Corporation Finance:
Risk and Investment

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All modern studies of investment decisions and their financing must build essentially upon Irving Fisher's *The Theory of Interest* [11], published more than a third of a century ago. As Kuh [22] has remarked, "the foundations of capital theory in its modern form have been best articulated [in this work]. . . . It is a theory at the level of microeconomic choice and at the level of total market price determination which has the major ingredients, correctly related to each other, that a capital theory should have." In this basic model of the capital markets, all the individual participants' perceptions of their "real" investment opportunities and their market opportunities to borrow or lend, on the one hand, and their "initial endowments" of income (or funds) and their personal time preferences (or utility functions), on the other, mutually interact to provide Pareto-optimal, stable equilibrium market prices (interest rates).

The theory assumed that "investment opportunities," along with utility functions and initial endowments, were given; its rigorous analysis was confined to comparative statics under certainty; and even in this context, it was at best ambiguous in multiple-period cases with non-constant interest rates (Hirshleifer [17]). Much work remained to be—and, happily, has since been—done on the rational derivation of relevant (real) "investment opportunity functions" from more basic considerations and data; on the dynamics of adjustment processes; and, quite

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¹ Numbers in brackets refer to Bibliography at end of paper.
recently, on the development of a rigorous analysis of the crucial role and effects of uncertainty on investment and financing decisions and on market equilibrium. But a granite cornerstone in the foundations of future work had been securely put in its place. Subsequent efforts ignore or disregard Fisher at their peril.

In this paper we undertake to identify the variables required in a structural equation to explain investment outlays. In section I we assume that Fisher’s perfectly competitive model under certainty is an adequate theoretical model of investment behavior. We examine the various forms of accelerator-capacity relationship in the light of certain real-world complications consistent with certainty of expectations, and point up additional variables required by Fisher’s framework even under the restrictions involved in the assumption of certainty.

In section II, we examine the modifications required when the Fisher framework is extended to incorporate the fact of uncertainty. We retain under uncertainty the assumption that firms are optimizing at all times by making the decisions that will maximize the market value of their equity (given the level of “the market” as measured, say, by the Standard & Poor or Dow-Jones index); and continue to assume that all securities markets are purely competitive with no frictions or imperfections whatsoever. Nevertheless, to determine optimal capital outlays in this rigorous neo-Fisherian model, the fact of uncertainty is shown to require the inclusion of several financial and risk variables (such as leverage and retained funds) that would have no place at all if the world were really characterized by prescience so that the traditional certainty models would be adequate. The latter part of this section summarizes the author’s model of investment and financial policy under uncertainty. Using the assumption that bonds (as well as stocks) are risk assets to their owners, we show that investment is inversely related to leverage—in contrast to the implications of the MM model based on an assumption that corporate debt is a riskless asset to its owners.

Section III of the paper then specifies a statistical model of investment outlays based on the foregoing theoretical-economic model; briefly discusses certain important problems of estimation, and of identification, in working with data generated under conditions of uncertainty; and presents the empirical results of applying the model to explain the plant and equipment expenditures of manufacturing corporations over the period 1953-63. Not only are correlations unusually high, but t-ratios are uniformly good, and forecasts into 1964 are reassuring.
I. Analysis Under Certainty

THE ROLE OF ACCELERATOR-CAPACITY VARIABLES IN THE FISHER MODEL UNDER CERTAINTY

The initial formulation of the accelerator relationship held that net investment (change in capital stock) would not only be proportional to the actual current change in output,

\[ I_t = K \Delta O_t \]  

(1)

but also that its elasticity to capital stock would be unity. Even if we retain Fisher's classical assumption of certainty for the nonce and also defer our consideration of all factors impounded in ceteris paribus, it is clear that reliance on this rigid form of the accelerator involves accepting certain underlying assumptions about the factual situation. In particular, even with other things equal under certainty, this particular formulation assumes that (a) production functions are homogeneous of order one, (b) the existing capital stock of potential buyers of plant and equipment is at the desired level at the beginning of the period, (c) suppliers of capital goods (in contrast) have sufficient excess capacity to meet all demands promptly at existing prices, and (d) the elasticity of expectations of capital goods buyers is zero.

Some of these restrictive assumptions are more serious than others. The linear homogeneity of production functions is probably an adequate approximation for present purposes. Also, lack of excess capacity on the part of suppliers and long production periods for capital goods merely serve to introduce a distributed lag of actual investment expenditures trailing the initiating change in outputs. (See Eisner and Strotz [10].) While this reduces the short-run elasticities, the long-run elasticity of capital to output would still be unity if the other relevant conditions were satisfied. If the elasticity of expectations of capital goods buyers is greater than zero, the elasticity of capital stocks to output will generally be increased and a variable measuring growth in output may need to be added.

If the remaining critical assumption is not accepted, more substantial modifications are required. It has long been recognized (indeed, since 1917 by J. M. Clark himself) that equation (1) will not hold whenever the existing capital stocks of the potential buyers of plant and equipment were already in excess of desired levels. An alternative formulation of the basic "acceleration" approach, free of this defect, substitutes an assumption that investment will be determined by the discrepancy between the (known) current capital stock and a desired level which,
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in turn, is assumed proportional to (actual or expected) output. A still more flexible formulation of this "capacity" version of the "acceleration family" of models introduces a distributed lag of actual investment, after assuming realistically that only some fraction of an existing discrepancy between desired and actual capital can be (or will be) acquired in any particular time interval. This more flexible version of the general "capacity" variant of the acceleration family of models thus relies on an equation of the form:

\[ I_t = \gamma (BO_t - K_{t-1}), \]

where \( \gamma \) is an adjustment coefficient equal to the fraction of the "capacity gap" eliminated per period, and \( B \) is the fixed (desired) capital-output ratio.

Still more flexible versions of the accelerator can, of course, be obtained by dropping the a priori requirement that the capital-output elasticity coefficient \( K \) in equation (1) is unity, and by combining this more flexible version of the initial accelerator model with its capacity variant, to specify investment as a function of the level of output, change in output, and the capital stock at the beginning of the period:

\[ I_t = f(O_t, \Delta O_t, K_s). \]  

(3)

The relation between current output and beginning capital stock may, of course, be subsumed in some measure of the utilization of capacity along the lines introduced by de Leeuw [4] and others.

OTHER FACTORS REQUIRED EVEN UNDER CERTAINTY AND PERFECT CAPITAL MARKETS

It is clear that at least the more flexible forms of accelerator relationships form an essential part of any valid theory of investment. They represent essential determinants of the position—and of the shifts in the position—of Fisher's investment opportunity function. But quite apart from considerations introduced by the fact of uncertainty, it is clear that in the context of Fisher's basic framework, even the most sophisticated and flexible versions of a generalized "accelerator principle"—such as equation (3)—would a priori be expected to be an adequate representation of the true structure of empirical investment behavior only under severely restrictive ceteris paribus assumptions. If the ceteris of these accelerator-capacity formulations are not, in fact, paribus to an acceptable degree of approximation, reliance on these models as an adequate representation of the structure of investment decision-making behavior would involve a certain degree of myopia.
Many more elements in the economy were allowed—and expected—to vary in Fisher's model than in even the most flexible version of the accelerator genus. Fisher's model of the real investment and financial sectors was (implicitly to be sure) embedded in a general equilibrium model of the entire economy including factor and product markets. While the immediately relevant product prices and factor costs were taken to be exogenously given at any one time, they were determined by interactions with the rest of the economy and subject to change over time. Changes in wage rates relative to labor productivity, for instance, or changes in the efficiency of economic life or other factors affecting the real cost of physical capital will directly affect the position and shape of the investment opportunity function, even if all accelerator-capacity factors were unchanged. Similarly, changes in marginal labor costs relative to marginal real capital costs per unit of output, by inducing substitutions of capital for labor along a given production function, will shift the position of the investment opportunity function even under certainty. If data on these factors have in fact been changing in the real world over the time period of interest, adherence to Fisher's model would require that such variables be added to the list of accelerator-capacity variables in statistical analyses of investment behavior.

Correspondingly, in Fisher's framework—even under conditions of prescient certainty and blissfully perfect markets throughout—the position and shape of the investment opportunity function determine the amount of investment only in conjunction with the line of "financial market opportunities." As is well known, the amount of investment is determined by moving along the (real) investment opportunity function to the point at which net increments in present value are no longer positive—in the continuous case, the point of tangency between the real investment opportunity function and the financial market opportunity line. Fisher was as sure that a change in the market interest rate—the relevant "cost of funds" in his model under certainty—would change the amount of investment without changing the position or shape of the real investment opportunity function as he was that a change in the latter would alter the amount of real investment when market opportunity lines and the rate of interest were unchanged.

To rely on traditional accelerator-type models as complete explanations of real investment behavior would, in the strictest theoretical terms, be equivalent to relying on one blade of Marshall's famous scissors. To rely on unaugmented accelerator-type models as an empirically adequate representation of real investment behavior would be to act upon a presumption that changes in the financial markets, in financial (as
opposed to real) investment opportunities, and in the explicit (or implicit “opportunity”) cost of funds, are uniformly ignored by all business decision-makers, and thereby to deny the relevance of “the main body of the maximization principle in economic theory.” In the specification of the statistical models fitted in the latter part of this paper, I consequently include variables for the labor and (real) capital costs of output, for output growth, and for financial costs (interest rates), along with the levels and change in level of output, capital stock, and capacity utilization.

Our structural equations must include all these variables if they are to be consistent with the basic Fisher framework of investment decisions and capital markets. (Whether the effects of all these variables come through loud and clear in the face of uncertainty and noise in the data is, of course, an empirical question which we defer to the latter parts of this paper.) Although our list of variables is already substantially longer than called for by accelerator-capacity models, it should be emphasized that up to this point our analysis has been conducted entirely within Fisher's own assumptions of perfect certainty and perfectly competitive market structures. For this reason, it will be noted, we have as yet not introduced certain other variables often included in investment equations—notably, current (or lagged) dollar profits or retained funds, current (or lagged) profit rates, initial endowments of funds, or assets. Within Fisher's idealized world of perfect competition and certainty, these factors affect the amounts of borrowing and lending and thereby the level of interest rates; but, given the level of interest rates, they do not affect the amount of investment, and with certainty none of them would affect the expectations of future profitability on incremental real investments incorporated in the investment opportunity function.

Before turning to the effects of uncertainty, there is one further real-world factor which must be considered briefly. Fisher did not develop the effects of corporate taxes upon optimal investment criteria, but the existence of a modern corporate tax structure is perfectly consistent with standard assumptions of certainty. Jorgenson's recent work has elegantly filled in this gap in the theory. By maximizing net worth subject to a general neoclassical production function and the constraint that the growth in capital stock is investment less replacement, he shows [19] that the marginal productivity of capital is equal to the ratio of the user cost of capital to the price of output, where the user cost in turn is proportional to the cost price of capital goods. The factor of proportionality is a weighted sum of the depreciation rate and the market

2 The slightly ungrammatical phrase is taken from Eisner [9], p. 139.
interest rate. Both weights involve the corporate tax rate. The other factor in the depreciation term is the proportion of replacement chargeable against income for tax purposes, and the other factor affecting the impact of the interest rate is the proportion of interest charges in total financial costs. Because of the latter term, leverage would affect optimal investment decisions, even in a world of certainty whenever there was a modern corporate income tax and, because of the favored treatment of the interest expense in such a tax, both the optimal capital stock and the level of current investment would vary directly with leverage. The Jorgenson marginal productivity of capital term, involving the appropriate combination of tax, interest and depreciation rates, and leverage, is included in our later statistical analysis.

II. Analysis Under Uncertainty

THE FORMATIONS OF EXPECTATIONS:
MODIFICATIONS AND ADDITIONAL FACTORS
SUGGESTED BY THE FACT OF UNCERTAINTY

All the factors which would affect investment decisions under idealized conditions of certainty obviously must be included in the structural specification of investment behavior under uncertainty. The variables representing these factors may or may not be different under uncertainty: such items as beginning-of-period capital stock or interest rates can be introduced as known data; the variables representing other such crucially important elements as outputs may need to be modified or supplemented because it is the uncertain expectations of the future output levels (and associated operating costs) that determine the expectations of future profitability which directly affect current investment decisions.

