussion of whether G represents "preferences" or "technology." In some of my subsequent discussion I will consider other reasons to use rental rates in an econometric analysis.

Adam, Kuang, and Marcet follow prior research by Kiyotaki and Moore (1997) and others in which a financing constraint is introduced:

$$b_t \leq \theta \frac{E_t q_{t+1}}{R} H_t = \theta \frac{E_t q_{t+1}}{q_t R} V_t, \tag{2}$$

where R is the interest rate assumed to be constant and given externally to the model, q_t the price of a homogeneous unit of housing, and $V_t = q_t H_t$ is the value of the housing stock. Prospective limits to the amount of borrowing induces a collateral value to the housing stock. Increases in the value of a house enlarges the amount of permissible debt, and this adds to the value to home owning beyond the direct consumption of housing services.

The asset-pricing formula for the stock of housing must account for its value as a source of collateral. Adam et al. follow previous literature by imposing an incentive for borrowing. The gross rate of interest R, assumed to be constant and determined outside the model, is less than $1/\delta$. As a result, the borrowing constraint binds and a house gains additional value by allowing for additional borrowing. The motivation for the inequality relating the subjective rate of discount to the interest rate is the time series data on the current account for the United States. With these simplifying assumptions on R, the relevant discount factor for housing is:

$$\rho = \delta(1 - d) + \theta\left(\frac{1}{R} - \delta\right),$$

where d is the depreciation rate. The second term captures the collateral value of the house. This term increases the discount factor ρ and thus adds to the value of house.

The model in Adam et al. focuses on rental rates induced by random preference shocks as the source of fluctuations in housing values. Is this really the most important source of variation in housing values? Recent dynamic stochastic equilibrium models feature other sources of fluctuations. For instance, temporal variation in θ has been used to capture changes in borrowing environments. See, for instance, Jermann and Quadrini (2011). Of course, such an approach leads naturally to the question of what might induce such fluctuations. Interest rates are typically not constant in such models, and interest rate fluctuations are of particular interest for understanding fluctuations in housing prices. More generally, stochastic discount factor variation induced by risk averse investors, and perhaps amplified by market imperfections, could be critical to understanding the behavior of expected returns, including returns to investing in the housing market.[1] The lack of interest variability and the absence of time variation in risk prices closes down two important channels for asset price determination. As we will see, the linearized version of the equation that determines housing prices becomes a version of the present-value model that LeRoy and Porter (1981) and Shiller (1981) challenged empirically in their study of stock prices. In the macro/finance literature, part of this puzzling finding has been attributed to time-variation in risk premia induced by changes in risk prices or exposures. I suspect these additional channels for variation in asset values are important for understanding housing values. Adam et al. explore an alternative channel that is also interesting: distorted expectations. I will have more to say about that channel later. To the authors' credit, they consider the impact of changing real interest rates in experiments in which investors presume before and after the change that interest rates will remain constant. The overall model may be linear enough that the calculations have more general validity as an interesting approximation. Nevertheless, a more complete attempt at shock accounting for housing-market would be a useful complement to the current results in this paper.

Model Solution

I now expand on a pedagogically useful simplification of the model when G is linear and borrowing constraints are known to bind. Of course this simplification misses some potentially interesting nonlinearities, but it gives us a valuable starting point. The model solution proceeds as follows.

1. Solve for the housing price q_t as a function of the preference shock (rental rate).
2. Use the housing price solution to infer the housing stock H_t from supply considerations.
3. Use the binding financing constraint to infer the aggregate debt b_t.

Absent this linearity of the transformation G, we are compelled to solve for (q_t, H_t) simultaneously (combine steps 1 and 2). Absent a binding borrowing constraint, all three steps must be done simultaneously. While the authors have the more ambitious model as their target, the actual analysis does not drift far away from the simplified version.

The basic housing price formula is:

$$q_t = \sum_{j=0}^{\infty} \rho^j E[\xi_{t+j} G'(H_{t+j}) | \mathcal{F}_t].$$

When $G'(H) = g$ this formula simplifies to:

$$q_t = \left(\frac{g}{1-\lambda}\right)\xi_t$$

where

$$\lambda = \rho \exp\left(g + \frac{\sigma_\varepsilon^2}{2}\right).$$

While the distinction between ρ and λ plays no role in this paper, equality could be induced by setting $g = -(\sigma_\varepsilon^2/2)$ in contrast to the $g = 0$ specification posed in this paper. The variance adjustment does not show up in the first-order approximation in the shock exposure, consistent with some of the calculations that follow, so I will also ignore the variance adjustment in this section; but I will include g when I consider a more general specification of the process ζ. Recall that in the house-price formula, ρ includes both the usual discount and depreciation rate adjustments intertwined with an adjustment for the collateral value of housing.

