

This PDF is a selection from a published volume from the National Bureau of Economic Research

Volume Title: NBER Macroeconomics Annual 2011, Volume 26

Volume Author/Editor: Daron Acemoglu and Michael Woodford, editors

Volume Publisher: University of Chicago Press

Volume ISBN: 978-0-226-00214-9 (cloth); 978-0-226-00216-3, 0-226-00216-0 (paper)

Volume URL: <http://www.nber.org/books/acem11-1>

Conference Date: April 8-9, 2011

Publication Date: August 2012

Chapter Title: Comment on "Natural Expectations, Macroeconomic Dynamics, and Asset Pricing"

Chapter Author(s): George W. Evans

Chapter URL: <http://www.nber.org/chapters/c12406>

Chapter pages in book: (p. 61 - 71)

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## Comment

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### Introduction

Expectations clearly play a central role in modern macroeconomics. Households and firms are assumed to be dynamic optimizers, making decisions about work, consumption, savings, production, and investment, based in part on current economic conditions, but also to a great extent on the future state of the economy. Thus, in particular, household saving and portfolio decisions depend on expected future interest rates, inflation, and taxes and on the likely future trajectory of equity dividends and prices. Because of the key role of expectations in economics and finance, theories of expectations have been central to modern economic theory. Since the rational expectations (RE) revolution of the 1970s—associated with John Muth, Robert Lucas, and Thomas Sargent—the benchmark theory has been that expectations are formed rationally, in the sense that they are consistent with the true model and yield forecast errors that are orthogonal to agents' information sets.

In their paper in this volume, Andreas Fuster, Benjamin Hebert, and David Laibson present an asset-pricing model in which RE is replaced by natural expectations (NE). Under NE, agents misspecify the time-series model in a “natural” way: they chose a parsimonious model of dividends that omits longer lags. This captures short-run dynamics but misses long-run mean reversion. An earlier paper, Fuster, Laibson, and Mendel (2010), made similar arguments about other macroeconomic time series.<sup>1</sup> Taken together, the two papers suggest the even bolder possibility of NE as a general stylized description of expectation formation.

In the current paper, Fuster et al. insert an NE dividend forecasting equation into a Lucas-type consumption-based asset-pricing model

with constant absolute risk aversion (CARA) preferences and habit persistence. Using this setup, Fuster et al. can replicate a number of stylized facts and puzzles about asset price data and consumption. These include the findings of excess volatility of stock prices, that excess returns are negatively predicted by lagged excess returns, price to earnings ratios, and consumption growth, and the existence of a large equity premium.

### Outline of Their Argument

Fuster et al. consider a variation of the Lucas-type “tree” model of asset prices. In keeping with Lucas (1978) there is an endowment economy with a single risky asset, trees, which provide an exogenous stochastic dividend of the perishable, homogeneous consumption good. The principal variation is that Fuster et al. consider an open economy version in which agents can borrow or lend internationally at a fixed interest rate  $R$ . In addition, exponential (CARA) preferences with habits are used. Finally,  $\Delta d_t$ , the first difference in dividends, is assumed to follow to follow a stationary AR( $p$ ) process for some  $p > 0$ .

The exponential preferences give a type of mean-variance setup, and Fuster et al. show that the asset price  $p_t$  satisfies

$$p_t = \sum_{s=1}^{\infty} R^{-s} E_t d_{t+s} - \frac{R\alpha \times \text{Var}_t \Delta c_{t+1}}{(1 - R^{-1}\gamma)(R - 1)^2}.$$

Fuster et al. also show how to obtain closed-form expressions for  $p_t$  and for consumption,  $c_t$ , given beliefs about the AR( $p$ ) process for  $\Delta d_t$ . Here  $R$  is the inverse of the discount factor,  $\alpha$  is the CARA measure of risk aversion, and  $\gamma$  is the habit-persistence parameter.

