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A Model of Household Investment in Financial Assets

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I. Introduction

The theory of consumer behavior, generally considered to include household behavior, is elaborately presented in a static, strictly consuming framework in most economic texts. Various more sophisticated aspects of household behavior have been given specialized attention in some texts and journal articles, but for the most part these have been concerned with elucidating some fine theoretical points and providing a research framework for a very limited range of empirical applications.[1]

To a considerable extent, the lack of more ambitious attempts to examine household behavior can be explained by the limitations of available data. More fundamentally, attempts to create richer models run afoul of mathematical intractability or of gross distortion of the richness of alternatives available to humans in their economic activities.[2]

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[1] See the references cited in the Bibliography at the end of the paper.

[2] Especially compare the articles by Bear [1] and Miller [6] cited in the Bibliography.

The Social Systems Research Institute at the University of Wisconsin is developing a library with extremely rich data, especially on a very large shifting panel of households over a period of thirteen or more years, with emphasis on assets and incomes data matched from various sources.[3] To take full advantage of these data when they become completely available for analysis, we consider it important to have a fairly detailed model of household behavior with respect to the available variables.

In particular, we wish to focus on explaining the choices individuals make with respect to ownership of financial assets, and with respect to use of their time. We need a model that simultaneously determines the size, composition, changes, and management of portfolios, as well as other productive expenditures of effort, all at a given time (or short period) and also over time. We wish to explore the cumulative effect of time spent in various activities such as portfolio management, the effects of aging, and cross-sectional as well as longitudinal differences in behavior associated with different demographic positions at one time and over time.

For example, increased effort on portfolio management may result in the investor choosing more rationally among alternative investments in the capital market. It is also true that the effort and attention required to manage a portfolio in an optimal fashion are substitutes for other income-producing activities of the individual, such as overtime or secondary employment. Indeed, to the extent that tax considerations increase the relative rewards of portfolio management and induce persons to devote more attention to it, the increased rationality of their decisions *may* increase real productivity enough to compensate for the loss of product entailed in their foregone alternative work plus any ill effects of tax-induced misallocation of financial resources.

We consider our "model investor" to be an expected utility maximizer with a horizon extending to his death. More shortsighted individuals can be subsumed in the model by giving them a high rate of discounting future satisfactions. A person's utility depends upon his consumption, upon the use he makes of his time, and upon his estate value at time of death. To maximize his expected utility, he makes a plan for his entire

[3] The Wisconsin Assets and Income Studies within the Institute are bringing together detailed income tax records with sample survey data, social security data, and other sources, matched up by name, address, and social security number. Tax data for 1947–59 have been collected and are in the final stages of processing. A large, stratified subsample of Wisconsin taxpayers has been interviewed and the data are nearly ready for integration with tax data.

future, but this plan is subject to constant revision as his circumstances and information vary.

In formulating this model, it became clear that the more or less discrete behavior of individuals existing in continuous time raised problems that neither continuous-action models nor discrete-period models could handle adequately. For those activities in which timing is of the essence, such as trading in assets, both the discreteness and timing of actions are variables that must be determined within a model which also recognizes the continuity of time and the near continuity of many personal actions or external events (such as price quoting on a stock exchange). To capture these features, we have dichotomized the model. Section II below is concerned with the long-range planning of broadly conceived over-all decision problems, while section III presents a model of short-run suboptimizing behavior with regard to portfolio composition changes. In so doing, we have been more concerned to explicate than to manipulate the model and its two complementary parts. In section IV we make some comments on features of the model and tentatively advance some potential hypotheses that the model may produce.

We have tried to embrace in our model many of the features of some of the more limited models in the recent literature, the most important of which are listed in the Bibliography at the end of the paper. To avoid drawing attention to the violence we may have done to the work of other scholars in the process of our synthetic translation, we make no specific references or parallels with these works. Our indebtedness to others in this present endeavor is nonetheless warmly acknowledged.

II. The Household's Over-All Decision Problems

In general, we will not distinguish single-person from multiple-person households. For the latter, we disregard the problem of combining the preferences of the individual household members into a single, well-behaved preference function for the household as a unit. For the major economic decisions we wish to discuss, this appears to be an acceptable simplification. The principal income earner is likely to be the principal investment decision-maker and is likely to be the most heavily insured family member. It is this "head of the family," born at $t = 0$, to whom we refer principally in what follows when we speak of a "person's" decisions.

This does not mean that we disregard the fact that the family head's decision will be affected by the structure of the household. For any one household at any one time, we take as given the age, sex, and relationship composition of that household. Similarly we take the head's principal occupation and job as given, as well as his age and education cohort, and his history of the use of his time and of his past investments. These things are very important in explaining cross-sectional variations and will also vary over time for a given household. Such uncertainty as the planner may have with respect to these changes over time will not be considered to affect his current plans.

Further, we ignore problems of the composition of consumption, at any one time and over all time, by assuming that the allocation of consumption expenditures among commodities can be treated as an independent suboptimizing problem. We will, however, define a price index of consumption commodities for any time τ, $P_c(t, \tau)$, which is assumed to be computed with weights corresponding to the amounts of commodities the particular household actually consumes or plans to consume at time τ, relative to the base prices for this same bundle of goods at time t. If we then define $C(t, \tau)$ as the total amount of the household's planned consumption expenditures at τ, the plan being made at t, the "real" consumption expected for τ is

$$\gamma(t, \tau) = \frac{C(t, \tau)}{\bar{P}_c(t, \tau)}, \tag{1}$$

where $\bar{P}_c(t, \tau)$ is the household's expected value of the future price index. The latter results from integrating the household's price index over all possible future consumption good price configurations (each computed with different weights as the consumption pattern changes appropriately), each such index multiplied by the subjective probability density that the household assigns to that particular price configuration.

Finally, we will assume that problems of evaluating, investing in, and subsequently consuming the services of durable consumers goods are completely subsumed in the above suboptimization process. This means, among other things, that the decision to live in and enjoy the services of a particular home is being considered as independent of the decision to invest in the title to the home: 200 year leases which lapse at the death of the head (or of his surviving spouse) are available as alternative ways of acquiring the services of the home. The investment in the title to the home is thus a separate financial investment.

In addition to deciding on consumption expenditure, the household chooses an allocation of time amongst a variety of activities. The principal occupation of the head being given, the household decides on other employment (including both overtime or a second job for the head, as well as employment of the other household members), on home management (including maintenance), on portfolio management, on time spent on education, and on pure leisure time, including time spent in pure consumption. In addition, the household may hire labor services from nonhousehold members, to aid in either home or portfolio management. If we let $L(t, \tau)$ be the vector of planned (at t) allocation of time (at τ) amongst all possible uses, and $\bar{y}(t, \tau)$ the vector of expected direct financial remunerations for the corresponding elements of $L(t, \tau)$, then the expected budget restraint for τ will include $L(t, \tau)' \bar{y}(t, \tau)$. Elements of $\bar{y}$ for employment outside the home will be positive, those for purchasing of outside labor will be negative, and those for pure consumption will be zero. Those for education may have any real number as a value, depending upon whether the receipts of scholarship or fellowship awards exceed tuition and other educational costs or not. Elements of $\bar{y}$ for portfolio management will be zero, although there exists a shadow price or wage rate based on the effect such activity has on the income from the portfolio itself.

In addition to its role in the budget constraint, $L(t, \tau)$ enters into another set of constraints which recognize that total time is limited, that some opportunities are not available, and that some activities overlap or otherwise interact. Formally, we define $R(t, \tau)$ as a matrix of constraint coefficients and require that

$$R(t, \tau)\, L(t, \tau) \leq \underset{\sim}{0}, \tag{2}$$

where $\underset{\sim}{0}$ is the null vector. In order for this formulation to be satisfactory, we require that the first element of the L be the constant "one" (so that the first element of $\bar{y}$ is "zero"), to allow for a vector of restraint constants as the first column of R. If L is an $n \times 1$ vector, R is $m \times n$, and $m > n$.

We regard the restraints in (2) as of considerable importance for our model and wish to provide some additional analysis with respect to the matrix R. For any given arbitrary interval $d\tau$, the integral of $\iota' L(t, \tau)$ is restricted to the total amount of time available to all household members over the interval, where ι is the $n \times 1$ vector of ones for household members' times and zeros for purchased times and for the initial element. One row of R will, therefore, be ι' with the initial element being replaced by a number whose integral is the negative of that total amount of available time. Naturally, that initial element will itself vary with the composition of the household and so is variable over time. If a subset of elements of L relates to a given member of the household, a similar restraint in R could be formulated for that person alone. If there is one such special restraint for each household member (as there should be), then the row of R described in the second and third sentences of this paragraph would simply be the summation of the individuals' restraints, redundant (not independent), and could be dispensed with.

In addition, there may be effective upper and lower limits to the quantity of time that can be spent by one household in any activity. This is probably the case with home management, so that there will be one row of R with a negative first element and positive coefficients for each person's home management function, including those of hired servants, plumbers, etc., and also a row of R with a positive first element (smaller than the absolute value of the negative first element in the row just discussed) and with negative coefficients for each person's home management activity. The first of these rows recognizes the upper limit, the second recognizes the lower limit on the amount of home management needed. The sizes of the upper and lower limits are determined by the size of the household, its aggregate consumption, etc., all of which vary over time. Similarly, the coefficients will be different from one another to the extent that persons in the household are not equally efficient in carrying out home management tasks. Because these relative efficiencies will not be constant for all household tasks, separate restraints of this nature are required for each task for which the coefficients are recognized to be different. Furthermore, the distribution of capability for performing these tasks varies over time as children mature and are trained, etc., so that the coefficients themselves are variable over time.

If a person is completely unqualified for a particular type of activity, he has no opportunity to use his time in that activity. For such a person, there is a row of R with all zeros except for a positive coefficient in the column corresponding to the location of the element of L for that activity by that person. If more than one person in the household is similarly limited with respect to a particular occupation, a similar single restraint will handle all of them at once since a subset of the restraints are nonnegativity restraints (requiring all elements of L to be nonnegative). If a person becomes qualified by education and experience for a job he could not have held before, his separate restraint changes (acquires a negative first element) as a function of this past use of his time in education and work.

Even the restraints recognizing a limited amount of total time available need not have unitary coefficients. Activities can be performed simultaneously (e.g., a routine home management task such as paying the bills may take place concurrently with such pure consumption activities as listening to a Bartok composition and consuming gin and bitter lemon). An added complication is the interaction of these activities (excessive consumption of gin might prevent the person from seeing the bills for which he is trying to write checks). Such nonlinear terms are not directly incorporated into the restraint in (2) above. Since we have already noted that R is a function of past activities, however, such interactions can be incorporated into an explicit expression for this effect:

$$R(t, \tau) = \Gamma[L(\nu, \tau); t; \tau], \tag{3}$$

which incorporates within it a lag operator or other integral of elements of L over the interval $0 < \nu < \tau$.

We have been using a notation which indicates that consumption and time use are continuous in time. In fact, of course, persons carry out more or less discrete activities in a continuous time space. Consumption and time use are not highly variable activities, for a given household over time, however, and we think it does little violence to reality to treat these activities as average flow rates over time. When it comes to trading in assets, however, discreteness of activity is an important feature of the decision processes we wish to describe. Between transactions, the holding of assets is a truly continuous activity. Asset prices vary over time in a very nearly continuous fashion. Buying and selling assets, however, are highly discrete activities in which timing is of the essence. Indeed, the interval between transactions is a variable which itself needs explanation. The major problems of portfolio management per se are postponed to the next section, where they are dealt with in a suboptimizing model. For the moment we are concerned only with the flow of cash income in τ in the form of interest and dividends, as anticipated in t—$D(t, \tau)$; the "flows" of net long- and short-term realized capital gains—$G(t, \tau)$ and $H(t, \tau)$ respectively; the flow of total appreciation due to price changes—$A(t, \tau)$; the total value of the portfolio in τ—$V(t, \tau)$; the flow of current (at τ) saving into the portfolio—$S(t, \tau)$; and the effort expended in portfolio management (an element, say $L_n(t, \tau)$, in the L vector).

All these scalars are intimately related. Furthermore, to the extent that they are future values, they are not known with certainty, but the decision-maker for the household has a subjective probability distribution over future prices and dividends that, evaluated in terms of his past portfolio management experience and current information, allows him to make subjective probability statements about certain rates of change or flow. The detailed portfolio management model of the next section elucidates the shorter-run, day-to-day activity of the household's "investment manager," but in the model of this section we are more concerned with the complete consumption and long-run investment plans of the household. Clearly these are interdependent models and must be consistent in any statements about the household's behavior.

For the purposes of this section, we regard the portfolio as a wholly owned mutual investment fund corporation, for which the investment manager of the household is the chairman of the board. The corporation will experience and realize capital gains and losses at unknown dates but at reasonably predictable long-term rates. The corporation's rate of dividend receipt is more predictable in both amount and time. In this section, we consider these rates to be net of all portfolio transactions costs except the capital gains tax.