I generalize the specification for ζ by representing its growth rate as an infinite-order moving average:

$$\log \xi_{t+1} - \log \xi_t = d + \eta\psi(L)\varepsilon_{t+1}$$

where $\psi(L)$ is an infinite-order row vector of polynomials in the the lag operator, L, and ε is a possibly multivariate i.i.d. shock process with mean zero and covariance I. For future reference, the "z-transform" of the moving-average coefficients is:

$$\psi(z) = \sum_{j=0}^{\infty} \psi_j (z)^j, \quad \sum_{j=0}^{\infty} |\psi_j|^2 < \infty,$$

which defines a function of the complex variable z that is well-defined as a power series for $|z| < 1$. The parameter η scales the shock exposure and is used as a way to compute an approximate solution.

Construct the deterministic counterpart as:

$$\log \xi_{t+1}^0 - \log \xi_t^0 = d,$$

and a "first derivative" component with respect to η as:

$$\log \xi_{t+1}^1 - \log \xi_t^1 = \psi(L)\varepsilon_{t+1}.$$

Of course, this is really not an approximation but a decomposition, because the first-order adjustment leads to an exact representation.

As is standard in the asset pricing literature with growth, it is convenient to work with the counterpart to the logarithm of a dividend-price ratio:

$$v_t = \log q_t - \log \xi_t.$$

Compute $v_t^0 = \bar{v}$ by solving for the deterministic $\eta = 0$ value:

$$\bar{v} = \frac{g}{1 - \lambda},$$

where $\lambda = \rho \exp(d)$. The derivative process for v satisfies the difference equation:

$$v_t^1 = \lambda E(v_{t+1}^1 \mid \mathcal{F}_t) + \lambda E(\log \xi_{t+1}^1 - \log \xi_t^1 \mid \mathcal{F}_t).$$

I solve this model using the same tools as in Hansen and Sargent (1980), Whiteman (1983), and others, by converting moving-average coefficients into power series of a complex variable z that converges for $|z| < 1$. This allows me to represent the solution in terms of a function of a complex variable. I guess a solution represented as an infinite-order moving average of current and past shocks:

$$v_t^1 = \upsilon(L)w_t.$$

Then the function

$$\upsilon(z) = \lambda\left[\frac{\upsilon(z) - \upsilon(0)}{z}\right] + \lambda\left[\frac{\psi(z) - \psi(0)}{z}\right] \tag{3}$$

for $|z| < 1$. The terms in square brackets are the z-transforms of the moving-average coefficients for the one-step-ahead forecasts. Rearranging terms of (3):

$$(z - \lambda)\upsilon(z) = -\lambda\upsilon(0) + \lambda[\psi(z) - \psi(0)].$$

By evaluating this equation at $z = \lambda$,

$$\upsilon(0) = \psi(\lambda) - \psi(0).$$

Therefore,

$$\upsilon(z) = \frac{\psi(z) - \psi(\lambda)}{z - \lambda}.$$

This formula gives the (first-order approximate) solution for the logarithm of the ratio of the price of the stock of housing to the rental rate. It shows how the stochastic dynamics of the exogenously specified process ζ get transmitted into the stochastic dynamics for the housing price. The formula for υ looks problematic because of the division by $z - \lambda$ suggesting that the function might be poorly behaved near $z = \lambda$. Notice that $\psi(z) - \psi(\lambda)$ is also zero at $\lambda = z$, so in fact there is an implicit cancellation that can be done and behavior near $\lambda = z$ is not unusual. This formula depicts the restrictions embedded in the present-value formula and essentially reproduces a result in Hansen and Sargent (1980).