The key assumptions of the Fuster et al. model concern the stochastic process *actually* followed by dividends and the *perceived* process followed by dividends. The true dividend process is assumed to be a high-order stationary AR( $p$ ) for the first difference in dividends  $d_t$ ; that is,  $\Delta d_t = \sum_{i=1}^p \rho_i \Delta d_{t-i} + \varepsilon_t$ , or

$$\Delta d_t = (1 - \Phi(L))^{-1} \varepsilon_t,$$

where  $\varepsilon_t$  is white noise and  $\Phi(L) = \sum_{i=1}^p \rho_i L^i$ . Furthermore,  $\Phi(L)$  is assumed to be such that there is a hump-shaped impulse response function for dividend levels  $\partial d_{t+j} / \partial \varepsilon_t$ . Put differently,  $d_t$  is assumed to have a unit root with dynamics that lead to long-run mean reversion.

Evidence for this is given in Fuster et al.'s figure 2, which presents  $\partial d_{t+j}/\partial \varepsilon_t$  based on empirical estimates of  $\Phi(L)$  for  $AR(p)$  processes with alternative values of  $p$ . (In the empirical work, Fuster et al. use earnings rather than dividends.) The long-run level of persistence is given by

$$\lim_{j \rightarrow \infty} \partial d_{t+j}/\partial \varepsilon_t = (1 - \Phi(1))^{-1},$$

and for  $p \geq 15$  we have  $0 < (1 - \Phi(1))^{-1} < 1$ . Thus there is *mean reversion* in the sense that an innovation  $\varepsilon_t$  has a reduced permanent impact. Fuster et al. assume that large values of  $p$  (e.g.,  $p = 40$ ) correspond to the truth.

In contrast, for  $p \leq 10$  estimates of long-run persistence are  $(1 - \Phi(1))^{-1} > 1$ . That is, based on low-order  $AR(p)$  estimates, one would come to the conclusion that one should *extrapolate* innovations in dividends—that the long-run effects are larger than the impact effect. Fuster et al. assume that low-order  $AR(p)$  estimates correspond to the *perceived* dividend process; that is, to the view held by economic agents.

The essence of Fuster et al.'s approach is thus that the beliefs of agents differ from the truth and do so in a particular way. Agents use simpler low-order time-series models that lead them to accentuate the importance of short-run trends and to neglect longer-run corrections in which dividends revert toward an underlying trend. This central feature leads to the empirical implications noted before.

What is the rationale for the discrepancy they assume between truth and perception? Fuster et al. give two types of argument—statistical/econometric and psychological. The statistical argument is that econometricians have often argued that in forecasting there is an advantage in using parsimonious models in preference to more complex models. Furthermore, standard statistical procedures for model selection based on Akaike Information Criterion (AIC) and, especially, Bayesian Information Criterion (BIC) often select low-order models. The psychological argument is that agents, for a variety of reasons, prefer to use simple, parsimonious models in preference to complex models when trying to understand the world and make decisions.

Fuster et al. are usually careful not to be too dogmatic on this point. In essence they say: there is some evidence, as seen in their table 1 and figure 2, that a higher-order  $AR(p)$  process of  $\Delta d_t$  might well be correct, while agents might plausibly believe in a low-order process. Their paper then explores the full implications of this assumption.

## Links to the Macro Learning Literature

If the truth is that  $p$  is large, but agents believe that  $p$  is small, then clearly agents do not have RE. There is a now extensive macro literature, which started around 1980, in which RE is replaced, for example, by adaptive or econometric learning (see Sargent 1993 and Evans and Honkapohja 2001, 2009).

A major argument for the adaptive learning approach is what might be called the *cognitive consistency principle*.<sup>2</sup> According to this principle, agents should be assumed to have the same level of rationality as the economic modeler or policymaker (in contrast to both old-style adaptive expectations and to RE). On the adaptive learning approach agents are assumed to make forecasts in the same way that econometricians do—formulating models, estimating their parameters, and updating estimated coefficients over time as new data become available. When parameters are updated using a form of least-squares, this is known as least-squares (LS) learning.