Let $x_d(t, \tau)$ be the rate of cash income yield from the portfolio in τ as anticipated in t. The household's planner has a cumulative subjective probability distribution function over all possible values of $x_d(t, \tau)$ which is implicitly dependent upon his experience and information and explicitly dependent upon his cumulative lifetime amount of effort at portfolio management to τ, the rate of flow of new savings into the portfolio to τ, the total value of the portfolio in τ, and the time interval $(\tau - t)$:

$$\text{Prob}\,[x_d(t, \tau) \leq x_d^*] = X_d\left[x_d^*; \int_0^\tau L_n(t, \nu)\,d\nu; \int_t^\tau S(t, \nu)\,d\nu; V(t, \tau); (\tau - t)\right] \quad (4)$$

The mean value of $x_d(t, \tau)$ is then

$$\bar{x}_d(t, \tau) = \int_{(\text{all } x_d^*)} x_d(t, \tau) dX_d[\ \ ]. \quad (5)$$

Let X_{di} represent the partial derivative of $X_d[\ \ ]$ with respect to the i^{th} argument as they are arranged in (4). We hypothesize that

$$X_{di}\begin{Bmatrix}\geq 0\\ < 0\end{Bmatrix} \text{ as } i = \begin{Bmatrix}1\\ 2, 3, 4\end{Bmatrix} \text{ and as, for } i = 5,\ x_d^*\begin{Bmatrix}< \bar{x}_d(t, \tau)\\ > \bar{x}_d(t, \tau)\end{Bmatrix} \quad (6)$$

in such a manner that, for $\bar{x}_{di}(t, \tau)$ being the partial derivative of $\bar{x}_d(t, \tau)$ with respect to the i^{th} argument of $X_d[\ \ ]$,

$$\bar{x}_{di}(t, \tau)\begin{Bmatrix}\geq 0\\ = 0\end{Bmatrix} \text{ as } i = \begin{Bmatrix}2, 3, 4\\ 5\end{Bmatrix} \quad (7)$$

and so that the variance of $x_d(t, \tau)$, defined in the usual way as

$$\operatorname{Var}[x_d(t, \tau)] = \int_{(\text{all } x_d^*)} [x_d(t, \tau) - \bar{x}_d(t,\tau)]^2 \, dX_d[\quad], \tag{8}$$

has partial derivatives with respect to the $i = 2, \ldots 5$ arguments of $X_d[\quad]$ of

$$\operatorname{Var}_i [x_d(t, \tau)] \begin{Bmatrix} > 0 \\ \leq 0 \end{Bmatrix} \text{ as } i = \begin{Bmatrix} 5 \\ 2, 3, 4 \end{Bmatrix}. \tag{9}$$

Finally, we also assume that all changes in the person's information state not subsumed under the second argument of $X_d[\quad]$ are anticipated to be purely random future shocks whose effects are already taken into account in the fact that the variance defined in (8) is positive for all $\tau > t$. The person is aware of his past and present experience, however, so that the variance in (8) is zero for $\tau \leq t$, and indeed,

$$\lim_{(t \to \tau;\ t < \tau)} \operatorname{Var}[x_d(t, \tau)] = 0. \tag{10}$$

At the present time we leave open the question as to whether or not, for larger $\tau - t > 0$, the effects of arguments 2–4 of $X_d[\quad]$ can offset the effects of argument 5 in (9) above: as $t \to \tau$, the approach of $\operatorname{Var}[x_d(t, \tau)]$ to zero may not be monotonic for all initial values of $\tau - t$. In part this reflects the fact that arguments 2–4 in $X_d[\quad]$ are controllable by the portfolio manager, within the limits of his constraints.

For a given value of $V(t, \tau) = V^*$ then, the flow of $D(t, \tau)$ is itself a variable with a subjective probability distribution, for

$$D(t, \tau) = x_d(t, \tau)V^*. \tag{11}$$

However, $V(t, \tau)$ is not a fixed value in general; its variability is expounded below. Before proceeding, however, we can define $x_a(t, \tau)$, $x_g(t, \tau)$, and $x_h(t, \tau)$ as rates of total appreciation and rates of net long- and short-term gain realization, respectively, similarly to $x_d(t, \tau)$ so that

$$\begin{aligned} A(t,\tau) &= x_a(t, \tau)V^*, \\ G(t,\tau) &= x_g(t, \tau)V^*, \text{ and} \\ H(t,\tau) &= x_h(t, \tau)V^*. \end{aligned} \tag{12}$$

With suitable change in subscripts from d to a, g, and h, all statements (4)–(10) can be repeated as statements (4)′–(10)′, (4)″–(10)″, and (4)‴–(10)‴ respectively, with precisely the same form and content (but not identical $X[\quad]$ functions) with the following exceptions:

$$X_{g3} \geq 0 \text{ so that} \tag{6a)''$$

$$\bar{x}_{g3}(t, \tau) \leq 0. \tag{7a)''$$

The statements in the preceding paragraphs are the result of much of the thinking that went into the formulation of the suboptimizing portfolio management model of section III below, and detailed justification is not called for in this section. They seem plausible enough to use without the suboptimizing model to justify their use in any event.

From a short-run point of view we do not regard the face or cash values of the household's insurance as part of its portfolio that needs management in the sense of our suboptimizing model. These values are thus not included in $V(t, \tau)$. Generally they represent long-run contractual arrangements. However, we do include any borrowings against the cash value in the debt portion of $V(t, \tau)$: any lender is interested in the net worth of the borrower, dead or alive, and the pledge of specific assets against particular borrowings is primarily a means of obtaining preferred interest rates, whether the asset be cash value of insurance, market value of a home, or marketable securities.

Denote by $N(t, \tau)$ the face value of the head's insurance contracts that he plans at t to have at τ, let

$$\eta(t, \tau) = \eta[N, t, \tau] \tag{13}$$

be the percentage of $N(t, \tau)$ that will be accumulated cash value rather than pure insurance at τ, and let

$$M(t, \tau) = m[\eta, t, \tau] \tag{14}$$

be the per dollar premium on N payable in τ. Naturally, if anyone could predict with certainty the day upon which they would become uninsurable (with the actual date of death as the extreme case for a person otherwise healthy), they would wait until the "last moment" before taking out insurance. Uncertainty with respect to insurability is of the essence of insuring. Finally, let $\theta(t, \tau)$ be the "pure (instant term) insurance" premium per dollar of face value minus cash value at τ. Thus the individual at t plans to pay a premium of $[M(t, \tau)N(t, \tau)]$ at τ, of which $\left\{\theta(t, \tau)[1 - \eta(t, \tau)]N(t, \tau)\right\}$ is the cost of the pure insurance he purchases at τ, the difference being an amount of saving by investing in cash value, in order to purchase a contract paying $N(t, \tau)$ in the event of death at τ, with $[\eta(t, \tau)N(t, \tau)]$ being the amount of $N(t, \tau)$ representing an accumulation of savings in the insurance contracts. In addition, the person receives nontaxable current (at τ) income, denoted $Y_N(t, \tau)$:

$$Y_N(t, \tau) = \left\{\frac{d\eta}{d\tau} - M(t, \tau) + \theta(t, \tau)[1 - \eta(t, \tau)]\right\}N(t, \tau). \tag{15}$$

This is simply the excess of the current increase of cash value above the current premium net of current pure insurance costs. (It is important to recall that t, τ, and ν are "age dates" rather than calendar dates, where $t = 0$ at the time of the birth of the head of household.)

If we let $Z_w[W(t, \tau)]$ represent the estate tax on a net-of-tax estate value of $W(t, \tau)$ which would be left by the head of household were he to die at τ, then this net estate bequeathed is

$$W(t, \tau) = V(t, \tau) + N(t, \tau) - Z_w[W(t, \tau)]. \tag{16}$$

Let

$$U_w(t, \tau) = U_w[W(t, \tau)] \tag{17}$$

represent the utility which the head of household foresees (at t) himself as receiving (at τ) by being able to leave a net estate worth $W(t, \tau)$ should he die at τ. Let $\beta(t, \tau)$ be a subjective discounting function for future bequesting utility to bring it to a current present value, and let $\pi(t, \tau)$ be the conditional probability density of dying at τ given that the person is alive at t. Then the conditional probability that the person will live to τ, given that he is alive at t, is

$$\Omega(t, \tau) = \int_{\nu=\tau}^{\infty} \pi(t, \nu)\, d\nu = 1 - \int_{\nu=t}^{\tau} \pi(t, \nu)\, d\nu. \tag{18}$$

This is the probability that the individual will be able to enjoy the household consumption and activity at τ, at which time he presently looks forward to receiving $U_c(t, \tau)$ utility, where

$$U_c(t, \tau) = U_c[\gamma(t, \tau); L(t, \tau)]. \tag{19}$$

Let $\alpha(t, \tau)$ be a subjective discounting function for future consumption and time use utility to bring it to a current present value. If T is an age beyond which the person cannot expect to live, then the present value at t of his expected utility for his remaining lifetime is

$$U_T(t) = \int_t^T \left\{ \Omega(t, \tau)\alpha(t, \tau)U_c(t, \tau) + \pi(t, \tau)\beta(t, \tau)U_w(t, \tau) \right\} d\tau. \tag{20}$$

The person seeks to maximize $U_T(t)$ by his choices of all decision variables subject to his constraints, and in the light of his anticipations of future changes in those variables over which he has no control. We now turn to the task of completing the formulation of those constraints.

A person alive at t and expecting to be alive at τ anticipates paying taxes (at τ) on his consumption purchases, property, ordinary income, and long-term capital gains income, in the aggregate amount of

$$\begin{aligned} Z(t, \tau) = {} & Z_c[C(t, \tau)] + Z_p[q(t, \tau); p(t, \tau)] \\ & + Z_y[Y_L(t, \tau) + Y_h(t, \tau) + D(t, \tau)] \\ & + Z_g[Z'_y(t, \tau); G(t, \tau); H(t, \tau)], \end{aligned} \tag{21}$$

respectively, where $q(t, \tau)$ is a vector of asset quantities and $p(t, \tau)$ is the corresponding vector of asset market values; where $Y_L(t, \tau)$ is household income from employment net of the deductible portion of outside labor wages paid; where $Y_h(t, \tau)$ is the amount of short-term capital gains income that is legally includable in net taxable income:

$$Y_h(t, \tau) = Y_h[H(t, \tau); G(t, \tau)]; \tag{22}$$

and where $Z'_y(t, \tau)$ is the derivative of the $Z_y[\ \ ]$ function (the marginal tax rate on ordinary income).

Recognizing total rather than just taxable income (except for imputed incomes which are instantly identically consumed), an "instantaneous" budget identity for a household at τ is

$$\begin{aligned} S(t, \tau) \equiv {} & L(t, \tau)'y(t, \tau) + A(t, \tau) + D(t, \tau) + Y_N(t, \tau) \\ & - Z(t, \tau) - C(t, \tau) - M(t, \tau)N(t, \tau) \end{aligned} \tag{23}$$

which also defines the anticipated time rate of savings net of insurance premiums as a function of other, previously defined flows. Since

$$\frac{d}{d\tau} V(t, \tau) = S(t, \tau) \quad \text{and} \tag{24}$$

$$\int_{\nu=t}^{\tau} \frac{d}{d\tau} V(t, \nu)\, d\nu = V(t, \tau) - V(t, t),$$

we have, for a person living through any given interval from t to τ,

$$\begin{aligned} V(t, \tau) = {} & V(t, t) + \int_t^{\tau} L(t, \nu)'y(t, \nu)\, d\nu + \int_t^{\tau} A(t, \nu)\, d\nu + \int_t^{\tau} D(t, \nu)\, d\nu \\ & + \int_t^{\tau} Y_N(t, \nu)\, d\nu - \int_t^{\tau} Z(t, \nu)\, d\nu - \int_t^{\tau} C(t, \nu)\, d\nu \\ & - \int_t^{\tau} M(t, \nu)N(t, \nu)\, d\nu. \end{aligned} \tag{25}$$

This expression is deceptively simple since the third, fourth, and sixth terms on the right-hand side of (25) all involve the intermediate values of $V(t, \nu)$. An expression such as (25) exists for each sequence of intermediate values, of course, and by substitution we may obtain a differential equation involving simply the household's initial asset position, its choice variables through time, and the unknown and exogenously determined price vectors. There is a joint probability density function over the values of the unknown price vectors; the conditional densities may be multiplied into the appropriate terms in (25) and the resulting expression integrated over all possible values of the price vectors for any given $V(t, t)$ and given set of time paths of the decision variables; this yields an expected value of the portfolio, $\bar{V}(t, \tau)$, as a function of $V(t, t)$, the time paths of the decision variables, and the moments of the future price density function. Given $V(t, t)$, therefore we can associate a distribution function on $V(t, \tau)$ with each time path of the decision variables. The household then chooses that time path to T of the decision variables that maximizes its total expected utility in (20) subject to the expected budget restraint derived from (25), the expectation of the bequest restraint in (16), the expected time use restraints in (2), and the additional set of nonnegativity restraints:

$$\left\{ \begin{array}{l} q(t, \tau) \geq \underline{0} \\ c(t, \tau) \geq \underline{0} \\ V(t, \tau) \geq 0 \\ N(t, \tau) \geq 0 \end{array} \right. \text{and} \left. \right\} \quad \text{for all } t \leq \tau \leq T, \tag{26}$$

where $c(t, \tau)$ is a vector of consumption quantities.