Unfortunately, we have relatively little firm knowledge of the way such expectations of uncertain future outcomes are formed. Presumably they reflect some amalgam of past experience adjusted by new information. As an empirical matter, it has become common to assume (following the lead of Friedman [13]) that these judgments of the relevant future magnitudes reflect current judgments of "permanent" or "true" output levels, which are free of the random or transient components in the unfiltered and unadjusted raw data on current outputs. In this

3 As shown in [19], p. 249, the optimal capital stock, given output and sales prices, is inversely proportional to the user cost, which in turn varies inversely with leverage, ceteris paribus. Jorgenson's cost variable is described in more detail in footnote 33 below.
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approach, it is generally considered reasonable to believe that, at least to a first-order approximation, these current estimates of “sustainable” outputs are formed in terms of a simple “learning” theory that reduces arithmetically to an exponential smoothing (or some similar weighted average) of past observed data on outputs. It may be noted, however, that this approach presumes that decisions are made on the basis of “point estimates” of future data, whereas all the modern neo-Bayesian economic-statistical theory of rational decision-making under uncertainty argues the relevance and importance of the entire probability distribution except in special cases. Since it is very unlikely that corporations (or their shareholders!) are “linear in money” with respect to their capital budgets and since relevant “loss functions” are surely not symmetrical, nor quadratic utilities adequate to reflect preferences [27, p. 18] and [35], it seems clear that some variable measuring risk or degree of uncertainty may well be required along with measures of expected values.

It might have been argued that, even in a world of certainty, the level of orders and of order-backlogs would be as useful a measure (particularly in durable goods industries) as current output levels in judging future sales. When uncertainty is admitted into the analysis, their a priori relevance is greatly strengthened, for these are current data which bear directly on the future. Moreover, changes and trends in the level of orders and order-backlogs can probably be expected to have as much or more expectational significance as changes and trends in outputs; and, as Hart [15] has recently proposed, the level of the ratio of orders to capacity may play an even more critical role in the formation of the expectations which determine investment decisions.

The presence of uncertainty also suggests the need for still other variables to measure the expectations that are relevant to investment decisions. In a maximizing world, investment decisions depend upon estimates of future profitability, which in turn depend upon operating costs and sales prices as well as output levels. In a world of certainty, future data on each would be known. Judgments of each are required in the real world of uncertainty. The current levels of (and recent changes and trends in) the ratio of product prices to unit labor costs, and in the ratio of product prices to the costs of real capital, thus properly belong in the specification of determinants of investment outlays. Alternatively, current levels, changes, and trends in profit margins may be used. As in the case of the judgment of future output levels, there are important empirical questions regarding the way in which these expectations are formed and how much effect they have in practice.
Nevertheless, the expectations of future profit margins are, in principle, relevant on a par with expectations of output levels.

THE IMPACT OF FINANCE AND LIQUIDITY ON INVESTMENT OUTLAYS UNDER UNCERTAINTY

Up to this point—except for the brief comment on the possible relevance of a risk or dispersion variable in output and margin expectations—our discussion of the effects of uncertainty has pointed up the probable need for different or additional variables to capture the future expectations relevant to investment decisions under uncertainty. These modifications or additions were all required to capture under uncertainty the expectations that managements hold with respect to the specific determining variables which we saw would determine investment decisions if the future were known exactly in advance. With the exceptions noted, the modifications and additions have related to the same list of determining factors that would be operative under certainty.

The changes and additions required in the finance sector of the model by the fact of uncertainty are much more fundamental. Under certainty, only the risk-free interest rate (adjusted for corporate taxes, if any) and the "tax shield" of depreciation allowances would be relevant to investment decisions. Current profits, dividends, retained funds, current assets or liabilities, funded debt, and net worth would—in strict theory—all be completely irrelevant. But once uncertainty—and certain other facts of life which would be of no consequence were it not for the fact of uncertainty—are admitted into the analysis, the situation is basically altered.

Suppose, for the moment, that there were no taxes and financial markets were really such that the Modigliani-Miller fundamental proposition [31], [32], and [33] was valid. The aggregate market value of all the securities (stocks and bonds combined) issued by any corporation would be independent of its debt-equity ratio or, more generally, of the mixture of claims upon its cash flows. This aggregate market value in turn would depend upon both the expected value and the risks associated with the corporations’ cash flows (before interest charges), and upon expectations of their future growth as a result of further investment outlays. In this simplified world without taxes, the minimum marginal expected return required to justify new investment (the company’s “cost of capital”) would be independent of its existing capital structure and of the mix of fund sources utilized to finance its current (or future) capital budgets, but, even so, it would depend upon risk as well as upon the pure interest rate.
The introduction of corporate taxes in this model, of course, makes the relevant cost of capital depend upon tax rates and leverage as well. But in this framework, with corporate debt assumed to be riskless, corporate taxes make the market value of the corporate entity a necessarily increasing function of the degree of leverage. Rather remarkably, therefore, within the MM model addressed to an uncertain world, the relevant cost of capital is inferred to be a declining function of the degree of leverage, just as it is implicitly in Jorgenson's model derived without introducing uncertainty. Within the MM framework, the more leverage a company has, the further down its marginal efficiency of capital schedule it can profitably and properly move—i.e., the larger the fraction of the projects submitted to its capital budgeting committee which should be approved. This MM model thus predicts a positive association between leverage and investment outlays under uncertainty, just as under certainty. This prediction is, of course, contrary to what many of us have observed in the field interviewing and working with corporate financial officers; it also does not accord with some recent statistical evidence [36]. The proposition is further tested later in this paper.

This surprising prediction, of course, follows logically from the rather severe assumptions of the MM model. But if, instead, one accepts the assumption made in my own work that corporate bonds as well as corporate stocks are risky assets to their owners and if, in particular, it is recognized and consequently assumed that, for any given stochastic EBIT stream, an increase in fixed financial charges exponentially increases the probability of default and risks of loss on forced refinancing, thereby reducing expected returns by more than the mere interest charge on the added debt—then the troublesome inferences of the MM model no longer follow in strict theory under uncertainty, even within purely competitive markets with no imperfection whatsoever.

In my own theoretical work, I have assumed that all risk assets are traded in a single purely competitive capital market in which all investors are risk-avers with explicit joint (Gaussian or log-Gaussian) probability distributions over the end-of-period outcomes of the n risk-assets in the market. By introducing these additional assumptions into Fisher's model of capital market equilibrium under certainty, the equilibrium vector of market values of all risk-assets are simultaneously determined. (See Lintner [27] and [28].) Within this Fisherian model adopted to

Note from equation (8) in [33], p. 442, that the required return net of tax declines linearly with leverage.

Presumably institutional restrictions take over before leverage approaches 100 per cent, but the model is silent on how or where.
uncertainty, I then regard the $i^{th}$ company's bonds and its stock as different risk assets (each with its own vector of attributes) being traded in the perfectly fluid and purely competitive market of all stocks and bonds. I allow for the fact that the risks of holding corporate bonds and stocks (even of the same company) differ substantially because they have different variances and because their returns are subject (to at least a significant degree) to different influences, including different intercorrelations with other risk assets and with different exogenous variables.

In this Fisher framework adapted to uncertainty, the market values of the $i^{th}$ company's bonds and of its stock will each reflect its own vector of attributes (including its own row or column of the master variance-covariance matrix). In particular, the differential effect on the market value of a company's common stock of a given action by the company is not determined by first finding its differential effect upon the aggregated "entity" value of the company and then merely subtracting bond values (as for MM), but rather is determined directly as its effect on the common stock itself. (Any action by a company, of course, affects both the market value of its bonds and of its equity in the competitive equilibrium, but the effect of the action shareholder's welfare is measured directly by the latter.)

Within this model (even with the vector of all future investment outlays and "earnings" invariant), maximization of equity value occurs with positive leverage (except in limiting cases of a priori low relevance) and the degree of optimal leverage is a unique function of the set of parameter values pertaining to any given case. Apart from the effects of taxes, the minimum marginal expectation of return on new real investment which will justify the issuance of debt for (some or all of) its financing—the marginal "cost of debt capital"—is a continuously rising function of the amount of debt or the degree of leverage. (This is true even in the simpler versions of the model in which the interest is not assumed to be a rising function of leverage. The introduction of rising interest costs merely compounds the effect.) This result reflects both the marginal effect of leverage on the expectations of earnings before interest charges noted above, as well as its more commonly recognized impact on variances of rates of returns. Both these considerations are directly attributable to the fact of uncertainty as such, and both are cumulative in their effect (i.e., both have positive second derivatives with leverage). In contrast, the favorable treatment accorded

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*For a more precise statement, see also item (d) on p. 227 below.*
interest expense in the tax laws—a factor common to the certainty case and our model under uncertainty—provides at most a linear offset. In the presence of a modern corporate tax structure, the marginal cost of debt capital under uncertainty must consequently be an orthodox U-shaped function, rising beyond some moderate amount of leverage. (This is true, incidentally, in both the short and long run.)

It follows immediately that, given any dividend or retention policy, there is an optimal degree of leverage. I have shown elsewhere [25] that, in the Fisher model of capital markets adapted to uncertainty, there is an optimal dividend payout policy, given probabilistic investment opportunities (and any given level of leverage). A fortiori, there is an optimal mix of retentions and leverage for each company in the market.

In finding what this optimum is under any given set of expectations, the firm is assumed to act in accordance with standard maximizing principles: at any point along the “expansion path” of standard production theory (i.e., as the tentative size of the firm’s capital budget is increased), it equates the marginal (not the average) cost of each source, retentions and debt. If the firm is at a point on its “funds-for-investment-outlays” production function which is not on the “expansion path,” these marginal costs in turn get equated by its using the cheaper source until it is no longer cheaper. Now for any given retention ratio, the marginal costs of debt are U-shaped when plotted against leverage (as noted above); but an optimizing firm will always be operating on the rising, right-hand side of this curve. The reason is simply that the marginal costs of retention are themselves a continuously rising function of the degree of leverage with which the retentions are associated. Since the maximizing firm uses debt instead of retentions as long as debt is cheaper, it will continue to use debt until marginal debt costs have become as large as marginal retention costs.

It must also be noted that these marginal debt costs are necessarily larger than the yield on the outstanding debt for four compounding

7 In this paper for simplicity, I am confining the analysis to the joint optimization of dividends-retentions policy, debt, and real investment. (See also footnote 18.) It should be noted, however, that there will also be an optimal dividends-retention policy vis à vis common stock financing as a result of differentials in tax rates on ordinary income and capital gains (as pointed out in both Modigliani and Miller [32] and Lintner [24]) if investor’s incomes differ, even if their probability distributions are identical and there are no issue or transfer costs. Relaxing the latter assumptions also introduces definite investor preferences for dividend policy even in the absence of taxes (see Lintner [24]).

8 The average is acceptable if and only if it is equal to the marginal cost.
reasons.\textsuperscript{9} The yield on outstanding debt is in the nature of an average return to the bond owner; the relevant costs to the company and its shareholders are higher\textsuperscript{10} because: (a) issue costs are often relatively substantial compared with buyer yields at the time of issue; (b) increasing leverage, other things being equal, increases lender's risks, which results (even in purely competitive markets with no imperfections) in interest rates that are an increasing function of borrower's leverage and which makes the relevant marginal cash flow costs to the borrower higher than the stated interest charge on the immediate increment of debt (this difference between the relevant marginal and the average interest cost is itself a rapidly increasing function of leverage); (c) as noted earlier, for any given stochastic EBIT stream, an increase in fixed financial charges exponentially increases the risks of default and risks of loss to the company and its shareholders on forced refinformings, thereby reducing expected returns by more than even the relevant issue and marginal interest charges on the added debt; and, finally, (d) increasing leverage increases the variance of the distribution of the present values\textsuperscript{11} of the relevant cash flows (associated with any given stochastic EBIT stream which reflects a given vector of investments) which crucially affect the current market values of equities in a purely

\textsuperscript{9} See footnote 12.