To understand the empirical challenge implied by a time series on housing prices, it is also of interest to "invert" this mapping. For this computation I take as a given the housing price dynamics and infer what rental rate processes are consistent with these dynamics. Suppose:

$$\log q_t^1 - \log q_{t-1}^1 = \phi(L)w_t$$

where

$$\phi(z) = (1 - z)\upsilon(z) + \psi(z) = \frac{\psi(z) - \lambda\psi(z) + z\psi(\lambda) - \psi(\lambda)}{z - \lambda}. \tag{4}$$

I take (4) and solve for the function, $\psi(z)$, used to represent the linear dynamics for ζ:

$$\psi(z) = \frac{1 - z}{1 - \lambda}\psi(\lambda) + \frac{z - \lambda}{1 - \lambda}\phi(z).$$

This equation does not pin the $\psi(\lambda)$, where

$$\psi(\lambda) = \sum_{j=0}^{\infty} \psi_j \lambda^j$$

is the discounted response of the housing services to a shock. Given $\psi(\lambda)$ and $\phi(z)$ we can infer a unique $\psi(z)$. What do I make of this? To the extent there is an empirical challenge posed by a rational expectations version of this model, it relies on explicit restrictions on the process ζ used model rental rates. The resulting empirical challenge may very much be similar in nature to the stock-price excess volatility puzzle analyzed initially by LeRoy and Porter (1981) and Shiller (1981). The puzzle they analyze is sensitive to the stochastic properties of stock prices and dividends, and not just stock prices alone.

While Adam et al. use some information on rental rates in their discussion, they posit a model in which $\psi(z)$ is constant. It follows that ϕ is equal to this same constant, as implied by (4). Thus the linearized rational expectations version of the model implies that growth rates in housing prices are not predictable. While this latter finding becomes a platform to explore the consequences of modifying the rational expectations assumption, a good complementary exercise is to consider alternative models of rental rates that might have some empirical validity. Such an analysis would provide a better and more general statement of the empirical shortcomings of the rational expectations version of their model.

A Digression on Rational Expectations Econometrics

Adam et al. model investors as using house prices to forecast future rental rates. Before discussing their approach, let me review previous literature related to relational expectations econometrics. My digression is meant to add clarity about the role of the restrictions across rental rates and housing prices for empirical analysis and to help place the approach to expectations taken in Adam et al. in the context of an earlier literature.

Formula:

$$\upsilon(z) = \frac{\psi(z) - \psi(\lambda)}{z - \lambda} \tag{5}$$

derived previously captures the cross-equation restrictions embedded in the assumption of rational expectations. The function ψ that governs rental rate dynamics, together with the discount factor λ, determine the dynamics for housing prices relative to rental rates. To apply this formula, I am compelled to say something about the information that is available to economic agents. This is reflected in part in the specification of number of shocks, but also by how much information is revealed by these shocks that is useful in forecasting future rental rates.

A feature of this linearized model is that we can allow economic agents to observe more than an econometrician. This linearized model is a special case of what Hansen and Sargent (1991) call an exact linear rational expectations model. Suppose that the econometrician observes both rental rates and housing prices. Include $\log \xi_t - \log \xi_{t-1}$ and $\log q_t - \log \xi_t$ as the first two components of the vector y_t observed by the econometrician. Other variables observed by economics agents may be included in this vector. Suppose that

$$y_t = \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j} + \mu \sum_{j=0}^{\infty} \text{trace}\Psi_j(\Psi_j)' < \infty$$

where ε is an i.i.d. sequence of shock vectors with mean zero and covariance matrix I. In addition, suppose that the dimension of ε agrees with the dimension of y, and that the matrix function $\Psi(z)$ is nonsingular for $|z| < 1$. This nonsingularity restriction guarantees that linear combinations of current past value of y_t generate the same information as linear combinations of current and past values of ε_t. On the other hand, linear combinations of current and past values of ε_t may generate more information germane for forecasting future values of ξ_t than linear combinations of current and past value of ε_t. Consistent with our assumption about what economic agents know, current and past values y_t are in the information set of economic agents, but economic agents might observe more.

Building on an insight in Shiller (1972) and applying the Law of Iterated Expectations, restrictions (5) continue to apply to the econometrician's specification of information, where I now let $\psi(z)$ be the first row of $\Psi(z)$ and $\upsilon(z)$ be the second row of $\Psi(z)$. In other words, the cross-equation restrictions are robust to an econometrician specifying too little information provided that the econometrician uses house prices (or in fact the ratio of house prices to rental rates) in the analysis. In some rational expectations models an important component of information for economic agents is determined endogenously (see Lucas 1972 for an initial example). In my discussion, however, house prices are being used by an econometrician, even though within the economic model itself agents have other information to forecast future rental rates. While prices *within the model* do not reveal new information to the agents, *outside the model* prices reveal information to an econometrician.