Among other things, the above model determines the flow of funds from current employment income into the portfolio's total value as

$$Q(t, \tau) = S(t, \tau) - A(t, \tau) - D(t, \tau), \tag{27}$$

the flow of portfolio management labor through time, $L_n(t, \tau)$, and the marginal tax rates applicable to short- and long-term capital gains. All of these quantities are taken as given in the short-run portfolio management model presented in the next section. The utility function used there is also considered to have been derived from that in (20), recognizing that accumulated assets may be used both for financing future consumption and for providing a larger bequest.

III. A Submodel of Dynamic Portfolio Adjustment

PRELIMINARY REMARKS

This section develops a model for exploring a particular subgroup of the household's economic decisions, namely, its selection of an investment portfolio. The investor *qua* household decision-maker has the problem of allocating a given stock of accumulated resources among alternative forms of wealth. At any point in time, the current allocation will depend on past decisions. The question the investor must continually answer is whether there is a transaction at the present moment which would improve upon the current allocation in terms of the long-run objectives of the household. The model that follows provides a particular structure for analyzing this set of decisions.

THE ELEMENTARY TRANSACTION DECISION IN THE PURE APPRECIATION CASE

The model will be developed from a relatively simple decision problem. An investor has a particular inventory of assets at time t, the present. These assets are continuously divisible, have market-determined prices which vary continuously through time. No asset provides a current flow of income, and no asset has a fixed price.[4] All alternative assets have these same characteristics. The investor has a set of probability beliefs about the prices in the future and he must decide to retain his current portfolio or make a specific transaction.

The current portfolio may be described by a vector $q(t)$ with nonnegative elements $q_i(t)$ denoting the number of units of asset i held at t. This vector remains constant over time except for discrete changes, $\Delta q(t)$, called transactions. The following identity holds for a sufficiently small ϵ:

$$q(t + \epsilon) = q(t) + \Delta q(t). \tag{28}$$

The prices of the assets at time t are described by the vector $p(t) = \{p_i(t)\}$. The current "value" of the portfolio, $V(t)$, may now be defined as:

$$V(t) = q'(t)p(t). \tag{29}$$

Except for the existence of transactions costs, any portfolio which has the same current value as $q(t)$ could be obtained by a suitable transaction. That is, any transaction, Δq, which satisfies $\Delta q'p(t) = 0$ and $\Delta q + q(t) \geq \underline{0}$ (all elements nonnegative) would be "feasible." If transaction costs are covered by the proceeds of the sale(s) involved in a transaction, the alternative transactions feasible at time t will satisfy

$$\Delta q'p(t) + K(\Delta q) = 0, \text{ and} \tag{30a}$$

$$\Delta q + q(t) \geq \underline{0}, \tag{30b}$$

[4] The market prices are expressed in terms of a bundle of consumption goods—they are "real" prices.

where $K(\Delta q)$ is the cost of transactions function. The function $K(\Delta q)$ is discontinuous at $\Delta q = \underline{0}$, i.e.:

$$K(\underline{0}) = 0 \tag{31a}$$

$$\lim_{k \to 0} K(k\Delta q) > 0, \tag{31b}$$

for all $\Delta q \neq \underline{0}$.

The cost function increases for larger transactions but not in proportion to the size. More precisely:

$$K(\Delta q) < K(k\Delta q) < kK(\Delta q) \tag{31c}$$

for all $\Delta q \neq \underline{0}$ and $k > 1$.

The investor's probability beliefs at time t about future prices are summarized in a sequence of subjective probability distributions:

$$F(p; t, \tau) = Pr[p(\tau) \leq p], \ \tau \geq t. \tag{32}$$

These probability distributions are conditioned by the investor's experience and the information available to him including, among many other things, the history of prices up to time t. At time $t + \triangle t$ new information will have been generated and the probability beliefs will be altered. The means of these distributions are specified to have the same property of continuity in time as the actual prices. This can be formalized as follows:

let
$$\bar{p}(t, \tau) = \int_{\text{all } p} p \, dF(p; t, \tau) \tag{32a}$$

and require that

$$\lim_{\tau \to t} \bar{p}(t, \tau) = p(t) \text{ and}$$

$$\lim_{t \to t'} \bar{p}(t, \tau) = p_t(t', \tau).$$

Any linear combination of future prices and, in particular, the future value of an arbitrary portfolio, q, will have mean

$$\bar{V}(t, \tau) = q'\bar{p}(t, \tau) \tag{32b}$$

and this will have similar continuity properties. The variance of the value of a portfolio in the subjective probability distribution is given by:

$$\text{Var}\,[V(t, \tau)] = \int_p [q'(p - \bar{p}(t, \tau))]^2 dF(p; t, \tau). \tag{32c}$$

The notion of increasing uncertainty about increasingly more remote events can be expressed through the further requirement that:

$$\frac{d \operatorname{Var}[V(t, \tau)]}{d\tau} > 0. \tag{32d}$$

An additional property, which is suggested by the notion of continuous price variation, is:

$$\lim_{\tau \to t} \operatorname{Var}[V(t, \tau)] = 0. \tag{32e}$$

The utility of the portfolio is a function of its value at each point in time and does not, given the value, depend on the composition. This utility function is derived from the over-all decision problem of the household in which the portfolio value is among the constraining resources for consumption and bequest plans. The utility of a given portfolio value V at time τ in the future will be denoted:

$$U(V; t, \tau). \tag{33}$$

This function is assumed to be monotonic, increasing in V for any given set of values for the other variables in the over-all optimization.

The expected utility at time τ from a constant portfolio q is simply:

$$\bar{U}(q; t, \tau) = \int U(q'p; t, \tau)\, dF(p; t, \tau). \tag{34}$$

The investor is assumed to focus on a particular average of these expected utilities:

$$\bar{\bar{U}}(q; a(h), t) = \int_0^\infty a(h)\bar{U}(q; t)\, t + h, dh, \tag{35}$$

where

$$a(h) \geq 0 \text{ for all } h \text{ and } \int_0^\infty a(h)\, dh = 1. \tag{36}$$

This is the criterion he uses for evaluating alternatives available to him. At time t the investor has the portfolio $q(t)$. He will choose the transaction $\triangle q$ which maximizes $\bar{\bar{U}}(q(t) + \triangle q; a(h), t)$ subject to the constraints (30a)–(30b). The discontinuity of the cost function, $K(\triangle q)$, prevents a simple use of calculus to describe the maximizing conditions. Most of the maximizing transactions will be at "corners," with all or most of the elements of $\triangle q$ equal to zero. As usual in such cases, there will be a certain range of price expectations, and utility functions for that matter, within which the maximizing value of $\triangle q$ will be unchanged.

The weighting function $a(h)$ may now be given a more complete rationalization. The investor operating in a world of uncertainty can be fairly confident of one thing—namely, that any portfolio he chooses at present will not continue to be the best available one as time passes and further information on prices and other matters accumulates. He may anticipate with confidence that sooner or later a further transaction, not now apparent, will be in order. One may interpret the $a(h)$ function as a subjective density function on the random interval from any point in time to the next time a transaction (other than the trivial no-change transaction) will become advantageous. He then chooses as a proximate goal to have as much value as possible to allocate at that time. Given this interpretation an average "horizon" may be defined as:

$$\bar{h} = \int_0^\infty ha(h)\, dh. \tag{37}$$

The distribution $a(h)$ is regarded as a characteristic of a particular investor. Further discussion of the relation between $a(h)$, or descriptive summaries of it, and other characteristics of the investor will be presented below.

Briefly, the investor is continuously revising his subjective probability distribution of future prices as more and more of the future passes into history. Other aspects of his economic situation may alter and hence change his utility of portfolio value functions. When these variations make it advantageous to carry out a transaction, he does so. Clearly this will happen only when the average expected utility is greater for some altered portfolio than for the current one in spite of the fact that the immediate result of carrying out the transaction will be a reduction of the real value of the portfolio by the amount of the transaction cost.

INCOME-PRODUCING ASSETS

So far the assets considered have been of a very simple sort. The prices of the assets and their relative variation provided the only basis for gain (or loss). It will be argued below that the addition of dividends does not alter the essential features of the model.

One of the assets, say, the first, can be considered to be cash. If the other assets yield dividends, they are assumed to be paid on periodic dates, say, each quarter. The dividend consists of a payment of cash to the holders of each asset in proportion to their holdings. It is clear that the value of each unit of each asset will fall by the amount of its respective dividend at the time the sales are made ex-dividend. This discontinuous break in the market prices does not, however, immediately affect the value of the investor's portfolio. His cash holding is larger and the total value of his dividend paying assets is smaller.

This suggests an alternative way of describing a dividend payment, i.e., as a cost-free and involuntary transaction in which a part of each asset is sold for cash. The investor who receives d_i dollars for each unit of his holding of asset i can regard the quantity of the i^{th} asset as having been reduced by a factor $[1 - (p_0 d_i/p_i)]$ where p_0 is the real value of a dollar. This viewpoint requires some adjustment in the interpretation of units and prices. In the previous section the prices were market prices per unit of the asset and the units corresponded to usual notions of corporate "shares." Now the unit of the asset is more elastic—it can be taken to correspond to the share between any adjacent dividend dates, but it must be redefined as a larger number of corporate shares when a dividend is paid. The price per unit becomes continuous through dividend dates by this process. For purposes of relating to actual market prices of corporate shares, the reinterpreted price corresponds roughly to the share price at time t plus all dividends paid since the unit was equal to the corporate share.

The investor at time t, while reviewing his holdings and considering alternatives, may be regarded now as holding probability beliefs about both prices and dividends at future dates. These probability beliefs can be viewed as the product of a conditional distribution of dividend payments given prices and the marginal distribution of prices (the continuous ones). The objective function can now be expressed in parts, each representing the interval between dividend dates, and the portfolio in each interval reflecting the effect of the involuntary transactions implied by the dividend payments.

An approximation can be obtained by replacing the uncertain dividends by their conditional expectations, given prices, and then integrating over the future prices, as before, to find the expected utility. It is argued that the residual uncertainty about the exact size of the fractional adjustments in portfolio composition occasioned by dividend payments is of minor importance and that the primary source of uncertainty is the unknown future prices, including any distribution of cash. Following this argument, one can reinterpret the price vector in such a way as to include income-producing assets in the basic model without any further modification.

TRANSFERS BETWEEN PORTFOLIO AND CURRENT ACCOUNTS

A primary function of the portfolio is to receive the resources currently earned by nonportfolio activities of the household in excess of current consumption (or to disburse for covering a deficiency). The amounts of such transfers are determined by the household in allocating its expected total resources over its lifetime, including its opportunity to earn a return on invested resources. The portfolio allocation model will regard the schedule of transfers as exogenously determined at the time the household is considering possible transactions. The same average expected utility can be maximized, with the additional feature that there are discrete additions to or subtractions from the cash component at particular points in future time. There is no essential change in the constraint function, except that the total value to be allocated will change discontinuously at transfer dates.

It does not follow, of course, that the portfolio composition will not depend upon the presence of such transfers. A flow of cash into the portfolio allows for a limited amount of adjustment of the composition of the portfolio at very low transaction cost because of the very low cost of transactions in cash. Transactions will tend to be more frequent or larger, or both, when there are positive transfers into the portfolio. Similarly negative transfers may affect the pattern of transactions.

FIXED-DOLLAR OBLIGATIONS AND BORROWING

Bonds, loans, and other assets that specify payments of specific dollar amounts at specific dates form a group which does not fit comfortably among the assets considered above. One reason is that an important source of uncertainty about the future prices of such assets is uncertainty about future interest rates. But, as one considers future prices at times approaching a bond's maturity date, this source of uncertainty becomes insignificant. This phenomenon raises the possibility that the subjective probability distribution will not have the property postulated in (32d).

Another reason for regarding the debt instruments as special is that there are a limited number of factors which determine a large fraction of the variation of their (real) prices, the value of money, the interest rate(s), etc. About the only element of uniqueness associated with a particular debt issue is the risk of default, and for many issues that can be negligible. Consequently, the structure of bond prices, to the extent that we understand it, should be used to place restrictions on the investor's subjective probability distribution of bond prices.