\textsuperscript{10} Because of the prevalence (among economists and elsewhere) of the belief that a "weighted average cost of capital" rule is appropriate under uncertainty, we should note that the "weighted average cost of capital" is usually a weighted average of the earnings yield on the stock and the market yield (or coupon rate) on debt (using equity and debt as weights). In terms of our model, this weighted average alternative is too low because (1) the relevant debt cost is necessarily (and usually very much) greater than the average debt yield used in the weighted average formula, and (2) the cost of retentions or new issues for equity is also higher than the earnings yield whenever we allow for the facts that the future is uncertain, that we know still less about the more distant future, and that most investors are "risk averters." (See Lintner [25].) In sum, the weighted average cost usually used is too low because, in the face of the uncertainties of life, it understates the required returns for the use of equity capital, and it doubly understates the marginal expected returns which must be required to justify the use of debt. Moreover, these understatements are mutually reinforcing in the sense that the greater the "spread" on the equity side (i.e., the difference between the marginal rates which are really required to justify the use of equity and the current earnings yield of the stock), the greater will be the corresponding spread (above marginal cash costs on a yield basis) on the debt, and vice versa.

\textsuperscript{11} Alternatively, and in more traditional language, we find that increased leverage increases the variance of profit rates, and a fortiori the variances of prospective growth rates. (Stock prices were shown to be inversely related to the variance of growth rates in Lintner [25].) The marginal prospective return on new investment financed with debt must be enough higher than the combined costs of (a), (b) and (c) above to offset this further variance effect on stock values.
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competitive capital market. (See Lintner [27].) The third and fourth factors together determine the relevant risk premium involved in debt financing in excess of issue costs and its relevant marginal cash costs.\(^{12}\)

The maximizing firm will increase the size of its capital budget until the marginal expected return no longer exceeds the marginal cost of capital along its expansion path—i.e., the equalized marginal costs of debt and of retentions. Under uncertainty, this optimal intersection of “m.e.c.” and “m.c.c.” will occur where the marginal cost curve is rising with increased budget size. It follows that the marginal cost of added retentions and of added leverage are not only equal but rising at the equilibrium point.\(^{13}\)

Finally, a distinction between long-run and short-run equilibrium is required. So far in my discussion of the derivations of the costs of capital and the optimal finance mix, I have implicitly been discussing the analogue under uncertainty of the “long-run equilibrium” analysis of the traditional “comparative statics (or dynamics)” under certainty. I firmly believe—and always have\(^{14}\)—that (target) dividend payout ratios (or retention rates), target or ex ante maximal debt-equity ratios, and capital budgeting criteria in the long-run planning context are mutually determined. Other things being given, relatively greater volumes of more promising and profitable investment opportunities on the average (or expected over the long pull) lead to greater retentions (lower dividend payouts) and greater leverage; while expected values pertaining to investments being the same, greater “business risks” or ex ante variances in average and marginal EBIT streams lead to lower retentions (higher cash dividend payouts) and lower leverage. In this way, ceteris paribus, the target dividend payout ratio and the ex ante expected average (or ex ante maximal) debt ratios are mutually determined along with the “cost of capital” (used to determine the optimal ex ante size of capital budgets) in terms of long-run expectations (in the economist’s sense).\(^{15}\) These long-run expectations, in turn, involve judgments

\(^{12}\) It should be noted that the sizes of (c) and (d) are increasing functions of factors (a) and (b); (c) and (d) themselves are additive; and (b) will also be an increasing function of the combined effect of (c) and (d).

\(^{13}\) For the no leverage case, see Models II–VI in Lintner [25]. The same conclusion that the marginal cost of retentions is necessarily rising at the optimum, of course, holds as well for any given degree of leverage. The costs of leverage are also rising at the optimum, since they are rising throughout the relevant range, ceteris paribus.

\(^{14}\) See my [23] and [26].

\(^{15}\) Liquidity considerations may also be included in this simultaneous long-run ex ante optimization along the lines laid out in Anderson [2], Chap. III.
of the company's investment opportunity functions and the operating characteristics of the business, along with the characteristics of competitive reactions in the firm's product and factor markets and the other properties of the stochastic processes within which it lives.

In terms of shorter-run equilibria, however, the situation is basically different in one important respect. Although the target dividend payout ratios (as well as the debt-equity ratios and criteria for determining the size of the capital budget) are simultaneously determined in the theoretical context of a comparative dynamic, stochastic, long-run equilibrium analysis, along with somewhat more flexible "speed-of-adjustment" coefficients\(^{16}\) (to control the response of cash dividends to cyclical and stochastic changes in earnings), cash dividend payments in line with these predetermined standards constitute a top priority claim on funds during short-run fluctuations. Given the fact of substantial uncertainty, the assumption that companies are seeking to maximize the value of their equity, the crucial dependence of market values on shareholders expectations, and the extraordinarily high "information content" of changes in dividend payments in the eyes of shareholders as they form their expectations—such action by companies is perfectly rational (i.e., maximizing behavior under uncertainty) in terms of strict theory. There is also substantial evidence that this theoretical formulation describes actual practice with respect to dividend payments very well.\(^{17}\)

In our theoretical model, consequently, short-run optimization occurs subject to the constraint of dividend outflows at each point in time in accordance with the established dividend policy (except for infrequent

\(^{16}\) These standards for speed-of-adjustment factors within each company are established in a similar manner on the basis of the mutual interrelationships expected between the volume of profitable investment opportunities and other parts of the sources and uses of funds flows over the cycle. For a fuller discussion of dividend determination, see Lintner [23] and [26]. It should be emphasized that the theoretical (and practical) conclusions drawn in the text here depend only on company's adherence to an established dividend policy; they do not depend upon the ultimate resolution of any differences there may be on whether the policy itself is denominated in terms of reported "earnings" or cash flows, for instance, as Brittain [3] has concluded.

\(^{17}\) See, for instance, the three references in the preceding footnote. Important additional confirmation is provided by Glauber [14]. In a canonical correlation analysis of all sources and uses of funds of forty-four chemical companies in three postwar years of extreme expansions and three years of contraction, he found that the dividend earnings relationship was uniformly the first and dominant factor in the analysis; it was also the most stable. Subject to this orthogonal factor, the residuals revealed a complex interaction among working capital, real investment, and liquidity decisions.
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crisis situations requiring immediate re-examination of the company's entire set of long-run expectations and policies). As in the traditional short-run theory of the firm (in which price and volume decisions are taken subject to the constraint of a fixed stock of real capital), our model of short-run adjustments assumes that firms simultaneously adjust and optimize over time the volume of their real investment, the amount (and forms) of their outside financing (if any), and their working capital and liquidity positions, all subject to predetermined requirements for dividend payments.

In this short-run context (quarter to quarter or year to year), the cost-of-funds factor in investment decisions works out as follows. The cost of the debt used will clearly be equal to its marginal costs, which in the relevant range, it will be recalled, rises with the degree of leverage in an essentially exponential manner, due not only to the rising marginal explicit cash costs of debt but even more to the rapidly increasing additional risk premiums required because of the greater "borrower's risk" being borne as leverage increases. Further, the opportunity cost of using some more retained funds to increase the current size of the capital budget is the foregone opportunity to repay debt, as Duesenberry [6] has emphasized. The cost of retained funds for investment is thus also given by the marginal cost-of-debt function. The cost of using either debt or retained funds to finance investments is thus higher than the quoted interest rate by amounts that increase rapidly with leverage.18

Note particularly that this cost of using either debt or retained funds to finance investment will, other things being the same, vary inversely with the amount of retained funds.19 For any given level of investment outlay, the availability of more retained funds enables the firm to have less debt outstanding at the end of the period. Alternatively, we can say that more retained funds permit the financing of a given level of invest-

18 Space does not permit any detailed discussion of the place of common stock financing in the model. Suffice it to say that such issues will be made only at such times as the (essentially long-term) standards of acceptable debt and liquidity positions are being strained by current (and prospective) profitable investment outlays and when market conditions are sufficiently favorable to bring the full costs of a new equity issue down to or below the costs of other funds (or of foregone investment). With the principal exceptions of utilities and small, rapidly growing, ambitious firms, such conditions apparently occur infrequently, and we will simply assume here that the costs of equity are equal to or greater than the cost of debt.

19 Note also that, given dividend policy, the relevant cost of funds in the short run also varies inversely with the amount of dollar profits. See following comment in the text.
ment at lower marginal costs than would otherwise be possible.\textsuperscript{20} It also follows that, for any given size (or schedule) of marginal costs of debt capital, the optimal scale of investment outlays will be an increasing function of the amount of retained funds currently available.\textsuperscript{21}

Given uncertainty and risk aversion on the part of shareholders, this conclusion that, other things being equal, the availability of greater retained funds should increase the real investment outlays of optimizing decision-makers is rigorously implied by the theoretical model. This is true without introducing tax considerations or any imperfections whatsoever (not even transactions costs), and it is true under uncertainty \textit{cum} risk aversion even though the result is foreign to classical doctrine based on the traditional assumption of prescience. Uncertainty and risk aversion do make a big difference, in theory as well as in practice!\textsuperscript{22}

Not only are profits and flow variables (which would be irrelevant under certainty) required as legitimate and essential elements in the neo-Fisherian theory of the structural determinants of optimal investment outlays under uncertainty; but entirely new content and values are introduced into the relevant “interest rate” variables.

\textsuperscript{20} The value to the firm of the availability of some finite increment of retained funds is thus \textit{greater} than indicated by the marginal cost of currently outstanding debt. For it is equal to the (integral of the) marginal costs of the \textit{larger} amount of debt that would have been outstanding had the marginal retained earnings not been available, and the relevant debt cost function is highly concave upward for reasons already given.

To avoid misunderstanding, the reader should keep in mind that we are here in a short-run context—the values involved are the \textit{shadow prices} of existing supplies of retained earnings; except within the limits of flexibility in “speed-of-adjustment” factors in dividend policy (this flexibility in practice is largely confined to “rounding up” or “rounding down” to even nickels or dimes in per-share dividends) we are not referring to adjustments in the amounts of retained earnings available.

\textsuperscript{21} This analysis has ignored issue costs, which make the marginal costs of increases in debt greater than those marginal savings of reductions in debt. The analysis has also assumed that managements' judgments and reactions are symmetrical with respect to the effects of increases or decreases in current debt levels on \textit{future} borrowing costs and “safety margins” or “flexibility.” The effect of all such considerations would be to make the favorable marginal effect of retained funds on investment outlays greater when they were short of capital requirements than when they were in excess of the volume of otherwise profitable investments.

\textsuperscript{22} In the same connection, the reader will recall earlier conclusions regarding the effects of leverage. The presence of uncertainty involves cumulatively rising costs which convert the linearly declining marginal cost of debt function (found with taxes under certainty) into a U-shaped curve; and optimizing firms will necessarily operate on the \textit{rising} portion of the curve.
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III. Statistical Analysis

In this section we present the results of a statistical analysis of the determinants of the plant and equipment outlays of all U.S. manufacturing companies over the forty-four quarters in 1953–63. The analysis, incorporating accelerator, financial, and risk variables, is based directly on the theory developed in the previous section. All financial data used were drawn from the F.T.C.-S.E.C. Quarterly Financial Reports, except that interest rates were taken from the Federal Reserve Bulletin and Standard and Poor's data on stock prices and seasonally adjusted earnings were used. Hickman's careful estimates [16] of constant dollar capital stocks and quarterly interpolations of his annual estimates of declining-balance depreciation rates provided the data for these series. Output was measured by the Federal Reserve Board index of manufacturing production, and capacity utilization by de Leeuw's index. Hart kindly made his data on the orders-to-capacity ratio available, and these were supplemented with O.B.E. data on backlogs. The series on labor cost per unit of output was taken from the Bureau of the Census Business Cycle Developments.

Several essentially statistical problems must be considered before we specify the particular models fitted and present the results. The first problem arises from the fact that investment decisions taken at any point in time—on the basis of the expectations as of that point in time and hence upon current and past data then available—result in expenditures over a succession of future periods. From the work of Koyck [21], Solow [36] and Jorgenson [19], we know that, in principle, the form of this distributed lag can be determined by including one or two lagged values of the dependent variable as explanatory variables in the equation being fitted. But given the character of the experiments which have been run for us in the real world and the available collinear data, these procedures run the serious risk that the autoregressive properties of the dependent variable itself will swamp the effects of other explanatory variables. In this event—even though very high $R^2$'s, good Durbin-Watson (D.W.) ratios, and low standard errors of estimate are obtained as a matter of course—the estimates obtained are essentially those of a pure autoregressive model, and the parameter values for the other variables of more theoretical interest can be quite unstable and misleading. In particular, if by chance the estimates obtained for the auto-

23 As an illustration, see comments of Jorgenson in [20], commenting on his own model.
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regressive terms differ from their true values, the estimates for the structural decision variables will almost inevitably be biased. Fortunately, other alternatives are available.\textsuperscript{24} The one utilized in the work reported here was the use of the independent estimates of the lag structure of manufacturers' investment outlays behind their appropriations developed in Almon's recent study [1].\textsuperscript{25} Use of these exogenous estimates of the relevant lag structure enables us to confine the list of explanatory variables in our equations to those representing the determinants of investment decisions, and thereby to avoid the ambiguities and risks involved in the use of lagged investment terms.