The early macro learning literature focused on whether or not LS learning would converge over time to RE in *self-referential* models, in which the variables being forecasted are affected by the forecasts. Conditions were worked out that determined whether or not REE (RE equilibria) were indeed stable under LS learning. Stability under learning could then be used as a selection criteria in models with multiple REE, since in some cases only a subset of REE were stable under learning.

More recently, another major strand has been to show how learning can generate transitory or persistent “learning dynamics”; that is, dynamics different from RE. Much of the recent macro learning literature has emphasized learning dynamics induced by one or more of the following factors: (a) misspecified forecasting models (misspecified “perceived laws of motion” or PLMs); (b) discounted LS learning (downweighting past data due to concern about unknown structural change); and (c) dynamic predictor selection (selecting between alternative PLMs based on past performance) or Bayesian model averaging. Applications of the approach that emphasize learning dynamics include: the rise and fall of inflation in the United States, hyperinflation, business cycles, output and inflation inertia, optimal monetary and fiscal policy, and asset price anomalies.

One useful concept from the recent macro learning literature has been that of a restricted perceptions equilibrium (RPE), in which agents make the best forecast they can, given their misspecified PLM.<sup>3</sup> A special case

of interest has been models that are underparameterized, either in terms of variables or lag lengths. The argument here has been precisely that econometricians recognize the value of parsimoniously specified models, and thus the cognitive consistency principle dictates that we should examine the implications of underparameterization. One can, for example, work out stability conditions for an RPE when agents use LS learning to update coefficients of a particular underparameterized model.

Thus the Fuster et al. approach fits well with the recent macro learning literature. The principal contribution of this paper, in this context, is that it posits a particular, plausible type of misspecification by agents, which can arguably explain several puzzling features of asset prices, and which may also be of more general applicability.

## Discussion

I certainly find plausible Fuster et al.'s key assumption that economic agents underparameterize their forecasting models. This assumption is consistent with the cognitive consistency principle, given that many applied econometricians place value on parsimony and recognize the likelihood of misspecification. This hypothesis also fits well with the observation that many economists believe there is long-run mean reversion that is nonetheless difficult to detect.<sup>4</sup> That is, the misspecification that Fuster et al. assume is particularly plausible.

Other aspects of the Fuster et al. model are also attractive: the closed-form solutions under CARA preferences, for the class of perceived dividend processes examined, is likely to be more generally useful, and the simultaneous fit of a range of stylized facts is impressive.

It is therefore hard not to like this paper: the model is both simple and powerful. However, of course the model has some weaknesses, several of which are noticeable from the macro learning viewpoint. This in turn suggests a number of natural extensions.

## *Criticisms*

In my critical discussion I will focus on three main issues. The first concerns the information set available to agents when making forecasts. By assumption,  $\Delta d_t$  is an exogenous univariate process, which leads Fuster et al. to examine alternative univariate forecasting models. However, within macroeconomics the norm is to consider multivariate forecast-

ing models, and this issue is pertinent to the question of long-run persistence and to the plausibility of the form of underparameterization assumed. For example, in the early discussion of long-run GDP persistence, Campbell and Mankiw (1987) focused on univariate techniques, and found persistence levels greater than one. However, both the unemployment rate and the consumption-output ratio Granger cause output growth, and lower levels of persistence, with mean reversion, are found in multivariate models (e.g., see Evans 1989 and Evans and Reichlin 1994). In the current context Timmermann (1994), for example, has argued that stock prices Granger cause dividends. Thus a simple bivariate forecasting model might lead to different persistence results. The issue is whether simple—that is, low-order vector autoregressions—might show long-run mean reversion more clearly, in which case this feature of the data would be less plausibly missed by economic agents. Of course, many agents might still in practice use “natural” low-order univariate models, but a heterogeneous expectations model might then be more realistic.