The portfolio adjustment model would seem, otherwise, to be able to accommodate this type of asset. While there may be differences, and differences worth exploring, in the properties of an investor's subjective probability beliefs about the two classes of assets, at a higher level of abstraction they are quite comparable.

Borrowing of the household is another portfolio item that should be included in the model. Household debt can be treated as a negative fixed-price asset. The evaluation of the average expected utility will not be affected nor will the total value restraint. The nonnegativity restraints will be reversed for the debt variables and there may be upper bounds on debt, either absolute or related to other components of the portfolio (collateral).

OTHER ASSETS

A further category of portfolio assets should be mentioned. A substantial part of most portfolios is represented by assets that are not divisible, do not have currently observed market prices, and are not fixed in (dollar) price. Such items as real estate, both residential and nonresidential, equity in unincorporated enterprise, shares of closely held corporations, professional practice, etc., do not fit into the framework as it stands. The nondivisibility is a minor problem since most transactions in this model are of a discrete nature anyway. Other properties of these assets are more troublesome.

For assets of this sort that are currently owned by the investor, it is reasonable to assert that he has subjective beliefs about their present and future prices. Here, in the absence of organized and informed markets, explicit assumptions about the process of selling such assets have to be made. But the investor's beliefs about alternatives to his current holdings are not so easy to specify—the alternatives are so numerous and varied. The extent of the investor's information becomes quite crucial here and that, in turn, depends upon a prior choice of the investor to spend time and effort on acquiring information.

For many of these assets, the utility of ownership is not satisfactorily captured in terms of the realizable value—they are directly involved in nonportfolio activities of the household. For this part of the portfolio, it becomes necessary to augment the argument of the partial utility function that appears in the objective function. If the utility function (33) is replaced by:

$$U(V, q; t, \tau), \tag{33a}$$

the model can make a formal allowance for these effects. In the broader model, this phenomenon appears as a restraint on alternative uses of time, which depend upon ownership of specific assets. For example, one cannot be the manager of one's own grocery store without owning it.

TAXATION OF PORTFOLIO RETURNS

The model developed above has purposely obscured the distinction between portfolio gains that are realized in cash by an involuntary transaction, such as an interest or dividend payment, and those that accrue, whether realized or not, via price appreciation. The income tax laws on the other hand draw sharp distinctions between these forms. Roughly speaking, the interest and dividends are taxed as regular income, at the marginal rate determined by the household's nonportfolio income, realized gains at half that rate (if they are long-term) up to a maximum of 25 per cent, and unrealized gains not at all. Since an understanding of household investment behavior must not ignore the presence of taxation, we must see how the tax structure fits into the model.

The taxation of interest and dividend income can be introduced by a minor change in the rule for defining the involuntary transaction implied by a dividend or interest payment. In addition to the other changes in asset quantities, a change in a liability—taxes owed—must be introduced equal to the product of the income receipt and the marginal tax rate in the bracket the investor expects his total income to attain.

The taxation of realized gains enters the model at a quite different point. Here the act of realization is a voluntary transaction. The immediate effect of the gains tax on the portfolio is similar to a transaction cost, and it seems appropriate to introduce it in the same way. The constraint function is augmented by an additional term showing the amount of capital gains tax payable as a function of the transaction. The calculation of this tax liability requires added information on the basis (purchase price) and holding period for each separately acquired lot of a divisible asset. This, in practice, requires a simple inventory, in sharp contrast to the clumsy notation that is needed to give it formal expression. It is sufficient to note that, by distinguishing among lots acquired at different times, the analysis remains essentially the same. The tax liabilities do occasion a reduction in the current value of the portfolio, in the same fashion as transactions costs, and similarly constrain the set of feasible transactions.

The appropriate treatment of the contingent tax liability on unrealized gains is much less obvious. It seems inappropriate to charge the portfolio for taxes on gains as the gains accrue because the amount of tax is, in part, a variable the investor can control. The alternative of evaluating future portfolios on the basis of their net after-tax liquidation value also proceeds on an unrealistic hypothesis. However, it is also wrong to ignore the contingent liability completely. When an asset is sold and the gains taxes paid, there is no longer any uncertainty about that particular component of the portfolio gain. There has been a reduction in contingent tax liabilities and the decision model should reflect this. As a tentative compromise, the objective function could be modified to include an estimate of the contingent liability eliminated by a transaction. Let $\phi(\triangle q)$ denote the investor's expectation of the amount of gains tax he would eventually pay on the sales included in $\triangle q$ if he did not sell them now. Among the other alternatives he considers are sales in years with lower rates either because of changes of statutory rates or lower nonportfolio income, holding the asset until death, donating it to a university, etc. The expected utility of the portfolio at time τ can now be written:

$$\bar{U}((q + \Delta q); t, \tau) = \int U[(q + \Delta q)'p + \phi(\Delta q)] \, dF(p; t, \tau). \quad (38)$$

The added term partly offsets the immediate reduction in the portfolio produced by taxes on realized gains.

CONCLUDING REMARKS

The portfolio adjustment model developed above has several basic features that might be emphasized at this point. It is concerned with the essentially dynamic process of a household's adjustment to imperfectly foreseen changes in its environment. It makes use of a form of the expected utility hypothesis in an objective function that is only a proximate goal, one that we hope is reasonably consistent with the ultimate objectives. Finally, the model gives a large role to the subjective probability belief of the investor household. This last feature provides a vehicle for considering a wide range of supplementary hypotheses about how the investor acquires these beliefs.

IV. Conclusion

The introduction noted the need for a relatively ambitious and comprehensive model of household economic behavior to provide a coherent unifying framework for analyzing increasingly detailed and extensive household data. The model presented in section II represents our attempt to fill this need. The submodel in section III elaborates, and to some degree isolates, the decisions of the household that are concerned with portfolio management, which involve most prominently the choices among financial investments. This submodel illustrates, in our opinion, the usefulness of having an explicit over-all model standing behind the necessarily partial models we devise for particular problems. The nature and the cost of the simplifications required to achieve the partial isolation are much more apparent and are of interest in themselves.

Both the model and the submodel are, at present, rather long on notation and short on results. Further analytical and empirical work must be done and is, indeed, proceeding. At this point, however, it is possible only to indicate a few specific features of the model which seem to us particularly interesting and worth further study.

The formation and continuous revision of the subjective probability distributions have already been mentioned as worthy of further analysis. Acquiring and absorbing current information undoubtedly requires time and effort that have alternative uses. Both the value of current information and the capacity for assimilating it efficiently probably depend on the accumulated experience of the investor [2]. The model outlined above provides a link between the precision of the investor's expectation, as determined by inputs of effort, and the frequency of transactions, both of which are observable. Implications concerning investor specialization in particular classes of investments could also be sought. Recent work on adaptive forecasting [8, 9] has provided a mechanism that could be incorporated in the model to generate the expectations of future prices on the basis of past price movements.

The proximity of the proximate goal used as the objective function in the submodel is another feature that is of interest. The $a(h)$ function which determines this was explained earlier as a distribution of the interval between portfolio transactions and implicitly regarded as a relatively stable characteristic of the investor. Why, then, do some investors carry out transactions frequently and others only occasionally? It is tentatively proposed that the frequency will depend on the joint effects of portfolio size (total value), the amount and variety of "risky" securities in the portfolio, and the precision of the investor's price expectations. Work of a more theoretical nature could be carried out to determine the conditions under which such a proximate goal provides a satisfactory substitute for a more complicated ultimate objective.

While not intended as a substitute for the hard work that remains to be done, the foregoing remarks are offered as an indication of the directions such work might take. It is hoped that these remarks, together with the models outlined in the body of the paper, provide, even at this stage, a basis for critical comment and suggestions.

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COMMENT

On Muth

BY LOUIS WINNICK, FORD FOUNDATION

As I see it, Muth's paper makes two contributions to urban economics. First, his regression analyses reaffirm the role of the automobile as a determinant of the development patterns of metropolitan areas. For a half century or more the technology of transportation has been recognized as the most powerful influence on the size and configuration of cities and their hinterlands. And since the 1920's Robert Murray Haig and others have singled out private automotive transport as the "exploder" of urban conglomerations. Given the growing ownership of automobiles and trucks, the speed of movement, and the economics of highway construction which permits lattice-type networks of movement impossible of attainment by any rail system, it was possible to foresee not merely the decentralization of population but its diffusion as well; that is, not only would there be a reduction in the density gradient moving outward from the center but there would also be an increase in the average distance between household and household, between establishment and establishment, and between households and establishments. Muth's work shows that higher car registrations are in fact accompanied by lower gradients—the higher the incidence of car use, the more our cities tend to look like Los Angeles rather than New York.

The second contribution that Muth makes is to show that gradient reductions take place inside as well as outside the city, that not only the suburbs but also the outer rings within municipal corporate lines have tended to become the destination of urban migrants. He rightly points out there has been too much loose talk about the "flight from blight" and about the dichotomy of city and suburb.

Where Muth falls short is in excusing himself from the task of relating population movements to investment behavior; the latter is, after all, the subject of our conference. I should like to make a few general remarks on this subject, mainly to point out a few reasons why investment gradients need not correspond to population gradients.

Plainly, a substantial portion of metropolitan investment has no discernible relation either to resident population or to population growth. A notable example of such investment is the very substantial office building booms of New York, Chicago, Washington, San Francisco, and Boston. The location of national or regional headquarters is based on factors other than population trends in the immediate rings of the new establishment.

This much is obvious, but what of residential construction? Surely, housing investment is sensitive to resident population. It ought to be and it is but, as described at an earlier National Bureau conference, the relation between population levels and the dollar volume of new housing is only a very general one and holds only over very long periods of time. A much better dependent variable is change in number of households. But would not change in households correlate closely with change in population? Here again, the answer is a qualified yes if one is looking at national or regional aggregates. But the relationship falls apart when one is looking at the intrametropolitan aspects of population, settlements, and residential investment.

Inherent in the nature of an urban population is that there are systematic differences in the locational choices made by different types and sizes of households. What this means is that changes in number of households need not be the same as changes in the number of people. We can illustrate this by beginning with the well-known fact that, at the same time as our largest cities were losing population, they were gaining households. Thus, as Table 1 shows, between 1950 and 1960 seven out of the ten largest cities decreased in population but increased in households. In New York City, for example, population declined by 110,000 but the number of households expanded by nearly 300,000. More to

TABLE 1

Change in Population and Households in Ten Largest Cities Between 1950 and 1960

(per cent)

City	Population	Households
New York City	− 1.4	+12.6
Chicago	− 1.9	+ 8.2
Los Angeles	+25.8	+33.0
Philadelphia	− 3.3	+ 7.4
Detroit	− 9.7	+ 2.7
Baltimore	− 1.1	+ 5.2
Houston	+57.4	+59.3
Cleveland	− 4.2	+ 4.4
Washington, D.C.	− 4.8	+13.3
St. Louis, Mo.	−12.5	− 1.2

TABLE 2

Average Household Size, by Area Inside the N.Y.–N.J. SMSA, 1960

1.	All SMSA	3.1
2.	New York City	2.9
3.	Manhattan	2.4
4.	Census tract 130 (Manhattan)	2.2

Note: SMSA = Standard Metropolitan Statistical Area.

the point, this increase in households was accompanied by the largest residential investment boom since the 1920's, new construction totaling some 314,000 dwelling units.

This phenomenon already noted in the 1920's (when Manhattan lost population but experienced a record apartment boom) reflects deep-seated life-cycle preferences. The suburbs attract relatively more of the large families in the child-rearing stage, the central city relatively more of the smaller households—the young and old adults without children. Further, similar locational patterns are evident within the central city. As one moves from the outer rings to the inner core, the proportion of one- and two-person households sharply increases. Thus, in 1960, average household size was 3.1 for the entire New York metropolitan area, but only 2.9 for New York City, 2.4 for Manhattan, and 2.2 for a typical census tract in which substantial new apartment construction occurred (see Table 2).

In the typical new center-city apartment house in New York, Chicago, or Philadelphia, 90 to 95 per cent of the apartments are occupied by single individuals or married couples without children. New York's Third Avenue, for example, where a mile of four- and five-story tenements were replaced with 20- to 30-story apartments actually experienced a population decline: working class families with children departed for the outer rings and were replaced by both well-to-do executives and not-so-well-to-do secretaries who paired up and accepted rent-to-income ratios of 30 per cent or more in order to live in a convenient location or at a fashionable address. It goes without saying that any meaningful analysis of intrametropolitan population movements or investment trends must place these life-cycle differentials in choice of housing type and location at the very forefront.