A somewhat similar problem is imposed by the common time trends in much of the data. Apart from the use of cross sections with which we are not now concerned, one quite effective device for handling this problem—much used in consumption studies following the pioneering work of Duesenberry [5] and Modigliani [30]—is, of course, to fit the model to the data in ratio form. This procedure was also followed in this paper. Specifically, investment outlays in real terms and all explanatory variables (not already in ratio form or having the dimensions of interest rates) were deflated by beginning-of-period capital stock.

The fact that the Almon weights represent the time pattern between appropriations and the subsequent expenditures on investment goods raises another troublesome issue.\textsuperscript{26} Clearly, the first period in Almon's lags dated from appropriations may not be the quarter in which the decision was taken, nor is it necessarily even the immediately following

\textsuperscript{24} See, for instance, Eisner [7] and de Leeuw [4]. The Eisner approach requires the use of large numbers of lags for each relevant explanatory variable; even when put in first difference form to reduce collinearities, the latter problem remains and the large number of lags in the explanatory variables rules out the approach as a practical matter in fitting the theory advanced in this paper. De Leeuw's approach of testing various previously defined weight patterns is directly used in this study, employing the Almon weights which were developed after de Leeuw's work.

\textsuperscript{25} These have also been used by Resek in [36].

\textsuperscript{26} There is clearly much to be said in favor of undertaking two separate studies, one to establish the set of factors which together determine investment decisions and their associated appropriations, the other to establish the structure and parameter values of the realization function relating the appropriated inputs (together with subsequent information and "surprises") to the later expenditures. Such research should surely be encouraged. It seemed, however, that the essential purposes of the present investigation could best be served by following the more usual practice of relating the theoretical model of decisions-to-invest directly to the resulting spending on investment goods. Within the restricted compass of the empirical work reported here, this was the primary tack taken. In the final stages of our work, however, some additional runs were made including measures of the current quarter's conditions, which we hoped would pick up cancellations and other effects of "surprises."
one. There are lags between decisions and appropriations, just as there are between the latter and actual spending. As in Anderson's earlier work [2], the appropriate time span between the conditions initiating decisions and the first period of spending was estimated empirically. Specifically, for the $i^{th}$ period of weighted-summed explanatory variables, the denominator of the dependent variable was beginning-of-year real capital stock, but differently dated numerators were tried. The dependent variable $I_{t+i}/K_{t-1}$ with $i = 1$ proved to be consistently superior to no lead ($i = 0$) or to a two- or three-period lead.

Another problem is reflected in the fact that all the initial "fits" to the data left unacceptably low Durbin-Watson statistics, generally in the range .4—.6. Much of this autocorrelation doubtless arises from the fact that Almon's best lag distribution for all manufacturing showed a D.W. statistic of only about .9 when investment outlays were predicted from appropriations. Further autocorrelation is introduced by our own treatment of the data. Our use of data with a given quarter's time index\textsuperscript{27} (as inputs into the Almon-weighted sums which are our explanatory variables) in effect assumes that these particular input data control the investment decisions made within the quarter. In point of fact, of course—as Hart [15] has aptly said—"deciding to invest is a process rather than an event." Consequently, it can be argued that the inputs into the Almon lags should themselves be moving averages rather than data for a single quarter. Our use of the latter inputs then involves using data with a form of autocorrelated "observational error."

Although autocorrelated residuals, as such, do not bias parameter estimates, they do reduce their efficiency. The use of lagged values of the dependent variable was ruled out for reasons given above.\textsuperscript{28} But the device recommended by Johnston [18] and Theil [39] to reduce autocorrelations and improve estimating efficiency was employed—all variables, dependent and independent, were converted to the form $X'_t = X_t - rX_{t-1}$. The residuals on the first pass suggested an $r$ of .8, which led to a substantial improvement. Successive recalculations using $r$ values from .4 through 1.0 indicated, however, that the optimum value

\textsuperscript{27} This does not, of course, mean that only data for the given quarter were used as inputs. For instance, the inputs into the Almon weights for, say, the second quarter of 1956 include the capacity utilization ratio, retained funds and leverage of that quarter, along with first differences as of that quarter, and lagged (i.e., beginning-of-quarter) interest rates, and so on. Also, exponentially smoothed values or trend values as of a given quarter were tested for some variables.

\textsuperscript{28} We should perhaps also note that use of the new unbiased estimating technique developed by Taylor and Wilson [38] was ruled out by the fact that the lagged dependent variable does not enter our model as an explanatory variable in its own right.
for \( r \) was .9, and this value was used in preparing the final estimates since it minimized both the standard error of estimate and the remaining autocorrelation, and did so consistently for the various equations presented below.

Finally, we have to recognize the fact that in a world of uncertainty, as noted earlier, rational investment decisions depend upon the \textit{expectations} (in the sense of economists) of future profitability, rather than upon known data on nonrandom outcomes. And there is as yet very little firm knowledge of how these expectations are generated.

Along with the current capital stock and real costs, the theoretical economic model tells us that expectations of such items as future outputs and profit margins are basic determinants of the position of the firm's real investment opportunity function; but economic theory does not tell us precisely what available numbers (reflecting current situations and past experience) are processed in precisely what ways to produce the inherently unobservable judgments regarding these theoretically relevant future magnitudes. Similarly, the theoretical economic model tells us that, in addition, financial and risk elements are required to determine the optimal point on the expected real investment opportunity function. It goes as far as to require the inclusion of interest rates, retained funds, and leverage (as a risk as well as a financial cost variable); but it does not tell us in advance, for instance, what particular form of the leverage variable will best measure the essentially subjective risk content of the variable. In the circumstances, one must experiment with real world series to see which ones (on standard statistical-econometric tests) perform best as empirical counterparts or representations of the theoretically required decision elements.29

Previous studies have established the great if not dominant role of capacity-accelerator considerations in empirical investment behavior,30 although it is not entirely clear which \textit{form} of the relationship is most significant and reliable. In view of the special focus of the present paper on the theoretical and empirical significance of long-term finance and risk considerations in investment decisions, we consequently structured our empirical work in the following way. Along with each of several

29 In part, this is because the \textit{economic} theory per se takes the primary probability judgments (like von Neumann-Morgenstein utility functions or the classical preference functions) as \textit{givens}, and merely derives optimal decision rules from these; in part, it is because the formation of these basic expectations doubtless involves personal and social psychological factors along with strictly economic considerations.

30 See Kuh's fine summary [22], especially pp. 264–267, and Meyer and Glauber [29].
standard (and some new) accelerator or capacity variables, interest rates and their smoothed first difference, as well as leverage (with and without allowance for retained funds), were added in different forms and combinations to the equations to be fitted. The resulting estimates of parameters and summary statistics were then examined to determine whether the finance and risk variables were consistently significant on standard statistical grounds regardless of the choice of variables to represent accelerator considerations. Given some uncertainty about the optimal choice of the latter variables, this research strategy clearly provides a stronger test of the sign and importance of the finance and risk variables of crucial interest to this paper than would results based on estimates of a single equation combining them with only one of the several plausible and theoretically motivated accelerator-capacity variables. Using two of the apparently better and more interesting accelerator-capacity variables, we also examined which representation of leverage seemed to produce the best over-all results. At a later stage, each of six variables representing real costs or profit margins in some form were then added to the best sets of variables from previous runs in order to determine the marginal effect of these further variables, but less systematic study was given to these variables in view of the focus of the present effort. Finally, since all the basic runs in the study were based on the use of data available at the decision-making stage to forecast subsequent investment outlays, some additional runs were made in the final stages of our work that also included measures of the current (i.e., outlay) quarter's conditions, which we hoped would pick up "surprises" and the falsifications of earlier expectations associated with cancellations. Apart from their own intrinsic interest, these runs provided a further test of the inherent stability and importance of the finance and risk considerations which are of primary concern to this study.

Although many of the input data were already in seasonally adjusted form, seasonal dummies were included in all runs. Although very small and uniformly insignificant (except for the second quarter), their continued inclusion represents an element of conservatism in the estimates of corrected $R^2$ and standard errors of estimate. As a general point of reference, it may also be noted that the total contribution of the three seasonal dummies to $R^2$ ranged narrowly about .035. In addition, time trends were included in all equations fitted in order to guard against the possibility that some of the other variables included were merely serving as proxies for excluded variables which were also associated

31 Since two needless degrees of freedom (for the third and fourth quarters) were allowed for in their calculation.
with "time." The contribution of the time trend variable to $R^2$ was generally less than the total for the seasonal dummies. This clearly reflected our earlier decision to estimate all equations in ratio form with $I_{t+1}/K_{t-1}$ as the dependent variable.

Before proceeding to the presentation and discussion of our detailed results, we should explain the symbols used and the measurement of the variables included in the statistical work and tables.

**SYMBOLS**

$I_t$ represents the seasonally adjusted plant and equipment outlays of manufacturing companies (S.E.C.-F.T.C. series), deflated by a price index constructed as a 2-1 weighted average of the Department of Commerce implicit GNP deflators for producers' durable equipment and nonresidential construction. $I_{t+1}$ indicates the value for the quarter following the reference date.

$K_t$ is a quarterly interpolation of Hickman's estimates of manufacturers' stocks of real capital in 1954 dollars. $K_{t-1}$ indicates the value at the beginning of the reference quarter.

$O_t$ is an unweighted average of the Federal Reserve Board's Index of Production for manufacturing over the three months in the indicated quarter.

$U_t$ is de Leeuw's index of the utilization of manufacturing capacity for the current quarter.

$(CR_t)$ represents capital requirements, $1.111 \cdot U_t - 1 + d_t$, where $d$ is the quarterly rate of (real) declining balance depreciation interpolated from Hickman's annual estimates. The first two terms give the percentage change in capacity needed to bring capacity to its optimum relation to present output (expressed as a percentage of present capacity) when the optimum rate of operations is taken to be 90 per cent. This is the measure of capital requirements used by de Leeuw in [4] without the inclusion of an additional term to reflect the needs associated with projected future growth.\(^3\)

$J$ is Jorgenson's ratio of the current price of output ($P_o$) to the (current dollar) user cost of capital services ($c$), which was computed according to Jorgenson's formulas and procedures as given in [19] and

\(^3\) Several experiments were run with "growth" included in a new $CR$ variable; but, with any of these "growth-CR" variables in the equation, the coefficient on the growth component entered (in addition) as a separate variable was always strongly negative. With the $CR$ variable defined as in the text, outlays reflecting steady expectations of growth will be reflected (with other things) in the constant term. I had no luck formulating a variable which would pick up varying expectations of growth.
It should particularly be noted that the user cost factor \(c\) is a function of the current quarter's effective income tax rate, and bookkeeping depreciation, as well as the price level of capital goods, the interest rate, and the debt-to-asset ratio. This \(J\) term was used as a multiplicative form with the above three accelerator or capacity variables.\(^{34}\) If, for instance, other things being equal, investment increases with capacity utilization \(U\), it is rational for the increment reflecting a favorable value of \(J\) to be larger as utilization is higher, and vice versa. Similarly with \(CR\). Indeed, the product term \(J(CR)\) is especially attractive inasmuch as it varies in the appropriate way the otherwise arbitrary assumption (used in computing \(CR\)) that the optimal capacity provides for a 90 per cent utilization ratio. In point of fact, the desired capital stock in relation to output will clearly be higher the more favorable the relation \(J\) between output prices and user costs of capital. Finally, we note that Jorgenson has shown \([19]\) that desired capital stock \(K^*\) will be proportional (in my notation) to the product \(J \cdot O\), say \(K^* = \alpha J O\). If investment is equal to a fraction \(\beta\) of the discrepancy between \(K^*\) and the existing capital stock, we have \(I_t = \beta(\alpha J O - K_{t-1})\) or \(I_t/K_{t-1} = \beta[\alpha J (O_t/K_{t-1}) - 1]\). This is the equation fitted in our work after reinterpreting the right side as inputs into the investment expenditures stream (i.e., applying Almon weights to the terms on the right-hand side) and allowing for the lag between decisions and initial outlays.