My second concern is the fixed parameter assumption of Fuster et al. Suppose first that we agree that agents plausibly underparameterize  $\Delta d_t$  as an AR(1). From the learning viewpoint this leads to the corresponding RPE as the appropriate equilibrium to which the system would, if stable, converge. However, the cognitive consistency principle suggests that agents would not know the parameters of this process a priori, but, like real-world econometricians, would estimate the parameters and update their estimates over time. Furthermore, if agents are concerned about potential structural change, they might discount older data, leading to persistent learning dynamics around the RPE. This particular issue could easily be addressed by simulations in which fixed parameter natural expectations were replaced by discounted LS learning with the same AR(1) PLM.

Related to both of the previous two points, if long-run estimates of persistence are crucial for good decision making in their portfolio choices, one might expect agents to focus on this issue in their choice of forecasting models. They might estimate mean reversion directly and allow for uncertainty concerning its value in their decisions. Alternatively, they might adopt decision-making rules that are robust to errors in this dimension, along the lines of Hansen and Sargent (2007).

The third issue, which is probably most central from the learning perspective, is that the Fuster et al. model is not self-referential. Agents simply forecast dividends, which is treated as an exogenous process, and do not have a forecasting model for stock prices. Some

learning models emphasize short-horizon decision making, in which the demand for stocks depends on short-horizon expected returns, and possibly also on the estimated conditional variance of returns. See, for example, Brock and Hommes (1998); Lansing (2010); Adam, Marcet, and Nicolini (2010); and Branch and Evans (2011). Indeed, one way to formulate the most basic risk-neutral model of stock prices is to assume that prices are determined by the sum of expected dividend and expected stock price in the coming period. These models are self-referential in the sense that asset prices today depend on the expected price tomorrow, so that the evolution of the variable being forecasted depends on the expectations themselves.

Self-referential models, because of this feedback, give a much greater role to expectations, and this makes more likely asset-price bubbles: self-fulfilling or nearly self-fulfilling asset price movements with complex dynamics in which prices can become detached from fundamentals for extended periods. My own view is that this dynamic plays a central role in asset prices.

*Example: A Simple Model of Bubbles*

An example of the scope for dramatic learning dynamics in self-referential asset price models is given in my work with William Branch, presented in Branch and Evans (2011). We use a simple mean-variance linear asset pricing model. The setup can come from an overlapping generations model in which agents have two period planning horizons, CARA preferences, and a choice between a risky stock and a risk-free asset. The central equation is

$$p_t = \beta E_t^*(p_{t+1} + d_{t+1}) - \beta \alpha \sigma_t^2 z_{st}$$

where  $z_{st}$  is the iid random supply of the risky asset,  $E_t^*$  denotes the subjective expectations of agents, and  $\sigma_t^2$  is their estimate of the conditional variance of returns. We assume the dividend process is known and that agents therefore need estimates of the price process to make their decisions.

With iid dividend and supply shocks, the REE for  $p_t$  is a constant + white noise. Under learning, agents forecast  $p_t$  as an AR(1) using discounted LS and they estimate  $\sigma_t^2$  using a simple recursive algorithm. Because agents discount past data, prices under learning will occasionally break free from their fundamentals and exhibit bubbles and crashes. This results from the self-referential feature of the model.