A second "institutional" fact that Muth tends to neglect in his research on population distribution is the legal constraint, such as zoning and building codes, that significantly affects the costs of particular housing types and locations. According to Muth, the locational decisions of households are influenced by the gradient in residential land values as one moves outward from the center of the city. The housing consumer is confronted with a price system for land which, in conjunction with his income and tastes, determines his ultimate choice. On this I have three comments. The first and least important is that gradients and slope lines are no longer adequate descriptors of the pattern of land values. Simple geographical configurations, such as pyramids, cones, and concentric circles, may have been at one time useful visual aids to an understanding of the distribution of rents and value. But, alas, the automobile which has so drastically changed the map of population has also apparently changed the map of land values. Instead of a single-peaked cone, there seem to be multipeaked irregular configurations full of unexpected bumps and valleys. Land values at many points in the business district are lower than in the secondary rings and high-rent neighborhoods and blocks are side by side with slums.

My more important comments are, first, that land values or land rents expressed on per acre or per square foot are not very revealing measures of final housing costs; and, second, that building costs cannot be assumed to be equal throughout a metropolitan area. The influence of land costs on housing costs is more pertinently measured as a relative, namely, the amount of land cost per room or per square foot of building space. The unit price of land must be multiplied by the quantity wanted or required by the zoning code for a given type of housing. A large apartment house in Manhattan built on land costing $40 per square foot may have a lower relative land price than a single family house in the suburbs built on land costing 50 cents a square foot. The former may require only 125 square feet per dwelling unit, the latter would typically require 6,000 square feet. In other words, the housing consumer can adjust to high land costs by sharply decreasing the quantity purchased. Just how high the coefficient of price elasticity may be would be difficult to calculate, one reason being that consumer responses are heavily constrained by zoning regulations, another reason being that apartment house sites and single-family house sites are far from being perfect substitutes, i.e., to a large extent, they are traded in different markets.

But at the same time as the housing consumer seeks to escape both transportation and land costs by choosing a central city apartment house, he is confronted by another cost—the relatively high cost of constructing high-rise fireproof structures that are customarily mandated in core areas by the building codes. The incremental costs for elevators and fireproofing are quite substantial. The latter type of structure may cost fully twice as much to build per room as a low nonfireproof frame house or garden apartment. Other things being equal and excluding the cost of the land, a garden apartment in New York City may have a construction cost of as low as $12 per square foot of building space, a high-rise fireproof building a minimum cost of $20 per square foot. This differential in building costs can be a much more significant determinant of final housing costs than are differential land prices.

In sum, my remarks therefore constitute a series of suggestions to Muth that he might find useful as he proceeds with his analysis of 1960 data. Even if he continues to confine himself to the population rather than the investment aspects of intrametropolitan change, he ought to build up both the demographic and the economic dimensions of his model, adding to the first by bringing in life-cycle subsets, adding to the latter by taking account of the effects on housing costs of building density—that is, the size of structures relative to the size of sites.

On Muth

BY VERNON G. LIPPITT, UNIVERSITY OF ROCHESTER

This study is an intriguing attempt to obtain empirical equations relating the distribution of population in urban areas of U.S. cities to factors determining that distribution. In the main, it is a cross-section study based on data for some forty-six cities in 1950, though the cross-section findings are applied to some time series data near the end of the report.

Perhaps because of my unfamiliarity with investigations in this field, I had to reread sections of the paper several times before they came clear. The organization and presentation of the material is not suitable for a beginner. The author's use of well-chosen abbreviations to designate his variables was a helpful practice which, it seems to me, could well be more widely adopted. In some instances it would have clarified the variables if units of measure (or dimensions) had been specified. Is k measured in miles? Is income per capita in dollars or thousand dollars? Such information is needed to assess the relative importance of some terms in equations. What are the units for q, the quantity of housing consumed? Is it a *flow,* perhaps dollars per year at 1950 prices? How are quality differences in housing units allowed for? What about density gradient D_1? Change in persons per square mile per mile of increase in distance from the center of the city suggests itself, but dimensional analysis of some of the equations indicates that it is percentage change in density per mile of distance.

Since I am not qualified to discuss the findings from familiarity with the problems analyzed and since Louis Winnick has commented so ably on the practical considerations involved, I shall limit my remarks to a couple of questions about the logic underlying the research.

In the first section of the report, Muth summarizes findings of an earlier study of his which indicated that population density declines roughly exponentially from the city center.

$$D = D_0 e^{-D_1 k}.$$

In developing the logic underlying this finding, the author relates density to factors involving housing.

$$D = \frac{\text{persons}}{\text{land area}} = \frac{\text{housing services per land area}}{\text{housing services per person}}.$$

Housing services per land area declines with k, because land is cheaper relative to costs of the building at larger k; so more land is used per house as k increases.

Housing services per person increases with k. It is the logic underlying this second conclusion about which I have some doubts.

For an individual household choosing a house in a metropolitan area, the condition for indifference among sites of various distances from the central business district (CBD) is apparently: housing expenditures + transportation expenditures to CBD = constant, or

$$pq + T = \text{constant for various } k, \tag{1}$$

where q = quantity of housing service consumed by an individual household, say, dollars per year at 1950 prices (including land rental value); p = prices per unit of housing, dollars per unit of q; T = transportation expenditure to CBD for all purposes, say, dollars per year; k = distance from the CBD, say, miles.

The distance gradient becomes

$$\frac{d(pq)}{dk} + \frac{dT}{dk} = 0, \tag{2}$$

or, in terms of percentage changes,

$$\frac{1}{pq}\frac{d(pq)}{dk} = -\frac{1}{pq}\frac{dT}{dk}. \tag{3}$$

Case 1: If q is constant for the household at various k, we have

$$\frac{1}{p}\frac{dp}{dk} = -\frac{1}{pq}\frac{dT}{dk} \tag{4}$$

which is Muth's equation (1).

Case 2: If the household considers letting q vary with k to help offset the change in T with k, then (3) becomes

$$\frac{1}{p}\frac{dp}{dk} + \frac{1}{q}\frac{dq}{dk} = -\frac{1}{pq}\frac{dT}{dk}. \tag{5}$$

The rise in T may be offset by a decrease in q or in p, or in some combination of the two which makes the left side of equation (5) negative and equal to the right side.

Perhaps we can say that q changes with k because of a price effect and an income effect, the latter arising from the increase in T as k increases. In terms of percentage changes

$$\frac{1}{q}\frac{dq}{dk} = E_p\frac{dp}{pdk} + E_Y\frac{dY}{Ydk}, \text{ where } dY = -dT. \tag{6}$$

Substitution of (6) into (5) yields

$$(1 + E_p)\frac{dp}{pdk} - E_Y\frac{dT}{Ydk} = -\frac{1}{pq}\frac{dT}{dk}. \tag{7}$$

Collecting like terms, we find

$$(1 + E_p)\frac{1}{p}\frac{dp}{dk} = \left(E_Y - \frac{Y}{pq}\right)\frac{dT}{Ydk}. \tag{8}$$

The right side is quite surely negative, since $E_Y < 1 < Y/pq$ normally. Hence, on the left, dp/dk must be opposite in sign to $(1 + E_p)$; i.e., dp/dk is negative if $0 > E_p > -1$ and positive if $E_p < -1$.

Substitution of this solution for $(1/p)(dp/dk)$ into (6) and noting $dY = -dT$, we find

$$(1 + E_p)\frac{1}{p}\frac{dq}{dk} = -\left(\frac{E_pY}{pq} + E_Y\right)\frac{dT}{Ydk} \tag{9}$$

The quantity in brackets on the right will normally be negative, since usually E_Y will be smaller in magnitude than E_p multiplied by Y/pq. Thus the right side is positive, and $(1/q)(dq/dk)$ will be of the same sign as

Figure 1

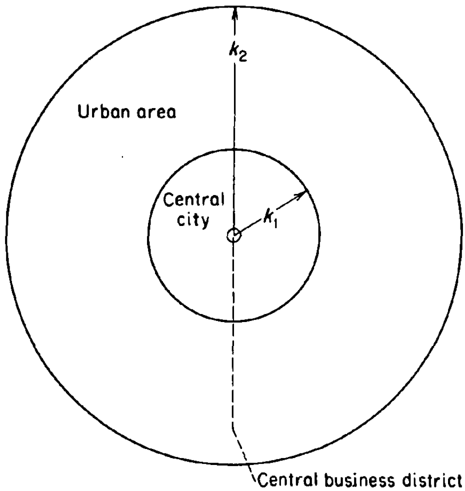


Figure 2

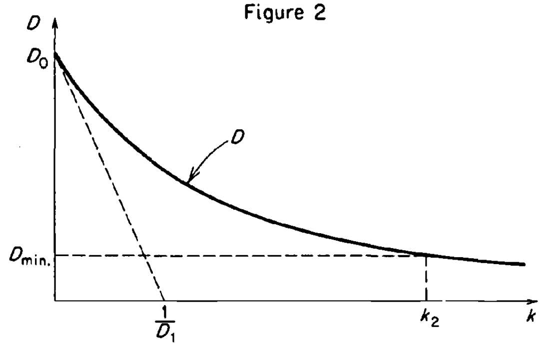
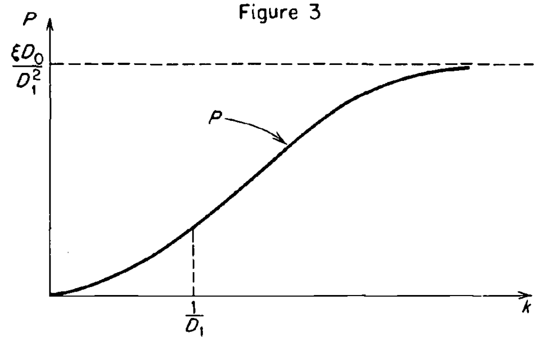


Figure 3



$(1 + E_p)$; i.e., dq/dk will be positive if $0 > E_p > -1$ and negative if $E_p < -1$.

In summary, it seems to me that Muth assumed in deriving his equation (1)—my equation (4)—that households would keep q constant as they decided where to locate. This leads to a negative price gradient because of the positive gradient in transportation costs. Then he concludes that the negative price gradient will lead to a positive gradient in q per capita because of price elasticity. It would seem better to permit both p and q to vary with k from the start of the analysis, with results indicated by my equations (8) and (9) above.

In the second section of his paper Muth analyzes "the distribution of population between the central city and its suburbs, and the total land used by urban areas."

The model used is that of a circular city with a small central business district, a central city of radius k_1 and population P_1, and an entire urban area of radius k_2 and population P_2 (Muth used P). (See Figure 1.)

Population density D is assumed to decline exponentially from the center of the city with constant gradient D_1 out to k_2.

$$D = D_0 e^{-D_1 k} \tag{10}$$

where D_1 is the percentage decrease in D per mile of increase in k (see Figure 2).

Total population within a circle of radius k is given by

$$P = \frac{\xi D_0}{D_1^2} f\{D_1, k\}, \tag{11}$$

where $f\{D_1, k\} = 1 - (1 + D_1 k)e^{-D_1 k}$ and $\xi = 2\pi$ times fraction of urban land area used for housing, assumed constant for all k (see Figure 3).

Urban land area used for housing is

$$L_2 = \tfrac{1}{2}\xi k_2^2 \quad \text{(Muth uses } L \text{ instead of } L_2.\text{)} \tag{12}$$

From these three equations and the Census Bureau specification of a constant population density at the boundary of an urban area (with serves to eliminate D_0 from the equations), Muth derives equations to explain differences in the values of P_1 and L_2 among U.S. cities in terms of differences in their values of P_2, D_1, k_1, and ξ. All variables are expressed as percentage changes (or logarithmic differentials) between cities.

$$dP_1^* = (1 - \gamma)\, dP_2^* + 2(\delta - \gamma)\, dD_1^* + 2\delta dk_1^* + \gamma d\xi^* \tag{13}$$

$$dL_2^* = \beta dP_2^* - 2(1 - \beta)\, dD_1^* + (1 - \beta)\, d\xi^* \tag{14}$$

where β and γ are functions of (D_1k_2) and δ is a function of (D_1k_1).

Four comments suggested themselves to me regarding this formulation.

1. The fraction (ξ) of urban land area used for housing is apparently assumed constant for all k. It would seem more logical to assume that ξ would increase with k, as the proportion of land area devoted to commercial and industrial purposes tapers off with distance from the center of the city.

2. In my equations (13) and (14) the Greek-letter coefficients are functions of (D_1k_2) and (D_1k_1). Yet in the fitting of equations in the cross-section analysis it would appear that they have been treated as constants. Since D_1k_2 and D_1k_1 are known for various cities, it would seem desirable to calculate the Greek-letter coefficients directly to check on the assumed constancy or to enter their values in the equations as additional explanatory variables.

3. In my equation (13) the term involving dP_2^* (logarithmic differential of urban area population) apparently accounts for 94 per cent of the variance in dP_1^* (the logarithmic differential of central city population). Probably the urban area population term dP_2^* in equation (5) accounts for a large part of the variance in dL_2^* also. One wonders if it might not be good strategy to deflate by this dominant variable and use the ratios P_1/P_2 and L_2/P_2 as the dependent variables for analysis. It might then be possible to determine the contributions of the other explanatory variables independently of the total population effect.