It should particularly be noted that Jorgenson's analysis was carried through under the assumption of certainty. His demonstration that the optimal capital stock is proportional to the product of \(J\) and an accelerator term suggests that we may regard the effects that interest rates, taxes, and leverage would have under certainty as being incorporated in such a "compound accelerator" term in our equations. Any further effects of interest rates and leverage variables, found when they are entered as

\(^{33}\) Ignoring capital gains (as he does), Jorgenson's formula for \(c\) is

\[c = q\left(\frac{1 - uw}{1 - u}\right) d + \left(\frac{1 - uw}{1 - u}\right) r,\]

where \(u\) is the ratio of direct taxes paid to corporate profits before taxes, the current quarter's effective tax rate; \(d\) is the declining balance depreciation rate; \(r\) is the interest rate; \(v\) is the ratio of capital consumption allowances currently taken to the current replacement cost of capital assets; and \(w\) is net monetary interest to total capital cost. Since Jorgenson is working in a world of certainty, total capital cost is the product of the interest rate and total capital so that \(w\) reduces to the debt-to-asset ratio; and current replacement cost is computed as the product \(dKq\), where \(q\) is the current price of capital goods.

\(^{34}\) Entering \(J\) (or its reciprocal) linearly in the equation did not result in significant coefficients, though the signs were generally appropriate.
additional separate variables in the equation, can thus reasonably be regarded as their impact upon risk under real world conditions of uncertainty.

*r* is the interest rate variable, measured by the average of the monthly Baa rates (Moody's) during the quarter, as given in the *Federal Reserve Bulletin*. This rate was used instead of the Aaa or long-term government bond rate generally used by other investigators because it provides a much better measure of the cost of funds to the average corporation. The Baa rate, of course, reflects both the current level of the fully risk-free rate and allowances for leaders' risk, but it reflects the costs to the borrowers which they expect with assurance to discharge. The latter is clearly the relevant matter for borrower's investment decisions, not the fully risk-free rate itself.

\( tr(\Delta r)_{t-1} \) is the trend in the Baa rate. This variable was measured by a weighted average of past quarterly changes in the Baa rate, using linearly declining weights for four quarters. With the Baa rate entered separately in the equation, this variable reflects expectations of costs of new funds and, in particular, it is an index of the difference between the average costs and the incremental costs of new debt.

*LTD* is long-term debt, measured by all debt due in more than one year to banks and other lenders, as shown in the F.T.C.-S.E.C. *Quarterly Financial Reports* for manufacturing corporations.

*RF* represents retained funds, measured by net profits after taxes, less cash dividends charged to surplus, plus depreciation and depletion.

*NW* is net worth, measured by total assets less total liabilities (current liabilities, plus LTD plus other noncurrent liabilities). The F.T.C.-S.E.C. reports do not segregate preferred stock, but this item is known to have been sufficiently small and stable as to introduce little bias.

*$K* is capital stock expressed in current dollars, computed by reflating Hickman's estimates of constant dollar capital stock by the price index of the prices of capital goods (indicated under *I* above).

*SP* is the market value of equity indexed by twice the Standard and Poor index of the market prices of 425 industrial securities (which is, of course, adjusted for all stock splits and dividends). Ideally, it would have been desirable to have made a further adjustment for net new issues, but this did not prove feasible on a careful basis within the time available for the present study.

\[ tr(\Delta r)_{t-1} = .4(\Delta r)_{t-1} + .3(\Delta r_{t-2}) + .2(\Delta r_{t-3}) + .1(\Delta r_{t-4}) = .4r_{t-1} - .1 \sum_{i=2}^{5} r_{t-i}. \]
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<th>$r$</th>
<th>$t(Ar)_{t-1}$</th>
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</tbody>
</table>
The leverage variables used in the study were computed by forming ratios with LTD and \((LTD - RF)\), respectively, in the numerator and using each of the variables \(NW\), \(SK\) and \(SP\) in the denominator.

**THE RESULTS**

Table 1 presents the regression coefficients, \(t\)-ratios, and partial correlation indexes obtained when the following equation was fitted to the data:

\[
\frac{I_{t+1}}{K_{t-1}} = (SD) + b_1 + b_2X_4 + b_3\frac{LTD - RF}{SP} + b_4r + b_5[tr(\Delta r)_{t-1}] + b_6T,
\]

where \((SD)\) indicates three seasonal dummies, and \(X_4\) represents the particular "accelerator-capacity" variable (see list Table 1) which was used. As previously indicated, in this and all other equations reported, all explanatory variables (except, of course, the \(SD\)'s and time) were the weighted sums of past data cumulated to the period in question using the Almon weights; also, all equations were fitted with all variables in differenced form. Incidentally, it should be kept in mind that the adjusted for degrees of freedom, which are given in the tables indicate the fraction of the variance of the quarterly differences which are accounted for by the explanatory variables. If, instead, these results are transformed into the corresponding fraction of the variance of \(I_{t+1}/K_{t-1}\) itself (without differencing) which is explained, the \(R_c^2\)'s, would be given much higher values. For instance, a standard error of estimate of .36 per cent implies an adjusted \(R_c^2\) of .935 on this alternative basis. Similarly, a \(\sigma\) (est) of .32 per cent corresponds to an \(R_c^2\) of .955, and a \(\sigma\) (est) of .30 per cent to an \(R_c^2\) of .959.

In terms of the theoretical model developed in section II, the rationale for the structure of equation (1) should be clear. Given any particular accelerator-capacity variable to measure the volume of new investment (as a percentage of existing capital stock) which would be undertaken *ceteris paribus* if financing and risk considerations were neutral, the other terms are included to measure the effects of the latter factors. Specifically, the Baa rate \((r)\) reflects the average cost of outstanding debt at current bond prices, which serves as the base for judging the (before-tax) cost of new debt and thus is one factor directly influencing the risk of debt and the costs of funds. In addition to reflecting expectations of

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86 The author is very grateful to George Schussel for his effective handling of all the computer work involved in this study.
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rates, the trend in the Baa rate \((tr(\triangle r)_{t-1})\) indexes the difference between the marginal costs of debt and the average cost since it is a rising linear function of the latter difference. (A negative value of \(tr(\triangle r)_{t-1}\), of course, does not necessarily imply that the marginal cash cost of new borrowing is below the average cost of outstanding, but rather that this differential is smaller than at other times when the interest-trend term has positive values.)

The theoretical analysis further showed that the difference between the marginal cash costs of debt and its average cost is also an increasing function of leverage. Our leverage variable is included in the equation to reflect both these important effects.

The tests of the significance of these fund-cost and risk factors, in conjunction with different choices of accelerator-capacity variables, which are reported in Table 1, utilizes leverage measured by \((LTD - RF)/SP\) because on a priori grounds this form of the leverage variable was expected to best reflect and incorporate the different elements involved in the theoretical content of the leverage variable. Specifically, the basic theoretical analysis showed that the marginal (opportunity) costs of the use of debt and retained funds for investment outlays should be the same. The underlying model of behavior thus points directly to the use of \(LTD - RF\) rather than long-term debt alone in the numerator of the leverage variable. Similarly, the whole analysis is based on the fact that the relevant risks (and hence the economic costs in the form of the minimum required expected returns) depend fundamentally on ex ante judgments of vectors looking into the future of expected values of earnings and cash flows, together with their stochastic properties, variances, and risks. Equity values reflect both these critical matters, and do so with a directness and degree of precision which is lacking in either book net worth or even the current dollar value of the net fixed capital stock. A further reason for believing that equity value is the superior and more relevant denominator in the leverage variable is that the impediment to new investment posed by any given stock of debt varies inversely with equity prices because of the alternative of issuing new shares on more favorable terms as market prices rise, and conversely.

Table 1 shows that, over these forty-four quarters of recent experience in manufacturing, leverage is very strongly and negatively related to investment, regardless of whether the output-capital stock ratio, de Leeuw's capacity utilization index, or his "capital requirements" measure—or any of the three multiplied by Jorgenson's measure of the marginal productivity of capital—was used to represent accelerator-
<table>
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<th>Leverage Variable Denoted by $X_i$</th>
<th>Constant</th>
<th>$O_t/K_{t-1}$</th>
<th>$r$</th>
<th>$X_i$ tr($\Delta r$)</th>
<th>Time</th>
<th>$R^2_{c}$</th>
<th>$\alpha_{est}$</th>
<th>D.W.</th>
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<td>$t_r(\Delta x)_{t-1}$</td>
<td>Time</td>
<td>$R_c^2$</td>
<td>$\sigma_{est}$</td>
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<td>.345</td>
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</table>
capacity considerations in the equation. In all six equations, the negative leverage coefficient had a t-ratio of over 4.3, and the partial correlation coefficients on the leverage variable ranged upward from .60. In both respects, incidentally, the leverage variable was actually “more significant” than the accelerator-capacity variable in any of the equations.

The coefficients on the Baa rate and its trend-change were both negative in all six equations (in spite of the inclusion of “time” as a separate variable). In the first three equations with “simple” accelerator-capacity variables, the negative coefficients on the two interest rate variables all had t-ratios of 1.45 or better, and one or the other showed a t of 3.0 or more in each equation. It will be recalled that the interest rate is an important component of Jorgenson’s marginal productivity of the capital term J used multiplicatively with the accelerators in the last three equations. Perhaps this partially explains the generally less satisfactory performance of the separate interest rate variables in these runs. It should be noted, however, that the separate interest rate variables have t-ratios of 2.1 and 3.0, respectively, when the product of J and (CR) is used to measure investment needs and profitability.

Perhaps the most surprising result in the table is that the combination of the marginal capital productivity term J with either $O_t/K_{t-1}$ or $U_t$ reduced the over-all $R^2$’s and D.W. statistics, as well as the t-ratios for the individual variables in the equation. But the over-all results when the product $J(CR)_t$ is used are quite good and compare favorably with those obtained when either capital requirements $(CR)$ or a simple accelerator $(O_t/K_{t-1})$ is used in the equation. Each of these three accelerator variables leads to equations with standard errors of estimate of .32 per cent and D.W. statistics of 1.7+ over the forty-four quarters of data.

Table 2 shows that in these data, when the equations are fitted using the simple accelerator $(O_t/K_{t-1})$ and both interest variables (and “time”), the impact of leverage on investment is consistently negative and “significant” according to the usual standards, regardless of which of the six leverage variables is chosen. The same conclusion holds (Table 3) when the compound marginal productivity of capital cum capital requirements term $J(CR)_t$ is substituted for the simple accelerator. And in both bases, both interest rate terms are consistently negative and generally “significant” regardless of the leverage variable.

The use of the excess of long-term debt over retained funds in the numerator of the leverage variable (rather than long-term debt itself) generally led to higher t-ratios on the leverage term, and higher D.W. statistics and lower estimating errors for the equation, as expected from theoretical considerations. With both “accelerators,” this was consistently
the case when either book net worth or the market value of the equity were used in the denominator of the leverage variable, although the comparison is reversed when the current value of the stock of real capital ($K) is used in the denominator.

As expected on theoretical grounds, the use of the market value of the equity as the denominator of the leverage ratio led to uniformly superior results to those obtained when either book net worth or the current value of net real capital stock was used as the "base" of the leverage measure. With both accelerators, and with LTD − RF as with LTD alone in the numerator, the D.W. ratio and the t-ratio on the leverage term was higher, and the estimating error of the equation was lower, with SP than with NW or $K. While none of the differences between individual pairs of regression coefficients using different numerators or different denominators in the leverage variable was "statistically significant," the over-all pattern and general consistency of the results, together with theoretical considerations, justify continued use of (LTD − RF)/SP as the preferred measure of the leverage variable.