An illustrative simulation is shown in figure 1 (see Branch and Evans

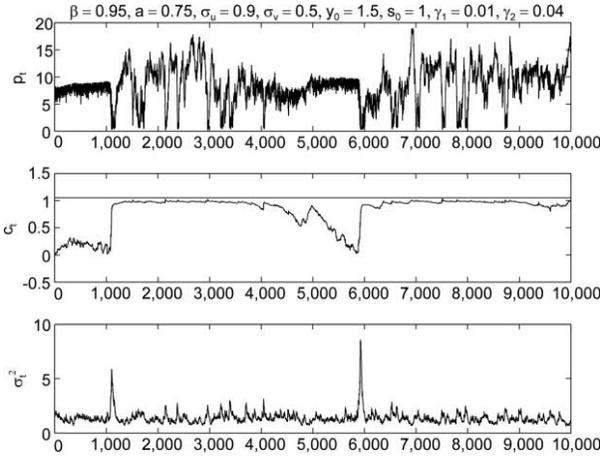


Fig. 1. Asset price dynamics in the Branch and Evans (2011) model

2011 for analysis and for other simulations). The figure shows the realized price  $p_t$  under learning and also the time series of estimates of two key learning parameters, the AR(1) coefficient  $c_t$  and the estimate of the conditional variance  $\sigma_t^2$ . The figure shows the price process initially very close to the REE, which is a constant plus white noise in our setup. However, under learning, asset prices occasionally break free into a bubble regime in which stock prices are believed to follow a pure random walk ( $c = 1$ ). In this regime  $p_t$  is particularly sensitive to changes in the estimate of risk  $\sigma_t^2$ .

In summary, self-referential learning models have great scope for generating some of the more extreme partially self-fulfilling movements of stock prices often described as bubbles and crashes. Intuitively, the reason for this is that the stock price  $p_t$  depends on expected price  $E_t^* p_{t+1}$  with a coefficient  $\beta < 1$  that is close to one.

#### *Other Types of Learning Dynamics*

In various settings learning dynamics have also been shown in self-referential models to lead to: (a) inertia of inflation and output, as in Orphanides and Williams (2007) and Milani (2007); (b) overshooting and nonmonotone impulse response functions (e.g., Eusepi and Preston 2011 and Evans, Honkapohja, and Mitra 2009); and (c) regime-switching and parameter drift (e.g., Sargent 1999, Cogley and Sargent 2005, and

Branch and Evans 2007). For numerous examples and references, see Evans and Honkapohja (2011).

With this in mind, consider again the question of whether it is plausible that agents believe  $\Delta d_t$  is a specific constant coefficient AR( $p$ ) process with known parameters. Parameter drift and regime switching appear to be standard features of the data, as emphasized by Sims and Zha (2006), Cogley and Sargent (2005), and Sargent, Williams, and Zha (2006). The cognitive consistency principle suggests that agents should therefore allow for the possibility of structural change in their parameter estimation and through model selection, model averaging, or robust decision making.

In the Fuster et al. setup, under RE the risk premium is very small. In the late 1990s some people argued (“Dow 30,000”) that the rise of the stock market was due to a recognition that the market risk premium was too high. An implication of Fuster et al. is that this view is fundamentally correct. Is it not, however, more plausible to believe that the risk premium reflects the uncertainty that economists and agents share?

## Conclusions

Although I have indicated a number of reservations, overall I find the Fuster et al. story very attractive. The setup is conceptually simple, and it is based on the plausible premise that agents underparameterize their forecasting model. This is in line with standard econometric advice to estimate parsimonious models, as well as evidence from psychology that people are inclined to make decisions based on simple heuristics. The Fuster et al. model is disciplined and delivers a number of important empirical implications that appear to be in line with the data.

I would prefer to extend the model to include additional insights from the adaptive learning literature, but even as it stands the Fuster et al. model provides an impressive but simple benchmark model of asset-price behavior, which is sure to receive considerable attention.

## Endnotes

For acknowledgments, sources of research support, and disclosure of the authors’ material financial relationships, if any, please see <http://www.nber.org/chapters/c12406.ack>.

1. Actually, in Fuster et al. (2010) the term “natural expectations” is used to denote an average between RE and what is called NE in the current paper.
2. See Evans and Honkapohja (2009, 2011).

3. Closely related concepts are those of “self-confirming equilibria” and “consistent expectations equilibria.”

4. Estimating long-run persistence  $(1 - \Phi(1))^{-1}$  is equivalent to estimating the spectrum of  $d_t$  at zero, and is understood to be subject to great uncertainty.

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