4. Finally, I wondered about the use of the findings from cross-section analysis in the discussion of changes over time. In view of the developments of suburban business districts and of urban renewal projects in recent years, it would seem that dynamic changes may reflect considerably different causal forces from those that operated to determine the patterns of urban population distribution in a cross section as of 1950.

On Sparks

BY VERNON G. LIPPITT, UNIVERSITY OF ROCHESTER

I feel this is a competent paper; it is clearly and logically presented. Variables are clearly defined in a statistical appendix, with units of measure and sources given, and data recorded.

I applaud the use of year-to-year changes for dependent and independent variables. This reduces serial correlation and forces the coefficients in regression equations to reflect short-term relations between the variables rather than relations between their trends. Of course, using years for the time units and postwar years only restricts the number of degrees of freedom available, so that in several instances R^2 for a regression equation (adjusted for degrees of freedom) increases when one or two variables are omitted. It also prevents any investigation of time lags or leads of a few months' duration, such as one might expect in comparing housing starts and demand for mortgage funds.

As data for more postwar years become available, it may well prove desirable to investigate whether the form of relation between the annual change variables is linear, as assumed in the regression equation. It might also be interesting to split the data into earlier and later years to test the stability of the coefficients. I suspect that this might be a revealing, though humbling, test of many of our time series models. As the author notes, the large error terms in recent years suggest that the calculated equation for change in housing starts may be drifting off calibration. Included variables may be shifting in relative importance, or new variables may be coming into play.

There is little effort in this paper to develop a theoretical model, aside from the discussion of the income shifts and the fixed-rate theory as explanations of the shift of funds from the mortgage market to the corporate bond market in times of good business. And these alternative hypotheses were not tested because of the lack of good empirical data for credit terms. The variables included in the regression model are a combination of variables from consumer demand theory (income, relative prices, need based on demographic factors, available inventory of unoccupied dwelling units, and credit terms) plus variables from financial investment theory (inflows of saving deposits, repayments of loans, financial ratios, and relative rates of interest). A few comments may be in order on the choice and specification of these variables.

1. The equation for housing starts and the one for demand for mortgage funds are the same in form and in the explanatory variables. I wonder if there might not be an appreciable difference in timing between the housing starts variable and the demand for mortgage funds (including advance commitments). Probably use of annual data rules out detection of lags here. More important, one of these variables is a physical flow and the other is a money flow. The mortgage funds demanded differ from housing starts by a price, or unit value, variable which probably has changed over the years covered by the analysis, but which has not been included in the analysis. Mortgage credit demanded per housing start may have changed because construction costs have risen, the mix of dwelling units has shifted to higher average values,

or the average amount borrowed from financial intermediaries per start may have changed. In recent years mortgage borrowing seems to have been used as a cheap source of credit for purposes other than housing expenditures. A plotting of the series for change in supply of mortgage funds ($\triangle MC + \triangle$FNMA) does indicate an upward drift from 1949 to 1964 compared with the series for $\triangle HS$.

2. I have some reservations about the way in which the variable for credit terms was eliminated from the housing starts equation. Equations for demand and supply of mortgage funds were set equal, and solved for the credit terms variable $\triangle Cr$. This solution for $\triangle Cr$ was then substituted back in the housing starts equation, at least conceptually. The solution for $\triangle Cr$, while logically correct, might involve rather large errors, it seems to me. It depends essentially on the difference between the contributions to demand and supply of mortgage funds from all factors other than credit terms. So $\triangle Cr$ would pick up the time pattern of any omitted variables or nonlinearities of the demand and supply equations. I wonder if it might not have been better to use some direct measure of credit terms, even if it were not precise.

Of course, in the final equation for change in housing starts, the above solution for credit terms was not used. Instead, the series for supply of mortgage funds as derived from data for financial intermediaries (excluding commercial banks) was entered in place of the series for $\triangle Cr$. Some theoretical justification or discussion of this substitution would seem desirable. It is not the replacement for the credit term which is indicated by the reduced form calculations.

3. In the equations for net increase in saving deposits ($\triangle SD$) by type of financial intermediary, personal financial saving was used as an explanatory variable. Personal disposable income would have seemed a preferable behavioral variable to use, since it is more likely to be included in the over-all model of which these equations are a part.

Finally, note that "the purpose of this paper is to develop a model of the residential construction sector of the U.S. economy with particular emphasis on the financial factors that provide a link between construction activity and the monetary sector." The final equation for $\triangle HS$ and year-by-year values of the explanatory terms in Table 4 suggest that interest rates made an appreciable contribution to explaining $\triangle HS$ in several postwar years. Beyond the question of statistical significance lies the question of forecasting significance.

Can the year-to-year changes in the relevant interest rates be forecast reliably enough to make a useful contribution to forecasting housing starts? If the Research Seminar in Quantitative Economics is developing a model in which these interest rates are endogenous variables, I can see that we have a competitor to the SSRC model springing up.

How stable will the coefficients of the predictor equation remain through time? As noted above, the demand for mortgage loans in recent years seems to have been based increasingly on uses of funds other than for residential purchases. And as housing supply has caught up with demand in the postwar period, I wonder if the availability of mortgage funds and mortgage credit terms have become less effective in making residential housing activity a countercyclical force in the economy.

In any event this paper represents a good start, it seems to me, in an area where monetary factors may be expected to affect real demand and supply. If this model is to be incorporated into the ongoing forecasting effort under Daniel Suits at the University of Michigan, there is hope that it will be subject to continuous review, revision, and evaluation. That is the ultimate test of a model from the forecasting point of view.

On Snowbarger and Suits

BY VERNON G. LIPPITT, UNIVERSITY OF ROCHESTER

The technique for successive dichotomous partitioning of a sample used in this paper to analyze what characteristics of buying units are related to purchase and ownership of consumer durables seems an ingenious one. The authors present this technique as an alternative to regression and related techniques used to analyze cross-section data—"an alternative by which the data can be scanned to identify the most important variables and their interactions." The claimed advantages of this technique are: (1) it provides "a better profile of consumer activity" in cross-section analysis where "the complex intercorrelations and interactions preclude a straightforward linear regression"; and (2) it does not "require the analyst to select the 'control' variables in advance"; it searches out the explanatory variables by uncovering significant differences in the dependent variable between dichotomous partitions on all recorded characteristics of the observation units.

A few comments on these claims would seem to be warranted.

First, it is not clear that this technique gets around the intercorrelation problem. The authors point out that: "In any split, some variables may be highly correlated with the partitioning variable, but not be sufficiently powerful to make the split." They suggest that "the relative discriminatory power of each variable is shown at every stage of the program" so that the investigator can analyze the splits that "almost" occurred. This deals with the problem qualitatively, but not quantitatively.

Second, regression techniques *can* handle some nonlinearities and interactions by use of dummy variables for each class on each characteristic and for subclasses determined by two-way classification.

Third, the fact that the AID program does not require advance specification of the explanatory variables indicates that the method does not involve the testing of a formal hypothesis of consumer behavior, except the hypothesis that the characteristics entered in the analysis do discriminate among buying units' probability of purchase. The technique is essentially a preliminary search technique, and should not be considered an alternative to regression analysis or analysis of variance techniques. As was indicated in the oral presentation, after using the AID program you "think up hypotheses for what the data show."

A review of the findings in this investigation reveals several interesting features.

1. In several cases a "successful" split involves breaking out a group of buying units characterized by two nonadjacent classes on some characteristic. See split 4 on television sets and washers (AIDPR levels of <9 per cent and 20–39 per cent), or split 18 on refrigerators (disposable income classes $4000–$6000 and $7500–$10,000), or split 6 on washers (residence established in 1955–1957 or in 1959). This may indicate significant nonlinearity in the influence of the classifying variable, but it does point up the incompleteness of this technique in not giving a picture of the way buying influences change when classes of that characteristics are placed in some rational order.

2. For split 4 on refrigerators we find that, among buying units who indicated they would not buy, purchases were most frequent among a group who answered "don't know" to the question "Do you expect to be better or worse off a year from now?" This is an odd explanatory variable. Perhaps it points to the need to find some characteristic distinguishing agnostic types or families beset by uncertainty whose buying behavior is most susceptible to outside influences.

3. It is noteworthy that relatively few further splits occur in the "high" group of the first split for household durables, i.e., among the group exhibiting high frequency of purchase. This group is characterized by their having expressed an intention to buy the appliance. Apparently this characteristic swamps the effects of all other characteristics. Given the intention to buy, few other characteristics discriminate strongly between the buying units *in this group* who do and do not carry out their intentions. Alternative explanations might be that in this

group with high probability of purchase a higher level of differentiating power (or *absolute* reduction of variance) is required for causing a split, or the small numbers of families may make it harder for a characteristic to be recognized as having a significant effect. If either of these explanations is true, I wonder about modifying the technique so that all of the buying units are searched together on successive splits after removing the influence of the characteristic found important on the prior split. The removal might be accomplished by subtracting from probability of purchase in the high group the difference between the proportion buying in the high and the low group.

Of course, it would be of interest to run the AID program on these buying units without including expectations or intentions variables also, to see how the demographic variables alone turn out.

4. It was interesting that time interval since marriage or since move to a new residence did not show up as very significant. Presumably this is either because newly married or recently moved buying units are too small in number to make their influence show up or because such units intend to buy and possession of that characteristic obscures the influence of the underlying factors motivating the buying intentions. Again the importance of analyzing the data with intentions data omitted is indicated, if the purpose is to derive demand determinants applicable to situations when intentions data are not available.

5. A significant omission from the variables used in analyzing the purchase data is a measure of the age of the given consumer durable at the time of the initial survey. It is sometimes stated that, among the group who do not express an intention to buy, breakdown or other enforced replacement is a frequent cause of purchase. Many market demand analyses give replacement demand an important place. It would have been interesting to see whether age of an appliance would have shown up as an important variable explaining unplanned purchases. It would be interesting to know, also, how omission of this variable may have affected the ability of other variables to cause splits.

6. The findings on second-car ownership are illuminating. With the intentions data omitted, we get more splits on the income and demographic variables among the high as well as low groups. The indication that the most relevant demographic factors are different for different income groups is, I think, an illustration of the most valuable type of insight with regard to interaction which the AID program is best suited to bring out. Such information would have marketing significance and some unlimited forecasting value. I say "limited" because the individual subclasses obtained after a few splits would be hard to forecast.

The forecasting of population by demographic characteristics becomes difficult if numbers in cells determined by several characteristics are required.

In summary, the AID technique of analysis used in this study can, it seems to me, provide useful information in a search for explanatory variables and in detecting some interactions. It should not be regarded as an alternative to regression or analysis of variance approaches in testing hypotheses or in evaluating quantitatively the over-all importance and systematic pattern of influence for an explanatory variable.

On Sparks, Snowbarger-Suits, Muth, and Miller-Watts

BY JAMES MORGAN, UNIVERSITY OF MICHIGAN

Gordon Sparks' paper is part of a program to introduce monetary factors in a meaningful way into the Klein-Suits-Goldberger-Suits-Locke Anderson-Suits Michigan model. He has selected a promising area where the supply of funds and interest rates should matter, namely, housing. He uses a two-stage analysis to explain first the flow of funds, and then uses that to help explain housing starts and residential construction expenditures. And he seems to have avoided using anything to explain itself: increase in dwelling units is not made a function of new family formation.

The paper by Marvin Snowbarger and Daniel Suits faces up directly to the problem that all our theorizing is accompanied by increasingly systematic and exhaustive exploration of the data, testing, and revision of models. Hence we are really not testing one hypothesis but selecting among many. The search procedure he uses loosens up the restrictions (additivity) with which we view data, and looks for what matters. It does it in a defined and reproducible way. His results should be looked upon as a first step toward the development of a better model or set of models. What is important about his findings is the evidence that both expenditures and income elasticities vary a great deal among different subgroups. The search for a single income elasticity may, therefore, be illusory. Economic policies which affect the incomes of different groups in different ways may well require knowing the separate income elasticities, not just the weighted average. The differences in level among subgroups matter. Even though some of the differences between subgroups may not lead to changes over time, others may. There may be substantial changes for instance in the number and proportion of families with working wives, teen-age children, or a dwelling in the suburbs.