Theoretical considerations also indicate that, under rational optimizing behavior, there will be significant interaction between the interest rate and leverage variables. Higher interest rates mean that the risks associated with any given degree of leverage are higher and also that the incremental risk associated with any increment in leverage will be greater. Similarly, higher leverage would in theory add to the deterrent effects of any given level (or increase) in interest rates. Unfortunately, it was not possible to get any meaningful measures of these incremental interaction effects by simply adding the new product term r[(LTD − RF)/SP] to the earlier equations because of the collinearities involved. As an alternative, the latter multiplicative form of compound interest-rate and leverage variable was introduced in place of the interest rate and leverage separately. As illustrated in Table 4, this procedure did raise the precision of the estimate of the regression coefficient as shown by the higher t-ratios, and this was true whether or not time was included as a separate variable in the equation. Not only did the t-ratios of the compound interest-leverage variable show unusually high values of 6.6 and 7.0, but the smoothed rate of change of the interest rate and the accelerator terms both had t values of 3.7 or better. These latter (clearly very satisfactory) values were, however, somewhat lower than those obtained when the interest rate and leverage were entered separately in the equation; and the standard errors of estimate over the observation period for the equations as a whole, and the values of the D.W. statistics, were about the same with both forms of the equation.
TABLE 4

Regression Results for Interest and Leverage as Separate Variables
and as a Product Term, with and Without Time Trend

<table>
<thead>
<tr>
<th>Constant</th>
<th>$O_t/K_{t-1}$</th>
<th>$r$</th>
<th>$LTD-RF$</th>
<th>$r[LTD-RF]$</th>
<th>$tr(\Delta r)_{t-1}$</th>
<th>Time</th>
<th>$R^2$</th>
<th>$\sigma$ est</th>
<th>D.W.</th>
</tr>
</thead>
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<td>- .3218</td>
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<td>.7356</td>
<td>.0031</td>
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Altogether, this evidence provides little basis for the choice between entering interest and leverage separately or as a single compound term. In either case, the D.W. statistic is a very satisfactory 1.7; the standard error of the equation is only about .31 per cent of the capital stock—or less than $250,000 of investment outlay at annual rates on the average in 1954 dollars—and the error of forecasts of investment for the second and third quarters of 1964 were consistent with this (percentage) figure.

Although these investigations were focused primarily upon the effects of financial and risk considerations (with accelerator effects) on investment outlays, we should also briefly note the results of including other terms in the equations presented to pick up the independent effect of certain factors reflecting relative real costs and profit margins, both as current data and (by changes or trends of changes) as elements of expectations. The comments can properly be brief because, whatever significance for future research our results may have, the findings were negative. Specifically, none of the separate variables for profit margins (either on sales or net worth) or their first differences or trends, the ratio of product price to labor cost or its first difference or trend, or the ratio of labor to real capital cost proved to be significant when added as an additional variable to our basic equations. In each case, the addition of the term produced a t-ratio of less than 1.2 on the variable and failed to improve the over-all equation. The same negative results were also obtained when stock price, equity earnings yields, or retained funds were added as separate variables (except that retained funds always had a positive sign and were often “significant” as a separate variable if LTD rather than the theoretically preferred form \((LTD - RF)\) was used in the leverage variable). In the same way (perhaps because of our use of the Almon weights on past values of all explanatory variables), the addition of either the first differences or the smoothed rate of change of any of the accelerator variables shown in Table 1 frequently produced “wrong” signs on these terms and no “significant” and “right” signs on these added variables in any of the equations tested. Neither did the orders-capacity variable, which has been suggested by Hart [15], prove to be a significant additional variable in our equations—a finding that may be due to our use of data for all manufacturing (rather than the durable sector separately) or to the period since 1953 when supply bottlenecks were of less importance than they had been in earlier years.

\footnote{This did not merely reflect the order in which the variables were introduced. In several tests, these variables were put in early and “fell out” as financial-risk variables were added.}
Finally, it should be recalled that all of our basic runs and tests were undertaken with a view to explaining investment outlays in terms of the fundamental determinants of the decisions to make real investments. To this end, in view of the spreading of the actual resulting expenditures over time, all explanatory variables consisted of data which had been cumulated to the given index quarter using Almon’s weights (and the explanatory data cumulated to the \( t \)th period were used to determine investment in the \( (t + 1) \)th quarter to allow for the so-called “decision lag”). But not all decisions or initial commitments result in final expenditures on a fixed time schedule without-speed-up, deferral, or cancellation. This is clearly established by studies of the “realization function” relating appropriations to actual subsequent outlays and also by studies of the differences between the expenditures planned for future quarters, as reported in S.E.C. surveys, and the actual amounts later expended (see, e.g., [34] and [12]).

In the final stages of our work, consequently, some additional runs were made that also included a measure of current conditions in the \( t \)th quarter, which we hoped would pick up cancellations and other positive and negative effects of “surprises” on the actual investment outlays in the \( (t + 1) \)th quarter. Since previous studies have found that the discrepancy between actual and “expected” sales is the factor most closely related to differences between actual and expected investment outlays, the variable used was the percentage difference between actual output and “expected” output in the \( t \)th quarter. (“Expected” output was estimated by the exponentially smoothed value of the stream of past quarterly outputs with a smoothing constant of .1). For the results see Table 5.

This “surprise” or “sales discrepancy” variable proved to be positive and significant with a \( t \)-ratio of 2.37 or better. The statistical results over the observation period were improved. The D.W. statistic was raised to over 1.9, and the standard error of estimate of \( I_{t+1}/K_{t-1} \) was reduced to .30 per cent or less. The forecasting errors for \( I_{84:2} \) and \( I_{84:3} \) were also somewhat improved by the addition of this “surprise” term. Most relevant to the basic concerns of the present paper, however, is that the addition of the current sales “surprise” variable produced little change in the regression coefficients of the leverage variables \((LTD - RF)/SP \) or \( r[(LTD - RF)/SP] \) and that their \( t \)-ratios remained high (4.6 or more for the former, and 5.6 or more for the latter). The addition of the surprise term raised the \( t \)-ratio on the Baa rate as a separate variable to more than 3.0 when a time trend term was not included in the equation; with “time” included separately, the coefficient on the Baa rate was still
negative and "significant" at about the .1 level. The addition of the sales surprise term reduced the $t$-ratios on the smoothed rate of change of the Baa rate, but its $t$-ratios still remained above 2.15. It will also be noted that the addition of the $t^{th}$ periods sales surprise raised the precision of the estimate of the coefficient on the basic accelerator variable $O_t/K_{t-1}$ which is cumulated with the Almon weights.

In view of Hickman's [16] earlier finding of negative time trends in investment equations, which appeared to imply a gradual weakening of investment demand due to technical progress, attention should also be called to the fact that all the time trends in our equations are essentially zero or positive. The models fitted here include financial and risk variables, while no such terms were included in his equations. Our results consequently suggest that the negative time trends he obtained probably reflect the powerful effects of these other factors upon which we have focused rather than any persistent marginal depressing effect of technological change. In particular, they suggest that the ratio of new investment to capital stock will be maintained if output is kept strong in relation to real capital stock and finance-risk factors are held neutral.

**IV. General Conclusions**

As developed in section II, the neo-Fisherian model of purely competitive security markets of risky bonds and stocks traded by risk-averse investors leads directly to a unified theory of rational corporate financial and investment policy, under the traditional assumption that management's objective is to maximize the market value of the equity of its shareholders. In this theory, as in Fisher's earlier model under certainty, both financial and accelerator or capital-requirements variables are required by the optimizing economic theory itself in the specification of the statistical model intended to represent the structure of investment decisions and behavior. In keeping with this theory, the present paper has sought to throw light on the simultaneous influence of both sets of considerations upon investment behavior. Rather than undertaking a race (or alternative choice test) between allegedly competing subtheories of investment, the statistical work has sought to test and implement the unified structure suggested by the theory.

Since the empirical work was confined to data for all manufacturing for the eleven years 1953–63, with limited forecasts beyond that period, the econometric results are not conclusive at this stage. Further work with individual industries, longer forecast periods, different time spans, and other data will be required for that. But the results suggest the
**TABLE 5**

Regression Results with a "Surprise" Variable, with and Without Time Trend

<table>
<thead>
<tr>
<th>Constant</th>
<th>$O_t/K_{t-1}$</th>
<th>$r$</th>
<th>$\frac{LTD-RF}{SP}$</th>
<th>$r\frac{LTD-RF}{SP}$</th>
<th>$\frac{tr(\Delta r)}{t-1}$</th>
<th>$S$</th>
<th>Time</th>
<th>$R^2_c$</th>
<th>$o_{est}$</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0086</td>
<td>.1571</td>
<td>-1.714</td>
<td>.2707</td>
<td>-</td>
<td>-7.214</td>
<td>.0488</td>
<td>-</td>
<td>.7665</td>
<td>.0029</td>
<td>1.92</td>
</tr>
<tr>
<td>5.81</td>
<td>-3.08</td>
<td>-4.75</td>
<td>-</td>
<td>-2.35</td>
<td>2.40</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>.701</td>
<td>- .461</td>
<td>- .626</td>
<td>-</td>
<td>- .369</td>
<td>.375</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td>.1514</td>
<td>-</td>
<td>-</td>
<td>-7.265</td>
<td>-7.144</td>
<td>.0484</td>
<td>-</td>
<td>.7729</td>
<td>.0029</td>
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</tr>
<tr>
<td>5.72</td>
<td>-</td>
<td>-10.50</td>
<td>-</td>
<td>-2.44</td>
<td>2.50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>.690</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.735</td>
<td>.377</td>
<td>.384</td>
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<td>-</td>
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<tr>
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<td>.1648</td>
<td>-1.459</td>
<td>.2750</td>
<td>-</td>
<td>-7.642</td>
<td>.0498</td>
<td>- .0002</td>
<td>.7601</td>
<td>.0030</td>
<td>1.93</td>
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<tr>
<td>4.11</td>
<td>-1.30</td>
<td>-4.59</td>
<td>-</td>
<td>-2.18</td>
<td>2.38</td>
<td>-</td>
<td>- .045</td>
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<td>.576</td>
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<td>-</td>
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<td>.378</td>
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<tr>
<td>.0027</td>
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<td>- .112</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>.609</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.687</td>
<td>.377</td>
<td>.373</td>
<td>- .019</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: S, the "surprise" variable, is measured by the percentage difference between actual output and expected output, the latter being computed as the exponentially smoothed value of the stream of past quarterly outputs with a smoothing constant of .1.
Financial Aspects

desirability of further work along these lines, since the quarterly data for all manufacturing over this decade do quite consistently bear out the expectations of the theory.

In terms of these data, the most important finding is the fact that leverage (and one or more interest variables) were highly significant and negatively related to investment outlays. This was consistently true with different accelerator-capacity variables in the equation; with long-term debt alone, or its excess over retained funds, used in the numerator of the leverage variable; and with either numerator used with any of three denominators in the leverage variable. As suggested by the theory, the ratio \((LTD - RF)/SP\) seemed to be clearly superior to the other leverage variables regardless of the accelerator-capacity component of the equation. No strong basis was found for choosing between entering the interest rate and leverage-retained earning variable separately in the equation or using one variable which is a multiplicative combination of them, although the \(t\)-ratios on the latter leverage variable were higher \((5.6+)\). The smoothed rate of change of the Baa rate remained highly significant and negative regardless of the form of the leverage variable and regardless of whether the level of the Baa rate was entered separately or as part of a compound interest-leverage term.

All explanatory variables were cumulated to the index quarter, using Almon's weights to allow for the distributed lag of investment outlays after decisions and appropriations. We also found that all explanatory variables in these basic equations retained their significance and were in general little changed by the addition of a current (i.e., noncumulated) actual-to-expected sales-discrepancy term to measure cancellations and other positive and negative effects of "surprises" (see Table 5). With this additional term, the Durbin-Watson statistics were 1.9+, and standard errors of estimate were under .30 per cent of \(I_{t,1}/K_{t-1}\), which is the equivalent of an adjusted \(R^2\) on investment itself of \(.96+\). Time trend terms were nonnegative in all equations incorporating finance and leverage terms and were virtually zero in the final equations presented.

In one important respect, the results did not conform to those expected on the basis of theory: like other investigators, we failed to find significant effects and coefficients for any of several relative real cost variables tested as separate variables added to the equation.

BIBLIOGRAPHY


The Miller and Modigliani paper is a curious combination of impres-
sive formal analysis resting on inconsistent and mutually contradictory
premises. Drawing on their earlier work, they make assumptions and
define terms so that they may conclude that financial policy does not
affect the firm's cost of capital under certainty and with no income taxes.

With income taxes, a subsidy is provided to debt. The consequence
would be that firms would use the maximum amount of debt. But the
observed facts indicate that this is obviously not true.