Distorted Beliefs

Adam et al. push back on rational expectations to generate interesting low frequency movements in house prices. As I argued, the rational expectations version of the model has no scope for prices to reveal new information, although it may provide a useful summary statistic about future beliefs when growth rates in the rental process are predictable. Adam et al. entertain the notion that economic agents use prices to help them make forecasts and this can provide an intriguing way to modify the rational expectations assumption.

They suppose that economic agents form beliefs based on:

$$E[\log q_{t+1} - \log q_t \mid \mathcal{F}_t] = \log m_t$$

$$\log m_{t+1} - \log m_t = \gamma(\log q_t - \log q_{t-1} - \log m_t),$$

where $0 < \gamma < 1$. In the limiting case in which $\gamma = 0$, $\log m_t$ is invariant over time, but smaller values of γ allow for deviations that depend on past price movements. This allows for temporal dependence in the growth rate of house prices, even when it will be absent in the rational expectations counterpart economy.

To the extent that such a specification is successful, how do we rule out low frequency movements in rental rates? A more general claim might be that even with a more flexible rental rate specification constrained to be empirically plausible, one cannot generate the necessary price movements. As I mentioned previously, this empirical claim is not formally addressed in the paper. Nevertheless, we may have other good reasons to explore deviations from rational expectations. Since the Adam et al. model of the housing market has similar aims to models with speculative bubbles, there should be scope for making some interesting comparisons to such models.

In defense of their approach, Adam et al. argue:

> [E]ven expert economists rarely agree on the correct economic model linking fundamentals to prices. Therefore, it appears of interest to relax the assumption that agents know the correct model of prices and to consider instead agents who do not know exactly how prices behave. We assume that agents express their uncertainty about the true process by formulating a perceived joint distribution over prices and fundamentals.

While I like very much this motivation, the paper falls short of explaining how we jump to the last sentence. Other work on uncertainty aversion, robustness, and belief fragility put more structure on this problem. For example, see Hansen (2007) and Hansen and Sargent (2010). In this literature the expression of uncertainty and concerns about model misspecification have important implications for asset pricing, as reflected in a fluctuating uncertainty premia. Such fluctuations could be an additional source of asset pricing dynamics.

Potential Nonlinearities

My discussion has focused on a simplified version of the model and has abstracted from some potentially important sources of nonlinearities that could be explored. While the model specification given in Adam et al. allows for some nonlinearities, the actual calculations from the model do not seem to move far from a linear specification.

I have already discussed the transformation:

$$h_t = G(H_t),$$

where G can be nonlinear. I prefer to think of this as a way to put curvature in the utility function for housing services.

A second source of nonlinearity that could be intriguing to explore is the potential for endogenous regimes whereby the financing constraint only binds some of time. These fluctuations could be induced by interest rate variation or time series variation in θ, a parameter that plays an essential role in determining the collateral value of a house. Such generalizations result in a model that is harder to solve and would likely render perturbation type methods of solution inappropriate. Allowing for endogenous changes in the financing regimes could, however, allow for the study of much longer time series. Time series data could be analyzed that included episodes in which financing was severely limited, along with episodes in which such restrictions are much less important. Conceptually minor (but not computationally minor) changes in the model could lead to a rich extension of the current analysis.

Conclusions

In summary, my comments suggest some important next steps for this line of research.

• Introduce empirically plausible persistence into the growth rate specification for the preference shock process. This will allow for a richer discussion of the empirical implications of the model and a more revealing comparison to rational expectations models. Such an analysis will be all the more valuable if it is accompanied by an extensive exploration of the information embedded in rental rates on houses.

• Engage in a quantitative analysis of multiple shocks within the context of an extended version of this model. In addition to the preference shock considered here, stochastic fluctuations in the exogenous input to collateral value of housing, and an explicit stochastic analysis of interest rates, could be part of a comparison of the roles shocks play in accounting for the time series evidence.

• Modify the model to accommodate interesting fluctuations in stochastic discount factors and hence fluctuations in risk or uncertainty prices, perhaps motivated by investors' struggles with potential model misspecification.

The paper provides much food for thought.

Endnotes

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1. See Hansen and Renault (2010) for several examples of stochastic discount factor models explored in the macro/asset pricing literature.

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