Richard Muth's basic model implies that people have a general desire to be as close to the center of the city as possible, and are deterred by an increasing rent gradient. My disagreement is with these basic assumptions of the model rather than with the statistics. Bernard J. Frieden's new book *The Future of Old Neighborhoods*[1] shows a wide variety of rent gradients from one city to the next. Recent studies at the Survey Research Center have shown that more people would like to be farther out than the reverse. They want to have more space, to own their own homes, to have good schools, and to have access to outdoor recreation. The younger they are and the more outdoor activities they like, the greater their preference for living farther out. Even among those already fifteen miles or more from the center, 20 per cent would like to be farther out and only 10 closer in.[2]

Actual moving plans are in the same direction. In large cities of a million and a half or more, fewer than 40 per cent go downtown for purposes other than work even as often as once a month.[3] It seems likely that there are two groups, one of which is mostly made up of people without children who like being close to downtown, and the other made up of people who are only kept from real country-estate living by the costs of land and of the journey to work. There are certainly differences between cities as to the amount of employment available without even going downtown.

The frequency of passenger car ownership seems likely to be a result of past decisions about streets and public transport rather than a cause of urban spread. The Lansing-Mueller monograph shows that the age of a city, reflected by its population in 1900, is a major determinant of the proportion who use a car to get to work. Ranking the twelve largest metropolitan areas according to their population in 1900, one finds that the percentages using a car to get to work vary from 36 in New York and 63 in Chicago and 67 in Philadelphia to 79 in San Francisco, 86 in Detroit, 73 in Washington, D.C., and 91 in Los Angeles.[4] The use of two-stage least squares may avoid bias but it does not remove the doubt about whether cars are a cause or a result of urban spread.

Finally, the explanations of the two significant (but opposite in sign) coefficients for the median income in the central city and the median

[1] Cambridge, Mass., 1964.

[2] See John B. Lansing and Eva Mueller, *Residential Location and Urban Mobility*, Survey Research Center, University of Michigan, Ann Arbor, 1964, p. 24.

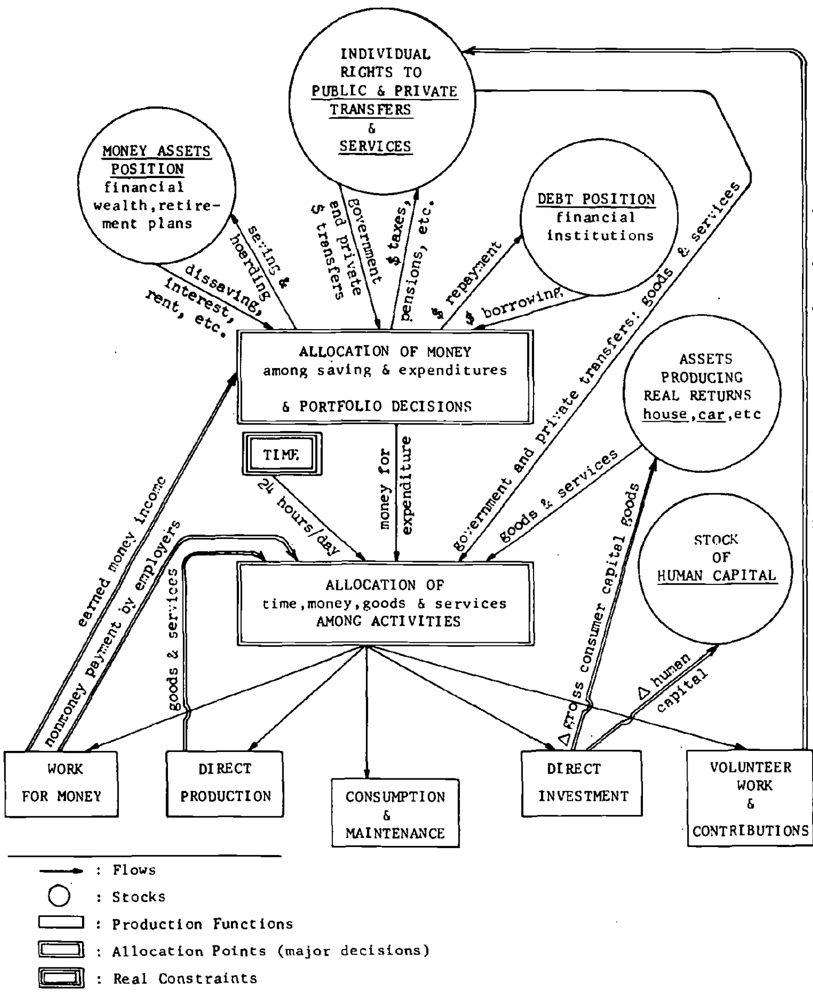
[3] *Ibid.*, p. 40.

[4] *Ibid.*, p. 92.

income in the urban area are ingenious, but since the two are correlated .94 I have the uneasy feeling that this may be one more case where multiple regression provides an excellent predictive mapping but does not provide good explanations. Close examination may even show that the result depends on one or two cases of "outlyers."

FIGURE 1

A Theory of Time Use



The paper by Roger Miller and Harold Watts is an excellent, tersely written attempt to formulate a stochastic and dynamic theory of the household and to attach a subtheory about its portfolio investment decisions. It repays close reading since it at least touches upon most of the main problems in such a theoretical model. It sees the household as making decisions about the allocation of its time, the spending or saving of its income, and the investing of the accumulated saving. I have only two suggestions and, unfortunately, they are for still more complexity before we can start simplifying and making it operational:

First, rather than one stock, the financial portfolio, there are really several: investments in financial assets; direct investments in real estate and business, which require more management time; debt (not connected with earning assets), which has motives and constraints of its own; consumer assets (houses, cars, etc.), which provide services directly and involve a commitment to a future stream of consumption (real behavioral problems are buried by treating all this like renting the service from oneself); human capital, whose value increases with investment in education or health; equity rights to public services and to public and private transfers, including social security and private pensions.

It is not important just how these are regrouped, but they cannot be forgotten. Many of them are mentioned in the paper, but not all, and the paper does not pretend to develop models of behavior with respect to the others. We are engaged at the Survey Research Center in a study of people's economic time allocations and have developed a graphic model (Figure 1) which is reproduced here both for its contribution and to make it easier to see what the Miller-Watts paper is about. The rich Wisconsin panel data will enable the development and testing of models of behavior with respect to the money assets (upper left circle). Our study deals with the allocation of time between the five activities at the bottom of the figure. It is useful to think of a household first allocating its time, money, and goods to activities, then making money allocations partly determined by the previous decisions, and finally making portfolio decisions about the way in which stocks of assets are held.

Secondly, we need a model that allows for learning, growth, and changes in taste. We know that people only think about an estate for their heirs if and when they get old and rich. We know their motivation to save feeds on past success in accumulating. George Katona has a forthcoming monograph with some new data indicating that the addition of a private pension plan to social security encourages people to save still more.[5]

Reply to Winnick, Lippitt, and Morgan

by Muth

I was certainly gratified by the amount of attention devoted to my paper by the discussants. It would be nice to be able to believe that it reflects an increased interest in urban problems on the part of economists generally, for I have become increasingly convinced that economists, and not merely a small group of dedicated specialists, have a great deal to contribute to understanding and solving urban problems. In this regard I was especially impressed by the comments of Vernon Lippitt, since it is my understanding that he has not previously done any work in this field.

Louis Winnick is certainly correct that there is more to the matter of where, within a metropolitan area, investment or even investment in housing takes place than mere population movements. I would like to have been able to present some empirical results on other aspects of the problem but time, both for my research and for presenting my results here at the conference, is, like housing, an economic good. Population movements since World War II have almost certainly contributed greatly to the location of postwar residential construction, however. The other major factor affecting the aggregate expenditures for housing in different parts of cities and thus residential construction, of course, is expenditures per capita. I have examined the variation of housing expenditures within the central city and believe that they are highly predictable; the occupant's income level would appear to be by far the major factor affecting these expenditures. The variation of median income by census tract within the central city, in turn, seems to be closely associated with age of dwellings and previous income level of the tract. Thus I would anticipate but cannot yet demonstrate that the variation of housing expenditure per capita between the central city and its suburbs could be largely explained in terms of the factors noted above.

Winnick's comparison of the difference between the relative change in population and in households in the ten largest U.S. cities over the

[5] George Katona, *Private Pensions and Individual Saving,* Survey Research Center Monograph 40, Ann Arbor, 1965. The model must be simple, but it does need to allow for some change in people's goals.

past decade, while interesting, is practically irrelevant for my purposes. The fact that in every case the percentage increase in households exceeded that of population probably reflects the great increase in incomes that occurred during the period. Earlier establishment of separate households by the young, longer maintenance of them by the aged, and less frequent sharing of quarters by single adults are among the ways by which the per capita consumption of housing is increased. For this phenomenon to make a difference for my purposes, however, there would have to have been differences among urbanized areas in the differential (change) in households relative to population between the central city and its suburbs. And even this would make no difference if one were able to explain the differential (change) in *per capita* expenditures for housing between the central city and its suburbs in terms of factors such as I noted in the preceding paragraph.

I quite agree with Winnick, and with the similar observation of Morgan, that there is a tendency for the average size of household to increase with distance from the CBD. But I feel this phenomenon is not very important quantitatively.

In part of my work, as yet unpublished, I have examined the relative change in average household size with distance as well as the change in housing expenditures per household and production per square mile of land in U.S. central cities for 1950. If anything, the association between average household size and distance is closer than that between the other two components of population density and distance. Housing expenditures per family and production per square mile each vary on the order of 20 per cent per mile, but average household size tends to increase on the order of only 1 per cent per mile within a given city. I once thought of including various demographic characteristics, such as number of children per family and fraction of adults of age 65 or more, in my analysis of variations in density gradients among the various cities for reasons similar to those set forth by Winnick and Morgan. When I had gathered the data, however, there was so little variation among cities in these measures that it hardly seemed worthwhile to use up my already limited residual degrees of freedom by including them. Thus, I do not feel that my neglect of the demographic considerations of which my critics make so much is a very important omission.

I also agree with the substance of Winnick's remarks on the importance of the substitution of construction outlay for land or space in the production of housing in response to differences in the price per unit of land and construction. I certainly did not mean to assume, however, that construction "costs" interpreted as total construction outlay or as a fraction of the total value of land plus structure are constant throughout the city; rather I assume that the variation in the price per unit of construction is small relative to that of land rents per square foot. Toward the beginning of section II of my paper I expressly noted the substitution of land for other productive factors in producing housing as land rents (per square foot) decline with distance from the CBD. Winnick is mistaken, I believe, when he argues in the last sentence of his penultimate paragraph that the differential in building expenditure is in any relevant sense a "determinant" of the way residential land is used. I would argue that the variation in building expenditure results from the variation of land rents relative to construction costs, both on a per unit basis. Only if construction costs *per unit* vary with distance can one say that "building costs" are in any meaningful sense a determinant of residential land use.

I must apoligize to Lippitt, as well as to any other reader with similar questions, for not being sufficiently clear as to the units of measurement I used. Distance, k, is indeed measured in miles, and it should be obvious from my equation (1) that the density gradient, D_1, is the relative change in population density per unit change in distance. Since the log of D_1 was used in my regression analysis, however, the coefficients shown would have been the same if distance had been measured in kilometers or rods. Income was measured in dollars, but the exponent shown in the stub of the various tables converts the units in which the income coefficients are expressed to thousand dollars. This should also have been clear from my discussion of the quantitative impact of various changes in section V.

Since q is housing consumption, it unambiguously relates to a flow of services per unit time. Actual measurement of this flow might be a very complicated problem when one is concerned with differences among different parts of a given city, and partly for this reason I did not attempt it. In principle, measurement of housing consumption would be similar to the measurement of, say, food consumption. When one attempts to combine quantities of diverse physical things into a single magnitude, as it is so frequently convenient to do for analytic simplicity, one does so by using an index number with prices as weights, though with well-known index number problems. The resulting measure is, of course, expressed in constant dollars of expenditure (per unit of time). To test the theory I have developed or to estimate certain parameters, however, it is not necessary to have such an index of housing consumption. Magnitudes such as population density and actual dollars of expenditure, both per household and per square mile of land, are much easier to measure, and I have concentrated my attention on them in my empirical work.

Contrary to Lippitt's assertion, I have not assumed that households keep housing consumption, q, constant in choosing their location. Rather, I have assumed, though not spelled out in my paper, that households maximize a conventional ordinal utility function $U = U(x, q)$, where x is dollars of expenditure (per unit of time) on all other commodities. The consumer is subject to the budget constraint $x + p(k)q + T(k, y) = y$, where T is expenditure for transportation and is a function of location as well as money income, y, the latter to allow for the opportunity cost of time spent in travel. There is nothing in this formulation which requires that q remain constant in any part of the household's decision process.

The first-order or necessary conditions for the constrained utility maximization are, first, the usual requirement that housing be consumed in such an amount in relation to other commodities that the marginal utility per dollar of expenditure be the same for housing and all other commodities and, second, my equation (1). The latter requires that no small change in location can increase the real income or consumption of the household. One set of sufficient conditions consists of the usual curvature properties of indifference curves plus the restriction that the rate of decline of housing price with distance not be so great that the saving in expenditure on that quantity of housing purchased at the optimal location for the household exceed the increased transportation expenditure which results from a small increase in distance.