Miller and Modigliani resort to the concept of a “target debt ratio,”
denoted as $L$, to explain why observed debt ratios are not as high as
their model implies. A target debt ratio implies some policy on financing
mix by a firm. But this is precisely what traditional business finance has
argued and what Miller and Modigliani have denied. Their reference
to “the maximum permitted by lenders” is an imprecise way of saying
that the cost of debt at some point rises sharply with increased leverage
—implying an optimal debt-to-equity ratio.

The theoretical underpinnings of their empirical materials are there-
fore confused. Their development of the theoretical model is marred by
loose statements and unsupported assertions such as the following: “For
companies with reasonable access to the capital markets, as would cer-
tainly be true of those in our sample, investment and financing decisions
(including decisions to retire outstanding securities) are made con-
tinuously and largely independently over time.”

What does “reasonable access to the capital markets” signify in
analytical terms? What is their evidence that investment and financing
decisions are made “continuously and largely independently over time”? Surely such critical assertions should not be made without evidence or
reference to other studies which provide evidence.

After an extended discussion of theoretical models, the equation
which they use in their statistical tests turns out to be a rough empirical
approach. The value of the firm is related to the factors generally con-
sidered important in the business finance literature. These factors include
earnings, dividend policy, growth rate, leverage, and size of firms. Their
results, then, critically depend on how the variables are measured and the nature of their sample.

The Miller-Modigliani study is focused on the electric utility industry. A number of characteristics of the regulated industry can influence the generality of their results and even interpretation of their significance for the electric utility industry. First, if the earnings of a utility company exceed or fall below some range, a rate adjustment may take place. Second, a review of the opinions of utility commissions indicates a preference for investment financed from outside equity compared with retained earnings. This attitude certainly influences dividend policy. Third, the effect of regulation is to create a tendency toward uniformity in both leverage and dividend policies. For example, if higher debt ratios lead to higher earnings, the higher profits may be reduced by rate adjustments. On the other hand, if earnings are low because leverage ratios are low, commissions are unlikely to make compensatory rate adjustments. The influence of regulation is to make for conformity in financial policies—a consequence which should greatly influence the interpretation of the empirical results.

In this connection, the empirical measurements of Lintner present another set of problems. Lintner studies the plant and equipment outlays of all U.S. manufacturing corporations over the forty-four quarters of 1953–63. The Miller-Modigliani studies focus on cross-section analysis of an industry for which product homogeneity is emphasized. Lintner employs a time series analysis covering a wide variety of industries. This substantially departs from the Miller-Modigliani concept of industries representing homogeneous risk classes.

It is interesting to note that the underlying theoretical models differ as well. Miller and Modigliani seek to explain the value of the firm, while Lintner seeks to explain investment outlays. He emphasizes that financing decisions and investment decisions are mutually determined. This strong belief is in direct contrast with the counter assertion of Miller and Modigliani.

Lintner argues that the marginal cost of financing is represented by the marginal cost of debt. He suggests that the marginal cost of debt is likely to rise exponentially for a number of reasons. But since the equity holders are in a junior position to the creditors of a company, the factors that cause the cost of debt function to rise sharply are likely to cause the cost of equity function to rise even more.

Lintner objects to the use of the weighted average method of calculating the cost of capital. In theory the marginal cost of capital should be measured as we measure the marginal cost of any inputs. We formu-
late a function and take the partial derivatives with respect to the inputs being varied. Lintner argues that the weighted average cost method of calculating the cost of capital is wrong because the average cost is used rather than the marginal cost of debt and equity financing. But this objection can be met conceptually by using the marginal costs rather than the average costs.

Lintner's objection to the use of weighting appears to confuse (1) increasing the size of the capital budget by the use of debt or equity and (2) changing the debt-equity mix. This goes to the heart of the difficulty in calculating the cost of financial inputs. For calculating the marginal cost of physical (real) labor and capital inputs, the points of tangency of the isoproduct and isocost lines for different levels of output are connected. The isoquants representing different levels of value of the firm for different combinations of debt and equity unfortunately involve interdependence. To change the debt-to-equity mix not only influences the cost of debt, but influences the cost of equity as well. Altering the financing mix changes both the earnings of the company and the appropriate capitalization rates to be employed. The theoretical problem is formidable and the measurement problem appears to require some approximation methods for working purposes.

Lintner asserts that the opportunity cost of using more retained funds to increase the current size of the capital budget is the foregone opportunity to repay debt, "as Duesenberry has emphasized." Neither Duesenberry nor Lintner provides evidence that the opportunity cost of increasing the size of the capital budget is the foregone opportunity to repay debt. Why is not retirement of equity another alternative use of retained funds? Would not the decision to retire debt or to retire equity be a function of the marginal decreases in the cost of funds resulting from applying retained earnings in the one direction or the other?

Particularly in the latter section of the paper where Lintner is formulating statements about the behavior of the cost of capital function, it would be useful to have the arguments presented in a more precise analytical framework. Without a clear statement of the complete model that is being used, it is not certain what the linkage is between the data and model, between measurement procedures and the meaning of the results.

ON MILLER-MODIGLIANI

BY IRWIN FRIEND, UNIVERSITY OF PENNSYLVANIA

The most interesting part of the paper by Miller and Modigliani consists of an econometric implementation of their basic model of firm
valuation and cost of capital, which is applied to cross-section data for electric utilities in 1954, 1956, and 1957. This model assumes that the cost of capital for a firm in a given risk class is, apart from the very important tax complication (for which they adjust explicitly), invariant to capital structure.

Their theory leads to a basic structural equation in which the value of the firm, apart from tax (and size of firm) effects, is determined by two terms. The first is the capitalization of current earnings before interest, represented as the product of such earnings (which can be measured directly) and the reciprocal of the cost of equity capital, which cannot be measured directly but is assumed to be a constant for all firms of given size in the industry. The second term is the value placed on the growth potential, expressed empirically as the product of expected growth in the book value of assets (based on the most recent five-year growth rate) and another “constant,” which in theory is an inverse function of the cost of capital and a direct function of the rate of return on new investment and of the duration of the period for which profitable new investment opportunities are expected to persist. The cost of financing for an all-equity structure, which is different from the total cost of financing in the Miller-Modigliani model only as a result of the tax savings associated with debt, is estimated as the reciprocal of the regression coefficient of the current earnings variable in this structural equation.

My reaction to such an approach to estimation of the cost of financing is that, while it is intriguing, it is also questionable. If the measure of expected growth is a poor one, the current earnings variable (expressed as a ratio of the book value of assets) may act as a proxy for such growth so that its coefficient is biased upward and hence the estimated cost of capital would be biased downward. More important, there is a much more direct approach to the estimation of the cost of capital which avoids both this statistical difficulty and the theoretical necessity of assuming the irrelevance of capital structure, apart from tax savings, to the over-all valuation and cost of financing of the firm. This alternative approach, which appears preferable both on statistical and theoretical grounds, consists of estimating the cost of equity (which is no longer the cost for a pure equity stream) as the sum of a dividend yield plus an estimated growth rate in earnings and dividends, and obtaining the

1 For statistical reasons, all variables are divided by the book value of assets in the equations actually fitted.

2 This is the cost of capital for an all-equity structure in that risk class.

3 The Miller-Modigliani approach permits the incorporation of leverage variables into their basic structural equation, but this may aggravate the statistical difficulties, particularly if the irrelevance assumption is justified.
over-all cost of capital as the market-value weighted sum of the costs of equity and senior capital (adjusted for tax effects). There are, of course, major problems involved in estimating the expected growth rate—including decisions on the use of the past objective record, forecasts used by financial analysts, the duration of the period for which growth is to be projected, etc.—but these same problems exist in the implementation of the Miller-Modigliani econometric model. I (as well as others) have used this approach in the past, and it is my impression that it would yield significantly higher estimates of the cost of capital for the electric utilities than the Miller-Modigliani figures ranging from 3.6 per cent in 1954 to 4.6 per cent in 1957, which seem to me to be implausibly low (and indeed are not much higher than the yield on AAA public utility bonds).

In estimating the regressions coefficients in their basic structural equation, Miller and Modigliani substitute normalized earnings for reported earnings in order to eliminate measurement errors from the key earnings variable in the equation. Their method of normalizing earnings is to regress reported earnings on five different variables including dividends, with normalized earnings measured as the expected value of the dependent variable in this regression. Of the variables tested, the unabridged version of their paper seems to indicate that only dividends exert a major influence on normalized earnings, though one other variable—strangely enough, the ratio of preferred stock to assets—was also significant. It is not clear in their analysis that dividends alone would not have sufficed as a proxy for normalized earnings. The authors do not discuss other alternatives which have been used for normalizing earnings, some of which seem preferable to me. Actually, the change in the coefficients of the earnings variable in their basic structural equation, as a result of the substitution of normalized for reported earnings, does not appear to be very large. The only noteworthy difference between the normalized and reported earnings coefficients is that the former imply an increase in the cost of capital from 1954 to 1956 and 1957 in both regressions fitted, i.e., with and without a constant term, while the latter yield almost the identical results in the regression without a constant term but point to a decrease in the cost of capital in the regression with a constant term (which is the regression the authors like least). Intuitively, in spite of an increase in interest rates, stability or even some decrease in the cost of capital over this period does not seem

implausible to me in view of the reasons for believing there was a reduction in the required risk differential on equity.⁵

Though they devote considerable attention to the rise in their estimates of the cost of capital from 1954 to 1957, I do not really understand the explanation of this phenomenon offered by Miller and Modigliani. The average cost of capital according to them rises from 3.6 to 4.6 per cent, while the yields on AAA public utility bonds rise from 3.1 to 4.3 per cent. Disregarding what I consider to be an unreasonably low figure for the cost of capital at the beginning of this period, the Miller-Modigliani estimates appear to imply that there was no lowering of the risk differential on equity over this period—a result which, to me at least, seems dubious.

In view of these and other reservations (including a feeling that the authors have been less than charitable in their references to the state of the arts), my over-all reaction to the paper is that it is a stimulating but unsuccessful attempt to measure an elusive but highly important economic parameter, the cost of capital. As the authors point out, the goal is extremely important and difficult of attainment. It is no reflection on the ingenuity displayed in this paper to suggest that the goal has yet to be attained.

**REPLY TO FRIEND AND WESTON**

**BY MILLER AND MODIGLIANI**

We are grateful to Friend for pointing out so specifically, succinctly, and frankly those aspects of our approach to measuring the cost of capital about which he has serious misgivings. It is apparent that his objections spring largely from his qualms about our actual numerical estimates of the cost of capital, which strike him as implausibly low and as moving over time in the wrong direction. These qualms, quite understandably, have led him to search for possible shortcomings in our procedures that might be responsible for estimates seemingly so wide of the mark. In

⁵ Similarly, in the authors' unabridged version of their paper, which incorporates dividend policy as an additional explanatory variable in their basic structural equation, I do not understand the authors' preference for two-stage or normalized results vs. direct least-squares results. In the latter, dividend policy is significant in 1954, with higher payout associated with higher market value, but not at all in 1956 and 1957, while in the former the payout ratio is not significantly related to market value in any year, but there is an upward drift in its impact on market value over this period. Intuitively again, the direct least-squares results (including the other regression coefficients in the equation) seem fully as plausible as the two-stage results.
what follows we shall try to indicate why we do not share his doubts and why we persist in believing that our estimates, properly interpreted, are plausible and sensible. Before doing so, however, it may be useful first to consider his methodological objections.

As one possible reason for the low estimates he points out that “the current earnings variable . . . may act as a proxy for [expected] growth . . . and hence the estimated cost of capital would be biased downward.” We have acknowledged in our paper that our measure of expected growth opportunities is only an approximation and that, to the extent that it is measured subject to error, biases in the other coefficients may well creep in. But we have some grounds at least for believing that any likely bias from the correlation between earnings and growth in the present case could hardly be large enough to change the picture dramatically. Not only is there no particular reason to expect a priori any systematic correlation between the level of current earnings (relative to assets) and future growth potential in a regulated industry such as this one, but the data themselves give no hint of such correlation. The simple correlations between earnings and our growth variable (see Appendix B, Table 2, of the unabridged version) are virtually zero or negligibly negative in all three years. Granted that our measure of growth is far from perfect, and hence that we cannot be absolutely sure that there is no correlation between earnings and the true measure, our measure of growth cannot really be so bad (judging by how well it does perform in the valuation regressions) that it would fail to pick up any correlation between growth and earnings strong enough to have a noticeable effect on the results.

Friend’s second possible explanation of the peculiar results is that we have used a “questionable” and certainly too roundabout method of estimation in preference to a “much more direct” method of estimating the cost of capital. By direct, he means first estimating the cost of equity capital as the sum of an estimate of the expected dividend yield and some estimate of the expected growth rate of dividends or earnings, and then obtaining the over-all cost of capital as a weighted average of this cost of equity, current debt, and preferred stock yields (adjusted for tax effects). In his view, this approach is superior to ours “both on statistical and theoretical grounds.” It is also his impression (based on his own work and that of others) that his method “would yield significantly higher estimates of the cost of capital for the electric utilities.”

This is, indeed, a major point of contention, since we would have regarded as one of the major methodological contributions of our paper precisely that it provided an effective alternative to the traditional
piecemeal approach to the cost of capital that Friend describes and recommends.

By "preferable on theoretical grounds," he means, apparently, that his approach "avoids the theoretical necessity of assuming the irrelevance of capital structure, apart from tax savings, to the over-all valuation of the firm." It is true, of course, that our estimates were computed on that assumption and that such an assumption is also implied in our model of rational valuation. But this has nothing to do with our method of estimation. Before discussing results based on the assumption, we first tested that assumption and found that it was indeed consistent with the data in our sample (see Table 4 of the unabridged version). If these tests had turned out differently, so would the final results we presented.

It is also true, of course, that his "direct" method avoids the necessity of considering explicitly the relation between capital structure and valuation. But it does so only at the cost of having a less useful and less informative result. At best, his approach will provide an estimate of a single point on the cost of capital function, namely, the point corresponding to the actual current sample value of the capital structure (and the current levels of any other variables that affect valuation). Hence, in contrast to our approach, the results of the piecemeal approach cannot be used for policy-making or policy evaluation purposes by individual firms or regulatory bodies.

Even more puzzling is Friend's claim that his method is "statistically superior." He certainly cannot mean by this that his concept of growth is a simpler one than ours or that it presents fewer problems to the estimator. Indeed, the opposite is closer to the truth. His growth component, after all, is nothing more or less than the expected capital gain on a share of common stock, and this expectation must reflect not only growth in our sense (i.e., of advantageous future investment opportunities) but also expected financial policy (i.e., the extent to which future asset expansion will be financed from retained earnings). If, nevertheless, Friend still feels more comfortable with his approach to growth rather than with ours, that is, of course, his privilege; but other investigators should at least not be misled into thinking that his approach offers any clear-cut advantage over ours.

As to whether the Friend method, properly implemented, will indeed turn out to yield substantially higher estimates than ours, that is anybody's guess. We have our doubts since we do not really think our estimates are unreasonably low for reasons to be indicated later. In any event, we too, in another connection, happen to have made some computations of expected equity yields along the lines he describes for a
number of companies, including some also in our utility sample. It was not our experience that such yields, after correction for leverage, were substantially and systematically higher than the pure equity yields implied by the results in the present paper. We wonder whether Friend may not have derived his impression that equity yields in this period were quite high from the reported figures on realized yields or rates of return. These ex post yields were indeed rather high on the whole; but since they contain a very substantial amount of what must surely be regarded as unanticipated capital appreciation, they are hardly likely to be good measures of the kind of ex ante or required yields that he and we are concerned with.¹

Friend's third methodological objection is that we have based our main conclusions on one particular set of test results involving a very special "method of normalizing earnings," i.e., our two-stage, instrumental variable approach to estimation. On this point, a casual reader of his comments is likely to come away with the impression that our use of the two-stage estimates is simply a peculiar matter of taste on our part. We seem to prefer two-stage estimates with constant suppressed to direct least-squares estimates with or without constant. He, on the other hand, would prefer our direct least-squares estimates and would presumably be even happier with estimates derived from another "normalizing" approach developed by him and Marshall Puckett.

Actually, of course, we base our discussion of the cost of capital on the estimates without a constant term not because we enjoy suppressing constants or feel that those estimates are more in line with our prejudices but because the basic theoretical specification requires the constant to be so treated and because the tests of that specification presented in our paper give no evidence that the constant term is other than zero. We used a two-stage instrumental variable approach because, under our assumptions, such estimates would at least have the property of consistency whereas the direct least-squares estimates are open to bias from measurement errors in the key earnings variable (and particularly so if a constant term were to be included). It is for the same reason, of course, that we "preferred" our approach to the alternative method of Friend and Puckett. Ingenious as their method may be, it cannot be expected to yield consistent estimates, as they themselves have acknowledged.²

¹ Our suspicions about the unanticipated nature of much of the realized capital gains over this period are strongly reinforced by the findings of F. Ardetti in his as yet unpublished dissertation, "Risk and The Required Return on Equity," Massachusetts Institute of Technology, 1965, esp. Chapter IV.

In any event, anticipating that our instrumental variable procedure might be unfamiliar to some and might cause just such uneasiness as Friend has shown, we took the precaution of providing some alternative "calibrating" estimates derived from a more conventional method. This approach, which represents what we called the "yield formulation," does rely on familiar direct least-squares estimation. It can be expected to be free at least from bias stemming from the measurement error in earnings, though subject to certain other sources of bias that should tend on balance to underestimate the cost of capital. Comparison of the estimates derived from these very different procedures (see Table 6 and surrounding text in the unabridged version) shows that they are quite close and stand in the predicted relation to each other. If, therefore, our two-stage estimates are "implausibly low," this cannot be attributed to any serious distortions introduced through our "normalization" procedure.\(^3\)

Having failed to find anything in Friend's methodological objections that could account for the "implausibility" of the estimates, a few words may now be in order in defense of their reasonableness. We suspect that some of Friend's misgivings stem from his concentration on our estimates of the so-called weighted average cost of capital or tax-adjusted required rates of return given in Table 4. The tax-adjusted rate is a measure about which it is difficult to make intuitive judgments because the concept of income on which it is based does not correspond to any of the more commonly used accounting concepts. Furthermore, because of the way we controlled for size of firm, the estimates presented apply directly only to the largest firms (strictly speaking, to firms of infinite size). Adjusting to average size our estimates of the tax-adjusted required rates of return for 1956 and 1957 come to 4.6 and 4.8 per cent, respectively. If we next convert from a tax-adjusted to the more familiar before-tax basis, these estimates imply before-tax required rates of return for companies of average size in this industry of 9.5 and 10 per cent, respectively. Such values certainly do not strike us as being preposterously low.

Nor is it meaningful to compare, as Friend does, our tax-adjusted average rate with the before-tax bond yield since our required rate is so substantially affected by the sizable tax subsidy on the very high degree of leverage that is found in this industry. The more sensible comparison,

\(^3\) There is equally little ground for Friend's concern over the fact that, in the first-stage regressions on the instrumental variables, the dividend variable alone seems to account for most of the correlation. As we pointed out in the unabridged version, virtually identical estimates (though, of course, with somewhat higher standard errors) are obtained when dividends are not included among the instrumental variables in the first stage.
if one is to be made, would be between the bond yield and our estimated cost of equity capital or tax-adjusted required return for an unlevered firm. The latter figure, again adjusting for size, we find to be 5.3 per cent in 1954, compared with a bond yield of 3.1 per cent, implying a "risk premium" of some 70 per cent. By 1957, the required equity yield had risen to 6.4 per cent, compared with a bond yield of 4.3 per cent, implying a risk premium of just below 50 per cent. Premiums of this size do not strike us as unreasonably low for an industry like the electric utilities consisting of regulated but protected monopolies, none of which suffered any losses (or even sizable declines in earnings) either during the sample period or for several prior years, despite the fact that some of these were years of recession. Note also that our estimates, properly interpreted, do seem to indicate a falling trend in the risk premium of the kind that Friend's intuition led him to suspect. If anything, this fall in the "risk differential on equity" strikes us as rather remarkably large for the period 1954-57. For, though we share his feeling that a substantial downward adjustment in risk premiums probably took place between the late 1940's and the middle 1950's (reflecting the growing confidence in the economy's resistance to severe depression), our guess would have been that more of the adjustment had occurred before the end of 1954.

There are still other ways to put our estimates in a more meaningful perspective from which to bring intuition to bear. For example, starting from our 1957 estimate of the average tax-adjusted cost of capital, one finds that after adjusting for average size of firm and average leverage the required rate of return or yield on levered common stock implied by our results is on the order of 8.5 per cent, which, again, hardly seems preposterously low.4 As a further and particularly telling consideration suggesting that our estimates of the tax-adjusted cost of capital are not substantially too low, we may point out that the conventional 6 per cent return after taxes on the rate base is equivalent to about 5.2 per cent when converted to our tax-adjusted basis (see section III of our paper). Since there seems to be clear and undisputed evidence that investors during this period were willing to pay significant premiums for growth potential, they must presumably have been projecting a tax-adjusted cost of capital or required yield that was less than 5.2 per cent. With an implied ceiling of this order of magnitude, our estimates of 4.6 and 4.8 per cent as the required yields in 1956 and 1957 can hardly be con-

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4 Curiously enough, our implied yields on levered equity shares of utilities in this period are actually somewhat higher than those reported by Professor Friend in his paper with Puckett.
considered as implausibly low. On the contrary, we suggest that it will be the Friend estimates, if they turn out to be very substantially above ours, that will have to be regarded as implausibly high!

Summing up, then, we can find no basis for Friend's repeated assertions that our estimates of the tax-adjusted cost of capital for the utility industry are far too low. And since we certainly respect his judgment in matters pertaining to the capital markets, we are led to suspect that he did not dig deeply enough into our perhaps overlong paper to appreciate what our estimates really mean or how they relate to other, more familiar concepts of yield.

Turning now briefly to Weston's critical remarks it should be noted that, with one exception to be discussed below, his comments really have little bearing on the problem of main concern in this paper, namely, how to estimate the cost of capital. His quarrel is rather with the presumed "theoretical underpinning" of our model of valuation. Since these underpinnings have been discussed at length in several of our previous papers and will be further explored in sequels to the present paper, there is no point in rehashing the matter here. Suffice it to say, for the sake of the record, that we do not agree that our target leverage ratio is the same as the optimum financing mix of traditional discussions of finance—an optimum such that deviations in either direction would adversely affect the value of the shares. As for his strictures on our failure to present any direct evidence for our working assumptions of rational behavior and perfect capital markets, we were frankly astonished to see this sort of ploy in what purports to be a serious critique. We would have thought it had long been accepted among economists that standard working assumptions of this kind can only be effectively tested by testing their consequences. And that, of course, is precisely what most of our paper is about! We also feel that his comments on this matter come with particular ill grace since he has interlarded his own remarks with a number of dubious, unsupported assertions about electric utilities—assertions much more susceptible to direct check than our working assumptions.

Weston's one substantive point is that external regulation of earnings and financial policies in the electric utility industry may reduce the "generality" and "significance" of our results, particularly those bearing on the relation between financial policy and the cost of capital. There are really two distinct kinds of questions involved here. First, does the mere fact that the financial policies may have been imposed on the managements of some firms create problems of estimation or interpretation? Here the answer is clearly no. Our concern is solely with what investors
are willing to pay for the different packages of earnings and financial policy that actually do come onto the market for sale. For establishing the empirical relations between the prices of various packages, the questions of why they differ or who really makes them up are wholly irrelevant. On the other hand, a serious estimation problem might arise if regulation of financial policy was so pervasive and so uniform that all the packages on the market were virtually alike. When we first began the study, this was indeed our greatest fear about the sample. As it turns out, however, there does happen to be considerable variation within the industry—less so certainly than we would like, but enough to rule out the possibility that our zeroish results for debt and dividend policy reflect merely a fit to a single point with some random variation in its immediate neighborhood. Weston could have discovered this himself either by looking at the table of means and standard deviations of the variables presented in the appendix of the unabridged version, or by looking at the results of the direct least-squares regressions where there was apparently enough variability, in some years at least, for the financial policy variables to show up as highly significant and in the classical direction. All this is not to say, of course, that we regard our assumptions, procedures, or results as unexceptionable; but we do think such severe critics should present us at least with a bill of particulars.