Thus, in my formulation the appropriate value of q for the household to examine in deciding whether or not to move from a given location varies with distance because of the variation of housing prices with distance. When comparing different households, all of whom are assumed to be in their equilibrium locations, the sufficient conditions outlined above imply that those households in locations more distant from the CBD consume more housing relative to other commodities because housing prices are lower relative to other commodity prices at greater distances from the CBD. Lippitt's contention that my equation (1) holds only if q is invariant with distance follows directly from his assumption that the sum of housing expenditures and transportation expenditure is constant, an assumption for which there is certainly no economic rationale. Indeed, in my formulation constancy of this sum holds if and only if housing demand is perfectly inelastic. I submit, therefore, that Lippitt's contention is simply not correct.

With respect to Lippitt's other comments, there are certainly no logical grounds, though there could be empirical ones, which require one to assume that the fraction of land devoted to residential uses increase with k. I say "could be empirical ones" because other investigations I have not discussed here fail to reveal any evidence of increasing ξ. In particular I would expect a generally increasing ξ to impart a negative curvature to the relation between the log of gross population density and distance. In results reported in my earlier paper, such a curvature was not generally observed. And the direct inspection I have made for the south side of Chicago also fails to reveal any appreciable variation of ξ with distance or other measures of accessibility.[1] Lippitt is, of course, correct that the Greek-letter coefficients in my equations (10) through (14) may vary from city to city, and indeed I did calculate their values to obtain the average values used for interpreting my regression coefficients. I did not use the estimated city coefficients to weight the values of the various explanatory variables for the different cities as my equations (12) and (14) would suggest doing, however, because my D_1 values are sample estimates which are subject to appreciable sampling error. Had I used these in weighting the explanatory variables, I would have introduced appreciable measurement errors, and correlated ones at that, into all of the explanatory variables. It seemed to me that the specification error of not weighting is almost certainly likely to be less serious than the problem of correlated measurement errors in all the explanatory variables.

Since in my regressions I used logs for P_1, L, and P_2, it would seem to make little difference whether or not I deflate by P_2 as Lippitt suggests, especially since my computed regression equations give little indication of homoscedasticity. In comparing the increase in explained variance when all explanatory variables are included relative to that which is unexplained when size alone is included as I have done, a perfectly valid measure of the contribution of the other variables is obtained. Finally, as regards the propriety of the use of cross-section findings to explain changes over time, I would argue that consistency of regression estimates of the type I have presented with additional data, especially of a different type, is the best defense against the many objections to which an analysis such as mine might be subject. In fact, the so-called "dynamic changes" which Lippitt mentions are probably incorporated in my set of explanatory variables. The development of

[1] "The Variation of Population Density and Its Components in South Chicago," paper presented at the Regional Science Association Meetings, Ann Arbor, Mich., November 15, 1964.

suburban shopping centers is certainly not an exogenous force but is probably explainable in large part by the development of automobile transportation and increasing city sizes. To the extent that urban renewal has had any effects at all it would change the relative income level as between the central city and its suburbs, the racial composition of the central city population, the age and quality of the central city's housing stock, etc. Indeed, equations such as I have developed may well be useful in appraising the effects of urban renewal.

In first writing out my reaction to Morgan's comments, I was struck by how little disagreement there is between us. I will certainly agree that a wide variety of rent (presumably per unit of housing service) gradients, which I have argued are closely related to density gradients, exists in different cities. Indeed, I pointed this out in my earlier paper, delivered three years before the work Morgan cites; that paper and the present one are partly attempts to explain these differences. Although people may "want" to live further from the city center, in seeking to explain why they do not, Morgan mentions precisely the factors included in my equation (*1*)—transport costs and housing prices. Still another example is Morgan's expression of doubt as to the causal effect of car ownership on urban spread. I explicitly stated in my earlier paper that I used CAREGS as a surrogate variable for the many possible factors, some of which Morgan mentions explicitly, which may lead both to greater car ownership and urban spread.

There are a few differences between us, however. Morgan's remark about job opportunities not in the CBD seemingly implies that the effects of travel costs downtown on housing prices must certainly be weak. I tried to control for differences in the location of job opportunities by including the MANCIT variable, admittedly imperfect but the only such variable available for as many cities as I wished to study. More importantly, however, it appears to me that, in every city with which I am familiar, CBD and locally employed workers live throughout the city in significant numbers. Under such conditions, the two types of households must pay the same price (per unit) for housing at any given location. The fact that housing prices decline with distance from the CBD for locally employed workers implies that the money wages received by locally employed workers must likewise decline with distance. The fact of non-CBD employment by no means need weaken the effect of transport costs on housing prices.

Morgan's comments on the high intercorrelation between URBINC and CITINC might seem to imply that the coefficients of these variables are somehow tainted. I will readily admit that my explanation for the opposite signs of these two income variables in my equations (19) and (21) is a tentative one and not borne out by any additional evidence. From my Table 1 it appears quite certain that for these data the two income variables measure pretty much the same thing so far as the central city density gradient is concerned. But including CITINC in equations (19) and (21) actually corrected what appeared to me to be the wrong signs of the URBINC coefficients. This seems to me to be a step in the right direction, at least, and if due to "outlyers" I am grateful for them.

In closing I would like to make one comment upon what I perceive to be a common objection of my three critics. This is the belief that if something conceivably important is omitted from the analysis the latter is necessarily suspect. I would argue just the reverse. A necessary, though certainly not sufficient, condition of a usable analysis is that it explain the phenomena it attempts to explain in terms of a limited set of data. The more limited the set of explanatory variables and, of course, the more consistent the model with observed data on the phenomena to be explained, the more useful is the analysis. If I were to attempt the risky business of criticizing my own work, I would be inclined to argue that, if anything, my explanation of urban population distribution requires too many variables rather than too few. To the comment that I have omitted other things from my explanation, I can only shout "hooray"!

Reply to Lippitt

BY SNOWBARGER

I would like to thank Vernon Lippitt for his penetrating and insightful analysis of our paper. His comments will allow me to discuss, in more detail, some features of the AID program and its use in statistical anaylsis.

1. The AID program adds to the collection of statistical techniques that can be used to study the characteristics of empirical data. It is logically prior to the use of regression analysis and can be used to search data for interactions. An interaction variable can then be constructed and entered into a regression model. The AID program is not designed to supplant regression analysis.

The AID program can be used as an independent study device, or as a preliminary exploratory device within the framework of a complex statistical study. We have stressed the former feature in our study of multiple-car ownership (MCO). Without introducing regression techniques, a reasonably complete picture of MCO, and the dynamics of entry into the MCO market, is obtained. The MCO study could be placed into a regression framework and tested on other data.

Consumer expenditure on selected durables are explained by some interesting variables and their interactions. A consumer durable regression model can be constructed. The model might be used to predict individual household purchases. Or it might be aggregated to produce macropredictions.

The ideal use of the AID program allows the analyst to blend these two features into a complete statistical analysis. The AID program provides a profile of the data and uncovers the interactions. The complementary use of regression analysis provides significance tests, individual predicton models, macroprediction models, or both. The prediction may be attempted on a single cross section or on another panel. As an example of panel prediction, a model may be designed from one panel and then tried on another panel. In this manner, hypotheses may be tested. But before they can be tested they must be formulated. And this is precisely the advantage of the AID program when it is used prior to a regression analysis. It allows the researcher to design a hypothesis to fit or explain what he observes in the AID tree. The resulting hypothesis, within the framework of a complete regression model, may be tested on other data.

In summary, the AID program increases the efficiency of the research process. The application of the program to a set of data will reveal its structure. As the program is applied to data over time, stable relations may become apparent. The trees assist the researcher in formulating the hypothesis. The advantages are that better hypotheses may be designed, and premature testing (i.e., in regression) of hypotheses can be reduced. After the hypothesis is formulated, it should be tested on new data. The strength of the model will be revealed in its predictive ability.

2. There are several technical features of the AID program that increase its usefulness and flexibility. It can scan many more variables than it actually uses. Hence, a minimum of hunches and conditions needs to be imposed on the analysis. In addition, all variables may be entered unconstrained. When the variables enter the analysis unconstrained, nonadjacent groups may be selected or a split may occur containing a small number of cases. Neither problem should cause any concern. The computer output gives detailed information on the size of all groups. If nonadjacent categories are grouped together, and

there are a small number of cases, the regrouping can be disregarded. It might have occurred as a result of sampling error, etc. But if the regrouping contains a large number of cases, it might indicate an important result that should be considered by the analyst.

Splits that occur on a small number of cases (e.g., split 4 on refrigerators) are no problem. The order of splits is not important and these may be overlooked. The computer output should be studied carefully, however. When groups with a small number of cases are produced by the program, this is simply an indication that the program is working correctly. Individual behavior is not homogeneous and extreme variations are common. The program is looking for variation and does not stop to analyze the implications of the split. The research analyst must do this. (Small splits are actually an advantage because they warn the researcher of extreme skewness in the data.) This is why the problems of nonadjacent groups and small splits should cause no worry. The analyst must read and interpret all the output and not expect the tree itself to provide the answers.

There are several ways to get around the problems of nonadjacent groups and small splits. One solution is to not publish these "messy" things and carefully edit the tree to produce esthetically pleasing economic results. But if another person seeks to build on the analysis, he should be able to get the same results with the same data. The implication of these remarks is that there is a virtue in being able to reproduce another's work. Our trees are entirely reproducible by simply putting the data and the program on the computer.

Another way to avoid the problems of nonadjacent groups and small splits is to constrain and aggregate the variables that are to be used by the program. When a variable is constrained, the program is unable to split it on nonadjacent groups. A variable may be aggregated by reducing the number of code categories. (E.g., disposable income might be a ten-category variable with a range from $1000 to $50,000. It can be aggregated to five categories with the same range.) The effect of aggregation is to create a smaller number of code categories with a larger number of households in each category. Of course, to the extent that the researcher manipulates the variables in either of these two ways, he is dictating the "optimal" combinations to the program. (Carried to the limit, it means the AID analysis is superfluous. The researcher can use his personal feelings to design his own interactions and regroupings. The interactions thus designed are unlikely to be found in the data, however.) The result of doing either, or both, of these things is to reduce the flexibility of the program, reduce the variation explained by the tree, alter the structure of all interactions subsequent to the use of the constrained or aggregated variable, and, perhaps, even prevent the variable from being used.

The third way to handle the problems of nonadjacent groups or small splits is to enlarge the data set. This tends to reduce the impact of extreme variation and allows the program to isolate the dominant and pervasive characteristics of the data. This method will not entirely remove these problems, but it helps. We have used this technique in our paper in sections II, III. In section II we took the panel spanning two years and, rather than analyzing each year separately, we grouped the two years. The grouping had the effect of doubling our sample size. We, nevertheless, encountered these problems as the trees materialized. There are three reasons for this. First, the total two-year panel sample size of 2118 is not large, and the proportion of households purchasing a durable in a given year is small. Second, the purchase behavior of consumers for any of the five separate durables studied was not undergoing any large-scale change. Therefore, it was difficult to distinguish purchasers from nonpurchasers. Third, we did not have information on the age distribution of the current stock of durables. It seems certain that the stock characteristics of households are very important to purchase decisions of new durables.

In section III we grouped 1962 and 1963 data to increase our sample to over 4000 cases. The large number of sample points and the intensive growth of MCO allowed the program to isolate the dominant interactions with a minimum of extraneous splitting.

3. The output of the AID program can be carried to extremes. It is theoretically possible to continue splitting until every single data point is isolated. The point is that the concept of a final group, prior to this actual limit, is arbitrary. The researcher can call a group "final" at his discretion. One criterion that should be followed in defining a final group is that it be reasonably large in relation to the over-all sample. Hence, many seemingly final groups in the trees in our paper are not final at all. We have the computer output specifying many more splits on every final group in the trees. But we show no further splits because the group itself is small, or the mean of the group is small, or our analysis of the subsequent splits indicated that the program was unable to produce a dominant partition (i.e., a partition that contained a reasonably large number of cases).

4. Purchase intentions cause problems when they are used with other "objective" variables or predictors. Intentions may be part of a different stage in the causal process. They are unquestionably the

most efficient interaction term since they embody the objective economic determinants.

The purchase intentions variable was in the predictor list on every run on the panel data. It was the dominant variable in the purchase of the five durables. Purchase intentions were not part of the predictor list for the trees explaining the structure of MCO for 1957 and 1962–63. It would be meaningless to use current attitudes or intentions to explain ex-post MCO status.

We intend to re-examine the panel data and omit the purchase intentions variable. The sequential procedure of the AID program will then provide an even better picture of the influence of the objective economic variables. We also plan to use the panel data to examine purchase intentions as a dependent variable. The AID program will reveal the structure of interactions explaining these intentions.